

# The Modeling and Dynamic Characteristics of a Variable Speed Wind Turbine

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## Abstract

In this paper, a functional structure of a wind energy conversion system is introduced, before making a comparison between the two typical wind turbine operating schemes in operation, namely constant-speed wind turbine and variable-speed wind turbine. In addition, the modeling and dynamic behavior of a variable speed wind turbine with pitch control capability is explained in detail and the turbine performance curves are simulated in the MATLAB/simulink.

**Keywords:** Drives, energy conversion, modeling, power systems, turbines, wind energy, velocity control.

## 1. Introduction

One way of addressing the rising energy demands and growing environmental concerns, is to harness green sources of power. Among these, tapping wind energy with wind turbines appears to be the most promising source of renewable energy. Wind energy conversion systems are used to capture the energy available in the wind to convert into electrical energy. A schematic diagram of a wind energy conversion system is presented in the next section along with a detailed description of wind turbine and its modeling. The functionality of other system components is also discussed briefly. In the last section, simulation results, obtained for the variable speed wind turbine, are presented to give a better understanding of the wind turbine dynamics.

## 2. Functional Structure of Wind Turbine

A wind energy conversion system is a complex system in which knowledge from a wide array of fields comprising of aerodynamics, mechanical, civil and electrical engineering come together. The principle components of a modern wind turbine are the tower, the rotor and the nacelle, which accommodates the transmission mechanisms and the generator. The wind turbine captures the wind's kinetic energy in the rotor consisting of two or more blades mechanically coupled to an electrical generator. The main component of the mechanical assembly is the gearbox, which transforms the slower rotational speeds of the wind turbine to higher rotational speeds on the electrical generator side. The rotation of the electrical generator's shaft driven by the wind turbine generates electricity, whose output is maintained as per specifications, by employing suitable control and supervising techniques. Besides monitoring the output, these control systems also include protection systems to protect the overall system.

Two distinctly different design configurations are available for a wind turbine, the horizontal axis configuration and the vertical axis configuration. The vertical axis machine has the shape of an egg beater, and is often called the Darrieus rotor after its inventor. However, most modern turbines use horizontal axis design [1]. In this paper the dynamic model of a horizontal axis turbine is developed and simulated in the MATLAB/Simulink, based on the turbine performance curves.

The functional structure of a typical wind energy conversion system is as shown in Figure 1:

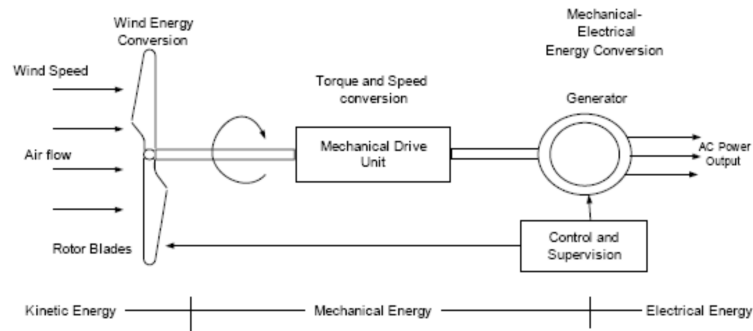


Figure 1. Power Transfer in a Wind Energy Converter.

### 3. Wind Turbine Modeling

As noted above, a wind energy conversion system is a complex system converting wind energy to rotational energy and then to electrical energy. The output power or torque of a wind turbine is determined by several factors like wind velocity, size and shape of the turbine, etc. A dynamic model of the wind turbine, involving these parameters, is needed to understand the behavior of a wind turbine over its region of operation. By studying its modeling, it is possible to control a wind turbine's performance to meet a desired operational characteristic. In the following pages we will look at different performance characteristics and variables that play an important role in wind power generation, by deriving the speed and power relations. The control principles of the wind turbine are also discussed in this section.

#### 3.1 Inputs and Outputs of a Wind Turbine

The inputs and output variables of wind turbine can be broken into the following:

1. The independent input quantity *wind speed*, determines the energy input to the wind turbine.
2. Machine-specific input quantities, arising particularly from rotor geometry and arrangement (i.e., different configurations like horizontal axis or vertical axis turbines, area of the blades, etc.).
3. Turbine speed, rotor blade tilt, and rotor blade pitch angle, arising from the transmission system of the wind energy conversion system.
4. Turbine output quantities, namely Power or Drive torque, which may be controlled by varying the above three input quantities.

