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# Nonlinear Thermal Analysis of Functionally Graded Plates Using Higher Order Theory

Suresh Kumar. Jyothula
Department of Mechanical Engineering
J.N.T.U.H. College of Engineering, Kukatpally-85, Hyderabad, India.
Tel: +919866953636, E-mail: jyothula_1971@gmail.com

Sidda Reddy. Bathini (Corresponding author)
School of Mechanical Engineering, RGMCET, Nandyal-518 501
Kurnool (D. t), Andhra Pradesh, India.
Tel: +91 9440 844 600, E-mail: bsrrgmcet@gmail.com

Eswara Reddy. C
Director, Sri Padmavathi Mahila Visvavidyalayam, Tirupati, A.P, India
Tel: +919963564789, E-mail: ce_reddy@yahoo.com

Vijaya Kumar Reddy. Kontakkagari
Department of Mechanical Engineering
J.N.T.U.H. College of Engineering, Kukatpally-85,Hyderabad, India.
Tel: +919866953636, E-mail: kvijayakumarreddy@gmail.com

**Abstract**

In this paper, the nonlinear thermal analysis of functionally graded material (FGM) plate with material variation parameter (n), boundary conditions, aspect ratios and side to thickness ratios is investigated using higher order displacement model. The derivation of equations of motion for higher order displacement model is obtained using principle of virtual work. The nonlinear simultaneous equations are obtained by Navier's method considering certain parameters, loads and boundary conditions. The nonlinear algebraic equations are solved using Newton Raphson iterative method. The effect of shear deformation and nonlinearity response of functionally graded material is investigated.

**Keywords:** Nonlinear thermal analysis, FGM plates, higher order theory, Navier's method, Newton Raphson method.

## 1. Introduction

In Conventional laminated composite materials usually have an abrupt change in mechanical properties across the interface where two different materials are bonded together at some extreme loading conditions; this can result in cracking and large interlaminar stresses leading to delamination. One way to solve these problems is to employ functionally graded materials. A functionally graded material (FGM) is a material in which the composition and structure gradually change resulting in a corresponding change in the properties of the material. This FGM concept can be applied to various materials for structural and functional uses (Miyamoto *et al.* 1996, Masoud Tahani & Seyed Mahdi Mirzababaee 2009). The behavioral analysis of functionally graded composite materials is an important field of research owing to the interest for a wide range of applications: thermal barrier coatings for turbine blades (electricity production), armor protection for military applications, fusion energy devices, biomedical materials including bone and dental implants, space/aerospace (space vehicles, aircraft, aerospace engines, rocket heat shields) industries, automotive applications, etc. because of their superior advantages such as high resistance to temperature gradients, capability to withstand to high loads and high temperature fields and high durable properties, reduction in residual and thermal stresses, high wear resistance, and an increase in strength to weight ratio when compared to the other engineering materials. Hence, the non-linear behavior of functionally graded plates has to be understood for their optimum design. Reddy (2000) proposed the theoretical formulation; Navier's solutions of rectangular plates and finite element models based on third order shear deformation theory and presented the analysis through thickness of functionally graded plates. Hui-Shen Shen (2002) presented nonlinear bending analysis for a simply supported functionally graded rectangular plate subjected to transverse uniform or sinusoidal load. Galerkin technique is employed to determine load deflection and load bending deflection and load bending moments. Ashraf (2006) presented the static response for simply supported functionally graded rectangular plates subjected to a transverse uniform load. The equilibrium equations of a functionally graded plates are given, are based on a generalized shear deformation plate theory. The influences based on shear deformation, plate aspect ratio, side to thickness ratio and volume fraction distributions are investigated. Praveen & Reddy (1998) investigated the static response of functionally graded material plates by varying the volume fraction of the ceramic and metallic constituents using a power law distribution and the numerical results for the deflections and stresses are presented. Lee *et al.* (1989) proposed higher order theory for studying the bending response of functionally graded plates. The Von Karman theory is used for obtaining the approximate solutions for nonlinear bending. Sasaki and Watanabe (1989) developed some techniques for fabricating the FGMs. Fukui & Yamanaka (1992) investigated the effect of the gradation of the composition on the strength and deformation of the thick walled FGM tubes. Victor Birman and Larry Byrd (2007) presented the principal developments in functionally graded materials with an emphasis on the recent work published since 2000. Diverse areas relevant to various aspects of theory and applications of FGM are reflected in this paper. They include homogenization of particulate FGM, heat transfer issues, stress, stability and dynamic analyses, testing, manufacturing and design, applications, and fracture. Aboudi *et al.* (1999) presented the major findings obtained with the one- and two-directional versions of the higher order theory. The results illustrate the response of composites and technologically important applications. A major issue discussed was the applicability of the classical homogenization theory in the analysis of functionally graded materials. Hirai (1996) provides micro-structural details that are varied by non uniform distribution of the reinforcement phase. It is accomplished by using reinforcements with different properties, sizes, and shapes, as well as by interchanging the roles of the reinforcement and matrix phases in a continuous manner. Tanigawa (1995) compiled a comprehensive list of papers on the analytical models of thermoelastic behavior of functionally graded materials. Yang *et al.* (2005) investigated the stochastic bending response of moderately thick FGM plates. The parametric effects of the material gradient property n, boundary conditions, thickness-to-radius ratio and shear deformation on the nonlinear bending of functionally graded plates are investigated for both first order shear deformation theory and third order shear deformation theory(Golmakani & Kadkhodayan 2010). Huang *et al.* (2006) studied the nonlinear static and dynamic analysis of functionally graded plates using first order shear deformation theory. They used the quadratic extrapolation technique for linearization, finite double Chebyshev series for spatial discretization of the variables and Houbolt time marching scheme for temporal discretization. Lee *et al.* (1989) proposed higher order theory for studying the bending response of functionally graded plates. The Von Karman theory is used for obtaining the approximate solutions for nonlinear bending. Fuchiyama (1993) examined the study of thermal stresses and stress intensity of factors of FGMs with cracks and he also concluded that temperature dependant properties should be considered in order to obtain the more realistic results. Tanaka *et al.* (1993) designed FGM property profiles using sensitivity and optimization methods based on the reduction of thermo stresses. Obata and Nosa (1992) explained in-plane and thermo stress distribution due to a temperature distribution in the thickness of a plate. Takeuti *et al.* (1981) considered a plate under a thermal shock. He analyzed the quasi static coupled thermo-elastic problem and the uncoupled dynamical thermo-elastic problem for plate. The results of his analysis are that the thermal coupling has considerable effects on the temperature and stress distributions. Cho and Oden (2000) studied thermo-elastic characteristics of FGMs using Galerkin method and explained that FGMs show considerable improvement in the temperature and thermal stress distribution.

