Journal of Natural Sciences Research
ISSN 2224-3186 (Paper) ISSN 2225-0921 (Online)
Vol.5, No.4, 2015
www.iiste.org

Comparison of Beta, Gamma Weibull Distributions for Characterising Tree Diameter in Oluwa Forest Reserve, Ondo State, Nigeria

Ogana, F.N* Osho, J.S.A. Gorgoso-Varela, J.J
Department of Forest Resource Management, University of Ibadan, Ibadan, Nigeria
E-mail: ogana_fry@yahoo.com

Abstract

This study compared the accuracy of the Beta, 2-parameter Gamma (2P) and 3-parameter Weibull (3P) distributions, fitted with the method of moments, for characterising the tree diameter of the reserve. Comparison was based on the Kolmogorov-Smirnov statistic (K-S), bias, mean absolute error (MAE), and mean square error (MSE). Distributions with location parameter were fixed as the minimum inventoried diameter of each plot. A total of eight (8) temporary sample plots (TSPs) of size 50m x 50m were laid in the natural stand of the reserve. Systematic line transect was used in the laying of the plots. All trees with DBH $\geq$ 10.0cm in the selected plots were enumerated, identified and measured. The results from the goodness-of-fit statistics revealed that the Weibull (3P) distribution performed slightly better than the Beta distribution used in this study. The mean values for the K-S, bias, MAE, and MSE of the Weibull distribution were 0.11449, 0.00015, 0.00847, and 0.00022, respectively; as such ranked best. The Gamma (2P) distribution provided the worst fit to the dataset, with relatively large values for the goodness-of-fit statistics. It fits for the entire plot were far from the reverse J-shaped of natural forests, which implies that the Gamma (2P) distribution is inappropriate for determining the structure of the natural stand.

Keywords: diameter characterisation, probability distribution, moments, natural forest

1. Introduction

Diameter distribution modelling has been an intrinsic part of forest management planning and research in the recent times. This has often been used in bridging the gap between crude stand-level simplification and complex individual tree models (Thomas and Cao, 2006). Size distribution information is essential for analysis of the structure of forest stand and making management decisions. Diameter distribution modelling in natural forest has always been a herculean task owing to the complex nature of the tropical forest which is characterised by diverse species composition and indeterminate age structure.

The first study on size distribution was done by De Liocourt in 1898 when he observed that plotting the number of stems against equal-diameter classes as a frequency histogram results in a reverse J-shaped. In recent forest practice, several models of probability density of tree diameters in stand have been verified e.g. Beta (Loetsch *et al*., 1973; Gorgoso *et al*., 2008, 2012), Gamma (Nelson, 1964; Mohammed, *et al*., 2009; Zheng and Zhous, 2010; Eslami *et al*., 2011), Johnson S_B (Johnson and Kitchen, 1971; Knoebel and Burkhart, 1991), Lognormal (Sheykholeslami *et al*., 2011), Normal (Nanang, 1998) and Weibull distribution (Bailey and Dell, 1973;Zutter *et al*., 1986; Maltamo *et al*., 1995; Palahi *et al*., 2007; Ajayi, 2013). While most of these distributions, for example, the Weibull distribution has gained prominence and has enjoyed elaborate application in quantitative forestry in Nigeria and other parts of the world, little studies have been done on Beta and Gamma distribution model for characterising tree diameter of natural forest. This is not far-fetched from the complexity of computation in estimating the parameters of the Beta and Gamma distributions and few literatures abound on the subject, fortunately these distribution have the capability of assuming a wide variety of shapes to suit structure of natural forest. It is towards this end that the study seeks to explore and compare the effectiveness of Beta, Gamma and Weibull distributions for characterising tree diameter of Oluwa Forest Reserve, Nigeria.

2. Methodology

2.1 The Study Area

This study was carried out in Oluwa Forest Reserve located in the moist tropical rainforest zone of Nigeria. It occupies an area of about 629km^2 with much of it lying approximately between 300 and 600m above sea level (Ogunjemite *et al.,* 2006). The natural forest covers about 8km^2 (approximately 800ha) of the Forest Reserve. The Reserve is situated in Odigbo Local Government Area of Ondo State, Nigeria and lies between Latitude 6.83° - 6.91°N and Longitude 4.52° - 4.59°E (see Fig. 1). Annual rainfall ranges from 1700 to 2200 mm. Annual mean temperature in Oluwa is 26 °C. The relative humidity is high and uniform, ranging from 75% (afternoon) to 95% (morning).The natural vegetation of the area is tropical rainforest characterised by emergent with multiple canopies and lianas.

