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Non-Linear Effort dynamics for Harvesting in a Predator- Prey System

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Abstract
In this paper, a non-linear harvesting of prey is considered in a prey-predator system. The predator is considered to be of modified Leslie- Gower type. The effort is taken as dynamic variable. The steady states of the system are determined and the dynamical behavior of the system for its all steady states is discussed under certain conditions. Necessary condition for global stability of the system is analyzed at the positive interior equilibrium point. Numerical simulations are carried out to explore the dynamics of the system for the suitable choice of parameters.
Keywords: Modified Leslie-Gower predation, Nonlinear Harvesting, stability, Numerical Simulations.

1. Introduction

The ecological non-linear models of interacting populations have been studied extensively by many authors .In recent years, the need for more food/ resources is growing at large scale, which has resulted in over exploitation of several biological resources. Even there is a global concern to protect the ecosystem from exploitation at large scale. Therefore, there is a need for a sustainable development policy in various spheres of human activity to protect ecosystems. In particular, bio-economic modeling is concerned with scientific management of the exploitation of renewable resources like fisheries and forestry. Hence harvesting of ecosystems has been of interest to economists and ecologists. A harvesting policy refers to the management of biological resources by systematically controlling the period, intensity and type of harvesting. The primary objective here is to maximize productivity without depleting or driving the stocks to extinction. In recent years, many works on optimal management of renewable resources are done. An excellent introduction to optimal management of renewable resources is given by Clark [3] and also policies related to bionomic exploitations of renewable resources are discussed by him. Harvesting has a strong impact on the dynamics of biological resources. The severity of the impact depends on the nature of the implementation of harvesting strategy. Basically three types of harvesting strategies are reported in literature (i) constant harvesting (ii) proportional harvesting, $h(x) = qEx$ and (iii) nonlinear harvesting, $H(x) = qEx/(m1E + m2x)$ (Holling type-II. Many researchers have analyzed mathematical models using non-linear harvesting by considering different type of growth depending upon the species and their interactions [5, 6, 12] etc.

Zhu and Lan [16] studied Leslie- Gower model with constant harvesting in prey. Though there are numerous works on predator- prey system incorporating the harvesting of the species. Leslie Gower predator prey model and modified Leslie Gower predator-prey under the assumption of the nonlinear harvesting on prey species has been studied by Gupta and Banerjee et al. [5]. But in this paper, we consider a dynamical reaction model of modified Leslie Gower predator- prey incorporating the non-linear harvesting on prey where effort is taken as a dynamic variable. The present paper deals with a dynamic reaction model in the case of a prey-predator type fishery system, while the model we study here is especially based on a modified version of the Leslie Gower scheme, where only the prey species is subjected to non- linear harvesting. The main aim of this paper is to examine the effect of harvesting on such kind of prey - predator system with the effort dynamic which can give the best possible benefit through harvesting to the society while preventing extinction of the species.

2. The Mathematical Model

Let x(t), y(t) and E(t) are the densities of prey, predator population and the harvesting effort at a time ’t’. The Holling type- II functional response and non-linear harvesting is considered for a logistically growing prey species and predator is assumed to be modified Leslie- Gower type. The mathematical model for the dynamics of the system governed by the following system of differential equations:

(2.1)
$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \frac{\alpha xy}{\alpha + x} - \frac{qEx}{m_1E + m_2x} = xf(x, y, E),$$
$$\frac{dy}{dt} = sy\left(1-\frac{\beta y}{a + x}\right) = yg(x, y),$$
$$\frac{dE}{dt} = \eta E\left(\frac{qpx}{m_1E + m_2x} - c\right) = Eh(x, E),$$

x(0) >0, y(0)> 0, E(0)>0.

The constants r, s, k, α, β, p, c, and η are the parameters assuming only positive values. The parameters r, s, k, α, β, p, c, and η represents intrinsic growth rate of prey population, growth rate for the predator, environmental carrying capacity, en-counter rate of predator to prey, maximum rate of the reduction of predator population, price and cost per unit mass and stiffness parameter, respectively.

