Innovative Systems Design and Engineering
ISSN 2222-1727 (Paper) ISSN 2222-2871 (Online)
Vol.4, No.14, 2013
www.iiste.org

Errors Analysis in Distance Relay Readings with Presence of FACTS Devices

Gaber El-Saady[1*] Rashad M.Kamel[2] Essam M.Ali[3]
Electrical Engineering Department, Assiut University, Assiut, Egypt, gaber1@yahoo.com

Abstract

This paper presents studying the performance of distance impedance relay with the presence of FACTS devices .Also the measured impedance at the relaying point in the presence of series FACTS device SSSC , and shunt FACTS device called STATCOM are obtained. A detailed model of both SSSC and STATCOM is introduced and then the faulty system is studied analytically , where the errors in the measured impedance of distance impedance relay are introduced as a result of the presence of series and shunt FACTS devices. The analysis results show the effect of impacting the FACTS devices location, the values of inserted SSSC and STATCOM voltages (operational conditions) and also the fault resistance values.

Keywords: Distance Relay, FACTS, STATCOM, SSSC.

1. Introduction

The measured impedance at the relaying point is the basis of distance protection operation. In addition to the power system parameters, the fault resistance could greatly influence the measured impedance in the case of single phase to ground, double phase to ground and three phase to ground faults, in such a way that for zero fault resistance the power system parameters do not affect the measured impedance (Kazemi & Jamali 2009). FACTS devices can be broadly classified into three types, (a) Shunt (b) Series and (c) composite series and shunt (Tarlochan *et al.* 2005).

The use of power electronics devices to improve power transfer capability of long transmission lines forms the basis of the concept of FACTS. The Static Synchronous Series compensator (SSSC) is a series device of the FACTS family using power electronics to control power flow and improve power oscillation damping on power grids (Khederzadeh 2009).

More than 70% of transmission line faults are single phase to earth faults, and phase to phase faults are the next common fault type . Double phase to earth and three phase faults are less common in power transmission systems (Jamali & Shateri,2011). The operating principle of distance protection is based on the fact that , from any measuring point in power system the line impedance to a fault in that system can be determined by measuring the voltage and current at the measuring point (Khederzadeh 2009).

Some researchers have been done to evaluate the performance of a distance relay for transmission line in the presence of FACTS controllers. The work in (Kazemi & Jamali 2009) presented a steady state analysis on impacting STATCOM and SSSC and compared between SSSC and STATCOM impacting effects on the measured impedance by distance relay. The work in (Khederzadeh 2009) presented analytical and simulation results of the application of distance relays for the protection of transmission employing Static Synchronous Series Compensator (SSSC).

This paper studied the effects of the installation of SSSC and STATCOM on a doubly fed circuit transmission line in the case of single line to ground fault by means of presenting the errors in measured impedance at the relaying point and the distance relay ideal tripping impedance values. The measured impedance is presented for two cases of SSSC and STATCOM exclusion and inclusion in the fault loop. For three installation points of SSSC, i.e. at near end, mid-point, and far end, the ideal tripping characteristic is investigated. Also the operational conditions of FACTS that may be affected by loading condition are considered in this work.

2-FACTS devices models

The Facts devices under study have different types; the model of each type is described in the following sections.

2.1. SSSC model

The SSSC is placed in a group of series connected FACTS devices. As shown in figure (1) SSSC consists of a voltage source connected in series with the transmission line through voltage source inverter (VSI) and coupling transformer.

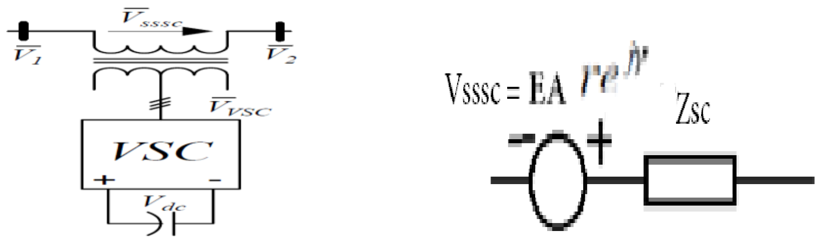


Figure (1) (a) Basic construction (b): an equivalent circuit of SSSC

2.2. STATCOM model

A STATCOM is a controlled reactive power source and provides voltage support, which is its primary duty, by generating or absorbing reactive power at the point of common coupling without the need of large external reactors or capacitor banks (Prof. Detlef 2007).

The STATCOM basically consists of a step down transformer Tr, reactance LF, a three phase GTO or IGBT based multi- pulse VSC and a DC capacitor as shown in figure (2a). STATCOM can be modelled as a shunt branch consisting of an impedance Zsh due to the coupling transformer, and a voltage source Esh which is in phase with the voltage of its connection point (Kazemi & Jamali 2009).

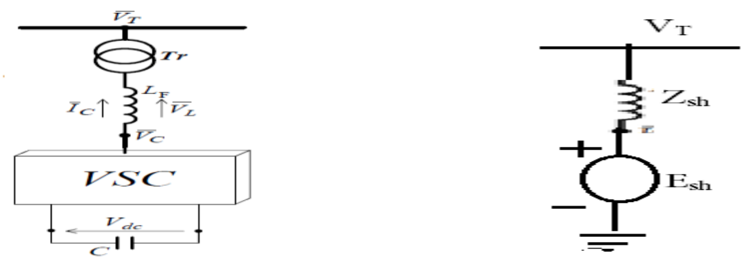


Figure (2) (a) STATCOM model construction (b) STATCOM representation

2.3. The system model

The system model under studying here is a single circuit transmission line fed doubly from two generators E_A and E_B where $E_B=E_Ahe^{-j\delta}$. Our FACTS devices are inserted between bus A and bus B to control the transferred power, voltage and current of line, etc. The distance relay is placed at bus A, so its function is protecting the transmission line till point B see figure (3).

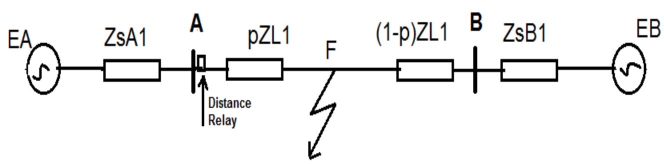


Figure (3): the model of the system consists of doubly fed transmission line

3. Fault analysis

Operating times of protective relays are typically 1.0–3 cycles and the tripping times of breakers are 1.5 – 3 cycles, giving a total fault tripping time of 2.5– 6 cycles (50 –120 ms at 50 Hz, 42–100 ms at 60 Hz) . The relays are installed on each feeder at a substation, to which the secondary voltages and currents of the VT (voltage transformers) and CT (current transformers) of the associated feeder line are applied as shown in Fig4. The signal voltage V (t) and current I (t) from the secondary terminals of the associated PT and CT are applied to the relay, and the new quantities u (t), v' (t) compose the detected part of the relay circuit as shown in the figure (4b). Then the phase angular difference between u (t) and v' (t) is continuously compared in the phase comparator part of the relay circuit (Xia *et al.*1994) and (Yi Hu *et al.* 2002).

In the absence of FACTS devices with zero fault resistance, the measured impedance by a distance relay only depends on the length of the line section between the fault location and the relaying points. In Figure (3) this impedance is equal to pZL1, where p is per unit length of the line section between the fault and the relaying points, and ZL1 is the line positive sequence impedance in ohms. In the case of a non-zero fault resistance, The measured impedance is not equal to the impedance of the line section between the relaying and the fault points. In this case, the structural and operational conditions of the power system affect the measured impedance (Kazemi & Jamali 2009).

The operational conditions prior to the fault instance can be represented by the load angle of the line δ and the ratio of the voltage magnitude at the line ends h or E_B / E_A = h $e^{-j\delta}$. The power system before the fault at point F can be drawn as a symmetrical circuit in the 1–2–0 domain as shown in figure (5). The fault calculations for impedance readings will derive now from the first principles case of SLG fault and it is given in appendix A .

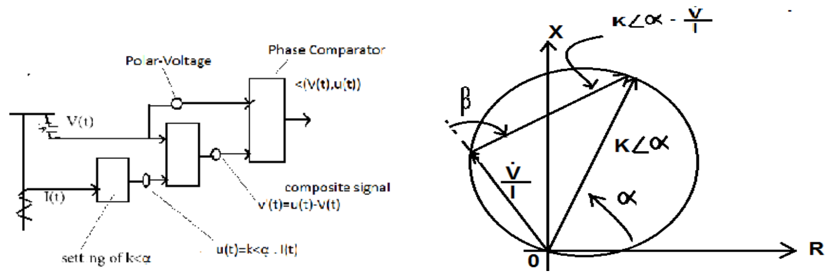


Figure (4): (a) principle of fault detection (b) MHO relay characteristics

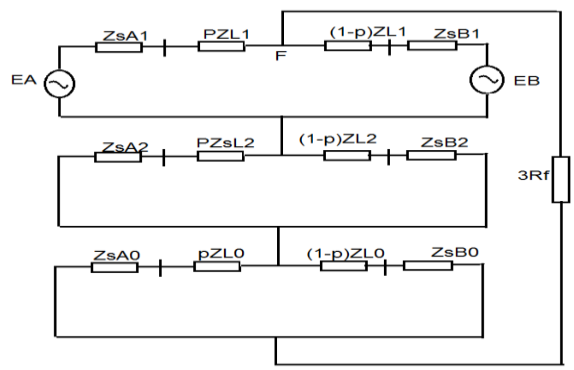


Figure (5): the 1-2-0 domain of the model case of fault with no FACTS connection.

3.1. Fault impedance calculation case of no FACTS

See appendix A for more details.

