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Generalized and Subset Integrated Autoregressive Moving Average Bilinear Time Series Models

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Abstract

Generalized integrated autoregressive moving average bilinear model which is capable of achieving stationary for all non linear series is proposed and compared with subset generalized integrated autoregressive moving average bilinear model using the residual variance to see which perform better. The parameters of the proposed models are estimated using Newton-Raphson iterative method and Marquardt algorithm and the statistical properties of the derived estimates were investigated. An algorithm was proposed to eliminate redundant parameters from the full order generalized integrated autoregressive moving average bilinear models. To determine the order of the models, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) were adopted. Generalized integrated autoregressive moving average bilinear models are fitted to Wolfer sunspot numbers and stationary conditions are satisfied. Generalized integrated autoregressive moving average bilinear model performed better than subset generalized integrated autoregressive bilinear model.
Keywords: Stationary, Newton-Raphson, Residual Variance, Marquardt Algorithm and Parameters.

1. Introduction

The bilinear time series models have attracted considerable attention during the last years. An overview of bilinear models and their application can be found in (Subba Rao 1981), (Pham & Tran 1981), (Gabr & Subba Rao 1981), (Rao et al. 1983), (Liu 1992), (Gonclaves et al. 2000), (Shangodoyin & Ojo 2003), (Wang & Wei 2004), (Boonaick et al.2005), (Bibi 2006), (Doukhan et al. 2006), (Drost et al. 2007), (Usoro & Omekara 2008) and (Ojo 2009). The bilinear modes studied by the above researchers could not achieve stationary for all nonlinear series. One-dimensional bilinear time series model that could achieve stationary for all non linear series was developed, for details see (Shangodoyin et al. 2010) and (Ojo 2010). In this paper we proposed generalized integrated autoregressive moving average and subset integrated autoregressive moving average bilinear time series models that could achieve stationary for all non linear real series.

2. Proposed Generalized Integrated Autoregressive Moving average Bilinear Time Series Models

We define generalized bilinear (GBL) and generalized subset bilinear (GSBL) time series models as follows:

Model 1(M1)

$$\psi(B)X_t = \phi(B)\nabla^d X_t + \theta(B)e_t + \sum_{k=1}^{r}\sum_{l=1}^{s} b_{kl}X_{t-k}e_{t-l}$$, denoted as GBL (p, d, q, r, s)

where $\phi(B) = 1-\phi_1 B-\phi_2 B^2 .....-\phi_p B^p$, $\theta(B) = 1-\theta_1 B-\theta_2 B^2 .....-\theta_q B^q$ and

$$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + e_t - \theta_1 e_{t-1} - ..... - \theta_q e_{t-q} + b_{11}X_{t-1}e_{t-1} + ... + b_{rs}X_{t-r}e_{t-s}$$

$\phi_1,...,\phi_p$ are the parameters of the autoregressive component; $\theta_1,...\theta_q$ are the parameters of the associated error process; $b_{11},.....,b_{rs}$ are the parameters of the non-linear component and $\theta(B)$ is the moving average operator; *p* is the order of the autoregressive component; *q* is the order of the moving average process; *r, s* is the order of the nonlinear component and $\psi(B) = \nabla^d \phi(B)$ is the generalized autoregressive operator; ∇^d is the differencing operator and *d* is the degree of consecutive differencing required to achieve stationary. e_t are independently and identically distributed as *N* $(0, \sigma_e^2)$ and the models are assume to be invertible.

Model 2 (M2)

$$X_t = \sum_{i=1}^{l} \psi_{p_i+d} X_{t-p_i-d} - \sum_{j=1}^{m} \theta_{q_j} e_{t-q_j} + \sum_{k=1}^{n} b_{r_k s_k} X_{t-r_k} e_{t-s_k} + e_t ,$$

The above equation is denoted as GSBL (p, d, q, r, s) and p_i is the order of subset autoregressive integrated component, q_j is the order of subset moving average component and $r_k s_k$ is the order of subset nonlinear component. In the models above, e_t are independently and identically distributed as *N* $(0, \sigma_e^2)$ and the models are assume to be invertible.