The Present work is concerned with the determination of nonlinear behavior and static characteristics of functionally graded material plate with material variation parameter (n), boundary conditions, aspect ratios and side to thickness ratios using means of higher order displacement model.

## 2. Higher- Order Theory for Displacement Model

Consider a functionally graded rectangular plate made of mixture of metal and ceramics of thickness *h*, side length *a* in the *x*-direction, and *b* in the *y*-direction and the location of the rectangular Cartesian coordinate axes used to describe deformations of the plate are given in Figure 1. It may be assumed that a state of plane strain exists. Hence, in formulating the higher-order shear deformation theory, a rectangular plate of $0 \le x \le a$; $0 \le y \le b$ and $-\frac{h}{2} \le z \le \frac{h}{2}$ is considered.

In order to approximate 3D-elasticity plate problem to a 2D one, the displacement components u (x, y, z, t), v(x, y, z, t) and w (x, y, z, t) at any point in the plate are expanded in terms of the thickness coordinate. The elasticity solution indicates that the transverse shear stress varies parabolically through the pate thickness. This requires the use of a displacement field, in which the in-plane displacements are expanded as cubic functions of the thickness coordinate. The displacement field which assumes w (x, y, z) constant through the plate thickness is expressed as (Kant and Pandya 1998, Kant and Manjunath 1990, Kant and Mallikarjuna 1989, Kant and Swaminathan 2001, Marur and Kant 1997):

$$\left.\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned}\right\} \quad (1)$$

Where $u_o$, $v_o$, $w_o$ denote the displacements of a point (x, y) on the mid plane.

$\theta_x$, $\theta_y$ are rotations of the normal to the mid plane about *y* and *x*-axes

$u_0^*$, $v_0^*$, $\theta_x^*$, $\theta_y^*$ are the higher order deformation terms defined at the mid plane.

All the generalized displacements ($u_o$, $v_o$, $w_o$, $\theta_x$, $\theta_y$) are functions of x, and y.

In the present work, analytical formulation and solutions are obtained without enforcing zero transverse shear stress conditions on the top and bottom surfaces of the plate.

From strain-displacement relations appropriate for infinitesimal deformations, we obtain the following relations as:

$$\begin{aligned} \varepsilon_x &= \varepsilon_{xo} + zk_x + z^2\varepsilon_{xo}^* + z^3k_x^* \\ \varepsilon_y &= \varepsilon_{yo} + zk_y + z^2\varepsilon_{yo}^* + z^3k_y^* \\ \varepsilon_z &= 0 \\ \gamma_{xy} &= \varepsilon_{xyo} + zk_{xy} + z^2\varepsilon_{xyo}^* + z^3k_{xy}^* \\ \gamma_{yz} &= \phi_y + z\varepsilon_{yzo} + z^2\phi_y^* \\ \gamma_{xz} &= \phi_x + z\varepsilon_{xzo} + z^2\phi_x^* \end{aligned} \quad (2)$$

### *2.1 Constitutive relations*

Consider a FGM plate, which is made from a mixture of ceramics and metals. It is assumed that the composition properties of FGM vary through the thickness of the plate. The variation of material properties can be expressed as:

$$P(Z) = (P_t - P_b)V + P_b \quad (3)$$

Where *P* denotes a generic material property like modulus, $P_t$ and $P_b$ denotes the corresponding properties of the top and bottom faces of the plate, respectively, and *n* is a parameter that dictates the material variation profile through the thickness. Also *V* in Eq. (3) denotes the volume fraction of the top face constituent and follows a simple power-law as:

$$V = \left(\frac{Z}{h} + \frac{1}{2}\right)^n \quad (4)$$

Where *h* is the total thickness of the plate, *z* is the thickness coordinate and *n* is a parameter that dictates the material variation profile through the thickness. Here it is assumed that moduli E and G vary according to Eq. (3) and the Poisson's ratio is assumed to be a constant. The linear constitutive relations are:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12} & Q_{22} & Q_{23} \\ Q_{13} & Q_{23} & Q_{33} \end{bmatrix} \begin{Bmatrix} \in_x - \alpha_x\Delta T \\ \in_y - \alpha_y\Delta T \\ \gamma_{xy} - \alpha_{xy}\Delta T \end{Bmatrix} \quad (5)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

Where $\sigma = (\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{xz})^t$ are the stresses

$\varepsilon = (\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})^t$ are the strains

$\alpha = (\alpha_x, \alpha_y, \alpha_{xy})$ are transformed thermal coefficients of expansion.