3. Uniform Boundedness

Theorem: All the solutions of the system (2.1) which start in the region R3 are uniformly bounded.

Proof: let us consider a function φ (t) such that

$$\phi(t) = x(t) + y(t) + \frac{1}{\eta p}E(t)$$

$$\frac{d\phi(t)}{dt} = x'(t) + y'(t) + \frac{1}{\eta p}E'(t)$$

$$= rx\left(1-\frac{x}{k}\right) - \frac{\alpha xy}{\alpha + x} - \frac{qEx}{m_1E + m_2x} + sy - \frac{s\beta y^2}{a + x} + \frac{qpx}{m_1E + m_2x} - \frac{cE}{\eta p}$$

$$\frac{d\phi(t)}{dt} \le \left(rx - \frac{rx^2}{k}\right) + sy' - \frac{\beta sy^2}{a + k} - \frac{cE}{\eta p}$$

Introducing, a positive constant N and rewrite the above equation, we get

$$\frac{d\phi(t)}{dt} + N\phi(t) \le \left((r+N)x - \frac{rx^2}{k}\right) + \left((s+N)y - \frac{\beta sy^2}{a+k}\right)' - \frac{(c-N)E}{\eta p}$$

Let us c > N, Then we have:

$$\frac{d\phi(t)}{dt} + N\phi(t) \le -\frac{r}{k}\left(x - \frac{k(r+N)}{2r}\right)^2 - \frac{s\beta}{a+k}\left(y - \frac{(s+N)(k+a)}{2s\beta}\right)^{2'} + M$$

$$\frac{d\phi(t)}{dt} + N\phi(t) \le M; \qquad M = \frac{k^2(r+N)^2}{4r^2} + \frac{(s+N)^2(k+a)^2}{4s^2\beta^2}$$

Solving above inequality, we get

$$0 < \lim_{t\to\infty} \phi(t) \le \frac{M}{N}$$

All the solutions of the system (2.1) which start in the region R3 are confined in the region:

$$R = \left\{(x, y, E) \in R; 0 < x(t) + y(t) + \frac{1}{\eta p}E(t) \le M + \phi, \phi > 0\right\}$$

This proves the result.

4.Existence of Equilibrium Points

For the model (2.1), there exists six non-negative equilibrium points which are given below:

1. P0 (0, 0, 0) is a trivial equilibrium point.
2. P1 (k, 0, 0) is the axial equilibrium point on x-axis.
3. P2 (0, a/β, 0) is the axial point on y-axis in the absent of prey and harvesting.
4. $P_3\left(\bar{x}, \bar{y}, 0\right)$ is boundary equilibrium point on xy-plane in the absence of harvesting effort. The

equilibrium level $\bar{x}$ and $\bar{y}$ are the solution of the following equations:

$$r\bar{x}\left(1-\frac{\bar{x}}{k}\right)-\frac{\alpha\bar{x}\bar{y}}{\alpha+\bar{x}}=0;\left(1-\frac{\beta\bar{y}}{a+\bar{x}}\right)=0$$

Solving, we get, $\bar{x}=r\left(1-\frac{\alpha}{r\beta}\right)$ $\bar{y}=\frac{a+\bar{x}}{\beta}$

Point $P_3\left(\bar{x},\bar{y}.0\right)$ exists for $r\beta>\alpha.$

5. $P_4\left(\hat{x},0,\hat{E}\right)$ is boundary equilibrium point on xy- plane in the absence of harvesting effort. These equilibrium levels are the solution of the following equations:

$$r\hat{x}\left(1-\frac{\hat{x}}{k}\right)-\frac{\alpha\hat{x}\hat{y}}{\alpha+\hat{x}}-\frac{q\hat{E}\hat{x}}{m_1\hat{E}+m_2\hat{x}}=0$$

$$\frac{qpx}{m_1\hat{E}+m_2\hat{x}}-c=0$$

Solving, we get the values as: $\hat{x}=k\left(1-\frac{qL}{r(m_1L+m_2)}\right)$ and $\hat{E}=L\hat{x}$; where $L=\frac{pq-cm_2}{cm_1}$

