Innovative Systems Design and Engineering
ISSN 2222-1727 (Paper) ISSN 2222-2871 (Online)
Vol.5, No.10, 2014
www.iiste.org

Study of Linear Quadratic Regulator (LQR) Control Method and Its Application to Automatic Voltage Regulator (AVR) System

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Abstract

Automatic Voltage Regulator (AVR) play a great role in terminal voltage profile design process in a generator. Load state changing as dynamic system behavior will change the current flow in the generator system that result in armature voltage and terminal voltage change.
This final project try to make a generator terminal voltage profile control mechanism using Linear Quadratic Regulator method. This method implemented in *Automatic Voltage Regulator* (AVR) model system. This Model control system is very cheap because it does not need a complex control mechanism that tends to like PID controller. In *Linear Quadratic Regulator* (LQR) method decided Q and R weight matrix to gain the optimal control signal from feedback state to raise AVR performance in maintaining system stability.

Keywords : AVR, LQR, Stability, Bandwidth, Settling Time.

I. BACKGROUND

Quality of electrical energy for some parameters such as frequency fluctuations, voltage fluctuations, flicker, harmonics and continuity of energy services. Of all the parameters above, the voltage fluctuations that most often get the attention from the experts and operators of power systems. Some of developed countries such as France, Japan, and USA gave special attention and treatment in voltage stability problems. Voltage instability will lead to instability of the whole power system, especially the security system, the quality and the ability to transfer power from the generator to the consumer, and the most severe conditions is load shedding mechanism or brown out. There are several factors that cause voltage instability (voltage collapse) that is, the performance of the AVR, the increase in load transmission line, reactive power constraint settings, dynamics OLTC (on load tap changer) transformer and load characteristics. This study is the first phase of designing a linear control system to control *Automatic Voltage Regulator* (AVR) system in the form of simulation. The condition for using the mhetod above is a model system *Automatic Voltage Regulator* (AVR) must be linear. To obtain a linear model, we must linearized the model of the system at a particular operating point.

This research was conducted with several conditions :

1. System *Automatic Voltage Regulator* (AVR) which to be controlled is a system that is linear and continuous.
2. Performance analysis is done with the time domain and frequency domain.
3. Determination of Q and R weighting functions performed by trial and error.
4. Simulation was done using MATLAB software 2010.

II. Literature Review

A. Modeling System

System Automatic Voltage Regulator (AVR) generator is an equipment which charge to maintaining the stability and reactive power generator to keep the value of the desired work. In Figure 1 below it can be seen from a simple schematic diagram of the AVR.

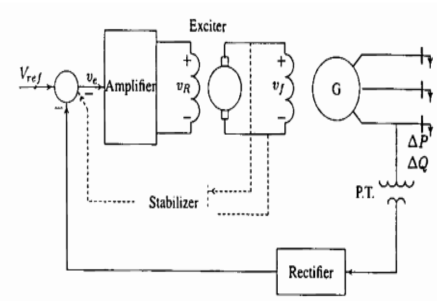

Figure 1. Simple schematic diagram AVR system. [12]

The parameters that must be considered in the design automatic Voltage Regulator which include Gain amplifier, Gain Exciter, Gain sensor, Gain Generator, and Gain Stabilizer. [12].

1. Amplifier Model

In the excitation system, the amplifier can be a magnetic amplifier, rotating amplifier or amplifier electronic systems. Amplifier model represented in a first-order system with an amplification factor and time constant. Amplifier transfer function is shown in equation (1)

$$\frac{V_R(s)}{V_E(s)} = \frac{K_A}{1+\tau_A} \qquad (2.1)$$

K_A value has ranges from 10-400, while the time constants amplifier has very little value which is 0.02 s - 0.1 s.[16]

2. Exciter Model

In Figure 2.5 above shows that the excitation source is using the source of the main generator output which then converted by using a rectifier circuit. The output voltage of the exciter system is non-linear and is a function of the voltage field caused by magnetic core saturation effect, so it can be seen that the relation between the terminal voltage and the voltage of the exciter field is very complex. There are so many models are available with varying levels of accuracy available in IEEE publications. Modern exciter model can be approximated by eliminating the effect of saturation but with great attention to the time constant [12]. For the amplifier transfer function is shown in equation (2) below.

$$\frac{V_F(s)}{V_R(s)} = \frac{K_E}{1+\tau_E s} \qquad (2.2)$$

3. Generator Model

Emf that generated by the synchronous generator is a function of the magnetization, while the terminal voltage depending on the load. For the linear model of a generator can be approximated by a first-order system, which has a constant amplification factor KG and TG time constant [16]. The generator model is expressed in equation (3) below.

$$\frac{V_R(s)}{V_E(s)} = \frac{K_G}{1+\tau_G s} \qquad (2.3)$$

4. Sensor Model

An increase in reactive power load side will result the decrease of terminal voltage magnitude. Decrease in terminal voltage will then be screened by a potential transformer.

