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Fixed point Results with Soft G- Metrc spaces

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Abstract
In the present paper, we prove fixed point results of mapping defined on soft G-metric space.

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2 INTRODUCTION & PRELIMINARIES

In the year 1999, Molodtsov [8] initiated a novel concept of soft sets theory as a new mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential, applications of soft set theory in other disciplines and real life problems are progressing rapidly. Maji et al. [5,6] worked on soft set theory and presented an application of soft sets in decision making problems.

Guler et. Al. [4] introduced the concept of soft G-metric space according to a soft element and obtained some of its properties. Then, they defined soft G-convergence and soft G-continuity, they proved existence and uniqueness of fixed pints in soft G-metric spaces.

Our aim of this article is to present fixed point theorems in soft G-metric space satisfying a new rational contractive condition.

Definition 2.1: Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X if and only if X is a mapping from E into the set of all subsets of the set X, $i.e. F: E \to P(X)$, where $P(X)$ is the power set of X.

Definition 2.2: The intersection of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. This is denoted by $(F, A) \cap (G, B) = (H, C)$.

Definition 2.3: The union of two soft sets (F, A) and (G, B) over X is the soft set, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \varepsilon \in A - B \\ G(\varepsilon), & if\ \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F, A) \cup (G, B) = (H, C)$.

Definition 2.4: The soft set (F, A) over X is said to be a null soft set denoted by Φ if for all $\varepsilon \in A, F(\varepsilon) = \phi$ (null set)

Definition 2.5: A soft set (F, A) over X is said to be an absolute soft set, if for all $\varepsilon \in A, F(\varepsilon) = X$.

Definition 2.6: The difference (H, E) of two soft sets (H, E) and (H, E) over X denoted by $(H, E) \setminus (H, E)$, is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 2.7: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by

$(F,A)^c = (F^c, A)$ where $F^c: A \to P(X)$ is mapping given by $F^c(\alpha) = X - F(\alpha), \forall \alpha \in A$.

Definition 2.8: Let $\Re$ be the set of real numbers and $B(\Re)$ be the collection of all nonempty bounded subsets of $\Re$ and E taken as a set of parameters. Then a mapping $F: E \to B(\Re)$ is called a soft real set. It is denoted by (F, E). If specifically (F, E) is a singleton soft set, then identifying (F, E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

$\bar{0}, \bar{1}$ are the soft real numbers where $\bar{0}(e) = 0, \bar{1}(e) = 1$ for all $e \in E$, respectively.

Definition 2.9: For two soft real numbers

(i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(e) \leq \tilde{s}(e)$, for all $e \in E$.

(ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(e) \geq \tilde{s}(e)$, for all $e \in E$.

(iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(e) < \tilde{s}(e)$, for all $e \in E$.

(iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(e) > \tilde{s}(e)$, for all $e \in E$.

Definition 2.10: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \phi, \forall e' \in E \setminus \{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 2.11: Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = e'$ and $P(e) = P(e')$ i.e. $x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition 2.12: A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$, is said to be a soft metric on the soft set $\tilde{X}$ if $\tilde{d}$ satisfies the following conditions:

(M1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \tilde{\geq} \bar{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \bar{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

(M3) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = (\tilde{y}_{e_2}, \tilde{x}_{e_1})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M4) $\tilde{d}(\tilde{x}_{e_1}, \tilde{z}_{e_3}) \tilde{\leq} \tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2}, \tilde{z}_{e_3})$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$.

Definition 2.13 (Cauchy Sequence): A sequence $\{\tilde{x}_{\lambda,m}\}_n$ of soft points in $(\tilde{X}, \tilde{d}, E)$ is considered as a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\varepsilon} \tilde{\geq} \bar{0}, \exists m \in N$ such that $d(\tilde{x}_{\lambda,i}, \tilde{x}_{\lambda,j}) \tilde{\leq} \tilde{\varepsilon}, \forall i, j \geq m$, i.e. $d(\tilde{x}_{\lambda,i}, \tilde{x}_{\lambda,j}) \to \bar{0}$, as $i, j \to \infty$.

Definition 2.14 (Soft Complete Metric Space): A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called complete, if every Cauchy Sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.