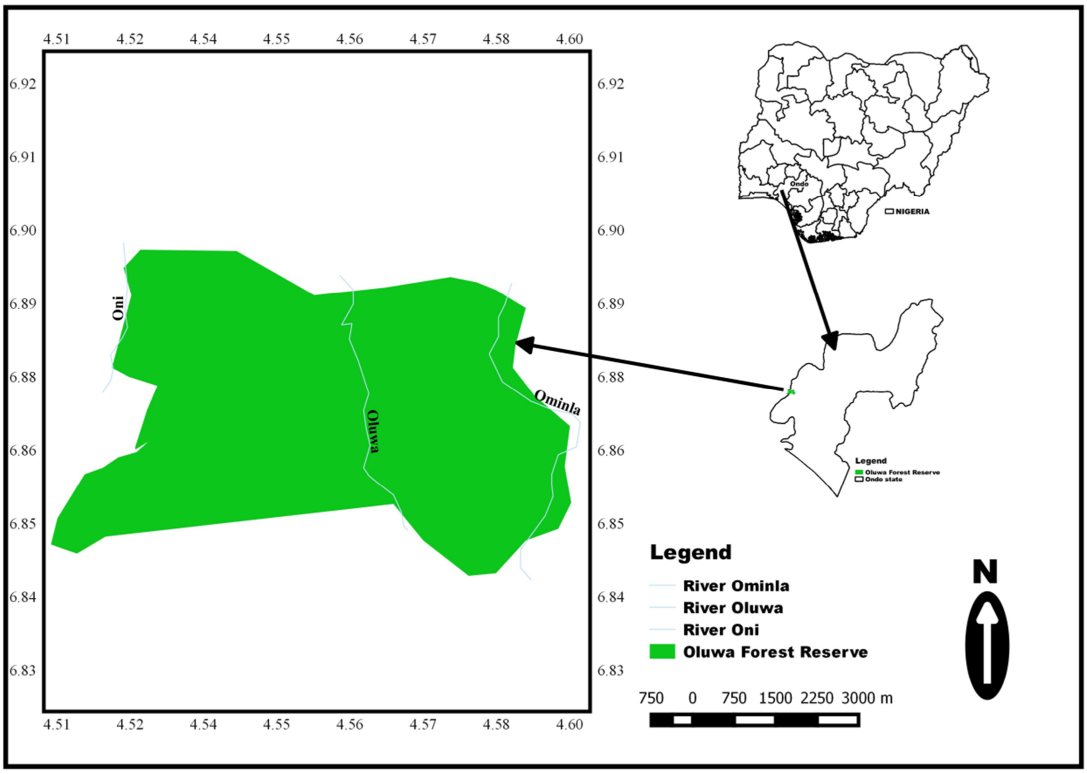


Fig. 1: Map of Oluwa Forest Reserve located in Ondo State, Nigeria (Source: Ogana, 2015)

2.2 Sampling Procedure, Data Collection and Processing

In this study, systematic sampling technique was used in the laying of the temporary sample plots (TSPs) in the 8km^2 natural forest. Two transects of 500m in length with a distance of 200m between the two parallel transects were laid. Sample plots of 50m x 50m in size were established in alternate position along each transect at 100m interval; summing up to 4 sample plots per 500m transect and a total of 8 sample plots in the study area, with a total area of 20,000m^2 (i.e. 2ha). All living trees with Dbh ≥10.0cm in the selected plots were enumerated and identified by their botanical name using Trees of Nigeria (Keay, 1989) and measured. The data collected were grouped into species and families, and the following stand variables were computed from the inventory data: mean diameter, minimum diameter, maximum diameter, number of trees per hectare and basal area. The summary statistics of the dataset used for this study are presented in Table 1.

Table 1. Summary statistics of the data from the sample plots

	Statistics			
Variables	**Mean**	**Maximum**	**Minimum**	**Standard deviation**
No of Species = 58				
No of Family = 26				
Dbh (cm)	24.7	118.5	10.0	16.2
Basal area (m^2/ha)	18.28	31.72	8.12	7.31
Density (tree/ha)	267.5	352.0	196	60.0
Dominant Ht (m)	33.3	46.9	26.7	8.3

2.3 *Model Specification and Fitting Method*

2.3.1 The Beta function

The general beta distribution function used by Loetsch *et al.* (1973) and more recently by Palahi *et al.* (2007) and Gorgoso *et al.* (2008 and 2012) was used for this study. It is expressed as

$$f(x) = c.(x\text{-}L)^{\alpha}.(U\text{-}x)^{\gamma} \quad L \leq x \leq U \tag{1}$$

Where: *f(x)* is the probability density associated with diameter at breast height (Dbh) *x*, L and U are the limits of the distribution (a<b) i.e. Lower and Upper limit respectively, *c* is the scaling factor of the function, and *α* and *γ* are the first and second exponents that determine the shape of the distribution respectively.

2.3.2 The Gamma function

Also, the 2-parameter Gamma distribution function proposed by Krishnamoorthy (2006) was used. It is expressed as:

$$f(x) = \frac{(x)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e\left(-\left(\frac{x}{\beta}\right)\right) \tag{2}$$

Where: $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t}\, dt \qquad (\alpha > 0)$ (3)

α = shape parameter ($\alpha > 0$), β = scale parameter ($\beta > 0$), and x = diameter (Dbh).

