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Bicriteria in n x 2 Flow Shop Scheduling Problem under Specified Rental Policy, Processing Time, Setup Time Each Associated with Probabilities Including Job Block Criteria and Weightage of Jobs

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Abstract

This paper is an attempt to obtain an optimal solution for minimizing the bicriteria taken as minimization of the total rental cost of the machines subject to obtain the minimum makespan for n-jobs, 2-machine flow shop scheduling problem in which the processing times and independent set up times are associated with probabilities including job block criteria. Further jobs are attached with weights to indicate their relative importance. The proposed method is very simple and easy to understand and also provide an important tool for the decision makers. A computer programme followed by a numerical illustration is given to justify the algorithm.

Keywords: Flowshop Scheduling, Heuristic, Processing Time, Setup Time, Job Block, Weighs of jobs

1. Introduction

Scheduling is one of the optimization problems found in real industrial content for which several heuristic procedures have been successfully applied. Scheduling is a form of decision making that plays a crucial role in manufacturing and service industries. It deals with allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives. The majority of scheduling research assumes setup as negligible or part of processing time. While this assumption adversely affects solution quality for many applications which require explicit treatment of set up. Such applications, coupled with the emergence of product concept like time based competitions and group technology, have motivated increasing interest to include setup considerations in scheduling theory. A flow shop scheduling problems has been one of the classical problems in production scheduling since Johnson (1954) proposed the well known Johnson's rule in the two stage flow shop makespan scheduling problem. Smith (1967) considered minimization of mean flow time and maximum tardiness. Yoshida & Hitomi (1979) further considered the problem with setup times. The work was developed Sen & Gupta (1983), Chandasekharan (1992), Bagga & Bhambani (1997) and Gupta Deepak et al. (2011) by considering various parameters. Maggu & Das (1977) established an equivalent job-block theorem. The idea of job block has practical significance to create a balance between a cost of providing priority in service to the customer and cost of giving service with non priority. In the sense of providing relative importance in the process Chandermouli (2005) associated weight with the jobs. The algorithm which minimizes one criterion does not take into consideration the effect of other criteria. Thus, to reduce the scheduling cost significantly, the criteria like that of makespan and total flow time can be combined which leads to optimization of bicriteria.

Gupta & Sharma (2011) studied bicriteria in n × 2 flow shop scheduling under specified rental policy, processing time and setup time associated with probabilities including job block. This paper is an attempt to extend the study made by Gupta & Sharma (2011) by introducing the Weightage in jobs, Thus making the problem wider and more practical in process / production industry. We have obtained an algorithm which gives minimum possible rental cost while minimizing total elapsed time simultaneously.

2. Practical Situation

Many applied and experimental situations exist in our day-to-day working in factories and industrial production concerns etc. The practical situation may be taken in a paper mill, sugar factory and oil refinery etc. where various qualities of paper, sugar and oil are produced with relative importance i.e. weight in jobs, hence Weightage of jobs is significant. Various practical situations occur in real life when one has got the assignments but does not have one's own machine or does not have enough money or does not want to take risk of investing huge amount of money to purchase machine. Under such circumstances, the machine has to be taken on rent in order to complete the assignments. In his starting career, we find a medical practitioner does not buy expensive machines say X-ray machine, the Ultra Sound Machine, Rotating Triple Head Single Positron Emission Computed Tomography Scanner, Patient Monitoring Equipment, and Laboratory Equipment etc., but instead takes on rent. Rental of medical equipment is an affordable and quick solution for hospitals, nursing homes, physicians, which are presently constrained by the availability of limited funds due to the recent global economic recession. Renting enables saving working capital, gives option for having the equipment, and allows upgradation to new technology. Further the priority of one job over the other may be significant due to the relative importance of the jobs. It may be because of urgency or demand of that particular job. Hence, the job block criteria become important.