Definition 2.15[4]: Let X be a nonempty set and E be the nonempty set of parameters. Let $\tilde{G}: SE(\tilde{X}) \times SE(\tilde{X}) \times SE(\tilde{X}) \to \mathbb{R}(E)^*$ be a function satisfying the following axioms:

$(\tilde{G}_1)$ $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = 0$ if $\tilde{x} = \tilde{y} = \tilde{z}$

$(\tilde{G}_2)$ $\tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) > 0$ for all $\tilde{x}, \tilde{y} \in SE(\tilde{X})$ with $\tilde{x} \neq \tilde{y}$

$(\tilde{G}_3)$ $\tilde{G}(\tilde{x}, \tilde{x}, \tilde{y}) \leq \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$ with $\tilde{y} \neq z$

$(\tilde{G}_4)$ $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) = \tilde{G}(\tilde{x}, \tilde{z}, \tilde{y}) = \tilde{G}(\tilde{y}, \tilde{z}, x) = \cdots$ (Symmetry in all three variables)

$(\tilde{G}_5)$ $\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \leq \tilde{G}(\tilde{x}, a, a) + \tilde{G}(a, \tilde{y}, z)$ for all $\tilde{x}, \tilde{y}, \tilde{z}, a \in X$ (Rectangle inequality)

Then the function $\tilde{G}$ is called a soft generalized metric or soft G-metric on $\tilde{X}$ and $(\tilde{X}, \tilde{G}, E)$ is called a soft G-metric space.

Definition 2.16: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space, let $\{\tilde{x}_n\}$be a sequence of soft points of $\tilde{X}$, a soft point $\tilde{x} \in \tilde{X}$ is said to the limit of the sequence $\{\tilde{x}_n\}$, if $lim_{n\to\infty} G(\tilde{x}, \tilde{x}_n, \tilde{x}_m) = 0$. Then $\{\tilde{x}_n\}$ is G-convergent to $\tilde{X}$.

Proposition 2.17[4]: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space, then for a sequence $\{\tilde{x}_n\} \subseteq \tilde{X}$ and a soft point $\tilde{x} \in \tilde{X}$. The following are equivalent

(i) $\{\tilde{x}_n\}$ is soft G-convergent to $\tilde{x}$.
(ii) $\tilde{G}(\tilde{x}_n, \tilde{x}_n, \tilde{x}) \to 0$ as $n \to \infty$
(iii) $\tilde{G}(\tilde{x}_n, \tilde{x}, \tilde{x}) \to 0$ as $n \to \infty$
(iv) $\tilde{G}(\tilde{x}_m, \tilde{x}_n, \tilde{x}) \to 0$ as $m, n \to \infty$.

Definition 2.18: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space, then the sequence $\{\tilde{x}_n\}$ is said to be soft G-Cauchy if for every $\varepsilon > 0$ there exists a positive integer N such that $\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_l) < \varepsilon$ for all $n, m, l \geq N$ i.e. $\tilde{G}(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Definition 2.19: A soft G-metric space $(\tilde{X}, \tilde{G}, E)$ is said to be soft G-complete space if every soft G-Cauchy sequence in $(\tilde{X}, \tilde{G}, E)$ is G-convergent in $(\tilde{X}, \tilde{G}, E)$.

Proposition 2.20[4]: Let $(\tilde{X}, \tilde{G}, E), (\tilde{X}', \tilde{G}', E')$ be two soft G-metric spaces, then a function $f: \tilde{X} \to \tilde{X}'$ is soft G-continuous at a soft point $\tilde{x} \in SE(\tilde{X})$ if and only if it is soft G-sequentially continuous at $\tilde{x} \in SE(\tilde{X})$; i.e. whenever $\{\tilde{x}_n\}$ is soft G-convergent to $\tilde{x}$, $\{f(\tilde{x}_n)\}$ is soft G-convergent to $f(\tilde{x})$.

3 MAIN RESULTS

Our main results of this article are as follows.

Theorem 3.1: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$a\,\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) \leq b \begin{Bmatrix} \tilde{G}(\tilde{x}, R\tilde{y}, R\tilde{y}) \\ +\tilde{G}(\tilde{y}, R\tilde{z}, R\tilde{z}) \\ +\tilde{G}(\tilde{z}, R\tilde{x}, R\tilde{x}) \end{Bmatrix} + c\,\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$$

…(3.1.1)

Where $a, b, c > 0$ and $3b + c < a$. Then R has a unique fixed point $\tilde{u}$ and R is G-continuous at $\tilde{u}$.

Proof: Let $x_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$T\tilde{x}_0 = \tilde{x}_1, T\tilde{x}_1 = \tilde{x}_2, T\tilde{x}_2 = \tilde{x}_3, \ldots\ldots\ldots\ldots T\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n$, $\tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.1.1) then we get

$$a\,\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \leq b \begin{Bmatrix} \tilde{G}(\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \\ +\tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) \\ +\tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_n, R\tilde{x}_n) \end{Bmatrix} + c\,\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\,\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq b \begin{Bmatrix} \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \\ +\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \\ +\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \end{Bmatrix} + c\,\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{(b+c)}{(a-2b)}\,\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq K\,\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \quad Where\ \frac{(b+c)}{(a-2b)} < 1.$$

On continuing this process $(n+1)$ times

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) \leq K^{n+1}\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}) \leq K^{n}\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)$$

Next we show that $\{x_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in N$, $n < m$ we have

$$\begin{aligned}\tilde{G}(\tilde{x}_n,\tilde{x}_m,\tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) + \ldots + \tilde{G}(\tilde{x}_{m-1},\tilde{x}_m,\tilde{x}_m)\\ &\leq (K^n + K^{n+1} + \cdots \ldots + K^{m-1})\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)\\ &\leq \frac{K^n}{1-K}\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)\end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X},\tilde{G},E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to $\tilde{u}$.

