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Two-Warehouse Partial Backlogging Inventory Model For Deteriorating Items With Ramp Type Demand

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Abstract

This paper deals with two warehouse system for deteriorating items with ramp type demand. In this inventory model initially demand is considered to be linear function of time and it became constant after a finite time parameter. Holding cost assume to be constant in both warehouse. Partial backlogging is allowed. The proposed model is developing to minimize the total inventory cost which includes holding cost, backlogging cost, lost sale cost, and deterioration cost. Here three cases are taken into consideration depending on time where demand becomes constant. This is only an analytic approach towards the model.

Keywords: - Two warehouse inventory, ramp type demand, holding cost, deteriorating items

1. Introduction

It can be easily observe that these days as consumption of everything is increasing day by day, so to fulfill the demands any enterprises or company or organization have to always ready. So generally they used to purchase everything in bulk. Sometimes extra discount a leads them to bulk purchase and if you are doing this you surely need extra space to hold all those stuff. This concept of extra storage gives birth to name or word rented warehouse (RW) that they rent for some time. The storage that enterprises have of their own is known as owned warehouse (OW). Also the cost of rented warehouse is always more than that of owned warehouse. So to reduce the total cost to hold an inventory it is better to release the goods of RW as earlier as possible. As a result, the stock in OW won’t be used until the stocks in rented warehouse are exhausted. Hartley [1976] gave a two warehouse model under the assumption that holding cost of RW is greater than OW. On the other hand, Sarma (1983) developed a deterministic inventory model with an infinite replenishment rate..Goswami and Chaudhuri (1992) further developed the model with or without shortages with the assumption that the demand varies over time with a linearly increasing trend, Pakkala and Achary (1992a,b) further considered the two-warehouse model for deteriorating items with finite replenishment rate and shortages, taking time as a discrete and continuous variable, respectively. Subsequently, many authors such as Bhunia and Maiti (1994, 1998), Benkherouf (1997), Zhou (1998), Kar, Bhunia, and Maiti (2001), Zhou and Yang (2005),Sarma K.V.S(1987), Das, Maity, and Maiti (2007), Dye, Ouyang, and Hsieh (2007), Niu and Xie (2008), Rong, Mahapatra, and Maiti (2008) and many others have worked in the area of two-warehousing under different scenarios.

Demand also affects the inventory. Many researchers worked with different kind of demand like constant demand, linear time varying demand, parabolic demand, Exponential demand, stock dependent demand etc. later on , it was observed that the demand does not remain same for a certain item as there always been some new launch e.g. newly launched fashion items, food items, garments, automobile, cosmetic, technology etc. these items comes in the market, their demand increase but after a certain time it become constant. This kind of demand is known as Ramp Type Demand, which depicts that a demand increase up to a certain level and after become stabilize or constant. Gupta and Vrat [7] developed an inventory model where demand rate is replenishment size (initial stock) dependent.

Deterioration is very common these days so it is not easy to neglect its effect. Whether it is food items, other household stuff, and technology etc., deterioration affect everything. Wee HM (1993) is the first one who define deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on by the passing time. Traditionally, it was considered that the items can preserve their characteristics while they kept stored in inventory. But it is not true for all. Considering this fact, now a days it's a great challenge to control and maintain the inventory of deteriorating items for the decision makers. The first EOQ inventory model was developed by Harris (1915), which was further generalized by Wilson (1934) to obtain formula for economic order quantity. After Within (1957) , Ghare and Schrader (1963) Dave and Patel (1981),Chung and Ting (1993); Wee (1995) Goyal and Giri(2001) ,Ouyang and Cheng [2005] , Alamri and Balkhi(2007) gave their respective in the field of inventory of deteriorating items for different demand rate. Karmakark, Dutta choudhury gave an inventory model with ramp-type demand for Deteriorating items with partial Backlogging and time-varying holding cost**.** Chandra K. Jaggi developed a Two-warehouse partial backlogging inventory model for deteriorating items with linear trend in demand under inflationary conditions

Here we develop a model for deteriorating items with ramp type demand in two warehouse RW and OW. RW has unlimited capacity and Own warehouse can store 'w' units at the initial stage. Firstly unit stored in RW used and then OW inventory. Holding cost is constant in both the warehouse. Model is developed to minimize the cost.

