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Generation and Stress Analysis in New Version of Novikov Helical Gear Combining Double Circular Arc and Crowned Involute Profiles

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Abstract

New version of Novikov helical gear combining double circular arc profile with crowned involute profile has been proposed in this paper. Generation of this gear is carried out using the required equations for these two profiles which are programmed using SOLIDWORK. To compare the resulting contact and bending stresses, three models of gear pairs are generated and investigated using finite element software package (ANSYS). The profiles of these models are as following: double circular arc for first model, crowned involute for second model, while the third model formed by combining double circular arc with involute crowned profiles. The results of stress analysis show that the generated stresses are lower in the proposed (combined) gear especially when the contact in the circular arc side.

Keywords: Generation of gears, versions of Novikov Gears, circular arc gear, crowned involute gear, stress analysis in gears.

1. Introduction:

Gears are mostly used to transmit power between shafts in several mechanical applications such that the rotary motion of the driven shaft is perfectly uniform relative to that of driving shaft [1]. In several mechanical applications gearing system is the preferred drive. As we know the toothed gearing has many advantages in compare with other mechanical drives such as its applicability for a wide range of torques and speed ratios, long service life and high reliability, can give constant speed ratio when there is no slipping as well as its small size [2]. These advantages illustrate the importance of developing gear profile.

Today, three basic types of teeth profiles are used in gears for power transmission. They are involute, Noviko (or circular arc) and cyc1odial [3, 4].

One of the important causes of gear tooth failure is the presence of large contact and bending stresses in loaded gear tooth. These stresses lead to reduce the overall gear life and can result in tooth failure under loaded conditions [5]. A small reduction in the stresses leads to increase in the fatigue life of the gears considerable. Therefore it is important to find out the method of reducing induced stresses in the gear to increase gears life.

Various improvements in gear design have taken place during the last few years such as use of material with improved strength, hardening the surfaces selectively with heat treatment and carburization, and improve the surface finish by shot peening. Many efforts such as using the asymmetric teeth, altering the pressure angle, introducing stress relief feature and using the gear with high contact ratio have been made to improve the strength of the gear [5].

Circular arc gears and crowned profile gears are superior compare with other types of gear profiles and they have higher load carrying capacity. In comparison with involute profile gear, crowned profile gear has a larger contact area, lower noise and vibration, and has a stronger tooth form [6]. Circular arc gears can take three to five times the load on the tooth flanks (in compare with involute gears) without detrimental pitting or wear, They more efficient in power transmission, and They retain lubricants between mating teeth more easily and form a thick oil film. Thus, the wearing of tooth flank is slower [7].

In this paper the asymmetric tooth profiles of two versions of Novikov gears will be used to reduce generated stress and increases gears life. These two profiles are double circular arc and crowned involute profiles.

Double circular arc gear has higher bending and contact strength in compare with involute gear for the same parameters. This gear has been widely used in industry machinery [8].

2. Gear Dimensions:

The dimensions of involute and double circular arc helical gears can be written in terms of the normal pressure angle (α_n), normal module (m_n), helix angle (β) and number of teeth (N) as follows [9, 10]:-

$$R_{Pi} = \frac{1}{2} m_t N_i = \frac{N_i}{2 * m_n * \cos\beta}$$

Where R_p is the radius and i =1, 2 for pinion and gear respectively.

$$m_t = \frac{m_n}{\cos\beta}$$

$$p = \pi * m_n$$

$$p_n = \frac{1}{m_n}$$

$$p_t = \frac{p_t}{\cos\beta}$$

$$\text{Center distance (E')} = \frac{N_1 + N_2}{2 * m_1 * \cos\beta}$$

Total tooth high (h) = addendum (h_a) + dedendum (h_d)

3. Crowned Profile Gears:

The normal section of pinion rack cutter is shown in figure (1). The profile of the basic tooth of the rack cutter in the normal section is symmetric about x_{cp}. The normal section of the pinion rack cutter is represented in the coordinate system S_{cp} by the equations:

$$r_{cp}(u_c) = \begin{bmatrix} x_{cp} \\ y_{cp} \\ z_{cp} \\ 1 \end{bmatrix} = \begin{bmatrix} -u_c \cos\alpha_n - a_c u_c^2 \sin\alpha_n \\ u_c \sin\alpha_n - a_c u_c^2 \cos\alpha_n - a_m \\ 0 \\ 1 \end{bmatrix} = M_{cpb} r_b(u_c) \qquad \ldots (1)$$

where $r_b(u_c) = [-u_c \quad -a_c u_c^2 \quad 0 \quad 1]^T$, $a_m = \dfrac{\pi}{4 p_n}$, (a_c) is the parabolic coefficient and

$$M_{cpb} = \begin{bmatrix} \cos\alpha_n & \sin\alpha_n & 0 & 0 \\ -\sin\alpha_n & \cos\alpha_n & 0 & -a_m \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

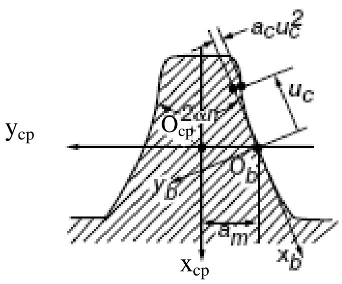
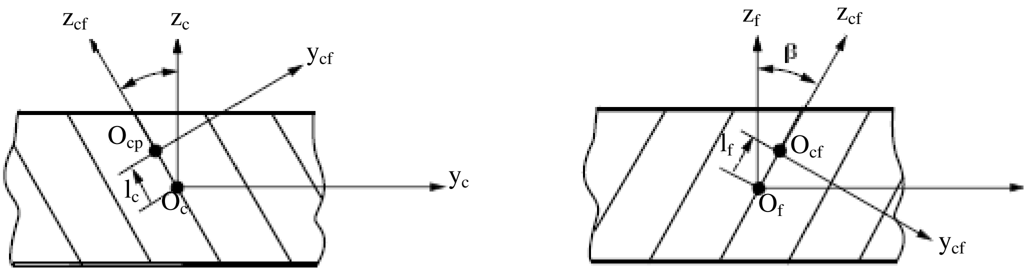


Figure1: Normal section of rack cutters for crowned profile helical gear



(a) (b)

Figure 2: Derivation of rack cutter surface for crowned involute helical gear.
(a) pinion rack cutter. (b) gear rack cutter.