$P_4\left(\hat{x},0,\hat{E}\right)$ is positive if $r(m_1L+m_2)>qL$ and $c<\frac{pq}{m_2}$

6. $P_5\left(x^*,y^*,E^*\right)$ is the unique interior point of the following equations:

$$rx^*\left(1-\frac{x^*}{k}\right)-\frac{\alpha x^*y^*}{\alpha+x^*}-\frac{qE^*x^*}{m_1E^*+m_2x^*}=0;sy^*\left(1-\frac{\beta y^*}{a+x^*}\right)=0$$

$$\eta E^*\left(\frac{qpx^*}{m_1E^*+m_2x^*}-c\right)=0$$

$$x^*=k\left(1-\frac{\alpha}{r\beta}-\frac{qL}{r(m_1L+m_2)}\right)\qquad y^*=\frac{a+x^*}{\beta}\quad\text{and}\quad E^*=Lx^*$$

x^* and y^* are positive for $\frac{\alpha}{r\beta}+\frac{qL}{r(m_1L+m_2)}<1$ and E^* is positive for $c<\frac{pq}{m_2}$.

6. Local Stability of Various Equilibrium Points

Here we discuss the local stability conditions for feasible equilibrium points of the system (2.1) based upon the standard linearization technique and then using the well-known Routh- Hurwitz criterion to determine the nature of eigenvalues of the Jacobian matrix evaluated at the equilibrium point[10]. The Jacobian matrix of the system (2.1) at any point (x, y, E) is given by:

$$J(x,y,E)=\begin{bmatrix} x\frac{\partial f}{\partial x}+f & x\frac{\partial f}{\partial y} & x\frac{\partial f}{\partial E} \\ y\frac{\partial g}{\partial x} & y\frac{\partial g}{\partial y}+g & y\frac{\partial g}{\partial E} \\ E\frac{\partial h}{\partial x} & E\frac{\partial h}{\partial y} & E\frac{\partial h}{\partial E}+h \end{bmatrix}$$

$$J(x,y,E)=\begin{bmatrix} x\left(-\frac{r}{k}+\frac{\alpha y}{(a+x)^2}+\frac{qEm_2}{(m_1E+m_2x)^2}\right)+f & \frac{-\alpha x}{a+x} & m_2q\left(\frac{x}{m_1E+m_2x}\right)^2 \\ \frac{\beta s y^2}{(a+x)^2} & \frac{\beta s y}{a+x}+g & 0 \\ \frac{m_1\eta pqE^2}{(m_1E+m_2x)^2} & 0 & -\frac{m_1\eta pqEx}{(m_1E+m_2x)^2}+h \end{bmatrix}$$

6.1. Stability of point (0, 0, 0);

The Jacobian matrix evaluated at the equilibrium point P_0 (0, 0, 0) is given by :

$$J_0(0,0,0)=\begin{bmatrix} -r & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & -\eta c \end{bmatrix}$$

The eigen values of J_0 evaluated at equilibrium point (0, 0, 0) are $\lambda_1=-r<0, \lambda_2=s>0$ and $\lambda_3=-\eta c<0$. Hence the origin (0, 0, 0) is a saddle point with unstable manifold in y direction and stable manifold in x as well as in E-direction.

6.2. Stability of point (k, 0, 0);

The Jacobian matrix evaluated at the equilibrium point P_1 (k, 0, 0) is given by:

$$J_1(k,0,0)=\begin{bmatrix} -r & \frac{-\alpha k}{a+k} & \frac{-q}{m_2} \\ 0 & s & 0 \\ 0 & 0 & \eta\left(\frac{pq}{m_2}-c\right) \end{bmatrix}$$

The eigen values of J_1 evaluated at equilibrium point (k, 0, 0) are $\lambda_1=-r<0, \lambda_2=s>0$ and $\lambda_3=\eta\left(\frac{pq}{m_2}-c\right)$. The equilibrium point (k, 0, 0) is saddle point as $\lambda_2=s>0$ with unstable manifold in y- direction and stable manifold in x-direction and also in E-direction if $\frac{pq}{m_2}<c$.

