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A Novel Method for Evaluating the Dynamic Load Factor of An Involute Gear Tooth with Asymmetric Profiles

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Abstract
In this paper a new generation of asymmetric tooth profile gear is considered to enhance the dynamic behavior and vibroacoustic properties of toothed gear system. This paper presents a non linear dynamic model as a single degree of freedom equation for teeth meshing gear system which includes static and dynamic transmission error in order to investigate the influence of time varying mesh stiffness and periodic tooth errors on dynamic load factor for symmetric and asymmetric spur teeth profile. A new model of nonlinear time varying mesh stiffness is based on four types of deflections with consideration a small pressure angle for loaded tooth profile side and high pressure angle for another side. The complicated variation of meshing stiffness as a function of contact point along the mesh cycle is studied. Typical dynamic load factor equations are developed for symmetric and asymmetric tooth gear in single and double tooth contact by studied symmetrictooth with pressure angle ($20^0/20^0$) and two pairs of asymmetric teeth ($14.5^0/25^0$ & $20^0/25^0$). The effect of pressure of asymmetry and static transmitted load on transmission error and dynamic load factorare studied. The results indicate enhancement percentage in transmission error and dynamic load factor for asymmetric teeth profile compare with that symmetric tooth profile .
Keywords: Asymmetric spur gear, Transmission Error , Non-linear mesh stiffness

1. Introduction

For the combination of high speeds and heavy loads encountered in modern engineering applications of the toothed gear, a precise analysis of the gear dynamic behavior is imperative. A New generation of asymmetric teeth gear play important role to increase load capacity, endurance ,long life and reduction vibration and noise. Transmission error (Tm) which is mean the difference between theoretical and actual angular position of driven gear when driver gear operating at constant speed ,therefore transmission error represent major excitation source for vibration and noise in geared system ,and reduction in transmission error represent major aim for researchers many decades ago, moreover gear vibration and noise level arise due to other several reasons [1] such as the error in the gear teeth profile at the contact point , misalignment between shaft axes , impact between mating teeth ,backlash, sliding and rolling friction between mating surface of gears ,bearing and housing ….etc. Most efforts to reduce the vibration and noise generation at the mesh have been directed towards improving the accuracy of manufacture. But, experience proves that the improving of manufacturing accuracy does not reduce the vibration and noise level considerably [2]. Several studies in literature have been conducted on the design and stress analysis of asymmetric tooth gear, little of them transact this approach dynamically, kaplelevich[3] present analytical method to design a gear with asymmetric tooth side surface, he consider a high pressure angle for the drive side and low pressure angle for the coast side teeth , Yang [4] provide geometrical modeling to design the asymmetric helical gear meshing when assembly errors are present, he constructed Stress analysis for the helical and the cylindrical form ,Mallesh et al. [5,6] generate asymmetric spur gear tooth geometry for different pressure angles on drive and coast side using computer programme to create a finite element model of gear tooth and investigate the effect of bending stress at the critical section for different pressure angles, different number of teeth and module , Ekwaro-Osire et al. [7] employ the inverse problem technique for asymmetric gear teeth which include photo elastic experimental work , Wang et al.[8] extend the edge – based smoothed point interpolation method (ES-IPM) in the bending strength analysis of asymmetric gear with various drive pressure angles side which generated by a special rack cutter , Agrawal et al. [9] Had been tested an asymmetric gear virtually with ANSYS code under a predefined loading and it has been investigated how bending stress changes at the fillet region of the asymmetric gear .Karpat et al . [10] present dynamic analysis of spur gear with symmetric and asymmetric teeth gear ,they consider high pressure angle for the drive side and low pressure angle profile for the coast side teeth ,they develop a MATLAB-based virtual tool to analyze dynamic behavior of spur gears with asymmetric teeth. In This work a new mathematical model for nonlinear mesh stiffness and dynamic load factor formula are developed for symmetric & asymmetric teeth meshing gear system then investigate the influence of asymmetry on dynamic load factor and transmission error .

2. Mathematical Model

The simplest model, a single degree of freedom (SDOF) model of geared rotor systems shown in Figure (1) is considered in this paper ,where shafts and bearing are assumed to be rigid i.e. shafts , bearings are flexibilities ,the motor and load inertias are not considered in this model. As shown in Fig(1) geared system is modeled using two rotary inertia J1 , J2 , time varying mesh stiffness K m(t) , mesh damping Cm and e(t) denote periodic profile error. The governing differential equations of motion for the system can be derived by using Lagrange method for torsional model as :

$$J_1 \ddot{\theta}_1 + c_m \left(r_{b1} \dot{\theta}_1 - r_{b2} \dot{\theta}_2 - \dot{e}(t)\right) + k_m \left(r_{b1} \theta_1 - r_{b2} \theta_2 - e(t)\right) = T_1 \quad (1)$$

$$J_2 \ddot{\theta}_2 - c_m \left(r_{b1} \dot{\theta}_1 - r_{b2} \dot{\theta}_2 - \dot{e}(t)\right) - k_m \left(r_{b1} \theta_1 - r_{b2} \theta_2 - e(t)\right) = T_2 \quad (2)$$

Where r_{b1} & r_{b2} represent base radius for pinion and gear respectively , θ_1 & θ_2 represent angular displacement for gear and pinion respectively and T_1 & T_2 are external torque . reduction in two equation (1-2) was done to equations above which represent two (DOF) geared system as :

$$m_e \ddot{x} + c_m \dot{x} + k_m(t)\, x = F \quad (3)$$

Where: $$m_e = \frac{(J_1 \ J_2)}{(J_1 rb_2{}^2 + J_2 rb_1{}^2)} \quad (4.a)$$

$$F = \frac{T_1}{r_{b1}} = \frac{T_2}{r_{b2}} \quad (4.b)$$

It is a usual practice to introduce the variable *x* as:

$$x = r_{b1} \theta_1 - r_{b2} \theta_2 - e(t) \quad (5)$$

Equation (5) represent dynamic transmission error which is defined as the difference between the actual and ideal positions of driven gear which usually expressed as a linear displacement along the line of action . profile error can be defined as a periodic function :

$$e(t) = \sum_{n=1}^{\infty} E_n \sin(\omega_{mesh}\, t + \emptyset) \quad (6)$$

Where: ω_{mesh} mean mesh frequency which equal frequency (Hz) of rotation multiply by number of gear teeth and $\emptyset$ pressure angle.

