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A Mathematical Modelling Study Suitable for Bachelor's Education: The Example of Paying Lecturing Fees

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Abstract

In this study, a mathematical model has been designed to calculate the summer school lecturing fees of Dokuz Eylul University. The modelling in this case has been done for a multiple of one course hour of formal school fee which needs to be determined. Our model has been interpreted geometrically using real variable and real valued functions and analysed based on the directional derivatives of these functions. In addition, the computer programme of this model has been designed and the initial condition semi-differential equation has been formed. The problem status, has been generalized while being exemplified.

Keywords: Mathematics, Multivariable Real Mathematics Analysis, Mathematical Models, Mathematical Modelling

1. Introduction

It is necessary for people to be able to adapt mathematical knowledge gained in classrooms to other aspects of their life, and there is a need for studies conducted to this end. In this context, the principles and standards for school mathematics published by National Council of Teachers of Mathematics [NCTM], (2000) emphasizes the importance of students' using mathematics to solve the problems of the world that surrounds them. For this, mathematics education should be of the quality that enables students to develop a mathematical attitude towards issues and problems. One of the ways to achieve this is to conduct mathematical modelling studies within the classroom.

Mathematical modelling is the process of mathematically interpreting events and the connections between them, thus finding patterns between these events (Verschaffel, Greer & De Corte, 2002). Niss (1999) describes mathematical modelling as the mathematical knowledge or the combination of knowledge chosen to represent real-life situations and the relationships between these situations. According to Galbraith and Clatworthy (1990), mathematical modelling is the adaptation of mathematics to real-life problems that are unstructured. In short, mathematical modelling is the process of solving difficulties and problems faced in real-life through mathematical means. According to Lingefjard (2006) this process involves observing the phenomenon, uncovering the relationships, conducting mathematical analyses, finding results, and the reinterpretation of the model. In the mathematical modelling process, a real-life subject that exist outside of pure mathematics is taken and turned into mathematical expression, which enables the use of mathematical techniques to shed light on the subject (Blumm & Niss, 1989).

Modelling studies enable students to see, if just for a moment, that mathematical knowledge is about and adaptable to the real world (Sriraman, 2005). Yet many teachers and pre-service teachers are not prepared to reduce pure mathematics to modelling activities (Lange, 1989). Yet, the "mathematical modelling and problem solving" skill and proficiency comes first among the mathematical skills and proficiencies students must gain according to the mathematics class curriculum, changed to be used first with 9th grade students and applied gradually thereon, first coming into effect in the 2013-2014 academic year in Turkey (Ministry of National Education [MoNE], 2013, s. IV). In addition, NCTM also emphasized that mathematical modelling studies should be included more in school mathematics (1989, 2000). Tekin and Bukova Güzel (2011), in their study on middle school mathematics teachers' views on mathematical modelling, express that practicing teachers do not utilize mathematical modelling in their lessons because they do not know how to. For teachers and pre-service teachers to develop mathematical modelling skills of their students, it is necessary for them to first be proficient themselves and predisposed to bachelor's level mathematical modelling studies. Therefore, the aim of our study is to reach the solution of a real-life problem using mathematical modelling, and present an example of interpreting a solution through different approaches.

2. Problem Status

A certain fee is expected to be paid to teaching staff who serve in summer schools that happen in various faculties of universities. This fee is specified in relation to the formal education fee for one course hour of over-time. In this relation, the summer school fee is a multiple of the fee for one additional course hour in formal education (for example, for 2014 it cannot be over 6 times). This designated multiple is not always enough for calculating the instruction fee (such as at times where there are not enough students enrolled). Generally, (or up until now), this has been the situation, and a new multiple has been needed to be determined. It was seen that this multiple has been determined by those involved, through trial and error, which is the situation that lies behind the inception of our study.