2. Assumption & Notation:-

Assumptions

1. Demand rate is ramp type and is given by $D(t) = D_0\left[t-(t-\mu)H(t-\mu)\right]$ where

$$H(t-\mu) = \begin{cases} 0 & t < \mu \\ 1 & t \geq \mu \end{cases}$$

2. Shortages are allowed.
3. Replenishment rate is infinite.
4. Lead time is zero.
5. The OW has a fixed capacity 'w' and RW has unlimited capacity.
6. Firstly the units kept in RW are used and then is of OW.

Notations:-

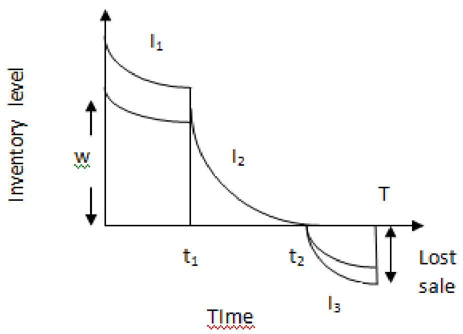
1. $\mathbf{I_1(t)}$ represents the inventory for the rented warehouse(R.W)
2. $\mathbf{I_2(t)}$ is the inventory for the owned warehouse(O.W)
3. $\mathbf{I_3(t)}$ is the inventory for the backorder.
4. w **i**s the capacity of the owned warehouse .
5. **Z** is the total inventory level at initial level.
6. **A** is the deterioration rate in the O.W with $0<\alpha<1$.
7. **B** is the deterioration rate in the R.W with $0<\beta<1$
8. γ is the backlogging rate.
9. **μ** is the parameter at which demand become Stable.
10. t_1 **i**s the time where inventory on RW will become Zero.
11. t_2 is the time where inventory on OW will become Zero.
12. **T** is the time where shortage level reaches to the lowest point due to replenishment.
13. **A** is the replenishment cost per order for two warehouse system.
14. **C** is the purchasing cost per unit.
15. $\mathbf{H_O}$ is the holding cost per unit per unit time in the O.W
16. $\mathbf{H_R}$ is the holding cost per unit per unit time in the R.W.

17. **HC_R** represents the total holding cost in R.W.
18. **HC_O** represents the total holding cost in O.W.
19. **BC** represents the backorder cost.
20. **LSC** represents the lost sale cost.
21. **TC** represents the total cost.

3. Mathematical Formulation:-

At time t =0, the inventory level is Z units. From these w units are kept in owned warehouse (OW) and rest are in rented warehouse (RW). Firstly the units kept in rented warehouse (RW) are consumed and then of owned warehouse (OW). Due to demand continue consumption will takes place and with combined effect of demand and deterioration at time $t=t_1$ the inventory in RW will reach to zero. Then the OW units will be used. And again with the same effects at time $t=t_2$ the OW will also become zero. After this back ordering will takes place and backlogging is also allowed, so this will affect the inventory also.

For the two warehouse inventory model the inventory levels in RW and OW during time period $[0,t_2]$ are given by the following differential equations:-



Graphical Representation of Two Warehouse Inventory System

$$\frac{dI_1(t)}{dt}+\beta I_1(t)=-D(t)\ ,\quad 0\le t\le t_1 \qquad \text{.... (1)} \qquad \text{With boundary condition } I_1(t_1)=0$$

$$\frac{dI_2(t)}{dt}+\alpha I_2(t)=-D(t)\ ,\quad 0\le t\le t_2 \qquad \text{..... (2)} \qquad \text{With boundary condition } I_2(t_2)=0$$

Also as partial backlogging is allowed, the negative inventory level is given by

$$\frac{dI_3(t)}{dt}=\gamma D(t)\ ,\quad t_1\le t\le T \qquad \text{... (3)} \qquad \text{With boundary condition } I_3(t_2)=0$$

As we have taken ramp type demand in this model. The solution of the above equations will be affected by the demand. To find the final result we will discuss this model into three different cases (1) $\mu\le t_1$ **(2)** $t_1\le\mu\le t_2$ **(3)** $t_2\le\mu\le T$