3. Stationary and Convergence of GBL (p, d, q, r, s)

For general S, it is not easy to provide an infinite series representation for each $\mathbf{X}_t$. For this general case, we exhibit the process $\{X_t, t \in Z)$ as an almost sure limit of a sequence $\{\{\mathbf{S}_{\mathbf{n},t}, t \in Z\}, n \geq 1\}$ of stationary processes.

Theorem

Let $\{e_t, t \in Z\}$ be a sequence of independent identically distributed random variables defined on a probability space (Ω, IR, P) such that E $e_t = 0$ and $Ee_t^2 = \sigma^2 < \infty$. Ψ, Θ, B_1, B_2,….,B_q be q+1 matrices each of order p x p.

$\Gamma_1 = \left(\Psi \otimes \Psi + \sigma^2((\Theta \otimes \Theta + B \otimes B) - 2\Theta \otimes B)\right)$

$$\Gamma_i = \sigma^2[B_i \otimes (\Psi^{j-1}B_1 + \Psi^{j-2}B_2 + ...+ \Psi B_{j-1})$$
$$+ (\Psi^{i-1}B_1 + \Psi^{i-2}B_2 + ...+ \Psi B_{i-1}) \otimes B_j$$
$$+ B_i \otimes (\Theta^{j-1}B_1 + \Theta^{j-2}B_2 + ...+ \Theta B_{j-1})$$
$$+ (\Theta^{i-1}B_1 + \Theta^{i-2}B_2 + ...+ \Theta B_{i-1}) \otimes B_j$$
$$+ (B_j \otimes B_j)], \quad j = 2, 3,\ldots\ldots s.$$

Suppose all the eigenvalues of the matrix

$$\underset{p^2q \times p^2q}{L} = \begin{pmatrix} \Gamma_1 & \Gamma_2 & ..... & \Gamma_{q-1} & \Gamma_q \\ I_{p^2} & 0 & ..... & 0 & 0 \\ 0 & I_{p^2} & ..... & 0 & 0 \\ 0 & 0 & I_{p^2} & ..... & 0 \end{pmatrix}$$

have moduli less than unity, i.e, $\rho(L) = \lambda < 1$. Let $\underset{px1}{C}$ be a given column vector. Then there exists a vector valued strictly stationary process $\{X_t, t \in Z\}$ conforming to the bilinear model

$$\mathbf{X}_t = \Psi \mathbf{X}_{t-1} - \Theta e_{t-1} + \sum_{l=1}^{s} \mathbf{B}_l \mathbf{X}_{t-1} e_{t-1} + \mathbf{C} e_t \text{ for every t in Z.}$$

Proof

Let the process $\{S_{n,t,}\, n, t \in Z\}$ be defined as follows.

$S_{n,t} = 0,$ if n < 0,

= Ce_t, if n = 0,

$= Ce_t + (\Psi - \Theta e_{t-1} + B_1 e_{t-1}) S_{n-t,t-1} + B_2 S_{n-2,t-2} e_{t-2} + ... + B_q S_{n-q,t-q} e_{t-q}$, if n>0

for every t in Z.

$\lim_{n\to\infty} S_{n,t}$ exists almost surely for every t in Z. If X_t is the almost sure limit of $\{S_{n,t,} n \geq 1\}$ for every t in Z, then it is obvious that the process $\{X_t, t \in Z\}$ conforms to the bilinear model

$$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} + e_t - \theta_1 e_{t-1} - ..... - \theta_q e_{t-q} + b_{11} X_{t-1} e_{t-1} + ... + b_{rs} X_{t-r} e_{t-s}$$

Also, for every fixed n in Z, $\{S_{n,t,} t \in Z\}$ is a strictly stationary process.

Let $s_{n,t} = S_{n,t} - S_{n-1,t,} t \in Z.\ E|(s_{n,t})_i| \leq K\lambda^{n/2}$ for every $n \geq 0$ and i = 1, 2, ….,p, where K is a positive constant. Since $\lambda < 1,$ this then implies that $\{S_{n,t,} n \geq 1\}$ converges almost surely for every t in Z.

