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Determination of Natural Frequencies of a Cantilevered Discontinous Beam with Aligned Neutral Axes: Receptance-Graphical Approach

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Abstract
This work presents a graphical approach to find the natural frequencies of a fixed-free discontinuous Beams with aligned neutral axes based on different boundary condition. A receptance model is introduced to analyze the problem of discontinuous beams, using the differential equation of motion based on the Euler-Bernoulli beam equation. The natural frequencies equation are obtained from the receptance model. The graphical methods gives the approximate solution, but need a lot of effort on calculation. The solution obtained are compared to the result from experimental model testing.
Keywords: Receptance function, discontinuous Beam, Graphical method, natural frequency.

1. Introduction

Discontinuous beams are stepped beams with a break in continuous direction or charge in cross-sectional area along continuous direction. These beam are structures containing joints, connections, notches and are frequently used in aeronautical, mechanical and civil engineering. Discontinuous beam-like structures are widely used in various engineering fields, such as Robot arms, helicopter rotor blades, spacecraft antennae, airplane wings, and tall building etc. The study of natural frequencies of a beam-like structures helps in monitoring structural failures. Studies on the natural frequencies of discontinuous beams are carried out by a number of researchers. Jang and Bert (1989) formulated the exact and numerical solution for natural frequencies of discontinuous beams for various boundary conditions. De Rosa (1994) carried out a study for the vibration of a stepped beam with elastic end supports. Naguleswaran (2002) extended De Rosa work and presented an analytical approach to obtained the frequencies of beams on elastic supports with three step changes in cross-section. Wang (1991) presented the vibration of stepped beam on elastic foundations. Jawarski and Dowell (2008) studied the free vibration of a cantilevered beam with multiple steps and compared the results of the proposed analytical solutions with experiment. Lee and Bergman (1994) used Green's function to find the solutions of the free vibration problem of stepped beams and rectangular plates. Kulda and Zamojska (2007) presented the solution of frequency analysis of axially loaded discontinuous beams by Green's function method. Lu et al (2009) analyze the free and forced vibration of stepped beams using composite element method and theoretical results compared with experimental results. Mao and Pietrozico (2010) investigated the vibrations of a two-stepped beams, by different boundary conditions using Adomain decomposition method. Luay and Al-Arjari 2012) used three models Rayleigh model, modified Rayleigh model and finite elements model) to calculate the natural frequency of cantilever stepping beam compound from two parts. Mostly recently, Koplow (2005) formulate the receptance model of free- free discontinuous beam with aligned neutral axes using Euler – Bernoulli beam theory but provided no numerical solution of Natural Frequencies from the frequency equation. The analytical models are compared to receptance coupling substructure analysis and experiment. In this paper a graphical numerical approach is been used for determining the Natural Frequencies of a cantilevered discontinuous beam and the results are compared with experimental modal testing.

2. Problem Formulation

The Euler – Bernoulli equation for a transverse vibration of uniform beam is

$$\frac{\partial^4 \mathrm{x}\ (x)}{\partial \times^4} - \lambda^4 \times (x) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$where \quad \lambda^4 = \frac{w^2 \ell A}{EI} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where E is young's modulus, I is the second moment of area of the beam cross – section, ℓ is the mass density and A is the cross- sectional area of uniform beam.
Hence, the general mode shape solution $\times (x)$ is

$$\times (x) = Asin\lambda x + Bcos\ \lambda x + Csinh\lambda x + Dcos\ h\lambda x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Figure 1 shows the geometry of the beam under study. The material consists of aluminum alloy. From the properties of material manual, the mass density (ℓ) and young's modulus (E) of the aluminum beams are $2.7\times 10^3 kg/m^3$ and $7.1\times 10^{10} N/m^2$ respectively.

