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Edges Detection Based on Renyi Entropy with Split/Merge

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Abstract

Most of the classical methods for edge detection are based on the first and second order derivatives of gray levels of the pixels of the original image. These processes give rise to the exponential increment of computational time, especially with large size of images, and therefore requires more time for processing. This paper shows the new algorithm based on both the Rényi entropy and the Shannon entropy together for edge detection using split and merge technique. The objective is to find the best edge representation and decrease the computation time. A set of experiments in the domain of edge detection are presented. The system yields edge detection performance comparable to the classic methods, such as Canny, LOG, and Sobel. The experimental results show that the effect of this method is better to LOG, and Sobel methods. In addition, it is better to other three methods in CPU time. Another benefit comes from easy implementation of this method.

Keywords: Rényi Entropy, Information content, Edge detection, Thresholding

1. Introduction

Edge detection is a popular method for image segmentation. It is widely used in many image processing applications such as optical character recognition [1], infrared gait recognition [2], automatic target recognition [3], detection of video changes [4], medical image applications [5], machine vision and automated interpretation systems, satellite television, magnetic field resonance imaging, and geographical information system [6,7,8]. The detection results benefit applications such as image enhancement, morphing, compression, retrieval, watermarking, hiding, recognition, restoration, and registration etc [9,10, 11,12,13].

Edge detection is a process in which a digital image is partitioned into its constituent bounders or regions. For successful solution of image processing problems a robust system is necessary, but automatic edge detection is still a big challenge for the researchers of recent times. In this regard, thresholding technique being important application in edge detection [14].

There are many image thresholding studies in the literature. In general, thresholding methods can be classified into parametric and nonparametric methods. For parametric approaches, the gray-level distribution of each group is assumed to obey a Gaussian distribution, and then the approaches attempt to find an estimate of the parameters of Gaussian distribution that best fits the histogram. Wang et al. [15] integrated the histogram with the Parzen window technique to estimate the spatial probability distribution. Fan et al. [16] approximated the histogram with a mixed Gaussian model, and estimated the parameters with an hybrid algorithm based on particle swarm optimization and expectation maximization. Zahara et al. [17] fitted the Gaussian curve by Nelder-Mead simplex search and particle swarm optimization. To resolve the histogram Gaussian fitting problem, Nakib et al. used an improved variant of simulated annealing adapted to continuous problems [18]. Nonparametric approaches find the thresholds that separate the gray-level regions of an image in an optimal manner based on some discriminating criteria. Otsu's criterion [19], which selects optimal thresholds by maximizing the between class variance, is the most popular method. However, inefficient formulation of between class variance makes the method quite time-consuming in multilevel threshold selection. To overcome this problem, Chung et al. [20] presented an efficient heap and quantization based data structure to realize a fast implementation. Huang et al. [21] proposed a two-stage multi-threshold Otsu method. Wang et al. [22] proposed applying an improved shuffled frog-leaping algorithm to the three dimensional Otsu thresholding. Besides the criterion of the between class variance, other criteria are also investigated. Hamza [23] proposed a non-extensive information-theoretic measure called Jensen-Tsallis divergence for image edge detection.

Thresholding technique being most simple is applied mostly in edge detection. Thresholding of images is done by mainly two ways: global thresholding and local thresholding. For global thresholding technique, there is a unique threshold value for the entire image, whereas, in local thresholding, number of thresholds selected equals number of the local regions. Many thresholding techniques are reported in literature for last thirty years [24, 25, 26, 27,28]. Many operators have been introduced in the literature, for example Roberts, Sobel and Prewitt [29, 30, 31, 32, 33]. Edges are mostly detected using either the first derivatives, called gradient, or the second derivatives, called Laplacien. Laplacien is more sensitive to noise since it uses more information because of the nature of the second derivatives.

Most of the classical methods for edge detection based on the derivative of the pixels of the original image are Gradient operators, Laplacian and Laplacian of Gaussian (LOG) operators [34]. Gradient based edge detection methods, such as Roberts, Sobel and Prewitts, have used two 2-D linear filters to process vertical edges and horizontal edges separately to approximate first-order derivative of pixel values of the image. Marr and Hildreth achieved this by using the Laplacian of a Gaussian (LOG) function as a filter [35]. The paper [36] classified and comparative studies of edge detection algorithms are presented. Experimental results prove that Boie-Cox, Shen- Castan and Canny operators are better than Laplacian of Gaussian (LOG), while LOG is better than Prewitt and Sobel in case of noisy image. The paper [37] used 2-D gamma distribution, the experiment showed that the proposed method obtained very good results but with a big time complexity due to the big number of constructed masks.

To solve these problems, we introduced a method based on Tsallis entropy in [11]. Here, another study proposed a novel approach based on information theory, which is entropy-based thresholding (Rényi entropy) with split and merge techniques. The proposed method is decrease the computation time. The results were very good compared with the well-known Sobel gradient [38] and Canny [39] gradient results.