The next step is to represent the rack cutter tooth in the three dimensional space defined by the coordinate system Sc (figure (2)), depending on the following consideration [11]:-

1. Coordinate system Scp in the normal section of the pinion rack cutter performs a translational motion along straight line joining O_{cp} with O_c. So that the new location of O_{cp} in coordinate system S_c is defined by the variable parameter $l_c = \overline{O_{cp}O_c}$.
2. Straight line $\overline{O_{cp}O_c}$ makes angle β which is the helix angle with the axis of the gear that is parallel to the Z_c-axis.

So, the surface of the rack cutter tooth in the three dimensional system can be represented as:

$$r_c(u_c, l_c) = M_{ccp} r_{cp}(u_c) \qquad \dots \qquad (2)$$

where M_{ccp} is the 4x4 matrix used for coordinate transformation from system S_{cp} to S_c which has the following values

$$M_{ccp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta & -l_c \sin\beta \\ 0 & -\sin\beta & \cos\beta & l_c \cos\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After using matrix transformation we obtain the following result

$$r_c(u_c, l_c) = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} -u_c \cos\alpha_n - a_c u_c^2 \sin\alpha_n \\ (u_c \sin\alpha_n - a_c u_c^2 \cos\alpha_n - a_m)\cos\beta - l_c \sin\beta \\ (-u_c \sin\alpha_n + a_c u_c^2 \cos\alpha_n + a_m)\sin\beta + l_c \cos\beta \\ 1 \end{bmatrix} \qquad \dots (3)$$

The unit normal to the pinion rack cutter surface is represented as:

$$n_c = \frac{N_c}{|N_c|} \quad , \quad N_c = \frac{\partial r_c}{\partial l_c} \times \frac{\partial r_c}{\partial u_c} \qquad \dots \qquad (4)$$

Thus, the final resulting equations for unit normal to pinion rack cutter surface Σc is

$$n_c(u_c) = \begin{bmatrix} n_{xc} \\ n_{yc} \\ n_{zc} \end{bmatrix} = \frac{1}{\sqrt{1+4a_c^2u_c^2}} \begin{bmatrix} -\sin\alpha_n + 2a_cu_c\cos\alpha_n \\ -(\cos\alpha_n + 2a_cu_c\sin\alpha_n)\cos\beta \\ -(\cos\alpha_n + 2a_cu_c\sin\alpha_n)\sin\beta \end{bmatrix} \quad \ldots (5)$$

Now the equation of meshing must be represented as:

$$f(u_c, l_c, \psi_1) = 0 \quad \ldots \quad (6)$$

Where ψ_1 is the angle of rotation of the pinion in the process for generation. The derivation of equation of meshing is based on the theorem that the common normal to Σ_c and Σ_1 must pass through the instantaneous axis of rotation [11] and [12]. Thus we have

$$\frac{X_c - x_c}{n_{xc}} = \frac{Y_c - y_c}{n_{yc}} = \frac{Z_c - z_c}{n_{zc}} \quad \ldots \quad (7)$$

where Xc =0 and $Y_c = -Rp_1\ \psi_1$. After transformation, we obtain the following equation of meshing

$$f(u_c, l_c, \psi_1) = R_{p1}\psi_1 - l_c\sin\beta - a_m\cos\beta + \frac{u_c(1+2a_cu_c^2)\cos\beta}{\sin\beta_n - 2a_cu_c\cos\alpha_n} = 0 \quad \ldots (8)$$

The generated surface of the pinion Σ1 is represented by the family of lines of contact between the rack cutter surface Σc and the pinion tooth surface Σ1 being generated. Surface Σ1 is represented in coordinate system S1 by the equations:

$$r_1(u_c, l_c, \psi_1) = M_{1n}M_{nc}r_c(u_c, l_c), \qquad f(u_c, l_c, \psi_1) = 0 \quad \ldots (9)$$

where M_{nc} and M_{1n} are the matrices of transformation from S_c to S_n and S_n to S_1 respectively, where

$$M_{nc} = \begin{bmatrix} 1 & 0 & 0 & R_{P1} \\ 0 & 1 & 0 & R_{p1}\psi_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad M_{1n} = \begin{bmatrix} \cos\psi_1 & \sin\psi_1 & 0 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus

$$r_1 == \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} (-u_c\cos\alpha_n a_cu_c^2\sin\alpha_n)\cos\psi_1 + \{(u_c\sin\alpha_n - a_cu_c^2\cos\alpha_n - a_m)\cos\beta \\ -l_c\sin\beta\}\sin\psi_1 + (\cos\psi_1 + \psi_1\sin\psi_1)R_{p1} \\ \\ (u_c\cos\alpha_n + a_cu_c^2\sin\alpha_n)\sin\psi_1 + \{(u_c\sin\alpha_n - a_cu_c^2\cos\alpha_n - a_m)\cos\beta \\ -l_c\sin\beta\}\cos\psi_1 + (-\sin\psi_1 + \psi_1\cos\psi_1)R_{p1} \\ \\ (-u_c\sin\alpha_n + a_cu_c^2\cos\alpha_n + a_m)\sin\beta + l_c\cos\beta \end{bmatrix} \ldots (10)$$

In the same manner gear tooth surface can be generated.
Using Eq.(10) in SOLIDWORK program, the 3-dimensional crowned profile helical gear can be generated as shown in Figure (3):