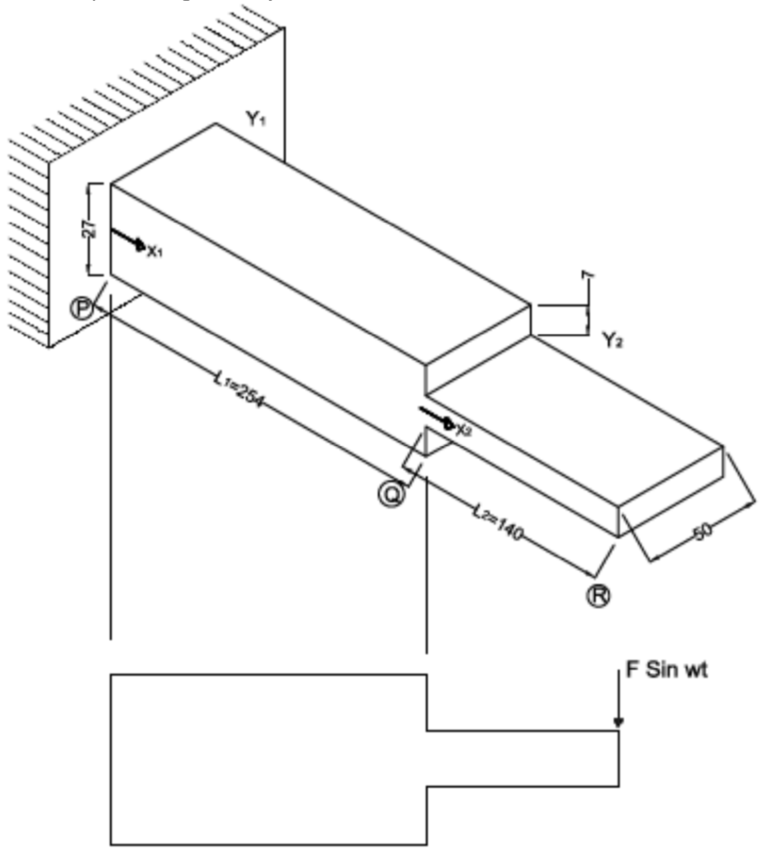

***Fig 1:** Beam dimensions of aligned neutral axes under stud and applied force at free end.*

From figure 1, the solution for the next section (P-Q) is given by

$$x_1(x_1) = a_1 \sin \lambda_1 x_1 + a_2 \cos \lambda_1 x_1 + a_3 \sin h\lambda_1 x_1 + a_4 \cos h\lambda_1 x_1 \quad \ldots\ldots (4)$$

Where $\lambda_1 = \frac{w^2 \ell_1 A_1}{E_1 I_1}$

At fixed end (point P), $y_1(0) = \frac{\partial y_1(0)}{\partial x_1} = 0$

Where y is beam deflection

So, $a_4 = -a_2 \; and \; a_3 = -a_1$

Hence, $X_1(x_1) = a_1(\sin \lambda_1 x_1 - \cos h\lambda_1 x_1) + a_2(\cos \lambda_1 x_1 - \cos h\lambda_1 x_1)$ ………………. (5)

The solution for the second beam section (Q – R) is given by

$$X_2(x_2) = a_5 \sin \lambda_2 x_2 + a_6 \cos \lambda_2 x_2 + a_7 \sin h\lambda_2 x_2 + a_8 \cos h\lambda_2 x_2 \quad \ldots\ldots (6)$$

Where $\lambda_2 = \frac{w^2 \ell_2 A_2}{E_2 I_2}$

At continuity (point Q)

$$y_1(L_1) = y_2(0) \; (deflection) \ldots\ldots (7a)$$

$$\frac{\partial y_1(L_1)}{\partial x_1} = \frac{\partial y_2(0)}{\partial x_2} (slope) \ldots\ldots (7b)$$

$$E_1 I_1 \frac{\partial^2 y_1(L_1)}{\partial {x_1}^2} = E_2 I_2 \frac{\partial^2 y_2(0)}{\partial x_2} (bending\ moment) \ldots\ldots (7c)$$

$$E_1 I_1 \frac{\partial^3 y_1(L_1)}{\partial {x_1}^3} = E_2 I_2 \frac{\partial^3 y_2(0)}{\partial {x_2}^3} (shear\ force) \ldots\ldots (7d)$$