This paper is organized as follows: in Section 2 presents some fundamental concepts of the mathematical setting of Rényi entropy, and information. Section 3, we describe threshold value which use in the proposed method. And we describe the proposed algorithm in Section 4. In Section 5, we report the effectiveness of our method and compare results of the algorithm against several leading edge detection methods, such as Canny, LOG, and Sobel method. At last conclusion of this paper will be drawn in Section 6.

2. Rényi Entropy, and Information

The seminal work of Shannon[40], based on papers by Nyquist [41, 42] and Hartley [43], rationalized these early efforts into a coherent mathematical theory of communication and initiated the area of research now known as information theory. The set of all source symbol probabilities is denoted by P, $P=\{p_1, p_2, p_3, ..., p_k\}$. This set of probabilities must satisfy the condition $\sum p_i=1$, $0\le p_i \le 1$. The average information per source output, denoted $S(P)$, Shannon entropy may be described as [44]:

$$S(P) = -\sum_{i=1}^{k} p_i \ ln \ p_i \tag{1}$$

being k the total number of states.

Rényi [45, 46] was able to extend Shannon entropy to a continuous family of entropy measures.There is extensive literature on the applications of the Rényi entropy in many fields from biology, medicine, genetics, linguistics, and economics to electrical engineering, computer science, geophysics, chemistry and physics. The Rényi's entropy measure of order α of an image, $H_\alpha(P)$ is defined as (see Refs. [45,47]):

$$H_\alpha(P) = \frac{1}{1-\alpha} ln \sum_{i=1}^{k} p_i^\alpha \tag{2}$$

where $\alpha\neq 1$ is a positive real parameter.

Theorem 1: *Shannon entropy measure is a special case of the Rényi entropy for $\alpha \to 1$.*

At $\alpha \to 1$ the value of this quantity is potentially undefined as it generates the form 0/0. In order to find the limit of the Rényi entropy, we apply l'Hopital's Theorem $lim_{\alpha\to 1}\{f(\alpha)/g(\alpha)\}= lim_{\alpha\to 1}\{f'(\alpha)/g'(\alpha)\}$, where in this case $a = 1$. We put $g(\alpha)=1-\alpha$. Then $g'(\alpha)=-1$ and $f(\alpha)= ln\sum(p_i)^\alpha$, $i=1,2,\ldots,k$. The form a^x can be differentiated w.r.t. x by putting $d/dx(a^x)= d/dx(e^{x \ ln \ a})= a^x \ ln(a)$. Therefore $f'(\alpha)= d/dx\{ln\sum(p_i)^\alpha\}= \sum(p_i)^\alpha.ln(p_i)$. Letting $\alpha \to 1$, we have $H(P)=-\sum p_i.ln(p_i)$ which is the Shannon entropy. □

Theorem 2: *The Rényi entropy and information content converge to the Shannon entropy for $\alpha\to 1$.*

Kendall [48] defines the information content of a probability distribution in the discrete case as:

$$I_\alpha(P) = -\sum_{i=1}^{k} \frac{p_i^\alpha}{\alpha-1} + \frac{1}{\alpha-1} = \frac{1}{\alpha-1}(1-\sum_{i=1}^{k} p_i^\alpha) \tag{3}$$

In order to find the limit of $I_\alpha(P)$, we apply l'Hopital's Theorem. We put $g(\alpha)=\alpha-1$, and $f(\alpha)=1- ln\sum(p_i)^\alpha$. Then $g'(\alpha)=1$, and $f'(\alpha)= -d/dx\{ln\sum(p_i)^\alpha\}= -\sum(p_i)^\alpha.ln(p_i)$. Letting $\alpha \to 1$, we have $I(P)=-\sum p_i.ln(p_i)$ which is the Shannon entropy. □

From (2) the Rényi (cross) entropy of order α of is derived:

$$H_\alpha(P,Q) = \frac{1}{1-\alpha} ln \sum_{i=1}^{k} \frac{p_i^\alpha}{q_i^{\alpha-1}} \tag{4}$$

where P and Q are two discrete distributions. The K-L (Kullback-Leibler [49]) distance is a special case of the cross entropy of (4) for when $\alpha \to 1$. One important property of the cross entropy is that if $P = Q$ then H_α=0. In a measure which is symmetric, i.e. $H_\alpha(P,Q)= H_\alpha(Q,P)$. If α =0.5 in (4), then the symmetric case of cross entropy become :

$$H_{0.5}(P,Q) = 2\ ln\sum_{i=1}^{k}\sqrt{p_i\,q_i} \tag{5}$$

This relation can be used for tracking between two consecutive scenes in video files, or change in networks.