Next we will show that $\tilde{u}$ is fixed point of R.

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.1.1) then

$$a\,\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) \leq b\begin{Bmatrix}\tilde{G}(\tilde{x}_n, R\tilde{u}, R\tilde{u})\\ +\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\\ +\tilde{G}(\tilde{u}, R\tilde{x}_n, R\tilde{x}_n)\end{Bmatrix} + c\,\tilde{G}(\tilde{x}_n,\tilde{u},\tilde{u})$$

$$a\,\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq b\begin{Bmatrix}\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\\ +\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\\ +\tilde{G}(\tilde{u},\tilde{u},\tilde{u})\end{Bmatrix} + c\,\tilde{G}(\tilde{u},\tilde{u},\tilde{u})$$

$$\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq \frac{2b}{a}\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ *i.e.* $\tilde{u}$ is fixed point of R.

To prove uniqueness, suppose that $\tilde{u}$ and $\tilde{v}$ are two fixed points of R. Then by inequality (3.1.1) we have

$$a\,\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) \leq b\begin{Bmatrix}\tilde{G}(\tilde{u}, R\tilde{v}, R\tilde{v})\\ +\tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v})\\ +\tilde{G}(\tilde{v}, R\tilde{u}, R\tilde{u})\end{Bmatrix} + c\,G(\tilde{u},\tilde{v},\tilde{v})$$

$$a\,\tilde{G}(\tilde{u},\tilde{v},\tilde{v}) \leq b\begin{Bmatrix}\tilde{G}(\tilde{u},\tilde{v},\tilde{v})\\ +\tilde{G}(\tilde{v},\tilde{v},\tilde{v})\\ +\tilde{G}(\tilde{v},\tilde{u},\tilde{u})\end{Bmatrix} + c\,G(\tilde{u},\tilde{v},\tilde{v})$$

So we deduct that $(a-b-c)\,\tilde{G}(\tilde{u},\tilde{v},\tilde{v}) \leq b\,\tilde{G}(\tilde{v},\tilde{u},\tilde{u})$. This implies that $\tilde{G}(\tilde{u},\tilde{v},\tilde{v}) \leq \frac{b}{(a-b-c)}\tilde{G}(\tilde{v},\tilde{u},\tilde{u})$ and by repeated use of the same argument we will find $\tilde{G}(\tilde{v},\tilde{u},\tilde{u}) \leq \frac{b}{(a-b-c)}\tilde{G}(\tilde{u},\tilde{v},\tilde{v})$. Therefore we get $\tilde{G}(\tilde{u},\tilde{v},\tilde{v}) \leq \left\{\frac{b}{(a-b-c)}\right\}^2\tilde{G}(\tilde{u},\tilde{v},\tilde{v})$, Since $3b + c < a$, this contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at $\tilde{u}$. Let $\{\tilde{y}_n\}$ be a sequence of soft elements in $\tilde{X}$ such that $\{\tilde{y}_n\} \to \tilde{u}$ then we can deduce that

Using (3.3.1), we have

$$a\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \leq b\begin{Bmatrix}\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)\\ +\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n)\\ +\tilde{G}(\tilde{y}_n, R\tilde{u}, R\tilde{u})\end{Bmatrix} + c\,\tilde{G}(\tilde{u},\tilde{y}_n,\tilde{y}_n)$$

$$a\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \leq b\begin{Bmatrix}\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)\\ +\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n)\\ +\tilde{G}(\tilde{y}_n,\tilde{u},\tilde{u})\end{Bmatrix} + c\,\tilde{G}(\tilde{u},\tilde{y}_n,\tilde{y}_n)$$

Taking the limit as $n \to \infty$ from which we see that $(a-2b)\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \to 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at $\tilde{u}$.

Theorem 3.2: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$a\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) + \text{b min}\left\{\begin{array}{c}\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), \tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}),\\ \tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y}), \tilde{G}(\tilde{z}, R\tilde{z}, R\tilde{z})\end{array}\right\} \leq c\, \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \qquad \ldots (3.2.1)$$

Where $a, b, c > 0 \; with \; c - b < a$. Then R has a unique fixed point $\tilde{u}$ and R is G-continuous at $\tilde{u}$.

Proof: Let $\tilde{x}_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$T\tilde{x}_0 = \tilde{x}_1, T\tilde{x}_1 = \tilde{x}_2, T\tilde{x}_2 = \tilde{x}_3, \ldots\ldots\ldots\ldots T\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$ for each $n \in N \cup \{0\}$.

