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Minimizing Rental Cost under Specified Rental Policy in Two Stage Flowshop Set Up time, Processing Time Each Associated with Probabilities Including Break-down interval and Job Block Criteria

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Abstract: This paper is an attempt to study two stage flow shop scheduling problem in which the processing times and independent setup times each associated with probabilities of the jobs are considered to minimize rental cost under specified rental policy including break-down interval and equivalent job-block criteria. The study gives an optimal schedule for the problem. The method is justified with the help of numerical example and a computer program.

Keywords: Equivalent-Job, Rental Policy, Makespan, Elapsed Time, Idle Time, Breakdown Interval, Utilization Time

1. Introduction

Practical machine scheduling problems are numerous and varied. They arise in diverse areas such as flexible manufacturing systems, production planning, computer design, logistics, communication etc. A scheduling problem is to find sequence of jobs on given machines with the objective of minimizing some function of the job completion times. A variety of scheduling algorithms have been developed over the past years to address different production system. In flow shop scheduling it is generally assumed that the jobs must be processed on the machines in the same technological or machine order. One of the earliest results in flow shop scheduling theory is an algorithm given by Johnson (1954). The work was developed by Ignall and Scharge (1965), Bagga. (1969), Szwarch (1977), Anup (2002), Singh (2005), Gupta (2006) by deriving the optimal algorithm for two, three or multistage flow shop problems taking into account the various constraints and criteria. The basic concept of equivalent job for a job – block has been investigated by Maggu & Das (1977) and established an equivalent job – block theorem. Breakdown of machine due to failure of electric current or due to non - supply of raw material or any other technical interruptions have a significant role in production concern. Adiri (1989), Akturk and Gorgulu (1999), Smith (1956), Szwarc (1983), Chandramouli (2005), Singh (1985) have discussed the various concepts of breakdown of machines.

Gupta & Sharma (2011) studied n× 2 flow shop problem to minimize rental cost under pre–defined rental policy in which probabilities have been associated with processing time including breakdown interval and job – block criteria. The present paper is an attempt to extend the study made by Gupta & Sharma (2011) by introducing the concept of set up time separated from processing time, each associated with probabilities including job block criteria and breakdown interval. We have developed an algorithm for minimization of utilization of 2nd machine combined with Johnson's algorithm to solve the problem

2. Practical Situations

A lot of practical situations have been observed in real life when one has got the assignments but does not have one's own machine or does not want to take the risk of investing a huge amount of money to purchase, under such situation the machine has to be taken on rent to complete the assignment. As in the starting of his professional career, the medical practitioner does not buy expensive machine say X-ray machine, ultrasound machine etc but instead makes a contract on rent basis. Renting enables saving working capital, gives option for having the equipment and allows upgradation of new technology. Sometimes priority of one job over other is preferred. It may be because of urgency or demand of its relative importance, the job block criteria become important. Another event which is most considered in models is breakdown of machines. . The break down of the machines (due to delay in material, changes in release and tails date, tool unavailability, failure of electric current, the shift pattern of the facility, fluctuation in processing times, some technical interruption etc.) have significant role in the production concern. Setup includes work to prepare the machine, process or bench for product parts or the cycle. This includes obtaining tools, positioning work-in-process material, return tooling, cleaning up, setting the required jigs and fixtures, adjusting tools and inspecting material and hence significant.

3. Notations

S : Sequence of jobs 1,2,3,...,n

M_j : Machine j, j=1,2,.....

A_i : Processing time of i^{th} job on machine A

B_i : Processing time of i^{th} job on machine B

A'_I : Expected processing time of i^{th} job on machine A

B'_I : Expected processing time of i^{th} job on machine B

p_i : Probability associated to the processing time A_i of i^{th} job on machine A

q_i : Probability associated to the processing time B_i of i^{th} job on machine B

S_i^A : Set up time of i^{th} job on machine A

S_i^B : Set up time of i^{th} job on machine B

r_i : Probability associated to the set up time S_i^A of i^{th} job on machine A

s_i : Probability associated to the set up time S_i^B of i^{th} job on machine B

S_i : Sequence obtained from Johnson's procedure to minimize rental cost

β : Equivalent job for job block

L : Length of breakdown interval

$A'_{\alpha i}$: Processing time of i^{th} job after breakdown effect on machine A