3. Notations

S : Sequence of jobs 1,2,3,….,n

S_k : Sequence obtained by applying Johnson's procedure, k = 1, 2 , 3, -------

M_j : Machine j, j= 1,2

M : Minimum makespan

a_{ij} : Processing time of i^{th} job on machine M_j

p_{ij} : Probability associated to the processing time a_{ij}

s_{ij} : Set up time of i^{th} job on machine M_j

q_{ij} : Probability associated to the set up time s_{ij}

A_{ij} : Expected processing time of i^{th} job on machine M_j

S_{ij} : Expected set up time of i^{th} job on machine M_j

A'_{ij} : Expected flow time of i^{th} job on machine M_j

w_i : weight of i^{th} job

A''_{ij} :Weighted flow time of i^{th} job on machine M_j

B : Equivalent job for job – block

$L_j(S_k)$: The latest time when machine M_j is taken on rent for sequence S_k

$t_{ij}(S_k)$: Completion time of i^{th} job of sequence S_k on machine M

t'_{ij} :Completion time of i^{th} job of sequence S_k on machine M_j when machine M_j start processing jobs at time $L_j(S_k)$

$I_{ij}(S_k)$: Idle time of machine M_j for job i in the sequence S_k

$U_j(S_k)$:Utilization time for which machine M_j is required, when M_j starts processing jobs at time $E_j(S_k)$

$R(S_k)$: Total rental cost for the sequence S_k of all machine

C_i : Rental cost of i^{th} machine

3.1 Definition

Completion time of i^{th} job on machine M_j is denoted by t_{ij} and is defined as :

$$t_{ij} = \max (t_{i-1,j} + s_{(i-1)j} \times q_{(i-1)j} , t_{i,j-1}) + a_{ij} \times p_{ij} \quad \text{for} \quad j \geq 2.$$

$$= \max (t_{i-1,j} + S_{(i-1),j} , t_{i,j-1}) + A_{i,j}$$

where $A_{i,j}$= Expected processing time of i^{th} job on j^{th} machine

$S_{i,j}$= Expected setup time of i^{th} job on j^{th} machine.

3.2 Definition

Completion time of i^{th} job on machine M_j starts processing jobs at time L_j is denoted by t'_{ij} and is defined as

$$t'_{i,j} = L_j + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j} = \sum_{k=1}^{i} I_{k,j} + \sum_{k=1}^{i} A_{k,j} + \sum_{k=1}^{i-1} S_{k,j}$$

Also $t'_{i,j} = \max(t'_{i,j-1}, t'_{i-1,j} + S_{i-1,j}) + A_{i,j}$.

4. Rental Policy

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required. .i.e. the first machine will be taken on rent in the starting of the processing the jobs, 2^{nd} machine will be taken on rent at time when 1^{st} job is completed on 1^{st} machine.

5. Problem Formulation

Let some job i (i = 1,2,……..,n) are to be processed on two machines M_j (j = 1,2) under the specified rental policy P. Let a_{ij} be the processing time of i^{th} job on j^{th} machine with probabilities p_{ij} and s_{ij} be the setup time of i^{th} job on j^{th} machine with probabilities q_{ij}. Let w_i be the weight of i^{th} job. Let A_{ij} be the expected processing time and $S_{i,j}$ be the expected setup time of i^{th} job on j^{th} machine. Our aim is to find the sequence $\{S_k\}$ of the jobs which minimize the rental cost of the machines while minimizing total elapsed time.

The mathematical model of the problem in matrix form can be stated as:

Jobs	Machine M_1				Machine M_2				Weight of job
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	w_i
1	a_{11}	p_{11}	s_{11}	q_{11}	a_{12}	p_{12}	s_{12}	q_{12}	w_1
2	a_{21}	p_{21}	s_{21}	q_{21}	a_{22}	p_{22}	s_{22}	q_{22}	w_2
3	a_{31}	p_{31}	s_{31}	q_{31}	a_{32}	p_{32}	s_{32}	q_{32}	w_3
4	a_{41}	p_{41}	s_{41}	q_{41}	a_{42}	p_{42}	s_{42}	q_{42}	w_4
5	a_{51}	p_{51}	s_{51}	q_{51}	a_{52}	p_{52}	s_{52}	q_{52}	w_5

Table 1

Mathematically, the problem is stated as

Minimize $U_j(S_k)$ and

Minimize $R(S_k) = \sum_{i=1}^{n} A_{i1} \times C_1 + U_j(S_k) \times C_2$

Subject to constraint: Rental Policy (P)

Our objective is to minimize rental cost of machines while minimizing total elapsed time.

6. Theorem

The processing of jobs on M_2 at time $L_2 = \sum_{i=1}^{n} I_{i,2}$ keeps $t_{n,2}$ unaltered:

Proof. Let $t'_{i,2}$ be the completion time of i th job on machine M_2 when M_2 starts processing of jobs at L_2. We shall prove the theorem with the help of mathematical induction.

