CORE Metadata, citation and similar papers at core.ac.uk
Provided by International Institute for Science, Technology and Education (IISTE): E-Journals

Innovative Systems Design and Engineering www.iiste.org
ISSN 2222-1727 (Paper) ISSN 2222-2871 (Online)
Vol 3, No.11, 2012

An Analytical Design: Performance Comparison of MMSE and ZF Detector

Pargat Singh Sidhu[1], Gurpreet Singh[2] , Amit Grover[3*]

1. Department of Electronics and Communication Engineering, Shaheed Bhagat Singh State Technical Campus, Moga Road (NH-95), Ferozepur-152004, India.
2. Department of Electronics and Communication Engineering, Shaheed Bhagat Singh State Technical Campus, Moga Road (NH-95), Ferozepur-152004, India.
3. Department of Electronics and Communication Engineering, Shaheed Bhagat Singh State Technical Campus, Moga Road (NH-95), Ferozepur-152004, India.

[*]Email of the corresponding author: amitgrover_321@rediffmail.com

Abstract

By using multiple antennas at transmitter and receiver sides, the performance of the system can be enhanced in terms of high data rates by applying the concept of multiplexing and diversity as compared to single antenna systems. In this article we will study and compare the performance of BLAST architecture with different detectors like Zero Forcing (ZF), Minimum Mean Square Error (MMSE). Furthermore, we introduced OSIC schemes to improve the independent coded BLAST system and to combat the error propagation. We have also analyzed the BER performance of these MIMO schemes in Rayleigh and Rician fading channel. Finally we observed that the performance of BPSK and QPSK modulation techniques is almost same in BLAST architecture, while using the given detection techniques in both the channels and 16-QAM modulation technique gives the worst result.

Keywords: Binary Phase Shift Key (BPSK), Bit Error Rate (BER), Multiple input multiple output (MIMO),Maximum Likelihood (ML), Minimum mean square error (MMSE), Zero Forcing (ZF), Ordered Successive Interference Cancellation (OSIC), Quardrature Phase Shift Keying (QPSK), Quadrature Amplitude Modulation (QAM), Independent identically distributed (i.i.d), Bell Laboratories Layered Space-Time (BLAST)

1. **Introduction**

The use of multiple antennas at both the transmitter and the receiver sides can drastically improve the channel capacity and data rate [1].The study of the performance limits of MIMO system [9] becomes very important since it will give lot of ideas in understanding and designing the practical MIMO systems [10]. Bell Laboratories Layered Space-Time (BLAST) Architecture and first practical implementation of this architecture on MIMO wireless communications to demonstrate a spectral efficiency as high as 40bits/s/Hz in real time in the laboratory [8]. Many schemes have been proposed to explode the high spectral efficiency of MIMO channels, among which BLAST [8] is relatively simple and easy to implement and can achieve a large spectral efficiency. In BLAST [3] at the transmitter de-multiplexes the input data streams into 'n' independent sub-streams, which are transmitted in parallel over the 'n' transmitting antennas. At the receiver end, antennas receive the sub-streams, which are mixed and superimposed by noise. Detection process [3] mainly involves three operations: Interference Suppression (nulling), interference cancellation

(Subtraction) and Optimal Ordering. The optimal Ordering is the last process that ensures the detected symbol has highest Signal to noise ratio (SNR). So, BLAST algorithm [8] integrates both linear and non-linear algorithms presented in the interference nulling and interference cancellation with 'N' transmitting antennas and 'M' receiving antennas respectively in Ricean Flat fading channel [7].In this we will considered receiving antennas are greater than or equal to transmitting antennas (M≥N), the first detected sub-stream has a diversity gain of only M-N+1 [5].

2. MIMO Channel Model

By considering a communication system with 'N' number of transmitting antennas and 'M' number of receiving antennas in Ricean Flat Fading channel [7], we adopted a correlation-based channel model [11] which can be expressed as

$$H \sim R_{Rx}^{\frac{1}{2}} H_w \left(R_{Tx}^{1\backslash 2}\right)^{T} \tag{1}$$

3.Rayleigh Fading and Rician Fading Channel Channel

The fading effect is usually described statistically using the Rayleigh distribution [11]. Ricean Fading and the presence of a fixed (possibly line-of-sight or LOS) component in the channel will result in Ricean fading [7].

4.Decoding Algorithm for BLAST System

The optimal detection order in the decoding algorithm of BLAST System is from the strongest symbol to the weakest one [11] with the condition of number of receive antennas are more than the number of transmit antennas, that is M x N.

4.1 Zero Forcing Nulling

Zero Forcing nulling can be done through multiplying r_n by an M × 1 vector W_n that is orthogonal to interference vectors $H_{n+1}, H_{n+2}, \ldots . H_N$ but not orthogonal to H_n .In other words, W_n should be such that

$$H_i . W_n = 0, \quad i = n+1, n+2 \ldots \ldots . N \tag{2}$$

$$H_n . W_n = 1 \tag{3}$$

W_n= Zero-Forcing Nulling vector with minimum norm.

Such a vector is uniquely calculated from the channel matrix H. To calculate W_n from H, for M ≥ N first we should replace the rows 1, 2...., n − 1of H by zero.

Let us denote the resulting matrix by Z. Then, W_n is the nth column of Z^+ the Moore–Penrose generalized inverse, pseudo-inverse, of Z

Using the error-free detection formula for r^n in w^n in (3), we have

$$r_n W_n = c_n + N W_n \tag{4}$$

The noise in (4) is still Gaussian and the symbol c_n can be easily decoded. The decoded symbol $\hat{c}_n$ is the closest constellation point to $r_n . W_n$. The noise enhancing factor using (4) is

$$E[(N.W_n)^H . N . W_n] = W_n^H . E[N^H . N] W_n \tag{5}$$

$$= N_0 \|W_n\|^2 \tag{6}$$

We know that zero forcing is given by

$$W_{ZF} = (H^{*}H)H \tag{7}$$

Comparing (6) with (7) demonstrates why adding an interference cancelation step improves the performance. Using the combination of canceling and nulling in a ZF-DFE structure enhances the noise by a factor of $||Wn||^2$. Vector W_n is orthogonal to N − n rows of the channel matrix H. On the other hand, using a pure interference nulling method like ZF, the corresponding vector that detects the nth symbol, the n^{th} column of the pseudo-inverse, is orthogonal to N − 1 rows of the channel matrix H. Using the Cauchy–Schwartz inequality, it can be shown that the norm of a vector is larger if it has to be orthogonal to a greater number of rows. Therefore, the enhancing factor for the case of nulling alone, ZF, is more than that of the canceling and nulling, ZF-DFE. For the first vector, n = 1, the two cases are identical.