$B'_{\alpha i}$: Processing time of i^{th} job after breakdown effect on machine B

A''_I : Expected processing time of i^{th} job after breakdown effect on machine A

B''_I : Expected processing time of i^{th} job after breakdown effect on machine B

C_j : Rental cost per unit time of machine j

U_i : Utilization time of B(2^{nd}machine) for each sequence S_i

$t_1(S_i)$:Completion time of last job of sequence S_i on machine A

$t_2(S_i)$:Completion time of last job of sequence S_i on machine B

$R(S_i)$:Total rental cost of sequence S_i of all machines

$CT(S_i)$:Completion time of 1^{st} job of each sequence S_i on machine A

4. Problem Formulation

Let n jobs say $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are processed on two machines A & B in the order AB. A job α_i (i=1, 2, 3, ...,n) has processing time A_i & B_i on each machines respectively with probabilities p_i & q_i s.t. $0 \le p_i \le 1$, $\sum p_i = 1$, $0 \le q_i \le 1$, $\sum q_i=1$. Let S_i^A & S_i^B be there setup times separated from their processing times associated with probabilities r_i & s_i s.t. $0 \leq r_i \leq 1$, $\sum r_i = 1$, $0 \leq s_i \leq 1$, $\sum s_i = 1$ on each machine. Let an equivalent job block β is defined as (α_k , α_m) where α_k , α_m are any jobs among the given n jobs such that α_k occurs before job α_m in the order of job block (α_k , α_m) .Our objective is to find an optimal schedule of the jobs which minimize the utilization time of machine and hence the total rental cost of the machines.

5. Assumptions

1. We assume the rental policy that all machines are taken on rent as and when they are required and are returned as when they are no longer required for processing.
2. Job are independent to each other.
3. Machine breakdown interval is deterministic, i.e. breakdown intervals are well known in advance. This simplifies the problem by ignoring the stochastic cases where breakdown interval is random.
4. Pre-emption is not allowed, i.e. once a job is started on a machine, the process on that machine can't be stopped unless job is completed.
5. It is given to sequence k jobs i_1, i_2 ....i_k as a block or group job in the order (i_1, i_2 ....i_k) showing priority of job i_1 over i_2 etc

6. Algorithm

To obtain optimal schedule we proceed as:

Step 1: Define expected processing time A'_i & B'_i on machine A & B respectively as follows :

$A'_i = A_i \times p_i - S_i^B \times s_i$

$B'_i = B_i \times q_i - S_i^A \times r_i$

Step 2: Define expected processing time of job block $\beta = (k,m)$ on machine A & B using equivalent job block given by Maggu & Das. i.e. A'_β and B'_β are as follows.

$A'_\beta = A'_k + A'_m - \min(B'_k , A'_m)$

$B'_\beta = B'_k + B'_m - \min(B'_k , A'_m)$

Step 3: Apply Johnson's (1954) technique to find an optimal schedule of given jobs.

Step 4: Prepare a flow time table for sequence obtained in step 3 and read the effect of break down interval (a,b) on different jobs on lines of Singh T.P (1985).

Step 5: Form a reduced problem with processing time $A'_{\alpha i}$ and $B'_{\alpha i}$ If breakdown interval (a,b) has effect on job i^{th} then .

$A'_{\alpha i} = A_i \times p_i + L$,

$B'_{\alpha i} = B_i \times q_i + L$ where $L = b - a$, length of break down interval.

If breakdown interval (a,b) has no effect on i^{th} job then.

$A'_{\alpha i} = A_i \times p_i$

$B'_{\alpha i} = B_i \times q_i$

Step 6: Find revised processing time after the effect of breakdown (6,10) ie

$A''_i = A'_{\alpha i} - S_i^B \times s_i$

$B''_i = B'_{\alpha i} - S_i^A \times r_i$

Step 7: Find the processing time A''_β and B''_β of job block β (k , m) on machine A and B using equivalent job block β as in step 2 .