Putting equation (7) into Equations (4) and (5)

$$a_5 = a_1 N_1 + a_2 S_2 \ldots\ldots (8a)$$

$$a_6 = a_1 N_1 + a_2 S_2 \ldots\ldots (8b)$$

$$a_7 = a_1 N_3 + a_2 S_3 \ldots\ldots (8c)$$

$$a_8 = a_1N_4 + a_2S_4 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8d)$$

Where

$$N_1 = \frac{H_4}{2\lambda_{21}} + \frac{H_3}{2{\lambda_{21}}^2} \ldots\ldots\ldots\ldots\ldots (9a)$$

$$N_2 = \frac{H_2}{2} + \frac{H_1}{2{\lambda_{21}}^2 I_{21}} \ldots\ldots\ldots\ldots\ldots (9b)$$

$$N_3 = \frac{H_4}{2\lambda_{21}} - \frac{H_3}{2{\lambda_{21}}^3 I_{21}} \ldots\ldots\ldots\ldots\ldots (9c)$$

$$N_4 = \frac{H_2}{2} - \frac{H_3}{2{\lambda_{21}}^3 I_{21}} \ldots\ldots\ldots\ldots\ldots (9d)$$

$$S_1 = \frac{-H_1}{2\lambda_{21}} - \frac{H_2}{2{\lambda_{21}}^3 I_{21}} \ldots\ldots\ldots\ldots\ldots (9e)$$

$$S_2 = \frac{H_4}{2} - \frac{H_3}{2{\lambda_{21}}^3 I_{21}} \ldots\ldots\ldots\ldots\ldots (9f)$$

$$S_3 = \frac{-H_1}{2\lambda_{21}} - \frac{H_2}{2{\lambda_{21}}^3 I_{21}} \ldots\ldots\ldots\ldots\ldots (9g)$$

$$S_4 = \frac{H_4}{2} - \frac{H_3}{2{\lambda_{21}}^3 I_{21}} \ldots\ldots\ldots\ldots\ldots (9h)$$

And

$$H_1 = \sin\lambda_1 L_1 + \sin h\lambda_1 L_1 \ldots\ldots\ldots\ldots (10a)$$

$$H_2 = \sin\lambda_1 L_1 - \sin h\lambda_1 L_1 \ldots\ldots\ldots\ldots (10b)$$

$$H_3 = \cos\lambda_1 L_1 + \cos h\lambda_1 L_1 \ldots\ldots\ldots\ldots (10c)$$

$$H_4 = \cos\lambda_1 L_1 - \cos h\lambda_1 L_1 \ldots\ldots\ldots\ldots (10d)$$

Where $I_{21} = \frac{E_1 I_2}{E_1 I_1}$ *and* $\lambda_{21} = \frac{\lambda_2}{\lambda_1}$

Substituting Eau (8) into eau (6)

$$X_2(X_2) = a_1(N_1\sin\lambda_2x_2 + N_2\cos\lambda_2x_2 + N_3\sin h\lambda_2x_2 + N_4\cos h\lambda_2x_2) + a_2(S_1\sin\lambda_2x_2 + S_2\cos\lambda_2x_2 + S_3\sin h\lambda_2x_2 + S_4\cos h\lambda_2x_2)$$

……………… (11)

Finally, at free end (point R)

$$\frac{\partial^2 y_2(L_2)}{\partial x_2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

$$E_2I_2\frac{\partial^3 y_2(0)}{\partial {x_2}^3} = -F\,Sinwt \ldots\ldots\ldots\ldots (7c)$$