$Q_{ij}$'s are the plane stress reduced elastic constants in the plate axes.The superscript t denotes the transpose of a matrix.

$\Delta T$ = Temperature increment

*2. 2 Equations of motion*

The work done by actual forces in moving through virtual displacements, that are consistent with the geometric constraints of a body is set to zero to obtain the equation of motion and this is known as energy principle. It is useful in deriving governing equations and the boundary conditions and obtaining approximate solutions by virtual methods. Energy principles provide alternative means to obtain the governing equations and their solutions. The principle of virtual work is used to derive the equations of motion of FGM. The governing equations of displacement model in Eq. (1) will be derived using the dynamic version of the principle of virtual displacements, i.e.

$$\int_0^T (\delta U + \delta V - \delta K)\, dt = 0 \tag{6}$$

Where $\delta U$ = virtual strain energy

$\delta V$ = virtual work done by applied forces

$\delta K$ = virtual kinetic energy

$\delta U + \delta V$ = total potential energy.

The virtual strain energy, work done and kinetic energy are given by:

$$\delta U = \int_A \int_{-h/2}^{h/2} \left[\sigma_x \delta\epsilon_x + \sigma_y \delta\epsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz}\right] dz\, dx\, dy$$

$$\delta V = \int_A q\,\delta w_0\, dx\, dy$$

$$\delta K = \int_A \left\{ \int_{-h/2}^{h/2} \rho_0 \Big[ \left(\dot{u}_0 + Z\dot{\theta}_x + Z^2 \dot{u}_0^* + Z^3 \dot{\theta}_x^*\right)\left(\delta\dot{u}_0 + Z\delta\dot{\theta}_x + Z^2 \delta\dot{u}_0^* + Z^3 \delta\dot{\theta}_x^*\right) + \left(\dot{v}_0 + Z\dot{\theta}_y + Z^2 \dot{v}_0^* + Z^3 \dot{\theta}_y^*\right)\left(\delta\dot{v}_0 + Z\delta\dot{\theta}_y + Z^2 \delta\dot{v}_0^* + Z^3 \delta\dot{\theta}_y^*\right) + \dot{w}_0\delta\dot{w}_0 \Big] dz \right\} dx\, dy \tag{7}$$

where q = distributed load over the surface of the functionally graded plate.

$\rho_0$ = density of plate material

$\dot{u}_0 = \partial u_0 / \partial t$, $\dot{v}_0 = \partial v_0 / \partial t$ etc. indicates the time derivatives

The equation of motion is obtained by substituting the δU, δV and δK from Eq. (7) in to the virtual work statement in Eq. (6) and integrating through the thickness of the functionally graded plate. Upon substitution of force and moment resultants and integration, the stress strain relation is obtained as:

$$\begin{Bmatrix} N \\ N^* \\ M \\ M^* \\ -- \\ Q \\ Q^* \end{Bmatrix} = \begin{bmatrix} A & | & B & | & 0 \\ B^t & | & D_b & | & 0 \\ 0 & | & 0 & | & D_s \end{bmatrix} \begin{Bmatrix} \in_0 \\ \in_0^* \\ K \\ K^* \\ -- \\ \phi \\ \phi^* \end{Bmatrix} - \begin{Bmatrix} N^T \\ N^{*T} \\ M^T \\ M^{*T} \\ -- \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

Where,

$N = [N_x \ \ N_y \ \ N_{xy}]^t$ ; $N^* = [N_x^* \ \ N_y^* \ \ N_{xy}^*]^t$

N, $N^*$ are called the in-plane force resultants

$M = [M_x \ \ M_y \ \ M_{xy}]^t$ ; $M^* = [M_x^* \ \ M_y^* \ \ M_{xy}^*]^t$

M, $M^*$ are called as moment resultants

$Q = [Q_x \ \ Q_y]^t$ ; $Q^* = [S_x \ \ S_y \ \ Q_x^* \ \ Q_y^*]^t$

Q, $Q^*$ denotes the transverse force resultant.

The thermal moments are:

$$\{M^T\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]\ \{\alpha\}\ \Delta T\, z\, dz$$

$$\{M^{*T}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]\ \{\alpha\}\ \Delta T\, z^3\, dz$$

$\{N^T\} = \{N_x^T \ N_y^T \ N_z^T \ N_{xy}^T\}^t$ $\{N^{*T}\} = \{N_x^{*T} \ N_y^{*T} \ N_z^{*T} \ N_{xy}^{*T}\}^t$

$\{M^T\} = \{M_x^T \ M_y^T \ M_z^T \ M_{xy}^T\}^t$ $\{M^{*T}\} = \{M_x^{*T} \ M_y^{*T} \ M_z^{*T} \ M_{xy}^{*T}\}^t$