Step 8: Now repeat procedure to obtain optimal schedule of given jobs as in step 3

Step 9: Observe processing time of 1^{st} job of S_i on 1^{st} machine A let it be α .

Step 10: Obtain all jobs having processing time on A greater than α . Put this job one by one in 1st position of sequence S_i in same order. Let these sequences be S_2 , S_3 , S_4 ,....., S_r .

Step 11: Prepare In-out flow table only for those sequences S_i (i=1,2,....,r) which have job block β (k,m) and evaluate total completion time of last job of each sequence ie t_1 (S_i) & t_2 (S_i) on machine A & B respectively.

Step 12: Evaluate completion time CT(S_i) of 1st job of each of above selected sequence S_i on machine A .

Step 13: Calculate utilisation time U_i of 2nd machine for each of above selected sequence S_i as :

$U_i = t_2(S_i) - CT(S_i)$ for i=1,2,3,....,r

Step 14: Find min {U_i} , i=1,2,....,r let it be corresponding to i = m .Then S_m is optimal sequence for minimum rental cost .

Min rental cost = $t_1(S_m) \times C_1 + U_m \times C_2$

Where C_1 & C_2 are rental cost per unit time of 1st & 2nd machines respectively.

7. Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
void display();
void schedule(int,int);
void inout_times(int []);
void update();
void time_for_job_blocks();

float min;int job_schedule[16];int job_schedule_final[16];int n;
float a1[16],b1[16],a11[16],b11[16],s11[16],s21[16];float a1_jb,b1_jb;
float a1_temp[15],b1_temp[15];int job_temp[15];int group[2];//variables to store two job blocks
int bd1,bd2;//break down interval
float a1_t[16], b1_t[16],a11_t[16],b11_t[16];float a1_in[16],a1_out[16];float b1_in[16],b1_out[16];
float ta[16]={32767,32767,32767,32767,32767},tb[16]={32767,32767,32767,32767,32767};
void main()
{

clrscr();int a[16],b[16],s1[16],s2[16];
float p[16],q[16],r[16],s[16];int optimal_schedule_temp[16];int optimal_schedule[16];
float cost_a,cost_b,cost;
float min; //Variables to hold the processing times of the job blocks
cout<<"How many Jobs (<=15) : ";cin>>n;
if(n<1 || n>15)
{cout<<"Wrong input, No. of jobs should be less than 15..\n Exitting";getch();exit(0);}
cout<<"Enter the processing time and their respective probabilities ";
```

```
for(int i=1;i<=n;i++)
{cout<<"\nEnter the processing time,set up time and their probability of "<<i<<" job for machine A : ";
cin>>a[i]>>p[i]>>s1[i]>>r[i];
cout<<"\nEnter the processing time and its probability of "<<i<<" job for machine B : ";
cin>>b[i]>>q[i]>>s2[i]>>s[i];
//Calculate the expected processing times of the jobs for the machines:
a11[i] = a[i]*p[i];b11[i] = b[i]*q[i];s11[i] = s1[i]*r[i];s21[i] = s2[i]*s[i];
a1[i] = a11[i]-s21[i];b1[i] = b11[i]-s11[i];}
for(i=1;i<=n;i++)
{cout<<"\n"<<i<<"\t\t"<<a1[i]<<"\t\t"<<b1[i];}
cout<<"\nEnter the two job blocks (two numbers from 1 to "<<n<<") : ";
cin>>group[0]>>group[1];cout<<"\nEnter the break down intervals : ";cin>>bd1>>bd2;
cout<<"\nEnter the Rental cost of machine A : ";cin>>cost_a;
cout<<"\nEnter the Rental cost of machine B : ";cin>>cost_b;
//Function for expected processing times for two job blocks
time_for_job_blocks();
int t = n-1;schedule(t,1);
//Calculating In-Out times
inout_times(job_schedule_final);
//Calculating revised processing times for both the machines
//That is updating a1[], and b1[]
update();
//Repeat the process for all possible sequences
for(int k=1;k<=n;k++)     //Loop of all possible sequences
{for(int i=1;i<=n;i++)
{optimal_schedule_temp[i]=job_schedule_final[i];}
int temp = job_schedule_final[k];optimal_schedule_temp[1]=temp;
for(i=k;i>1;i--)
{optimal_schedule_temp[i]=job_schedule_final[i-1];}
//Calling inout_times()
int flag=0;
for(i=1;i<n;i++)
{if(optimal_schedule_temp[i]==group[0] && optimal_schedule_temp[i+1]==group[1])
{flag=1;break;}}
if(flag==1)
{inout_times(optimal_schedule_temp);
ta[k]=a1_out[n]-a1_in[1];tb[k]=b1_out[n]-b1_in[1];
if(tb[k]<tb[k-1])
{//copy optimal_schedule_temp to optimal_schedule
for(int j=1;j<=n;j++)
```