2.3.3 The Weibull function

The 3-parameters Weibull distribution (Weibull 1951) was used for this study. It is expressed as:

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} exp\left[-\left(\frac{x-a}{b}\right)^{c}\right] \tag{4}$$

Where: x = tree diameter, a, b and c are the location, scale and shape parameters of the distribution respectively.

2.4 The Method of Moment (MOM)

In this study, the method of moments (MOM) was used to estimate the parameters of the distributions. It is based on the relationship between the parameters of the functions and the first and second moment of the diameter distribution (i.e. arithmetic mean diameter and variance, respectively). This method was selected for this study because it is computationally simple and consistent. More so, because only the method of moment has been used in forestry studies to estimate the beta parameters; this will provide a good platform for their comparison.

2.4.1 MOM for beta

The method of moment used by Palahi *et al.* (2007) and Gorgoso *et al.* (2012) was used to estimate the beta parameters in equation 1 above.

$$\alpha = Z.(\alpha_2 + 1) - 1 \tag{5}$$

$$\gamma = \frac{Z}{\frac{s_{rel}^2.(Z+1)^2}{Z+1} - 1} - 1 \tag{6}$$

Where:

$$Z = \frac{x_{rel}}{1 - x_{rel}} \tag{7}$$

$$x_{rel} = \frac{\bar{d} - L}{U - L} \tag{8}$$

$$s_{rel}^2 = \frac{s^2}{(U-L)^2} \tag{9}$$

The parameter *c* is estimated as: $C = \frac{1}{\int_L^U (x-L)^{\alpha}.(U-x)^{\gamma}\, dx}$ (10)

Integrating the above function will yield:

$$C = \frac{1}{\frac{(-L+U)^{1+\gamma}.\Gamma(1+\alpha).\Gamma(1+\gamma)}{\left(\frac{1}{-L+U}\right)^{\alpha}.\Gamma(2+\alpha+\gamma)}} \tag{11}$$

L and *U* are the lower and upper limit were considered as the minimum and maximum diameter in each plot to be inventoried, respectively, and $\underline{d}$, is the arithmetic mean diameter and s^2 is the variance, Γ(*i*) is the Gamma function in the point *i*.

2.4.2 MOM for Gamma

The general method of moment for estimating the Gamma parameters was used for this study. It is expressed as:

$$\alpha = \frac{\bar{d}^2}{s^2} \tag{12}$$

$$\beta = \frac{s^2}{\bar{d}} \tag{13}$$

The variables and parameters in the equations are previously defined

2.4.3 MOM for Weibull

The method of moment used by Stankova and Zlatanov, (2010); and Gorgoso *et al.* (2012) was used to estimate the Weibull parameters in equation 4 above. Expressed as:

$$b = \frac{\bar{d}-a}{\Gamma\left(1+\frac{1}{c}\right)} \qquad (14)$$

$$\sigma^2 = \frac{(\bar{d}-a)^2}{\Gamma^2\left(1+\frac{1}{c}\right)}\left[\Gamma\left(1+\frac{2}{c}\right) - \Gamma^2\left(1+\frac{1}{c}\right)\right] \qquad (15)$$

Where: *a* which is the location parameter was taken as the smallest diameter of the plot, $\bar{d}$ is the arithmetic mean diameter of the distribution, σ^2 is the variance and $\Gamma(i)$ is the Gamma function.

2.5 Diameter Characterisation

After the parameters of the distributions have been estimated using the method of moments, the parameters were fitted to the distribution functions. This was used to obtain the class probabilities (P_i) and subsequently used to compute the diameter-class frequencies for each plot.

∴ Predicted Number of tree per class (N_i) = N *x* P_i (16)

Where: N_i = estimated number of trees per class, N = number of trees per ha and P_i = class probability.

2.6 Model Comparison

The following goodness-of-fit indices were used to test the ability of the beta, Gamma and Weibull distributions to smooth the diameter distribution of the trees in Oluwa natural forest stand.

***2.6.1 Kolmogorov-Smirnov (KS) test*:** this was used to compare the cumulative estimated frequency with the observed frequency. The most striking difference between the two distributions was the D_n statistic value of the KS test:

$$D_n = Supx|F(x_i) - F_0(x_i)| \qquad (17)$$

Where: sup*x* is the supremum value for *x*:

$$D_n = max\left\{max_{1\le i\le n_i}\left[F_n(x_i) - F_0\left(x_j\right)\right], max_{1\le i\le n_i}\left[F_o\left(x_j\right) - F_n(x_{i-1})\right]\right\} \qquad (18)$$

$F(x_i)$ is the cumulative frequency distribution observed for the sample x_i (i = 1, 2,…, n)

$F_0(x_i)$ is the probability of the theoretical cumulative frequency distribution.

The fitting method consistency was also evaluated by the Bias, mean absolute error (MAE) and mean square error (MSE). The statistics were calculated for each fit in mean relative frequency of trees per one for all diameter classes and plots.