4.2 Minimum Mean Square Error Nulling (MMSE-Interference nulling)

Another approach for interference nulling is MMSE. Let us assume that the trans-mitted vector is a zero-mean random vector that is uncorrelated to the noise. Considering the received vector r in $r = C \cdot H + N$
as a noisy observation of the input C, the linear least-mean-squares estimator of C is

$$M = H^{H}.\left(\frac{I^{N}}{\gamma} + H.H^{H}\right)^{-1} \tag{8}$$

Note that in the nth stage of the algorithm, the effects of $c_1, c_2, \dots, c_{n-1}$ have been canceled. Therefore, similar to the ZF nulling, to calculate c_n , first we should replace the rows $1, 2, \dots, n-1$ of H by zero. Let us denote the resulting matrix by Z as we did in the ZF case. Now, to find the best estimate of the nth symbol, that is $\hat{c}_n$, we replace H with Z in (9) to calculate the best linear MMSE estimator at stage n as

$$M = Z^{H}.\left(\frac{I^{N}}{\gamma} + Z.Z^{H}\right)^{-1} \tag{9}$$

Then, the nth column of M, denoted by M_n is utilized as the MMSE nulling vector for the n^{th} symbol. In other words, the decoded symbol $\hat{c}_n$ is the closest constellation point to $r_n.M_n$

4.3.Zero Forcing with SIC

OSIC is basically based on subtraction of interference of already detected elements of **s** from the receiver vector r which results in a modified receiver vector with a few interferers. In other words, SIC is based on the subtraction of interference of already detected elements **s** from the received vector **x which** results in a modified receiver vector with a few interferers. When Successive Interference Cancellation (SIC) is applied, the order in which the components of **s** are detected is important to the overall performance of the system. To determine a good detection order, the covariance matrix of the estimation error $s - s_{est}$ is used.
We know that the covariance matrix is given by

$$Q = E[\in . \in^{H}] = \sigma_n^2 (H^H H)^{-1} \tag{10}$$

$$Q = E[(s - s_{est})(s - s_{est})^{H}] = \sigma_n^2 (H^H H)^{-1} \equiv \sigma_n^2 P \tag{11}$$

Where P $=H^{+}(H^{+})^{H}$

Let $(s_{est})_p$ be the p[th] entry of s_{est}, then the "best" is the one for which P_{pp}(i.e., the p-th diagonal element of **P**) is the smallest. Because this is estimate with the smallest error variance. From the eq^{n} (11) it becomes clear that P_{pp} is equal to the squared length of row p of H^{+}. Hence, finding the minimum squared length row of H^{+} is equivalent.

Summarizing, the decoding algorithm consist of three parts:

- Ordering
- Interference Nulling
- Interference Cancellation

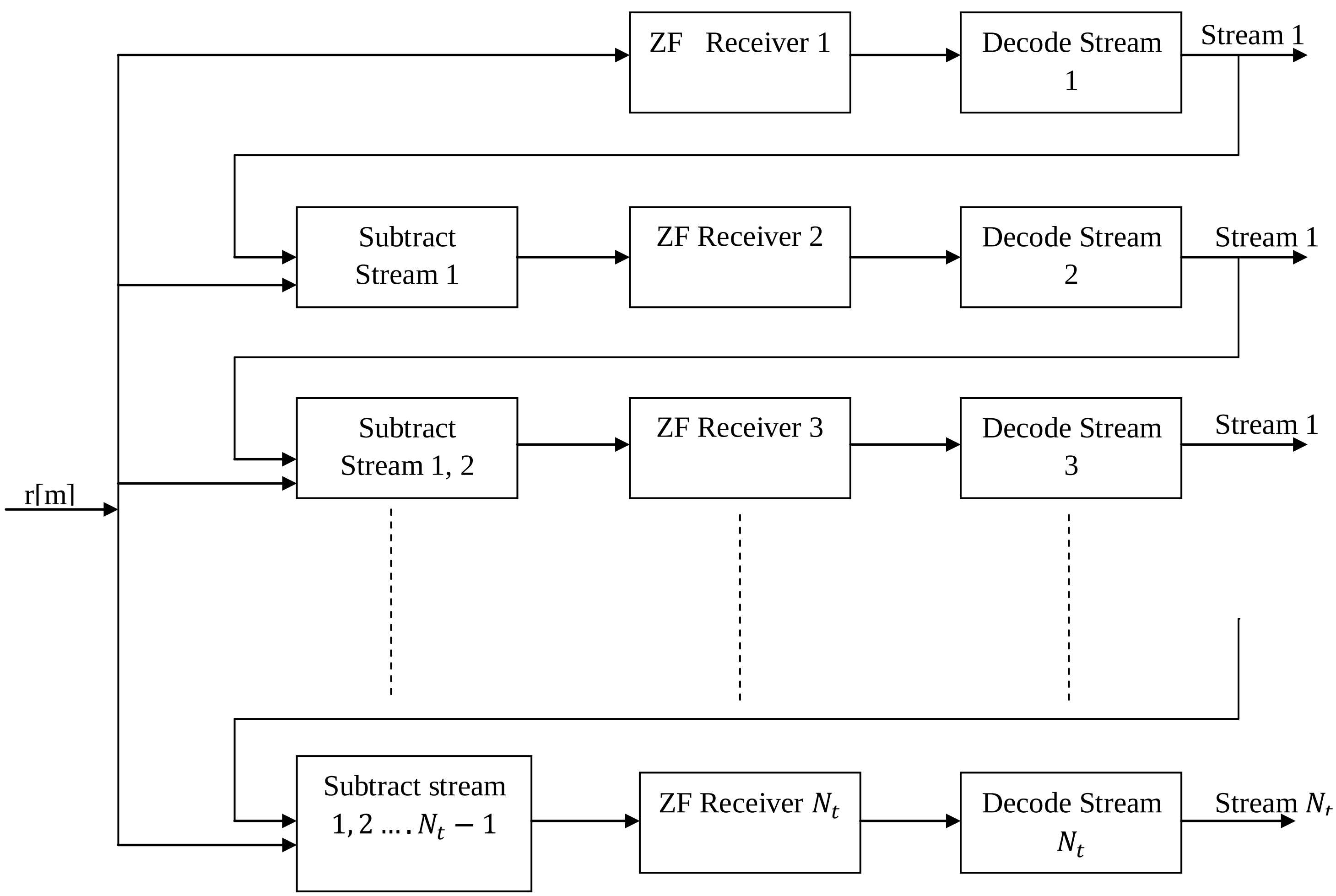

Figure.1 SIC Zero Forcing Detector

We use the first Zero-Forcing detector to detect the data stream $s_1(m)$ decode it and then subtract this decoded stream from the received vector. Assuming the first stream is successfully decoded, and then the second Zero-Forcing detector only needs to deal with $s_3 \dots \dots \dots s_{N_t}$ as interference, since s_1 has been correctly subtracted off. Thus, the second Zero-Forcing detector projects onto a subspace which is orthogonal to $h_3 \dots \dots \dots . h_{N_t}$.This process is continued until the last Zero-Forcing detector does not have to deal with any interference from the other data streams. We assume subtraction is successful in all preceding stages. This SIC (Successive Interference Cancellation) Zero-Forcing detector architecture is illustrated in Figure.1 so we can see here with respect to ZF, the ZF with OSIC algorithm introduces extra complexity.

4.4 The Minimum Mean Square Error

The MMSE suppresses both the interference and noise components, whereas ZF receiver removes only the interference components. This implies that the mean square error between the transmitted symbols and the estimate of the receiver is minimized. Hence MMSE is superior to ZF in the presence of noise. At low SNR, MMSE becomes matched filter and at high SNR, MMSE becomes Zero Forcing (ZF). For MMSE-BLAST, the nulling vector for the i^{th} layer is

$$w^i = \left(H_i H_i^* + \frac{1}{SNR} I\right)^{-1} h_i, \quad i = 1,2 \dots \dots . N \tag{12}$$

Where $H_i = C^{M \times i}$ consists of the first I columns of H. Denote h_i the i-th column of H

Therefore

$$W_{MMSE} = \left(H^* H + \frac{1}{SNR} I\right)^{-1} H^* \tag{13}$$

Where

$H \in C^{M \times N}$ is the Rayleigh fading channel with independent, identically distributed (i.i.d.)