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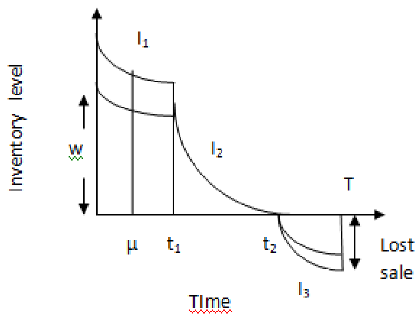
4. CASE I:- $\mu \le t_1$



$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -D_0 t \quad \text{for} \quad 0 \le t \le \mu \qquad \text{......}\textbf{(4.1)}$$

With boundary condition $I_1(\mu_-) = I_1(\mu_+)$

$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -D_0 \mu \quad \text{for} \quad \mu \le t \le t_1 \qquad \text{......}\textbf{(4.2)}$$

With boundary condition $I_1(t_1) = 0$

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = 0 \quad \text{for}\, 0 \le t \le t_1 \qquad \text{......}\textbf{(4.3)}$$

With boundary condition $I_2(0) = w$

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = -D_0 \mu \quad \text{for}\, t_1 \le t \le t_2 \qquad \text{......}\textbf{(4.4)}$$

With boundary condition $I_2(t_2) = 0$

$$\frac{dI_3(t)}{dt} = -D_0 \mu \gamma \quad \text{for}\, t_2 \le t \le T \qquad \text{......}\textbf{(4.5)}$$

With boundary condition $I_3(t_2) = 0$

Solution of above equations is as follows respectively:-

$$I_1(t) = \frac{-D_0}{\beta}\left[\left(t - \frac{1}{\beta}\right) - \mu e^{\beta(t_1 - t)} + \frac{1}{\beta} e^{\beta(\mu - t)}\right] \qquad 0 \le t \le \mu \qquad \text{......}\ (4.6)$$

$$I_1(t) = \frac{-D_0 \mu}{\beta}\left[1 - e^{\beta(t_1 - t)}\right] \qquad \mu \le t \le t_1 \qquad \text{......}\ (4.7)$$

$$I_2(t) = w e^{-\alpha t} \qquad 0 \le t \le t_1 \qquad \text{......}\ (4.8)$$

$$I_2(t) = \frac{-D_0 \mu}{\alpha}\left[1 - e^{\alpha(t_2 - t)}\right] \qquad t_1 \le t \le t_2 \qquad \text{......}\ (4.9)$$

$$I_3(t) = D_0 \mu \gamma (t_2 - t) \qquad t_2 \le t \le T \qquad \text{......}\ (4.10)$$

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Backordering quantity is given by $I_3(t) = Q - Z$ Where Q is the ordering quantity and Z is the initial inventory at for the period which will be equal to the sum of inventory in the OW and RW at t=0. i.e. $Z = w + I_1(0)$

$Z = w + \frac{D_0}{\beta}\left[\left(\frac{1}{\beta}\right) + \mu e^{\beta t_1} - \frac{1}{\beta}e^{\beta\mu}\right]$ Ordering Quantity is given by $Q = Z + I_3(T)$; $Q = Z + D_0\mu\gamma(t_2 - T)$ .Using continuity of $I_2(t)$ at time $t = t_1$. $we^{-\alpha t_1} = \frac{-D_0\mu}{\alpha}\left(1 - e^{\alpha(t_2 - t_1)}\right); t_2 = \frac{1}{\alpha}\log\left(\frac{w\alpha}{D_0\mu} + e^{\alpha t_1}\right)$ Now, the various cost involved in this model are holding cost for RW and OW, backlogging cost, Lost Sale cost, Replenishment Cost and deterioration cost. These are given as follows.