Let P(n) : $t'_{n,2} = t_{n,2}$

Basic step: For n = 1, j =2;

$$t'_{1,2} = L_2 + \sum_{k=1}^{1} A_{k,2} + \sum_{k=1}^{1-1} S_{k,2} = \sum_{k=1}^{1} I_{k,2} + \sum_{k=1}^{1} A_{k,2} + \sum_{k=1}^{1-1} S_{k,2}$$

$$= \sum_{k=1}^{1} I_{k,2} + A_{1,2} = I_{1,2} + A_{1,2} = A_{1,1} + A_{1,2} \quad = t_{1,2} \,,$$

$\therefore$ P(1) is true.

Induction Step: Let P(m) be true, i.e., $t'_{m,2} = t_{m,2}$

Now we shall show that P(m+1) is also true, i.e., $t'_{m+1,2} = t_{m+1,2}$

Since $t'_{m+1,2} = \max(t_{m+1,1}, t'_{m,2} + S_{m,2}) + A_{m+1,2}$

$$= \max\left(t_{m+1,1}, t_{m,2} + S_{m,2}\right) + A_{m+1,2} \qquad \text{(By Assumption)}$$

$$= t_{m+1,2}$$

Therefore, *P(m+1)* is true whenever *P(m)* is true.

Hence by Principle of Mathematical Induction P(n) is true for all n i.e. $t'_{n,2} = t_{n,2}$ for all n.

***Remark*:** If M_2 starts processing the job at $L_2 = t_{n,2} - \sum_{i=1}^{n} A_{i,2} - \sum_{i=1}^{n-1} S_{i,2}$, then total time elapsed $t_{n,2}$ is not altered and M_2 is engaged for minimum time. If M_2 starts processing the jobs at time L_2 then it can be easily shown that. $t_{n,2} = L_2 + \sum_{i=1}^{n} A_{i,2} + \sum_{i=1}^{n-1} S_{i,2}$.

7. Algorithm

Step 1: Calculate the expected processing times and expected set up times as follows

$A_{ij} = a_{ij} \times p_{ij}$ and $S_{ij} = s_{ij} \times q_{ij}$ $\forall i, j$

Step 2: Calculate the expected flow time for the two machines M1and M_2 as follows

$$A'_{i1} = A_{i1} - S_{i2} \quad \text{and} \quad A'_{i2} = A_{i2} - S_{i1} \ \forall i$$

Step 3: If min (A'_{i1}, A'_{i2})= A'_{i1} , then $G_i = A'_{i1} + w_i$, $H_i = A'_{i2}$ and

If min (A'_{i1}, A'_{i2})= A'_{i2} , then $H_i = A'_{i2} + w_i$, $G_i = A'_{i2}$.

Step 4: Find the weighted flow time for two machine M1 and M_2 as follows

$$A''_{i1} = G_i / w_i \quad \text{and} \quad A''_{i2} = H_i / w_i \quad \forall i$$

Step 5: Take equivalent job $\beta(k,m)$ and calculate the processing time $A''_{\beta 2}$ and $A''_{\beta 2}$ on the guide lines of Maggu and Das [6] as follows

$$A''_{\beta 1} = A''_{k1} + A''_{m1} - \min(A''_{m1}, A''_{k2})$$
$$A''_{\beta 2} = A''_{k2} + A''_{m2} - \min(A''_{m1}, A''_{k2})$$

Step 6: Define a new reduced problem with the processing times A''_{i1} and A''_{i2} as defined in step 3 and jobs (k,m) are replaced by single equivalent job β with processing time $A''_{\beta 1}$ and $A''_{\beta 2}$ as defined in step 4.

Step 7: Using Johnson's technique [1] obtain all the sequences S_k having minimum elapsed time. Let these be S_1, S_2, ----------.

Step 8 : Compute total elapsed time $t_{n2}(S_k)$, $k = 1,2,3$,----, by preparing in-out tables for S_k.

Step 9 : Compute $L_2(S_k)$ for each sequence S_k as follows

$$L_2(S_k) = t_{n,2}(S_k) - \sum_{i=1}^{n} A_{i,2}(S_k) - \sum_{i=1}^{n-1} S_{i,2}(S_k)$$

Step 10 : Find utilization time of 2nd machine for each sequence S_k as $U_2(S_k) = t_{n2}(S_k) - L_2(S_k)$.