3.1Algorithm for Fitting One-dimensional Autoregressive Integrated Moving Average Bilinear Time Series Models

For the sake of simplicity, we will break the algorithm down into the following steps.

Step 1

Fit various order of autoregressive integrated moving average model of the form

$$X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} - \theta_1 e_{t-1} - ... - \theta_q e_{t-q} + e_t$$

Step 2

Choose the model for which Akaike Information Criterion (AIC) is minimum among various order fitted in step 1.

Step 3

Fit possible subsets of chosen model in step 2 using $2^q - 1$ subsets approach (Hagan & Oyetunji 1980).

Step 4

Choose the model for which AIC is minimum among the fitted models in step 3 to have the best subset model and the parameters of this model form the initial values.

Step 5

Fit various order of generalized autoregressive integrated moving average bilinear model of the form $X_t = \psi_1 X_{t-1} + ... + \psi_{p+d} X_{t-p-d} - \theta_1 e_{t-1} - .... - \theta_q e_{t-q} + b_{11} X_{t-1} e_{t-1} + ... + b_{rs} X_{t-r} e_{t-s} + e_t$

Step 6

Fit possible subsets of chosen model in step 5 using $2^q - 1$ subsets approach (Shangodoyin & Ojo 2003)

Step 7 The model with the minimum AIC is the generalized subset autoregressive integrated moving average bilinear model.

3.2 Estimation of the Parameters of Generalized Bilinear Models Proposed

The joint density function of $(e_m, e_{m+1},....,e_n)$ where m = max (r, s) is given by

$$\frac{1}{(2\pi\sigma_e^2)^{(n-m+1)/2}}\exp(\frac{1}{-2\sigma_e^2}\sum_m^n e_t^2)$$

Since the Jacobian of the transformation from $(e_m, e_{m+1},....,e_n)$ to $(X_m, X_{m+1},...., X_n)$ is unity, the likelihood function of $(X_m, X_{m+1},...., X_n)$ is the same as the joint density function of $(e_m, e_{m+1},....,e_n)$. Maximising the likelihood function is the same as minimizing the function $Q(G)$,

where $Q(G) = \sum_{i=m}^{n} e_{t,}^2$ (1)

with respect to the parameter $G^{'} = (\psi_1,....,\psi_p;\theta_1,\theta_2,....,\theta_q;B_{11},....,B_{rs})$

Then the partial derivatives of Q(G) are given by

$$\frac{dQ(G)}{dG_i} = 2\sum_{t=m}^{n} e_t \frac{de_t}{dG_i} \qquad \text{(i = 1, 2,…..,R)} \qquad (2)$$

$$\frac{d^2Q(G)}{dG_i dG_j} = 2(\sum_{t=m}^{n} e_t \frac{de_t}{dG_i}\frac{de_t}{dG_j} + \sum_{t=m}^{n} e_t \frac{d^2e_t}{dG_i dG_j})$$

where these partial derivatives of e(t) satisfy the recursive equations

$$\frac{de_t}{d\psi_i} + \sum_{j=1}^{s} W_t(t)\frac{de_{t-j}}{d\psi_i} = \begin{cases} 1, \text{ if } i = 0 \\ X_{t-i,} \text{ if } i = 1, 2, ..., p \end{cases} \qquad (3)$$

$$\frac{de_t}{d\theta_i} + \sum_{j=1}^{s} W_t(t)\frac{de_{t-j}}{d\theta_i} = e_{t-i,} \quad \text{if } i = 1, 2,….., q \qquad (4)$$

$$\frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{de_{t-j}}{dB_{kmi}} = -X_{t-k}e_{t-m} \quad (k=1,2,…,r\ ;\ m_i = 1,2,…,s) \qquad (5)$$

$$\frac{d^2e_t}{d\psi_i d\psi_i^{'}} + \sum_{j=1}^{s} W_j(t)\frac{d^2e_{t-j}}{d\psi_i d\psi_i^{'}} = 0 \text{ (i, i' = 0, 1, 2, …, p)} \qquad (6)$$

$$\frac{d^2e_t}{d\theta_i d\theta_i^{'}} + \sum_{j=1}^{s} W_j(t)\frac{d^2e_{t-j}}{d\theta_i d\theta_i^{'}} = 0 \quad \text{(i, i' = 0, 1, 2, …, q)} \qquad (7)$$

$$\frac{d^2e_t}{d\psi_i dB_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{d^2e_{t-j}}{dB_{kmi} d\phi_i} + X_{t-k} \frac{d^2e_{t-mi}}{d\psi_i} = 0$$