From the equations given in the appendix A the predicted measured impedances with changing fault location can be given by the following equations:

$$Z_{A1} = Z_{SA1} + p\, Z_{L1} \tag{1}$$

$$Z_{B1} = Z_{SB1} + (1-p)\, Z_{L1} \tag{2}$$

$$Z_{A0} = Z_{SA0} + p\, Z_{L0} \tag{3}$$

$$Z_{B0} = Z_{SB0} + (1\text{-}p)\, Z_{L0} \tag{4}$$

Z_1=Z_2 each positive sequence impedance equal negative sequence one. Where Z_{A1}, Z_{A2}, Z_{A0} are positive, negative and zero sequence impedances looking into the left-hand side at point F.

Z_{B1}, Z_{B2}, Z_{B0} are positive, negative and zero sequence impedances looking into the right-hand side at point F.

Now the two measured quantity Vrelay and Irelay at point A

The relaying current is as follows:

$$I_{relay} = I_A + K_{OL}\, I_{A0} = (C_d + 2\, C_1 + C_0\, (1 + 3K_{0L}))\, I_f^{(1)} \tag{5}$$

The relaying voltage

$$V_{relay} = V_A = V_F + I_{Af}\,(p\,Z_{L1}) = 3\,I_f^{(1)}\,R_F + I_{relay}\,(p\,Z_{L1}) \quad (6)$$

From equations (5) and (6) Where $Z_{relay} = V_{relay} / I_{relay}$, we found that:

$$Z_{relay} = pZ_{L1} + \frac{3R_F}{C_d + 2C_1 + C_0(1+3K_{0L})} \quad (7)$$

Where

$$C_1 = Z_{B1} / (Z_{A1} + Z_{B1}) \quad (8)$$

$$C_0 = Z_{B0} / (Z_{A0} + Z_{B0}) \quad (9)$$

$C_1 = C_2$ because $Z_1 = Z_2$

$$Z_e = Z_{A1} + Z_{B1} \quad (10)$$

$$K_{0L} = (Z_{L0} - Z_{L1}) / (3Z_{L1}) \quad (11)$$

$$K_d = 1 - h\,e^{-j\delta} \quad (12)$$

$$D = Z_{B1} + Z_{A1} h\,e^{-j\delta} \quad (13)$$

$$C_d = (Z_e + 3\,R_F)\,K_d / D \quad (14)$$

It can be seen when the fault resistance is equal to zero, the measured impedance at the relaying point is equal to the impedance of the line section between the relaying and the fault points, and otherwise the measured impedance deviates from its actual value depending on the power system conditions. Note K_{0L} is Zero sequence compensated factor.

3.2. Relay impedance with presence of SSSC

Once a FACTS devise is installed on the transmission line, the measured impedance at the relaying point is affected. The device is installed at the length of m per unit from the relaying point. In the following, the measured impedance in the presence of SSSC and STATCOM is presented. Inserting SSSC depending on its exclusion or inclusion in the fault loop, the measured impedance would change. And the following equations are derived with the same previous sequence. Firstly, some factors are defined:

$$C_{SC1} = Z_{se} / Z_{L1} \quad (15)$$

$$C_{SC0} = Z_{se} / Z_{L0} \quad (16)$$

$$Z_{Am1} = Z_{SA1} + mZ_{L1} \quad (17)$$

$$Z_{Am0} = Z_{SA0} + mZ_{L0} \quad (18)$$

Inserting SSSC may be at any point on the line and also the fault, so the measured impedance will be affected by both the FACTS and fault location. Let us check impedance calculations at two locations (1) nearer to far end than fault point (i.e. SSSC out of fault loop) (2) nearer to near end than fault point (i.e. SSSC inside the fault loop).

3.2.1. SSSC nearer to far end (out of fault loop)

Some equations are modified according to variation of V_{SSSC} and Z_{se}. Fault distance p is smaller than inserting location m (m > p).

$$Z_{Bm1} = Z_{SB1} + (1 - m + C_{SC1})\,Z_{L1} \quad (19)$$

$$Z_{Bm0} = Z_{SB0} + (1 - m + C_{SC0})\,Z_{L0} \quad (20)$$

$$Z_{Af1} = Z_{SA1} + pZ_{L1} \quad (21)$$

$$Z_{mF1} = |m - p|\,Z_{L1} \quad (22)$$

$$Z_{mF0} = |m - p|\,Z_{L0} \quad (23)$$

$$Z_{A1} = Z_{Af1} \quad (24)$$

$$Z_{B1} = Z_{mf1} + Z_{Bm1} \quad (25)$$

$$Z_{Af0} = Z_{SA0} + pZ_{L0} \quad (26)$$

$$Z_{Am0} = (m - p)\, Z_{L0} \tag{27}$$

$$Z_{A0} = Z_{Af0} \tag{28}$$

$$Z_{B0} = Z_{mf0} + Z_{Bm0} \tag{29}$$

$$D_n = Z_{Bn1} + Z_{A1}\, [h\, e^{-j\delta} - re^{\,j\Upsilon}] \tag{30}$$

$$K_{dn} = 1 + r\, e^{j\gamma} - h\, e^{-j\delta} \tag{31}$$

Using the same equation (10) of measured impedance here, the measured impedance in the case of zero fault resistance is equal to the impedance of the line section between the relaying and the fault points as will be shown later.

3.2.2. SSSC at near end (inside of fault loop)

In this case the fault location p is larger than FACTS inserting point m (m < p). The equations in previous case will be modified as follow:

$$Z_{mf1} = (p - m + C_{SC1})\, Z_{L1} \tag{32}$$

$$Z_{A1} = Z_{Am1} + Z_{mf1} \tag{33}$$

$$Z_{B1} = Z_{Bf1} \tag{34}$$

$$Z_{mf0} = (p - m + C_{SC0})\, Z_{L0} \tag{35}$$

$$Z_{A0} = Z_{Am0} + Z_{mf0} \tag{36}$$

$$Z_{B0} = Z_{Bf0} \tag{37}$$

$$D_n = [Z_{Am1}\, (1 + re^{\,j\gamma}) + Z_{mf1}]\, h\, e^{-j\delta} + Z_{Bf1}\, (1 + re^{\,j\gamma}) \tag{38}$$

$$C_Z = (C_{SC0} - C_{SC1})\, C_0\, (1 + 3K_{L0})\, Z_{L1} \tag{39}$$

$$Cv = -\, (Z_e + 3R_F)\, (Z_{An1} + Z_{B1})\, re^{\,j\gamma} / D_n \tag{40}$$

$$Z_{relay} = (p + C_{SC1})\, Z_{L1} + \frac{C_Z + C_V + 3\, R_F}{C_{1d} + 2C_1 + C_0(1 + 3\, K_{0L})} \tag{41}$$

It can be seen obviously that when ($R_F = 0$) no fault resistance , the measured impedance will not be the same as pZ_{L1} because of the effect FACTS presence.

3.3. Relay impedance with presence of STATCOM

Inserting STATCOM in the line depending on its position with respect to fault location (inside or outside fault location) affect the measured impedance, so some equations are modified with STATCOM with STASTCOM voltage = E_{sh} and an impedance Z_{Sh} as follows :

$$Z_{Am1} = Z_{SA1} + mZ_{L1} \tag{42}$$

$$Z_{Bm1} = Z_{SB1} + (1 - p)\, Z_{L1} \tag{43}$$

$$Z_{Am0} = Z_{SA0} + m\, Z_{L0} \tag{44}$$

$$Z_{Bm0} = Z_{SB0} + (1 - p)\, Z_{L0} \tag{45}$$

$$Z_{mf1} = |m - p|\, Z_{L1} \tag{46}$$

$$Z_{mf0} = |m - p|\, Z_{L0} \tag{47}$$

These equations specify the behaviour of no fault impedances independently of inserting FACTS inside or outside fault loop.

3.3.1. STATCOM nearer to far end (out of fault loop)

The following equations are modified also according to insertion point location.

$$Z_{A1} = Z_{Af1} \tag{48}$$

$$Z_{B1} = Z_{mf1} + \frac{Z_{sh}\, Z_{Bm1}}{Z_{sh} + Z_{Bm1}} \tag{49}$$

$$Z_{A0} = Z_{Af0} \tag{50}$$

$$Z_{B0} = Z_{mf0} + \frac{Z_{sh}\,Z_{Bm0}}{Z_{sh} + Z_{Bm0}} \quad (51)$$

$$D_n = Z_{Bn1} - [C_{B1} h\, e^{-j\delta} + (1 - C_{B1}) e_{sh}] Z_{A1} \quad (52)$$

$$K_{dn} = 1 - e_{sh}(1 - C_{B1}) - C_{B1}\, h\, e^{-j\delta} \quad (53)$$

And using equation (10) it can be observed that in case of zero fault resistance, the relay measured impedance is equal to pZ_{L1}. Otherwise the relay impedance will have impedance different from pZ_{L1}.

3.3.2. STATCOM nearer to near end (inside fault loop)

Some modifications are performed according to point m and p locations with respect to each other.

$$D_n = Z_{An1} + Z_{B1} - (C_{A1} + e_{Sh}(1 - C_{A1}) - h\, e^{-j\delta})(C_m Z_{Am1} + Z_{An1}) \quad (54)$$

$$K_{dn} = 1 - h_n\, e^{-j\delta} \quad (55)$$

$$C_{dn} = (Z_e + 3R_F) K_{dn} / D_n \quad (56)$$

$$C_{sh} = Z_{mf1}(-C_V + (1 - C_{A1})(C_{dn} + 2C_1) + C_0(1 - C_{A0})(1 + 3K_{0L})) \quad (57)$$

$$Z_{relay} = pZ_{L1} + \frac{3R_F + C_{sh}}{C_V + C_{A1}(C_{dn} + 2C_1) + C_{A0}\, C_0\,(1 + 3K_{0L})} \quad (58)$$

It is obvious also that when RF = 0, the relay impedance is not equal to pZL1 as will be discussed later.