4. Double Circular Arc Gears:

The normal section of the rack-cutter is shown in figure (4). The profile of the basic tooth of the rack-cutter in the normal section is symmetric about $y_b^{(p)}$. Each side of the basic tooth consists of three circular arcs. The normal section of pinion rack-cutter is represented in coordinate system $S_b^{(p)}$ by the following equations:

$$r_b^{(p)} = \begin{bmatrix} x_b^{(p)} \\ y_b^{(p)} \\ z_b^{(p)} \end{bmatrix} = \begin{bmatrix} \rho_p \cos\theta_p + x_{op} \\ \rho_p \sin\theta_p + y_{op} \\ 0 \end{bmatrix} \qquad \ldots (11)$$

Where ρ_p is the radius of circular arcs; (x_{op}, y_{op}) are the arc center coordinates; θ_p is the variable parameter (angle) and the subscribed p =a, f, g. Circular arcs ρ_a and ρ_f generates the working surfaces of the pinion, and ρ_g generates the fillet surface.

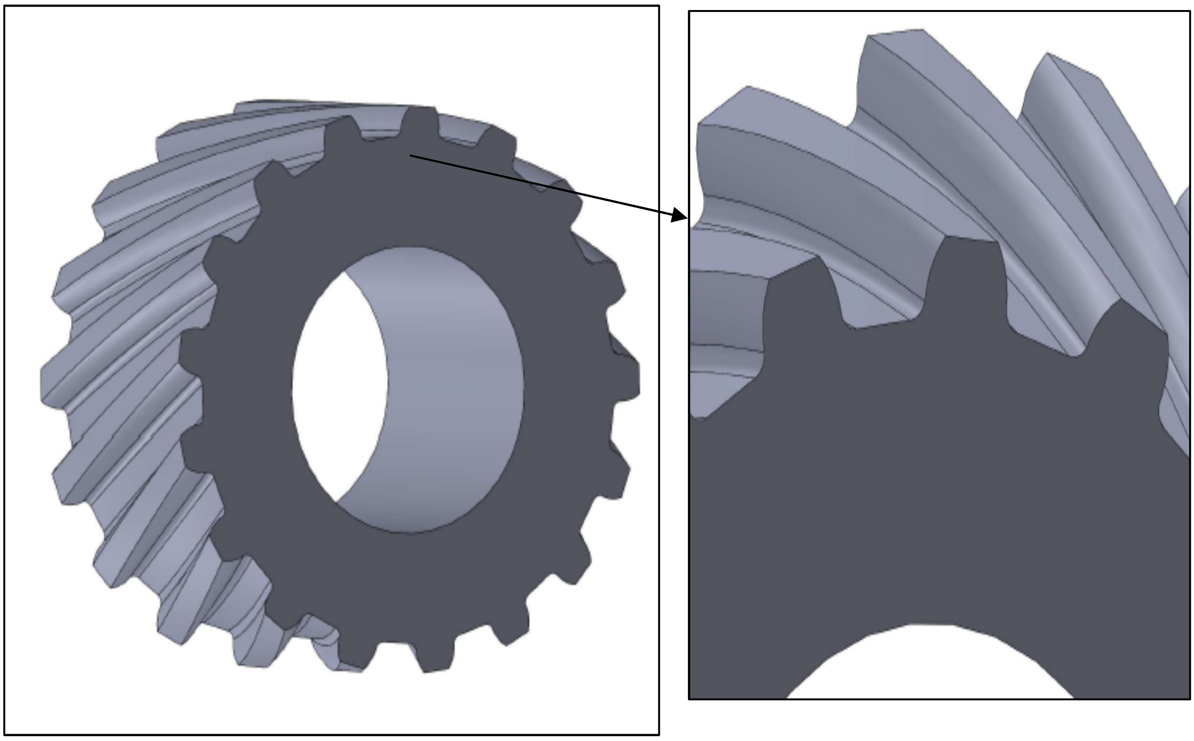


Figure 3: Crowned profile helical gear generated using SOLIDWORK program.

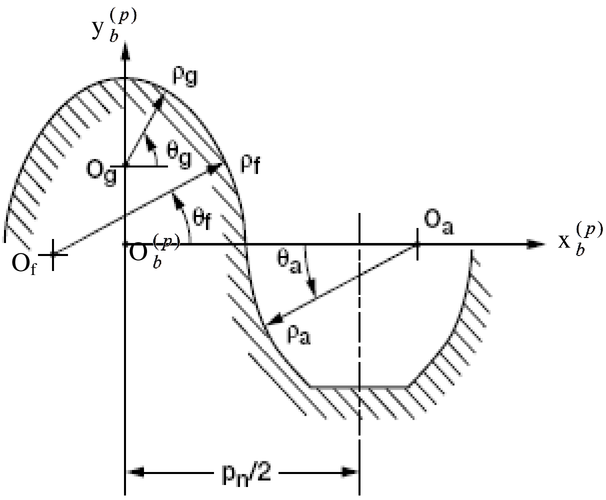


Figure 4: Derivation of normal section of pinion rack-cutter.

To represent the pinion rack-cutter tooth surface in the three dimensional system, the following consideration must be taken into account (figure 5):-

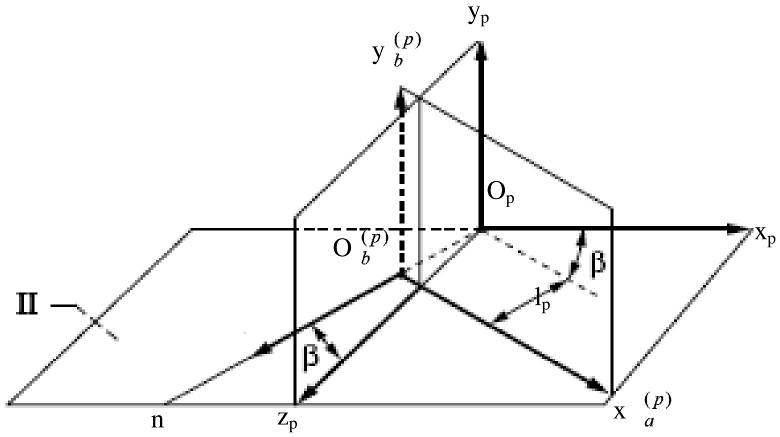


Figure 5: Derivation of rack-cutter surface for double circular arc helical gear.