#### 3.2. Power Extraction from the Air Stream

With the identification of the wind turbine's input and output variables, now it is possible to derive an expression relating these two values. The relation between the power and wind speed is derived as follows [1]:

The kinetic energy in air of mass  $m$  moving with speed  $V$  is given by the following:

$$\text{Kinetic energy} = \frac{1}{2} \cdot m \cdot V^2 \text{ Joules} \quad (1)$$

The power in moving air flow is the flow rate of kinetic energy per second.

$$\text{Power} = \frac{1}{2} \cdot (\text{mass flow rate per second}) \cdot V^2 \quad (2)$$

The actual power extracted by the rotor blades is the difference between the upstream and the downstream

wind powers. Therefore, equations (2) results in;

$$P = \frac{1}{2} \cdot (\text{mass flow rate per second}) \cdot (V^2 - V_o^2) \quad (3)$$

Where:

$P$  is the Mechanical Power extracted by the rotor in watts.

$V$  is the upstream wind velocity at the entrance of the rotor blades in m/s

$V_o$  is the downstream wind velocity at the exit of the rotor blades in m/s

Let  $\rho$  be the air density in ( $\text{kg}/\text{m}^3$ ) and  $A$  is the area swept by the rotor blades in ( $\text{m}^2$ ); then, the mass flow rate of air through the rotating blades is given by multiplying the air density with the average velocity.

$$\text{Mass flow rate} = \rho \cdot A \cdot \frac{V + V_o}{2} \quad (4)$$

From [3] and [4], the mechanical power extracted by the rotor is given by:

$$P = \frac{1}{2} \left[ \rho \cdot A \left( \frac{V + V_o}{2} \right) \right] (V^2 - V_o^2) \quad (5)$$

After algebraic rearrangement of the terms we have:

$$P = \frac{1}{2} \rho \cdot A \cdot V^3 \cdot C_p \quad (6)$$

Where:

$$C_p = \frac{\left( 1 + \frac{V_o}{V} \right) \left( 1 - \left( \frac{V_o}{V} \right)^2 \right)}{2}$$

is the fraction of the upstream wind power, which is captured by the rotor blades and has a theoretical maximum value of 0.59. It is also referred as the *power coefficient of the rotor* or the *rotor efficiency*. In practical designs, the maximum achievable  $C_p$  is between 0.4 and 0.5 for high-speed, two-blade turbines and between 0.2 and 0.4 for slow-speed turbines with more blades [1].

From equation (VI), we see that the power absorption and operating conditions of a turbine are determined by the effective area of the rotor blades, wind speed, and wind flow conditions at the rotor. Thus, the output power of the turbine can be varied by effective area and by changing the flow conditions at the rotor system, which forms the basis of control of wind energy conversion system.

### 3.3. Tip Speed Ratio

The tip speed ratio  $\lambda$ , defined as the ratio of the linear speed at the tip of the blade to the free stream wind speed and is given by the following expression [1]-[4]:

$$TSR = \lambda = \frac{\omega R}{V}$$

(7)

Where:

R is the rotor blade radius in meters

$\omega$  Is the rotor angular speed in rad/sec.

TSR is related to the wind turbine operating point for extracting maximum power. The maximum rotor efficiency  $C_p$  is achieved at a particular *tip speed ratio* TSR, which is specific to the aerodynamic design of a given turbine. The rotor must turn at high-speed at high wind, and at low-speed at low wind, to keep *tip speed ratio* TSR constant at the optimum level at all times. The larger the *tip speed ratio* TSR, the faster is the rotation of the wind turbine rotor at a given wind speed. High (rotational) speed turbines are preferred for efficient electricity generation. From [7], for a particular value of wind speed V, turbines with large blade radius R result in low rotational speed  $\omega$ , and vice versa. For operation over a wide range of wind speeds, wind turbines with high tip speed ratios are preferred [3].

#### 3.4. Typical Wind Turbine Operating Systems

There are mainly two kinds of wind energy conversion systems in operation; *fixed-speed* or *constant speed wind turbines* which operate at a nearly constant speed, predetermined by the generator design and gearbox ratio, and *variable speed wind turbines*.