Step 11 : Find minimum of $\{(U_2(S_k)\}$; $k = 1,2,3,\ldots$

Let it for sequence S_p. Then S_p is the optimal sequence and minimum rental cost for the sequence S_p is

$$R(S_p) = t_{n,1}(S) \times C_1 + U_2(S_p) \times C_2.$$

8. Programme

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
#include<process.h>
int n,j;
float a1[16],b1[16],g[16],h[16],g1[16],h1[16],g12[16],h12[16],sa1[16],sb1[16];
```

```
float macha[16],machb[16],cost_a,cost_b,cost;
int f=1;
int group[16];//variables to store two job blocks
float minval,minv,maxv;
float gbeta=0.0,hbeta=0.0;
void main()
{
	clrscr();
	int a[16],b[16],sa[16],sb[16],j[16],w[16];
	float p[16],q[16],u[16],v[16];float maxv;
	cout<<"How many Jobs (<=15) : ";cin>>n;
if(n<1 || n>15)
	{cout<<endl<<"Wrong input, No. of jobs should be less than 15..\n Exitting";
	getch();exit(0);}
for(int i=1;i<=n;i++)
	{j[i]=i;
	cout<<"\nEnter the processing time and its probability, Setup time and its probability of "<<i<<" job for machine A : ";
	cin>>a[i]>>p[i]>>sa[i]>>u[i];
	cout<<"\nEnter the processing time and its probability, Setup time and its probability of "<<i<<" job for machine B : ";
	cin>>b[i]>>q[i]>>sb[i]>>v[i];
	cout<<"\nEnter the weightage of "<<i<<"job:";cin>>w[i];
	//Calculate the expected processing times of the jobs for the machines:
	a1[i] = a[i]*p[i];b1[i] = b[i]*q[i];
	//Calculate the expected setup times of the jobs for the machines:
	sa1[i] = sa[i]*u[i];sb1[i] = sb[i]*v[i];}
	cout<<"\nEnter the rental cost of Machine A:";cin>>cost_a;
	cout<<"\nEnter the rental cost of Machine B:";cin>>cost_b;
	cout<<endl<<"Expected processing time of machine A and B: \n";
for(i=1;i<=n;i++)
{cout<<j[i]<<"\t"<<a1[i]<<"\t"<<b1[i]<<"\t";cout<<sa1[i]<<"\t"<<sb1[i]<<"\t"<<w[i];
	cout<<endl;}
	//Calculate the final expected processing time for machines
	cout<<endl<<"Final expected processing time of machin A and B:\n";
for(i=1;i<=n;i++)
	{g1[i]=a1[i]-sb1[i];h1[i]=b1[i]-sa1[i];}
for(i=1;i<=n;i++)
```

```
    {if(g1[i]<h1[i])
    {g12[i]=g1[i]+w[i];h12[i]=h1[i];}
else
    {h12[i]=h1[i]+w[i];g12[i]=g1[i];}}
for(i=1;i<=n;i++)
    {g[i]=g12[i]/w[i];h[i]=h12[i]/w[i];}
for(i=1;i<=n;i++)
    {cout<<"\n\n"<<j[i]<<"\t"<<g[i]<<"\t"<<h[i]<<"\t"<<w[i];cout<<endl;}
    cout<<"\nEnter the two job blocks(two numbers from 1 to "<<n<<"):";
    cin>>group[0]>>group[1];
  //calculate G_Beta and H_Beta
if(g[group[1]]<h[group[0]])
    {minv=g[group[1]];}
else
    {minv=h[group[0]];}
    gbeta=g[group[0]]+g[group[1]]-minv;hbeta=h[group[0]]+h[group[1]]-minv;
    cout<<endl<<endl<<"G_Beta="<<gbeta;cout<<endl<<"H_Beta="<<hbeta;
int j1[16];float g11[16],h11[16];
for(i=1;i<=n;i++)
    {if(j[i]==group[0]||j[i]==group[1])
    {f--;}
else
    {j1[f]=j[i];}f++;}
    j1[n-1]=17;
for(i=1;i<=n-2;i++)
    {g11[i]=g[j1[i]];h11[i]=h[j1[i]];}
    g11[n-1]=gbeta;h11[n-1]=hbeta;
    cout<<endl<<endl<<"displaying original scheduling table"<<endl;
for(i=1;i<=n-1;i++)
    {cout<<j1[i]<<"\t"<<g11[i]<<"\t"<<h11[i]<<endl;}
    float mingh[16];char ch[16];
for(i=1;i<=n-1;i++)
    {if(g11[i]<h11[i])
    {mingh[i]=g11[i];ch[i]='g';}
else
    {mingh[i]=h11[i];ch[i]='h';}}
for(i=1;i<=n-1;i++)
    {
```