1. Coordinate system $S_b^{(p)}$ in the normal section of the pinion rack-cutter performs a translational motion along the straight line $\overline{O_p n}$. Thus, the new location of the origin $O_b^{(p)}$ in coordinate system S_p is defined by the variable parameter $l_p = \overline{O_p O_b^{(p)}}$.
2. Straight line $\overline{O_p n}$ makes angle β which is the helix angle with the pinion axis that is parallel to the z_p-axis.

So, the surface of the pinion rack-cutter tooth in S_p can be represented as

$$r_p(\theta_p, l_p) = M_{pb} r_b^{(p)} \qquad \ldots (12)$$

$$M_{pb} = \begin{bmatrix} \cos\beta & 0 & -\sin\beta & -l_p \sin\beta \\ 0 & 1 & 0 & 0 \\ \sin\beta & 0 & \cos\beta & l_p \cos\beta \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

After derivation the following equations are obtained:

$$r_p(\theta_p, l_p) = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} (\rho_p \cos\theta_p + x_{op})\cos\beta - l_p \sin\beta \\ \rho_p \sin\theta_p + y_{op} \\ (\rho_p \cos\theta_p + x_{op})\sin\beta + l_p \cos\beta \end{bmatrix} \quad \dots (13)$$

The unit normal to the pinion rack-cutter surface is represented as:

$$n_p = \frac{N_p}{|N_p|}, \qquad N_p = \frac{\partial r_p}{\partial \theta_p} \times \frac{\partial r_p}{\partial l_p} \quad \dots (14)$$

After derivation we obtain the following equations

$$n_p(\theta_p) = \begin{bmatrix} n_{yp} \\ n_{xp} \\ n_{zp} \end{bmatrix} = \begin{bmatrix} \cos\theta_p \cos\beta \\ \sin\theta_p \\ \cos\theta_p \sin\beta \end{bmatrix} \quad \dots (15)$$

To derive the equation of meshing, consider the movable coordinate systems S_p and S_1 are rigidly connected to the pinion rack-cutter and the pinion, respectively. The fixed coordinate system S_n is rigidly connected to the frame of the cutting machine as shown in figure (6).
The equation of meshing must be represented as

$$f(l_p, \theta_p, \psi_1) = 0 \quad \dots (16)$$

Where ψ_1 is the angle of rotation of the pinion in the process for the generation. The derivation of equation of meshing is based on the theorem [11] and [12] that yields

$$\frac{X_p - x_p}{n_{xp}} = \frac{Y_p - y_p}{n_{yp}} = \frac{Z_p - z_p}{n_{zp}}$$

Where $X_p = R_{p1}\psi_1$ and $Y_p = 0$. Thus:

$$f(l_p, \theta_p, \psi_1) = (R_{p1}\psi_1 + l_p \sin\beta - x_{op}\cos\beta)\sin\theta_p + y_{op}\cos\theta_p \cos\beta = 0 \quad \dots (17)$$

The generated surface of the pinion Σ_1 is represented by the family of lines of contact between the rack-cutter surface and the surface of the pinion being generated. Surface Σ_1 is represented in S_1 by the equations (figure (6)):

$$r_1 = M_{1n} M_{nq} r_p(\theta_p, l_p), \qquad f(l_p, \theta_p, \psi_1) = 0 \quad \dots (18)$$

$$M_{np} = \begin{bmatrix} 1 & 0 & 0 & -R_{p1}\psi_1 \\ 0 & 1 & 0 & R_{p2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad M_{1n} = \begin{bmatrix} \cos\psi_1 & -\sin\psi_1 & 0 & (R_{p1}+R_{p2})\sin\psi_1 \\ \sin\psi_1 & \cos\psi_1 & 0 & -(R_{p1}+R_{p2})\cos\psi_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

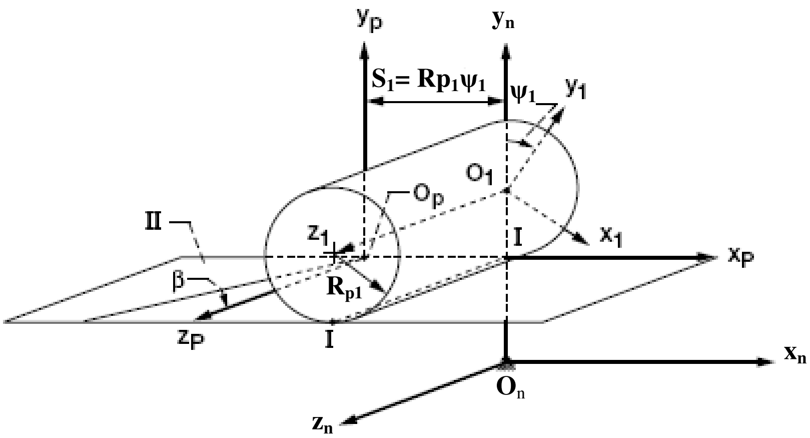


Figure 6: Derivation of coordinate transformation for pinion generation.