2.1. *Static & Dynamic transmission error*

Table (1) recognize significantly the difference between loaded and unloaded static & dynamic transmission error cases , dynamic transmission error can be obtained by solving governing differential equation analytically or approximately , also we can find loaded static transmission error (x_s) by deleting derivatives terms in equation (3) to :

$$x_s(t) = \frac{F}{k_m(t)} + e(t) \quad (7)$$

2.2 Dynamic mesh load F_d

An alternative formulation for the equation of motion can be given by expressing the dynamic mesh load as [11]:

$$F_d = c_m \dot{x} + k_m(t)\, x \quad (8)$$

Dynamic load factor which define as the ratio between the maximum dynamic load to static load and can be found from equation as :

$$DLF = \frac{F_{d\,max}}{F} \quad (9)$$

The solution of governing differential equation (3) was obtained by numerical analysis. Classical fourth order Runge kutta method for initial value problem is used after transform equation (3) into *state –space* equations . This method is of simplicity and facility for obtaining transient response , and even response can be obtained for long time calculation by using Matlab code as well as FFT method can be used to obtain amplitude spectrums in frequency domain .It should be noted [12] that the above equations are valid when there is contact between two gears .when separation occurs between two gears .because the relative vibrations and backlash between the teeth of gears ,the dynamic gear load will be zero and the equation of motion will be:

$$m_e \ddot{x} = F \quad (10)$$

3. Gear Teeth compliance

When geared system loaded and rotate, many components of deflections occur at mating teeth gear such as bending ,shear , hertizian and foundation deflection as a result of transmitted load. nonlinearity in mesh stiffness is considered in this paper to investigate a real perception of dynamic behavior of mating gear.

Symmetric as well as asymmetric spur gear teeth compliance based on bending deflection (positive & negative) ,shear deflection , Hertizian deflection and foundation deflection is presented in this work. The overall deflection for mating gear in the direction of applied load

$$\delta_t = \delta_p + \delta_f + \delta_h + \delta_s \quad (11)$$

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3.1 Bending & shear deflection

Bending and shear deflections are calculated based on strain energy theory ,we assume that the tooth is an elastic beam based on a rigid foundation fig (2) ,and From strain energy theory [13]:

Total energy due to external work is :

$$U_t = \frac{1}{2}\int\left(\frac{M^2}{E.I}+\frac{fn^2}{E.A}+\frac{ft^2}{G.A}\right)dy \tag{12}$$

Where:

M = Bending moment .

$ft =$ Tangential force

$fn =$ Normal force

$I =$ Moment of inertia

$A = b.t$

$k = \frac{12+11.v}{10+10.v}$

also:

E = modules of elasticity.

G = shear modules of elasticity.

K = shape form factor.

$\emptyset$ = pressure angle for loaded side

3.2 Strain energy for symmetric teeth profile

We may consider symmetric gear tooth as a very short cantilever beam with the part inside the base circle modeled as a rectangular beam , and the part outside the base circle as triangular beam as in fig (2) .

From gear tooth definition , we have

$$y_{bj} = r_{bj} - r_{dj} \tag{13}$$

$$y_{dj} = r_{aj} - r_{dj} \tag{14}$$

j Denote gear teeth pair . You should be noted that amount of thickness at each tooth section above the base circle proportional with the height of point of contact.

After integrated equation (12) and simplified the result , we find:

$$\delta_{b,p} = \frac{12\,F.\cos^2\emptyset\ .y_{bi}^{\ 2}}{E\ .b.tb^{\ 3}}\left(y_{ci}^{\ 2}+\frac{y_{bi}^{\ 2}}{3}-y_{ci}y_{bi}\right)+\frac{6\,F.\cos^2\emptyset\ .(Z_i-y_{bi})^3}{E.b\ .tb^{\ 3}}\times\left[\frac{Z_i-y_{ci}}{Z_i-y_b}\left(4-\frac{Z_i-y_{ci}}{Z_i-y_{bi}}\right)\right.$$
$$\left.-2\ \ln\frac{Z_i-y_{ci}}{Z_i-y_{bi}}-3\right] \tag{15}$$

$$\delta_{b,n} = \frac{F\ \sin\emptyset^2}{E\ .b\ .t_b}\left[y_b+(Z_i-y_{bi})\ln\frac{Z_i-y_{bi}}{Z_i-y_{ci}}\right] \tag{16}$$

$$\delta_s = \frac{F\ .K.\ \cos\emptyset^2}{G.b\ .t_b}\left[y_b+(Z_i-y_{bi})\ln\frac{Z_i-y_{bi}}{Z_i-y_{ci}}\right] \tag{17}$$

3.3 Strain energy for asymmetric teeth profile

As in previous section asymmetric gear tooth is assumed as a very short cantilever beam with the part inside the base circle modeled as a rectangular beam , and the part outside the base circle as triangular beam as in fig (2) .