3. Threshold Value

Let $p_i = p_1, p_2, \ldots, p_k$ be the probability distribution for an image with k =255 gray-levels. From this distribution, we derive two probability distributions, one for the object (class A) and the other for the background (class B), given by:

$$p_A : \frac{p_1}{P_A}, \frac{p_2}{P_A}, \ldots, \frac{p_t}{P_A} \quad , \qquad p_B : \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, \ldots, \frac{p_k}{P_B} \tag{6}$$

and where

$$P_A = \sum_{i=1}^{t} p_i \quad , \qquad P_B = \sum_{i=t+1}^{k} p_i \tag{7}$$

The Rényi entropy of order α for each distribution is defined as:

$$H_\alpha^A(t) = \frac{1}{1-\alpha} ln\sum_{i=0}^{t}\left(\frac{p_i}{P_A}\right)^\alpha \quad \text{and} \quad H_\alpha^B(t) = \frac{1}{1-\alpha} ln\sum_{i=t+1}^{255}\left(\frac{p_i}{P_B}\right)^\alpha \tag{8}$$

$H_\alpha(t)$ is parametrically dependent upon the threshold value t for the foreground and background. We try to maximize the information measure between the two classes (object and background). When $H_\alpha(t)$ is maximized, the luminance level t that maximizes the function is considered to be the optimum threshold value.

$$t^*(\alpha) = Arg \quad \max_{t\in G}[H_\alpha^A(t) + H_\alpha^B(t)]. \tag{9}$$

Take α =0.5 , the optimum threshold value is

$$t^*(0.5) = 2Arg\max_{t\in G}[ln\sum_{i=0}^{t}\sqrt{p_i / P_A} + ln\sum_{i=t+1}^{255}\sqrt{p_i / P_B}] \tag{10}$$

Let $f(x,y)$ be the gray value of the pixel located at the point (x, y). In a digital image $\{ f(x,y) | x\in\{1,2,\ldots,M\}, y\in\{1,2,\ldots,N\} \}$ of size $M\times N$, let the histogram be $h(a)$ for $a\in\{0,1,2,\ldots,255\}$ with f as the amplitude (brightness) of the image at the real coordinate position (x, y). For the sake of convenience, we denote the set of all gray levels $\{0,1,2,\ldots,255\}$ as G. Global threshold selection methods usually use the gray level histogram of the image. The optimal threshold t^* is determined by optimizing a suitable criterion function obtained from the gray level distribution of the image and some other features of the image.

Let t be a threshold value and $B = \{b_0, b_1\}$ be a pair of binary gray levels with $\{b_0, b_1\}\in G$. Typically b_0 and b_1 are taken to be 0 and 1, respectively. The result of thresholding an image function $f(x, y)$ at gray level t is a binary function $f_t(x, y)$ such that $f_t(x, y) =b_0$ if $f_t(x, y) \le t$ otherwise, $f_t(x, y) =b_1$. In general, a thresholding method determines the value t^* of t based on a certain criterion function. If t^* is determined solely from the gray level of each pixel, the thresholding method is point dependent [44].

$$t^*(1) = Arg \quad \max_{t\in G}[S^A(t) + S^B(t)]. \tag{11}$$

When $\alpha \to 1$, the threshold value in Equation (2), equals to the same value found by Shannon's method. Thus this proposed method includes Shannon's method as a special case. The following expression can be used as a criterion function to obtain the optimal threshold at $\alpha \to 1$.

The *Threshold* procedure to select suitable threshold value $t^*$and α =0.5 for grayscale image f can now be described as follows:

Procedure *Threshold*,

Input: A grayscale image f of size $r \times c$.

Output: t^* of f , with α =0.5.

Begin

1. Let $f(x, y)$ be the original gray value of the pixel at the point (x, y), x=1..r, y=1..c .

2. Calculate the probability distribution $0 \leq p_i \leq 255$.
3. For all $t \in \{0,1,\ldots,255\}$,
    i. Calculate p_A, p_B, P_A, and P_B, using Eq.s (6 and 7).
    ii. Find optimum threshold value t^*, where $t^*(0.5)=2 Arg\ max[\ ln \sum_{i=1}(p_i/P_A)^{0.5} + ln \sum_{i=t+1}(p_i/P_B)^{0.5}]$.

End.

The technique consists of treating each pixel of the original image and creating a new image, such that $f_t(x, y)=0$ if $f_t(x, y) = t^*(\alpha)$ otherwise, $f_t(x, y) =1$ for every $x \in \{1,2,\ldots,M\}$, $y \in \{1,2,\ldots,N\}$.

4. The proposed algorithm

Geometric properties of a binary image such as connectivity, projection, area, and perimeter are important components in binary image processing. An object in a binary image is a connected set of 1 pixels. The following definitions related to connectivity of pixels in a binary image are important.