6.3. Stability of point (0, a/β, 0);

The Jacobian matrix evaluated at the equilibrium point P_2 *(0, a/β, 0)* is given by:

$$J_2(0,\frac{a}{\beta},0)=\begin{bmatrix} r-\frac{\alpha}{\beta} & 0 & 0 \\ \frac{-s}{\beta} & -s & 0 \\ 0 & 0 & -\eta c \end{bmatrix}$$

The eigen values of J_2 evaluated at equilibrium point (k, 0, 0) are $\lambda_2=-s<0, \lambda_3=-\eta c<0$ and $\lambda_1=r-\frac{\alpha}{\beta}$. If λ_1 <0, then the equilibrium point P_2 *(0, a/β, 0)* is locally asymptotically stable. Otherwise, it is a saddle point with an unstable manifold in x-direction and stable manifold in y as well as in E direction.

6.4. Stability of point $P_3\left(\bar{x},\bar{y}.0\right)$:

The Jacobian matrix evaluated at the equilibrium point $P_3\left(\bar{x},\bar{y}.0\right)$ is given by:

$$J_3(\bar{x},\bar{y},0)=\begin{bmatrix} \bar{x}\left(-\frac{r}{k}+\frac{\alpha\bar{y}}{(a+\bar{x})^2}\right) & \frac{-\alpha\bar{x}}{a+\bar{x}} & \frac{-q}{m_2} \\ \frac{\beta s y^2}{(a+\bar{x})^2} & \frac{\beta s\bar{y}}{a+\bar{x}} & 0 \\ 0 & 0 & \eta\left(\frac{pq}{m_2}-c\right) \end{bmatrix}$$

The characteristic equation associated to the matrix $J_3(\bar{x},\bar{y},0)$ yields the eigen values as follows:

$$\lambda_{1,2}=\left(\frac{\alpha}{\beta}-r-s\right)+\frac{s\beta\bar{x}\bar{y}}{a+\bar{x}}\pm\sqrt{\left(\left(\frac{\alpha}{\beta}-r-s\right)+\frac{\alpha\bar{x}}{a+\bar{x}}\right)^2-4\frac{s\beta\bar{x}\bar{y}}{a+\bar{x}}} \quad \text{and} \quad \lambda_3=\eta\left(\frac{pq}{m_2}-c\right)$$

(i) If $\frac{s\beta\bar{x}\bar{y}}{a+\bar{x}}$ and $r+s-\frac{\alpha}{\beta}$, $\frac{pq}{m_2}<c$, then the equilibrium point P_3 is locally asymptotically stable. It is a saddle point with an unstable manifold in E-direction for $\frac{pq}{m_2}>c$. For $\frac{pq}{m_2}=c$, there is a chance of bifurcation.

6.5. Stability of equilibrium point $P_4\left(\hat{x},0,\hat{E}\right)$

The Jacobian matrix evaluated at the equilibrium point $P_4\left(\hat{x},0,\hat{E}\right)$ is given by:

$$J_4(\hat{x},0,\hat{E})=\begin{bmatrix} \hat{x}\left(-\frac{r}{k}+\frac{q\hat{E}m_2}{(m_1\hat{E}+m_2\hat{x})^2}\right) & \frac{-\alpha\hat{x}}{a+\hat{x}} & -m_2q\left(\frac{\hat{x}}{m_1\hat{E}+m_2\hat{x}}\right)^2 \\ 0 & s & 0 \\ \frac{m_1\eta pq\hat{E}^2}{(m_1\hat{E}+m_2\hat{x})^2} & 0 & -\frac{m_1\eta pqE\hat{x}}{(m_1E+m_2\hat{x})^2} \end{bmatrix}$$