2.6.2 Bias: $$Bias = \frac{\sum_{i=1}^{N} Y_i - \hat{Y}_i}{N} \qquad (19)$$

2.6.3 Mean Absolute Error (MAE) : $$MAE = \frac{\sum_{i=1}^{N} |Y_i - \hat{Y}_i|}{N} \qquad (20)$$

2.6.4 Mean Square Error (MSE): $$MSE = \frac{\sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2}{N} \qquad (21)$$

Where: Y_i is the observed value, $\hat{Y}_i$ is the theoretical value predicted by the model and N is the number of data points.

All statistical analysis was performed using SAS for Windows Version 9.1.

3. Result

Graphical analyses of the observed number of trees (N/ha) and the fitted distributions (beta, Gamma and Weibull) was no doubt typical of a natural forest, where a larger proportion of trees are found in the smallest diameter classes with decreasing frequency as the diameter increases; given rise to reverse J-shaped structure (see Fig. 2A-C). The skewness and kurtosis of the distributions were positive which also suggested that the number of small-size trees dominated the stand. From the results, it can be seen that the Weibull distribution did not show much differences from the beta distribution in fitting the diameter distribution of the entire plots assessed in this study. The expected number of trees (N/ha) produced by the Weibull and beta distributions showed slight variation with the observed distribution, however, the superiority of the Weibull distribution over the Beta distribution can still be seen in some of the diameter classes. A different scenario was observed in the case of the 2-parameter Gamma distribution. Poor fits were observed for the Gamma distribution for the eight (8) Plots assessed. In fact, it fits were characterised by under and overestimation of the N/ha for the individual diameter classes, which suggested that the 2-parameter Gamma distribution was inappropriate in fitting the diameter distribution of the natural stand.

Clear differences in the performance of the candidate distribution functions could be identified (Table 2). The overall ranking in terms of mean values of bias, mean absolute error (MAE), mean square error (MSE), and mean and standard deviation of the Kolmogorov-Smirnov (K-S) statistic summarizes the overall accuracy of the distribution functions as comparison criteria. The results showed that the Weibull distribution had the smallest K-S mean value of 0.11449, smallest MSE, MAE and bias of 0.00022, 0.00847, and 0.00015, respectively. This proved the superiority of the Weibull distribution function over the beta and Gamma distribution in characterising the diameter of the natural stand. This was followed by beta distribution which had a closer mean K-S value of 0.15501 to the Weibull distribution. Lastly, in the ranking order was the 2-parameter Gamma whose K-S statistic (0.30787) and bias revealed poor fits.

Table 2. Goodness of fit test for Beta, Gamma and Weibull distributions

Distribution	**Bias**	**MAE**	**MSE**	**K-S (D_n)**
Beta	0.00063	0.00963	0.00032	0.15501 [0.04864]
Gamma	0.00192	0.00913	0.00027	0.30787 [0.04639]
Weibull	0.00015	0.00847	0.00022	0.11449 [0.02145]