$G(x_1, x_2, ..., x_n) = 0$ is an algebraic equation with n number of unknowns. Assuming that everything is related, compatible and harmonious in the universe, it seen that an algebraic equation is an expression that contains all of the connections together. If we assign a corresponding phenomenon to each of the unknowns in this expression, it will be revealed that the types of relations between the phenomena is examined using an equation. If there is a relation like $G(x_1, x_2, ..., x_n) = 0$ between n number of phenomena, f_j (j=1,2,3,..,n) functions such as,

$$x_1 = f_1(x_2, x_3, ..., x_n), \quad x_2 = f_2(x_1, x_3, ..., x_n) \quad ... \quad x_n = f_n(x_1, x_2, x_3, ..., x_{n-1})$$

will be found. These functions show that every one thing in the universe is related to the other things. As such, the conclusion is reached that relations can be examined using figures (Bulut, 1988).

If the set of real numbers is IR, $f : IR^n \to IR, \ w = f(x_1, x_2, ..., x_n)$ is called a function with n real variables and real values. The graphic of this function is in the IR^{n+1} dimensional space. If we wish to visualize, we will conduct the geometrical interpretation for n<3 (Özer, Çakır, Çoşkun, Diker & Gürçay, 1996).

Since our study is unique to the subject, with $A, B, C, D, E, F \in IR^+$ as the constants; the closed function;

$$G(x_1, x_2, ..., x_5) = Ax_1 + Bx_2 + Cx_3 + Dx_4 + Ex_5 + F = 0$$

will be used, as well as the $f_j(j = 1,2,3,4,5)$ open functions, written with the G relation and with the general expression of $f_j : IR^4 \to IR$, $x_j = f_j(x_1, ..., x_{j-1}, x_{j+1}, .., x_5) \quad (j = 1,2,3,4,5)$, which are given in sequence; $x_1 = f_1(x_2, x_3, x_4, x_5)$, $x_2 = f_2(x_1, x_3, x_4, x_5)$, $x_3 = f_3(x_1, x_2, x_4, x_5)$, $x_4 = f_4(x_1, x_2, x_3, x_5)$, $x_5 = f_1(x_1, x_2, x_3, .x_4)$

As mentioned in the introduction of our study, determining a multiple of the one additional course hour fee for formal education and calculating the total lecturing fee for every lecturer working during summer school constitutes the research problem. The teaching staff, who teach courses (and summer school courses) in universities, can have the academic titles of professor, associated professor, assistant professor, lecturer, and instructor. There is an existing multiple of formal education additional fees specified by the Dokuz Eylul University senate. This number multiplies the lecture fee for one additional course hour of formal education. Let's call the number specified by the senate **M**. Concerning, the members of teaching staff giving summer school courses, which qualifies as additional, one of the three situations below will be encountered

SITUATION 1

The money to be distributed to all teaching staff = **M** x (All the teaching staff's additional fee for formal summer school)

SITUATION 2

The money to be distributed to all teaching staff > **M** x (All the teaching staff's additional fee for formal summer school)

SITUATION 3

The money to be distributed to all teaching staff < **M** x (All the teaching staff's additional fee for formal summer school)

The mathematical expression for all three situations can be written as such;

If, T = the net money to be distributed to all teaching staff and P_i (i=1,2,3,4,5) is the total formal lecturing fee for every member of the teaching staff by sequence of title, then we would write;

SITUATION 1

$$T = P_1+P_2+P_3+P_4+P_5 = \sum_{i=1}^{5} P_i \qquad \text{(I)}$$

SITUATION 2

$$T > P_1+P_2+P_3+P_4+P_5 = \sum_{i=1}^{5} P_i \qquad \text{(II)}$$

SITUATION 3

$$T < P_1+P_2+P_3+P_4+P_5 = \sum_{i=1}^{5} P_i \qquad \text{(III)}$$

Of these three situations, the third (which is most common in the Buca Educational Faculty), is the most problematic, because it requires a multiple smaller than **M**. Naturally this new multiple can be determined through trial and error. Yet this is both time consuming, and this **new multiple,** which is to be a positive real number, can often not be determined accurately.