- *Connected Pixels*: A pixel f_0 at (i_0,j_0) is *connected* to another pixel f_n, at (i_n ,j_n) if and only if there exists a path from f_0 to f_n, which is a sequence of points (i_0,j_0), (i_1,j_1),…, (i_n ,j_n), such that the pixel at (i_k ,j_k) is a neighbor of the pixel at (i_{k+1} ,j_{k+1}) and $f_k = f_{k+1}$ for all, $0< k < n -1$.
- *4-connected*: When a pixel at location (i, j) has four immediate neighbors at $(i +1, j)$, $(i-1, j)$, $(i, j+1)$,and $(i, j-1)$, or four immediate neighbors at $(i +1, j+1)$, $(i-1, j+1)$, $(i+1, j-1)$,and $(i-1, j-1)$ they are known as,*4-connected*. Two four connected pixels share a common boundary as shown in Figure (1-a,1-b).
- *8-connected*: When the pixel a t location (i, j) has. in addition to above two types of four immediate neighbors, together, they are known as *8-connected*. Thus two pixels are eight neighbors if they share a common corner. This is shown in Figure (1-c).
- *Connected component*: A set of connected pixels (4 or 8 connected) forms a *connected component.* Such a connected component represents an object in a scene as shown in Figure (1-d).

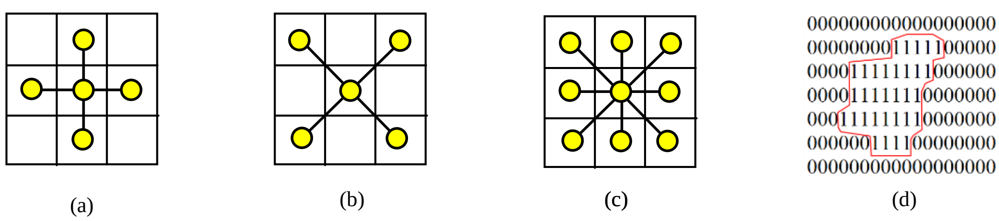


Figure 1. (a) 4-connected, (b) Diagonal 4-connected, (c) 8-connected, and (d) Connected component.

In order to edge detection, firstly classification of all pixels that satisfy the criterion of homogeneousness, and detection of all pixels on the borders between different homogeneous areas. In the proposed scheme, first create a binary image by choosing a suitable threshold value using Rényi entropy, using of the *Threshold* procedure. Region labeling in this system is done using 4-neighbor or 8-neighbor connectivity. A common alternative would be to use four neighbor connectivity instead (Figure 1).

The EdgeDetection Procedure can now be described as follows: $f_t(x, y)=0$ if $f_t(x, y) = t^*(\alpha)$ otherwise, $f_t(x, y) =1$

Procedure *EdgeDetection*;

Input: A grayscale image A of size $r \times c$ and t^*.

Output: The edge detection image g of A.

Begin

Step 1: Create a binary image: For all x, y, If $f_t(x, y) \leq t^*(\alpha)$ then $f_t(x, y)=0$ Else $f_t(x, y)=1$.

Step 2: Create an $r \times c$ output image, g: For all x and y, Set $g(x, y) = 0$.

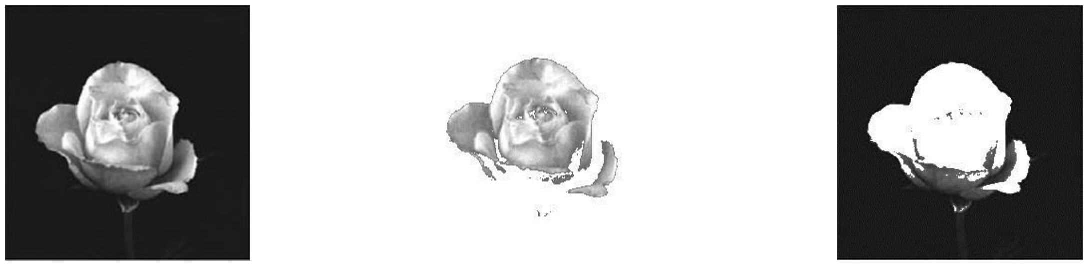
Step 3: Checking for edge pixels:

For all $1< j< r$, and $1< i< c$ do

$$\delta 1 = \left| f_{j,i} - f_{j,i-1} \right| + \left| f_{j,i} - f_{j,i+1} \right|, \quad \delta 2 = \left| f_{j,i} - f_{j-1,i} \right| + \left| f_{j,i} - f_{j+1,i} \right|,$$
$$\varepsilon 1 = \left| f_{j,i} - f_{j-1,i-1} \right| + \left| f_{j,i} - f_{j+1,i+1} \right|, \quad \varepsilon 2 = \left| f_{j,i} - f_{j-1,i+1} \right| + \left| f_{j,i} - f_{j+1,i-1} \right|,$$

If $\delta 1+\delta 2=0$ or $\varepsilon 1+\varepsilon 2=0$ then $g_{j,i}=1$.