Bending moment for mating asymmetric teeth gear teeth will be :

$$\delta_{b,p} = \frac{12\ F}{E\ .btb^2}\left(\cos^2\emptyset\ \ .\ y_b^{\ 2}\left(y_{ci}^{\ 2}+\frac{h^2{}_b}{3}-y_{ci}\,y_{bi}\right)+\frac{e.y_b}{t_b^3}\ .\sin\emptyset^2-\sin2\emptyset\left(\frac{e.y_b^{\ 2}}{2}-e.y_{bi}.y_{ci}\right)\right)$$
$$+\frac{12\ F\ .\ \emptyset\ .(z-y_b)^3}{E.b\ .tb^{\ 3}}\times\left(e.\sin2\emptyset\left(\frac{1}{z-y_b}-\frac{1}{z-y_{ci}}+\frac{y_c}{2(z-y_b)^2}-\frac{y_c}{2(z-y_{ci})^2}\right)\left[\frac{Z_i-y_c}{2(Z_i-y_b)}\left(4-\frac{Z_i-y_c}{Z_i-y_{bi}}\right)-\ln\frac{Z_i-y_{ci}}{Z_i-y_{bi}}-1.5\right]+\right.$$
$$\left.\frac{e.\sin\emptyset^2}{2}\left(\frac{1}{(Z_i-y_{bi})^2}-\frac{1}{(Z_i-y_{ci})^2}\right)\right) \tag{18}$$

Where: $e = \frac{t_u-t_l}{2}$ (19)

From involutemetry relationships , we can find as in figure (3)

$$e = rb\ (\ inv\ \emptyset_{pu}\ -inv\ \emptyset_{pl}\ ) \tag{20}$$

3.4 Hertizian & foundation deflection

In this work we consider Hertizian deflection which calculated by Yang & Sun [14]

$$\delta_h = \frac{4\,F.(1-\upsilon^2)}{Eb\pi} \tag{21}$$

Nakada and Utagawa according to ref [15] investigate foundation deflection which its consider here as:

$$\delta_f = 24\,F\ \cos^2\emptyset\ y_{ci}{}^2 / Eb\pi\ tb^2 \tag{22}$$

4. Tooth Contact Regions

Mesh stiffness for mating tooth gear varies from one region to another depend on type of contact , usually mating gear has single and double tooth contact ,the definition of each region represent major aim to investigate the nonlinearity behavior of mesh stiffness in this region ,let AB represent line of action for mating gear as shown in figure (4) which divided into multi regions , the segment between c & d represent tooth contact region . point c would be the intersection point between base circle of gear 1 with addendum circle of gear 2 and point d will be the intersection point between base circle of gear 2 with addendum circle of gear 1 .Distance between c & f equal to base pitch. distance . During one mesh cycle for spur gears with low-contact ratio , there is two cases of contact one tooth pair in contact and two tooth pairs in contact, occurring separately . The overall mesh stiffness modeled as two spring in series during single tooth contact, and modeled as two spring in parallel during double tooth contact as:

$Km = K_I$ single tooth contact (23)

$Km = K_I + K_{II}$ double tooth contact (24)

All following relations for the property of gear involute depend on rotation angle φ and pressure angle Φ which effect on thickness of tooth gear. , Return to deflection equations we note that all parameters are constant expect *(yc)* where:

$$y_{cj} = r_j - r_{dj} \tag{25}$$

The radius *rj* in above varies from an addendum radius *raj* to adendum radius *rdj*, also The radius *rj* depend directly on rotation angle and base circle as:

$$r_j = r_{bj}\ \ sec\,\theta \tag{26}$$

$$\alpha_{ij} = tan\,\theta \tag{27}$$

Angle *αij* difference from first meshing gear tooth to second meshing gear tooth , i.e. the relation between rotation angle of the first meshing gear 1 identified by angle α_{11},at the same time for gear 2 identified by α_{21}, for backside contact $\alpha_{12}$and α_{22}[15]

All parameters in deflections equations are ready to substitute, *yci* value will be obtain for single and double tooth contact. We can find mesh stiffness in gear for single and double tooth contact which model as

$$k_I = \frac{k_{11}k_{21}}{k_{11}+k_{21}} \tag{28}$$

$$k_{II} = \frac{k_{12}k_{22}}{k_{12}+k_{22}} \tag{29}$$

Dynamic load factor for symmetric and asymmetric tooth profile can be obtained by sub overall mesh stiffness in equation (8) to find dynamic load factor equation for both symmetric and asymmetric tooth profile and for single and double tooth contact.

$$DLF = \left(\frac{c_m}{F}\right)\dot{x} + \frac{1}{F}\left(\frac{k_{11}k_{21}}{k_{11}+k_{21}}\right)x \quad \text{Single tooth contact} \tag{30}$$

$$DLF = \left(\frac{c_m}{F}\right)\dot{x} + \frac{1}{F}\left(\frac{k_{11}k_{21}}{k_{11}+k_{21}} + \frac{k_{12}k_{22}}{k_{12}+k_{22}}\right)x \quad \text{Double tooth contact} \tag{31}$$

4.1 Dynamic load factor for symmetric tooth profile

Dynamic load factor can be recognized for symmetric tooth profile by substitute unit force per summation of deflections components equations (15-16-17 -21-22) with consideration value of hc_{ij} for each tooth pair of contact and for each type of contact(single & Double) i.e. $(yc_{11}, yc_{12}, yc_{22}, yc_{21})$

, then k_{ij} for symmetric tooth contact :

$$kij = \frac{12\,F.\cos^2\emptyset\ .y_b{}^2}{E.btb^{\,3}}\left(y_{cij}{}^2 + \frac{h^2{}_b}{3} - y_{cij}y_{bi}\right) + \frac{6\,F.\cos^2\emptyset\ .(Z_i-y_b)^3}{E.b.tb^{\,3}} \times [\ \frac{z_i-y_{cij}}{z_i-y_b}\left(4 - \frac{z-y_{cij}}{z-y_{bi}}\right) - 2\ln\frac{z_i-y_{cij}}{z_i-y_{bi}} - 3] + \frac{F\ \sin\emptyset^2}{E.b.t_b}\left[h_b + (z_i - y_{bi})\ln\frac{z_i-y_{bi}}{z_i-y_{cij}}\right] + \frac{F.K.\ \cos\emptyset^2}{G.b.t_b}\left[y_{bi} + (z_i - y_{bi})\ln\frac{z_i-yh_{bi}}{z_i-y_{cij}}\right] + \frac{4\,F.(1-\upsilon^2)}{Eb\pi} + \frac{24\,F\ \cos^2\emptyset\,y_{cij}{}^2}{Eb\pi\ tb^2} \tag{32}$$