4. Case Study and Results

We choose in our study three important points of impacting of FACTS through transmission lines (1) at near end (2) at far end (3) at midpoint impacting. Power system calculations gave these impedance values of the system shown in fig .3.

$Z_{L1} = 0.01133 + j\,0.3037$ Ω/KM

$Z_{L0} = 0.1535 + j\,1.1478$ Ω/KM

$Z_{SA1} = 1.3945 + j\,15.9391$ Ω

$Z_{SA0} = 7.454 + j\,27.8187$ Ω

$Z_{SB1} = 0.6972 + 7.9696$ Ω

$Z_{SB0} = 3.7270 + j\,13.9093$ Ω

h= 0.96

$\delta = 16^\circ$

The ideal tripping impedance readings without FACTS are shown in figure (6a) and the errors in readings due to presence of fault resistance as in figure (6b).

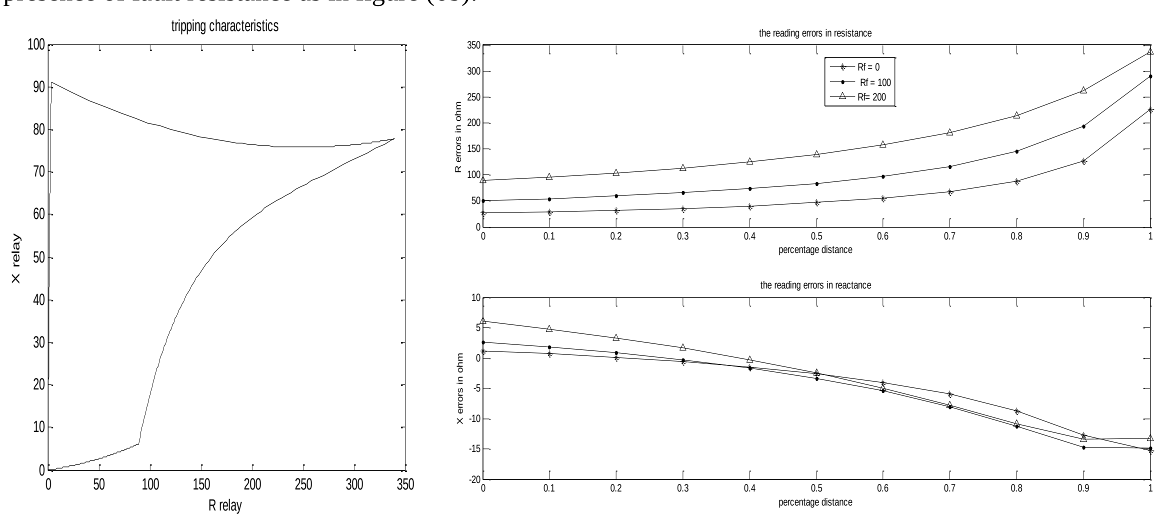


Figure (6) (a) tripping characteristics (b) errors due to R_F presence

From the above figure it is observed that when a bolt fault occurred, the angle of measured impedances were constant and the magnitudes only changed or say that both the measured resistance and reactance values changed with the same percentage with changing the fault location. Unlike the previous case, case of fault occurrence through resistance Rf≠0 both the angles and magnitudes changed. As shown in figure 6 the resistance increased but the reactance decreases compared with case of bolt fault.

4.1. FACTS at far end (outside fault loop)

4.1.1. SSSC impacting

From equation (10) , there is no error at any value of V_{SSSC} when a bolt fault occurred at any value of SSSC injected voltage. The results in fig.7 shows the errors occurred in impedance (resistance and reactance) readings case of fault occurrence through resistance Rf = 50 ohm . With changing the fault location, the errors in resistance were negative what means that resistance decreased compared with original case in fig6, also the reactance decreased with small changes, but at the end of the line fault occurrence the error was larger. With increasing the values of injected voltage we observe that readings error increased negatively in both resistance and reactance.

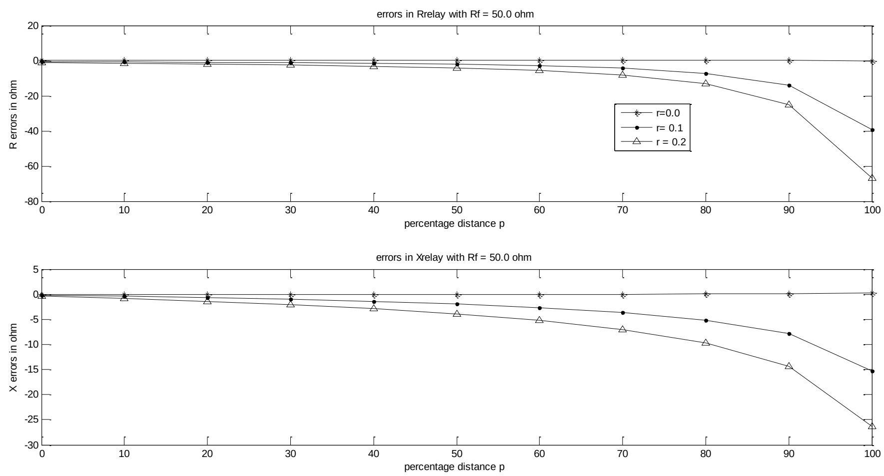


Figure (7) errors in readings with SSSC insertion out of loop with R_F =50 Ω

With increasing the fault resistance Rf, Comparing fig8 with fig.7 we can note that readings error increased comparing leading SSSC with inactive one . In some details, the errors in resistances increased unlike reactance errors. Resistances are lower values compared with original case.

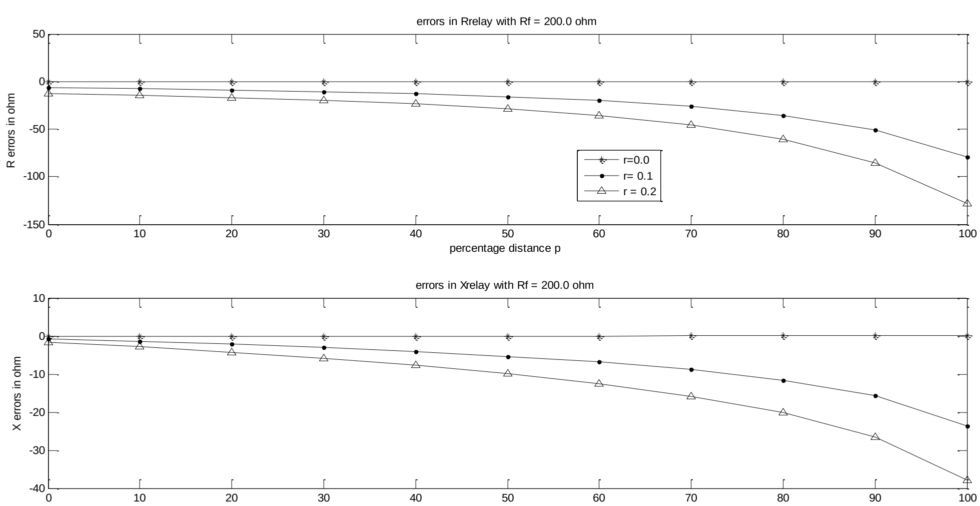


Figure (8) errors in readings with SSSC insertion out of loop with R_F =200 Ω

4.1.2 STATCOM impacting

From equation (10), it is obvious that case of a bolt fault the errors in readings disappeared at any value of STATCOM injected current (or voltage). The results in figure (9) shows the errors in impedance relay readings case of fault occurrence through resistance Rf = 50 Ώ. The errors in resistances are positive what shows that the resistance readings are higher than original case. but the reactance errors are negative values although the errors are smaller compared with SSSC impacting .

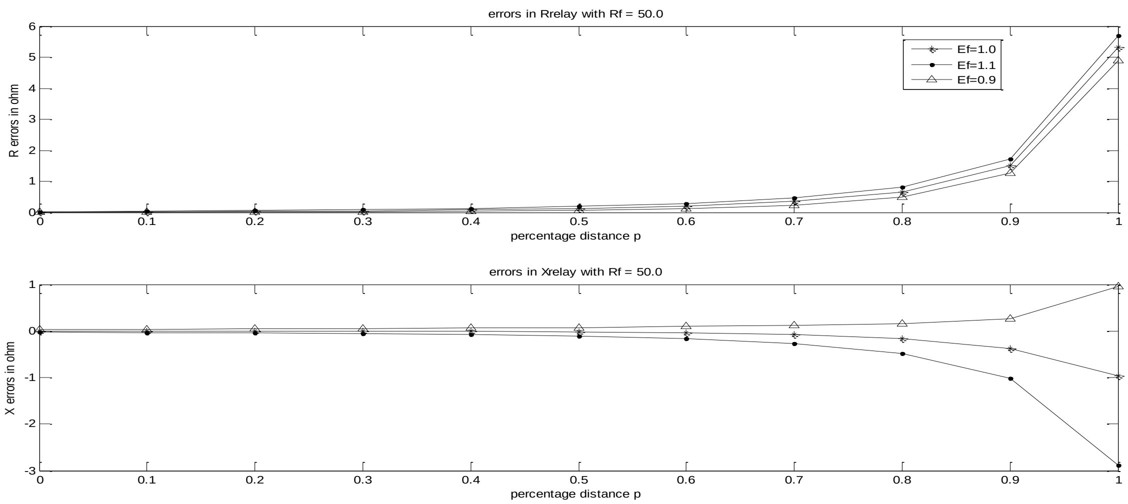


Figure (9) errors in readings with STATCOM insertion out of loop with R_F =50 Ώ

Giving a careful consideration to figure 10 that shows the errors with changing fault location and STATCOM voltage (Esh) but at higher fault resistance (Rf = 200 Ώ). We can observe that impedance readings diverge slowly from the original case at inactive STATCOM. Comparing leading STATCOM with inactive one we can say that the errors are within a convergent values but errors in reactance are higher than resistance one.