Replenishment cost =A

Holding Cost In Rented Warehouse:- $HC_R = H_R\int_0^{t_1} I_1(t)dt$

$$HC_R = \frac{H_R D_0}{2\beta^3}\left[2 - 2e^{\beta\mu} + 2\mu\beta e^{\beta t_1} - 2t_1\mu\beta^2 + \mu^2\beta^2\right]$$
......(4.11)

Holding Cost In Owned Warehouse:- $HC_O = H_O\int_0^{t_2} I_2(t)dt$

$$HC_O = H_O\left[\frac{w}{\alpha}\left(1 - e^{-\alpha t_1}\right) - \frac{D_O\mu}{\alpha}\left\{(t_2 - t_1) + \frac{1}{\alpha}\left(1 - e^{\alpha(t_2 - t_1)}\right)\right\}\right]$$
......(4.12)

Backlogging Cost:- $BC = C_B\int_{t_2}^{T} -I_3(t)dt$

$$BC = C_B D_0\mu\gamma\frac{(T - t_2)^2}{2}$$ ...... (4.13)

Total Deterioration Cost In RW and OW:- $DC = C\left[\beta\int_0^{t_1} I_1(t)dt + \alpha\int_0^{t_2} I_2(t)dt\right]$

$$DC = C\left[\frac{D_0}{2\beta^2}\left[2 - 2e^{\beta\mu} + 2\mu\beta e^{\beta t_1} - 2t_1\mu\beta^2 + \mu^2\beta^2\right] + \left[w\left(1 - e^{-\alpha t_1}\right) - D_0\mu\left\{(t_2 - t_1) + \frac{1}{\alpha}\left(1 - e^{\alpha(t_2 - t_1)}\right)\right\}\right]\right]$$
...... (4.14)

Lost sale Cost:- $LSC = C_L\int_{t_2}^{T}(1 - \gamma)D(t)\,dt$

$$LSC = C_L D_0\mu(1 - \gamma)(T - t_2)$$ ...... (4.15)

Total cost include in the inventory= Replenishment cost+ holding cost (RW) HC_R ***+ holding cost (OW)*** HC_O

+backlogging cost BC ***+ total deterioration cost*** DC ***+ lost sale cost*** LSC

$$TC = A + \frac{H_R D_0}{2\beta^3}\left[2 - 2e^{\beta\mu} + 2\mu\beta e^{\beta t_1} - 2t_1\mu\beta^2 + \mu^2\beta^2\right] +$$

$$H_O\left[\frac{w}{\alpha}\left(1-e^{-\alpha t_1}\right) - \frac{D_O\mu}{\alpha}\left\{(t_2-t_1) + \frac{1}{\alpha}\left(1-e^{\alpha(t_2-t_1)}\right)\right\}\right] +$$

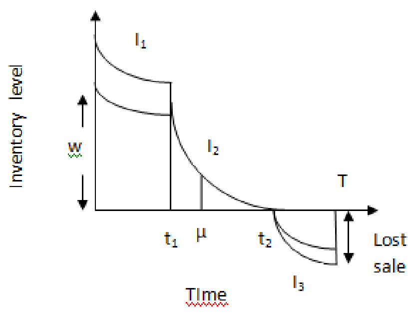
$$C_B D_0 \mu\gamma\frac{(T-t_2)^2}{2} +$$

$$C\left[\frac{D_0}{2\beta^2}\left[2-2e^{\beta\mu} + 2\mu\beta e^{\beta t_1} - 2t_1\mu\beta^2 + \mu^2\beta^2\right] + \left[w\left(1-e^{-\alpha t_1}\right) - D_0\mu\left\{(t_2-t_1) + \frac{1}{\alpha}\left(1-e^{\alpha(t_2-t_1)}\right)\right\}\right]\right]$$

$$+ \ C_L D_0 \mu(1-\gamma)(T-t_2) \qquad \text{...... (4.16)}$$

This equation will represent the total cost of the inventory. To minimize the cost the optimal solution of t_1 and T will be given by solving the following two differential equations:- $\frac{\partial TC}{\partial t_1} = 0$ And $\frac{\partial TC}{\partial T} = 0$ .Also if it will satisfy $\left(\frac{\partial^2 TC}{\partial^2 t_1}\right)\left(\frac{\partial^2 TC}{\partial^2 T}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0$; $\frac{\partial^2 TC}{\partial {t_1}^2} > 0$; $\frac{\partial^2 TC}{\partial T^2} > 0$ Using the above result the minimum average cost can be obtained.