```
    for(int j=1;j<=n-1;j++)
    if(mingh[i]<mingh[j])
        {float temp=mingh[i]; int temp1=j1[i]; char d=ch[i];
        mingh[i]=mingh[j]; j1[i]=j1[j]; ch[i]=ch[j];
        mingh[j]=temp; j1[j]=temp1; ch[j]=d;} }
    // calculate beta scheduling
    float sbeta[16];int t=1,s=0;
for(i=1;i<=n-1;i++)
    {if(ch[i]=='h')
    { sbeta[(n-s-1)]=j1[i]; s++;}
else if(ch[i]=='g')
    {sbeta[t]=j1[i];t++;}}
    int arr1[16], m=1;
    cout<<endl<<endl<<"Job Scheduling:"<<"\t";
for(i=1;i<=n-1;i++)
    {if(sbeta[i]==17)
    { arr1[m]=group[0]; arr1[m+1]=group[1];
    cout<<group[0]<<" " <<group[1]<<" ";m=m+2;continue;}
else
    {cout<<sbeta[i]<<" ";arr1[m]=sbeta[i];m++;}}
//calculating total computation sequence
    float time=0.0,macha1[15],machb1[15];macha[1]=time+a1[arr1[1]];
    for(i=2;i<=n;i++)
    {macha1[i]=macha[i-1]+sa1[arr1[i-1]];
    macha[i]=macha[i-1]+sa1[arr1[i-1]]+a1[arr1[i]];}
    machb[1]=macha[1]+b1[arr1[1]];
//displaying solution
cout<<"\n\n\n\n\n\t\t\t       #####THE SOLUTION##### ";
cout<<"\n\n\t***************************************************************";
cout<<"\n\n\n\t       Optimal Sequence is : ";
for(i=1;i<=n;i++)
cout<<" "<<arr1[i];
cout<<endl<<endl<<"In-Out Table is:"<<endl<<endl;
cout<<"Jobs"<<"\t"<<"Machine M1"<<"\t"<<"\t"<<"Machine M2"<<endl;
cout<<arr1[1]<<"\t"<<time<<"--"<<macha[1]<<"                    \t"<<"\t"<<macha[1]<<"--"<<machb[1]<<"
\t"<<"\t"<<endl;
for(i=2;i<=n;i++)
    {if((machb[i-1]+sb1[arr1[i-1]])>macha[i])
```