Standard deviation is enclosed in square brackets

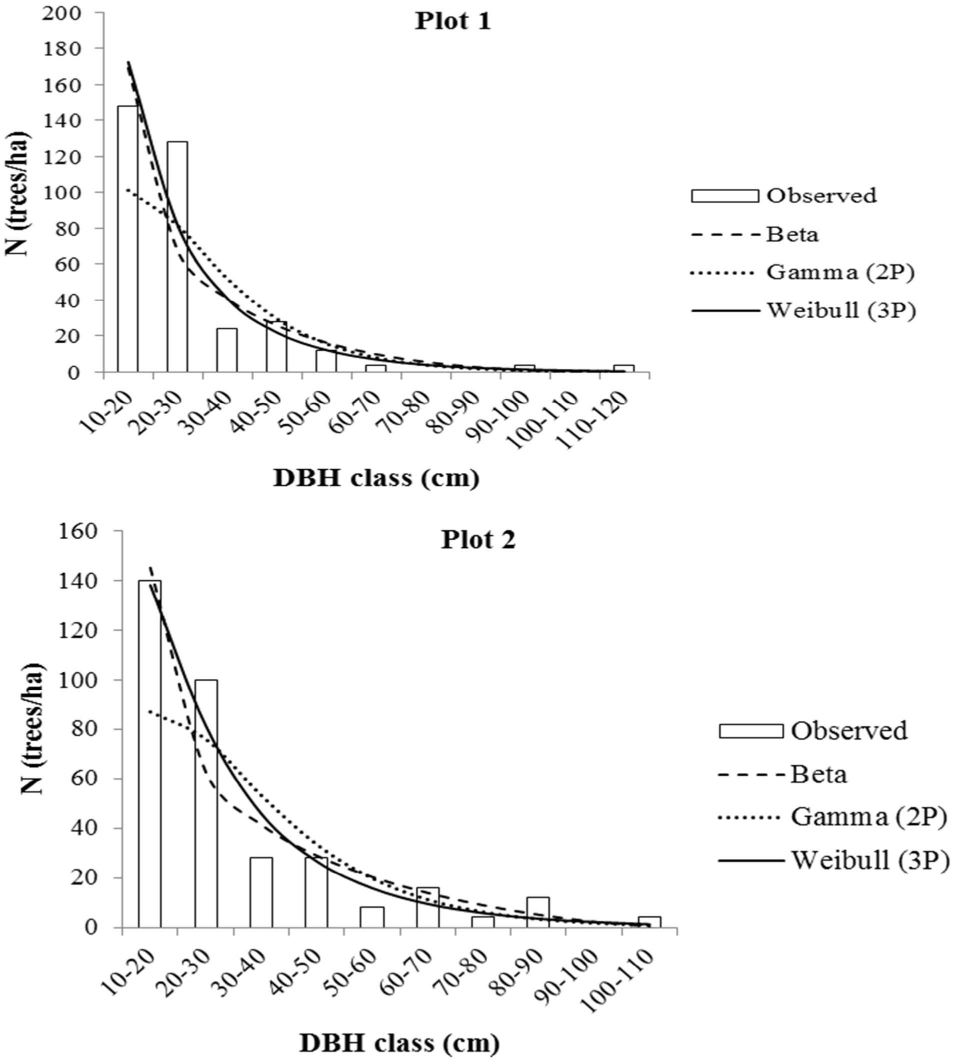
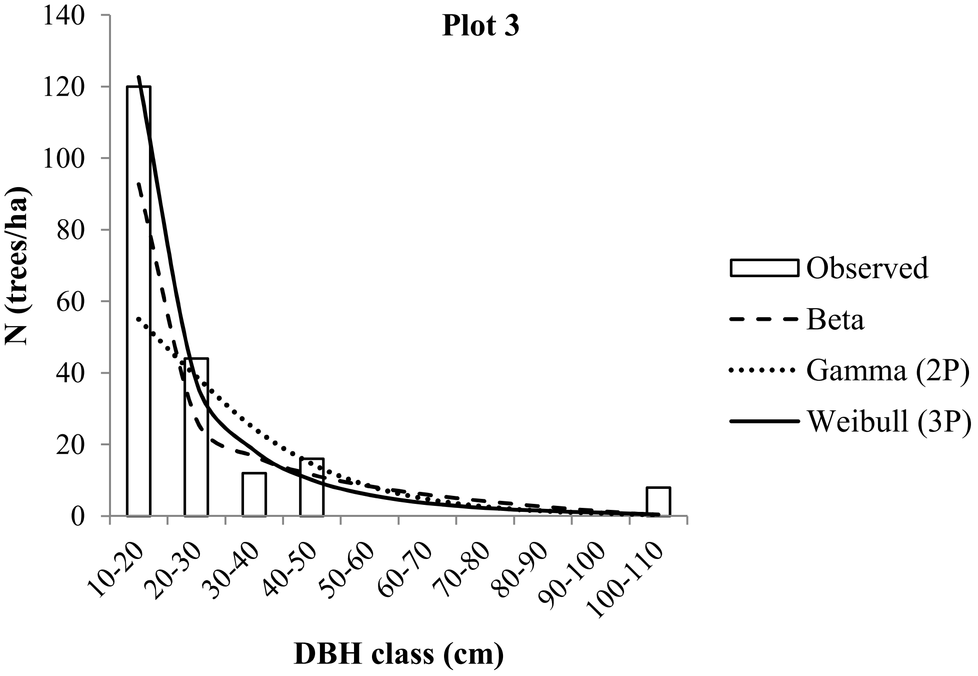


Fig. 2A: Observed diameter distributions, fitted beta, Gamma and Weibull distributions by moments approach in in number of trees per ha of plot 1 and 2