Substituting Eau (12) into Equ (11)

$$a_1 = \frac{K_1F}{E_2I_2(K_1K_4 - K_2K_3)} \ldots\ldots\ldots\ldots\ldots (13a)$$

$$a_2 = \frac{-F}{E_2I_2(K_1K_4 - K_2K_3)} \ldots\ldots\ldots\ldots\ldots (13b)$$

Therefore, putting Equ (13) into equation (11) yields the frequency response (Receptance function) solution;

$$\frac{y}{F} = \frac{I}{E_2I_2(K_1K_4 - K_2K_3)}\begin{bmatrix} K_2(N_1\sin\lambda_2x_2 + N_2\cos h\lambda_2x_2 + N_3\sin h\lambda_2x_2 + N_4\cos h\lambda_2x_2 \\ -K_1(S_1\sin\lambda_2x_2 + S_2\cos\lambda_2x_2 + S_3\sin h\lambda_2x_2 + S_4\cos h\lambda_2x_2) \end{bmatrix}$$

……………… (14)

Where;

$$K_1 = \lambda_2^2(-N_1\sin\lambda_2L_2 - N_2\cos h\lambda_2L_2 + N_3\sin h\lambda_2L_2 + N_4\cos h\lambda_2L_2)$$

$$K_2 = \lambda_2^2(-S_1\sin\lambda_2L_2 - S_2\cos\lambda_2L_2 + S_3\sin h\lambda_2L_2 + S_4\cos h\lambda_2L_2)$$

$$K_3 = \lambda_2^3(-N_1\cos\lambda_2L_2 + N_2\sin\lambda_2L_2 + N_3\cos h\lambda_2L_2 + N_4\sin h\lambda_2L_2)$$

$$K_4 = \lambda_2^3(-S_1\cos\lambda_2L_2 + S_2\sin\lambda_2L_2 + S_3\cos h\lambda_2L_2 + S_4\sin h\lambda_2L_2)$$

From Equ (14), the denominator $K_1K_4 - K_2K_3 = 0$,

Forms the frequency equation whose roots are the natural frequencies of the system.

Then, the roots of $K_1K_4 - K_2K_3 = 0$ are determine as follows:

If $K_1K_4 - K_2K_3 = 0 \therefore\ K_1K_4 - K_2K_3$

Therefore; $\frac{K_1}{K_3} = \frac{K_2}{K_4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$

The value of K_1, K_2, K_3 *and* K_4 are functions of $N_1, N_2, N_3, N_4, S_1S_2, S_3, S_4,\ \lambda_1, \lambda_2, L_1, L_2$ and the values of

$N_1, N_2, N_3, N_4, S_1S_2, S_3S_4$ are function of $H_1, H_2, H_3, H_4, I_{21}, \lambda_{21}$.

3. Determination of Natural Frequencies

Before the analysis, let find the relationship between λ_1 *and* λ_2 (see dimensions of figure 1)

So $\lambda_1^4 = \frac{w^2 \ell_1 A_1}{E_1 I_1}$ *and* $\lambda_2^4 = \frac{w^2 \ell_2 A_2}{E_2 I_2}$

Then, $\lambda_1 = \sqrt[4]{w^2} \times \sqrt[4]{\frac{\ell_1 A_1}{E_1 I_1}}$ *and* $\lambda_2 = \sqrt[4]{w^2} \times \sqrt[4]{\frac{\ell_2 A_2}{E_2 I_2}}$

So, let $\sqrt[4]{w^2} = x$

$$\text{Therefore,} \quad \lambda_1 = x\left(\sqrt[4]{\frac{\ell_1 A_1}{E_1 I_1}}\right) \text{ and } \lambda_2 = x\left(\sqrt[4]{\frac{\ell_2 A_2}{E_2 I_2}}\right)$$