End Procedure.



(a) Original image (b) Part1 (c) Part2

Figure 2. Original image , and its parts, Part1 and Part2.

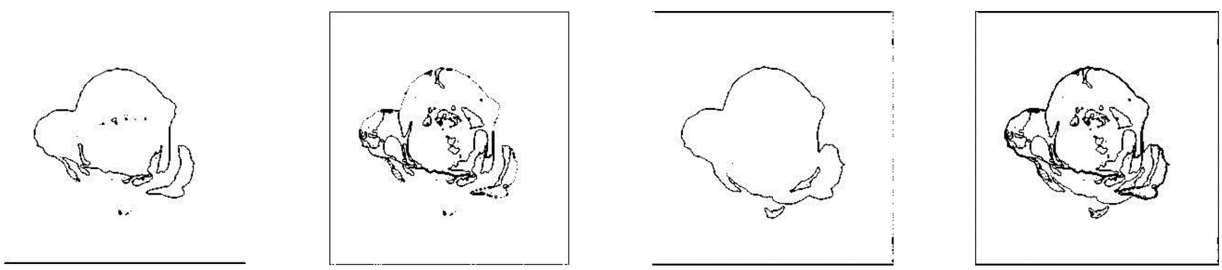
The steps of proposed algorithm are as follows:

Step 1: We use Shannon entropy, the equation (11), to find the global threshold value (t_1). The image is segmented by t_1 into two parts, the object and the background. See Figure 2.

Step 2: We use Rényi entropy, the equation (10) , α=0.5. Applying the equation (10), to find the locals threshold values (t_2) and (t_3) of Part1 and Part2, respectively.

Step 3: Applying *EdgeDetection* Procedure with threshold values t_1, t_2 and t_3. See Figure 3 .a-c

Step 4: Merge the resultant images of Step 3 in final output edge image. See Figure 3.d



(a) At threshold value t_1 (b) At threshold value t_2 (c) At threshold value t_3 (d) final edge image.

Figure 3 Edge images of original image , its parts, Part1 and Part2 and final output of edge image.

5. Results and Discussions

In order to test the method proposed in this paper and compare with the other edge detectors, common gray level test images with different resolutions and sizes are detected by Canny, LOG, and Sobel and the proposed method respectively. The performance of the proposed scheme is evaluated through the simulation results using MATLAB. Prior to the application of this algorithm, no pre-processing was done on the tested images.

The proposed algorithm used the good characters of each Shannon entropy and Rényi entropy, together, to calculate the global and local threshold values. Hence, we ensure that the proposed algorithm done better than the algorithms that based on Shannon entropy or Rényi entropy separately.

We run the Canny, LOG, and Sobel methods and the proposed algorithm 20 times for each image with different sizes. As shown in Figures 4-6, The charts of the test images and the average of run time for the classical methods and proposed scheme. It has been observed that the proposed edge detector works effectively for different gray scale digital images as compare to the run time of LOG, and Sobel methods.

Some selected results of edge detections for these test images using the classical methods and proposed scheme are shown in Figures 7-15. From the results; it has again been observed that the proposed method works well as compare to the previous methods, LOG and Sobel (with default parameters in MATLAB).

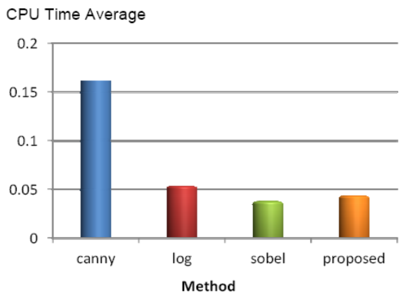


Figure 4. CPU time with 256×256 pixel test images

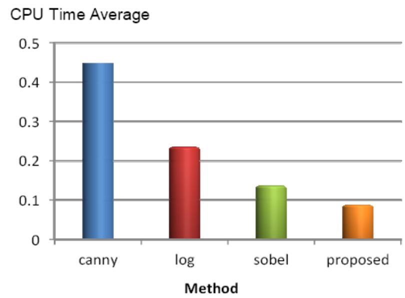


Figure 5. CPU time with 512×512 pixel test images

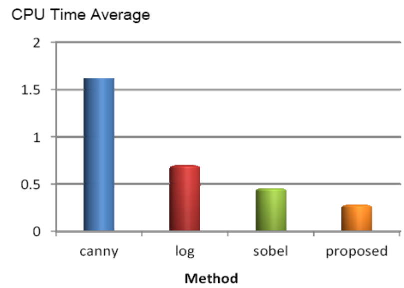


Figure 6. CPU time with 1024×1024 pixel test images

6. Conclusion

The hybrid entropic edge detector presented in this paper uses both Shannon entropy and Rényi entropy with α=0.5, together. It is already pointed out in the introduction that the traditional methods give rise to the exponential increment of computational time. However, the proposed method is decrease the computation time with generate high quality of edge detection. Experiment results have demonstrated that the proposed scheme for edge detection works satisfactorily for different gray level digital images. Another benefit comes from easy implementation of this method.