Thus

$$r_1 = \begin{bmatrix} x_1 \\ \\ y_1 \\ \\ z_1 \end{bmatrix} = \begin{bmatrix} \{(\rho_p \cos\theta_p + x_{op})\cos\beta - l_p \sin\beta - R_{p1}\psi_1\}\cos\psi_1 \\ -(\rho_p \sin\theta_p + y_{op} + R_{p2})\sin\psi_1 + (R_{p1} + R_{p2})\sin\psi_1 \\ \\ \{(\rho_p \cos\theta_p + x_{op})\cos\beta - l_p \sin\beta - R_{p1}\psi_1\}\sin\psi_1 \\ +(\rho_p \sin\theta_p + y_{op} + R_{p2})\cos\psi_1 - (R_{p1} + R_{p2})\cos\psi_1 \\ \\ (\rho_p \cos\theta_p + x_{op})\sin\beta + l_p \cos\beta \end{bmatrix} \quad \dots (19)$$

Gear tooth surface can be generated in the same manner from gear rack cutter.
Eq.(19) can be programed in SOLIDWORK to obtain the 3-dimensional double circular arc helical gear as shown in Figure (7):

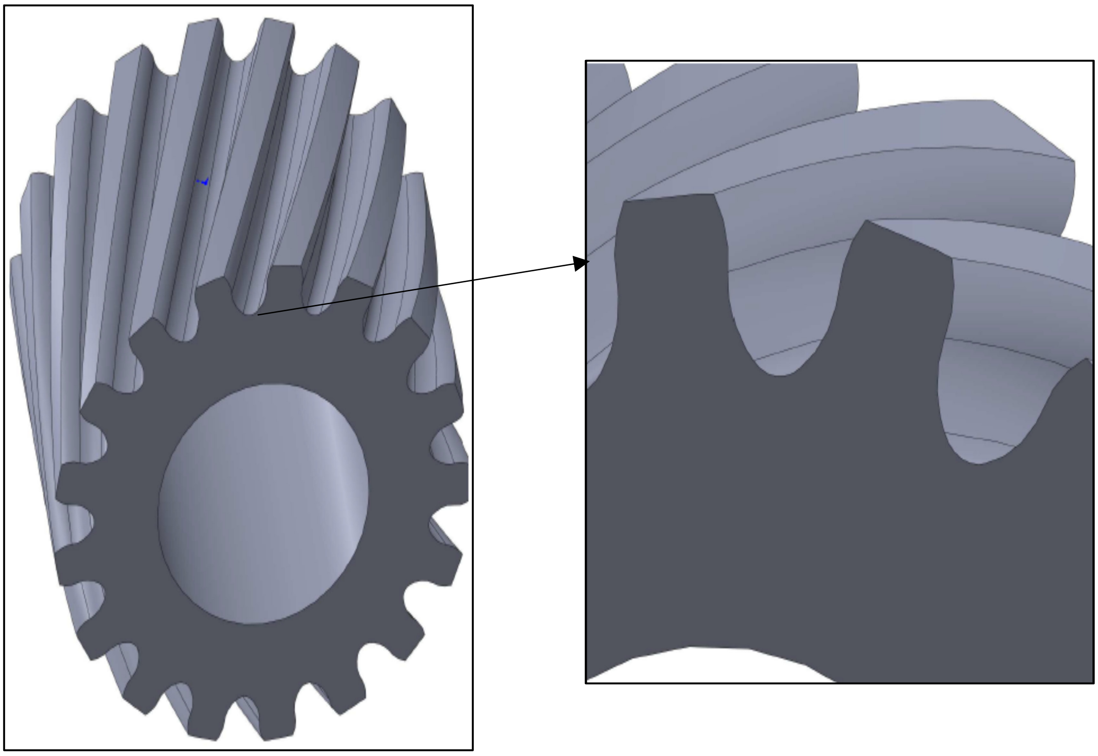


Figure 7: Double Circular Arc helical gear generated using SOLIDWORK program.

5. Combined circular arc–crowned involute gears:

This version of gears combining double circular arc with crowned involute profile in one tooth such that the circular arc profile forms one side of the tooth, while the crowned profile forms the other side. Generation of this gear may be carried out easily using Double circular arc profile (Eq.(19)) for one side and crowned involute profile (Eq.(10)) for the other side. Figure (8) shows the generated gear.