5. <u>Case II:-</u> $t_1 \le \mu \le t_2$



$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -D_0 t, 0 \le t \le t_1 \text{ ...... (5.1)} \qquad \text{with boundary condition } I_1(t_1) = 0$$

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = 0, 0 \le t \le t_1 \qquad \text{...... (5.2)} \qquad \text{with boundary condition } I_2(0) = w$$

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = -D_0 t, t_1 \le t \le \mu \text{ ...... (5.3)} \qquad \text{with boundary condition } I_2(\mu_-) = I_2(\mu_+)$$

$$\frac{dI_2(t)}{dt}+\alpha I_2(t)=-D_0\mu \ , \mu \le t \le t_2 \ ...... (5.4) \qquad \text{with boundary condition } I_2(t_2)=0$$

$$\frac{dI_3(t)}{dt}=-D_0\mu\gamma \ , t_2 \le t \le T \ ...... (5.5) \qquad \text{with boundary condition } I_3(t_2)=0$$

Solution of the above equations is as follows respectively:-

$$I_1(t)=\frac{-D_0}{\beta}\left[\left(t-\frac{1}{\beta}\right)-\left(t_1-\frac{1}{\beta}\right)e^{\beta(t_1-t)}\right] \qquad 0\le t\le t_1 \ ...... (5.6)$$

$$I_2(t)=we^{-\alpha t}, \qquad 0\le t\le t_1 \ ...... (5.7)$$

$$I_2(t)=\frac{-D_0}{\alpha}\left(t-\frac{1}{\alpha}\right)-\frac{D_0}{\alpha^2}e^{\alpha(\mu-t)}+\frac{D_0\mu}{\alpha}e^{\alpha(t_2-t)} \qquad t_1\le t\le \mu \ ...... (5.8)$$

$$I_2(t)=\frac{-D_0\mu}{\alpha}(1-e^{\alpha(t_2-t)}) \qquad \mu\le t\le t_2 \ ...... (5.9)$$

$$I_3(t)=-D_0\mu\gamma(t-t_2) \qquad t_2\le t\le T \ ...... (5.10)$$

Backordering quantity is given by $I_3(t)=Q-Z$ Where Q is the ordering quantity and Z is the initial inventory at for the period which will be equal to the sum of inventory in the OW and RW at t=0. i.e. $Z=w+I_1(0)$

$Z=w+\frac{D_0}{\beta}\left[\left(\frac{1}{\beta}\right)+\left(t_1-\frac{1}{\beta}\right)e^{\beta t_1}\right]$ Ordering Quantity is given by $Q=Z+I_3(T)$; $Q=Z+D_0\mu\gamma(t_2-T)$ Using continuity of $I_2(t)$ at time $t=t_1$. $we^{-\alpha t_1}=\frac{-D_0}{\alpha}\left(t_1-\frac{1}{\alpha}\right)-\frac{D_0}{\alpha^2}e^{\alpha(\mu-t_1)}+\frac{D_0\mu}{\alpha}e^{\alpha(t_2-t_1)}$

$t_2=\frac{1}{\alpha}\log\left(\frac{w\alpha}{D_0\mu}+\frac{1}{\mu}\left(t_1-\frac{1}{\alpha}\right)e^{\alpha t_1}+\frac{1}{\alpha\mu}e^{\alpha\mu}\right)$ The various cost includes are as follows:-

Replenishment cost =A

Holding Cost in Rented Warehouse (RW):- $HC_R=H_R\int_0^{t_1}I_1(t)dt$

$$HC_R=\frac{H_RD_0}{2\beta^3}\left[2+2\beta t_1e^{\beta t_1}-2e^{\beta t_1}-\beta^2t_1^2\right] ...... (5.11)$$

Holding Cost in Owned Warehouse (OW):- $HC_o=H_o\int_0^{t_2}I_2(t)dt$

$$HC_O = H_O\left[\frac{w}{\alpha}\left(1-e^{-\alpha t_1}\right)+\frac{D_0}{\alpha}\left(\frac{\mu^2}{2}+\frac{t_1^2}{2}-\frac{t_1}{\alpha}+\frac{\mu}{\alpha}e^{\alpha(t_2-t_1)}+\frac{1}{\alpha^2}-\frac{e^{\alpha(\mu-t_1)}}{\alpha^2}-\mu t_2\right)\right] \quad \ldots\ldots (5.12)$$