Substituting $\tilde{x} = \tilde{x}_n$, $\tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in **(3.2.1)** then we get

$$a\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + b \min\left\{\begin{array}{c}\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n),\\ \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1})\end{array}\right\} \leq c\, \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \min\left\{\begin{array}{c}\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}),\\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}),\\ \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})\end{array}\right\} \leq c\, \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b \min\left\{\begin{array}{c}\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}),\\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})\end{array}\right\} \leq c\, \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

Case I: If we take $\min\{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})\} = \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})$

Then $\quad a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq c\, \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c}{a+b}\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

Case II: If we take $\min\{\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})\} = \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

Then $\quad a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq c\, \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq \frac{c-b}{a} c\, \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

From **Case- I, II,** we have

$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq k\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq k\, \tilde{G}(\tilde{x}_{n-1}, \tilde{x}_n, \tilde{x}_n)$$

And $\quad \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq k^n\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$

Next we show that $\{x_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in N$, $n < m$ we have

$$\begin{aligned}\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \ldots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m)\\ &\leq (k^n + k^{n+1} + \cdots\ldots + k^{m-1})\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)\\ &\leq \frac{k^n}{1-k}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)\end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to $\tilde{u}$.

Next we will show that $\tilde{u}$ is fixed point of R.

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.2.1) then

$$a\,\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) + \mathrm{b}\,\min\left\{\begin{matrix}\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\end{matrix}\right\} \leq c\,\tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u})$$

As $n \to \infty$, we have

$$\mathrm{a}\,\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \mathrm{b}\,\min\left\{\begin{matrix}\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \\ \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\end{matrix}\right\} \leq c\,\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})$$

$$\mathrm{a}\,\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq 0 \qquad \text{Since } a > 0$$

so $R\tilde{u} = \tilde{u}.\ i.e.\ \tilde{u}$ is a fixed point of R.

To prove uniqueness, suppose that $\tilde{u}$ and $\tilde{v}$ are two fixed points of R. Then by inequality (3.2.1) we have

$$a\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) + b\,\min\left\{\begin{matrix}\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \\ \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v})\end{matrix}\right\} \leq c\,\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq \frac{c}{a}\,\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

This contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at $\tilde{u}$. Let $\{\tilde{y}_n\}$ be a sequence of soft elements in $\tilde{X}$ such that $\{\tilde{y}_n\} \to \tilde{u}$ then we can deduce that

Using (3.2.1), we have

$$\mathrm{a}\,\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + \mathrm{b}\,\min\left\{\begin{matrix}\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \\ \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n)\end{matrix}\right\} \leq c\,(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

$$\mathrm{a}\,\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + \mathrm{b}\,\min\left\{\begin{matrix}\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \\ \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)\end{matrix}\right\} \leq c\,(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

Taking the limit as $n \to \infty$ from which we see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \to 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at $\tilde{u}$.

Theorem 3.3: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$a\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}) + b\left[\frac{\min\{\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), \tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}), \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}), \tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y})\}}{\min\{\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), \tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}), \tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}), \tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y})\}}\right] \leq c\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z})$$

…(3.3.1)

Where $a, b, c > 0\ with\ c - b < a$. Then R has a unique fixed point $\tilde{u}$ and R is G-continuous at $\tilde{u}$.

Proof: Let $x_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$T\tilde{x}_0 = \tilde{x}_1, T\tilde{x}_1 = \tilde{x}_2, T\tilde{x}_2 = \tilde{x}_3, \ldots\ldots\ldots\ldots T\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n$, $\tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.3.1) then we get

$$a\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + b\left[\frac{\min\left\{\begin{matrix}\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1})\end{matrix}\right\}}{\min\left\{\begin{matrix}\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1})\end{matrix}\right\}}\right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + b\left[\frac{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})\end{matrix}\right\}}{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \\ \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}), \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2})\end{matrix}\right\}}\right] \leq c\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$a\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+\left[\frac{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{matrix}\right\}}{\min\{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\}}\right]\le c\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

(3.3.2)

Now following four cases are arise:

Case-I: If $\frac{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{matrix}\right\}}{\min\{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\}}=\frac{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+b\left[\frac{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}\right]\le c\,\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\le\frac{c}{a+b}\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

Case-II: If $\frac{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{matrix}\right\}}{\min\{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\}}=\frac{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+b\left[\frac{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}\right]\le c\,\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})\le\frac{c-b}{a}\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

Case-III: If $\frac{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{matrix}\right\}}{\min\{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\}}=\frac{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+b\left[\frac{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})}\right]\le c\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\le\frac{c}{a+b}\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

Case-IV: If $\frac{\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}).\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{matrix}\right\}}{\min\{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}),\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\}}=\frac{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}$

then condition (3.3.2) reduces to

$$a\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+b\left[\frac{\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1}).\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}{\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})}\right]\le c\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\le\frac{c-b}{a}\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