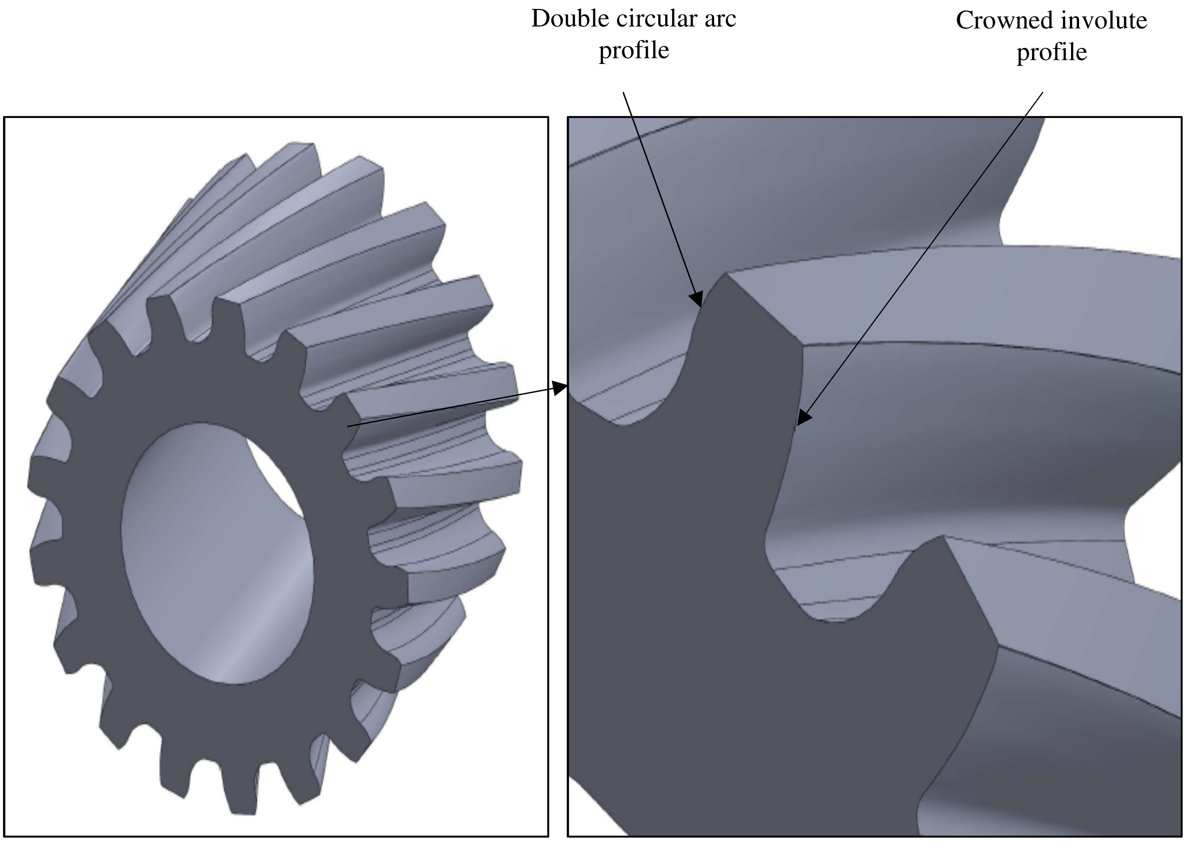


Figure 8: new gear version combining double circular arc with crowned involute profiles generated using SOLIDWORK program.

6. Results of Stress Analysis and Comparison:

To compare the resulting contact and bending stresses, three models has been used. The first model is of double circular arc profile, the second has crowned involute profile while the third model combines the first two profiles (double circular arc and crowned involute) in one tooth. The various design parameters of gear and pinion for all models are shown in table (1). The material used in the analysis is steel of 207Gpa Young modulus and 0.3 poison's ratio. The torque is applied to the pinion in all models with the same value of 200N.m. The finite element analysis has been achieved using Ansys program to find the resulting stresses.

Figure (9) shows the resulting Von Misses stresses for double circular arc gear model. The maximum stress induced is $1.6465*10^8$ pa (164.65Mpa). While figure (10) shows the Von Misses stresses induced in crowned involute gear model with maximum value of $4.2275*10^8$ pa (422.75Mpa).

For gear model combining circular arc with crowned involute profiles, the test achieved twice. First by taking the circular arc side as contact side (see figure (11)), the resulting maximum stress in this case is $1.0605*10^8$ pa (106.05Mpa) which is lower than that result when using double circular arc gear only. Then taking crowned involute side as contact side (see figure (12)), in this case the resulting maximum stress is $3.1816*10^8$ pa (318.16Mpa) which is also lower than that result when using crowned involute gear only.

Table (1): Parameters of tested gears for all gear models.

Normal Module, m_n (mm)	Normal Pressure Angle, α_n (Deg)	Helix Angle β (Deg)	Face Width b (mm)	No. of Pinion Teeth N_1	No. of Gear Teeth N_2
5	27	19.5	70	17	68

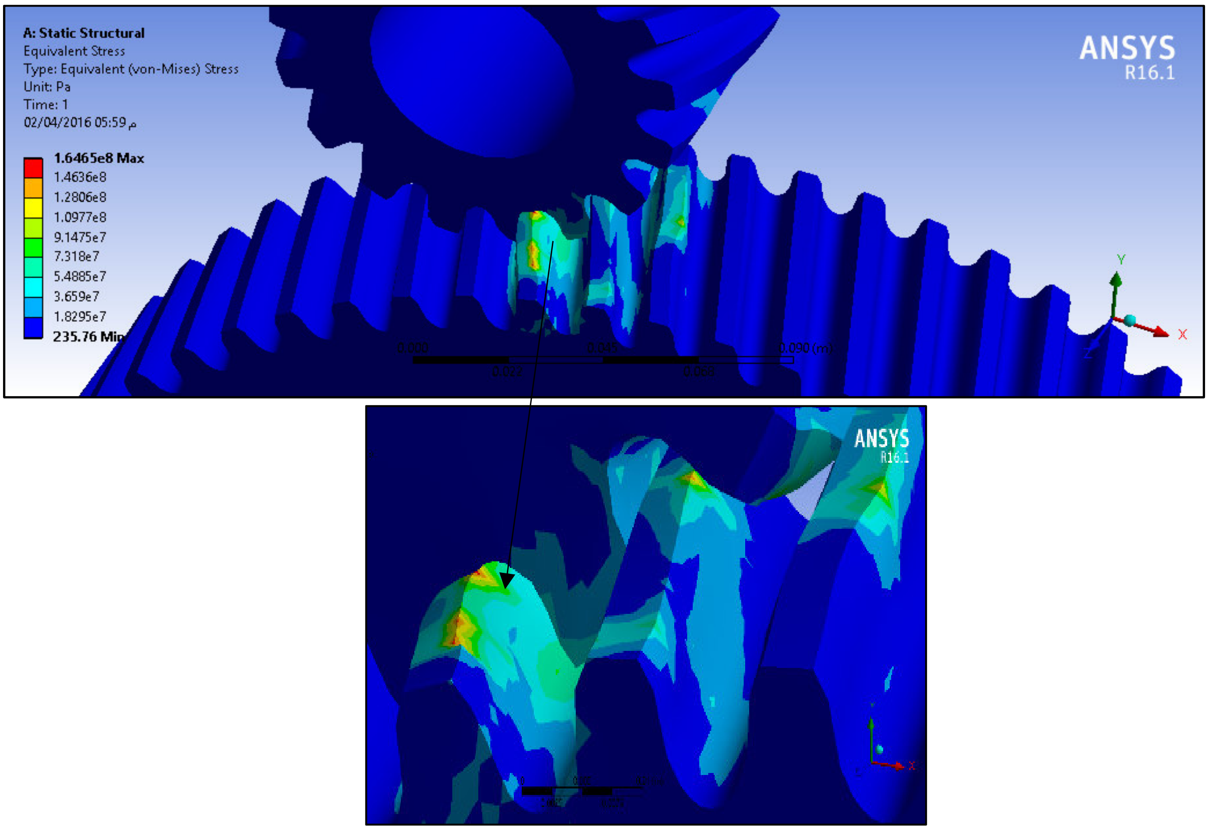


Figure 9: Von Misses stresses in double circular arc gear.