```
maxv=(machb[i-1]+sb1[arr1[i-1]]);
else
    maxv=macha[i];machb[i]=maxv+b1[arr1[i]];
    cout<<arr1[i]<<"\t"<<macha1[i]<<"--"<<macha[i]<<" "<<"\t"<<maxv<<"--"<<machb[i]<<endl;}
cout<<"\n\n\nTotal Elapsed Time (T) = "<<machb[n];cout<<endl<<endl<<"Machine A:";
for(i=1;i<=n;i++)
    {cout<<endl<<"Job "<<i<<" Computation Time"<<macha[i];}cout<<endl<<endl<<"Machine B:";
    for(i=1;i<=n;i++)
    {cout<<endl<<"Job "<<i<<" Computation Time"<<machb[i];}
float L2,L_2,min,u2;float sum1=0.0,sum2=0.0;
for(i=1;i<=n;i++)
    {sum1=sum1+a1[i];sum2=sum2+b1[i];}cout<<"\nsum1="<<sum1;L2=machb[n];
float sum_2,sum_3;arr1[0]=0,sb1[0]=0;
for(i=1;i<=n;i++)
    {sum_2=0.0,sum_3=0.0;
for(int j=1;j<=i;j++)
    {sum_3=sum_3+sb1[arr1[j-1]];}
for(int k=1;k<=i;k++)
    {sum_2=sum_2+b1[arr1[k]];}}
    cout<<"\nsum_2="<<sum_2;cout<<"\nsum_3="<<sum_3;L_2=L2-sum_2-sum_3;
    cout<<"\nLatest time for which B is taken on Rent="<<"\t"<<L_2;u2=machb[n]-L_2;
    cout<<"\n\nUtilization Time of Machine M2="<<u2;
    cost=(macha[n]*cost_a)+(u2*cost_b);
    cout<<"\n\nThe Minimum Possible Rental Cost is="<<cost;
cout<<"\n\n\t***************************************************************";
getch();
}
```

9. Numerical Illustration

Consider 5 jobs, 2 machine flow shop problem with weights of jobs, processing time and setup time associated with their respective probabilities as given in the following table and jobs 2, 5 are to be processed as a group job (2,5). The rental cost per unit time for machines M_1 and M_2 are 6 units and 7 units respectively. Our objective is to obtain optimal schedule to minimize the total production time / total elapsed time subject to minimization of the total rental cost of the machines, under the rental policy P.

Job	Machine M_1				Machine M_2				Weight of job
i	a_{i1}	p_{i1}	s_{i1}	q_{i1}	a_{i2}	p_{i2}	s_{i2}	q_{i2}	w_i
1	11	0.3	4	0.3	10	0.2	4	0.1	2

2	12	0.1	6	0.2	13	0.1	3	0.2	3
3	13	0.2	7	0.1	16	0.1	6	0.3	4
4	15	0.1	4	0.3	8	0.3	5	0.1	6
5	14	0.3	7	0.1	6	0.3	3	0.2	5

Table 2

Solution: As per step 1: Expected processing and setup times for machines M_1 and M_2 are as shown in table 3.

As per step 2: The expected flow times for the machines M_1 and M_2 are as shown in table 4.

As per step 3 : The weighted flow time for two machines M_1 and M_2 are as shown in table 5.

As per step 4: Here β = (2,5)

$A''_{\beta 1}=$ **0.2 + 0.72 – 0.72 = 0.2,** $A''_{\beta 2}$ **= 1.03 + 1.22 – 0.72 = 1.5.**

As per step 6 : Using Johnson's method optimal sequence is

S = β – 1 – 3 – 4 i.e.2–5–1–3– 4

As per step 7: The In-Out table for the sequence S is as shown in table 6.

Total elapsed time $t_{n2}(S_1)$ = 20.2 units

As per Step 8: The latest time at which Machine M_2 is taken on rent

$L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{i,2}(S)$ =20.2 – 9.1 – 3.4 = 7.7 units

As per step 9: The utilization time of Machine M_2 is

$U_2(S) = t_{n2}(S) - L_2(S)$ = 20.2 – 7.7 = 12.5 units

The Biobjective In – Out table is as shown in table 7.

Total Minimum Rental Cost = $R(S) = t_{n,1}(S)\times C_1 + U_2(S)\times C_2$ =16.6 × 6 + 12.5 ×7=187.1 units.

10. Conclusion

If the machine M_2 is taken on rent when it is required and is returned as soon as it completes the last job, the starting of processing of jobs at time $L_2(S) = t_{n,2}(S) - \sum_{i=1}^{n} A_{i,2}(S) - \sum_{i=1}^{n-1} S_{i,2}(S)$ on M_2 will, reduce the idle time of all jobs on it. Therefore total rental cost of M_2 will be minimum. Also rental cost of M_1 will always be minimum as idle time of M_1 is always zero. The study may further be extending by introducing the concept of transportation time, Breakdown Interval etc.

References

Bagga, P.C.(1969), "Sequencing in a rental situation", *Journal of Candian Operation Research Society* 7, 152-153.

Bagga, P.C.& Bhambani, A.(1997), "Bicriteria in flow shop scheduling problem", *Journal of Combinatorics, Information and System Sciences* 22, 63-83.