From **Case- I, II, III, IV,** we have

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\le k\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})$$

On continuing this process $(n+1)$ times

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\le k^{n+1}\tilde{G}(\tilde{x}_{0},\tilde{x}_{1},\tilde{x}_{1})$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})\le k^{n}\tilde{G}(\tilde{x}_{0},\tilde{x}_{1},\tilde{x}_{1})$$

Next we show that $\{x_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m\in N$, $n<m$ we have

$$\begin{aligned}\tilde{G}(\tilde{x}_{n},\tilde{x}_{m},\tilde{x}_{m})&\le\tilde{G}(\tilde{x}_{n},\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+\ldots+\tilde{G}(\tilde{x}_{m-1},\tilde{x}_{m},\tilde{x}_{m})\\&\le(k^{n}+k^{n+1}+\cdots\ldots+k^{m-1})\tilde{G}(\tilde{x}_{0},\tilde{x}_{1},\tilde{x}_{1})\\&\le\frac{k^{n}}{1-k}\tilde{G}(\tilde{x}_{0},\tilde{x}_{1},\tilde{x}_{1})\end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X},\tilde{G},E)$ is soft G-complete, there exists $\tilde{u}\in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to $\tilde{u}$.

Next we will show that $\tilde{u}$ is fixed point of R.

For this we take $\tilde{x}=\tilde{x}_n$ and $\tilde{y}=\tilde{z}=\tilde{u}$ in (3.3.1) then

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Form (3.3.1) we have

$$a\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) + b\left[\frac{\min\{\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}{\min\{\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n), \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}\right] \leq c\tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u})$$

$$a\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + b\left[\frac{\min\{\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}{\min\{\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}\right] \leq c\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})$$

$$a\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq 0 \qquad \text{Since } a \geq 0$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ *i.e.* $\tilde{u}$ is fixed point of R.

To prove uniqueness, suppose that $\tilde{u}$ and $\tilde{v}$ are two fixed points of R. Then by inequality (3.5.1) we have

$$a\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) + b\left[\frac{\min\{\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v})\}}{\min\{\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v})\}}\right] \leq c\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

$$a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) + b\left[\frac{\min\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{v}, \tilde{v})\}}{\min\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{v}, \tilde{v})\}}\right] \leq c\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq \frac{c}{a}\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \quad \textit{Since } \frac{c}{a} < 1.$$

This contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at $\tilde{u}$. Let $\{\tilde{y}_n\}$ be a sequence of soft elements in $\tilde{X}$ such that $\{\tilde{y}_n\} \to \tilde{u}$ then we can deduce that

Using (3.3.1), we have

$$a\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + b\left[\frac{\min\{\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}{\min\{\tilde{G}(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})\}}\right] \leq c\tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

$$a\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) + b\left[\frac{\min\{\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})\}}{\min\{\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})\}}\right] \leq c\tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n)$$

Taking the limit as $n \to \infty$ from which we see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \to 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at $\tilde{u}$.

Theorem 3.4: Let $(\tilde{X}, \tilde{G}, E)$ be a soft G-metric space and $R: (\tilde{X}, \tilde{G}, E) \to (\tilde{X}, \tilde{G}, E)$ be a mapping that satisfies the following condition for all $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{X})$

$$\min\left\{\begin{matrix}\tilde{G}(R\tilde{x}, R\tilde{y}, R\tilde{z}), [\tilde{G}(\tilde{x}, R\tilde{x}, R\tilde{x}) + \tilde{G}(\tilde{x}, R\tilde{y}, R\tilde{y})], \\ [\tilde{G}(\tilde{y}, R\tilde{y}, R\tilde{y}) + \tilde{G}(\tilde{y}, R\tilde{x}, R\tilde{x})]\end{matrix}\right\} \leq a\tilde{G}(\tilde{x}, \tilde{y}, \tilde{z}) \qquad \ldots(3.4.1)$$

Proof: Let $x_0 \in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$R\tilde{x}_0 = \tilde{x}_1, R\tilde{x}_1 = \tilde{x}_2, R\tilde{x}_2 = \tilde{x}_3, \ldots\ldots\ldots\ldots R\tilde{x}_n = \tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n \neq \tilde{x}_{n+1}$

Substituting $\tilde{x} = \tilde{x}_n$, $\tilde{y} = \tilde{x}_{n+1}$ and $\tilde{z} = \tilde{x}_{n+1}$ in (3.4.1) then we get

$$\min\left\{\begin{matrix}\tilde{G}(R\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}), \\ [\tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n) + \tilde{G}(\tilde{x}_n, R\tilde{x}_{n+1}, R\tilde{x}_{n+1})], \\ [\tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_{n+1}, R\tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, R\tilde{x}_n, R\tilde{x}_n)]\end{matrix}\right\} \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})], \\ [\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+1}, \tilde{x}_{n+1})]\end{matrix}\right\} \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

$$\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}), \\ [\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n, \tilde{x}_{n+2}, \tilde{x}_{n+2})]\end{matrix}\right\} \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \ldots(3.4.2)$$