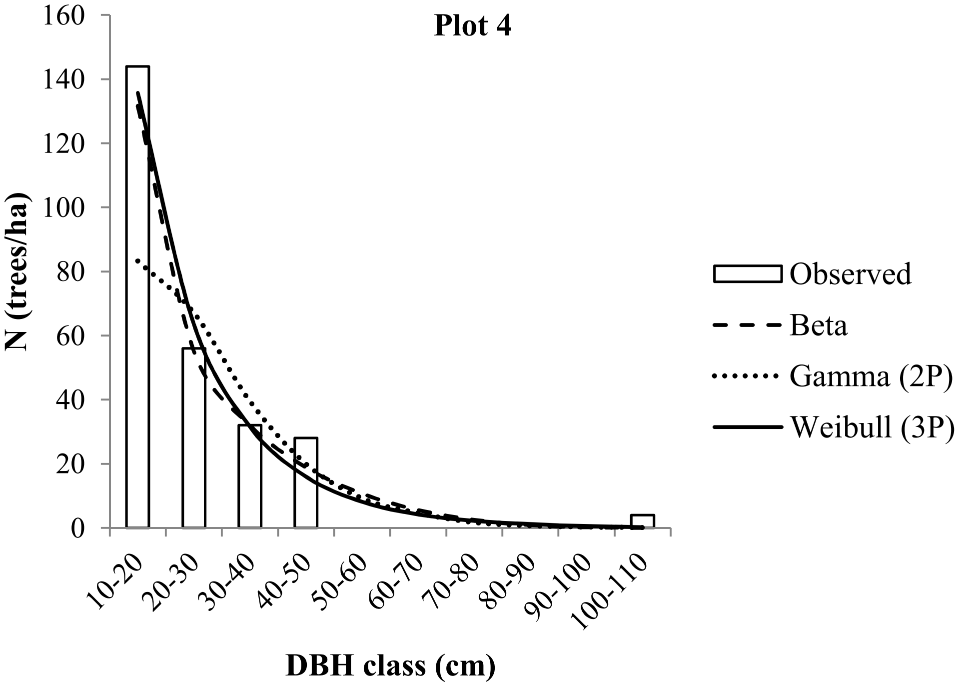


Fig. 2B: Observed diameter distributions, fitted beta, Gamma and Weibull distributions by moments approach in number of trees per ha of plot 3 and 4

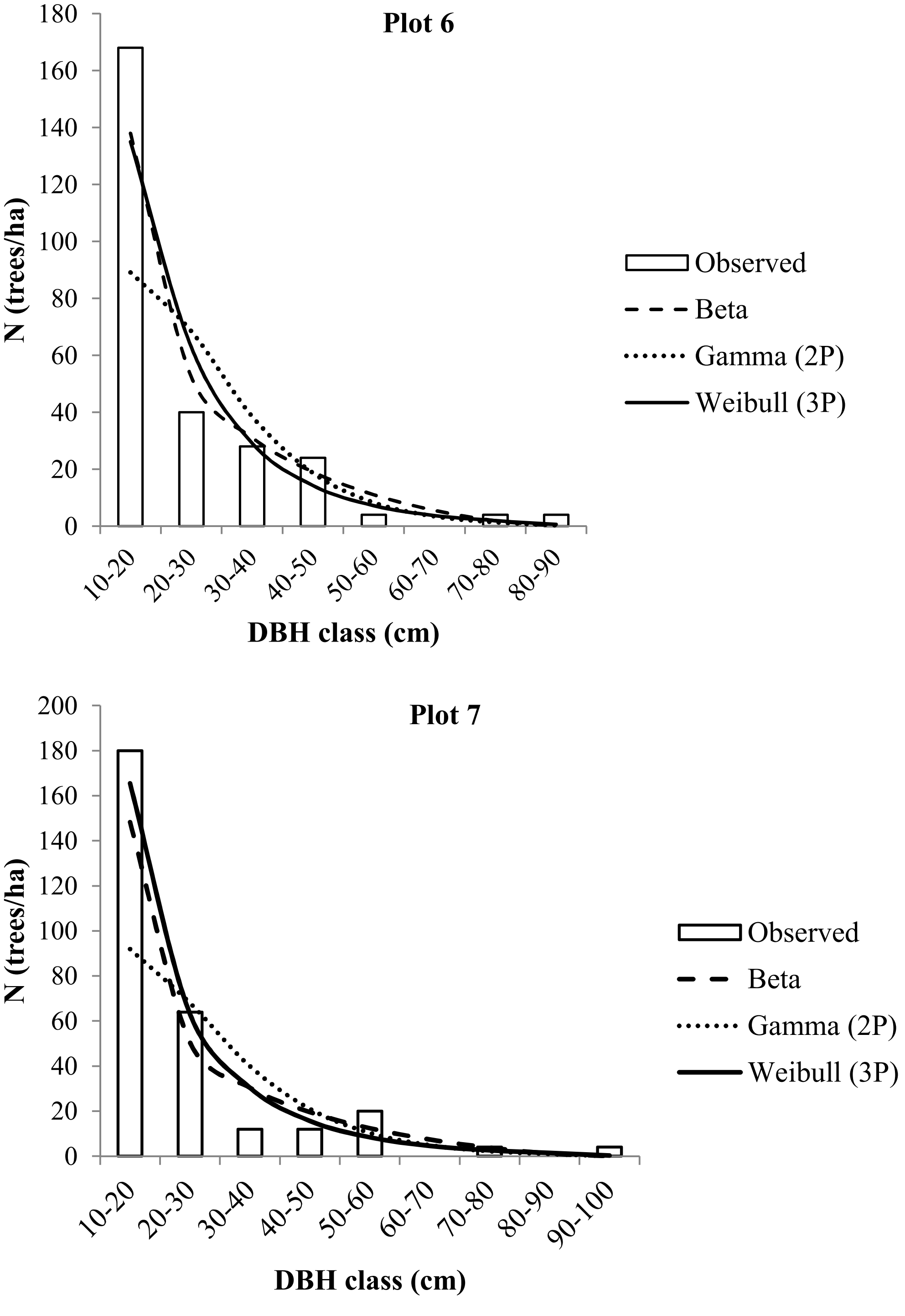


Fig. 2C: Observed diameter distributions, fitted beta, Gamma and Weibull distributions by moments approach in in number of trees per ha of plot 6 and 7