Hence,

$$\lambda_1 L_1 = x\left(\sqrt[4]{\frac{2.7\ x\ 10^3\ x\ 945\ x\ 10^{-6}}{7.1\ x\ 10^{10}\ x\ 57.41\ x\ 10^{-6}}}\right)0.254$$

$$\lambda_1 L_1 = 0.04018x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

$$\lambda_2 L_2 = x\left(\sqrt[4]{\frac{2.7\ x\ 10^3\ x\ 245\ x\ 10^{-6}}{7.1\ x\ 10^{10}\ x\ 1.0\ x\ 10^{-9}}}\right)0.14$$

$$\lambda_2 L_2 = 0.0435x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

$$Then, \quad \frac{\lambda_2 L_2}{\lambda_1 L_1} = \frac{0.435x}{0.0018x}$$

$$\therefore \lambda_2 L_2 = 1.083\lambda_1 L_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

$$Hence, \quad \lambda_2 L_2 = K\lambda_1 L_1 \text{ or } K = \frac{\lambda_2 L_2}{\lambda_1 L_1} \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

$ie\ K = 1.083$ (*For this analysis*)

The value of K varies, If the e, A, L E and I of the beam changes.

$$I_{21} = \frac{E_2 I_2}{E_1 I_1} = \frac{I_2}{I_1} = \frac{1.0\ x\ 10^{-9}}{5741\ x\ 10^{-9}} = 0.0174$$

$But\ \lambda_1 = 0.1582x\ \ and\ \lambda_2 = 0.31\ v\ 7x$

$$Then\ \frac{\lambda_2}{\lambda_1} = \lambda_{21} = \frac{0.031ox}{0.1582x} = 1.964$$

Hence,

$2I_{21}\,{\lambda_{21}}^3 = 0.264$ *and* $2I_{21}\,{\lambda_{21}}^2 = 0.1342$

The values for H_1, H_2, H_3, H_4 are from equation (10) and the values for N_1, N_2, N_3, N_4, S_1, S_2, S_3, S_4 from equation (9)

Then,

$$K_1 = -N_1 Sin\ \lambda_2 L_2 - N_2 Cos\ \lambda_2 L_2 + N_3 Sin\ h\lambda_2 L_2 + N_4 Cosh\ \lambda_2 L_2$$
$$K_2 = -S_1 Sin\ \lambda_2 L_2 - S_2 Cos\ \lambda_2 L_2 + S_3 Sin\ h\lambda_2 L_2 + S_4 Cosh\ \lambda_2 L_2$$
$$K_3 = -N_1 Cos\ \lambda_2 L_2 + N_2 Sin\ \lambda_2 L_2 + N_3 Cos\ h\lambda_2 L_2 + N_4 Sinh\ \lambda_2 L_2$$
$$K_4 = -S_1 Cos\ \lambda_2 L_2 - S_2 Sin\ \lambda_2 L_2 + S_3 Cos\ h\lambda_2 L_2 + S_4 Sinh\ \lambda_2 L_2$$

Hence, all these equations for this analysis are coded using a computer program and the required output are displaced using computer software (Microsoft Excel)

In the program,

$C_1 = 2I_{21}\,{\lambda_{21}}^3, C_2 = 2I_{21}\,{\lambda_{21}}^2$ *and* $C_3 = 2\lambda_{21}$

and $K = \frac{\lambda_2 L_2}{\lambda_1 L_1}$

Therefore, table 1 show the table of values for graph of K_1/K_3, K_2/K_4 against $\lambda_1 L_1$ from the Microsoft Excel. Where $0 \le \lambda_1 L_1 \le 450^0$.