3. Modelling the Status

Let's pick each titled member of teaching staff as a variable for our model. Since the number of titles is 5, this means 5 variables. Let's show these variables as, x1, x2, x3, x4, and x5. x1 standing for the number of professors, x2 for associated professors, x3 for assistant professors, x4 for lecturers, and x5 for instructors. Also since summer school fees have been linked to formal school lecturing fees, let's show the forma school fees as;

a_1=: One additional course hour fee for a professor
a_2 =: One additional course hour fee for an associated professor
a_3=: One additional course hour fee for an assistant professor
a_4=: One additional course hour fee for a lecturer
a_5=: One additional course hour fee for an instructor

These values are determined by regulations. In other words, they are constants in the model. In addition, let's show,

b_1=: Total course hours professors attend in summer school
b_2 =: Total course hours associated professors attend in summer school
b_3=: Total course hours assistant professors attend in summer school
b_4=: Total course hours lecturers attend in summer school
b_5=: Total course hours instructors attend in summer school

Once the course schedules are determined, these also become constant for the given year. These can also be treated as constants in the model.

(Note: A member of the academic staff cannot lecture for more than 30 hours additionally per week. If a person has over 30 hours of lecture time, this will be disregarded and calculated as 30. For lecturers and instructors the maximum is 32 hours.)

In addition;
K = the new multiple we aim to determine
T = the total money that is to be distributed to the teaching staff
According to these, the rations of fees according to T are as follows:
K. a_1. b_1. X_1=: The ration all professors will receive from summer school money,
K. a_2. b_2. X_2=: The ration all associated professors will receive from summer school,
K. a_3. b_3. X_3=: The ration all assistant professors will receive from summer school,
K. a_4. b_4. X_4=: The ration all lecturers will receive from summer school,
K. a_5. b_5. X_5=: The ration all instructors will receive from summer school money,
According to all this data, out main mathematical model is,

$$K.a_1.b_1.X_1 + K.a_2.b_2.X_2 + K.a_3.b_3.X_3 + K.a_4.b_4.X_4 + K.a_5.b_5.X_5 = T \qquad (1)$$

or in short,

$$\sum_{j=1}^{5} (a_j.b_j.x_j).K = T$$

From here, our mathematical model for K would be:

$$K = \frac{T}{a_1.b_1.x_1 + a_2.b_2.x_2 + a_3.b_3.x_3 + a_4.b_4.x_4 + a_5.b_5.x_5} \tag{2}$$

or in short;

$$K = \frac{T}{\sum_{j=1}^{5}(a_j.b_j.x_j)}$$

In addition, the (total) fees to be received by each member of the teaching staff would be,

Total fee to be received by 1 professor = K.a_1.total hours of lecture they attend at summer school (maximum 30 hours per week)

Total fee to be received by 1 associated professor = K.a_2.total hours of lecture they attend at summer school (maximum 30 hours per week)

Total fee to be received by 1 assistant professor = K.a_3.total hours of lecture they attend at summer school (maximum 30 hours per week)

Total fee to be received by 1 lecturer = K.a_4.total hours of lecture they attend at summer school (maximum 32 hours per week)

Total fee to be received by 1 instructor = K.a_5.total hours of lecture they attend at summer school (maximum 32 hours per week)

SPECIAL CIRCUMSTANCE: Let's have x_i (i=1,2,3,4,5) be the members of the teaching staff which haven't given courses in summer school. In this situation, the K multiple is found by writing $x_i = 0$ (i=1,2,3,4,5) in the (2) formula.

4. Analysis of Formulas (1) and (2)

4.1. Minimum and Maximum K

The formal school additional course hour fee for teaching staff titled professor is always higher than that of all other titles. In this case, if all courses in summer school are given by professors, the K multiple value in the (2) formula would be minimum. Let's represent this value with $\mathbf{K_{min}}$.

The teaching staff with the title instructor always has the smallest additional course hour fee of all titles. Therefore, if all courses in summer school are given by instructors the K multiple value in the (2) formula would be minimum. Let's represent this value with $\mathbf{K_{max}}$.

These are also the limit inferior and limit superior for the K value. Thus, $\mathbf{K_{min}} \le K$ and $\mathbf{K_{max}} \ge K$. And naturally $K \le$ M ve $K_{max} \le$ M. From here, we can write;

$$K_{min} \le K \le K_{max} \ (\le M) \tag{3}$$

Equality in formula (3) – $K_{min} = K = K_{max}$ – would only happen in the exceptional situation when each x_j (j=1, 2,3,4,5,6) share the same academic title.