```
{optimal_schedule[j]=optimal_schedule_temp[j];}}}}
float smalla = ta[1];float smallb = tb[1];
for(int ii=2;ii<=n;ii++)
{if(smalla>ta[ii])
smalla = ta[ii];
if(smallb>tb[ii])
smallb = tb[ii];}
clrscr();
cout<<"\n\n\n\n\n\n\n\n\t\t\t     #####THE SOLUTION##### ";
cout<<"\n\n\t***************************************************************";
cout<<"\n\n\n\t     Optimal Sequence is : ";
for (ii=1;ii<=n;ii++)
{cout<<optimal_schedule[ii]<<"     ";}
cout<<"\n\n\t     The smallest possible time span for machine A is : "<<smalla;
cout<<"\n\n\t     The smallest possible time span for machine B is : "<<smallb;
cost = cost_a*smalla+cost_b*smallb;
cout<<"\n\n\t     Total Minimum Rental cost for both the machines is : "<<cost;
cout<<"\n\n\n\t***************************************************************";
getch();}
void time_for_job_blocks()
{
//The expected processing times for two job blocks are
if(b1[group[0]]<a1[group[1]])
{min = b1[group[0]];}
else
{min = a1[group[1]];}
a1_jb = a1[group[0]]+a1[group[1]] - min; //(b1[k]<a1[m])?b1[k]:a1[m];
b1_jb = b1[group[0]]+b1[group[1]] - min; //(b1[k]<a1[m])?b1[k]:a1[m];
getch();}
void update()
{for(int i=1;i<=n;i++)
{if(a1_in[i]<=bd1 && a1_out[i]<=bd1 || a1_in[i]>=bd2 && a1_out[i]>=bd2)
{a1_t[i] =a1_t[i];a11[i] = a11[i];}
else
{a1_t[i] += (bd2-bd1);a11[i] += (bd2-bd1);}
if(b1_in[i]<=bd1 &&b1_out[i]<=bd1 || b1_in[i]>=bd2 && b1_out[i]>=bd2)
{b1_t[i] =b1_t[i];b11[i] = b11[i];}
Else
{b1_t[i] += (bd2-bd1);b11[i] += (bd2-bd1);}}
//Putting values of a1_t and b1_t into a1 and b1 with proper order of jobs
```