The overall operating strategy determines how the various components are controlled. For example, as part of the overall control strategy, the rotor torque can be controlled to maximize energy capture, or pitch angle control can help control the power output at high wind speeds. *Fixed-speed stall-regulated turbines* have no options for control input. In these turbines the turbine blades are designed with fixed pitch to operate near the optimal *tip speed ratio* TSR at a specific wind speed. As wind speed increases, so, too does the angle of attack, and an increasingly large part of the blade, starting at the blade root, enters the stall region resulting in the reduced rotor efficiency and limitation of the power output. A variation of the stall regulated concept involves operating the wind turbine at two distinct, constant operating speeds, by either changing the number of poles of the electrical generator or changing the gear ratio. The principal advantage of stall control is its simplicity, but there are significant disadvantages; for instance, the stall regulated wind turbine will not be able to capture wind energy in an efficient manner at wind speeds other than that it is designed for. *Fixed-speed pitch-regulated* turbines typically use pitch regulation for start-up and, after start-up only to control the power above the rated wind speed of the turbine. Variable speed wind turbines typically use generator torque control for optimization of power output. They use pitch control to control the output power, only above their rated wind speed. With variable speed, there will be 20-30% increase in the energy capture compared to the fixed-speed operation.

Typical curves for a constant speed and variable speed wind turbine are as shown in the Figure 2.

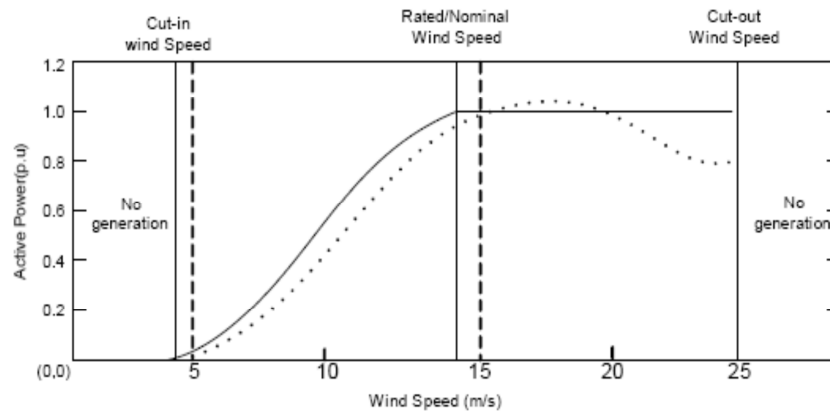


Figure 2. Typical curves for a constant speed, stall controlled (dotted) and variable speed pitch controlled (solid) wind turbine.

In a constant speed, stall-controlled wind turbine the turbine output power peaks somewhat higher than the rated limit, then decreases until the cut-out speed is reached. This feature provides an element of passive power output regulation, ensuring that the generator is not overloaded as the wind speed reaches above nominal values. With variable speed operation although the energy capture is more, the cost of variable speed control is added to the overall system cost. This tradeoff between the energy increase and cost increase has to be optimized in the system design.

The advantages of fixed-speed wind turbines are that they are; simple and robust, inexpensive electrical system, electrically efficient, fewer parts, hence high reliability, no frequency conversion, hence no current harmonics and lower capital cost. The disadvantages include; aerodynamically less efficient, mechanical stress and noisy. For a variable speed wind turbine, the advantages are; less mechanical stress, higher energy capture, aerodynamically efficient, low transient torque, mechanical damping not needed since the electrical system could provide the damping and no synchronization problems since stiff electrical controls can reduce voltage sags. The disadvantages are; electrically less efficient, expensive and sometimes involves complex control strategies.

A typical variable speed pitch-regulated wind turbine is as shown in Figure 3. Many parameters that characterize a variable-speed wind turbine are linked. For example:

1. The turbine power coefficient curve, the nominal rotor speed, and the rotor diameter determine the nominal wind speed for a wind turbine of a given nominal power.
2. The allowable amount of rotor over speeding and the rated power determines the parameters of the pitch controller.
3. The rotor inertia determines the turbine cut-in wind speed.