Now following two cases are arise:

Case-I: If $\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}),\\ [\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})]\end{matrix}\right\} = \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})$

then condition (3.4.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) \le a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

CaseII: If $\min\left\{\begin{matrix}\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}),\\ [\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})]\end{matrix}\right\} = \begin{bmatrix}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})\\ +\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\end{bmatrix}$

then condition (3.4.2) reduces to

$$\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2}) \le a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}) + [\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) - \tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})] \le a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) \le a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

From **Case- I, II,** we have

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) \le a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

On continuing this process $(n+1)$ times

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) \le a^{n+1}\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}) \le a^n\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)$$

Next we show that $\{x_n\}$ is Soft G-Cauchy sequence.

Then for all $n, m \in N$, $n < m$ we have

$$\begin{aligned}\tilde{G}(\tilde{x}_n,\tilde{x}_m,\tilde{x}_m) &\le \tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2}) + \ldots + \tilde{G}(\tilde{x}_{m-1},\tilde{x}_m,\tilde{x}_m)\\ &\le (a^n + a^{n+1} + \cdots\ldots + a^{m-1})\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)\\ &\le \frac{a^n}{1-a}\tilde{G}(\tilde{x}_0,\tilde{x}_1,\tilde{x}_1)\end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X},\tilde{G},E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to $\tilde{u}$.

Next we will show that $\tilde{u}$ is fixed point of R.

For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.4.1) then

$$\min\left\{\begin{matrix}\tilde{G}(R\tilde{x}_n,R\tilde{u},R\tilde{u}),[\tilde{G}(\tilde{x}_n,R\tilde{x}_n,R\tilde{x}_n)+\tilde{G}(\tilde{x}_n,R\tilde{u},R\tilde{u})],\\ [\tilde{G}(\tilde{u},R\tilde{u},R\tilde{u})+\tilde{G}(\tilde{u},R\tilde{x}_n,R\tilde{x}_n)]\end{matrix}\right\} \le a\tilde{G}(\tilde{x}_n,\tilde{u},\tilde{u})$$

$$\min\left\{\begin{matrix}\tilde{G}(\tilde{u},R\tilde{u},R\tilde{u}),[\tilde{G}(\tilde{u},\tilde{u},\tilde{u})+\tilde{G}(\tilde{u},R\tilde{u},R\tilde{u})],\\ [\tilde{G}(\tilde{u},R\tilde{u},R\tilde{u})+\tilde{G}(\tilde{u},\tilde{u},\tilde{u})]\end{matrix}\right\} \le a\tilde{G}(\tilde{u},\tilde{u},\tilde{u})$$

$$\tilde{G}(\tilde{u},R\tilde{u},R\tilde{u}) \le 0$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ *i.e.* $\tilde{u}$ is fixed point of R.

To prove uniqueness, suppose that $\tilde{u}$ and $\tilde{v}$ are two fixed points of R. Then by inequality (3.1.1) we have

$$\min\{\tilde{G}(\tilde{u},\tilde{v},\tilde{v}),[\tilde{G}(\tilde{u},\tilde{u},\tilde{u})+\tilde{G}(\tilde{u},\tilde{v},\tilde{v})],[\tilde{G}(\tilde{v},\tilde{v},\tilde{v})+\tilde{G}(\tilde{v},\tilde{u},\tilde{u})]\} \le a\tilde{G}(\tilde{u},\tilde{v},\tilde{v})$$

$$\tilde{G}(\tilde{u},\tilde{v},\tilde{v}) \le a\tilde{G}(\tilde{u},\tilde{v},\tilde{v})$$

Since $a < 1$, this contradiction implies that $\tilde{u} = \tilde{v}$.

To show that R is soft G-continuous at $\tilde{u}$. Let $\{\tilde{y}_n\}$ be a sequence of soft elements in $\tilde{X}$ such that $\{\tilde{y}_n\} \to \tilde{u}$ then we can deduce that

Using (3.4.1), we have

$$\min\left\{\begin{matrix}\tilde{G}(R\tilde{u},R\tilde{y}_n,R\tilde{y}_n),[\tilde{G}(\tilde{u},R\tilde{u},R\tilde{u})+\tilde{G}(\tilde{u},R\tilde{y}_n,R\tilde{y}_n)],\\ [\tilde{G}(\tilde{y}_n,R\tilde{y}_n,R\tilde{y}_n)+\tilde{G}(\tilde{y}_n,R\tilde{u},R\tilde{u})]\end{matrix}\right\} \le a\tilde{G}(\tilde{u},\tilde{y}_n,\tilde{y}_n)$$

$$\min\left\{\begin{array}{c}\tilde{G}(\tilde{u},R\tilde{y}_n,R\tilde{y}_n),\left[\tilde{G}(\tilde{u},\tilde{u},\tilde{u})+\tilde{G}(\tilde{u},R\tilde{y}_n,R\tilde{y}_n)\right],\\ \left[\tilde{G}(\tilde{y}_n,R\tilde{y}_n,R\tilde{y}_n)+\tilde{G}(\tilde{y}_n,\tilde{u},\tilde{u})\right]\end{array}\right\}\leq a\tilde{G}(\tilde{u},\tilde{y}_n,\tilde{y}_n)$$