4.2. Geometrical Interpretation

(i) After the K multiple is found, our model has a constant. On the other hand a_i and b_i (i=1,2,3,4,5) are also constant. In this case K.a_i.b_i (i=1,2,3,4,5) is also constant. Now, lets write K.a_i.b_i = A_i (i=1,2,3,4,5) into formula (1).

$$A_1.\ X_1+A_2.\ X_2+A_3.\ X_3+A_4.\ X_4+A_5.\ X_5 = T$$

or,

$$\sum_{j=1}^{5} A_j.x_j = T$$

or,

$$A_1.X_1+A_2.X_2+A_3.X_3+A_4.X_4+A_5.X_5-T=0 \tag{4}$$

The (4) equation in the context of a function with real variables and real values, will become the closed function:

$$G(x_1, x_2, ..., x_5) = A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5 - T = 0 \tag{5}$$

And through the G relation, lead to the following open functions, with the general expression,

$f_j : IR^4 \to IR$, $x_j = f_j(x_1,..., x_{j-1}, x_{j+1},.., x_5)$ $(j = 1,2,3,4,5)$, given in order:

$$x_1 = f_1(x_2, x_3, x_4, x_5, x_6) = \frac{T}{A_1} - \frac{A_2}{A_1}x_2 - \frac{A_3}{A_1}x_3 - \frac{A_4}{A_1}x_4 - \frac{A_4}{A_1}x_5,$$

$$x_2 = f_2(x_1, x_3, x_4, x_5, x_6) = \frac{T}{A_2} - \frac{A_1}{A_2}x_1 - \frac{A_3}{A_2}x_3 - \frac{A_4}{A_2}x_4 - \frac{A_5}{A_2}x_5,$$

$$x_3 = f_3(x_1, x_2, x_4, x_5, x_6) = \frac{T}{A_3} - \frac{A_1}{A_3}x_1 - \frac{A_2}{A_3}x_2 - \frac{A_4}{A_3}x_4 - \frac{A_5}{A_3}x_5,$$

$$x_4 = f_4(x_1, x_2, x_3, x_5, x_6) = \frac{T}{A_4} - \frac{A_1}{A_4}x_1 - \frac{A_2}{A_4}x_2 - \frac{A_3}{A_4}x_3 - \frac{A_5}{A_4}x_5,$$

$$x_5 = f_5(x_1, x_2, x_3, x_4, x_6) = \frac{T}{A_5} - \frac{A_1}{A_5}x_1 - \frac{A_2}{A_5}x_2 - \frac{A_3}{A_5}x_3 - \frac{A_4}{A_5}x_4,$$

If we interpret through graphics, the equation (5), and in relation the equation (1) should gain a notably special status. This notably special status, is at least two of the x_j (j=1,2,3,4,5) variables having a value of 0 (Because summer school opens, at least one of them must be other than 0). Accordingly, let's have **(a)** two components with the value 0. For example, $x_4=0$, $x_5=0$. Equation (5) could be expressed as;

$$G(x_1, x_2, x_3) = A_1x_1 + A_2x_2 + A_3x_3 - T = 0 \quad \text{(closed)},$$

or with the open functions;

$$x_1 = f_1(x_2, x_3) = \frac{T}{A_1} - \frac{A_2}{A_1}x_2 - \frac{A_3}{A_1}x_3 \tag{6}$$

$$x_2 = f_2(x_1, x_3) = \frac{T}{A_2} - \frac{A_1}{A_2}x_1 - \frac{A_3}{A_2}x_3 \tag{7}$$

$$x_3 = f_3(x_1, x_2) = \frac{T}{A_3} - \frac{A_1}{A_3}x_1 - \frac{A_2}{A_3}x_2 \tag{8}$$

For example, the graphic for (8) can be seen in Figure 1.

$$(x_1 > 0, x_2 > 0, x_3 > 0, \frac{T}{A_3} > 0, \frac{T}{A_2} > 0, \frac{T}{A_1} > 0)$$

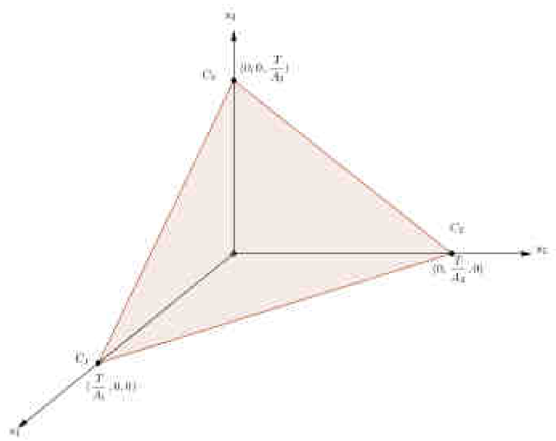

Figure 1. Plane segment, intersecting with and confined within the positive coordinates

The graphics for the functions x_1 and x_2 are also similar to Figure 1. Only the points intersecting with the coordinates are different.