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_z^T \\ N_{xy}^T \end{Bmatrix} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \begin{Bmatrix} N_{mn}^1(t) \\ N_{mn}^2(t) \\ N_{mn}^3(t) \\ N_{mn}^4(t) \end{Bmatrix} \sin\alpha x \sin\beta y \quad \begin{Bmatrix} N_x^{*T} \\ N_y^{*T} \\ N_z^{*T} \\ N_{xy}^{*T} \end{Bmatrix} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \begin{Bmatrix} N_{mn}^{*1}(t) \\ N_{mn}^{*2}(t) \\ N_{mn}^{*3}(t) \\ N_{mn}^{*4}(t) \end{Bmatrix} \sin\alpha x \sin\beta y \tag{9}$$

$$\begin{Bmatrix} M_x^T \\ M_y^T \\ M_z^T \\ M_{xy}^T \end{Bmatrix} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \begin{Bmatrix} M_{mn}^1(t) \\ M_{mn}^2(t) \\ M_{mn}^3(t) \\ M_{mn}^4(t) \end{Bmatrix} \sin\alpha x \sin\beta y \quad \begin{Bmatrix} M_x^{*T} \\ M_y^{*T} \\ M_z^{*T} \\ M_{xy}^{*T} \end{Bmatrix} = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \begin{Bmatrix} M_{mn}^{*1}(t) \\ M_{mn}^{*2}(t) \\ M_{mn}^{*3}(t) \\ M_{mn}^{*4}(t) \end{Bmatrix} \sin\alpha x \sin\beta y \tag{10}$$

Where

$$\{N_{mn}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]\{\alpha\}T_{mn}(z,t)dz$$
$$\{N^*_{mn}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q]\{\alpha\} \; z^2 T_{mn}(z,t)dz$$
$$\{M_{mn}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q] \; \{\alpha\} \; z \; T_{mn}(z,t)dz$$
$$\{M^*_{mn}\} = \sum_{l=1}^{n} \int_{h_{l-1}}^{h_l} [Q] \; \{\alpha\} \; z^3 T_{mn}(z,t)dz$$

**3. Analytical Solutions for Functionally Gradient Material Plates Using Displacement Model**

In the Navier method the displacements are expanded in a double trigonometric (Fourier) series in terms of unknown parameters. The choice of the trigonometric functions in the series is restricted to those, which satisfy the boundary conditions of the problem. Substitution of the displacement expansions in the governing equations should result in an invertible set of algebraic equations among the parameters of the displacement expansion; otherwise the Navier solutions cannot be developed for the problem. The Navier solutions are developed for rectangular plate with a set of simply supported boundary conditions.

The boundary conditions are given below.

At edges x = 0 and x = a

$v_o = 0$, $w_o = 0$, $\theta_y = 0$, $M_x = 0$, $v_o^* = 0$, $\theta_y^* = 0$, $M_x^* = 0$, $N_x = 0$, $N_x^* = 0$.

At edges y = 0 and y = b

$u_o = 0$, $w_o = 0$, $\theta_x = 0$, $M_y = 0$, $u_o^* = 0$, $\theta_x^* = 0$, $M_y^* = 0$, $N_y = 0$, $N_y^* = 0$.

The above simply supported boundary conditions are considered for solutions of the plates using higher-order shear deformation theory.

The thermal loads are also expanded in double Fourier sine series as:

$$\Delta T(x, y, z, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T_{mn}(t) \sin \alpha x \sin \beta y \tag{11}$$

Where

$$T_{mn}(z, t) = \frac{4}{ab} \int_0^a \int_0^b \Delta T(x, y, z, t) \; \sin \alpha x \sin \beta y \, dxdy$$

Where $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$

Substitution of Eq. (11) in governing equation of motions the displacements is obtained. The displacements are determined using Newton Raphson method.

**4. Results and Discussion**

In order to verify the accuracy and efficiency of the developed theories and to study the effects of transverse shear deformation, the following typical material properties are used in obtaining the numerical results.

Material 1: (Aluminium)

$$E = 70GPa; \upsilon = 0.3, \rho = 2{,}707Kg/m^3, \kappa = 204W/mK, \alpha = 23X10^{-6}/^0C$$

Material 2: (Zirconium)

$$E = 151GPa; \upsilon = 0.3, \rho = 3{,}000Kg/m^3, \kappa = 209W/mK, \alpha = 10X10^{-6}/^0C$$

The center deflection and load parameter are presented here in non-dimensional form using the following relations:

$$\bar{w} = w_0/h \; ; \; P = q_0 a^4 / E_m h^4$$

The solution procedures outlined in the previous section are applied to simply supported functionally graded material rectangular plate with above said material properties under transverse load for comparison of maximum deflection and stress-resultants for various material variation parameter (n). Figure 2 shows the Non-dimensionalized center deflection (w) Vs ceramic temperature for various values of volume fraction exponent. The results obtained in nonlinear analysis are shown through Figure 3-11.

## 5. Conclusions

The following conclusions are drawn from the results of functionally graded plates subjected to thermal loads:

- The geometric nonlinearity in functionally graded plates under thermal loading decreases the central deflections, transverse normal and shear stresses with increase in side to thickness ratio, aspect ratio. But normal stress ($\sigma_y$) increases as aspect ratio increases.
- The FGM plates experience less transverse deflections due to the thermal forces due to the fact that the thermal resultants than laminated composite plates.
- Another interesting observation is that the nonlinear deflections are larger under thermal loads for FGM plates. The geometric nonlinearity is negated by the thermal forces and moments, making the overall plate stiffness reduced.