4.2 Dynamic load factor for asymmetric tooth profile

Dynamic load factor can be recognized for symmetric tooth profile by substitute unit force per summation of deflections components equations(16-17 – 18 – 21 -22) with consideration value of hc_{ij} for each tooth pair of contact and for each type of contact(single & Double) i.e. $(yc_{11}, yc_{12}, yc_{22}, yc_{21})$, then k_{ij} for asymmetric tooth contact:

$$k_{ij} = \frac{12\,F}{E.btb^{2}}\left(\cos^{2}\emptyset\ .\ y_b^{\,2}\left(y_{ci}^{\,2}+\frac{h^{2}_{b}}{3}-y_{ci}\,y_{bi}\right)+\frac{e.y_b}{t_b^{3}}.\sin\emptyset^{\,2}-\sin 2\emptyset\left(\frac{e.y_b^{\,2}}{2}-e.y_{bi}.y_{ci}\right)\right)$$
$$+\frac{12\,F.\emptyset\ .(z-y_b)^{3}}{E.b.tb^{\ 3}}\times\left(e.\sin 2\emptyset\left(\frac{1}{z-y_b}-\frac{1}{z-y_{ci}}+\frac{y_c}{2(z-y_b)^{2}}-\frac{y_c}{2(z-y_{ci})^{2}}\right)\left[\frac{Z_i-y_c}{2(Z_i-y_b)}\left(4-\frac{Z_i-y_c}{Z_i-y_{bi}}\right)-\ln\frac{Z_i-y_{ci}}{Z_i-y_{bi}}-\right.\right.$$
$$\left.\left.1.5\right]+\frac{e.\sin\emptyset^{2}}{2}\left(\frac{1}{(Z_i-y_{bi})^{2}}-\frac{1}{(Z_i-y_{ci})^{2}}\right)\right)+\frac{F\ \sin\emptyset^{2}}{E.b.t_b}\left[h_b+(z_i-y_{bi})\ln\frac{z_i-y_{bi}}{z_i-y_{cij}}\right]+\frac{F.K.\cos\emptyset^{2}}{G.b.t_b}\left[y_{bi}+(Z_i-\right.$$
$$\left.y_{bi})\ln\frac{z_i-yh_{bi}}{z_i-y_{cij}}\right]+\frac{4F.(1-\upsilon^{2})}{Eb\pi}+\frac{24\,F\cos^{2}\emptyset\,y_{cij}^{\,2}}{Eb\pi\,tb^{2}}. \tag{33}$$

5. Results & Discussion

In order to verify the analytical equations derived in this paper , a computer programs by using Matlap Code is modeled to estimate nonlinear mesh stiffness of spur gear with symmetric and asymmetric teeth , Equations (3) solved numerically by using the classical fourth order Runge-Kutta method , the numerical solution of equations above , namely the relative vibration time domain response of gear is obtained to carry out dynamic transmission error and dynamic load factor of spur gear with symmetric and asymmetric teeth , A computer program was developing to simulate the dynamic characteristics of spur gear, the adaptive step size control is employed to assure convergence of the solution. In order to optimize the design with respect to gear design parameters , the effect of asymmetry and operating design load was investigated.

A sample gear pair given in table (2) is used to prove the accuracy of theoretical analysis in this paper for computing nonlinear mesh stiffness and compare result with shing [15] model.

Fig (6) shows non linear mesh stiffness for single and double tooth contact for properties listed in table (2) with symmetric tooth profile. This result match with a result obtained by shing[15] with percentage difference around 6 - 7 % along the line of contact , this difference between results due to the effect of component of bending deflection which investigated in this paper , fig (6)shows mesh stiffness varying with rotation angle φ , and this behavior can be varying with time by substitute this angle by the speed of mating gear in program script , nonlinearity indicted in fig (6) confirm that the time varying mesh stiffness represent major reason for noise and vibration in geared system.

In this study, three different gear pairs are considered to investigate the effect of asymmetry on mesh stiffness, static transmission error, dynamic transmission error and dynamic load factor , a new modified of asymmetric gear with low pressure angle for loaded side and high pressure angle for unloaded side which developed by Abdullah [16] is consider in this paper .

Fig (7) shows mesh stiffness varying with time for three gear pairs ,first is symmetric with 200 pressure angle for both sides of gear tooth , second is asymmetric with 200 pressure angle for loaded side and 250 for unloaded side, third is asymmetric with 14.50 pressure angle for loaded side and 250 for unloaded side, fig (7) shows increasing in mesh stiffness in asymmetric cases compare with that symmetric case due to increasing in tooth thickness and appear a new value (e) in asymmetric tooth profile as it investigated in theoretical analysis in this paper . The behavior of non linear mesh stiffness in double tooth contact for three cases is the same ,but this difference occur in single tooth contact , the length of line of contact for single and double tooth contact for two pairs ($20^0/20^0$ & $20^0/25^0$) is the same and difference appear in third pair ($14.5^0/25^0$) due to difference in contact ratio which is (1.46) for first two pairs and (1.64) for third pair .The length of line of contact for single tooth contact in first two pairs is longer that third pair and versa reverse.

Dynamic transmission error and dynamic load factor computational program should be verified with existing researches , Singh et el [17] predicate dynamic load factor analytically and compare them work with experimental results of Kubo. .experimental results and the relevant system parameters are extracted from recent paper by Ozguven and Houser [12] , properties of Ohio's model are listed in table(3) , minimum unloaded static transmission error $e(t)$ is obtained by tooth profile modification ,fig (8) shows predication of dynamic load factor based on theoretical analysis and Ohio's model , where results indicate accepted error ,there might be several reasons for difference like the activity of the computer model used to calculate unloaded static transmission error and little knowledge of some parameters such as tip relief amount and relief position , it is very important in fig (8) to note that the resonant frequency did not appear in fundamental frequency , the peak of curve occur in the other frequency named transition frequency [17] ,this frequency recognize when mating gear transits from no impact to single sided impact regimes below and above resonance ,respectively .