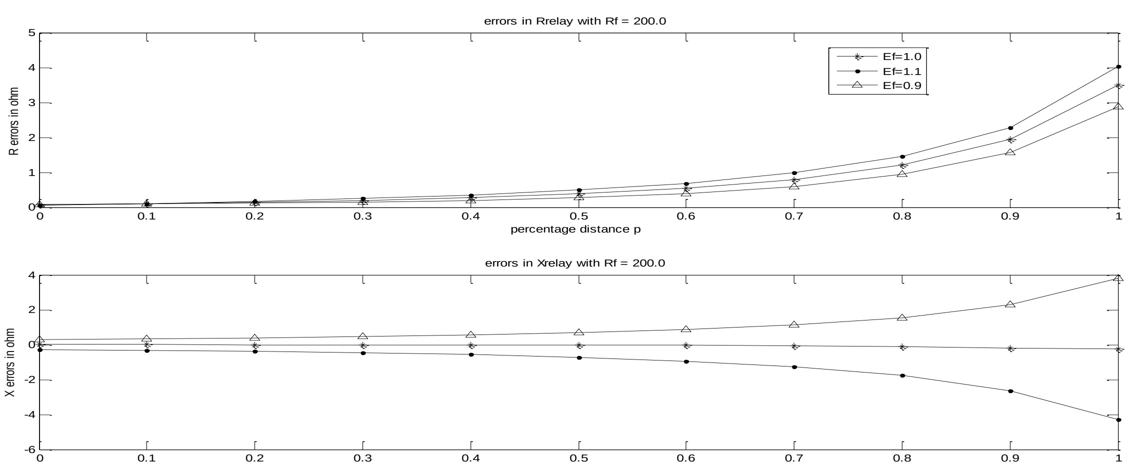


Figure (10) errors in readings with STATCOM insertion out of loop with R_F =200 Ώ

4.2. FACTS at near end

4.2.1. SSSC impacting

Back to equation (43) when a belt fault occurred, there are errors due to the presence of SSSC inside fault loop. By scrutiny in figure (11) the errors case of an inactive SSSC are very small in both resistance and reactance readings but these values differed obviously in leading SSSC devices by increasing the injected voltage as shown.

Increasing the injected voltage of SSSC resulted in observable values in impedance readings errors as seen in figure (11).

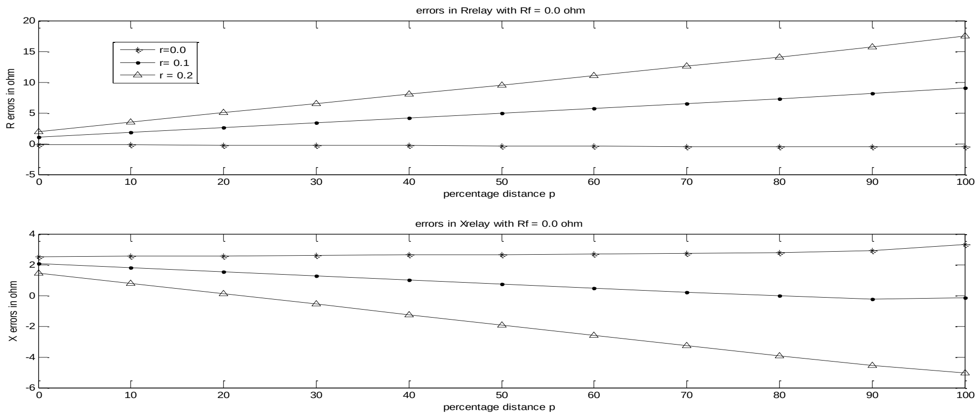


Figure (11) errors in readings with SSSC insertion in loop with R_F =0.0 Ω

By increasing the fault resistance the situation has became worst and the errors had larger values as obvious by comparing the results in figs 11 and 12. The errors commensurate with voltage injection were in a negative manner. The errors in impedance readings are at unacceptable levels in reactance and resistance especially where a fault occurs at far end side.

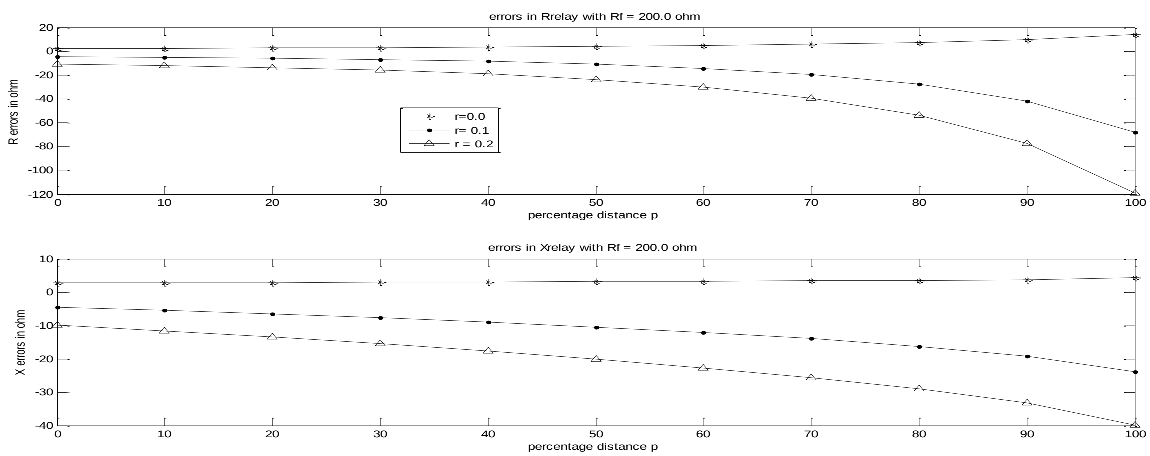


Figure (12) errors in readings with SSSC insertion in loop with R_F =200.0 Ω

4.2.2. STATCOM impacting

During a bolt fault in the presemce of STATCOM the errors in resistance can be neglected but in reactance; errors are considered as shown in fig13.

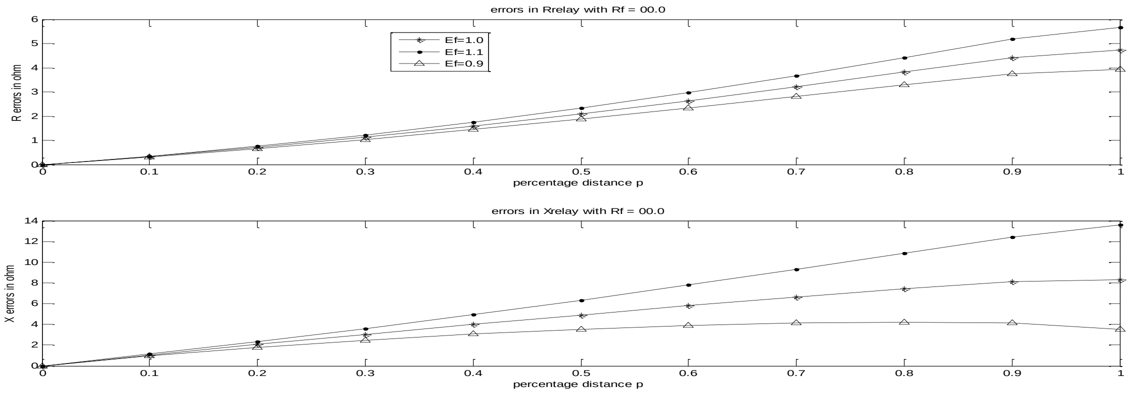


Figure (13) errors in readings with STATCOM insertion in loop with R_F =0.0 Ω

By increasing the fault resistance the resistance readings decreased but reactance readings increased slowly see figure 14.

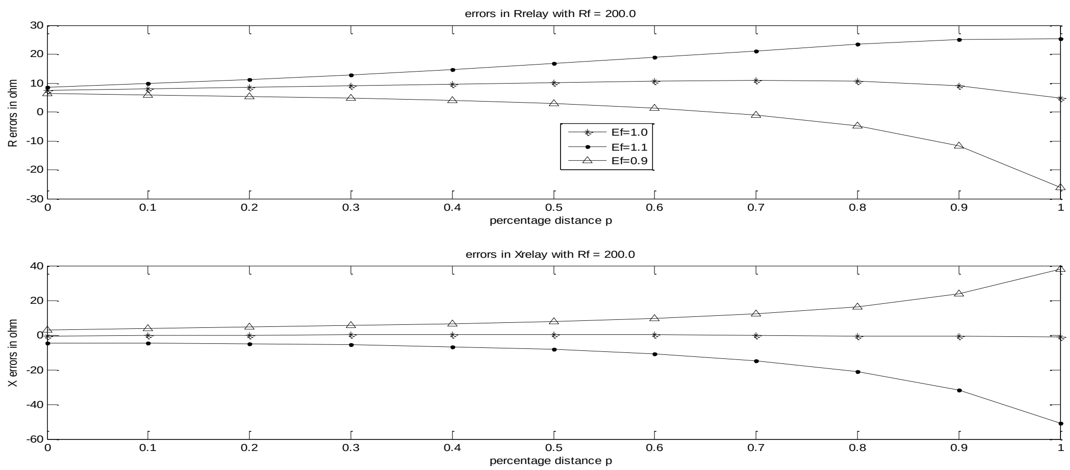


Figure (14) errors in readings with STATCOM insertion in loop with R_F =200.0 Ω

4.3. FACTS at midpoint

In this case, both SSSC and STATCOM are out of the fault loop for faults on the near half of the line, while they are introduced in the fault loop for faults on the far half one.

4.3.1. SSSC impacting

In the presence of SSSC at the midpoint the impedance errors are split into two parts. The right part (from p = 0.5 →1) is for the faults on the near half of the line, while the left part (p=0 → 0.5) is corresponding to the faults on the far half.