Furthermore, the terminal voltage will be rectified and compared to a reference value point. The amplifier error signal controlle will set the excitation voltage so the generator excitation voltage will increase. If the excitation voltage increases, the power that generated by the generator will also rise. For the transfer function of the sensor models can be seen from equation (4) below.

$$\frac{V_s(s)}{V_t(s)} = \frac{K_R}{1+\tau_R s} \qquad (2.4)$$

And the modeling for each from the above parameters and from simple schematic diagram that contained in Figure 2.5 above, then we get the model system of Automatic Voltage Regulator (AVR) without stabilizer then shown in Figure 2 below **[10],[16]**

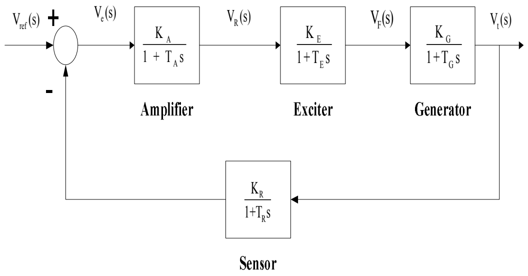

Figure 2 .. Model System Automatic Voltage Regulator (AVR) Conventional

Close loop transfer function which connect terminal voltage (V_t) and voltage reference (V_{ref}) expressed by equation (5) below.

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{K_A K_E K_G (1+T_R s)}{(1+T_A s)(1+T_E s)(1+T_G s)(1+T_R s)+K_A K_E K_G K_R} \qquad (5)$$

Furthermore, the Automatic Voltage Regulator (AVR) model system will be shown, the another conventional generator which consists of potential transformers function as a sensor terminal voltage, the rectifier and amplifier equipped with excitation voltage stabilizer shown in Figure 3 below.

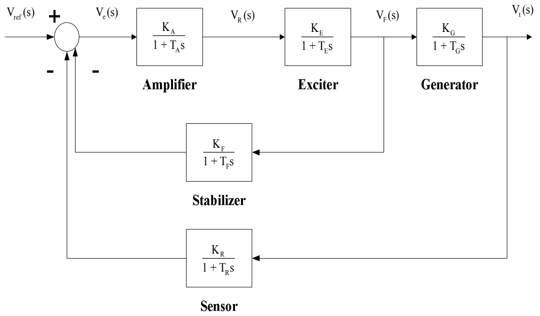

Figure 3. Model System Automatic Voltage Regulator (AVR) Conventional With Stabilizer [16]

For stabilizer represented by the transfer function in equation (6) below.

$$\frac{V_F(s)}{V_T(s)} = \frac{K_F}{1+\tau_F s} \qquad (6)$$

Close loop transfer function which connect terminal voltage and voltage reference (V_{ref}) (V_{ref}) expressed by equation (7) below.

$$\frac{V_t(s)}{V_{ref}(s)} = \frac{K_A K_E K_G (1+T_R s)(1+T_F s)}{(1+T_A s)(1+T_E s)(1+T_G s)(1+T_R s)(1+T_F s) + K_A K_E K_F (1+T_G s)(1+T_R s) + K_A K_E K_G K_R (1+T_F s)} \qquad (7)$$

B. *Linear Quadratic Regulator* (LQR)

For stabilizer expressed by *Linear Quadratic Regulator* (LQR) is a modern control techniques that use the equation of state (state space). transfer function in equation (6) follows. Control system which will be reviewed given by equation (8) below.

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (8)$$

where the matrix K will be determined from the optimal control vector in equation (9) below.

$$u(t) = -Kx(t) \qquad (9)$$

by minimizing the performance index in equation (10) and (11) below.

$$J = \int_0^\infty \left(x'(t)Q\,x(t) + u'(t)Ru(t)\right) dt$$

or

$$J = \int_0^\infty \left(y'(t)Q\,y(t) + u'(t)Ru(t)\right) dt$$

where Q is a real symmetric matrix positive definite (or positive semidefinite), and R is a real symmetric matrix positive definite. Q and R matrix determine the relative importance of the error and energy needs. In addition it is considered that the vector control $u(t)$ without constraints.

$$\dot{x}(t) = Ax(t) - BKx(t) = (A - BK)x(t) \qquad (11)$$

On the decline equations below, matrix $(A - BK)$ are considered stable.