Backlogging Cost:- $BC = C_B\int_{t_2}^{T} -I_3(t)\,dt$

$$BC = C_B D_0 \mu\gamma \frac{(T-t_2)^2}{2} \quad \ldots\ldots (5.13)$$

Total Deterioration Cost in RW and OW:- $DC = C\left[\beta\int_0^{t_1} I_1(t)\,dt + \alpha\int_0^{t_2} I_2(t)\,dt\right]$

$$DC = C\left[\frac{D_0}{2\beta^2}\left[2+2\beta t_1 e^{\beta t_1}-2e^{\beta t_1}-\beta^2 t_1^2\right]+\left[w\left(1-e^{-\alpha t_1}\right)+D_0\left(\frac{\mu^2}{2}+\frac{t_1^2}{2}-\frac{t_1}{\alpha}+\frac{\mu}{\alpha}e^{\alpha(t_2-t_1)}+\frac{1}{\alpha^2}-\frac{e^{\alpha(\mu-t_1)}}{\alpha^2}-\mu t_2\right)\right]\right]$$
...... (5.14)

Lost sale Cost= $LSC = C_L\int_{t_2}^{T}(1-\gamma)D(t)\,dt$

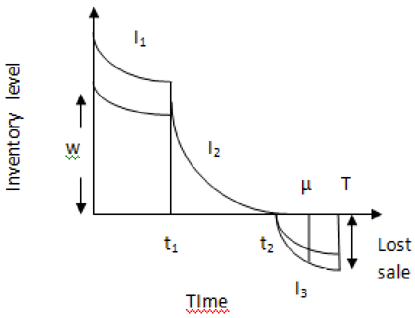
$$LSC = C_L D_0 (1-\gamma)(T-t_2) \quad \ldots\ldots (5.15)$$

Total cost include in the inventory= Replenishment cost+ holding cost (RW) HC_R **+ holding cost (OW)** HC_O **+backlogging cost** BC **+ total deterioration cost** DC **+ lost sale cost** LSC

$$TC = A + \frac{H_R D_0}{2\beta^3}\left[2+2\beta t_1 e^{\beta t_1}-2e^{\beta t_1}-\beta^2 t_1^2\right] + H_O\left[\frac{w}{\alpha}\left(1-e^{-\alpha t_1}\right)+\frac{D_0}{\alpha}\left(\frac{\mu^2}{2}+\frac{t_1^2}{2}-\frac{t_1}{\alpha}+\frac{\mu}{\alpha}e^{\alpha(t_2-t_1)}+\frac{1}{\alpha^2}-\frac{e^{\alpha(\mu-t_1)}}{\alpha^2}-\mu t_2\right)\right] + C_B D_0\mu\gamma\frac{(T-t_2)^2}{2} + C\left[\frac{D_0}{2\beta^2}\left[2+2\beta t_1 e^{\beta t_1}-2e^{\beta t_1}-\beta^2 t_1^2\right]+\left[w\left(1-e^{-\alpha t_1}\right)+D_0\left(\frac{\mu^2}{2}+\frac{t_1^2}{2}-\frac{t_1}{\alpha}+\frac{\mu}{\alpha}e^{\alpha(t_2-t_1)}+\frac{1}{\alpha^2}-\frac{e^{\alpha(\mu-t_1)}}{\alpha^2}-\mu t_2\right)\right]\right] + C_L D_0(1-\gamma)(T-t_2)$$

This equation will represent the total cost of the inventory. To minimize the cost the optimal solution of t_1 and T will be given by solving the following two differential equations:- $\frac{\partial TC}{\partial t_1}=0$ and $\frac{\partial TC}{\partial T}=0$ .Also if it will satisfy $\left(\frac{\partial^2 TC}{\partial^2 t_1}\right)\left(\frac{\partial^2 TC}{\partial^2 T}\right)-\left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2>0$; $\frac{\partial^2 TC}{\partial t_1^{\,2}}>0$; $\frac{\partial^2 TC}{\partial T^2}>0$ Using the above result the minimum average cost can be obtained.