(H^*) is the complex conjugate of H

N transmit antennas and M receiver antennas

We assume that the number of receive antennas is no less than the number of transmit antennas $M \geq N$

SNR is Signal to Noise Ratio

MMSE at a high SNR

$$W_{MMSE} = \left(H^* H + \frac{1}{SNR} I\right)^{-1} H^* \approx (H^H H)^{-1} H^H \tag{14}$$

At a high SNR MMSE becomes Zero Forcing

Hence MMSE receiver approaches the ZF receiver and therefore realizes (N-M+1)th order diversity for each data stream.

4.5 Minimum Mean Square Error (MMSE) with SIC

In order to do OSIC with MMSE, then the algorithm resulting as follows

Covariance matrix can be written as

$$Q = E[(s - s_{est})(s - s_{est})^H] = \sigma_n^2 (\alpha I + H^H H)^{-1} \equiv \sigma_n^2 P \tag{15}$$

Note that P is somewhat different from the case where ZF is used as estimation technique

Covariance matrix of the estimation error $(s - s_{est})$ will be used to determine good ordering for detection.

MMSE–SIC: a bank of linear MMSE receivers, each estimating one of the parallel data streams, with streams successively cancelled from the received vector at each stage. MMSE with OSIC is explained with block diagram explained in figure.2