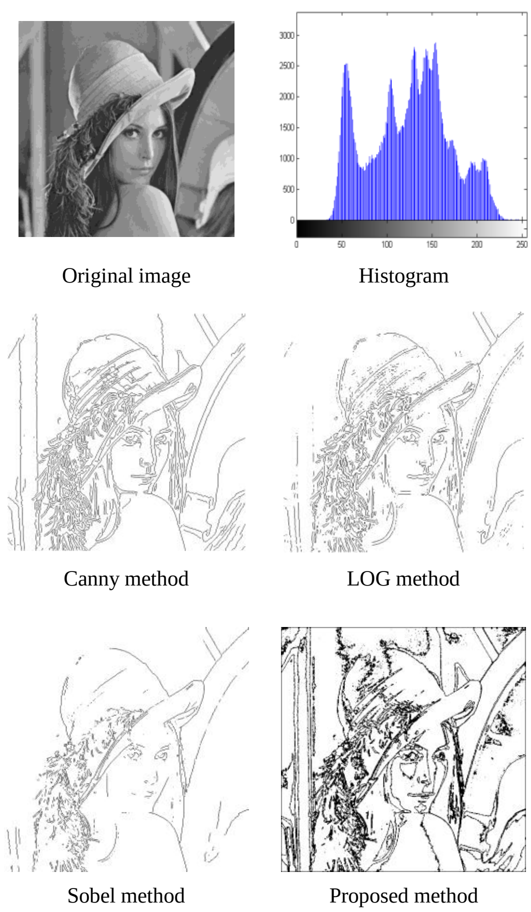


Figure 7. Lena image with 512×512 pixel

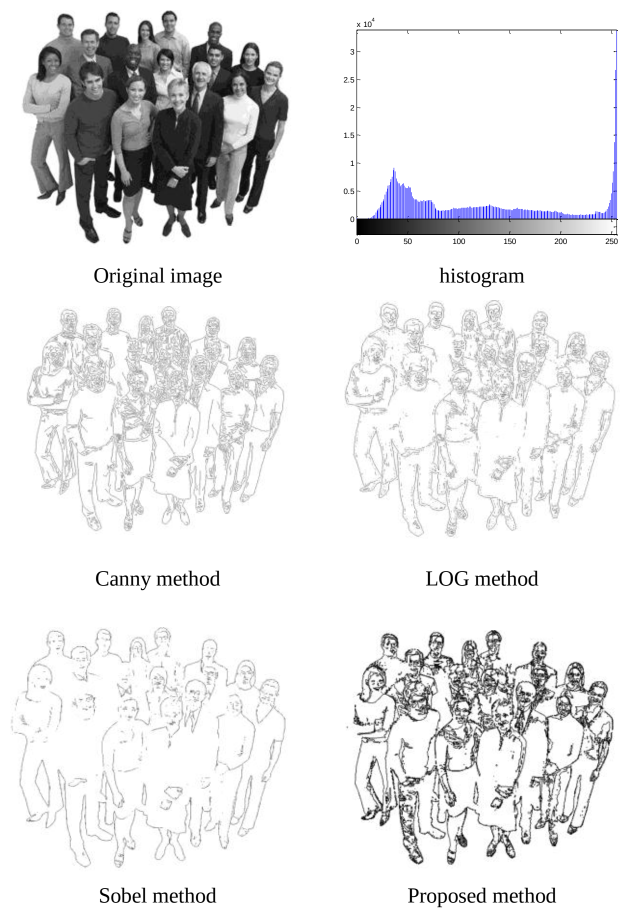


Figure 8. team image with 959×836 pixel

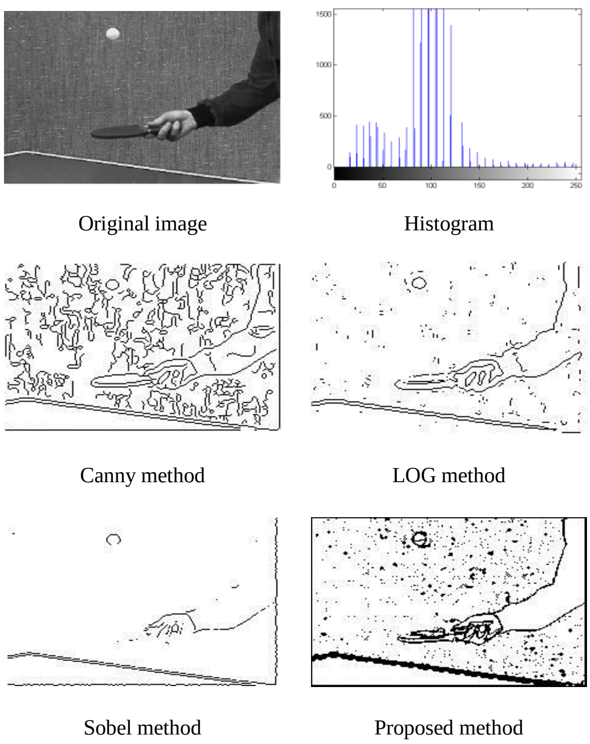


Figure 9. Sport image with 224×153 pixel .