6. CASE III:- $t_2 \le \mu \le T$



$$\frac{dI_1(t)}{dt} + \beta I_1(t) = -D_0 t, \quad 0 \le t \le t_1 \text{ ...... (6.1)}$$ with boundary condition $I_1(t_1) = 0$

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = 0, \quad 0 \le t \le t_1 \text{ ...... (6.2)}$$ with boundary condition $I_2(0) = w$

$$\frac{dI_2(t)}{dt} + \alpha I_2(t) = -D_0 t, \quad t_1 \le t \le t_2 \text{ ...... (6.3)}$$ with boundary condition $I_2(t_2) = 0$

$$\frac{dI_3(t)}{dt} = -D_0 t\gamma, \quad t_2 \le t \le \mu \text{ ...... (6.4)}$$ with boundary condition $I_3(t_2) = 0$

$$\frac{dI_3(t)}{dt} = -D_0 \mu\gamma, \quad \mu \le t \le T \text{ ...... (6.5)}$$ with boundary condition $I_3(\mu_-) = I_3(\mu_+)$

The solutions of the above equations are given as

$$I_1(t) = \frac{-D_0}{\beta}\left[\left(t - \frac{1}{\beta}\right) - \left(t_1 - \frac{1}{\beta}\right)e^{\beta(t_1 - t)}\right] 0 \le t \le t_1 \text{ ...... (6.6)}$$

$$I_2(t) = we^{-\alpha t} . 0 \le t \le t_1 \text{ ..... (6.7)}$$

$$I_2(t) = \frac{-D_0}{\alpha}\left[\left(t - \frac{1}{\alpha}\right) - \left(t_2 - \frac{1}{\alpha}\right)e^{\alpha(t_2 - t)}\right] t_1 \le t \le t_2 \text{ ...... (6.8)}$$

$$I_3(t) = -\frac{D_0\gamma}{2}\left(t^2 - t_2^2\right) t_2 \le t \le \mu \text{ ...... (6.9)}$$

$$I_3(t) = -D_0 \mu t \gamma + \frac{D_0\gamma}{2}\left(\mu^2 + t_2^2\right) \mu \le t \le T \text{ ...... (6.10)}$$

Backordering quantity is given by $I_3(t) = Q - Z$ Where Q is the ordering quantity and Z is the initial inventory at for the period which will be equal to the sum of inventory in the OW and RW at t=0. i.e. $Z = w + I_1(0)$

$Z = w + \frac{D_0}{\beta}\left[\left(\frac{1}{\beta}\right)+\left(t_1-\frac{1}{\beta}\right)e^{\beta t_1}\right]$ Ordering Quantity is given by $Q = Z + I_3(T)$;

$Q = Z + \left[-D_0\mu T\gamma + \frac{D_0\gamma}{2}\left(\mu^2+t_2^2\right)\right]$ Using continuity of $I_2(t)$ at time $t = t_1$;

$we^{-\alpha t_1} = \frac{-D_0}{\alpha}\left[\left(t_1-\frac{1}{\alpha}\right)-\left(t_2-\frac{1}{\alpha}\right)e^{\alpha(t_2-t_1)}\right]$; $t_2 = \left(\frac{w}{D_0}+\frac{1}{\alpha}\left(t_1-\frac{1}{\alpha}\right)e^{\alpha t_1}+\frac{1}{\alpha^2}\right)^{1/2}$

The various cost includes are:-

Replenishment Cost =A

Holding Cost For The Rented Warehouse (RW) Is Given By:- $HC_R = H_R\int_0^{t_1} I_1(t)dt$

$$HC_R = \frac{H_R D_0}{2\beta^3}\left[2+2\beta t_1 e^{\beta t_1} - 2e^{\beta t_1} - \beta^2 t_1^2\right] \quad ......(6.11)$$

Holding Cost For The Owned Warehouse (OW) Is Given By:- $HC_o = H_o\int_0^{t_2} I_2(t)dt$

$$HC_O = H_O\left[\frac{w}{\alpha}\left\{1-e^{-\alpha t_1}\right\}+\frac{D_0}{2\alpha^3}\left[2+2\alpha t_2 e^{\alpha(t_2-t_1)}+\alpha^2 t_1^2-\alpha^2 t_2^2-2\alpha t_1-2e^{\alpha(t_2-t_1)}\right]\right] \quad ......(6.12)$$