Fig (9) show the effect of symmetry on dynamic load factor, for normal operating speed (below 5000 rpm) the dynamic load factor for all pairs close to one ,this is mean that the dynamic load coincides with static load closely, fig(9) shows enhancement in dynamic load factor for gear pair ($14.5^0/25^0$) compare with gear pairs ($20^0/20^0$ & $20^0/25^0$) at operating speed (5000 – 1000 rpm) with enhancement up to 36% around resonant rotational speed (about 10000 rpm) . after rotational speed (11000 rpm) gear pairs ($20^0/20^0$ & $20^0/25^0$) show an advantage in dynamic load factor compare with gear pair ($14.5^0/25^0$) with difference around 50% , generally , the peak of dynamic load factor for asymmetric teeth increase with increasing pressure angle on the loaded and unloaded side, its (3.84) for asymmetric tooth ($20^0/25^0$) , and (4.51) for asymmetric tooth ($14.5^0/25^0$) ,while it is about (3.54) for classical symmetric tooth ($20^0/20^0$) , shifting between resonant rational speed (9605,10544,11493 rpm) due to algorithm of transition frequency [17] ,i.e. resonant frequency in fig (9) did not concedes fundamental frequency .

Fig (10) investigate the asymmetry effect on dynamic transmission error which varying with rotational speed , below(10000 rpm) each asymmetric teeth ($14.5^0/25^0$ & $20^0/25^0$) show enhancement comparing with symmetric tooth ($20^0/20^0$) with difference about 48% . as in dynamic load factor curves symmetric tooth ($20^0/20^0$) indicate low dynamic transmission error in high operating speed (up to 12000 rpm) ,but unlike dynamic load factor , the difference between dynamic transmission error for three gear pairs reduce and concedes after 14000 rpm closely. Fig (10) prove that asymmetric teeth show decreasing in dynamic transmission error at normal operating speed which is represent major reason for enhancement noise in geared system as many literature confirm [1].

Smith [1] developing Matlap program to generate static transmission error for spur and helical gear ,geometrical modification parameters such as tip relief, crowning with misalignment and interference considered in this program, smith program employed in this work to investigate the effect of asymmetry on static transmission error within mesh cycle , fig (11) shows static transmission error for three gear pairs , asymmetric tooth ($14.5^0/25^0$) recorded lower transmission error compare with other pairs , the decreasing between amplitude of static transmission error for asymmetric tooth ($14.5^0/25^0$) and amplitude static transmission error for symmetric tooth ($20^0/20^0$) was 12% , while the decreasing of amplitude static transmission error for asymmetric tooth ($20^0/25^0$) and amplitude static transmission error for symmetric tooth ($20^0/20^0$) was 6% . static transmission error shows that the peak of transmission error occur at the center of single tooth contact ,generally, enhancement of static transmission error by using asymmetric teeth leads to optimization problem of dynamic load factor and dynamic transmission error equations by reducing the fundamental harmonics of loaded static transmission error ,and this reduction increase with increasing of pressure angle for loaded and unloaded side .Table (4) shows the influence of asymmetry and static transmitted load on dynamic load factor ,it's obvious that the dynamic load factor change inversely with static transmitted load ,also asymmetric tooth ($14.5^0/25^0$) indicate less dynamic load factor compare with asymmetric tooth ($20^0/25^0$) and symmetric tooth ($20^0/20^0$) , percentage difference of dynamic load factor between three pairs decrease when static transmitted load increase, its about 16 % at 500 N ,and 6% at 2500 N.

7. Conclusions

1- Increasing in tooth mesh stiffness for asymmetric teeth ($20^0/25^0$ & $14.5^0/25^0$) compare with symmetric tooth (200/200) ,this increasing generate decreasing in static transmission error .
2- Asymmetric tooth ($14.5^0/25^0$) remarked better results for dynamic load factor and dynamic transmission error comparing with symmetric tooth (200/200) and asymmetric tooth ($20^0/25^0$) at operating rotational speed below 12000 rpm with percentage enhancement for dynamic load factor 36% around resonant rotational speed ,and percentage enhancement for dynamic transmission error 36% around resonant rotational speed about 48% around resonant rotational speed .
3- the peak of dynamic load factor for asymmetric teeth increase with increasing pressure angle on the loaded and unloaded side, its (3.84) for asymmetric tooth (& $20^0/25^0$) , and (4.51) for asymmetric tooth ($14.5^0/25^0$) ,while it is about (3.54) for classical symmetric tooth ($20^0/20^0$) .
4- Asymmetric tooth ($14.5^0/25^0$) remarked lowest static transmission error compare with other teeth , the decreasing between amplitude of static transmission error for asymmetric tooth ($14.5^0/25^0$) and amplitude of static transmission error for symmetric tooth (200/200) was 12% , while the decreasing of amplitude static transmission error for asymmetric tooth ($20^0/25^0$) and amplitude static transmission error for symmetric tooth ($20^0/20^0$) was 6% .
5- The dynamic load factor change inversely with static transmitted load ,where asymmetric tooth ($14.5^0/25^0$) remarked less dynamic load factor compare with other teeth , percentage difference of dynamic load factor between three pairs decrease when static transmitted load increase, it's about 16 % at 500 N ,and 6% at 2500 N.