Figure (15) shows that at a bolt fault in the right part; increasing the injected voltage has no effect on impedance relay readings. On the other hand in the right side the injected voltage magnitude resulted in decreasing the measured reactance and increasing the measured resistance.

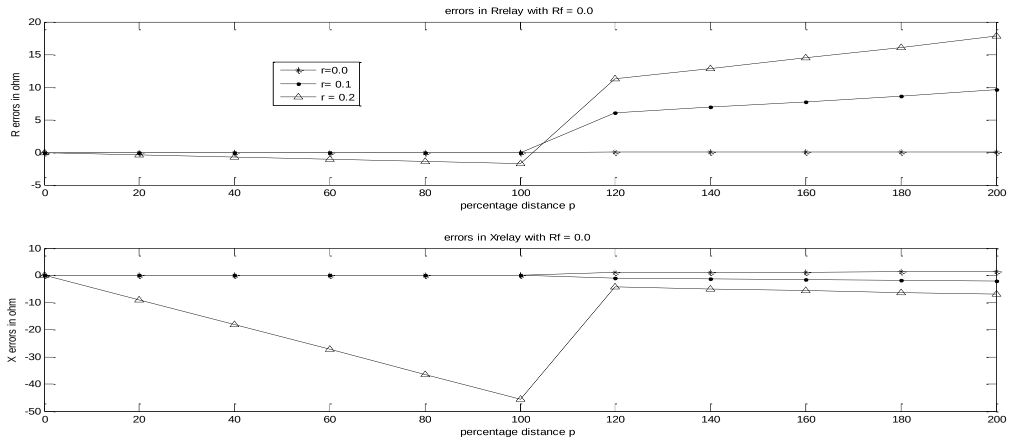


Figure (15) errors in readings with SSSC insertion at midpoint with R_F =0.0 Ω

Comparing fig 15 and 16, We can note that case of inactive SSSC the reactance and resistance readings in the left part are unnoticeable unlike in the right side where errors are more obvious . With increasing the injected voltage the errors are in negative values in both resistance and reactance readings change obviously.

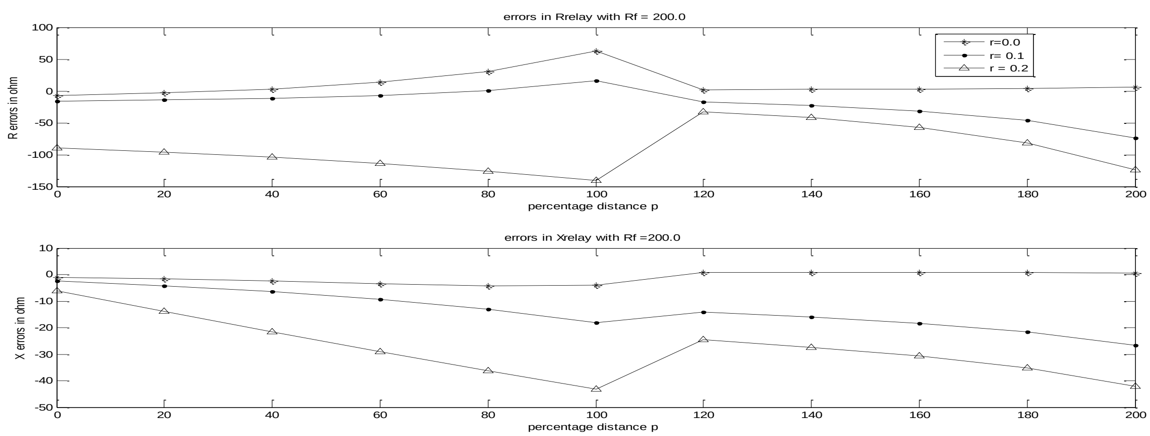


Figure (16) errors in readings with SSSC insertion at midpoint with R_F = 200.0 Ω

4.3.2. STATCOM Impacting

Figure (17) shows case of a bolt fault occurrence during impacting STATCOM at midpoint; in the left part increasing the injected voltage has no effect on impedance relay readings. On the other hand in the right part the injected voltage magnitude nearly has no effect on the errors in measured reactance and very small effect on the measured resistance.

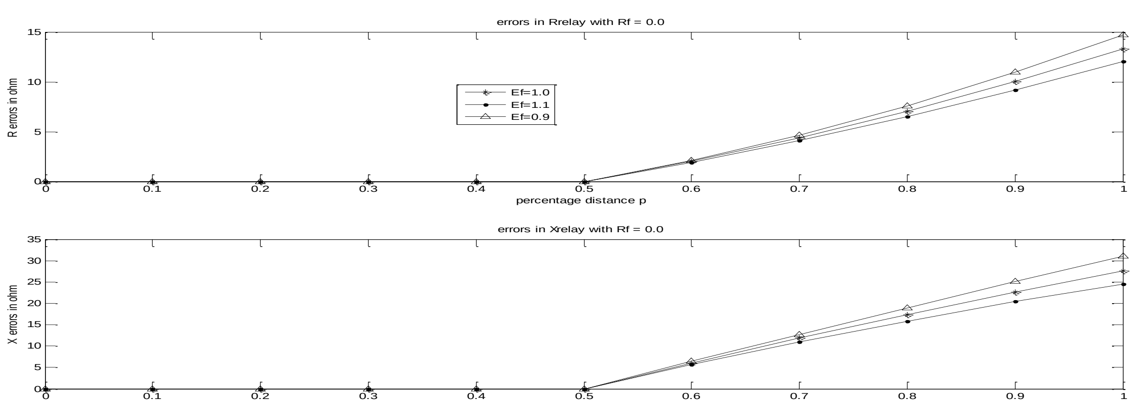


Figure (17) errors in readings with STATCOM insertion at midpoint with $R_F = 0.0\ \Omega$

By increasing the fault resistance; in the left part of fig18 the measured resistance errors increased slightly with increasing the injected voltage unlike reactance readings that decreased slightly. On the other hand at the right part with increasing the injected voltage (Esh), very small effects appeared on resistance also on reactance but with higher manner than resistances. Fortunately the errors in STATCOM are lower than that of SSSC.

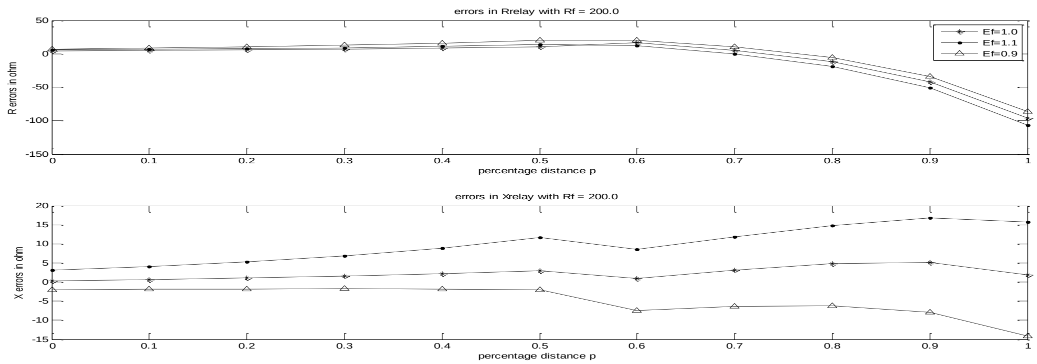


Figure (18) errors in readings with STATCOM insertion at midpoint with $R_F = 200.0\ \Omega$

5. Conclusion

The measured impedance has been studied for two cases of FACTS in and out of the fault loop for the installation points at near end , mid-point , and far end of a transmission line.

Comparing the errors in the measured impedance from its original value, the impedance of the line section between the relaying and the fault points in the presence of SSSC and STATCOM at the line ends, It can be concluded as follows :

1- In case of FACTS installed at the far end of the line

a- The measured impedance at absence of fault resistance is the original value. When there was a fault resistance some errors appeared that could be neglected for STATCOM but can’t for SSSC.

b- When a fault occurred through a resistance RF; increasing the injected voltage of SSSC affected negatively errors more than increasing STATCOM voltage with the same injected reactive power.

2- In case of FACTS devices at near end of the line

a- There were some errors in impedance readings in SSSC that disappeared with STATCOM case of a bolt fault occurrence.

b- By increasing the fault resistance the effects of FACTS are more obvious but SSSC effect was higher.

From the above two notes it can be said that the location of impacting FACTS devices from being inside or outside fault loop is an important uncontrollable factor that affect impedance readings. The operational conditions (changing E_{sh} and V_{SSSC}) of FACTS also play an effective role in appearance of impedance readings error.

3- In case of FACTS devices at midpoint of the line

When FACTS devices were inserted at midpoint the impedance readings changed obviously between two nearby points because one of them located inside fault loop and the other one was outside. This study may be considered a contribution to the protection engineering to easy modify the relay setting according to FACTS impacting locations and operational condition and that will be our future work (Automatic modification of Distance Relay Setting Avoiding the Effects of FACTS Devices Impacting).