(i=0,1,2,…,p ; k_i =1,2,…,r; m_i=1,2,…,s) (8)

$$\frac{d^2e_t}{d\theta_i dB_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{d^2e_{t-j}}{dB_{kmi} d\theta_i} + X_{t-k} \frac{d^2e_{t-mi}}{d\theta_i} = 0$$

(i=1,2,…,q ; k_i =1,2,…,r; m_i=1,2,…,s) (9)

$$\frac{d^2e_t}{d\psi_i d\theta_i} + \sum_{j=1}^{s} W_j(t) \frac{d^2e_{t-j}}{d\psi_i d\theta_i} = 0$$

(10)

$$\frac{d^2e_t}{dB_{kmi} dB'_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{d^2e_{t-j}}{dB_{kmi} dB'_{kmi}} + X'_{t-k} \frac{d^2e_{t-mi}}{dB_{kmi}} = -X_{t-k} \frac{de_{t-m}}{dB'_{kmi}}$$

(k, k' =1,2,…,r ; m_i m_i' = 1,2,…,s)

(11)

$W_j(t) = \sum_{j=1}^{s} B_{ij} X_{t-j}$ We assume e_t = 0 (t = 1, 2, …, m-1) and also

$$\frac{de_t}{dG_i} = 0, \frac{d^2e_t}{dG_i dG_j} = 0, \qquad \text{(i, j = 1, 2, …, R; t = 1, 2, …, m-1)}$$

From e_t=0 (t= 1, 2, …, m-1),

$$\frac{de_t}{dG_i} = 0, \frac{d^2e_t}{dG_i dG_j} = 0, \text{ and } \frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t) \frac{de_{t-j}}{dB_{kmi}} = -X_{t-k} e_{t-m} \text{ (k=1,2,…,r ; } m_i \text{ =1,2,…,s),}$$

it

follows that the second order derivatives with respect to ψ_i (i = 0, 1, 2, …, p) and θ_i (i = 0, 1, 2, …, q) are zero. For a given set of values {φi}, { θ_i } and {B_{ij} } one can evaluate the first and second order derivatives using the recursive equations 3, 4, 5 and 11. Now let

$$\mathbf{V}'(\mathbf{G}) = \frac{dQ(\mathbf{G})}{d\mathbf{G}_1}, \frac{dQ(\mathbf{G})}{d\mathbf{G}_2}, ..., \frac{dQ(\mathbf{G})}{d\mathbf{G}_k}$$

and let $\mathbf{H}(\mathbf{G}) = [d^2Q(\mathbf{G}) / d\mathbf{G}_i d\mathbf{G}_j]$ be a matrix of second partial derivatives as in (Krzanowski 1998). Expanding **V**(**G**), near $G = \hat{G}$ in a Taylor series, we obtain

$$V(\hat{G})_{\hat{G}=G} = 0 = V(G) + H(G)(\hat{G} - G) \qquad (12)$$

Rewriting this equation we get $\hat{G} - G = -H^{-1}(G)V(G)$, and thus obtain an iterative equation given by $G^{(k+1)} = G^{(k)} - H^{-1}(G^{(k)})V(G^{(k)})$ where $G^{(k)}$ is the set of estimates obtained at the k^{th} stage of iteration. The estimates obtained by the above iterative equations usually converge. For starting the

iteration, we need to have good sets of initial values of the parameters. This is done by fitting the best subset of the linear part of the bilinear model.