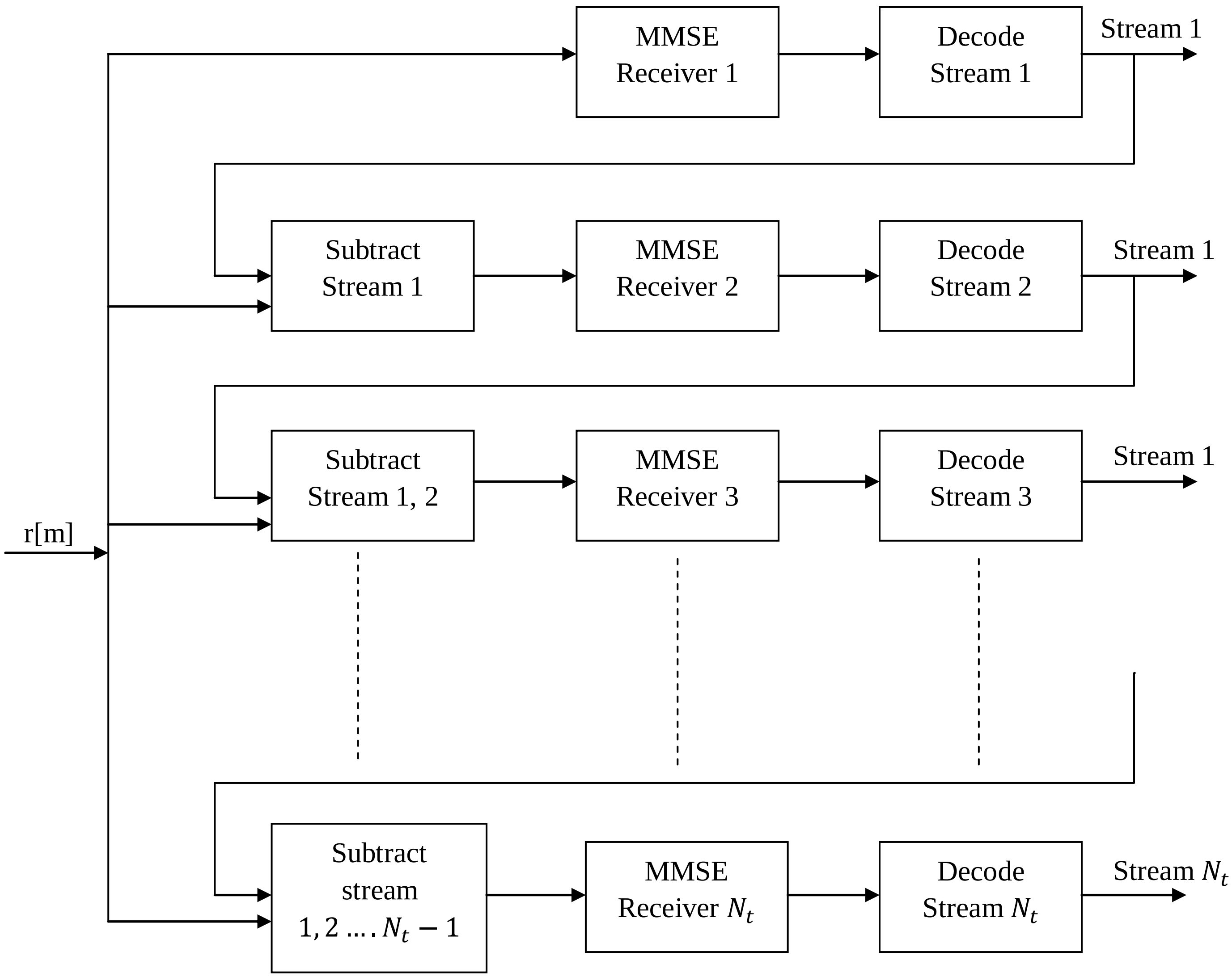

Figure.2 SIC MMSE detector

5. Simulation and Result

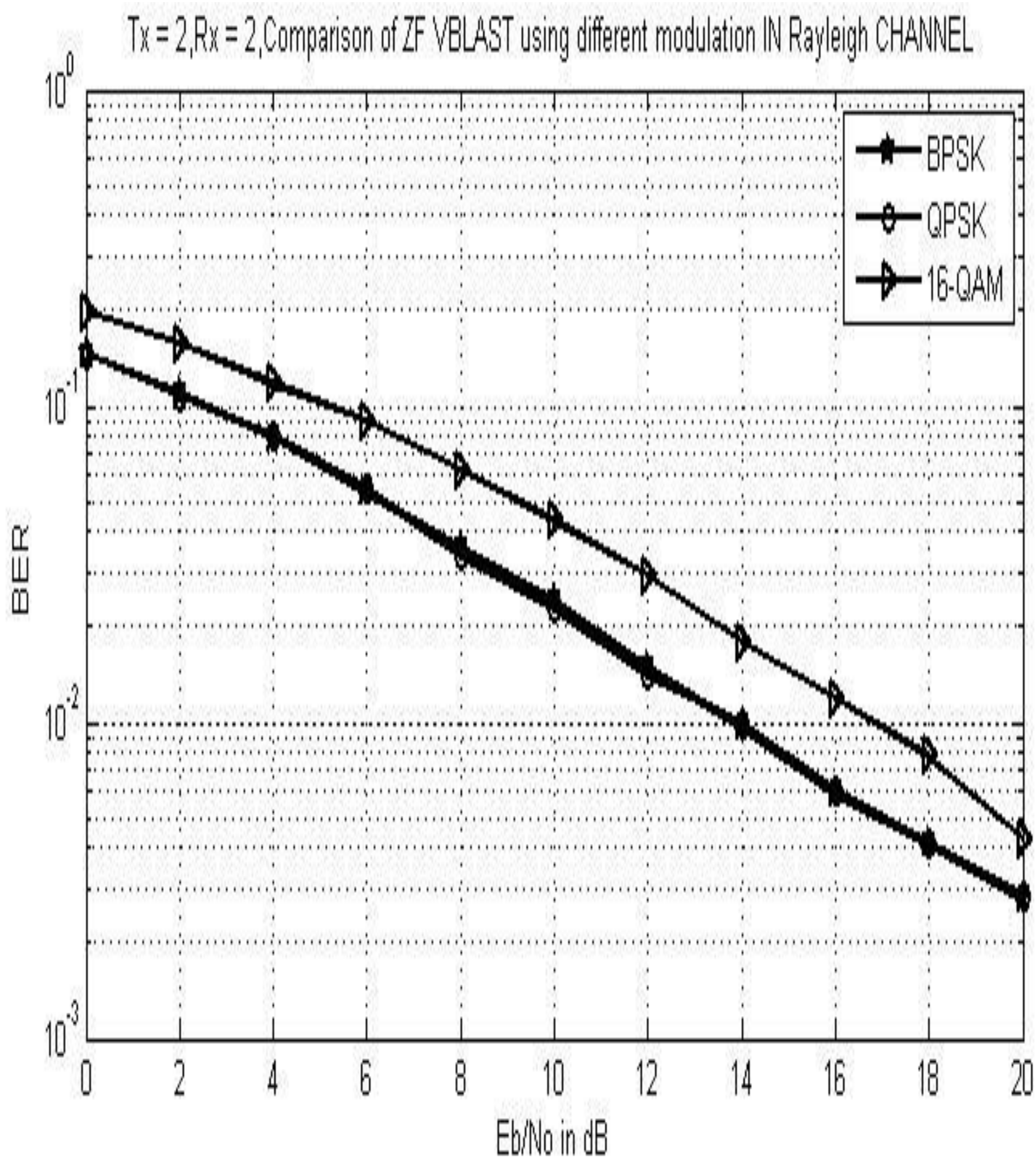

Figure.3 Comparison of ZF-BLAST using different modulations in Rayleigh Channel

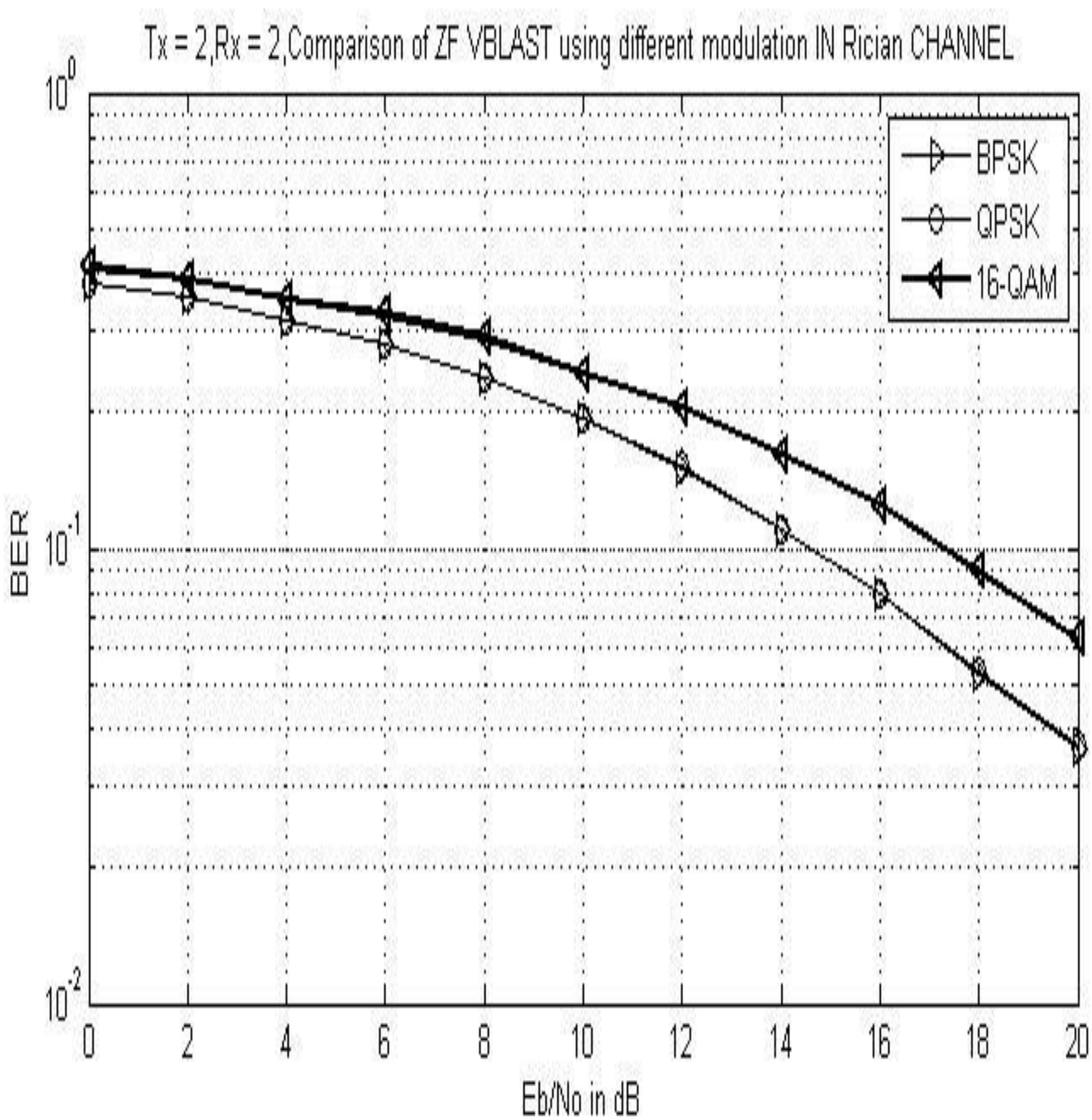

Figure.4 Comparison of ZF-BLAST using different modulations in Rician Channel

In Figure3, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.001, there is approximately 3 dB difference between the BPSK and16 QAM modulations in ZF in Rayleigh Channel.

In Figure4, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.01, there is approximately 3 dB difference between the BPSK and16 QAM modulations in ZF in Ricean Channel.