Appendix (A)

Derivations of relay impedance ZA formulas

1. without facts

Fault current can be calculated as follows:

$$I_{fa}^{(1)} = I_{fa}^{(2)} = I_{fa}^{(2)} = \frac{V_{f0}}{Z_1 + Z_2 + Z_0 + 3\,R_F} \quad \text{(A1)}$$

Where V_{f0}: pre-fault voltage

Z_1, Z_2, and Z_0: are positive, negative and zero sequence

Where R_F: fault impedance

$$Z_{A1,2} = Z_{SA1} + p\, Z_{L1,2} \quad \text{(A2)}$$

$$Z_{A0} = Z_{SA0} + p\, Z_{L0} \quad \text{(A3)}$$

$$Z_{B1,2} = Z_{SB1} + (1 - p)\, Z_{L1} \quad \text{(A4)}$$

$$Z_{B0} = Z_{SB0} + (1 - p)\, Z_{L0} \quad \text{(A5)}$$

$$Z_{1,2} = \frac{Z_{A1,2}\ x\ Z_{B1,2}}{Z_{A1,2} + Z_{B1,2}} \quad \text{(A6)}$$

$$Z_0 = \frac{Z_{A0}\ x\ Z_{B0}}{Z_{A0} + Z_{B0}} \quad \text{(A7)}$$

$$Z_e = Z_1 + Z_2 + Z_0 = 2\,\frac{Z_{A1,2}\ x\ Z_{B1,2}}{Z_{A1,2} + Z_{B1,2}} + \frac{Z_{A0}\ x\ Z_{B0}}{Z_{A0} + Z_{B0}} \quad \text{(A8)}$$

To complete the fault current equation, we need now to calculate the pre-fault voltage Vfo at point f. from kirchauf's voltage law (KVL) in figure (2), we can deduce these equations.

$$Vf_o = E_A - I_{Apf}\, Z_{A1} \quad \text{(A9)}$$

$$I_{Apf} = \frac{E_A - E_B}{Z_{A1} + Z_{B1}} = \frac{E_A(1 - h\, e^{-j\delta})}{Z_{A1} + Z_{B1}} = \frac{E_A\, K_d}{Z_{A1} + Z_{B1}} \quad \text{(A10)}$$

$$K_d = 1 - h\, e^{-j\delta} \quad \text{(A11)}$$

Using Equation (A11) in Equation (A9), V_{fo} is derived as follows:

$$V_{f0} = E_A - \frac{E_A\, K_d}{Z_{A1} + Z_{B1}}\, Z_{A1} =$$

$$= \frac{E_A[Z_{A1} + Z_{B1} - Z_{A1}] - E_B Z_{A1}}{Z_{A1} + Z_{B1}} = \frac{E_A[Z_{B1} + Z_{A1} h e^{-j\delta}]}{Z_{A1} + Z_{B1}} \quad \text{(A12)}$$

$$V_{f0} = \frac{E_A D}{Z_{A1} + Z_{B1}} \quad \text{(A13)}$$

$$D = Z_{B1} + Z_{A1}\, h\, e^{-j\delta} \quad \text{(A14)}$$

Standing on previous equations; we got $I_{fa}^{(1)}$.

$$I_{fa}^{(1)} = \frac{E_A D}{(Z_{A1} + Z_{B1})(Z_e + R_F)} \quad \text{(A15)}$$

As been discussed previously, the work of distance relays depends on two measured quantities; current and voltage measurable values. We now will deduce these two equations of relay voltage and current at bus A. Using Kirchhoff's current law (KCL) in figure (5), we can directly get both negative and zero sequence current values as follows.

$$I_{Af}^{(2)} = \frac{Z_{B1}}{Z_{A1} + Z_{B1}}\ I_{fa}^{(1)} = C_1\, I_{fa}^{(1)} \quad \text{(A16)}$$

$$I_{Af}^{(0)} = \frac{Z_{B0}}{Z_{A0} + Z_{B0}}\ I_{fa}^{(1)} = C_0\, I_{fa}^{(1)} \quad \text{(A17)}$$

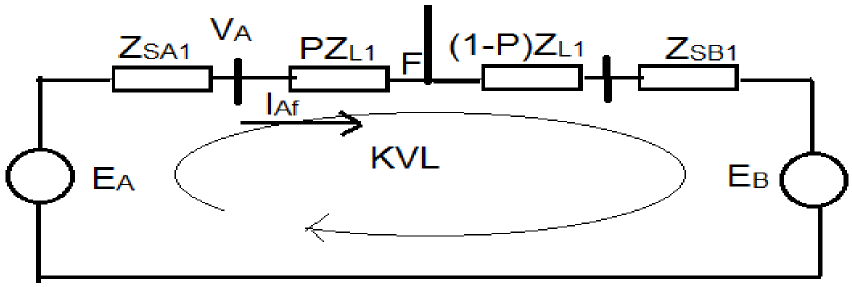
$$C_{1,0} = \frac{Z_{B1,0}}{Z_{A1,0} + Z_{B1,0}} \quad \text{(A18)}$$

KCL is applied only on zero and negative sequence currents only, but positive one has voltage sources. So KVL is used to get $I_{fa}^{(1)}$ as shown in figure (A3).

$$E_A - E_B = I_{Af}^{(1)}\, Z_{A1} - I_{Af}^{(1)}\, Z_{B1} = I_{Af}^{(1)}\, Z_{A1} - (I_{fa}^{(1)} - I_{Af}^{(1)})\, Z_{B1}$$

$$= I_{Af}^{(1)}\, [Z_{A1} + Z_{B1}] - I_{fa}^{(1)}\, Z_{B1} \quad \text{(A19)}$$

$$I_{Af}^{(1)} = C_1\, I_{fa}^{(1)} + \frac{E_A - E_B}{Z_{A1} + Z_{B1}} \quad \text{(A20)}$$



FigureA3. Positive sequence circuit

$$\frac{E_A - E_B}{Z_{A1} + Z_{B1}} = \frac{E_A\, K_d}{Z_{A1} + Z_{B1}} = \frac{E_A D}{(Z_{A1} + Z_{B1})(Z_e + R_F)} x \frac{K_d\,(Z_e + 3R_F)}{D} = C_d\, I_{fa}^{(1)} \quad \text{(A21)}$$

$$I_{Af}^{(1)} = (C_1 + C_d)\, I_{fa}^{(1)} \quad \text{(A22)}$$

$$C_d = \frac{Kd\ x\ (Ze + 3\, Rf)}{D} \quad \text{(A23)}$$

The relay current is:

$$I_{Af} = I_{Af}^{(1)} + I_{Af}^{(2)} + I_{Af}^{(0)} \quad \text{(A24)}$$

For single line to ground fault, the measured voltage will be as follows.

$$V_A = I_{Af}^{(1)} Z_1 + I_{Af}^{(2)} Z_2 + I_{Af}^{(0)} Z_0 = Z_1 (I_{Af}^{(1)} + I_{Af}^{(2)}) + Z_0 (I_{Af}^{(0)}) \quad \text{(A25)}$$

$$Z_0\, I_{Af}^{(0)} = Z_1\, I_{Af}^{(0)} - Z_1\, I_{Af}^{(0)} + Z_0\, I_{Af}^{(0)}$$

$$= Z_1\, I_{Af}^{(0)} + [Z_1 - Z_0]\, I_{Af}^{(0)} \quad \text{(A26)}$$

Back to Equation (A25), we can get.

$$V = Z_1 (I_{Af}^{(1)} + I_{Af}^{(2)} + I_{Af}^{(0)}) + [Z_1 - Z0]\, I_{Af}^{(0)}$$

$$= Z_1 (I_{Af}^{(1)} + I_{Af}^{(2)} + I_{Af}^{(0)} (1 + 3K_{0L})) \quad \text{(A27)}$$

Where K_{0L} is called zero sequence compensated factor. And it is used to modify zero sequence impedance to be positive sequence to treat with relay impedance as positive sequence Z1 only.

$$K_{0L} = 3\, \frac{Z_{L0} - Z_{L1}}{Z_{L1}} \quad \text{(A28)}$$

$$I_{Af} = (C_d + 2C_1 + C_0\, (1 + 3\, K_{0L}))\, I_{fa}^{(1)} \quad \text{(A29)}$$

After finishing deducing the first quantity (current one) formula, we now want to complete the relay measurable feedings.

From KVL in figure (2) the relay voltage can be calculated as follows.

$$V_A = V_F + I_{Af}\, p\, Z_{L1} = 3\, I_{fa}^{(1)} R_F + (C_d + 2C_1 + C_0\, (1 + 3\, K_{0L}))\, I_{fa}^{(1)} \quad \text{(A30)}$$

So after deducting the two quantity formulas, we can get predictable relay impedance at bus A.

$$Z_{relay} = \frac{V_A}{I_{Af}} = \frac{(3R_F + (C_d + 2C_1 + C_0\,(1 + 3K_{0L}))pZ_{L1}) I_{fa}^{(1)}}{(C_d + 2C_1 + C_0\,(1 + 3K_{0L})) I_{fa}^{(1)}}$$

$$Z_{relay} = PZ_{L1} + \frac{3R_F}{C_d + 2C_1 + C_0\,(1 + 3K_{0L})} \quad \text{(A31)}$$

A.2. with the presence of SSSC

Regardless the location of SSSC the following equations are valid

$$C_{SC1,\,0} = Z_{SC} / ZL_{1,\,0} \quad \text{(A32)}$$

$$ZA_{f1,\,0} = Z_{SA1,\,0} + p\, Z_{L1,\,0} \quad \text{(A33)}$$

$$Z_{Bf1,\,0} = Z_{SB1,\,0} + (1 - p)\, Z_{L1,\,0} \quad \text{(A34)}$$

A.2.1. Outside fault loop

According to figA:

$$Z_{mf\,1,\,0} = (m - p)\, Z_{L1,\,0} \quad (A35)$$

$$Z_{Am1,\,0} = Z_{SA1} + m\, Z_{L1} \quad (A36)$$

$$Z_{Am1,\,0} = Z_{SA1,\,0} + m\, Z_{L1,\,0} \quad (A37)$$

$$Z_{A1} = Z_{Af1} \quad (A38)$$

$$Z_{A0} = Z_{Af0} \quad (A39)$$

Calculating the pre-fault current

$$I_{Apf} = \frac{E_A - E_B + V_{SSSC}}{Z_{A1} + Z_{Bn1}} = \frac{E_A(1 + re^{-j\gamma} + h\, e^{-j\delta})}{Z_{A1} + Z_{Bn1}} = \frac{E_A\, K_{dn}}{Z_{A1} + Z_{Bn1}} \quad (A40)$$