3.3 Estimation of the Parameters of Generalized Subset Bilinear Model Proposed

In the estimation procedure to be discussed in this section we assume that the sets of integers $\{k_1, k_2, ..., k_l\}$, $\{q_1, q_2, ..., q_r\}$ and $\{(r_1, s_1), (r_2, s_2), ..., (r_m s_m)\}$ are fixed and known. Proceeding as in (Subba Rao 1981), we can show that maximizing the likelihood function of $(X_{m1}, X_{m1+1}, ..., X_N)$ is the same as minimizing the function

$$Q(\theta) = \sum_{t=m1}^{N} e_t^2$$

with respect to the parameters $(\psi_{k_1}, \psi_{k_2}, ..., \psi_{k_l}; q_{r_1}, ...q_{r_z}; b_{r_1 s_1}, ..., b_{r_m s_m})$.

The partial derivatives of $Q(\theta)$ are $G_i = \frac{dQ(\theta)}{d\theta_i} = 2\sum_{t=m_1}^{N} e_t \frac{de_t}{d\theta_i}$,

$$h_{ij} = \frac{d^2 Q(\theta)}{d\theta_i d\theta_j} = 2\sum_{t=m_1}^{N}\left(\frac{de_t}{d\theta_i}\right)\left(\frac{de_t}{d\theta_j}\right) + 2\sum_{t=m_1}^{N} e_t \cdot \frac{d^2 e_t}{d\theta_i d\theta_j},$$

where the partial derivatives satisfy the recursive equations

$$\frac{de_t}{d\psi_{k_r}} = X_{t-k_r} - \sum_{j=1}^{m} b_{r_j s_j} X_{t-r_j} \frac{de_{t-s_j}}{\psi_{k_r}}, (r = 1,2,3,..., l)$$

$$\frac{de_t}{d\theta_{q_r}} = e_{t-q_r} - \sum_{j=1}^{m} b_{r_j s_j} X_{t-r_j} \frac{de_{t-1}}{de_{q_r}}, (r = 1,2,3,...., l)$$

$$\frac{de_t}{db_{r_q s_q}} = -X_{t-r_q} e_{t-s_q} - \sum_{j=1}^{m} b_{r_j s_j} X_{t-r_j} \frac{de_{t-s_j}}{db_{r_q s_q}},$$

In the calculation of these partial derivatives, we set $e_1 = e_2 = ... = e_{mo} = 0$ and

$$\frac{de_1}{d\theta_i} = \frac{de_2}{d\theta_i} = ... = \frac{de_{mo}}{d\theta_i} = 0, (i = 1,2,..., R)$$

Let $G^T(\theta) = (G_1, G_2, ..., G_R)$ and $H(\theta) = (h_{ij})$.

In evaluating the second order partial derivatives we approximate $h_{ij} = 2\sum_{t=m_1}^{N}\left(\frac{de_t}{d\theta_i}\right)\left(\frac{de_t}{d\theta_i}\right)$

as is done in Marquardt algorithm. Expanding $G(\hat{\theta})$ near $\hat{\theta} = \theta$ in a Taylor series, we obtain $0 = G(\theta) + H(\theta)(\hat{\theta} - \theta)$.

Rewriting this equation, we get $(\hat{\theta} - \theta) = -H^{-1}(\theta)G(\theta)$ and thus obtain the Newton-Raphson iterative equation

$$\theta^{(k+1)} = \theta^{(k)} - H^{-1}(\theta^{(k)})G(\theta^{(k)})$$

$$\theta^{(k)} = \theta^{(k+1)} + H^{-1}(\theta^{(k)})G(\theta^{(k)}) \quad (13)$$

where $\theta^{(k)}$ is the set of estimates obtained at the k[th] stage of iteration. For starting the iteration, we need to have good sets of initial values of the parameters. This is done by fitting the best subset of the linear part of the bilinear model.

4. Numerical Example

To present the application of the models proposed, we will use a real time series dataset, the Wolfer sunspot, available in (Box et al. 1994). The scientists track solar cycles by counting sunspots – cool planet-sized areas on the Sun where intense magnetic loops poke through the star's visible surface. It was Rudolf Wolf who devised the basic formula for calculating sunspots in 1848; these sunspot counts are still continued.

As the Wolfer sunspot data set represent a non-stationary series, the bilinear models proposed in this paper may be applied. The Wolfer sunspot data set is considered at sample size of 150 and 250. For the fitted model below we have used the algorithm and the estimation technique in the previous section.