Taking the limit as $n\to\infty$ from which we see that $\tilde{G}(\tilde{u},R\tilde{y}_n,R\tilde{y}_n)\to 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u}=\tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at $\tilde{u}$.

Theorem 3.5: Let $(\tilde{X},\tilde{G},E)$ be a soft G-metric space and $R:(\tilde{X},\tilde{G},E)\to(\tilde{X},\tilde{G},E)$ be a mapping that satisfies the following condition for all $\tilde{x},\tilde{y},\tilde{z}\in SE(\tilde{X})$

$$\tilde{G}(R\tilde{x},R\tilde{y},R\tilde{z})\leq a\max\left\{\begin{array}{c}\tilde{G}(\tilde{x},\tilde{y},\tilde{z}),\left[\tilde{G}(\tilde{x},R\tilde{x},R\tilde{x})+\tilde{G}(\tilde{x},R\tilde{y},R\tilde{y})\right],\\ \left[\tilde{G}(\tilde{y},R\tilde{y},R\tilde{y})+\tilde{G}(\tilde{y},R\tilde{x},R\tilde{x})\right]\end{array}\right\} \quad \ldots(3.5.1)$$

Where $0\leq a<1$. Then R has a unique fixed point $\tilde{u}$ and R is G-continuous at $\tilde{u}$.

Proof: Let $x_0\in SE(\tilde{X})$ be an arbitrary soft element and define the sequence $\{\tilde{x}_n\}$ by

$$R\tilde{x}_0=\tilde{x}_1,R\tilde{x}_1=\tilde{x}_2,R\tilde{x}_2=\tilde{x}_3,\ldots\ldots\ldots R\tilde{x}_n=\tilde{x}_{n+1}$$

Here we may assume that $\tilde{x}_n\neq\tilde{x}_{n+1}$

Substituting $\tilde{x}=\tilde{x}_n$, $\tilde{y}=\tilde{x}_{n+1}$ and $\tilde{z}=\tilde{x}_{n+1}$ in (3.5.1) then we get

$$\tilde{G}(R\tilde{x}_n,R\tilde{x}_{n+1},R\tilde{x}_{n+1})\leq a\max\left\{\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \left[\tilde{G}(\tilde{x}_n,R\tilde{x}_n,R\tilde{x}_n)+\tilde{G}(\tilde{x}_n,R\tilde{x}_{n+1},R\tilde{x}_{n+1})\right],\\ \left[\tilde{G}(\tilde{x}_{n+1},R\tilde{x}_{n+1},R\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_{n+1},R\tilde{x}_n,R\tilde{x}_n)\right]\end{array}\right\}$$

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq a\max\left\{\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}),\left[\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\right],\\ \left[\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})+\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+1},\tilde{x}_{n+1})\right]\end{array}\right\}$$

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq a\max\left\{\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}),\left[\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\right],\\ \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{array}\right\} \quad \ldots(3.5.2)$$

Now following three cases are arise:

Case-I: If $\max\left\{\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \left[\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\right],\\ \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{array}\right\}=\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$

then condition (3.5.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$
$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq a\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

Case-II: If $\max\left\{\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \left[\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\right],\\ \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{array}\right\}=\left[\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})\\ +\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\end{array}\right]$

then condition (3.5.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq a\left[\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\right]$$
$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq\frac{a}{1-a}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})$$

Case-III: If $\max\left\{\begin{array}{c}\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1}),\\ \left[\tilde{G}(\tilde{x}_n,\tilde{x}_{n+1},\tilde{x}_{n+1})+\tilde{G}(\tilde{x}_n,\tilde{x}_{n+2},\tilde{x}_{n+2})\right],\\ \tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\end{array}\right\}=\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})$

then condition (3.5.2) reduces to

$$\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})\leq a\tilde{G}(\tilde{x}_{n+1},\tilde{x}_{n+2},\tilde{x}_{n+2})$$

Which is contradication.
From **Case- I, II, III,** we have

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1})$$

On continuing this process $(n+1)$ times

$$\tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) \leq a^{n+1}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Similarly we can show that

$$\tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) \leq a^n \tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)$$