Corresponding to equilibrium point P_4, one of the eigen value is $\lambda_2=s>0$, and the other two can be obtained as the eigen values of the following 2×2 matrix:

$$J^*{}_4(\hat{x},0,\hat{E})=\begin{bmatrix} \hat{x}\left(-\frac{r}{k}+\frac{q\hat{E}m_2}{(m_1\hat{E}+m_2\hat{x})^2}\right) & -m_2q\left(\frac{\hat{x}}{m_1\hat{E}+m_2\hat{x}}\right)^2 \\ \frac{m_1\eta pq\hat{E}^2}{(m_1\hat{E}+m_2\hat{x})^2} & -\frac{m_1\eta pqE\hat{x}}{(m_1E+m_2\hat{x})^2} \end{bmatrix}$$

If (i) Tr($J^*{}_4$)< 0, the the point P_4 is a saddle point. (ii) If Tr($J^*{}_4$)=0, the we get pair of imaginary roots which shows that there is a case of hopf bifurcation.

6.6. Stability of Interior Equilibrium Point $P_5\left(x^*,y^*,E^*\right)$

Theorem: The unique positive interior equilibrium $\left(x^*,y^*,E^*\right)$ is asymptotically locally stable provided condition (6.1) is satisfied.

The Jacobian matrix evaluated at the equilibrium point $P_5\left(x^*,y^*,E^*\right)$ is given by:

$$J(x^*,y^*,E^*)=\begin{bmatrix} x\left(-\frac{r}{k}+\frac{\alpha y^*}{(a+x^*)^2}+\frac{qE^*m_2}{(m_1E^*+m_2x^*)^2}\right) & \frac{-\alpha x^*}{a+x^*} & m_2q\left(\frac{x^*}{m_1E^*+m_2x^*}\right)^2 \\ \frac{\beta s y^{*2}}{(a+x^*)^2} & \frac{\beta s y^*}{a+x^*} & 0 \\ \frac{m_1\eta pqE^{*2}}{(m_1E^*+m_2x^*)^2} & 0 & -\frac{m_1\eta pqE^*x^*}{(m_1E^*+m_2x^*)^2} \end{bmatrix}$$

The characteristics equations of the above Jacobian matrix about the equilibrium point $P_5(x^*,y^*,E^*)$ is given by:

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0,$$

where

$$A_1 = -(a_{11}+a_{22}+a_{33})$$
$$A_2 = a_{22}a_{33} + (a_{11}a_{33} - a_{13}a_{31}) + (a_{11}a_{22} - a_{12}a_{21})$$
$$A_3 = a_{11}a_{22}a_{33} - a_{12}a_{21}a_{33} - a_{13}a_{31}a_{22}$$

Using Routh- Hurwitz Criteria, the conditions for local stability of point P_5 are $A_1>0$, $A_2>0$ and $A_1A_2-A_3>0$

Note that $A_1>0$ if $\frac{-r}{k}-\frac{\alpha y^*}{(a+x^*)^2}-\frac{qE^*m_2}{(m_1E^*+m_2x^*)^2}<0$ (6.1)

Also, $A_2>0$ and $A_1A_2-A_3>0$ for the condition (6.1)

Thus, the interior equilibrium point P_5 is locally asymptotically stable for the sufficient condition (6.1).

7. Global Stability

Theorem: The interior equilibrium point (x∗, y∗, E∗) is globally asymptotically stable.

Proof: Consider a Lyaponov function V (x, y, E) such that:

$$V(x,y,E)=d_0\left[(x-x^*)-x^*\log\frac{x}{x^*}\right]+d_1\left[(y-y^*)-x^*\log\frac{y}{y^*}\right]+d_2\left[(E-E^*)-E^*\log\frac{E}{E^*}\right]$$

As V (x, y, E) is zero at the equilibrium point (x∗, y∗, E∗) i.e., $V(x^*,y^*,E^*)=0$ and positive for all other values of (x,y,E).