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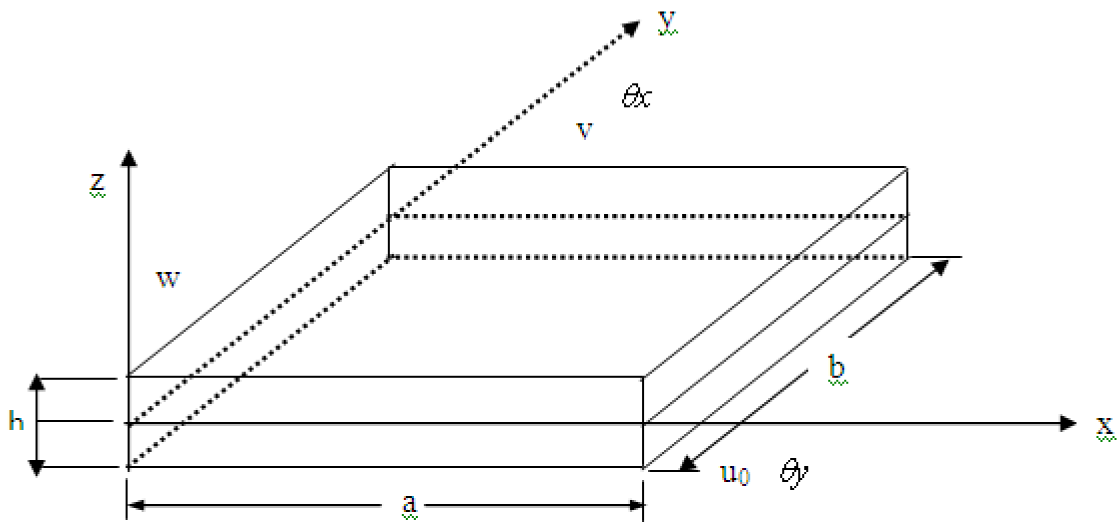


Figure 1. FGM Geometry with reference axes, displacement components.

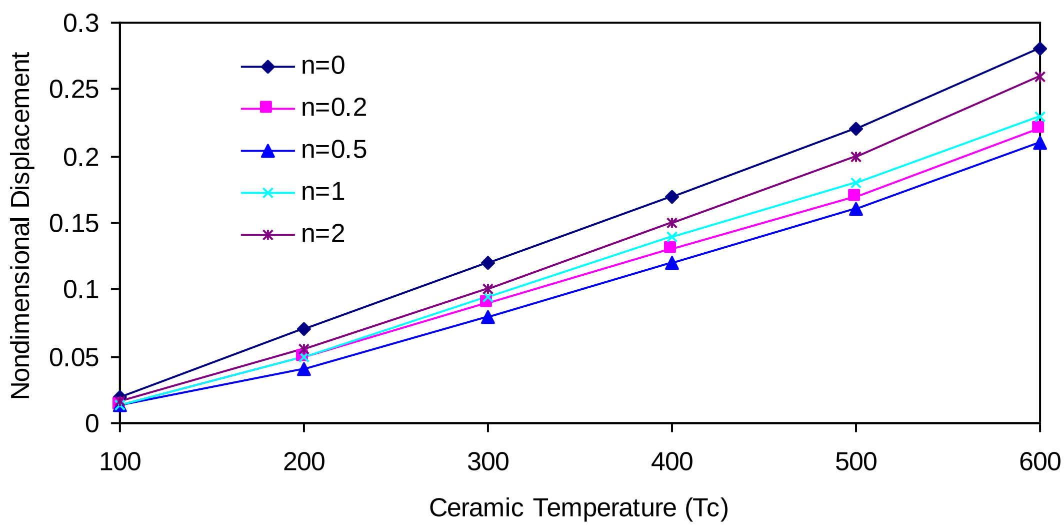


Figure 2. Non-dimensionalized center deflection (w) Vs ceramic temperature for a simply supported FGM plate for various values of volume fraction exponent

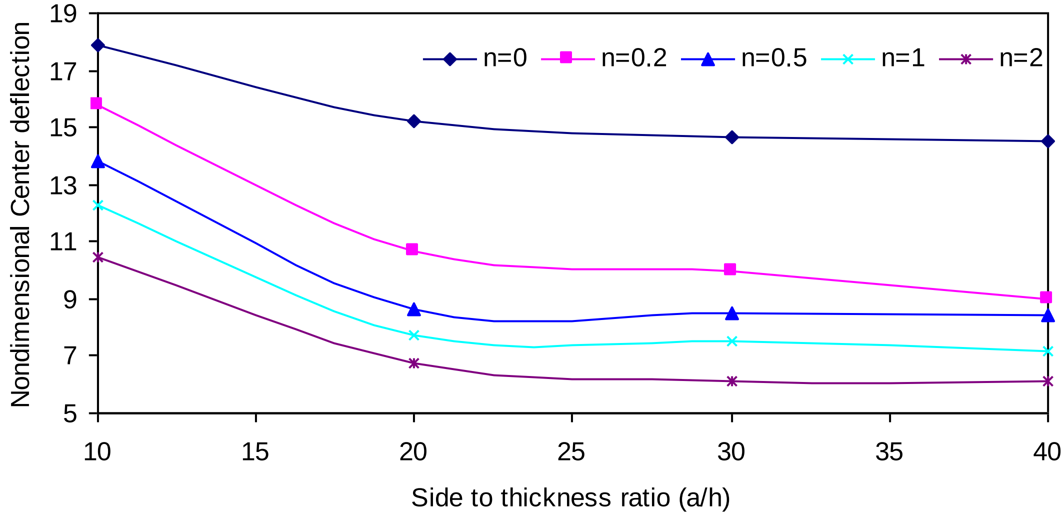


Figure 3. Non-dimensionalized center deflection (w) Vs Side to thickness ratio (a/h) for a simply Supported FGM plate for displacement model