Next we show that $\{x_n\}$ is Soft G-Cauchy sequence.
Then for all $n, m \in N$, $n < m$ we have

$$\begin{aligned}\tilde{G}(\tilde{x}_n, \tilde{x}_m, \tilde{x}_m) &\leq \tilde{G}(\tilde{x}_n, \tilde{x}_{n+1}, \tilde{x}_{n+1}) + \tilde{G}(\tilde{x}_{n+1}, \tilde{x}_{n+2}, \tilde{x}_{n+2}) + \ldots + \tilde{G}(\tilde{x}_{m-1}, \tilde{x}_m, \tilde{x}_m)\\ &\leq (a^n + a^{n+1} + \cdots\ldots + a^{m-1})\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)\\ &\leq \frac{a^n}{1-a}\tilde{G}(\tilde{x}_0, \tilde{x}_1, \tilde{x}_1)\end{aligned}$$

Therefore $\{\tilde{x}_n\}$ is soft G-Cauchy sequence. Since $(\tilde{X}, \tilde{G}, E)$ is soft G-complete, there exists $\tilde{u} \in SE(\tilde{X})$ such that $\{\tilde{x}_n\}$ soft G-converges to $\tilde{u}$.
Next we will show that $\tilde{u}$ is fixed point of R.
For this we take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{z} = \tilde{u}$ in (3.5.1) then

$$\tilde{G}(R\tilde{x}_n, R\tilde{u}, R\tilde{u}) \leq a \max \begin{Bmatrix} \tilde{G}(\tilde{x}_n, \tilde{u}, \tilde{u}), [\tilde{G}(\tilde{x}_n, R\tilde{x}_n, R\tilde{x}_n) + \tilde{G}(\tilde{x}_n, R\tilde{u}, R\tilde{u})], \\ [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{x}_n, R\tilde{x}_n)] \end{Bmatrix}$$

$$\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq a \max \begin{Bmatrix} \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})], \\ [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, \tilde{u}, \tilde{u})] \end{Bmatrix}$$

$$\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq a\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u})$$

$$(1-a)\,\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) \leq 0$$

This is the contraction, so $R\tilde{u} = \tilde{u}$ *i.e.* $\tilde{u}$ is fixed point of R.
To prove uniqueness, suppose that $\tilde{u}$ and $\tilde{v}$ are two fixed points of R. Then by inequality (3.5.1) we have

$$\tilde{G}(R\tilde{u}, R\tilde{v}, R\tilde{v}) \leq a \max \begin{Bmatrix} \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{v}, R\tilde{v})], \\ [\tilde{G}(\tilde{v}, R\tilde{v}, R\tilde{v}) + \tilde{G}(\tilde{v}, R\tilde{u}, R\tilde{u})] \end{Bmatrix}$$

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a \max \begin{Bmatrix} \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), [\tilde{G}(\tilde{u}, \tilde{u}, \tilde{u}) + \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})], \\ [\tilde{G}(\tilde{v}, \tilde{v}, \tilde{v}) + \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})] \end{Bmatrix}$$

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a \max\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})\} \qquad \ldots(3.5.3)$$

Case I: If $\max\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})\} = \tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$ then (3.5.3) we get

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\,\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$$

This is a contradiction implies that $\tilde{u} = \tilde{v}$.
Case II: If $\max\{\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}), \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})\} = \tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$ then (3.5.3) we get

$$\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$$

So we deduct that $\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a\,\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$. By repeated use of the same argument we will find $\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u}) \leq a\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v})$. Therefore we get $\tilde{G}(\tilde{u}, \tilde{v}, \tilde{v}) \leq a^2\,\tilde{G}(\tilde{v}, \tilde{u}, \tilde{u})$, Since $a < 1/2$, this contradiction implies that $\tilde{u} = \tilde{v}$.
To show that R is soft G-continuous at $\tilde{u}$. Let $\{\tilde{y}_n\}$ be a sequence of soft elements in $\tilde{X}$ such that $\{\tilde{y}_n\} \to \tilde{u}$ then we can deduce that
Using (3.5.1)

$$\begin{aligned}G(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) &= G(R\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)\\ &\leq a \max \begin{Bmatrix} \tilde{G}(\tilde{u}, \tilde{y}_n, \tilde{y}_n), [\tilde{G}(\tilde{u}, R\tilde{u}, R\tilde{u}) + \tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n)], \\ [\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) + \tilde{G}(\tilde{y}_n, R\tilde{u}, R\tilde{u})] \end{Bmatrix}\end{aligned}$$

$$\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n) \le a\tilde{G}(\tilde{y}_n, R\tilde{y}_n, R\tilde{y}_n)$$
$$(1-a)\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \le 0$$

We see that $\tilde{G}(\tilde{u}, R\tilde{y}_n, R\tilde{y}_n) \to 0$ and so, by proposition (2.17) we have that the sequence $R\tilde{y}_n$ is G – convergent to $R\tilde{u} = \tilde{u}$ therefore proposition (2.20) implies that R is G-continuous at $\tilde{u}$.

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