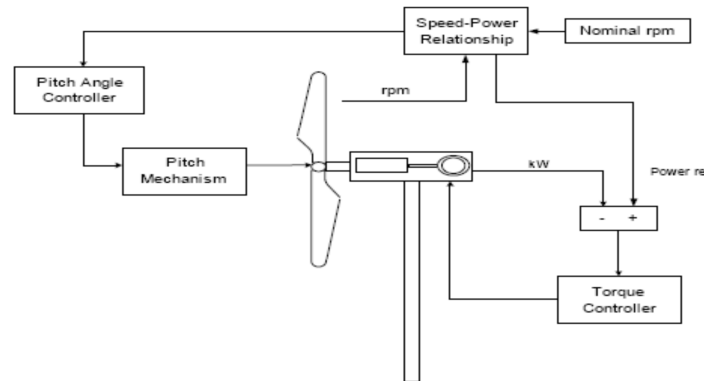


Figure 3. Typical pitch-regulated variable-speed wind turbine.

Variable speed pitch-regulated wind turbines have two methods for affecting the turbine operation, namely speed changes and blade pitch changes. In other terms, the control strategies employed in the operation of variable speed wind turbine system are:

1. *Power optimization strategy*, employed when the speed is below the rated wind speed, to optimize the energy capture by maintaining the optimum tip speed ratio. This can be achieved by maintaining a constant speed corresponding to the optimum tip speed ratio. If the speed is changed by controlling the electrical load, the generator will be overloaded for wind speeds above nominal value. To avoid such scenario, methods like generator torque control are used to control the speed.
2. *Power limitation strategy*, used above the rated wind speed of the turbine to limit the output power to the rated power by changing the blade pitch to reduce the aerodynamic efficiency, thereby reducing the wind turbine power to acceptable levels.

The regions of the above mentioned control strategies of a variable speed wind turbine system are as shown in the Figure 4:

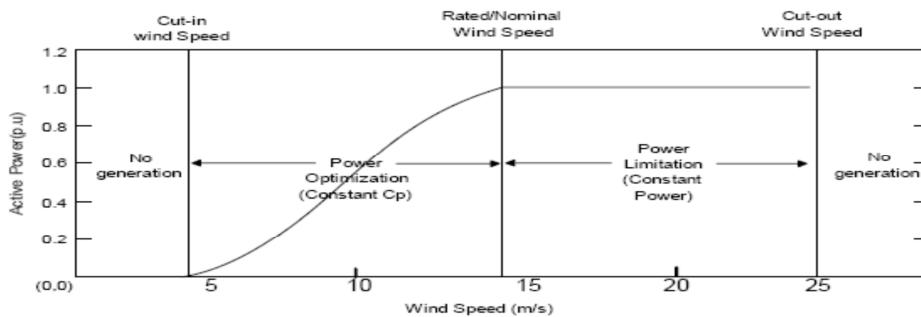


Figure 4. Variable Speed Pitch Controlled wind turbine operation

As mentioned in the previous subsection, pitch controller controls the wind flow around the wind turbine blade, thereby controlling the torque exerted on the turbine shaft. If the wind speed is less than the rated wind speed of the wind turbine, the pitch angle is kept constant at its optimum value. It should be noted that the pitch angle can change at a finite rate, which may be quite low due to the size of the rotor blades. The maximum rate of change of the pitch angle is in the order of 3 to 10 degrees/second. In this controller a slight over-speeding of the rotor above its nominal value can be allowed without causing problems for the wind turbine structure [6], [8].

The pitch angle controller used in this paper, employs a PI controller as shown below [7], [9]-[11]:

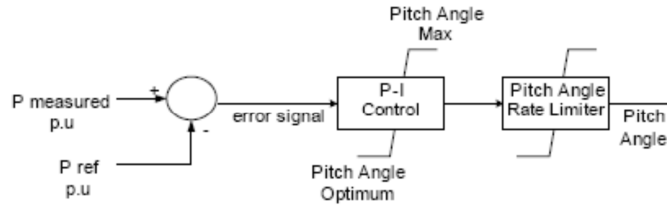


Figure 5. Pitch Angle controller

As long as the wind turbine output power is lower than the rated power of the wind turbine, the error signal is negative and pitch angle is kept at its optimum value. But once the wind turbine output power exceeds the rated power  $P_{ref}$ , the error signal is positive and the pitch angle changes to a new value, at a finite rate, thereby reducing the effective area of the blade resulting in the reduced power output. Inputs to the PI controller are in per-unit and the parameters for the controller are obtained from reference [11].