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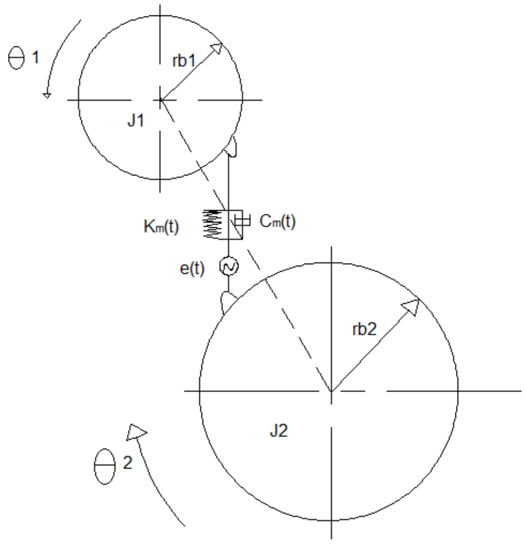
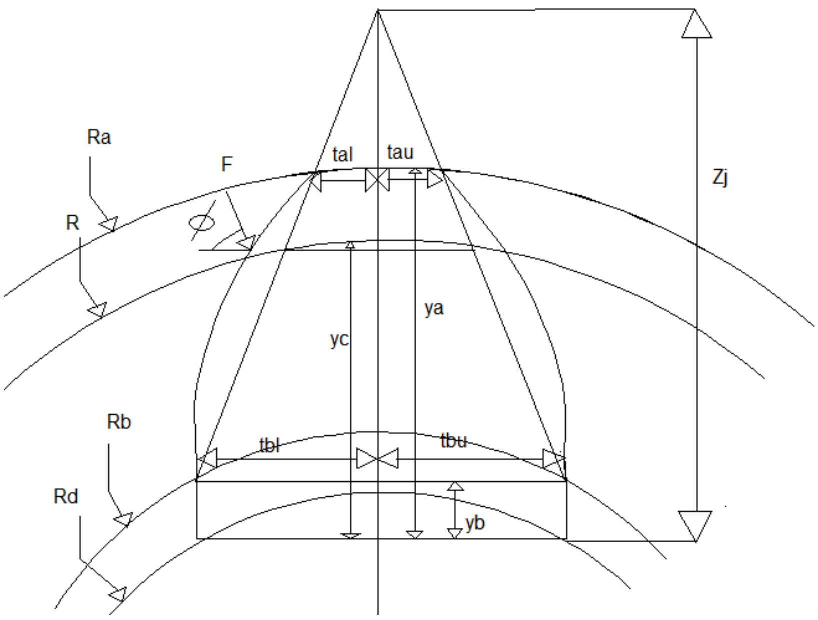


Figure (1) : a spur gear pair model



Figure(2) : Gear tooth Geometry

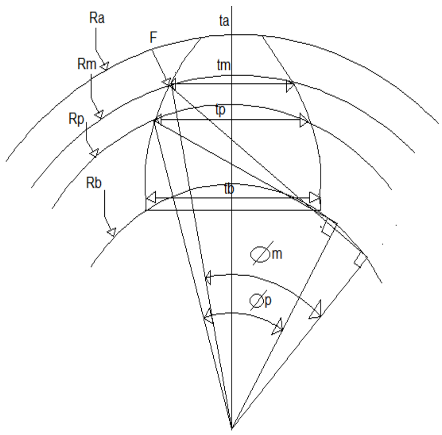


Figure (3) : derivations of tooth thickness

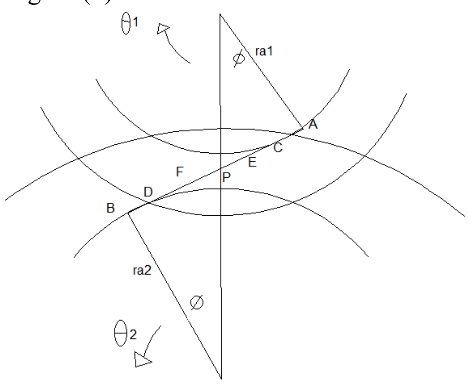


Figure (4) : Tooth contact regions

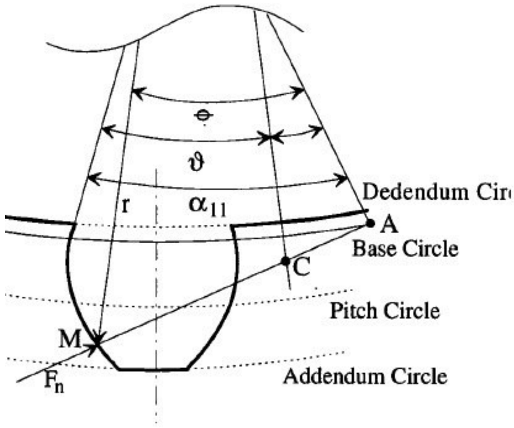


Figure (5) : Rotation angles relationship

	Low Torque	High Torque
Low Speed	Unloaded static transmission error	loaded static transmission error
High Speed	Unloaded dynamic transmission error	loaded dynamic transmission error

Table (1) : loaded and unloaded Static & Dynamic Transmission Error

Properties	Pinion	Gear
Module (mm)	2	
Pressure angle	20^0	20^0
No. of Teeth	20	80
Modules of elasticity (N/m^2)	2.06×10^{11}	2.06×10^{11}
Damping ratio	0.05	
Moment of inertia ($kg.\ m^2$)	1.528×10^{-5}	39×10^{-4}
Face width (mm)	10	10

Table (2) : Properties of Shing' s Gear pair [15]

Properties	Pinion	Gear
Module (mm)	3.75	
Pressure angle	20^0	20^0
No. of Teeth	25	25
Modules of elasticity (N/m^2)	2.06×10^{11}	2.06×10^{11}
Damping ratio	0.05	
Transmitted load ($kg.\ m^2$)	2295	
Static transmission error (micron)	1.92	
Mesh stiffness (N/m)	3.8E8	

Table (3) : properties of Ref[17] gear pair

Load (N) 6000rpm	Symmetric 20/20	Asymmetric 20/25	Asymmetric 14.5/25
500	2.57	2.33	2.15
1000	1.78	1.66	1.57
1500	1.52	1.44	1.38
2000	1.39	1.32	1.28
2500	1.31	1.26	1.23

Table (4) : The Effect of Transmitted load on Dynamic load Factor , mo = 7 mm , face width = 60 mm .