$$\frac{dV}{dt}=d_0(x-x^*)\frac{\dot{x}}{x}+d_1(y-y^*)\frac{\dot{y}}{y}+d_2(E-E^*)\frac{\dot{E}}{E}$$
$$=d_0(x-x^*)\left(r\left(1-\frac{x}{k}\right)-\frac{\alpha y}{a+x}-\frac{qE}{m_1E+m_2x}\right)+d_1(y-y^*)\left(s-\frac{\beta sy}{a+x}\right)+d_2(E-E^*)\eta\left(\frac{qpx}{m_1E+m_2x}-c\right)$$

$$\frac{dV}{dt}=d_0(x-x^*)^2\left[\frac{r}{k}-\frac{\alpha y^2}{(a+x)(a+x^*)}-\frac{qE^*}{(m_1E+m_2x^*)(m_1E^*+m_2x^*)}\right]+\frac{(x-x^*)(y-y^*)}{(a+x)}\left[-d_0\alpha x^*+\frac{d_1s\beta E^*}{a+x^*}\right]$$
$$+\frac{-d_0qx^*+d_2qpE^*}{(m_1E+m_2x)(m_1E^*+m_2x^*)}(x-x^*)(E-E^*)-\frac{d_1s\beta(y-y^*)^2}{a+x}-\frac{d_2qpx^*(E-E^*)^2}{(m_1E+m_2x)(m_1E^*+m_2x^*)}$$

$$\frac{dV}{dt}=-d_0(x-x^*)^2\left[\frac{r}{k}-\frac{\alpha y^2}{(a+x)(a+x^*)}-\frac{qE^*}{(m_1E+m_2x^*)(m_1E^*+m_2x^*)}\right]+\frac{d_1s\beta(y-y^2)^2}{a+x}-\frac{d_2qpx^*(E-E^*)^2}{(m_1E+m_2x)(m_1E^*+m_2x^*)}$$

Let $d_0=1$ $d_1=\frac{\alpha x^*(a+x^*)}{s\beta E^*}$ and $d_2=\frac{qx^*}{qpE^*}$

$$\frac{dV}{dt}<0 \quad \text{if} \quad \frac{r}{k}-\frac{\alpha y^*}{(a+x)(a+x^*)}-\frac{qE^*}{(m_1E+m_2x)(m_1E^*+m_2x^*)}>0 \qquad (7.1)$$

Thus, interior equilibrium point (x^*, y^*, E^*) is globally stable for the above condition (7.1).

8. Numerical Simulations

In this section, numerical simulations are carried out for different choice of parameters to investigate the dynamic behavior of the system. We start the numerical investigation keeping all the parameters fixed except 'c'. Let $r = 0.5$, $k = 100$, $\alpha = 0.005$, $m_1 = 0.5$, $m_2 = 0.5$, $q = 0.15$, $a = 3$, $s = 1$, $\beta = 0.15$, $\eta = 1$, $p = 5$;

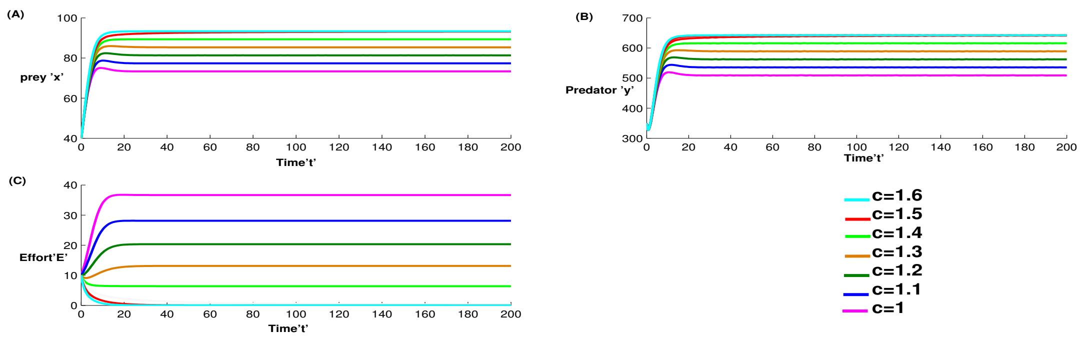

Figure 1: Time series analysis of prey population, predator population and Effort 'E' for different value of cost 'c' with initial level (40, 350, 10).