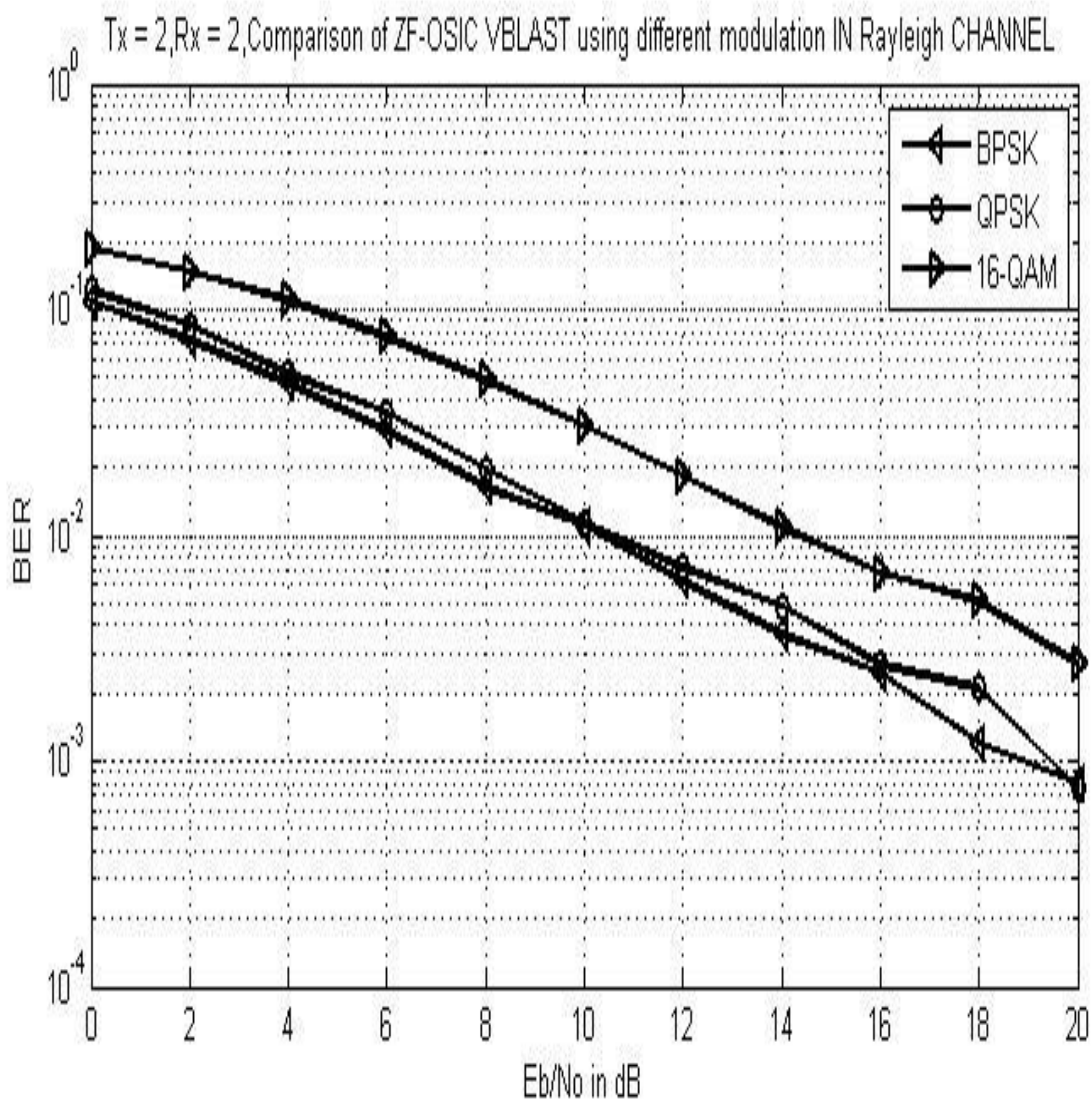

Figure5.Comparison of ZF-OSIC-BLAST using different modulations in Rayleigh Channel

In Figure5, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.001, there is approximately 4 dB difference between the BPSK and16 QAM modulations in ZF-OSIC in Rayleigh Channel.

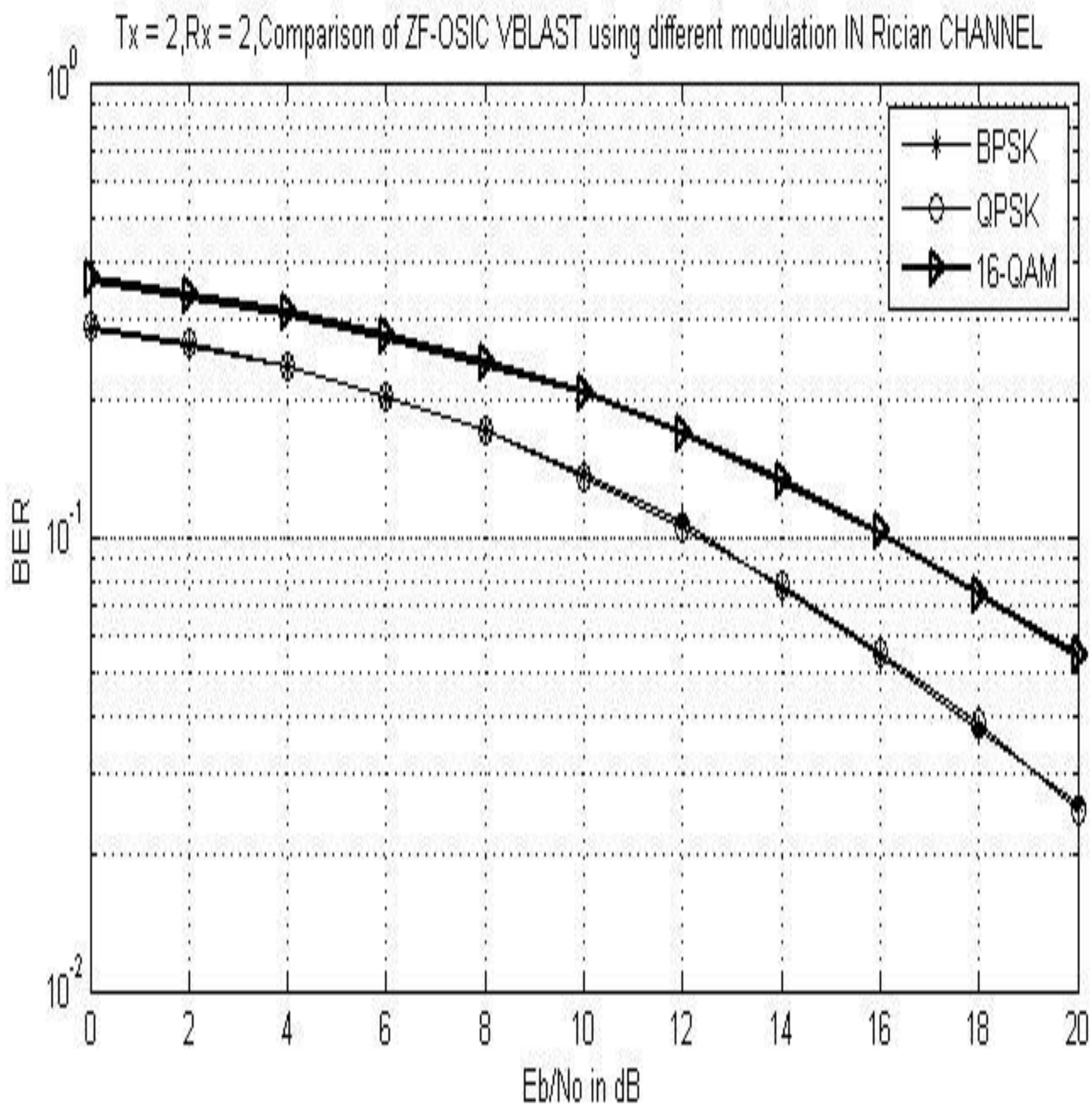

Figure6.Comparison of ZF-OSIC-BLAST using different modulations in Rician Channel

In Figure6, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.01, there is approximately 4 dB difference between the BPSK and16 QAM modulations in ZF-OSIC in Ricean Channel.