$\lambda_1 L_1 = 0^0, 30^0, 60^0, 90^0, 120^0, 150^0, 180^0, 210^0, 240^0, 270^0, 300^0, 330^0, 360^0, 390^0, 420^0, 450^0$

H_1	H_2	H_3	H_4	N_1	N_2	N_3	N_4
0.00	0.00	2.00	0.00	7.58	0.00	-7.58	0.00
1.05	-0.05	2.01	-0.27	7.53	7.78	-7.67	7.78
2.12	-0.38	2.10	-1.10	7.68	15.57	-8.24	15.57
3.30	-1.30	2.51	-2.51	8.87	23.95	-10.14	23.95
4.86	-3.13	3.62	-4.62	12.54	34.68	-14.90	34.96
7.32	-6.32	6.02	-7.76	20.85	51.37	-24.79	51.37
11.55	-11.55	10.59	-12.59	36.92	80.28	-43.33	80.23
19.02	-20.02	18.68	-20.41	65.56	131.71	-75.95	131.71
32.10	-33.83	32.48	-33.48	114.50	222.26	-131.55	222.26
54.65	-56.65	55.66	-55.66	196.68	378.93	-225.02	378.93
93.09	-94.82	94.46	-93.46	334.01	646.25	-381.60	646.25
158.11	-159.11	159.48	-157.74	563.92	1098.59	-644.24	1098.59
267.74	-267.74	268.75	-266.75	950.07	1861.25	-1085.89	1861.25
452.48	-151.48	452.85	-451.11	1600.48	3145.94	-1830.17	3145.94
763.85	-762.12	763.48	-762.48	2697.87	5310.81	-3086.10	5310.81
1288.99	-1286.99	1287.99	-1287.99	4550.83	8961.46	-5206.63	8961.46
3083.15	-3081.87	3081.74	-3083.27	10888.32	21433.37	-12458.22	21433.37

K_1	K_2	K_3	K_4	K_1/K_3	K_2/K_4
0.00	0.00	-15.15	0.00	0.00	0.00
-6.13	16.22	-14.81	55.37	0.41	0.29
1.69	65.44	-9.78	97.47	-0.17	0.67
35.26	138.14	12.13	132.95	2.91	1.04
110.82	198.33	74.50	163.65	1.49	1.21
271.28	155.57	228.50	124.81	1.19	1.25
644.11	-183.55	609.27	-186.13	1.06	0.99
1615.76	-1200.14	1615.80	-1180.66	1.00	1.02
4388.07	-3649.64	4471.52	-3630.06	0.98	1.01
12706.51	-9207.76	12940.82	-9192.71	0.98	1.00
38065.9	-22256.04	38497.35	-22221.71	0.99	1.00
115164.49	-55691.75	115719.55	-55619.58	1.00	1.00
347626.68	-149222.16	347959.54	-149158.32	1.00	1.00
1043426.68	-425391.01	1042772.01	-425485.95	1.00	1.00
3116817.51	-1258191.01	3113989.59	-1258677.05	1.00	1.00
9283134.86	-3775924.83	9276949.94	-3777013.89	1.00	1.00
57105237.62	-23626383.97	57094969.56	-23628164.96	1.00	1.00

Table 1: Table of Values for the graph of K_1/K_3, K_2/K_4 *against* $\lambda_1 L_1$

$From\ the\ table, the\ graph\ of\ \frac{k_1}{k_3}, \frac{k_2}{K_4}\ against\ \lambda_1 L_1$ as shown in Figure 2.