```
for(i=1;i<=n;i++)
{a1[job_schedule_final[i]] = a1_t[i];b1[job_schedule_final[i]] = b1_t[i];
a11[job_schedule_final[i]] = a11[i];b11[job_schedule_final[i]] = b11[i];}
time_for_job_blocks();
int t = n-1;schedule(t,1);}
void inout_times(int schedule[])
{for(int i=1;i<=n;i++)
{//Reorder the values ofa1[], a11[] and b1[], b11[] according to sequence
a1_t[i] = a1[i];b1_t[i] = b1[i];a11_t[i] = a11[i];b11_t[i] = b11[i];}
for(i=1;i<=n;i++)
{if(i==1)
{a1_in[i]=0.0;a1_out[i] = a1_in[i]+a11_t[i];b1_in[i] = a1_out[i];b1_out[i] = b1_in[i]+b11_t[i];}
else
{a1_in[i]=a1_out[i-1]+s11[i-1];a1_out[i] = a1_in[i]+a11_t[i];
if(b1_out[i-1]+s21[i-1]>a1_out[i])
{b1_in[i] = b1_out[i-1]+s21[i-1];b1_out[i] = b1_in[i]+b11_t[i];}
else
{b1_in[i] = a1_out[i];b1_out[i] = b1_in[i]+b11_t[i];}}}}
int js1=1,js2=n-1;
void schedule(int t, int tt)
{if(t==n-1)
{js1=1; js2=n-1;}
if(t>0 && tt==1)
{for(int i=1,j=1;i<=n;i++,j++) //loop from 1 to n-1 as there is one group
{if(i!=group[0]&&i!=group[1])
{a1_temp[j] = a1[i];b1_temp[j] = b1[i];job_temp[j] = I;}
else if(group[0]<group[1] && i==group[0])
{a1_temp[j] = a1_jb;b1_temp[j] = b1_jb;job_temp[j] = -1;}
Else
{j--;}}
//Finding smallest in a1
float min1= 32767;int pos_a1;
for(j=1;j<n;j++)
{if(min1>a1_temp[j])
{pos_a1 = j;min1 = a1_temp[j];}}
//Finding smallest in b1
float min2= 32767;int pos_b1;
for(int k=1;k<n;k++)
{if(min2>b1_temp[k])
{pos_b1 = k;min2 = b1_temp[k];}}
```