Pre-fault voltage

$$V_{f0} = E_A - I_{Apf}\, Z_{A1} = E_A - Z_{A1}\ x\ \frac{E_A - E_B + V_{SSSC}}{Z_{A1} + Z_{Bn1}} = \frac{E_A\,(Z_{Bn1} + [h\, e^{-j\delta} + re^{-j\gamma}]Z_{A1})}{Z_{A1} + Z_{Bn1}} = \frac{E_A\, D_n}{Z_{A1} + Z_{Bn1}} \quad (A41)$$

And with the same previous algorithm in (1), we found that

$$D_n = Z_{Bn1} + Z_{A1}\,[h\, e^{-j\delta} - re^{\,j\gamma}] \quad (A42)$$

$$K_{dn} = 1 + re^{-j\gamma} - h\, e^{-j\delta} \quad (A43)$$

A.2.2. Inside fault loop

$$Z_{mf1,\,0} = (p - m + C_{SC1,\,0})\, Z_{L1,\,0} \quad (A44)$$

$$Z_{mf1,\,0} = (p - m)\, Z_{L1,\,0} \quad (A45)$$

$$Z_{A1,\,0} = Z_{Am1,\,0} + Z_{mf1,\,0} \quad (A46)$$

$$Z_{B1,\,0} = Z_{Bf1,\,0} \quad (A47)$$

Now calculating the pre-fault voltage but noting that the pre-fault current didn't change because the series voltage location doesn't affect the line current.

$$V_{f0} = E_A + V_{SSSC} - I_{Apf}\, Z_{An1} \quad (A48)$$

$$V_{f0} = E_A + V_{SSSC} - \frac{E_A - E_B + V_{SSSC}}{Z_{An1} + Z_{B1}}\, Z_{An1} \quad (A49)$$

$$V_{f0} = \frac{E_A\,(Z_{B1}\,[1 + re^{-j\gamma}] - h\, e^{-j\delta} Z_{An1})}{Z_{An1} + Z_{B1}} \quad (A50)$$

$$D_n = Z_{B1}\,(1 + r\, e^{\,j\gamma}) + Z_{An1}\, he^{-j\delta} \quad (A51)$$

Calculating fault current

$$I_{fa}^{\,(1)} = \frac{E_A\, D_n}{Z_{An1} + Z_{B1}} \quad (A52)$$

Calculating the fault relay current. Firstly; using KVL in positive sequence circuit, the positive sequence is derived as follows:

$$E_A + V_{SSSC} - I_{Af}^{\,(1)}\, Z_{An1} = E_B - I_{Bf}^{\,(1)}\, Z_{B1} \quad (A53)$$

$$\text{But } I_{Bf}^{\,(1)} = I_{fa}^{\,(1)} - I_{Af}^{\,(1)}$$

$$I_{Af}^{\,(1)} = \frac{E_A - E_B + V_{SSSC} + I_{fa}^{(1)}\, Z_{B1}}{Z_{An1} + Z_{B1}}$$

$$= \frac{E_A - E_B + V_{SSSC}}{Z_{An1} + Z_{B1}} + C_1\, I_{fa}^{\,(1)} = \frac{E_A\, K_{dn}}{Z_{An1} + Z_{B1}} + C_1\, I_{fa}^{\,(1)}$$

$$\frac{E_A\, K_{dn}}{Z_{An1} + Z_{B1}} = \frac{E_A\, D_n}{(Z_{An1} + Z_{B1})(Z_e + 3\, R_F)}\ x\ \frac{K_{dn}\,(Z_e + 3\, R_F)}{D_n} = C_d\, I_{fa}^{\,(1)} \quad (A54)$$

Ending to eq (A14); the relay fault voltage can be calculated as follows:

$$V_A = V_F - V_{SSSC} + I_{AfC} (p + C_{SC1}) Z_{L1} \quad (A55)$$

IA_{fC}: is the compensated relay current due to using Z_{L1}.

$$V_A = 3R_F I_{fa}^{(1)} - V_{SSSC} + (C_d + 2C_1 + C_0 (1 + 3K_{0L})) I_{fa}^{(1)} (p + C_{SC1}) Z_{L1} - (C_{SC1} - C_{SC0}) C_0 (1 + 3K_{0L}) I_{fa}^{(1)} \quad (A56)$$

$$Z_{relay} = (p + C_{SC1}) Z_{L1} + \frac{3 R_F - V_{SSSC} / I_{fa}^{(1)} - 3 C_0 K_{0L} (p + C_{SC1}) Z_{L1}}{C_d + 2C_1 + C_0 (1 + 3K_{0L})} \quad (A57)$$

$$V_{SSSC} / I_{fa}^{(1)} = \frac{E_A re^{-j\gamma}}{E_A D_n / (Z_{An1} + Z_{B1})(Z_e + 3R_F)}$$

$$= (Z_e + 3 R_F) (Z_{An1} + Z_{B1}) re^{j\gamma} / D_n \quad (A58)$$

$$Z_{relay} = (p + C_{SC1}) Z_{L1} + \frac{3R_F + C_V + C_Z}{C_d + 2C_1 + C_0 (1 + 3K_{0L})} \quad (A59)$$

$$C_Z = - (C_{SC1} - C_{SC0}) C_0 (1 + 3K_{0L}) Z_{L1} \quad (A60)$$

$$C_V = - (Z_e + 3 R_F) (Z_{An1} + Z_{B1}) re^{j\gamma} / D_n \quad (A61)$$

A.3. with the presence of STATCOM

$$C_{Sh1,0} = Z_{SC} / Z_{L1,0} \quad (A61)$$

$$Z_{mf1,0} = |p - m| Z_{L1,0} \quad (A62)$$

$$Z_{Am1,0} = Z_{SA1,0} + m Z_{L1,0} \quad (A63)$$

$$Z_{Bm1,0} = Z_{SB1,0} + (1 - m) Z_{L1,0} \quad (A64)$$

A.3.1. STATCOM Outside fault loop

From figure (A2a), we derived the fault current using curtailment the E_{sh} and E_B sources to one source eq, the fault calculations are as follows:

$$C_{B1,0} = Z_{Sh} / (Z_{Sh} + Z_{Bm1,0}) \quad (A65)$$

$$Z_{B1,0} = Z_{mf1,0} + C_{B1,0} Z_{Bm1,0} \quad (A66)$$

$$E_{Bn} = \frac{E_B Z_{sh} + E_{sh} Z_{Bm1}}{Z_{sh} + Z_{Bm1}} \quad (A67)$$

$$Z_{Bm1,0n} = \frac{Z_{sh} + Z_{Bm1,0}}{Z_{sh} + Z_{Bm1,0}} \quad (A68)$$

$$h_n = \frac{E_{Bn}}{E_A} \quad (A69)$$

$$\delta_n = \angle E_A - \angle E_{Bn} \quad (A70)$$

$$Z_{A1,0} = Z_{SA1,0} + p Z_{L1,0} \quad (A71)$$

$$Z_{B1,0} = \frac{Z_{sh} + Z_{Bm1,0}}{Z_{sh} + Z_{Bm1,0}} + Z_{mF1,0} = Z_{Bm1,0n} + Z_{mF1,0} \quad (A72)$$

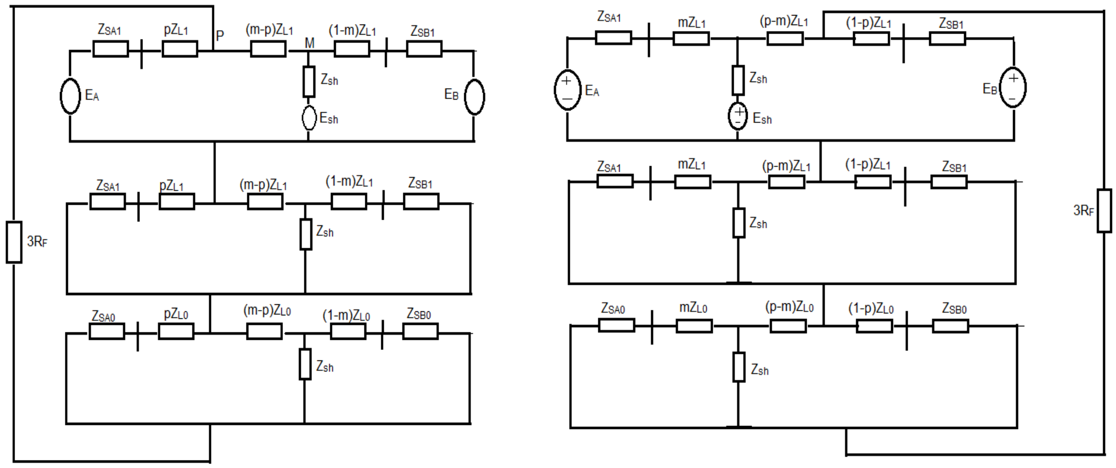


Figure (A2). The system circuit during fault (a) outside fault loops (b) inside fault loop

The pre-fault current I_{Apf} can be calculated as follows:

$$I_{Apf} = \frac{E_A - E_{Bn}}{Z_{A1} + Z_{Bn1}} = \frac{E_A - \frac{E_B Z_{sh} + E_{sh} Z_{Bm1}}{Z_{sh} + Z_{Bm1}}}{Z_{A1} + Z_{Bn1}}$$

$$= \frac{E_A - E_{sh}(1 - C_{B1}) - E_B\, C_{B1}}{Z_{A1} + Z_{Bn1}} = \frac{E_A\, K_{dn}}{Z_{A1} + Z_{Bn1}} \quad \text{(A73)}$$

$$K_{dn} = 1 - e_{sh}\,(1 - C_{B1}) - C_{B1}\, h\, e^{-j\delta} \quad \text{(A74)}$$

Eq (A73) is used to get the pre-fault voltage at point p.