$$J = \int_0^\infty x'(t)\left(Q + K'RK\right)x(t)\,dt \qquad (12)$$

Based on the derived parameter optimization problem that achieve equation (13) below.

$$x'(t)\left(Q + K'RK\right)x(t) = -\frac{d}{dt}\left(x'(t)Px(t)\right)$$

Finally obtained the following equation:

$$(A - BK)'P + P(A - BK) = -\left(Q + K'RK\right) \tag{14}$$

From the second method Liapunov if $(A - BK)$ is a stable matrix, then there is a positive definite matrix P that qualified equation (14). Furthermore, keeping in mind that the performance index is expressed in equation (15) follows

$$J = x'(0)P\,x(0) \tag{15}$$

C. Design Criteria

In control system design using the method of Linear Quadratic Regulator (LQR) must have fulfill the predetermined design. Criteria for the design of control systems by the method of Linear Quadratic Regulator (LQR) are:

For the performance of the system in the time domain is

- Maximum Overshoot is less than 5%
- The peak value for steps input is less than 1 second
- Rise time (*rise time*) (T_r) less than 0.500 second
- For the performance of the system in the frequency domain is
- The system is solid (robust) to disruption as indicated by the value of the maximum peak sensitivity (sensitivity) smaller than 2.
- The system has the ability to reduce the noise (noise) at high frequencies and have a rapid response to input as indicated by the value of the maximum peak sensitivity complementary (complementary sensitivity) less than 1.25.
- Close loop System (closed loop system) has a wide band (bandwidth) of less than 15 rad / sec.

III. Research Methodology

A. Parameters Data System Automatic Voltage Regulator (AVR)

Parameter	**Value**
Ka	10.0000
Ta	0.1000
Ke	1.0000
Te	0.4000
Kg	1.0000
Tg	1.0000
Kr	1.0000

B. Transfer Function of *Automatic Voltage Regulator* (AVR) system

By inserting the values in part (a) into the equation, then obtained the open loop transfer function stated in the following equation:

$$\frac{V_t}{V_{ref}} = \frac{10}{0.04s^3 + 0.54s^2 + 1.50s + 1}$$

For a closed loop transfer function expressed in the following equation:

$$\frac{V_t}{V_{ref}} = \frac{0.5000s + 10}{0.002s^4 + 0.067s^3 + 0.615s^2 + 1.550s + 11}$$

C. Controller Design Procedure

Controller design Linear Quadratic Regulator (LQR) procedure is as follows:

1. Enter the nominal value of object control matrix A, B, C and D.
2. Next it will be checked whether the system is under control (controllability) and observed (observability).
3. Furthermore, Q and R matrix selected by trial and error, which is useful to get feedback value (K) that is able to minimize the cost function J.

4. The performance of the controllers are designed to be tested in a variety of conditions in both the frequency and time.

IV. Result and Discussion

Comparison result of Design Automatic Voltage Regulator (AVR) Control System

Based on the analysis of the AVR system without the method of LQR, AVR system using LQR method, and with the addition of stabilizer AVR system, then the system can be compared to the response in the time domain, frequency domain, the robustness analysis for the three systems.

1. Analysis in Domaian Time.

Time domain analysis consists of an open loop performance and closed loop performance. Based on the analysis of AVR system without LQR method, the analysis of the system with LQR method, and the analysis of AVR system with the addition of stabilizer in the time domain, it can be seen the comparison to an open loop system in Table 1 below.

Tabel 1. Comparison of Open System Performance in the Time Domain.

Criteria	Without LQR method	With LQR method
System type	0 (null)	0 (null)
Error Position Constant (Kp)	10.0000	0.8812
Speed Error Constant (Kv)	0.0000	0.0000
Acceleration Error Constant (Ka)	0.0000	0.0000
Error Steady State for Step input	0.0909	0.5316
Error Position Constant (Kp)	Infinite	Infinite
Error Steady State for Parabolic input	Infinite	Infinite

comparisons between systems without LQR method, system LQR methods, and systems with the addition of stabilizer on performance of a closed loop system.

Tabel. 2. Comparison of Closed loop System Performance in the Time Domain.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
Rise Time (Tr)	0.2534 second	0.953 second	1.0115 second
Settling Time (Ts)	19.0812 second	3.5506 second	1.9127 second
Maximum Overshoot	82.7892 %	0.0000 %	0.0000 %

Based on the comparison of a closed loop system in Table 2, it appears that the settling time is the time required to reach a state of stability in the system, so it can be concluded that the system using LQR method has a settling time is faster than other systems.