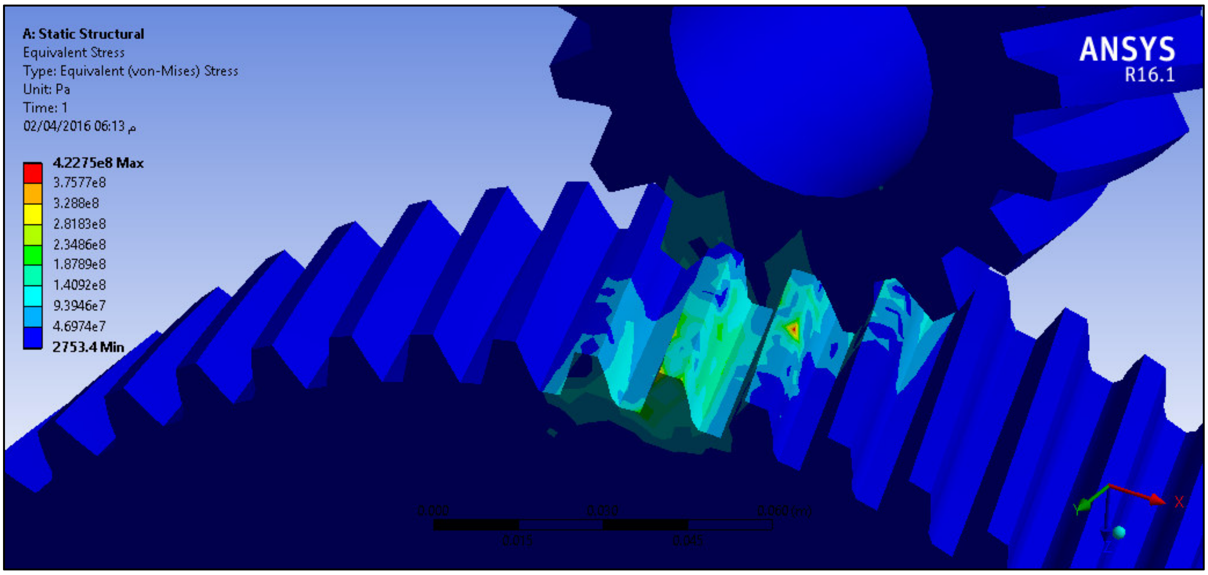


Figure 10: Von Misses stresses in crowned involute gear.

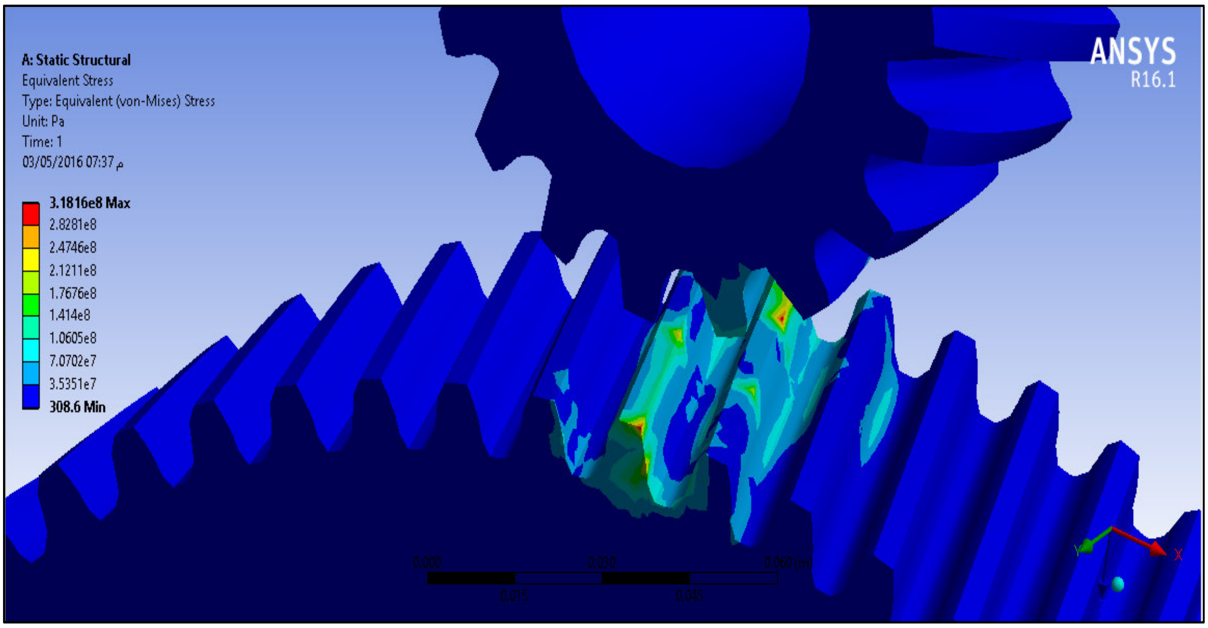


Figure 11: Von Misses stresses in combined gear when contact in crowned involute side.