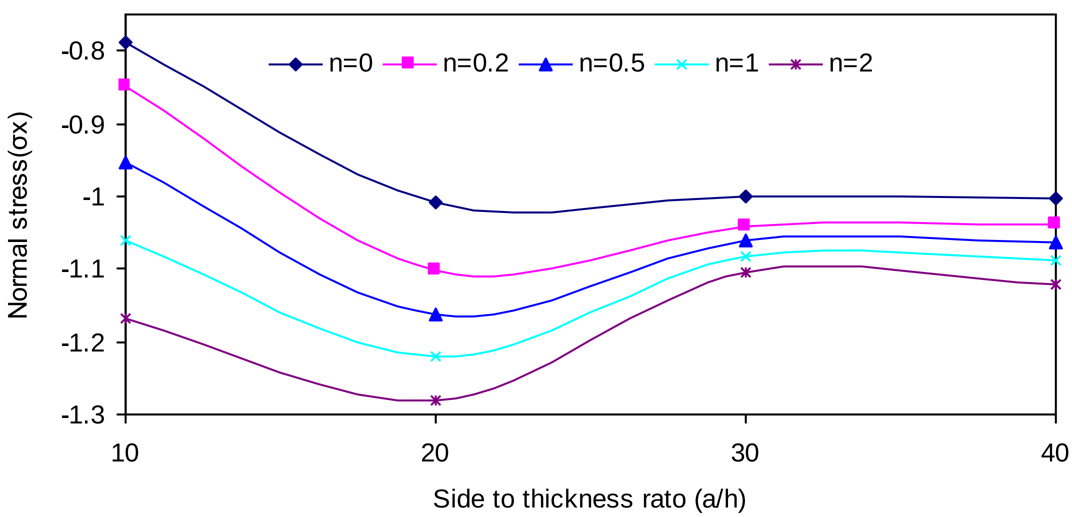


Figure 4. Non-dimensionalized normal stress ($\sigma_x$) Vs Side to thickness ratio (a/h) for a simply supported FGM plate for displacement model

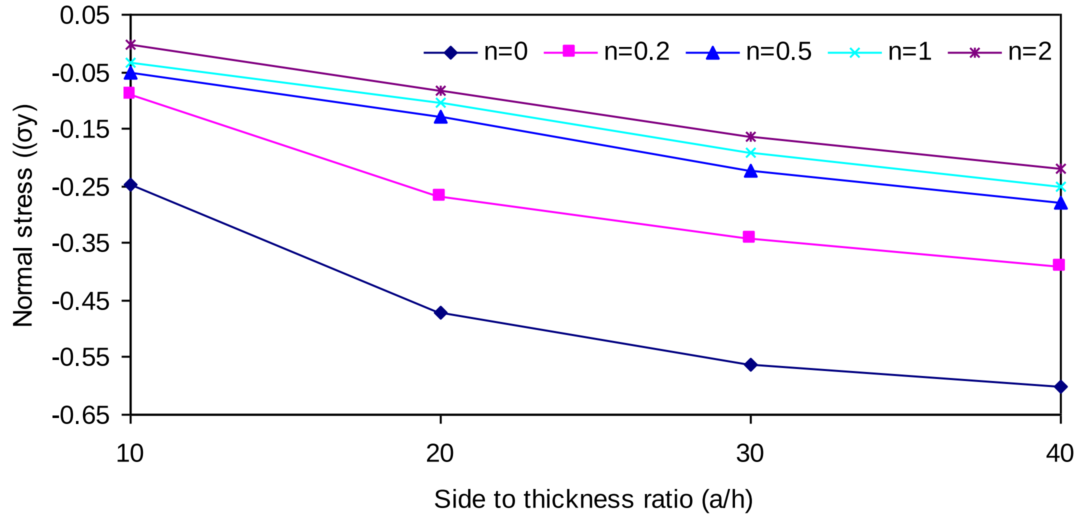


Figure 5. Non-dimensionalized normal stress ($\sigma_y$) Vs Side to thickness ratio (a/h) for a simply Supported FGM plate for displacement model

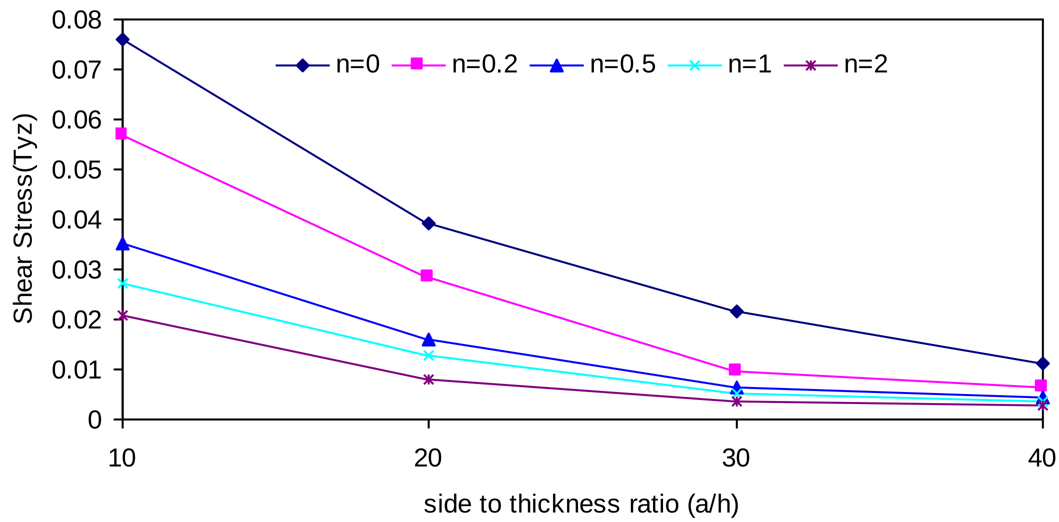


Figure 6. Non-dimensionalized shear stress ($\tau_{yz}$) Vs Side to thickness ratio (a/h) for a simply Supported FGM plate for displacement model