In addition to performance of open loop, in the time domain can also analyze the stability of the system using the equation of the transfer function characteristic of a closed loop system. In Table 3 shows the comparison of systems without LQR method, the system with LQR method, and with the addition of stabilizer systems based stability analysis using the characteristic equation.

Tabel 3. Stability Analysis in the Time Domain Using the Characteristic Equation.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
Characteristic Equation Roots	$S_1 = -0.2020 + i4.4800$ $S_2 = -0.2020 - i4.4800i$ $S_3 = -16.5000 + i0.4680$ $S_4 = -16.5000 - i0.4680$	$S_1 = -3.69+3.69i$ $S_2 = -3.69-3.69i$ $S_3 = -1.04$	$S_1 = -2.0800$ $S_2 = -4.6500 + i15.2000$ $S_3 = -4.6500 - i15.2000$ $S_4 = -19.4000$ $S_5 = -103.0000$
Damping factor (ς)	Root s_1 and s_2 is 0.0451 Root s_3 and s_4 is 1.0000	Root s_1 and s_2 is 0.707 Root s_3 and s_4 is 1.0000	Root s_1, s_4, and s_5 is 1.0000 Root s_2 and s_3 is 0.2920
Natural Frequency (ω_n)	Root s_1 and s_2 is 4.4800 rad/second Root s_3 and s_4 is 16.6000 rad/second	Root s_1 and s_2 is 5.22 Root s_3 is 1.04	Root s_1 is 2.0800 rad/second Root s_2 and s_3 is 15.9000 rad/second Root s_4 and s_5 each value is 19.4000 rad/second and 103.0000 rad/second

2. Analysis in Frequency Domain

Analysis in the frequency domain is also composed of open loop system and performance in closed loop system. Based on the discussion of the analysis for AVR system without using LQR method, analysis AVR system using LQR method, and analysis AVR system with the addition of stabilizer in the frequency domain, it can be compared to the response to an open loop system and a closed loop system responses in Table 4 and Table 5 below .

Tabel 4. Response Open loop Systems in the Frequency Domain.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
margin Strengthening	1.9250 dB	6.2883dB	11.2476 dB
Frequency Margin Strengthening	6.1238 rad/second	10.2082 rad/second	16.6823 rad/second
phase margin	18.5936^{0}	nan	nan
Frequency Phase Margin	4.4050 rad/second	nan	nan

Based on the analysis in Table 4, the performance of the system using the method is better because it has a margin of more than 6 dB amplifier.

Tabel 5. Response Close loop system.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
Bandwidth	0.9750 rad/second	4.72 rad/second	2.2700 rad/second
Peak Magnitude	----	----	----
Peak Frequency	----	----	----

Based on Table 5, a change in the value of the bandwidth (bandwidth) for the system without using LQR method is 0.9750 rad / sec, for a system using LQR method is 4.7200 rad / sec, and for the system with the addition of stabilizer is 2.2700 rad / sec. With increasing bandwidth values, it will accelerate the rise time (Tr) and settling time (Ts) of the closed loop system response in the time domain, so the system uses the method will accelerate the rise time (Tr) and settling time (Ts) of the system AVR.

3. Robustness Analysis System

Analysis of the robustness of the system using criteria that consist of a maximum peak value of the sensitivity maximum peak value and complementary sensitivity peak maximum. In Table 6 shows the comparison criteria between the maximum peak without using LQR method, system using LQR method, and the system with the addition of stablizer.

Tabel 6. Kriteria Puncak Maksimum.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
Maximum Peak Value Sensitivity (MS)	9.9795	1.3518	1.1098
Complementary Sensitivity Maximum Peak Value (MT)	9.6315	0.8812	0.4762

For a system that is robust to interference and noise attenuation at high frequencies, the value of the maximum peak sensitivity (MS) less than 2 (6 dB) and a maximum peak value of the complementary sensitivity (MT) less than 1.25 (2 dB), so that for the system without using LQR method is theoretically exceed the value so that it can be said the system is not robust to the disruption and to the system using LQR method and use stabilizer meets the theoretical value so that the system robust to interference and noise attenuation at high frequencies.

Based on the analysis of the AVR system without the method of LQR, AVR system using LQR method, and with the addition of stabilizer AVR system, then the system can be compared to the response in the time domain, frequency domain, the robustness analysis for the three systems.

1. Analysis in Time Domain

Time domain analysis consists of an open loop performance and closed loop performance. Based on the analysis of AVR system without LQR method, the analysis of the system with LQR method, and the analysis of AVR system with the addition of stabilizer in the time domain, it can be seen the comparison to an open loop system in Table 7 below.