(b) Let's have three components with a value of 0. For example, $x_3=0$, $x_4=0$, $x_5=0$. The equation (5) can be expressed with the function,

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$$G(x_1, x_2, x_3) = A_1x_1 + A_2x_2 - T = 0 \quad \text{(closed)},$$

or the open functions,

$$x_1 = f_1(x_2) = \frac{T}{A_1} - \frac{A_2}{A_1}x_2 \qquad (9)$$

$$x_2 = f_2(x_1) = \frac{T}{A_2} - \frac{A_1}{A_2}x_1 \qquad (10)$$

For the graphic representation of (10), see Figure 2.

$$(x_1 > 0, x_2 > 0, \frac{T}{A_2} > 0, \frac{T}{A_1} > 0)$$

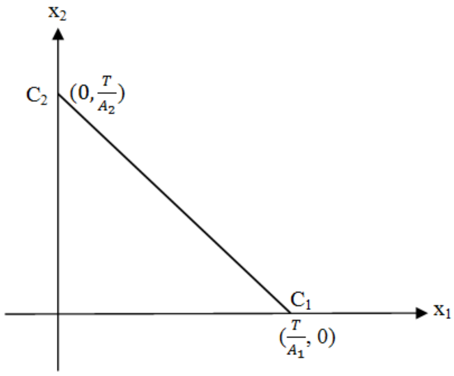

Figure 2. Line segment intersecting with and confined within the positive coordinates

The graphic for the function is also similar to Figure 2. Only the points intersecting with the coordinates are different.

(c) Let's have four components with a value of 0. For example, the equation (5) would become a relation within IR, as seen below:

$$G(x_1) = A_1x_1 - T = 0$$

Accordingly,

$$x_1 = \frac{T}{A_1} \qquad (11)$$

Is a point in IR^+. For the graphic of (11), see Figure 3.

Figure 3. A point on the positive coordinate

(ii) Let's take $a_1. b_1. X_1 + a_2. b_2. X_2 + a_3. b_3. X_3 + a_4. b_4. X_4 + a_5. b_5 = c$ (constant) in formula (2). Then,

$$K = \frac{T}{c} \qquad (12)$$

will be found. In relation (12), if K is thought of as the dependent variable, and T as the independent variable, the graphic would be the line graphic shown as Figure 4. This function is limited with M.

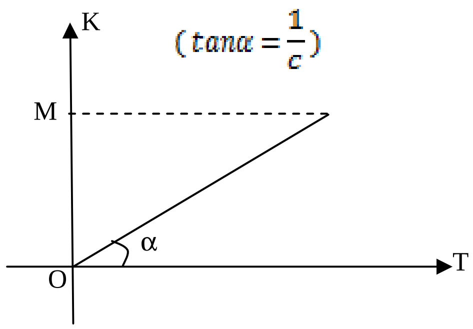

Figure 4. Limited line segment intersecting with the origin point and limited to the first quadrant.

If in the relation (12), T is thought of as the dependent variable and K the independent variable;

$$T = c.K \tag{13}$$

will be found. The graphic for (13) is the linear graphic shown in Figure 5.