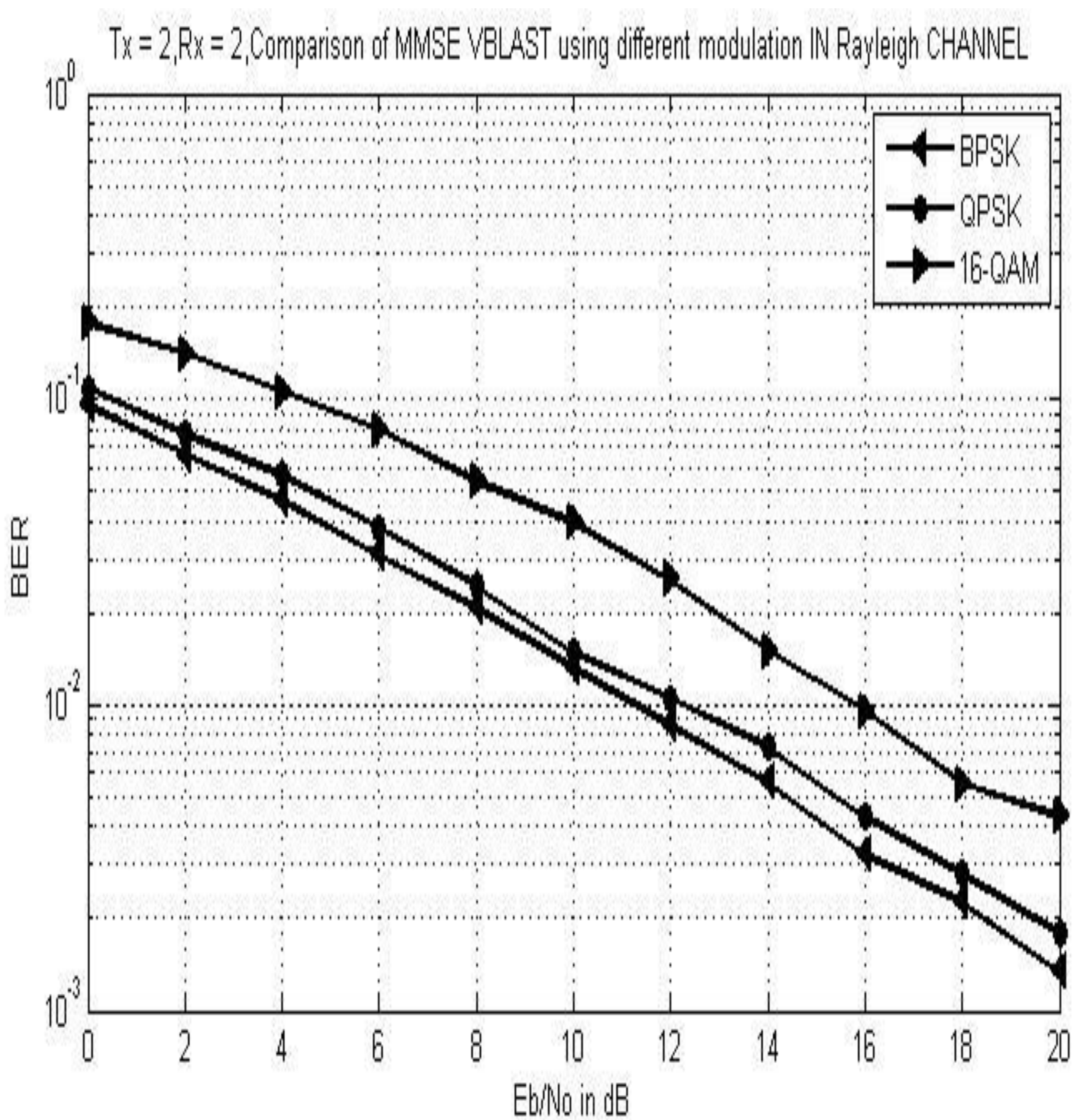

Figure7.Comparison of MMSE-BLAST using different modulations in Rayleigh Channel

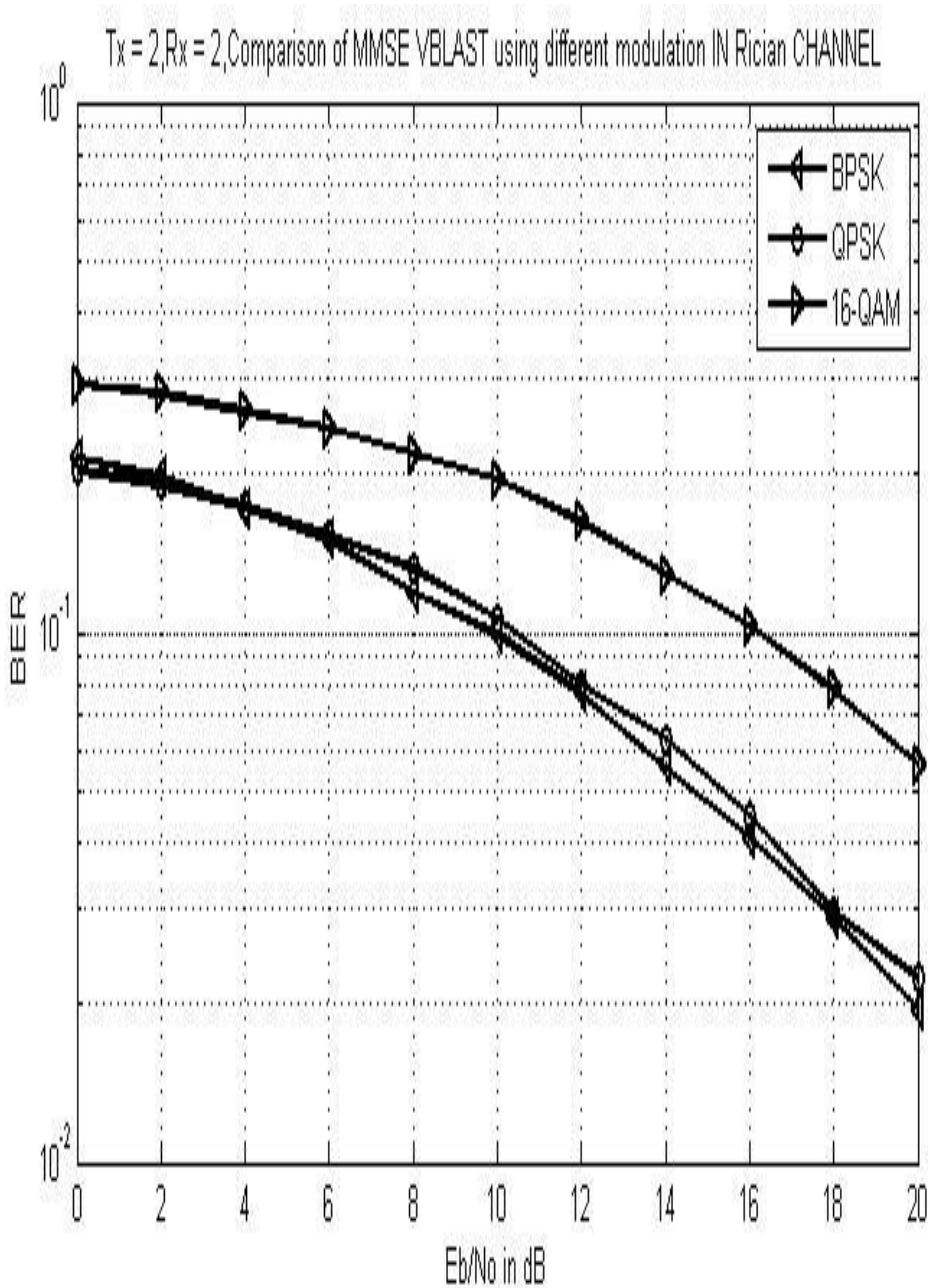
Tx = 2,Rx = 2,Comparison of MMSE VBLAST using different modulation IN Rician CHANNEL
BPSK
QPSK
16-QAM
BER
Eb/No in dB
10^0
10^-1
10^-2
0
2
4
6
8
10
12
14
16
18
20

Figure8.Comparison of MMSE-BLAST using different modulations in Rician Channel

In Figure7, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.001, there is approximately 5 dB difference between the BPSK and16 QAM modulations in MMSE in Rayleigh Channel.

