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Enhanced One Dimensional Modeling for Predicting Concentration of BOD in rivers

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Abstract

To maintain the river water quality it is necessary to predict the concentration of biochemical oxygen demand (BOD) in rivers. Various one dimensional models that are developed so far are applicable only after complete mixing of the pollutant across the cross-section is over which may take longer time for rivers with large width. Such type of situations is not represented effectively by the existing one dimensional model. Moreover, many of these one dimensional models do not account for the settle able part of BOD that invariably takes place when partially treated/ untreated waste enters these water bodies. A model is developed that is not more complicated than a one dimensional model but rationally predict the transport of BOD causing pollutant in almost 80% of initial period. The presented model can be used in conditions when partially treated/untreated waste is discharged in rivers with large width.

Keywords: Mathematical Model, BOD, Water Pollution

1. Introduction

It has been a common practice throughout the world to discharge waste material into the streams. In order to control contamination of a stream and predict the levels of accidental pollution an understanding is required of a rate at which a stream is capable of transporting pollutant. The contamination of organic matter into the river is determined by a non conservative parameter known as BOD. The first developed model by Streeter and Phelps in 1925[13], for the formulation of BOD includes only the dissolved part of BOD and does not account for the settleable part of BOD. Bhargava presented a model in 1983 [2] for accurate prediction of transport of organic matter containing both types of BOD settleable as well as dissolved. The model suggested that BOD was seemed to be removed in two forms, settleable part is removed by the linear settling law and non-settleable i.e. dissolved part follows the usual first order kinetics with a decay constant 'k'. Moreover the assumption of Streeter and Phelps that advection is the only relevent transport mechanism unnecessarily restricts the model's validity in the present era of digital computers

Various one dimensional models ([1],[3],[4],[7],[9],[11],[12],[14]) developed to date are valid only when mixing length is over i.e. they are not valid in the initial period but the fact is that mixing takes some time to complete which may be longer for rivers with large width and this fact is ignored while using these models to predict the BOD immediately after the point of discharge of pollutant.

Tyagi et. al.[15] developed a one dimensional model that takes into consideration the transport of both type of BOD(settleable as well as dissolved) into the rivers but this model is also applicable only after mixing length is over. Various two dimensional models ([5],[6],[8] and[16]) are available but they need a considerable amount of hydraulic data which in many cases in not available and has to be estimated or assumed which may lead to a partial loss of accuracy gained through multidimensional models.

An Enhanced one dimensional model is proposed by Reichert, P. and Wanner, O. [10] that is capable of predicting substance distribution for about 80% of the initial period. In the present work, the model suggested by Reichert, P. and Wanner O.[10] is modified to predict transport of BOD conditions in the river due to discharge of partially treated or untreated organic matter containing both types of BOD namely settleable and dissolved.

2. Physical problem

Consider the portion of river with a large width to length ratio. Let the partially treated/untreated waste is disposed into the river through a point source (such as disposal through a single drain). The entire BOD in this case consists of two parts viz. dissolved and settleable. Let the source strength vary with time at the outfall itself, which represents the case of malfunctioning of equipment or accidental spill of pollutant. It is required to predict the DO conditions in river due to the discharge of organic waste in the above stated situations.

The following velocity profile (in lateral direction) is assumed

$$u(y) = \begin{cases} 0 \quad ; & 0 \le y < p.\alpha/2 \\ \dfrac{v}{1-\alpha} \quad ; & p.\alpha/2 \le y \le p\left(1-\dfrac{\alpha}{2}\right) \\ 0 \quad ; & p\left(1-\dfrac{\alpha}{2}\right) < y \le p \end{cases}$$

Where u(y) = Depth –averaged velocity(L/T);p = width of river (L);y = space co-ordinate transverse to the flow direction (L);v = Cross-section mean velocity (L/T);α = Fraction of the wetted crossectional area where u is assumed to be zero.