Tabel 7. Comparison of Open System Performance in the Time Domain.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
System type	0 (nol)	0 (nol)	0 (nol)
Error Position Constant (Kp)	10.0000	0.8812	0.9091
Speed Error Constant (Kv)	0.0000	0.0000	0.0000
Acceleration Error Constant (Ka)	0.0000	0.0000	0.0000
Error Steady State for Step input	0.0909	0.5316	0.5238
Error Steady Stead for Rate input	Infinite	Infinite	Infinite
Error Steady State for Parabolic input	Infinite	Infinite	Infinite

Based on a comparison between the three systems AVR for closed loop in the time domain, it appears that the value of the position error constant (Kp) changed from 10.0000 to the system without LQR method to be 4.6522 for the system with LQR method and 0.9091 for the system with the addition of stabilizer. Steady state error value input unit steps is 0.9090 for the system without LQR method turned out to be 0.1769 for the system with LQR method and 0.5238 for the addition of stabilizer systems. Changes in error constants (Kp) and the steady state error coefficient AVR system with LQR method will improve the accuracy and performance of the excitation system.

Table 8 shows the comparison between systems without LQR method, system LQR methods, and systems with the addition of stabilizer on performance of a closed loop.

Tabel. 8. Comparison of System Performance Closed loop in the Time Domain.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
Rise Tine	0.2534 second	0.4639 second	1.0115 second
Settling Time (Ts)	19.0812 second	1.2353 second	1.9127 second
Maximum Overshoot (Mp)	82.7892 %	3.5506%	0.0000 %

Based on the comparison of a closed loop system in Table 7, it appears that the settling time (Ts) is 19.0812 seconds changes to the system without LQR method to be 1.2353 seconds for the system using LQR method and 1.9127 seconds for the system with the addition of stabilizer. Settling time is the time required to reach a state of stability in the system, so it can be concluded that the system using LQR method has a settling time is faster than other systems.

In addition to performance of open loop, in the time domain can also analyze the stability of the system using the equation of the transfer function characteristic of a closed loop system. In Table 8 shows the comparison of systems without LQR method, the system with LQR method, and with the addition of stabilizer systems based stability analysis using the characteristic equation.

Table 8. Stability Analysis in the Time Domain Using the Characteristic Equation.

Criteria	Without LQR method	With LQR method	Adding Sbalizer
Characteristic Equation Roots	S_1 = -0.2020 + i4.4800 S_2 = -0.2020 - i4.4800i S_3 = -16.5000 + i0.4680 S_4 = -16.5000 – i0.4680	S_1 = -3.69+3.69i S_2 = -3.69-3.69i S_3 = -1.04	S_1 = -2.0800 S_2 = -4.6500 + i15.2000 S_3 = -4.6500 - i15.2000 S_4 = 19.4000 S_5 = -103.0000
Damping factor (ς)	Root s_1 ands$_2$ is 0.0451 Root s_3 dan s_4 is 1.0000	Root s_1 and s_2 is 0.707 Root s_3 dan s_4 is 1.0000	Root s_1, s_4, and s_5 is 1.0000 Root s_2 dan s_3 is 0.2920
Natural Frequency (ω_n)	Root s_1 and s_2 is 4.4800 rad/second Root s_3 and s_4 is 16.6000 rad/ second	Root s_1 and s_2 sebesar 5.22 Root s_3 is 1.04	Root s_1 is 2.0800 rad/ second Root s_2 and s_3 is 15.9000 rad/ second Root s_4 and s_5 is 19.4000 rad/ second and 103.0000 rad/ second

V. Conclusion And Suggestion

5.1 Conclusion

Based on the overall results of the analysis that has been done in this thesis, it can be concluded that the implementation of the Linear Quadratic Regulator (LQR) control for the Automatic Voltage Regulator (AVR) system is well done. By looking at the results of simulations that have been conducted, several conclusions can be drawn as follows:

1. The results of optimization using LQR method has the most optimal response to the value of the weight matrix Q = [0.1 12:01 19.5] and R = 1
2. Effect of optimal control with LQR method on the control system Automatic Voltage Regulator (AVR) produces a faster response time. Comparison of settling time (settling time) before and after using LQR feedback, can be seen in table 9.

	T_s (*time settling*)
Without LQR method	19.0812 second
With LQR method	1.2353 second
Adding Sbalizer	1.9127 second

5.2 Suggestion

The selection of weight matrix Q and R in this thesis is done by trial and error (trial and error), which takes a long time to get results. Therefore, there need to develop a better methods to get the value of the matrix Q and R .

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