In Figure8, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.01, there is approximately 6 dB difference between the BPSK and16 QAM modulations in MMSE in Ricean Channel.

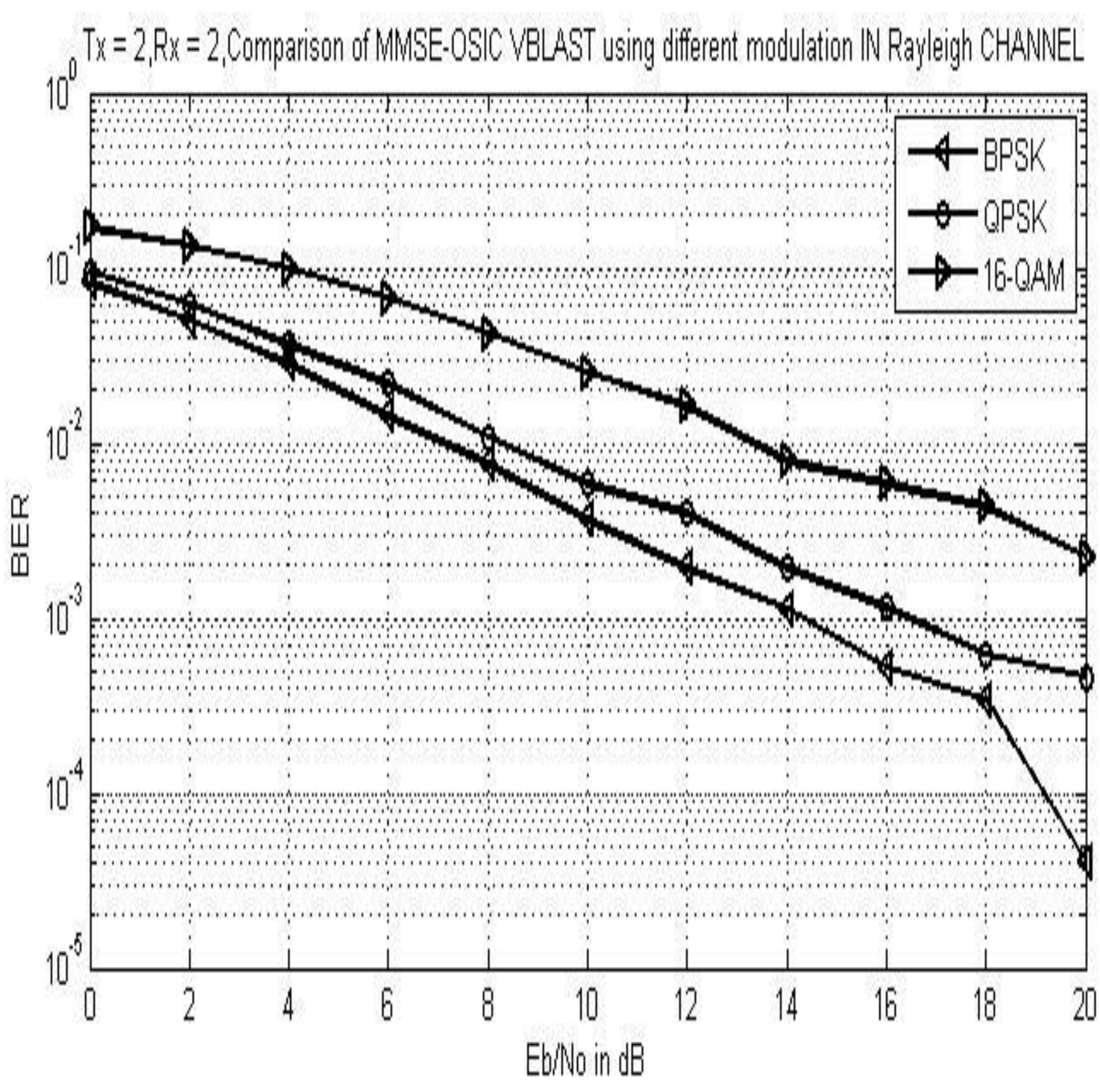

Figure9.Comparison of MMSE-OSIC-BLAST using different modulations in Rayleigh Channel