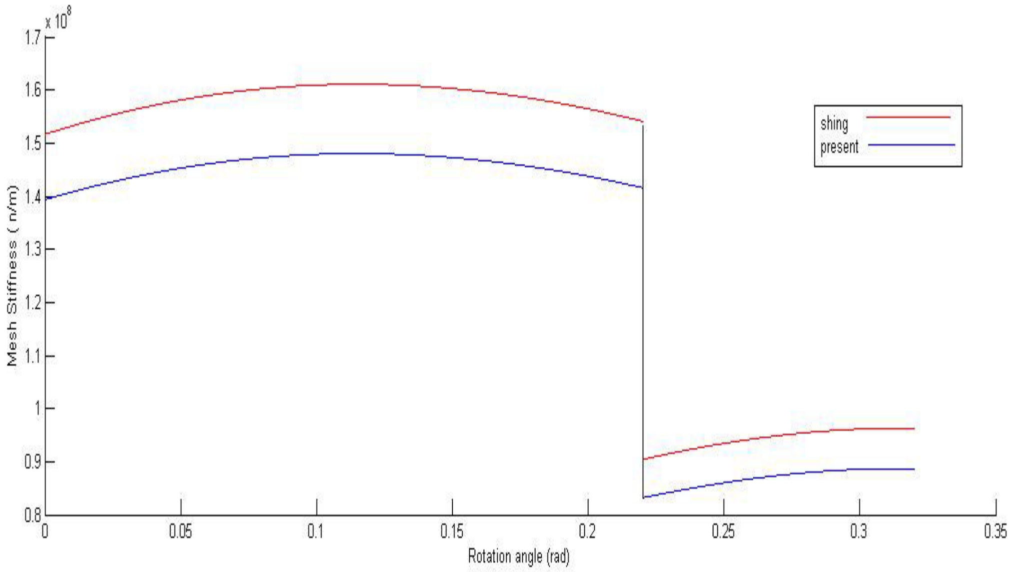


Figure (6) : Mesh Stiffness during Mesh cycle Vs. Rotation angle.

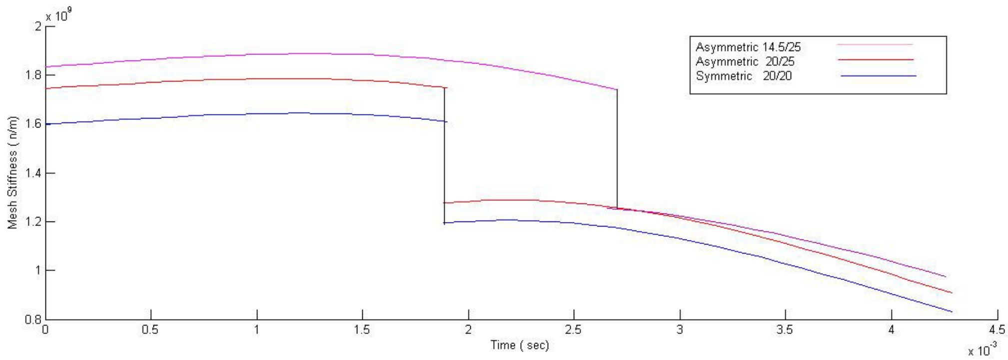


Figure (7) : Effect of Variation of Asymmetry on Mesh stiffness during Mesh cycle.