```
if(min1<min2)
{job_schedule[js1] = job_temp[pos_a1];js1++;a1_temp[pos_a1]=32767;b1_temp[pos_a1]=32767;}
Else
{job_schedule[js2] = job_temp[pos_b1];js2--;a1_temp[pos_b1]=32767;b1_temp[pos_b1]=32767;}}
else
if(t>0 && tt!=1)
{//Finding smallest in a1
float min1= 32767;int pos_a1;
for(int i=1;i<n;i++)
{if(min1>a1_temp[i])
{pos_a1 = i;min1 = a1_temp[i];}}
//Finding smallest in b1
float min2= 32767;int pos_b1;
for(i=1;i<n;i++)
{if(min2>b1_temp[i])
{pos_b1 = i;min2 = b1_temp[i];}}
if(min1<min2)
{job_schedule[js1] = job_temp[pos_a1];js1++;a1_temp[pos_a1]=32767;b1_temp[pos_a1]=32767;}
else
{job_schedule[js2] = job_temp[pos_b1];js2--;a1_temp[pos_b1]=32767;b1_temp[pos_b1]=32767;}}
t--;
if(t!=0)
{schedule(t, 2);}
//final job schedule
int i=1;
while(job_schedule[i]!=-1)
{job_schedule_final[i]=job_schedule[i];i++;}
job_schedule_final[i]=group[0];
i++;job_schedule_final[i]=group[1];i++;
while(i<=n)
{job_schedule_final[i]=job_schedule[i-1];i++;}}
```

8. Numerical Illustrations

Consider 5 jobs and 2 machine problem to minimize rental cost . The processing and set up time with their respective associated probabilities are given as follows. Obtain optimal sequence of jobs and minimum rental cost of complete set up , given that rental cost per unit time for machine M_1 & M_2 are 10 and 11 units respectively . Jobs 2,5 are to be processed as a group job (2,5) with breakdown interval (6,10) .

Jobs	Machine A				Machine B			
i	A_i	P_i	S_i^A	r_i	B_i	q_i	S_i^B	s_i
1	16	0.3	6	0.1	13	0.3	5	0.3

2	12	0.2	7	0.2	8	0.2	4	0.2
3	14	0.1	4	0.3	15	0.2	6	0.1
4	13	0.3	5	0.2	14	0.2	8	0.1
5	15	0.1	4	0.2	9	0.1	4	0.3

Table-1

Solution: Step 1: The expected processing time A'_i and B'_i are as shown in table 2.

Step 2: Using Johnson's two machine algorithm and job block criteria given by Maggu & Das (1977) , the optimal sequence S = 3 – 1 – 4 – β , .i.e. S= 3 – 1 – 4 – 2 – 5

Step 3: The in-out table for sequence S is as shown in table 3.

Step 5: On considering effect of breakdown interval (6, 10) revised processing times A''_i and B''_i of machines A & B are as shown in table 4.

Step 6: On repeating the procedure to get optimal sequence using Johnson's algorithm (1954) and Job block criteria by Maggu & Das (1977) ,We have the optimal sequence S_1 = 3 – 1 – 4 – 2 – 5.

Step 9: The processing time of 1st job of $S_1 = \alpha_1 = 0.8$

Step 10: Other optimal sequence for minimizing rental cost are

S_2 = 1 – 3 – 4 – 2 – 5 , S_3 = 2 – 3 – 1 – 4 – 5 , S_4 = 4 – 3 – 1 – 2 – 5

Step 11: In-out tables for sequences S_1, S_2 ,S_4 having job block (2,5) are shown in tables 5,6,7,8.

For sequence S_1= 3 – 1 – 4 – 2 – 5
Total elapsed time on machine A = $t_1 (S_1) = 26.2$

Total elapsed time on machine B = $t_2 (S_1) = 27.7$

Utilization time of 2nd machine B = $U_1 = 27.7 – 1.4 = 26.3$ units

For sequence S_2 = 1 – 3 – 4 – 2 – 5
Total elapsed time on machine A = $t_1 (S_2) = 26.2$

Total elapsed time on machine B = $t_2 (S_2) = 28.7$

Utilization time of 2nd machine B = U_1 =19.9 units

For sequence S_4 = 4 – 3 – 1 – 2 – 5
Total elapsed time on machine A = $t_1 (S_4) = 26.2$

Total elapsed time on machine B = $t_2 (S_4)$ =33

Utilization time of 2nd machine B = U_1=33.0 – 7.9 = 25.1 units

Total utilization time of machine A is fixed 26.2 units

Minimum utilization time of 2nd machine B is 19.9 units for the sequence S_2. Therefore the optimal sequence is S_2 = 1 – 3 – 4 – 2 – 5

Minimum rental cost is = 10 x 26.2 + 11 x 19.9 = 262 + 218.9 = 480.9 units.

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Remarks

1. In case set up times of each machine are negligible, the results tally with Gupta & Sharma (2011).
2. The study may be extended further for three machines flow shop, also by considering various parameters such as transportation time, weights of jobs etc.

Tables

Table 2: The expected processing time A'_i and B'_i are

Jobs	A'_i	B'_i
1	3.3	3.3
2	1.6	0.2
3	0.8	1.8

4	3.1	1.8
5	0.3	0.1

Table 3: The in-out table for sequence S is

Jobs	A	B
i	In – Out	In – Out
3	0 – 1.4	1.4 – 4.4
1	2.6 – 7.4	7.4 – 11.3
4	8.0 – 11.9	12.8 – 15.6
2	12.9 – 15.3	16.4 – 18.0
5	16.7 – 18.2	18.8 – 19.7

Table 4: The revised processing times A''_i and B''_i of machines A & B are

Jobs	A''_i	B''_i
1	7.3	7.3
2	1.6	0.2
3	0.8	1.8
4	7.1	1.8
5	0.3	0.1

Table 5: The In-out table for sequence S_1= 3 – 1 – 4 – 2 – 5

Jobs	A	B
i	In – out	In – out
3	0 – 1.4	1.4 – 4.4
1	2.6 – 11.4	11.4 – 19.3
4	12.0 – 19.9	20.8 – 23.6
2	20.9 – 23.3	24.4 – 26.0
5	24.7 – 26.2	26.8 – 27.7

Table 6: The In-out table for sequence S_2 = 1 – 3 – 4 – 2 – 5

Job	A	B
i	In – out	In – out
1	0 – 8.8	8.8 – 16.7
3	9.4 – 10.8	18.2 – 21.2
4	12 – 19.9	21.8 – 24.6
2	20.9 – 23.3	25.4 – 27.0
5	24.7 – 26.2	27.8 – 28.7

Table 7: The In-out table for sequence S_4 = 4 – 3 – 1 – 2 – 5

Job	A	B
i	In – out	In – out
4	0 – 7.9	7.9 – 10.7
3	8.9 – 10.3	11.5 – 14.5
1	11.5 – 20.3	20.3 – 28.2
2	20.9 – 23.3	29.7 – 31.3
5	24.7 – 26.2	32.1 – 33.0