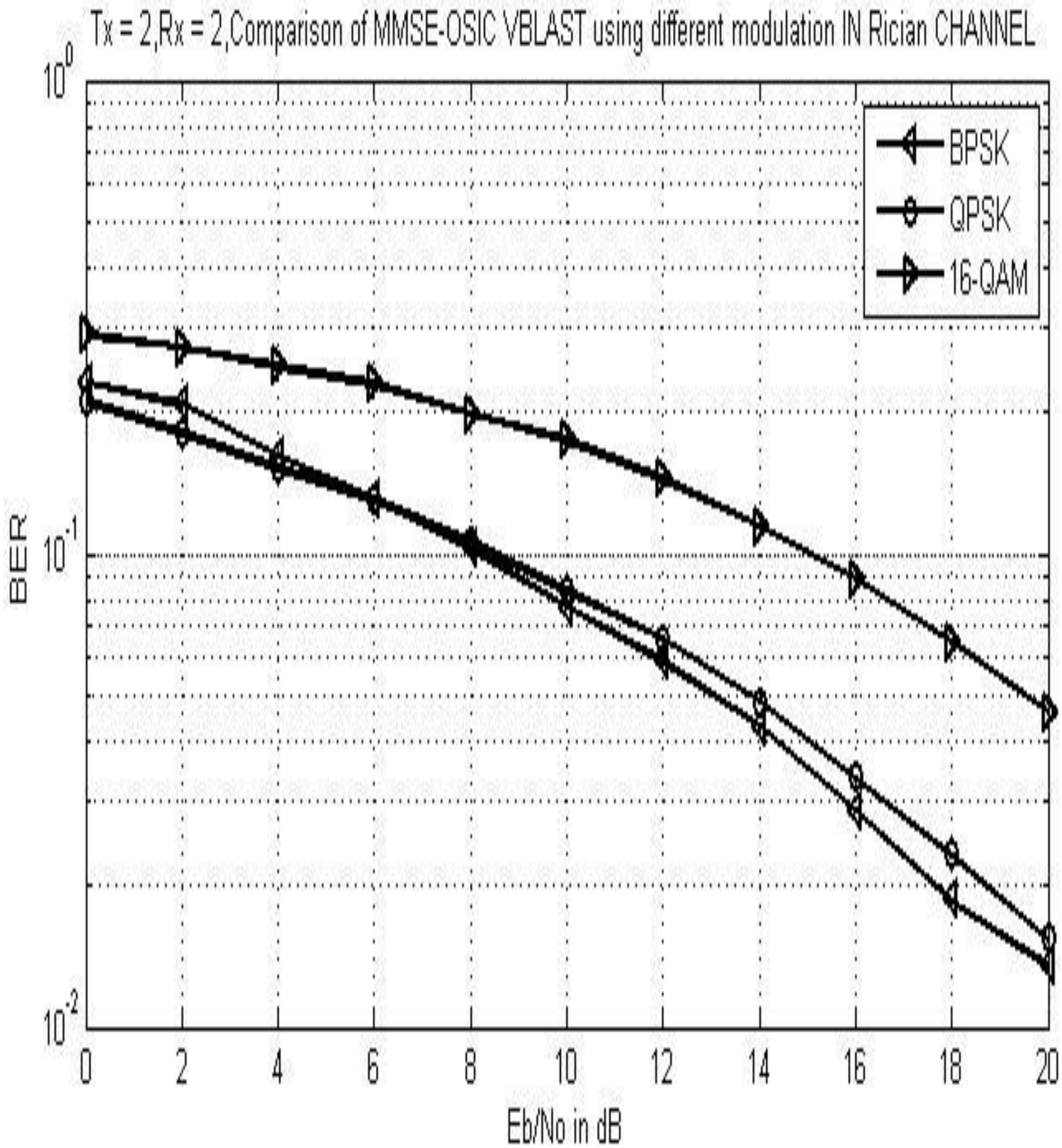

Figure10.Comparison of MMSE-OSIC-BLAST using different modulations in Rician Channel

In Figure9, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.001, there is approximately 7 dB difference between the BPSK and16 QAM modulations in MMSE-OSIC in Rayleigh Channel.

In Figure10, we have observed that BPSK and QPSK have almost the same results and 16 QAM has the worst result than BPSK and QPSK. At BER 0.01, there is approximately 8 dB difference between the

BPSK and16 QAM modulations in MMSE-OSIC in Ricean Channel.

6. Conclusions

Finally we conclude that by introducing the OSIC schemes the performance of BLAST architecture with these detectors like Zero Forcing (ZF), Minimum Mean Square Error (MMSE) has been improved. We have also observed that OSIC schemes improve the independent coded BLAST system by combating the error propagation; Furthermore we observed that BPSK and QPSK modulation techniques give the almost same results in BLAST architecture with these detection techniques in both Ricean and Rayleigh fading channel and 16-QAM modulation technique gives the worst results. When the SNR gets higher, the post detection of SNR is mainly affected by channel matrix H. By comparing the MMSE-OSIC and ZF-OSIC, at BER=0.001 using BPSK modulation there is an approximately 3 dB difference between these two detectors in Rayleigh channel and at BER=0.01 there is an approximately 4 dB difference between these two detectors in Rician Channel. By comparing the MMSE-OSIC and ZF-OSIC, at BER=0.001 using QPSK there is an approximately 2 dB difference in Rayleigh channel and at BER=0.01 there is an approximately 4 dB difference between these two detectors in Rician Channel. By comparing the MMSE-OSIC and ZF-OSIC, at BER=0.001 using 16 QAM there is an approximately 1.3 dB difference in Rayleigh channel and at BER=0.01 there is an approximately 1.3 dB difference between these two detectors in Rician Channel.

7. References

[1] I.E. Telatar (1999), "Capacity of multi-antenna Gaussian channels, "*European Transactions on Telecommunications, vol. 10, no.6,* pp.585-595.

[2] M. Varanasi and T. Guess (1997), "Optimum decision feedback multiuser equalization with successive decoding achieves the total capacity of the Gaussian multiple-access channel," *Conference Record of the Thirty-First Asilomar Conference on signals, Systems and computers, vol. 2,* pp. 1405-1409.

[3] G. D. Golden, G. J. Foschini, R.A. Valenzuela, and P. W. Wolniasky (1999), "Detection algorithm and initial laboratory results using the V-BLAST space-time communication architecture*," Electron Lett., vol.35, no.1,* pp.1415

[4] G. J. Foschini (1996), "Layered space–time architecture for wireless communication in a fading environment using multi–element antennas," *Bell-Labs Techn. J.,* pp. 41–59.

[5] G. Ginis and J. M. Cioffi (2001), "On the relationship between V-BLAST and GDFE," *IEEE Communications letters, vol. 5,* pp. 364-366

[6] Choi, J,, Yu, H., and Lee, Y.H.(2005), "Adaptive MIMO decision Feedback Equalization for Receivers with time varying channels", *IEEE transaction on signal processing, vol.55,* No. 7, pp.3405-3416.

[7] R.U. Nabar, H.Boleskei and A.J. Paulraj (2005)," Diversity and outage performance in Space Time Block Coded Rician MIMO Channels" *IEEE Trans. Wireless Commun. Vol. 4,* pp. 1102-1111.

[8] P. Wolniosky, G .J. Foschini, G. D. Golden and R.A. Valenzuela (1998)," V-BLAST: An Architecture for realizing very high data rates over rich scattering wireless channel" *URSI International Symposium on Signals, Systems and Electronics,* ISSSE 98.

[9] R. U. Nabar A. J. Paulraj, D. A. Gore and H. Bolcskei (2004), "An overview of MIMO communications—a key to gigabit wireless," *Proceedings of the IEEE, vol. 92, no. 2, pp. 198–218.*

[10] A. Paulraj and R. J. Heath (2001), "Characterization of MIMO Channels for Spatial Multiplexing Systems "*IEEE International Conference on Communications, vol.2*, no.11-14, pp-591-595.

[11] Sukhchain Singh, Gurpreet Singh, Amit Grover (2012), "Performance Evaluation of ML-VBLAST MIMO System using various antenna configurations with Ricean and Rayleigh Channel", vol. 3, no.10, pp. 62-71.

Biography

Er. Pargat Singh Sidhu is pursuing his Masters in the area of Electronics and Communication Engineering under the supervision of Mr. Amit Grover, Assistant Professor, Department of Electronics and Communication Engineering, Shaheed Bhagat Singh State Technical Campus, Moga road, Ferozepur, Punjab, India. Pargat Singh Sidhu received his B.Tech degree in the area of Electronics & Communication Engineering in 2011. His area of interest includes Signal processing, MIMO systems, Wireless mobile communications, High speed digital communications and 4G Wireless communications.

Gurpreet Singh The author place of birth is Faridkot, Punjab, India on 28th, August 1988. The author received M. Tech degree in Electronics and Communication Engineering from Jaypee University of Information and Technology, Solan, Himachal Pradesh, India in 2012 and received B. Tech degree in Electronics and Communication Engineering from Lovely Institutes of Technology, Phagwara, Punjab, India in 2010 with distinction. His area of interest is signal processing, MIMO Systems, Wireless mobile communications, High speed digital communications and 4G wireless mobile communications.

Amit Grover (M'06-SM'09-PI'11&12)The author became a Member (M) of Association ISTE in 2006, a Senior Member (SM) of society SELCOME in september 2009, and a Project-Incharge (PI) in august 2011 and in September 2012. The author place of birth is Ferozepur, Punjab, India on 27th, September 1980.The author received M. Tech degree in Electronics and Communication Engineering from Punjab Technical University, Kapurthla, Punjab, India in 2008 and received his B. Tech degree in Electronics and Communication Engineering from Punjab Technical University, Kapurthala, Punjab, India in 2001.Currently, he is working as an Assistant Professor in Shaheed Bhagat Singh State Technical Campus (Established by Punjab Government), Moga road, Ferozpur, 152004, Punjab, India. He has an experience of 11 years in teaching. His area of interest includes signal processing, MIMO Systems, Wireless mobile communications, High speed digital communications and 4G wireless communications.