### 3.5. Performance Curves

The performance of variable speed pitch-regulated wind turbines is determined by the characteristic curves relating the power coefficient  $C_p$ , tip speed ratio  $TSR$  and pitch angle  $\theta$ . Groups of  $C_p - \lambda$  curves obtained by measurement or by computation can also be approximated in closed form by non linear functions, which can be shown as:

$$C_p = C_1 (C_2 - C_3 \theta - C_4 \theta^x - C_5) e^{-C_6(\lambda_i)} \quad (8)$$

Where:

$\theta$  is the pitch angle and  $\lambda$  is  $\frac{1}{2}$  the tip speed ratio

The values chosen for  $C_1$  to  $C_5$  and  $\lambda_i$  in this paper are:

$$\frac{1}{\lambda} = \frac{1}{\lambda + 0.08\theta} - \frac{0.035}{\theta^3 + 1}$$

$$C_1 = 0.5, \quad C_2 = \frac{116}{\lambda_i}, \quad C_3 = 0.4, \quad C_4 = 0, \quad C_5 = 5, \quad C_6 = \frac{21}{\lambda_i}$$

According to the characteristics chosen, the coefficients  $C_1$  to  $C_5$  should be modified to obtain a close simulation of the machine in consideration. The differences between the curves of various wind turbines are small and can be neglected in dynamic simulations [3]. In [6], a comparison was made between the power curves of two commercial wind turbines using the general numerical approximation similar to equation (8).

### 3.6. Simulation Results

In this section, the simulations results of a variable speed pitch controlled wind turbine model are presented. All the simulations were carried out using MATLAB/Simulink. The dynamics of the wind turbine model can be represented with the help of a flow chart, as shown in Figure 6 followed by a brief description of the

simulated model:

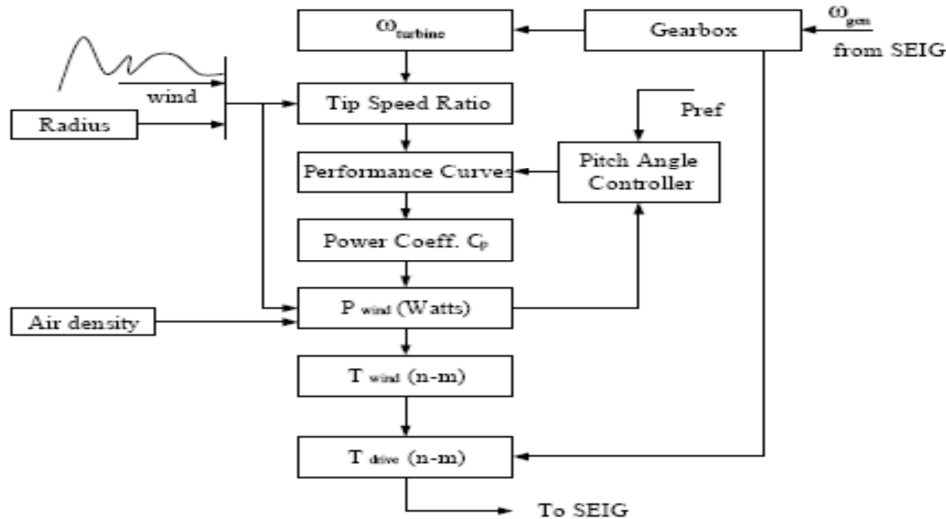


Figure 6. Simulated model of the variable speed pitch-regulated wind turbine.

The inputs for the wind turbine model are, wind speed, air density, radius of the wind turbine, mechanical speed of the rotor referred to the wind turbine side and power reference for the pitch angle controller. The output is the drive torque  $T_{drive}$  which drives the electrical generator. The wind turbine calculates the tip speed ratio from the input values and estimates the value of power coefficient from the performance curves. The pitch angle controller maintains the value of the blade pitch at optimum value until the power output of the wind turbine exceeds the reference power input.