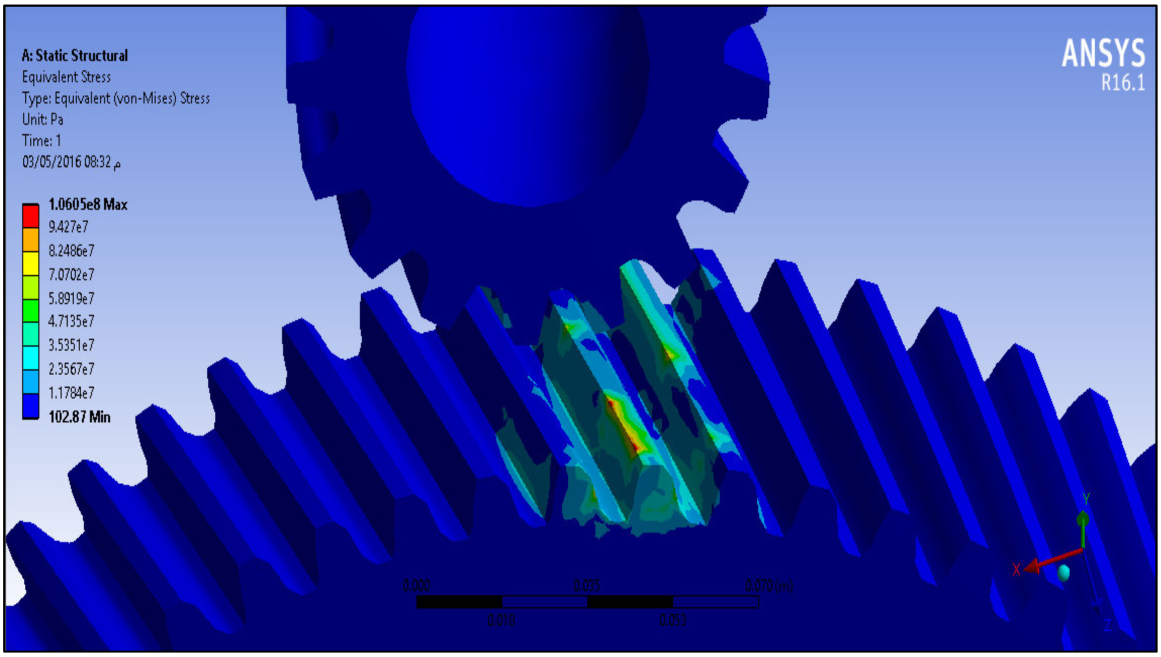


Figure 12: Von Misses stresses in combined gear when contact in circular arc side.

Reasons of the reduction in stresses belong to the method of organize gear elements which affect the form of the global stiffness matrix:

$$[K]\{u\}=\{F\}$$

Where [K] is the stiffness matrix, {u} represents displacement vector and {F} is the force vector. Since the contact problem is nonlinear and usually requires significant computer resources to solve, thus Ansys program used here to solve it.

Knowing that in the finite element method the stiffness matrix associated with contact elements and other element stiffness matrices of the body are formulated and assembled into the original finite element model. The solution is then obtained by solving the resulting set of nonlinear equations [13].

Thus, the resulting global stiffness matrix leads to decrease induced stresses in the new gear combines circular arc with crowned involute profiles for the proposed version.

It is known that the circular arc profile has a larger contact area than the crowned profile for the same design parameters, thus the resulting contact stresses are lower when the contact side is the circular arc in the combined version. Table (2) shows the resulting maximum Von Misses stresses in the tested models.

Table (2): Maximum Von Misses Stress in Tested Gears.

Gear Profile Type	Double Circular Arc	Crowned Involute	Combined Profile	
			Contact in the Crowned Involute Side	Contact in the Circular Arc Side
Maximum Von Misses Stress (Pa)	$1.6465*10^8$	$4.2275*10^8$	$3.1816*10^8$	$1.0605*10^8$

7. Conclusions:

From above discussion, it can be conclude that the contact stresses in the double circular arc helical gears are in general lower than generated stresses in the crowned profile helical gears for the same design parameters. When this two profiles combined in one tooth, the resulting stresses can be lowered by amount of 24.7% when the contact side is the crowned involute profile, and 35.5% when the contact side is the double circular arc profile. Thus, using combination of double circular arc and crowned involute profiles in one tooth leads to enhance the ability of gears to resist higher loads and increase gear life.

Nomenclatures:

a_c Parabolic coefficient of profile of pinion rack cutter for involute helical gear.
b Face width (m).
E' Center distance between gear and pinion (m).
f Equation of meshing between tooth surface and rack-cutter surface.
(l_i, u_j) Parameters of surface (j = c, f, p, t)-(m).
m_n Module in normal plane (m)
m_t Module in transverse plane (m).
Mi, Lij Matrices of coordinate transformation from coordinate system Si to Sj.
N_i Number of teeth on pinion (i=1) or for gear (i = 2).
n unit normal to any surface.
P Normal circular pitch (m).
P_n Normal diametral pitch (1/m).
P_t Transverse diametral pitch (1/m).
ri Position vector of appoint in coordinate system Si.
R_{pi} Radius of cylinder of pinion (i = 1) or for gear (i = 2)-(m).
S_i Coordinate system (i = c_p, b, c, n, 1, 2, c_f, p, f).
(x_i, y_i, z_i) Coordinates (i =c_p, c, 1, 2, c_f, f, p).
α_n Normal pressure angle (deg).
β Helix angle (deg).
Ψ_i Angle of rotation of pinion (i = 1) or the gear (i = 2) in the process of generation for involute and double circular arc helical gears (deg).
Σ_i Surfaces (i = c, f, 1 , 2, P, t) .
θ_i Variable parameter (angle) of double circular arc gear (i = p, t, c, f) (deg).
ρ_i Radius of double circular arc of double circular arc gears (i = 1, 2, a, f, g, p) -(m).

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