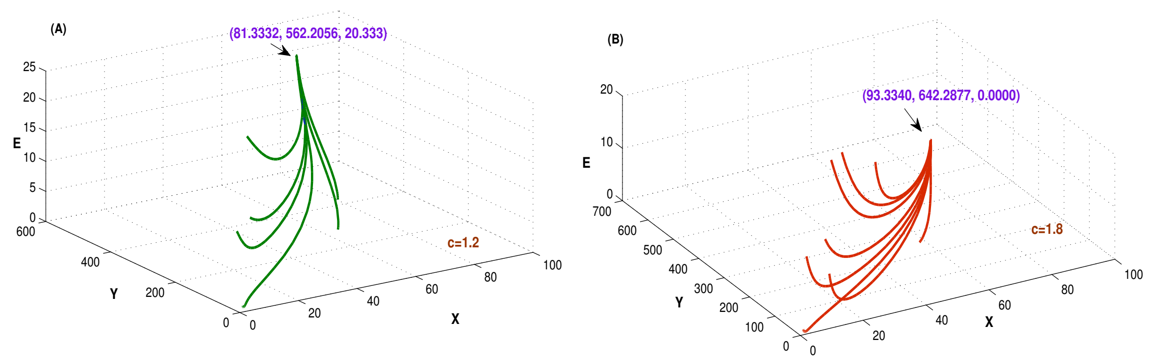

Figure 2: Phase portrait of prey population, predator population and Effort 'E' for different initial level corresponding (A).c=1.2 and (B). c=1.8

In the figure-1, diagrams (A), (B) and (C) give long term behavior of trajectories of prey- predator populations and effort 'E' w.r.t time 't'. which shows that for the initial condition (40, 350, 10) in the interior, for the different value of cost 'c' and keeping other parameters fixed, all the trajectories converges to its interior equilibrium point which means that interior equilibrium is asymptotically locally stable provided $0 < c < 1.5$. Also, we see that as the value of cost 'c' will start increase then eventually harvesting effort will start decrease and in resulted prey population increases as well predator population also. But, for $c > 1.5$ effort 'E' will tends to zero.

Figure-2(A) represents phase plane trajectories of species x, y and effort E with the different initial levels which represents that the interior point $(x^*, y^* E^*) = (81.3332, 562.2056, 20.333)$ is asymptotically global stable for $c = 1.2$ as $0 < c < 1.5$ for the existence of interior equilibrium point. The figure-2(B) represents phase plane trajectories of different biomasses with the different initial levels at the interior which converge to the point (93.3340, 642.2877, 0.0000) on the boundary plane i.e., x-y plane corresponding to $c = 1.8$ for the condition $c > 1.5$, keeping other parameters fixed.. Therefore, for the condition $c > 1.5$, it shows that for the every initial points on the x - y - E space converge to the point on x - y plane asymptotically which means that there is on harvesting of the prey population for this condition as it is not profitable to fishermen to do harvesting.

5. Conclusion

This paper is concerned with the study of nonlinear harvesting and the conservation of ecological resources for a two-dimensional prey predator type dynamical system. We have considered a prey predator model with Holling type-II functional response and nonlinear harvesting of prey. The harvesting effort is taken as a dynamic variable. The conditions for existence and local asymptotic stability of various equilibrium points have been examined. It is established that the coexistence of prey and predator population depends upon the proper harvesting strategies and hence we can avoid the risk of extinction or over exploitation of the species. By analytical and numerical results, we have examined that for fixed value of price ’p’ per unit mass and other parameters, as the value of cost ’c’ is increasing, then harvesting level will start to decrease. after a time, we will find a level of cost ’c’ where harvesting effort will tends to zero. So for the co-existence of prey- predator populations along with effort dynamics, there is a restriction on value of cost ’c’ so that there can co-exist all the species with optimal level of harvesting effort.

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