The performance curves used in this paper (from [8]) are as shown below:

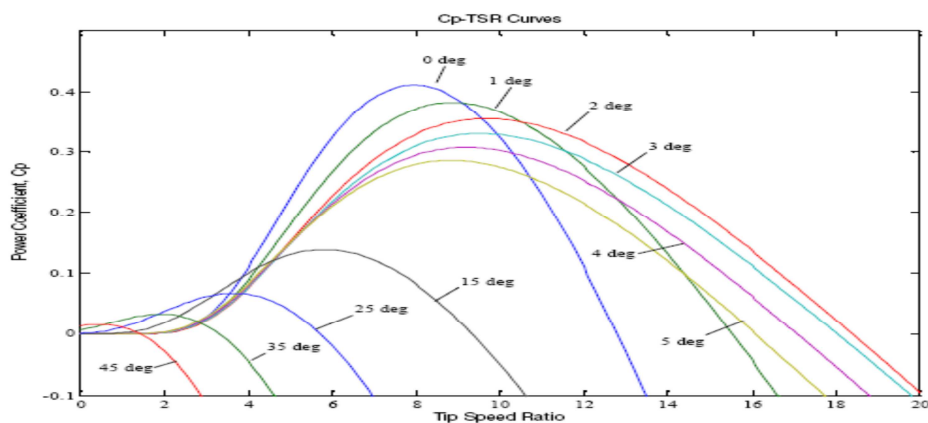


Figure 7.  $C_p - \lambda$  characteristics (blade pitch angle  $\theta$  as the parameter)

From the above set of curves (Figure 7), we can observe that when pitch angle is equal to 2 degrees, the tip speed ratio has a wide range and a maximum  $C_p$  value of 0.35, suitable for wind turbines designed to operate over a wide range of wind speeds. With an increase in the pitch angle, the range of tip speed ratio  $TSR$  and the maximum value of power coefficient decrease considerably.

The parameters used for the simulation are as follows:



Rated power of the wind turbine = 370 kW.

Radius of the wind turbine blade = 20 m.

Gearbox turns ratio = 1:20.

Air density = 1 kg/ m<sup>3</sup>

Figure 8 shows the wind turbine output power of the simulated model for different wind velocities. It can be observed that the output power is kept constant at higher wind velocities, even though the wind turbine has the potential to produce more power; this power limit is used and to prevent the over speeding of the rotor and to protect the electrical system.

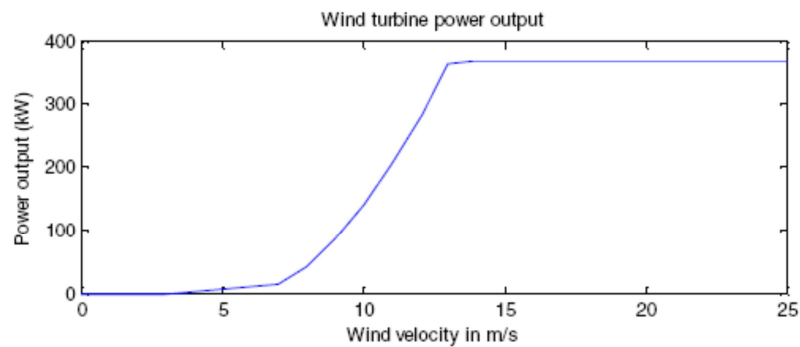


Figure 8. Output power of the wind turbine for different wind velocity

For the following simulation results, the wind turbine starts with an initial wind velocity of 11m/s at no-load, and load was applied on the machine at  $t=10$  seconds. At  $t=15$  seconds there was a step input change in the wind velocity reaching a final value of 14 m/s. In both cases the power reference remained the same (370 kW). The simulation results obtained for the above mentioned conditions are as follows:

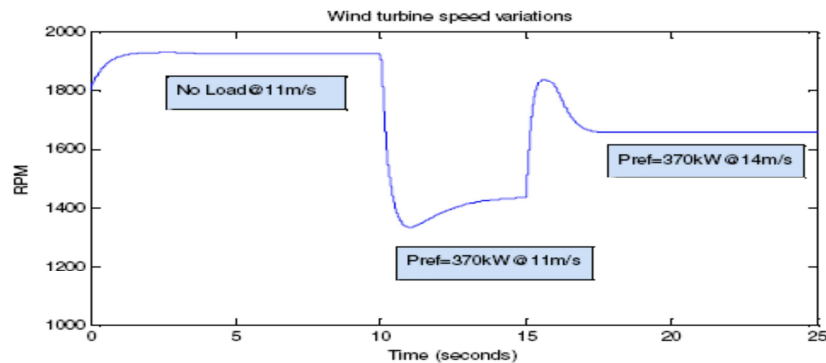


Figure 9. Wind turbine rotor speed variations with wind.