4. Discussion

The characterisation of the tree diameter of the natural stand of Oluwa Forest Reserve was the hallmark of this study. The effectiveness of beta, Gamma and Weibull distributions for diameter characterisation was compared in pursuant of the best distribution function that could determine the structure of the forest stand. The method of moments was used in the parameter estimation of the distribution functions wherein fitting was done. The assessment of the characterisation ability of the distribution functions based on the goodness of fit statistics (i.e. K-S, bias, MAE, and MSE) revealed that the 3-parameter Weibull distribution provided the best fit to the dataset for the eight (8) plots considered. The relative flexibility of the Weibull distribution could have influenced it performance. Attributed to this could also be as a result of the effects of the three-parameters that were used for the characterisation. This is because when the 2-parameter Weibull distribution was considered (though not documented in this write-up) provided a poor fit to data whose results were not better off than the 2-parameter Gamma distribution used in this study. This study is in agreement with the findings of Zhang *et al.* (2003) who opined that the results of the fitted distribution depend on the underlying assumptions used in the fitting process, and concluded that the assumption concerning some parameters could have been more refine. This result agreed with Zheng and Zhous (2010) who modeled diameter distribution of trees in natural stands managed on polycyclic cutting system using beta, Gamma, negative Exponential and Weibull distributions. They were of the opinion that Weibull distribution model fitted better than others regarding the structure of diameter distribution in natural forests managed on polycyclic cutting system.

The beta distribution whose performance is noteworthy, ranked second based the aforementioned fit statistics. It fits for the entire plots used for this study show little or no variation from the fitted 3-parameter Weibull distribution; which implies that the beta distribution could be used in lieu of the Weibull distribution for diameter characterisation. The finding agreed with Fallahchai *et al.* (2000) who investigated the structure of natural beech stands using different statistical distributions; found that the beta distribution was more appropriate for the diameter characterisation of the forest stands. However, this is not in line with Bullock and Boone (2007) who studied diameter distributions of loblolly pine trees, observed that sometimes none of the distributions may fit the dataset and concluded that in such situation Bayesian model, averaging distribution could be used. Furthermore, this finding agreed with Gorgoso *et al.* (2012) who compared the accuracy of the Weibull distribution, Johnson S_B and beta distributions for describing the diameter distributions in even-aged stands of three pines species; found that the Johnson S_B and Beta distribution were more superior than the Weibull distribution for two of the species.

The 2-parameter Gamma distribution used in this study did not provide good fits to the dataset. It fits for the eight (8) plots were far from the usual reverse J-shaped that is typical of natural forest. This suggests the fact that the 2-parameter Gamma distribution may be inappropriate for describing the stand structure of the natural forest. The fit provided by the Gamma distribution for this study also suggest to me that the 2-parameter Gamma could be adequate for even-aged stands. This study agreed with Eslami *et al.* (2011) who investigated the structure and distribution of diameter classes in beech forest; reported that the Gamma distribution did not fit the dataset, as such recommended the beta distribution model for the natural stand. However, this finding is not in line with Mohammad *et al.* (2009) who studied the diameter distribution in uneven-aged forest stand; observed that the Gamma distribution provided the best fit for the forest stand. From the foregoing discussion it seems that fitting the same dataset with 3-parameter Gamma distribution may provide a good fit. Since the number of parameters affect distribution performance.

5. Conclusion

This study has provided some baseline information on the diameter distribution of the natural stands of Oluwa Forest Reserve. The comparison on the effectiveness of the Beta, 2-parameter Gamma and 3-parameter Weibull distributions for characterising the tree diameter of the reserve has been made. The results from the goodness-of-fit statistics i.e. Kolmogorov-Smirnov (K-S), bias, mean absolute error (MAE), and mean square error (MSE) indicated that both the Weibull and Beta distribution were successful in fitting the data. However, the Weibull distribution was more consistent in all the diameter classes for the individual plots than the beta distribution. In the case of the Gamma distribution, poor fits and inconsistency were observed for the entire plots used for this study, which implies that the Gamma distribution cannot determine the true structure of the natural stands i.e. the reversed J-shaped.

Since poor fits were observed for the 2-parameter Gamma and 2-parameter Weibull distributions, the researchers suggested that further study be carried out on the likely effects of the number of parameter on the fitting performance of distributions vis-à-vis the type of forest stands (i.e. even-aged and uneven-aged).

Acknowledgement

This research work was financially supported by the family of Engr. Ndubuisi Ogana. Also, the authors wish to appreciate colleagues of the Department of Forest Resources Management, University of Ibadan, Nigeria for their assistance during the field work.