Backlogging Cost:- $BC = C_B\int_{t_2}^{T} -I_3(t)dt$

$$BC = \frac{-D_0\gamma C_B}{2}\left[\frac{\mu^3}{3}+\frac{2}{3}t_2^3+\mu T^2-\mu^2 T - T t_2^2\right] \quad ......(6.13)$$

Lost Sale Cost:- $LSC = C_L\int_{t_2}^{T}(1-\gamma)D(t)dt$

$$LSC = C_L(1-\gamma)D_0\left[\left\{\frac{\mu^2}{2}-\frac{t_2^2}{2}\right\}+\mu(T-\mu)\right] \quad ......(6.14)$$

Deterioration Cost:- $DC = C\left[\beta\int_0^{t_1} I_1(t)dt + \alpha\int_0^{t_2} I_2(t)dt\right]$

$$DC = \frac{CD_0}{2\beta^2}\left[2+2\beta t_1 e^{\beta t_1}-2e^{\beta t_1}-\beta^2 t_1^2\right] + C\left[w\left\{1-e^{-\alpha t_1}\right\}+\frac{D_0}{2\alpha^2}\left[2+2\alpha t_2 e^{\alpha(t_2-t_1)}+\alpha^2 t_1^2-\alpha^2 t_2^2-2\alpha t_1-2e^{\alpha(t_2-t_1)}\right]\right]$$

Total cost include in the inventory= Replenishment cost+ holding cost (RW) HC_R **+ holding cost (OW)** HC_O **+backlogging cost** BC **+ total deterioration cost** DC **+ lost sale cost** LSC

$$TC = A + \frac{H_R D_0}{2\beta^3}\left[2 + 2\beta t_1 e^{\beta t_1} - 2e^{\beta t_1} - \beta^2 t_1^2\right] + H_O\left[\int_0^{t_1} we^{-\alpha t}dt + \int_{t_1}^{t_2}\left(\frac{-D_0}{\alpha}\right)\left[\left(t - \frac{1}{\alpha}\right) - \left(t_2 - \frac{1}{\alpha}\right)e^{\alpha(t_2 - t)}\right]dt\right] +$$

$$\frac{-D_0 \gamma C_B}{2}\left[\frac{\mu^3}{3} + \frac{2}{3}t_2^3 + \mu T^2 - \mu^2 T - Tt_2^2\right] + C_L(1-\gamma)D_0\left[\left\{\frac{\mu^2}{2} - \frac{t_2^2}{2}\right\} + \mu(T - \mu)\right] +$$

$$\frac{CD_0}{2\beta^2}\left[2 + 2\beta t_1 e^{\beta t_1} - 2e^{\beta t_1} - \beta^2 t_1^2\right] +$$

$$C\left[w\left\{1 - e^{-\alpha t_1}\right\} + \frac{D_0}{2\alpha^2}\left[2 + 2\alpha t_2 e^{\alpha(t_2 - t_1)} + \alpha^2 t_1^2 - \alpha^2 t_2^2 - 2\alpha t_1 - 2e^{\alpha(t_2 - t_1)}\right]\right]$$

This equation will represent the total cost of the inventory. To minimize the cost the optimal solution of t_1 and T will be given by solving the following two differential equations:- $\frac{\partial TC}{\partial t_1} = 0$ and $\frac{\partial TC}{\partial T} = 0$ .Also if it will satisfy $\left(\frac{\partial^2 TC}{\partial^2 t_1}\right)\left(\frac{\partial^2 TC}{\partial^2 T}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0$; $\frac{\partial^2 TC}{\partial t_1{}^2} > 0$; $\frac{\partial^2 TC}{\partial T^2} > 0$ Using the above result the minimum average cost can be obtained.

Conclusion

Here we have developed a model for two warehouse system with ramp type demand and constant holding cost. The model is developed to minimize the total cost that include holding cost, backlogging cost, lost sale cost and deterioration cost. Here we find the various conditions which decision maker can keep in mind so as to make this model beneficiary to them. We are also working practically on this model by taking suitable numerical examples.

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