Chandrasekharan Rajendran (1992), "Two Stage flow shop scheduling problem with bicriteria", *Operational Res. Soc*, 43(9), 871-884.

Chakarvarthy K. & Rajendrah, C.(1999), "A heuristic for scheduling in a flow shop with bicriteria of makespan and maximum tardiness minimization", *Production Planning & Control*, 10 (7), 707-714.

Chandramouli, A.B. (2005),"Heuristic Approach for n-job,3-machine flow shop scheduling problem involving transportation time, breakdown interval and weights of jobs", M*athematical and Computational Applications*, 10(2), 301-305.

Gupta, D., Singh, T.P. & Kumar, R.(2007), "Bicriteria in scheduling under specified rental policy, processing time associated with probabilities including job block concept", *Proceedings of VIII Annual Conference of Indian Society of Information Theory and Application (ISITA),* 22-28.

Gupta, D.& Sharma, S.(2011), "Minimizing rental cost under specified rental policy in two stage flow shop, the processing time associated with probabilities including break-down interval and job – block criteria" , *European Journal of Business and Management* 3(2), 85-103.

Gupta, D., Sharma, S., Seema and Shefali (2011), "Bicriteria in n × 2 flow shop scheduling under specified rental policy ,processing time and setup time each associated with probabilities including job-block", Industrial Engineering Letters, 1(1), 1 -12.

Johnson, S.M.(1954), "Optimal two and three stage production schedule with set up times included", *Naval Research Logistics Quart.* 1(1), 61-68.

Maggu P.L. and Das G., "Equivalent jobs for job block in job scheduling", Opsearch, Vol 14, No.4, (1977), 277-281.

Narian,L. & Bagga, P.C.(1998), "Minimizing hiring cost of machines in n x 3flow shop problem", XXXI Annual ORSI Convention and International Conference on Operation Research and Industry, Agra[India].

Narain, L.(2006) , "Special models in flow shop sequencing problem", *Ph.D. Thesis, University of Delhi*, Delhi.

Sen, T. & Gupta, S.K.(1983),"A branch and bound procedure to solve a bicriteria scheduling problem", *AIIE Trans., 15*, 84-88.

Singh, T.P., Kumar, R. & Gupta, D.(2005) , "Optimal three stage production schedule, the processing and set up times associated with probabilities including job block criteria", *Proceedings of the national Conference on FACM,(2005)*, 463-470.

Smith, W.E.(1956), "Various optimizers for single stage production", *Naval Research Logistics* 3 , 59-66.

Smith, R.D.& Dudek, R.A.(1967) "A general algorithm for solution of the N-job, M-machine scheduling problem", *Operations Research*15(1) , 71-82.

Szwarc W., "A note on mathematical aspects of n × 3 job-shop sequence problem", Operational Research,Vol 25,(1974),70-77.

Van, L.N., Wassenhove & Gelders, L.F. (1980), "Solving a bicriteria scheduling problem", *AIIE Tran 15s.*, 84-88.

Yoshida and Hitomi (1979), Optimal two stage production scheduling with set up times separated. AIIE Transactions, Vol . II, 261-263

Tables

Table 3: The expected processing and setup times for machines M_1 and M_2 are

Job	Machine M_1		Machine M_2		Weight of job
i	A_{i1}	S_{i1}	A_{i2}	S_{i2}	w_i
1	3.3	1.2	2	0.4	2
2	1.2	1.2	1.3	0.6	3
3	2.6	0.7	1.6	1.8	4
4	1.5	1.2	2.4	0.5	6
5	4.2	0.7	1.8	0.6	5

Table 4: The expected flow times for the machines M_1 and M_2 are

Job	Machine M_1	Machine M_2	Weight of job
i	A'_{i1}	A'_{i2}	w_i
1	2.9	0.8	2
2	0.6	0.1	3
3	0.8	0.9	4
4	1.0	1.2	6
5	3.6	1.1	5

Table 5: The weighted flow time for two machines M_1 and M_2 are

Job	Machine M_1	Machine M_2
i	A''_{i1}	A''_{i2}
1	1.45	1.4
2	0.2	1.03
3	1.2	0.225
4	1.16	0.2
5	0.72	1.22

Table 6: The In-Out table for the sequence S is

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
2	0 – 1.2	1.2 – 2.5
5	2.4 – 6.6	6.6 – 8.4
1	7.3 – 10.6	10.6 – 12.6
3	11.8 – 14.4	14.4 – 16.0
4	15.1 – 16.6	17.8 – 20.2

Table 7: The Biobjective In – Out table is

Jobs	Machine M_1	Machine M_2
i	In - Out	In - Out
2	0 – 1.2	7.7– 9.0
5	2.4 – 6.6	9.6 – 11.4
1	7.3 – 10.6	12.0 – 14.0
3	11.8 – 14.4	14.4 – 16.0
4	15.1 – 16.6	17.8 – 20.2