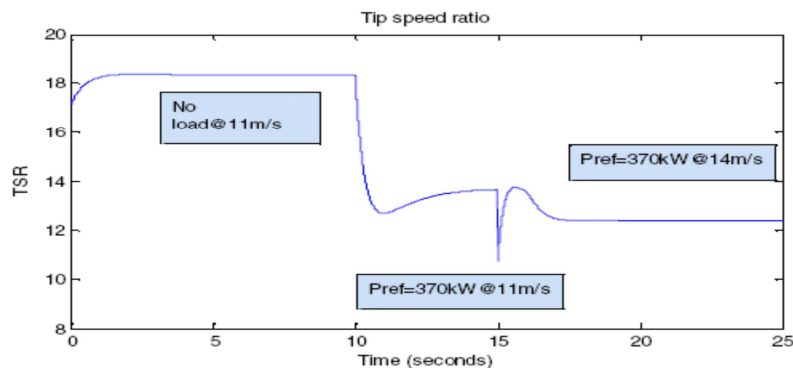


Figure 10. Tip speed ratio of the wind turbine

From Figures 9 and 10 we can see the changes in the tip speed ratio corresponding to the changes in the rotor speed for different wind and load conditions. During no-load operation, the wind turbine is not connected to the load and therefore rotates freely under the influence of wind attaining high angular velocities. From [7], which gives the relation between the tip speed ratio and angular velocity of the wind turbine, it can be concluded that higher angular velocities result in higher tip speed ratios. In this case as *tip speed ratio TSR* reaches a value greater than 18, the value of the power coefficient obtained from the set of performance curves is almost zero. As the rotor power coefficient value is zero, the energy captured by the rotor blades is also zero. Therefore, the torque exerted on the generator shaft also equals to zero.

As load is applied on the wind energy conversion system, the speed of the electrical generator drops which reduces the speed of the wind turbine (as they are coupled through a gear box, Figure 9). This drop in angular speed of the wind turbine reduces the value of *tip speed ratio TSR* resulting in a higher value of the power coefficient as can be observed from the power coefficient curves (Figure 7). Since rotor power coefficient is now a positive, non-zero, the rotor blades extract energy from the wind resulting in some output power.

Figure 11 shows the changes in the power coefficient with changes in the tip speed ratio. From Figure 7 it can be seen that, on the right hand side (after reaching the peak value) of the  $C_p$  Vs  $\lambda$  curves, with increase in the value of *tip speed ratio TSR* the value of  $C_p$  decreases. It also shows that lower wind velocities (higher *tip speed ratio TSR*, see [7]) result in high power coefficient, and higher wind velocities (lower *tip speed ratio TSR*) result in lower power coefficient values, so that the wind turbine output does not exceed its rated power.

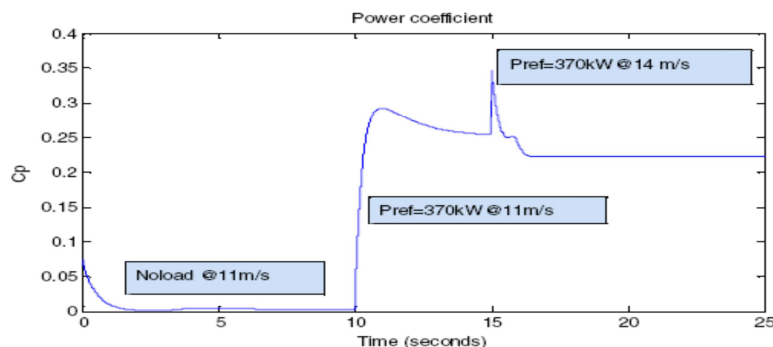


Figure 11. Variation of power coefficient with wind.