denoted by γ

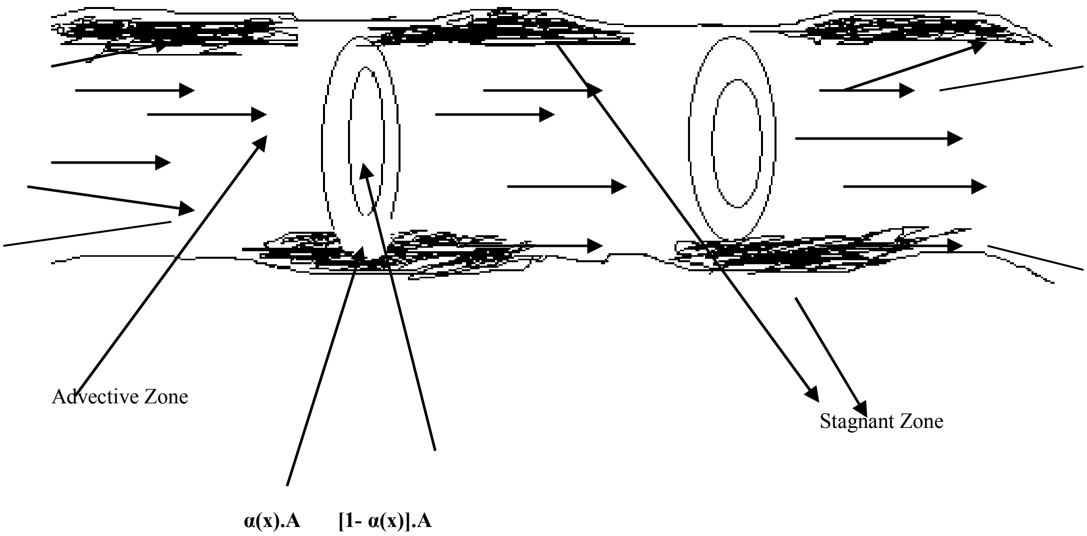

Figure1Conceptual model of crossectional velocity profile in the advective and stagnant zone of river.

3. Model Development

The cross-section of the river is divided into two zones namely advective zone in the centre of the river and stagnant zone along the two banks where the velocity is almost zero. A mathematical model is developed for the above stated river system based on the following assumptions:

- The entire BOD is in two forms namely settleable as well as dissolved forms. The dissolved part of BOD is decaying according to first order kinetics while the settleable part is being removed by linear law.

- The size of stagnant zone is αA_T and it consists of two parts located near the river banks while the size of advective zone is $(1-\alpha)\ A_T$, where A_T is the of cross-section of the river and α is fraction of wetted cross-sectional area of the stagnant zone however for the sake of simplicity we are taking $\alpha A_T = A_s$ and $(1-\alpha)\ A_T = A$.

- No transverse gradient exists within any of the two zones but there is exchange of mass between the two zones (viz. advective and stagnant) which is related to the difference in the respective concentration.

- In stagnant zone, only exchange of mass with the advective zone and reaction are considered.
- In advective zone, advection, reaction and exchange of mass are considered.
- The whole pollutant is being released into the advective zone only.

On the basis of the above assumptions the following equations representing the BOD mass balance equation in advective zone and stagnant zones are developed as follows:

3.1 Advective Zone

$$L = \begin{Bmatrix} L_o\left(1-\frac{v}{d}.\frac{x}{u}\right); x < x_1 \\ 0 \qquad ; x \geq x_1 \end{Bmatrix} \quad \ldots\ldots (1)$$

$$0 = -u\frac{\partial B_d}{\partial x} - kB_d - \frac{\gamma}{A}\left(B_d - B'\right) \quad \ldots\ldots (2)$$

3.2 Stagnant Zone

$$0 = \frac{\gamma}{A_s}\left(B_d - B'\right) - kB' \quad \ldots\ldots (3)$$

3.3 Boundary Conditions

The associated boundary conditions reflecting the release of the pollutants according to assumption mentioned at point 7:

$$B_d = B_{o-d} \; at \; x = 0$$

4. Model Development

Equation 1 is solved independently with known values of L_o, v, d, u to get the values of L at various distances downstream. To solve Equation 2 we need the values of B' which is obtained from Equation 3 which gives the value of B' in terms of B_d. The values of B' when substituted in Equation 2 gives a first order differential equation in B_d which is solved by variable separable method. The procedure is explained in the following steps.

Solving Equation 3, we get

$$B' = \frac{\gamma/A_s}{\left(\gamma/A_s + k\right)}B_d$$

$$or$$

$$B' = \frac{\beta'}{\left(\beta' + k\right)}B_d, where \beta' = \frac{\gamma}{A_s} \quad \ldots\ldots (4)$$

Using Equation 4 in Equation 2, We get

$$0 = -u\frac{\partial B_d}{\partial x} + [-kB_d] - \beta\left(B_d - \frac{\beta'}{(\beta'+k)}B_d\right) \quad where \beta = \frac{\gamma}{A}$$

Or

$$\frac{\partial B_d}{\partial x} = -\frac{k+\delta}{u}B_d, \text{ where } \delta = \frac{\beta k}{(\beta'+k)}$$

Or

$$\frac{\partial B_d}{\partial x} = -\mu B_d, \text{ where } \mu = \frac{k+\delta}{u} \qquad \text{.. (5)}$$

Solving with the boundary condition $B_d = B_{o-d}$ at x = 0, Equation.7 can be solved as

$$B_d = B_{o-d}\exp(-\mu x) \qquad \text{.. (6)}$$

The total BOD in the advective zone is then calculated by the following equation:

$$\left\{\begin{aligned} B &= B_d + L & x \le x_1 \\ &= B_d & x > x_1 \end{aligned}\right\} \qquad \text{.. . (7)}$$

Using Equation.6 in Equation.4 , we get

$$B' = \eta B_{o-d}\exp(-\mu x) \quad \text{where } \eta = \frac{\beta'}{\beta'+k} \qquad \text{.. (8)}$$