References

Ajayi, S. (2013). Diameter distribution for *Gmelina arborea* plantations in Ukpon River Forest Reserve, Cross River State, Nigeria, AFRREV STECH, 2 (1): 64-82.

Bailey, R. L., & Dell, T.R. (1973). Quantifying diameter distributions with the Weibull function, *For. Sci.* 19:97-104.

De Liocourt F.9, (1898). De l'amenagement des Sapiniers. Bul. Soc. For. Franche-Comptéet Belfort 4, 396-409

Eslami, A.R., Karimi, B., Payam, H. & Derakhshan, O.K. (2011). Investigation of the structure and distribution diameter classes models in beech forests of northern Iran, *African Journal of Agricultural Research*. 6(10):2157-2165.

Gorgoso, J.J, Rojo A, Camara-Obregon A. & Dieguez-Aranda U. (2012). A comparison of estimation methods for fitting Weibull, Johnson's S_B and beta functions to *Pinus pinaster, Pinus radiate* and *Pinus sylvestris* stands in northwest Spain. *Forest Systems,* 21(3), 446-459.

Gorgoso-Varela, J. J., A. Rojo-Alboreca, E. Afif-Khouri & M. Barrio-Anta (2008). Modelling diameter distributions of birch (*Betula alba* L.) and Pedunculate oak (*Quercus robur* L.) stands in northwest Spain with the beta distribution. Investigación Agraria: *Sistemasy Recursos Forestales* 17(3), 271-281.

Knoebel, B.R., Burkhart, H.E., & Beck, D.E., (1986). A growth and yield of thinned yellow poplar, *For. Sci. Monograph* 27, 41 p.

Krishnamoorthy, K. (2006). Handbook of statistical distributions with applications. London: Taylor & Francis, 2006. 346 p. (Statistics, a series of textbooks & monographs, 188).

Loetsch F., Zöhrer F. & Haller K.E. (1973). Forest inventory 2.Verlagsgesellschaft.BLV. Munich. 469 pp.

Maltamo M., Puumalainen J. & Päivinen R. (1995). Comparison of beta and Weibull functions for modelling basal area diameter distribution in stands of *Pinus sylvestris* and *Piceaabies*, Scan. *J. For. Res*. 10: 284-295.

Mohammad A.K.H, Zobeiri, M, Namiranian, M, Hourfar, A. & Marvy M.M. (2009). Fitness of distribution of diameter at breast height by using some statistic distribution models, *Iranian Journal of Forest and Poplar Research*, 17(1): 116 – 124.

Nanang, D.M. (1998). Suitability of the Normal, Log-normal and Weibull distributions for fitting diameter distributions of neem plantations in Northern Ghana. *For Ecol Manage*. 103, 1-7.

Nelson T.C. (1964). Diameter distribution and growth of loblolly pine, *Forest Sci*., 10: 105-115.

Ogunjemite, B.G., Agbelusi, E.A., Afolayan, T.A. & Onadeko, S. A. (2006)."Status Survey of Chimpanzee in the Forest Zone of Southwestern Nigeria", *Acta Zoological Sinica*, Vol. 52 No 4, 1009–1014.

Palahí M., Pukkala T., Blasco E. & Trasobares A. (2007). Comparison of beta, Johnson's SB, Weibull and truncated Weibull functions for modeling the diameter distribution of forest stands in Catalonia (north-east of Spain). *Eur. J. Forest Res* 126, 563-571.

SAS (2004). SAS Online Doc 9.1.3.SAS Institute Inc., Cary, North Carolina, USA.http://support.sas.com/onlinedoc/913/docMainpage.jsp

Sheykholeslami A., Kia Pasha, Kh. & Kia Lashaki A. (2011). A Study of Tree Diameter Distribution Classes in Natural Forests of Iran (Case Study: Liresara Forest), *Annals of Biological Research*. 2 (5):283-290.

Stankova, T.V. & Zlatanov T.M. (2010).Modeling diameter distribution of Austrian black pine (Pinusnigra Arn.) plantation: a comparison of the Weibull frequency distribution function and percentile-based projection methods, *Eur. J. Forest Res* 129, 1169-1179

Thomas N.L. & Cao Q.V. (2006).A Diameter Distribution model for Even-aged Beech in Denmark, *Forest Ecology and Management.* 231: 218-225.

Weibull, W. (1951).A Statistical distribution function of wide applicability. *J. Appl. Mech*. 18, 293-297

Zheng, L & Zhou, X. (2010). Diameter distribution of trees in natural stands managed on polycyclic cutting system, *For. Stud. China,* 2010, 12(1): 21–25.

Zutter, B.R., Oderwald, R.G., Murphy, P.A. & Farrar, R.M. (1986).Characterizing diameter distributions with modified data types and forms of the Weibull distribution, *Forest Science* 32(1): 37–48.

CALL FOR JOURNAL PAPERS

MORE RESOURCES

IISTE Knowledge Sharing Partners