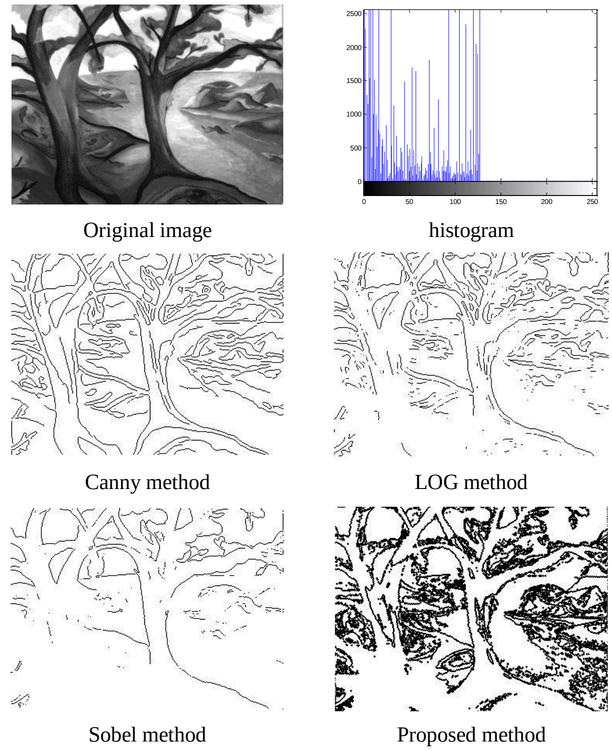


Figure 10. trees image with 350×258 pixel

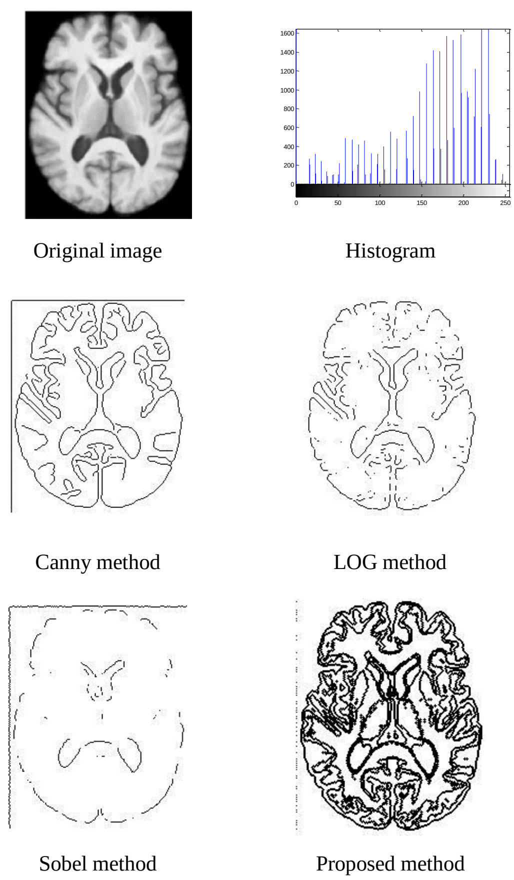


Figure 11. brain image with 180×224 pixel .