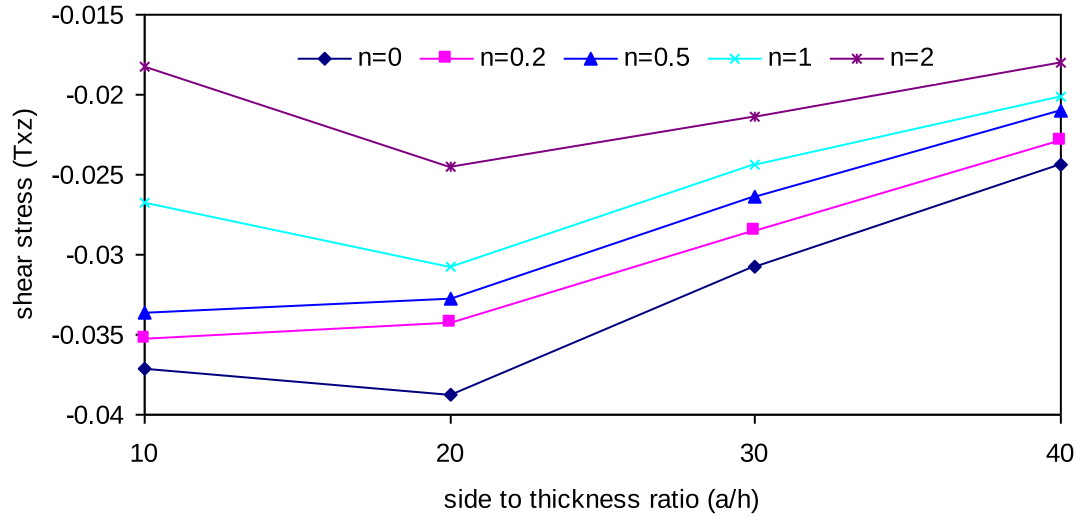


Figure 7. Non-dimensionalized shear stress ($\tau_{xz}$) Vs Side to thickness ratio (a/h) for a simply supported FGM plate for displacement model

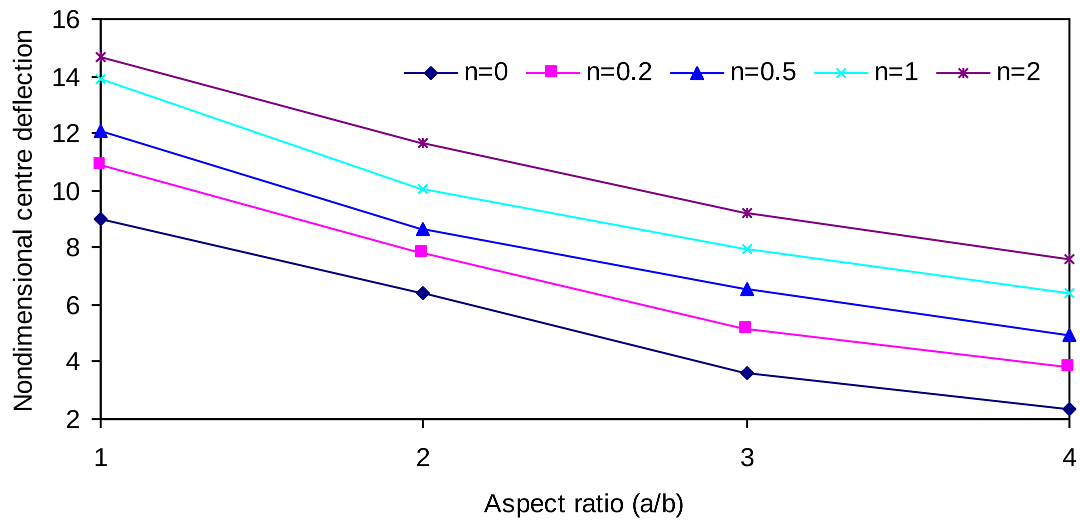


Figure 8. Non-dimensionalized center deflection (w) Vs Aspect ratio (a/b) for a simply supported FGM plate for displacement model