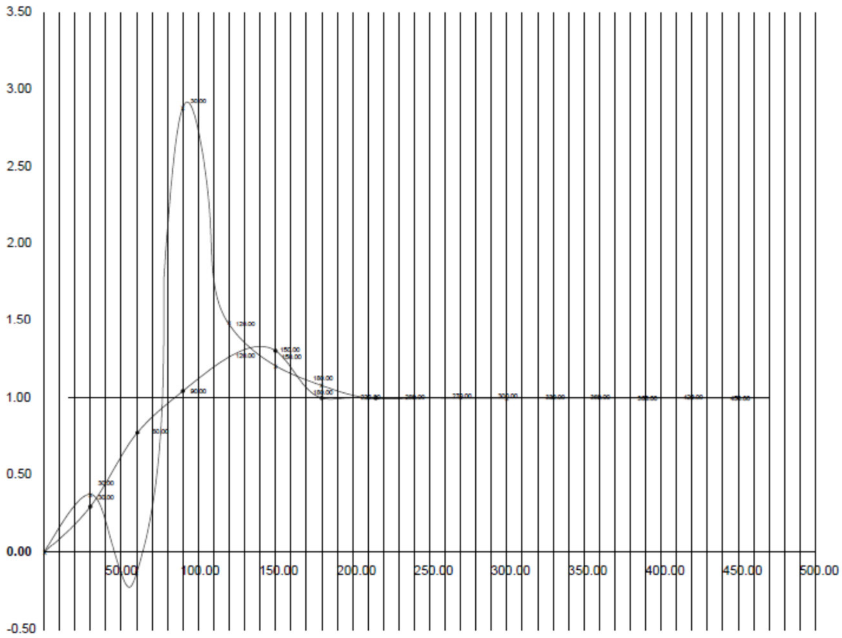

Figure 2: Graph of K_1/K_3, K_2/K_4 against $\lambda_1 L_1$

From the graph, the points of intersection of the two graphs are the solution of the natural frequencies, hence, this is at

$\lambda_{11}L_1 = 35^0 = 0.61094rad$

$\lambda_{12}L_1 = 70^0 = 1.2219rad$

$\lambda_{13}L_1 = 136^0 = 2.374\ and\ so\ on.$

Hence, the first modal shape natural frequency is

$\lambda_{11}L_1 = 0.61094$

$$Then\ \omega_1 = \sqrt{\frac{0.13931E_1L_1}{\ell_1\ A_1{L_1}^4}}$$

$$\omega_1 = \sqrt{\frac{0.13931\ x\ 7.1\ x\ 10^{10}\ x\ 57.41\ x\ 10^{-9}}{2.7\ x\ 10^3\ x\ 945\ x\ 10^{-6}\ x\ (0.254)^4}}$$

$\omega_1 = 231.233$

$$F_1 = \frac{231.233}{6.284} = 36.797Hz$$

The second modal shape natural frequency is

$\lambda_{12}L_1 = 1.2219$

$$\omega_2 = \sqrt{\frac{2.2292E_1L_1}{\ell_1\ A_1{L_1}^4}}$$

$$\omega_2 = \sqrt{\frac{2.2294x\ 7.1\ x\ 10^{10}\ x\ 57.41\ x\ 10^{-9}}{2.7\ x\ 10^3\ x\ 945\ x\ 10^{-6}\ x\ (0.254)^4}}$$

$\omega_2 = \sqrt{855587.6843}$

$\omega_2 = 924.9798$

$$F_2 = \frac{924.9798}{6.284} = 147.196Hz$$

The third modal shape natural frequency is

$\lambda_{13}L_1 = 2.374$

$$\omega_3 = \sqrt{\frac{31.763E_1L_1}{\ell_1\ A_1{L_1}^4}}$$

$$\omega_3 = \sqrt{\frac{31.763x\ 7.1\ x\ 10^{10}\ x\ 57.41\ x\ 10^{-9}}{2.7\ x\ 10^{3}\ x\ 945\ x\ 10^{-6}\ x\ (0.254)^{4}}}$$

$$\omega_2 = \sqrt{12190934.69}$$

$$\omega_3 = 3491.55$$

$$F_3 = \frac{3491.55}{6.284} = 555.63Hz$$

In summary, the first three natural frequencies of the discontinuous beam with aligned neutral axes are 36.797Hz, 147.196Hz and 555.63Hz.