Fitted Model M1 and M2 at sample size150

M1

$X_t = 0.217421X_{t-1} + 0.172224X_{t-3} - 0.518088X_{t-4} - 0.218600X_{t-5} - 0.135334X_{t-6} - 0.269434X_{t-7} + 0.630377e_{t-1} - 0.119139e_{t-2} - 0.763971e_{t-3} + 0.002651X_{t-1}e_{t-1} - 0.002651X_{t-1}e_{t-1} - 0.015220X_{t-1}e_{t-2} + 0.001332X_{t-1}e_{t-3} + 0.010671X_{t-2}e_{t-1} + 0.007194X_{t-2}e_{t-2} - 0.008443X_{t-2}e_{t-3} - 0.018346X_{t-3}e_{t-1} - 0.007363X_{t-3}e_{t-2} + e_t$

M2

$X_t = 0.217421X_{t-1} + 0.172224X_{t-3} - 0.518088X_{t-4} - 0.218600X_{t-5} - 0.135334X_{t-6} - 0.269434X_{t-7} + 0.630377e_{t-1} - 0.763971e_{t-3} - 0.017434X_{t-1}e_{t-2} + 0.014963X_{t-2}e_{t-1} + 009280X_{t-2}e_{t-2} - 0.007589X_{t-2}e_{t-3} - 0.019788X_{t-3}e_{t-1} - 0.008451X_{t-3}e_{t-2} + e_t$

Fitted Model M1 and M2 at sample size 250

M1

$X_t = - 0.712478X_{t-1} - 0.153047X_{t-2} + 0.032479X_{t-3} - 0.606080X_{t-4} - 0.351330X_{t-5} - 0.422284X_{t-6} - 0.407042X_{t-7} - 0.311950X_{t-8} + 0.809607e_{t-1} - 0.048903e_{t-2} - 0.673588e_{t-3} + 0.000174X_{t-1}e_{t-1} - 0.012392X_{t-1}e_{t-2} - 0.000523 + 0.008372X_{t-2}e_{t-1} + 0.002290X_{t-2}e_{t-2} - 0.004130X_{t-2}e_{t-3} - 0.010699X_{t-3}e_{t-1} + e_t$

M2

$X_t = - 0.712478X_{t-1} - 0.153047X_{t-2} + 0.032479X_{t-3} - 0.606080X_{t-4} - 0.351330X_{t-5} - 0.422284X_{t-6} - 0.407042X_{t-7} - 0.311950X_{t-8} + 0.809607e_{t-1} - 0.048903e_{t-2} - 0.673588e_{t-3} - 0.005131X_{t-1}e_{t-2} - 0.003221X_{t-2}e_{t-3} - 0.007347X_{t-3}e_{t-1} + e_t$

The fitted models' residual variances, coefficient of determination(*R*-squared) and F-statistic are given in table 1 below.

5. Conclusion

This study focused on generalized integrated bilinear models that could handle all non-linear series. Generalized bilinear models at different levels of sample sizes were considered using the non-linear real series. Generalized integrated bilinear model emerged as the better model when compared with subset model. And this is an improvement in the model proposed. Moreover, estimation of parameters witnessed a unique, consistent and convergent estimator that has prevented the models from exploding, thereby making stationary possible.

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Table 1. Goodness of fit of generalized and subset autoregressive integrated moving average bilinear models at sample sizes of 150 and 250. Two models are compared, namely M1: GBL (p, 1, q, r, s), M2: GSBL (p, 1, q, r, s). All models are significant at $p<0.001$.

Sample size	Sample size of 150		Sample size of 250	
Model	Full Bilinear	Subset Bilinear	Full Bilinear	Subset Bilinear
Residual Variance	193.4	194.2	293.7	300.1
R^2	0.61	0.60	0.54	0.53
F (Statistic)	31.22	43.97	47.0	136.91

We could see the performance of the two models above using the residual variance attached to each model. The residual variance of full bilinear model is smaller than that of subset model. The proposed model gave us the best model at full model which is an improvement. The usual convention is that the subset model is always better than the full model. But in this proposed model, testing all subsets of the models is not necessary.