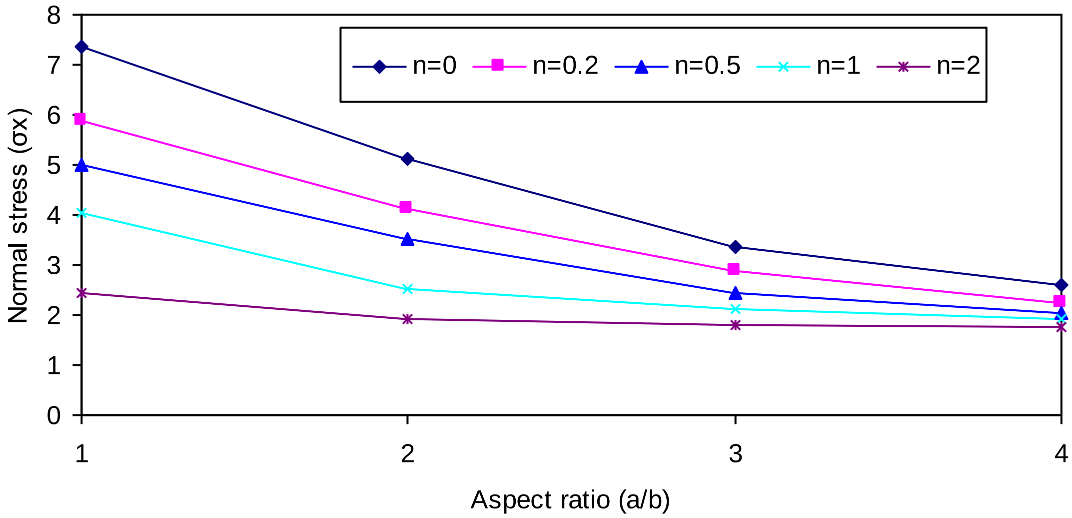


Figure 9. Non-dimensionalized normal stress ($\sigma_x$) Vs Aspect ratio (a/b) for a simply supported FGM plate for displacement model

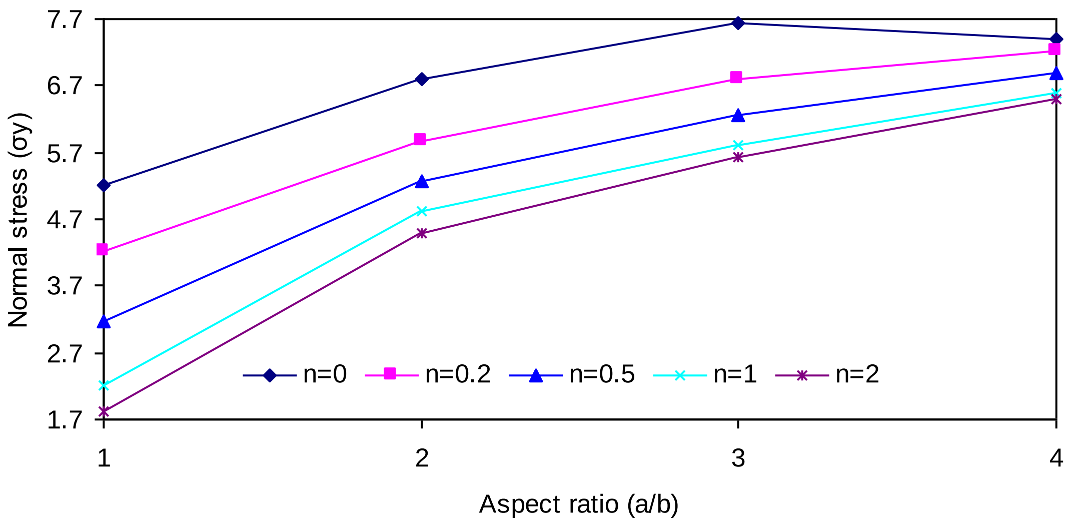


Figure 10. Non-dimensionalized normal stress ($\sigma_y$) Vs Aspect ratio (a/b) for a simply supported FGM plate for displacement model

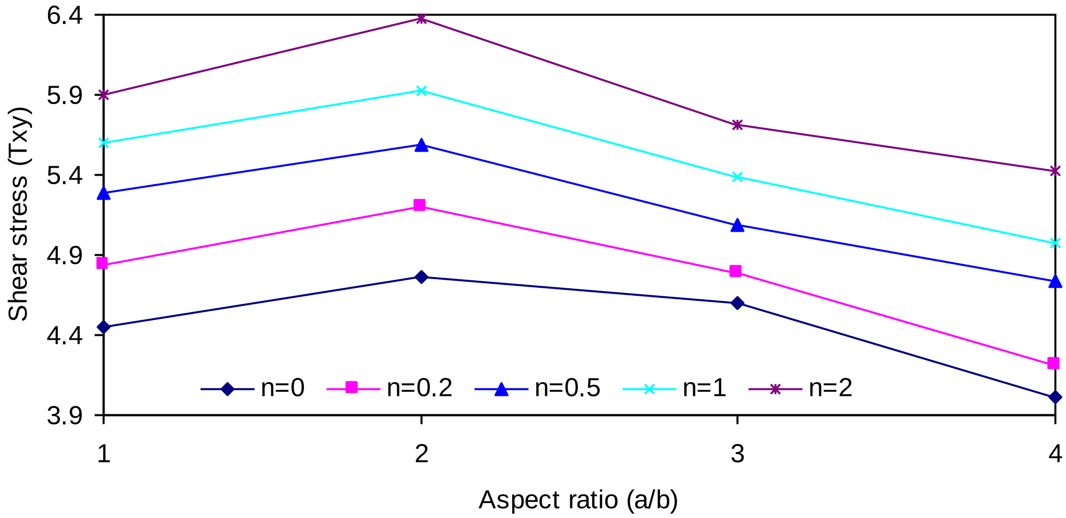


Figure 11. Non-dimensionalized shear stress ($\tau_{xy}$) Vs Aspect ratio (a/b) for a simply supported FGM plate for displacement model