$$V_{F0} = E_A - I_{Apf}\, Z_{A1} = E_A - \frac{E_A - E_{Bn}}{Z_{A1} + Z_{Bn1}}\, Z_{A1}$$

$$= E_A - \frac{E_A - \frac{E_B Z_{sh} + E_{sh} Z_{Bm1}}{Z_{sh} + Z_{Bm1}}}{Z_{A1} + Z_{Bn1}}\, Z_{A1}$$

$$= \frac{E_A\,(Z_{A1} + Z_{Bn1} - Z_{A1}) - [E_B C_{B1} + (1 - C_{B1}) E_{sh}]\, Z_{A1}}{Z_{A1} + Z_{Bn1}}$$

$$= \frac{E_A [Z_{Bn1} - [C_{B1} h\, e^{-j\delta} + (1 - C_{B1}) e_{sh}] Z_{A1}]}{Z_{A1} + Z_{Bn1}} = \frac{E_A D_n}{Z_{A1} + Z_{Bn1}} \quad \text{(A75)}$$

$$D_n = Z_{Bn1} - [C_{B1} h\, e^{-j\delta} + (1 - C_{B1}) e_{sh}] Z_{A1} \quad \text{(A76)}$$

$$C_{B1} = \frac{Z_{sh}}{Z_{Bm1} + Z_{sh}} \quad \text{(A77)}$$

$$I_{fa}^{(1)} = \frac{E_A D}{(Z_{A1} + Z_{Bn1})(Z_e + 3R_F)} \quad \text{(A78)}$$

$$I_{AF}^{(2)} = \frac{Z_{Bn1}}{Z_{Bn1} + Z_{A1}}\, I_{fa}^{(1)} = C_{1n}\, I_{fa}^{(1)} \quad \text{(A79)}$$

$$I_{AF}^{(0)} = \frac{Z_{Bn0}}{Z_{Bn0} + Z_{A0}}\, I_{fa}^{(1)} = C_{0n}\, I_{fa}^{(1)} \quad \text{(A80)}$$

KCL can be applied only on zero and negative sequence circuits only, but positive one has voltage sources. So we will use KVL to get $I_{fa}^{(1)}$ in positive sequence circuit.

$$E_A - E_{Bn} = I_{Af}^{(1)} Z_{A1} - I_{Af}^{(1)} Z_{Bn1} = I_{Af}^{(1)} Z_{A1} - (I_{fa}^{(1)} - I_{Af}^{(1)}) Z_{Bn1}$$

$$= I_{Af}^{(1)} [Z_{A1} + Z_{Bn1}] - I_{fa}^{(1)} Z_{Bn1} \quad \text{(A80)}$$

$$I_{Af}^{(1)} = C_{1n}\, I_{fa}^{(1)} + \frac{E_A - E_{Bn}}{Z_{A1} + Z_{Bn1}} \quad \text{(A81)}$$

$$\frac{E_A - E_{Bn}}{Z_{A1} + Z_{Bn1}} = \frac{E_A\, K_{dn}}{Z_{A1} + Z_{Bn1}}$$

$$= \frac{E_A\, D}{(Z_{A1} + Z_{Bn1})(Z_e + 3\,R_F)}\ X\ \frac{K_{dn}\,(Z_e + 3\,R_F)}{D} = C_{dn}\, I_{fa}^{(1)} \quad \text{(A82)}$$

$$I_{Af}^{(1)} = (C_{dn} + C_1)\, I_{fa}^{(1)} \quad \text{(A83)}$$

$$C_{dn} = \frac{K_{dn}\,(Z_e + 3\,R_F)}{D} \quad (A84)$$

The compensated relay current is:

$$I_{relay} = (C_{dn} + 2C_{1n} + C_{0n}\,(1 + 3K_{0L}))\,I_{fa}^{(1)} \quad (A85)$$

Now we want to calculate the second quantity (voltage quantity) with the same technique used previously.

$$V_A = V_F + I_{AF}^{(1)}\,P\,Z_{L1} + I_{AF}^{(2)}\,P\,Z_{L1} + I_{AF}^{(0)}\,P\,Z_{L0}$$

$$= 3\,R_F\,I_{fa}^{(1)} + P\,Z_{L1}\,[I_{AF}^{(1)} + I_{AF}^{(2)} + (1+3K_{0L})\,I_{AF}^{(0)}]$$

$$= 3\,R_F\,I_{fa}^{(1)} + P\,Z_{L1}\,(C_{dn} + 2C_{1n} + (1 + 3K_{0L})\,C_{0n})\,I_{fa}^{(1)} \quad (A86)$$

Now we can get the relay measured impedance.

$$Z_{relay} = PZ_{L1} + \frac{3\,R_F}{C_{dn} + 2C_{1n} + C_{0n}(1+3K_{0L})} \quad (A87)$$

A. 3.2. STATCOM Inside fault loop:

$$C_{A1,\,0} = Z_{Sh}\,/\,(Z_{Sh} + ZA_{m1,\,0}) \quad (A88)$$

$$Z_{A1,\,0} = Z_{mf1,\,0} + C_{A1,\,0}\,Z_{Am1,\,0} \quad (A89)$$

Following the same sequence in calculating fault current

$$C_V = \frac{(1 - e_{sh})\,(Z_{An1} + Z_{B1})(Z_e + 3R_F)}{(Z_{Am1} + Z_{sh})D_n} \quad (A90)$$

$$I_{Am}^{(1)} = C_V I_{fa}^{(1)} + C_{A1}\,I_{mf}^{(1)} = C_V + C_{A1}\,(C_{dn} + C_1)\,I_{fa}^{(1)} \quad (A91)$$

$$I_{Amc} = [C_V + C_{A1}\,(C_{dn} + 2C_1) + C_{A0}\,C_0\,(1 + 3K_{0L})]\,I_{fa}^{(1)} \quad (A92)$$

The relay voltage V_A is driven as follows:

$$V_A = V_F + I_{mf}\,Z_{mf1} + I_{Af}\,mZ_{L1}$$

$$= 3\,I_{fa}^{(1)}\,R_F + Z_{mf1}\,(C_{dn} + 2C_1 + C_0\,(1 + 3K_{0L}))\,I_{fa}^{(1)}$$

$$+ mZ_{L1}\,(C_V + C_{A1}\,(C_{dn} + 2C_1) + C_0\,C_{A0}\,(1 + 3K_{0L})\,I_{fa}^{(1)} \quad (A93)$$

$$Z_{mf1}\,(C_{dn} + 2C_1 + C_0\,(1 + 3K_{0L})) =$$

$$Z_{mf1}\,(C_V + C_{A1}\,(C_{dn} + 2C_1) + C_0\,C_{A0}\,(1 + 3K_{0L}))$$

$$+ Z_{mf1}\,(-\,C_V + (1 - C_{A1})\,(C_{dn} + 2C_1) + C_0\,(1 - C_{A0})\,(1 + 3K_{0L})) \quad (A94)$$

$$V_A = 3\,I_{fa}^{(1)}\,R_F + pZ_{L1}\,(C_V + C_{A1}\,(C_{dn} + 2C_1)$$

$$+ C_0\,C_{A0}\,(1 + 3K_{0L}))\,I_{fa}^{(1)} + C_{sh}\,I_{fa}^{(1)} \quad (A95)$$

$$C_{sh} = Z_{mf1}\,(-\,C_V + (1 - C_{A1})\,(C_{dn} + 2C_1) + C_0\,(1 - C_{A0})\,(1 + 3K_{0L}) \quad (A96)$$

From equations (A96) and (A93) by compensating the bus A current, we can get the relay measured impedance as follows:

$$Z_{relay} = pZ_{L1} + \frac{3R_F + C_{sh}}{C_V + C_{A1}(C_{dn} + 2C_1) + C_{A0}\,C_0\,(1+3K_{0L})} \quad (A97)$$

Where

$$D_n = Z_{An1} + Z_{B1} - (C_{A1} + e_{Sh}\,(1 - C_{A1}) - h\,e^{-j\delta})\,(C_m\,Z_{Am1} + Z_{An1}) \quad (A98)$$

$$C_m = \frac{(Z_{An1} + Z_{B1})(1 - e_{sh})}{(Z_{Am1} + Z_{sh})\,K_{dn}} \quad (A99)$$

References

A. Kazemi, S. Jamali, (Sep 2009) "Comparing Impacts of SSSC and STATCOM on Measured Impedance at Relaying Point" *IEEE transactions, power delivery*.

Tarlochan Singh Sidhu, Rajiv K. Varma, Pradeep Kumar Gangadharan and Fadhel Abbas Albasri, (JULY 2005), "Performance of Distance Relays on Shunt FACTS Compensated Transmission Lines "*IEEE transactions, power delivery*, VOL. 20, NO.3.

M. Khederzadeh, sep2009, "Impact of SSSC on the Digital Distance Relaying", *IEEE transactions, power delivery*.

S Jamali and H Shateri, (2011), "Locus of apparent impedance of distance protection in the Presence of SSSC "*EUROPEAN TRANSACTIONS ON ELECTRICAL POWER Euro. Trans. Electr. Power*.

Prof. Detlef Schulz, (2007), "Improvement in the Quality of Delivery of Electrical Energy using Power Electronics Systems", *Helmut Schmidt University*, pp27-39.

Yoshihide Has, (2007), "Handbook of Power System Engineering", pp315-325.

Y. Q. Xia, K. K. Li and A. K. David , (Jan 1994) ,"Adaptive Relay Setting for Stand-alone Digital Distance Protection", *IEEE Transactions on Power Delivery*, Vol. 9, No. 1,.

Yi Hu, M, Damir Novosel and Murari Mohan Saha, (Jan 2002), "An Adaptive Scheme for Parallel-Line Distance Protection ", *IEEE transactions, power delivery*, VOL. 17, NO. 1.