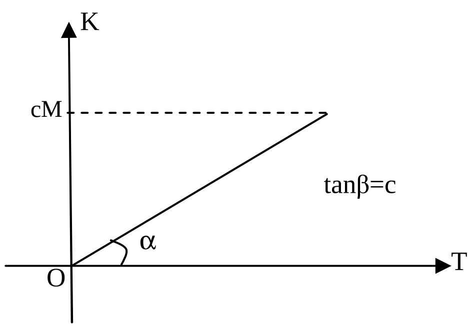

Figure 5. Limited line segment intersecting with the origin point and limited to the first quadrant

4.3. Analysis with the Concept of Directional Derivative

(i) Let's have the unit vectors of the Cartesian system consisting of the components be,

$$\vec{e_1}, \vec{e_2}, \vec{e_3}, \vec{e_4}, \vec{e_5} \quad x_j = f_j(x_1,..., x_{j-1}, x_{j+1},.., x_5) \quad (j = 1,2,3,4,5)$$

given in order. Now we'll calculate the directional derivatives of their functions in the direction of the unit vectors. First, let's calculate for j=1. Because,

$$x_1 = f_1(x_2, x_3, x_4, x_5) = \frac{T}{A_1} - \frac{A_2}{A_1}x_2 - \frac{A_3}{A_1}x_3 - \frac{A_4}{A_1}x_4 - \frac{A_4}{A_1}x_5$$

x_1 's derivative in the direction of $\vec{e_j}$ $(j = 2,3,4,5)$ is $D_{e_j} f_1 = \overrightarrow{grad} f_1.\vec{e_j}$. On the other hand,

$$\overrightarrow{grad} f_1 = -\frac{A_2}{A_1}.\vec{e_2} - \frac{A_3}{A_1}.\vec{e_3} - \frac{A_4}{A_1}.\vec{e_4} - \frac{A_5}{A_1}.\vec{e_5}$$

Which by, x_1 's directional derivative in the direction of the $\vec{e_2}, \vec{e_3}, \vec{e_4}, \vec{e_5}$ unit vectors can be found as

$$\overrightarrow{grad} f_1.\vec{e_2} = -\frac{A_2}{A_1}, \ \overrightarrow{grad} f_1.\vec{e_3} = -\frac{A_3}{A_1}, \ \overrightarrow{grad} f_1.\vec{e_4} = -\frac{A_4}{A_1}, \ \overrightarrow{grad} f_1.\vec{e_5} = -\frac{A_5}{A_1}, \tag{14}$$

in the given order. Similarly, let's look at the statuses j=2, j=3, j=4, and j=5. In the given order,

$$D_{e_i} f_2 = \overrightarrow{grad} f_2 . \vec{e_i} = -\frac{A_2}{A_i} \qquad (i = 1,3,4,5) \tag{15}$$

$$D_{e_i} f_3 = \overrightarrow{grad} f_3 . \vec{e_i} = -\frac{A_3}{A_i} \qquad (i = 1,2,4,5) \tag{16}$$

$$D_{e_i} f_4 = \overrightarrow{grad} f_4 . \vec{e_i} = -\frac{A_4}{A_i} \qquad (i = 1,2,3,5) \tag{17}$$

$$D_{e_i} f_5 = \overrightarrow{grad} f_5 . \vec{e_i} = -\frac{A_5}{A_i} \qquad (i = 1,2,3,4) \tag{18}$$

are found.

(ii) If the unit vectors of the TK-coordinate system are $\vec{e_1}, \vec{e_2}$, then K's derivative in the direction of $\vec{e_1}$, according to formula (12) will be

$$D_{e_1} K = \overrightarrow{grad} K . \vec{e_1} = \frac{1}{c} . \vec{e_1} . \vec{e_1} = \frac{1}{c} . \tag{19}$$

Similarly, let's have the unit vectors of the KT-coordinate system be $\vec{e_1}, \vec{e_2}$. Then, T's derivative in the direction of $\vec{e_1}$ would be found as,

$$D_{e_1} T = \overrightarrow{grad} T . \vec{e_1} = c . \vec{e_1} . \vec{e_1} = c \tag{20}$$

5. Conclusion and Discussion

The application of formula (I) expresses that, "The total net money will be distributed between the teaching staff participating in summer school. No money will be left for the organization (university)."

The application of the formula (II) expresses that, "The total net money is more than that what will be distributed between the teaching staff participating in summer school. Money will be left to the organization, and the amount of that money is $T - \sum_{i=1}^{5} P_i$ "

The application of formula (III), means that the teaching staff will be payed according to the K multiple found. The net total of money will be distributed between teaching staff. There is no money left to the organization.