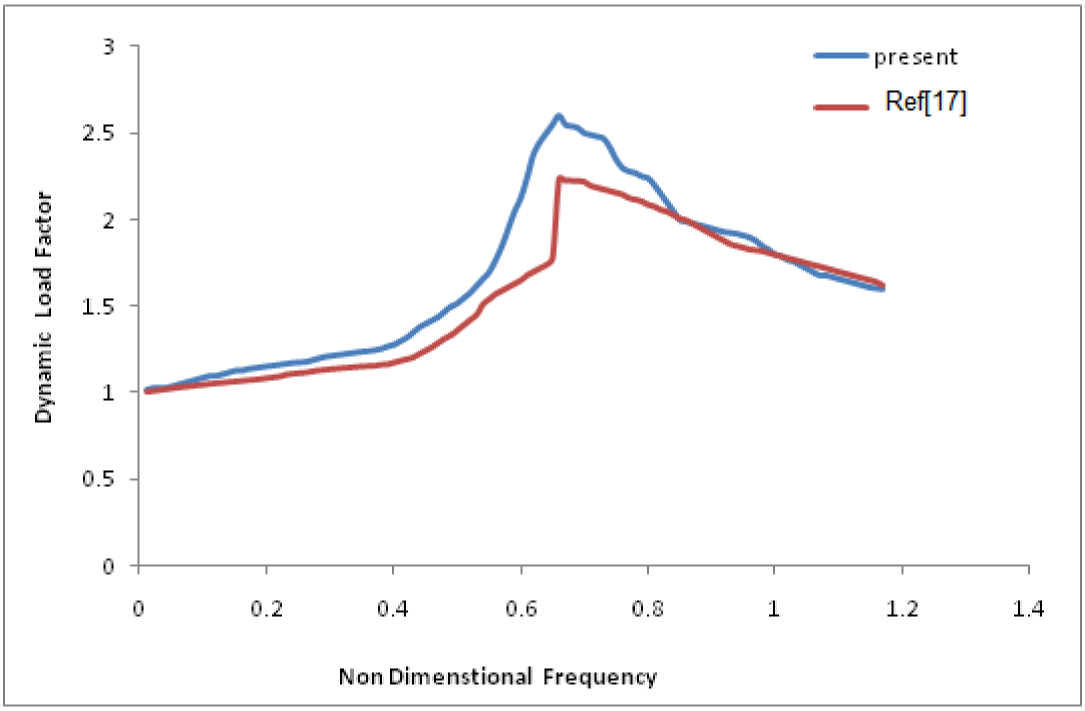


Figure (8): Comparison of Evaluated Dynamic load Factor

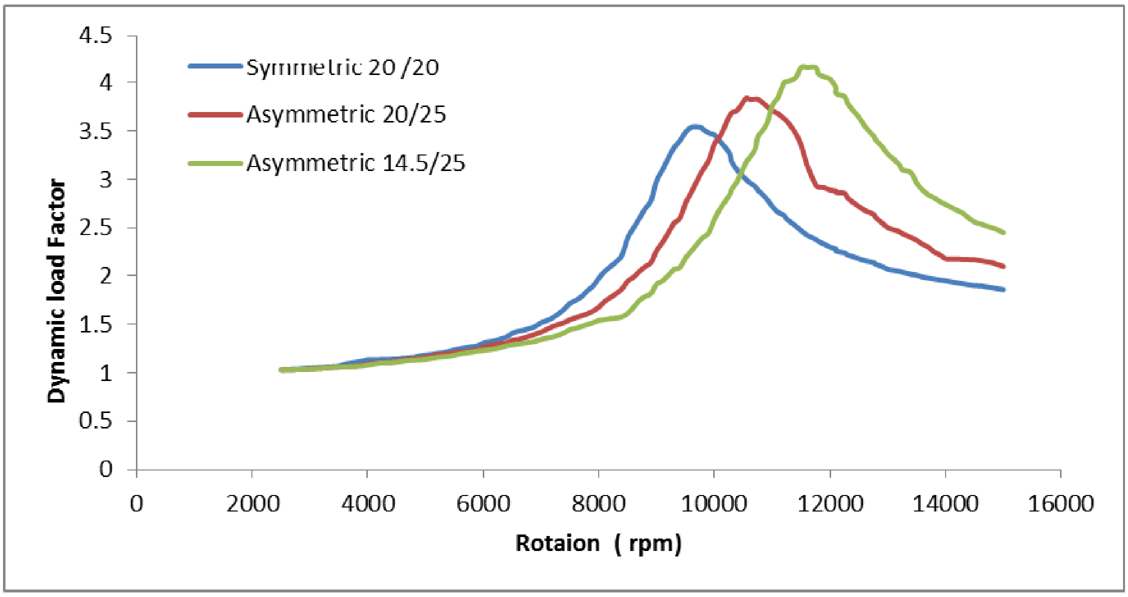


Figure (9): The Effect of Asymmetry on Dynamic load Factor

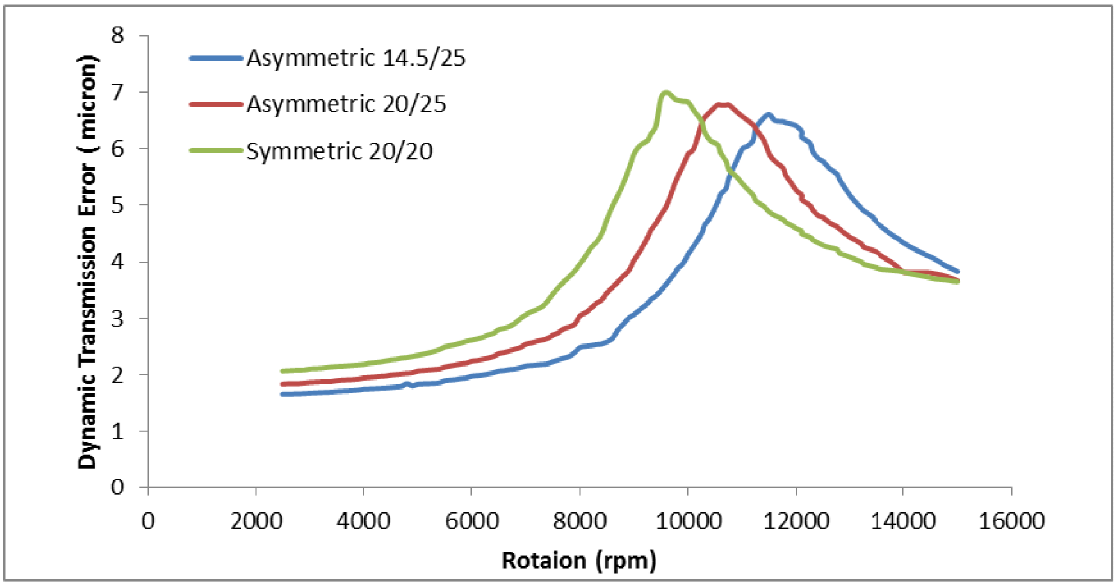


Figure (10): The Effect of Asymmetry on Dynamic Transmission Error

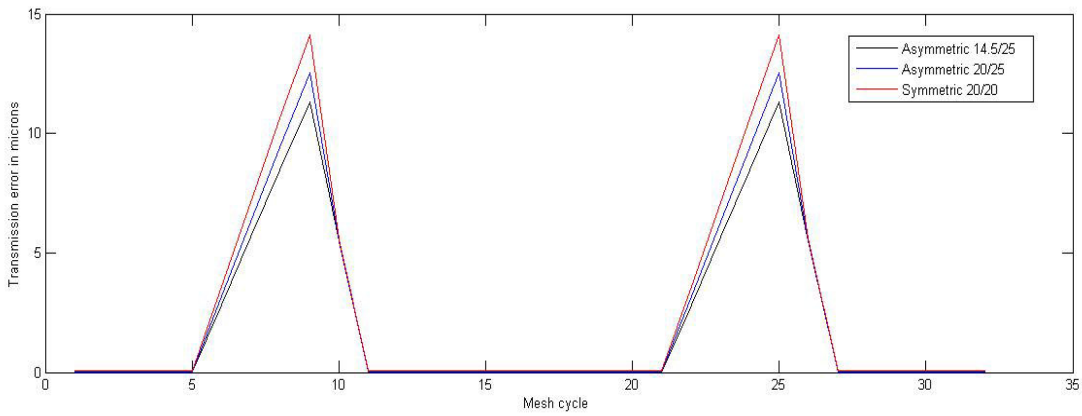


Figure (11): The Effect of Asymmetry on Static Transmission Error