4. Experimental set-up

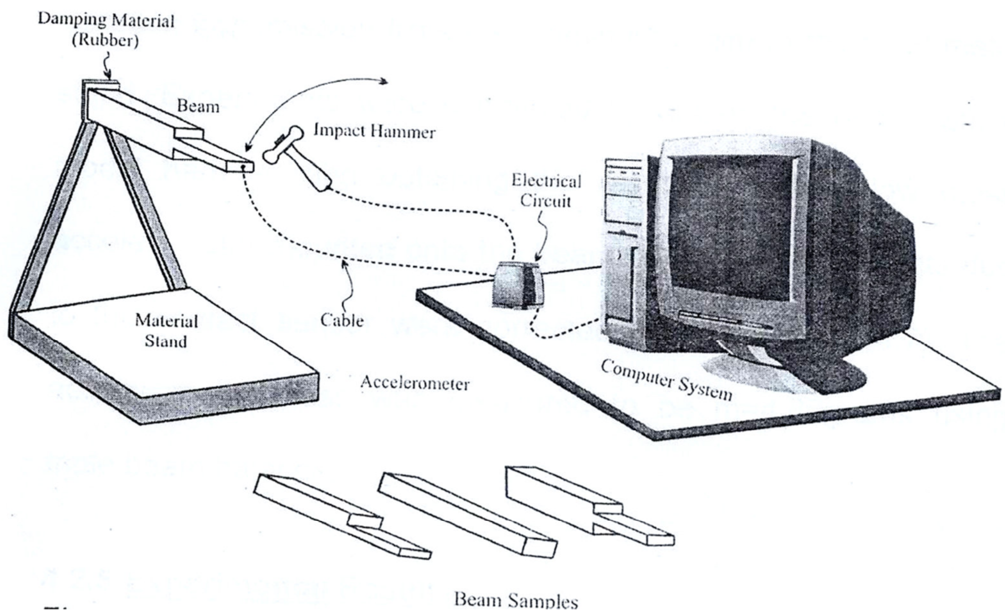

Figure 3: Schematic of Experimental set-up for FRF Testing

The experiment consists of an impact hammer, sample of aluminum beam used in this analysis, accelerometer, pure electrical digital circuit, cables, computer system with frequency analyzer software and rigid body or element for fixed-free boundary conditions. Fixed-free boundary conditions where obtained by hanging one end of the beam on a rigidly metal materials stand via a piece of rubber damping material, as shown in figure 3.

The rubber damping materials are used to reduce the vibration transmission from the aluminum beam to the fixed metal stands. Experiments were conducted by forcing the beam with a model hammer and obtaining the response with a low mass accelerometer mounted into the beam.

From the periodogram of the spectrum analyze software, the following natural frequencies were identified using peak picking method (PPM)

F_1 = 32.54Hz
F_2 = 138.14Hz
F_3 = 547.37Hz

5. Results and Discussion

Table 3 below shows the summary results of the analytical and experimental verifications of a cantilevered discontinuous beam

Natural frequency	$F_1(H_2)$	$F_2(H_2)$	$F_3(H_2)$
Discontinuous beam with Aligned neutral axes (analytical)	36.797	147.196	555.63
Discontinuous beam with Aligned neutral axes (experimental)	32.54	138.14	547.37
Error	4.257	9.056	826
%Error	11.5%	6.2%	1.5%

Table 3: Summary results for analytical and experimental models

The table above shows that the analytical results are in excellent agreement with the experimental predictions. But 0-12% errors was observed between the analytical and experimental natural frequencies.

6. Conclusion

In this paper, the natural frequencies of a cantilevered beam with aligned neutral axes is investigated using Receptance – Graphical Approach. It is observed that the method is a computationally efficient tool in determining the natural frequencies of beam with aligned neutral axes. This method is particularly attractive because of the case with which one choose the dimensions of the beams, other parameters and frequency graph can be displaced. The natural frequencies obtained by this method compare extremely with the experimental results. From the experimental analysis, it can also be recommended that more accurate result can be obtained if fast Fourier transform analyzer are used instead of the constructed electrical circuit system equipment.

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