The total BOD (TB) at any point x are then calculated by Equation 9 as follows:

$$TB = (1-\alpha)B + \alpha B' \qquad \text{... (9)}$$

5. Case Study

5.1

To analyse the capabilities of the presented model, it is applied to Uvas Creek for which the physical parameters estimated by Bencala and Walter for Uvas Creek California with their values are outlined in Table1, however, the total area of cross section is divided into two parts for the application of presented model. Some kinetic and chemical parameters are appropriately taken from the literature and their values are given in Table 2. A Comparison of concentration of BOD predicted by one dimensional model [11] and presented model is shown in Figure 2

Table 1. Parameter estimated by Bencala and Walter for Uvass Creek, California

Sr. No.	Parameter	Values	Units
1	Q	0.013	m^3/s
2	A	0.36	m^2
3	A_s	0.64	m^2
4	u	2800	m/day

Table 1 represents the values of flow rate denoted by Q,area of crossection of advective and stagnant zone denoted by A & As respectively and velocity of river in longitudinal direction denoted by u

Table 2. Chemical and Kinetic Parameters

Sr. No.	Parameter	Values	Units
1	k	3	day^{-1}
2	$k_r' = k_r = m$	10	day^{-1}
3	B_d	16	mg/L
4	L_o	12	mg/L
5	γ	8957.95	L^2/day

Table 2 contains the values decay coefficient of BOD denoted by k, coefficient of reaeration in advective and stagnant denoted by k_r and k_r' ,rate of degradation of settleable BOD part denoted by m,initial settleable BOD denoted by Lo, initial dissolved BOD denoted by Bo-d and exchange rate coefficient denoted by γ

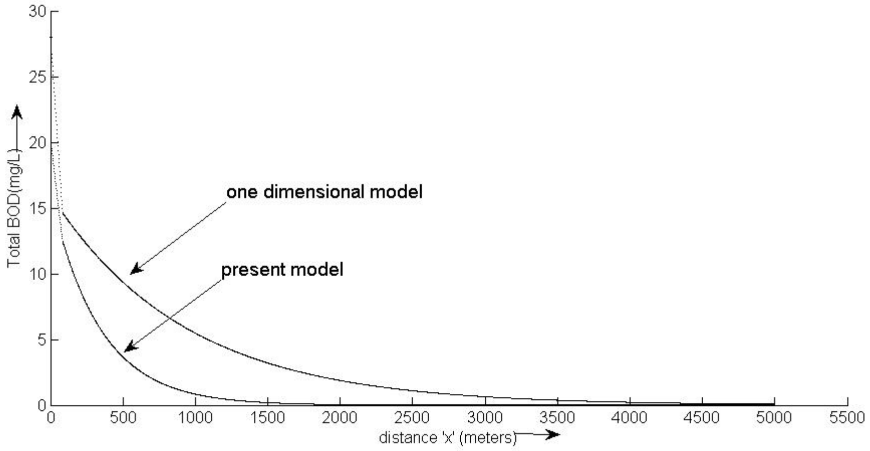

Figure 2 Concentration of BOD predicted by present model & one dimensional model in advective zone.

Figure 2. represents a comparison of BOD predicted by presented model and one dimensional model. It is observed that concentration of BOD obtained from present model is less than that predicted by one dimensional model. Since the settleable part is removed faster in present model, it assimilates more BOD and consequently the remaining BOD would be less with distance downstream.

5.2

To analyse the robustness of model, the values of BOD predicted by the present model are compared with the observed values of BOD in river Ganges along Kanpur summer is shown in Figure 3.

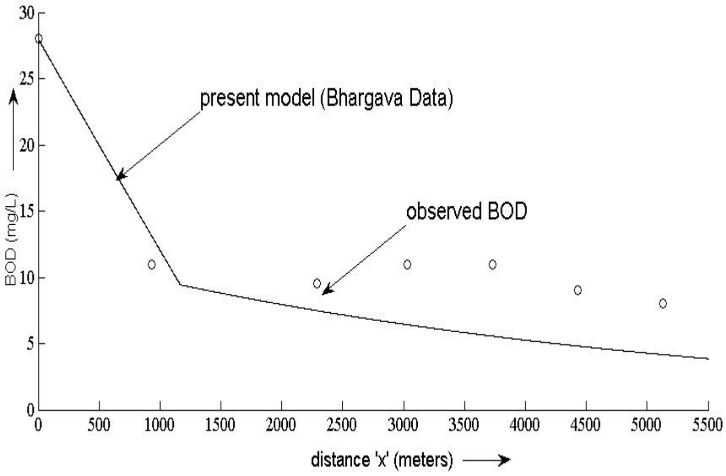

Figure 3 Comparison of predicted BOD by the present model with the observed values.

Figure 3 represents a comparison of BOD as predicted by the present model with the observed BOD values. A fair agreement of observed and predicted values of BOD establishes the robustness of model in the situation when partially treated and untreated waste is discharged into the river.

6. Conclusion

The presented model can effectively represent the situation and accurately predict the concentration of BOD in rivers with large width when partially treated or untreated waste is discharged into the river. The robustness of the model is established by comparing the predicted values of BOD with the observed values for a real river data. The accurate prediction of BOD values by the presented model may assist engineers and planners in the formulation of strategies for pollution control caused by BOD containing pollutant in the above stated conditions.

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