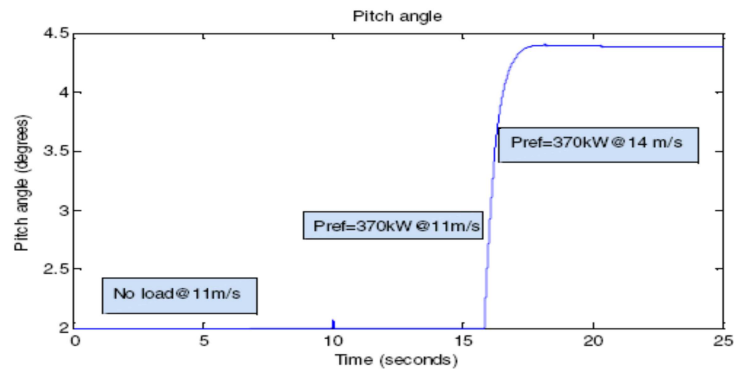


Figure 12. Pitch angle controlled response to the wind speed change

Observe that the pitch angle is kept constant by the pitch controller at an optimal value of 2 degrees (Figure 12) until the wind turbine reaches above the nominal wind speed (13.5 m/s), where it has the capability to produce more power than the rated power of the system. In this region pitch control alters the pitch of the blade, at a finite rate, thereby changing the airflow around the blades resulting in the reduced efficiency of wind turbine rotor. Also, from Figure 7 we see that with increase in the pitch angle the range of *tip speed ratio TSR* which produces a positive value for rotor power coefficient reduces sharply.

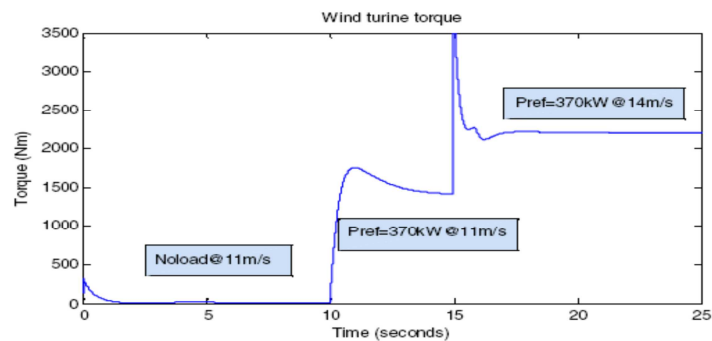


Figure 13. Wind turbine output variation with changes in wind speed

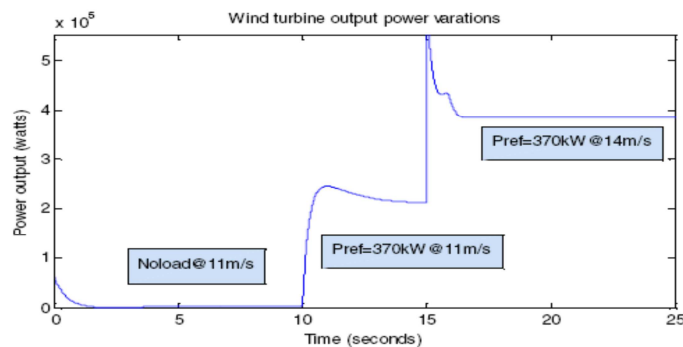


Figure 14. Wind turbine torque variation with wind

From Figures 12 and 13 we see that the output of the wind turbine is about 220kW at 11 m/s and has the ability to supply more than rated power of 370 kW at 14 m/s. As explained above, the wind turbine output power is limited to the rated power by the pitch controller.

The driving torque produced by the wind turbine as observed in Figure 14, varies based on the input (wind) and load conditions. When the system is not loaded the driving torque produced the wind turbine is zero, since the energy captured by the rotor is zero. When load is applied at  $t=10$  seconds, we can see a rise in the value of the torque produced by the wind turbine. With a step increase in wind velocity (to 14 m/s), the torque increases from its previous value, and delivers a higher torque to the electrical generator. Torque seen here is obtained by dividing the power extracted from the wind by the angular velocity of the rotor and then referring it on to the generator side.

#### 4. Conclusion

In this paper a variable speed wind energy conversion system has been presented. Emphasis has been laid on the wind turbine part of the total system. A comparison was made between the two typical existing wind turbine systems after a brief introduction of each system. A variable speed wind turbine model was simulated based on the power coefficient curves. The response of the simulated model was observed with dynamic load and changing wind conditions. It was observed with the help of the simulation results, that the power limitation strategy can be successfully enforced by using a pitch controller.

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