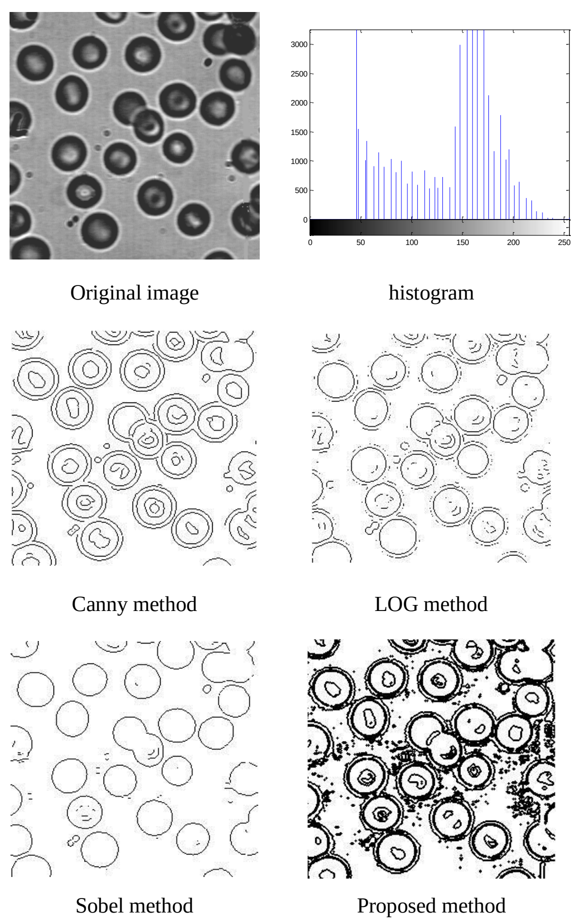


Figure 12. blood1 image with 272×238 pixel

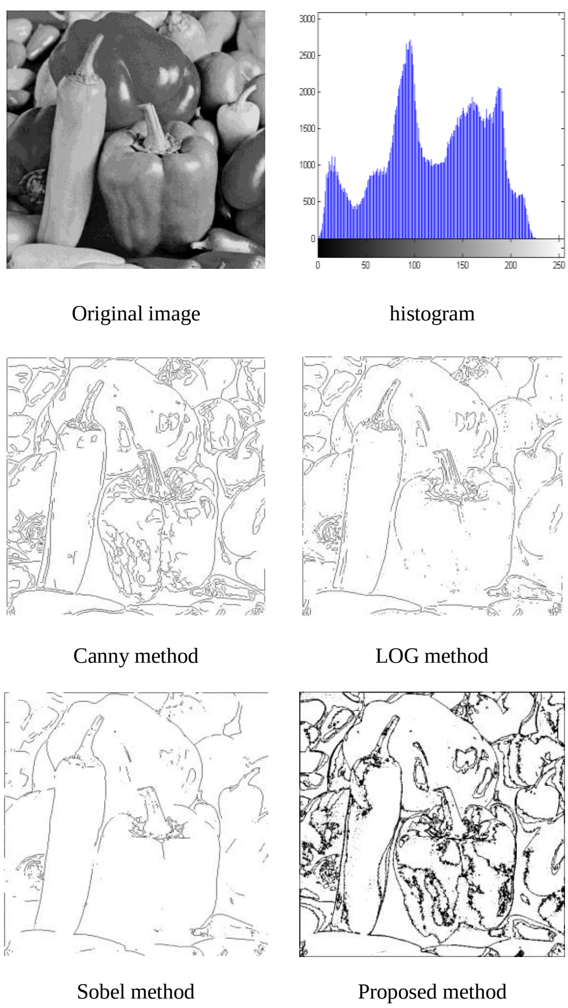


Figure 13. Peppers image with 512×512 pixel.

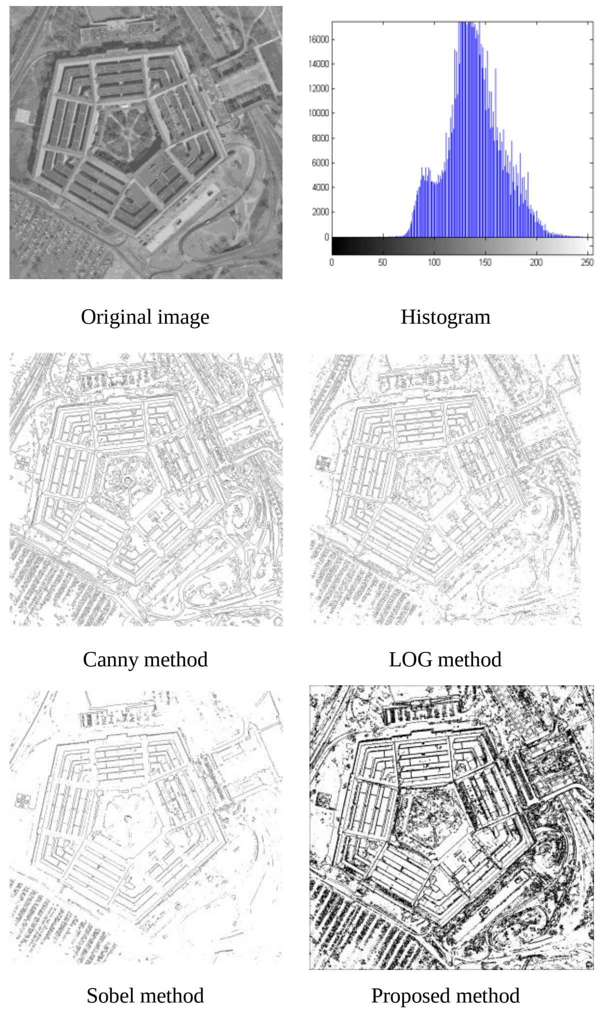


Figure 14. pentagon image with 1024×1024 pixel.

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