The number of students attending summer school can be thought of as a variable, because whether this number is high or low affects the income teachers will receive. But this number also affects the total net money (T) and therefore loses significance. Thus, it is not a necessary variable.

When the formulas (14) to (18) are examined, it is seen that each directional derivative is in the negative direction. This shows that each independent variable decreases with the increase of other independent variables. We can explain this negativity as such: The T value in formula (I) is unchanging (constant). Therefore when one of the values for the variables x_j (j=1,2,3,4,5) increases, then the value of at least one of the others must decrease. We could also reach these results through a) the partial derivatives of the closed functions b) partial derivatives of the corresponding open functions. Through the graphics shown in Figure 1 and Figure 2, it can be said that the points intersecting with the coordinates are related to T. Therefore, the higher the amount of net total money, the further away these points will be from the origin point. Inversely, the smaller the net total of money, the closer the points intersecting with the coordinates are to the origin.

Approaching formulas (19) and (20) we can see that each directional derivative is in the positive direction. This shows that K increases as T does and vice versa. The higher the c value, the lower K will be. Inversely, the smaller c is, the higher T is.

If we take a step further, we encounter differential equations with partial derivatives. For example, the partial derivative differential equation for $x_1=f_1(x_2,x_3,x_4,x_5)$, from equation (5) would be an initial condition partial differential equation:

$$\frac{\partial x_1}{\partial x_2} = -\frac{A_2}{A_1}, \quad \frac{\partial x_1}{\partial x_3} = -\frac{A_3}{A_1},$$

$$\frac{\partial x_1}{\partial x_4} = -\frac{A_4}{A_1}, \quad \frac{\partial x_1}{\partial x_5} = -\frac{A_5}{A_1},$$

$$x_1(0,0,0,0) = f_1(0,0,0,0) = \frac{T}{A_1}$$

Let's write A_i = K. a_i. B_i (i=1,2,3,4,5), into this equation. The partial derivative differential equation we find would be another mathematical model of the model in question. If the fee for one additional course hour is the same for lecturers and instructors, the total number of lecturers and instructors can be taken as a single variable, reducing the number of variables within the model by 1. Accordingly, all formulas, relations and interpretations will be made with 4 variables.

Because the lecturing fee for one additional course hour is the same for lecturers and instructors in Dokuz Eylul University, the computer programming has been done accordingly.

5.1. Programming the Mathematical Model

The coding of our model in the "Visual Studio" software is given in Figure 6.

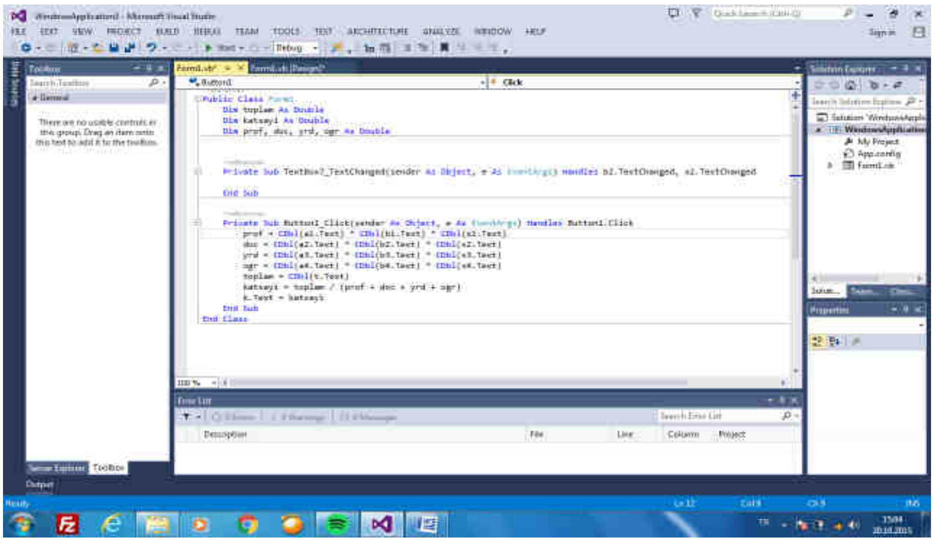

Figure 6. The writing of the model in basic programming language

The screenshot after the programme is run is given in Figure 7.

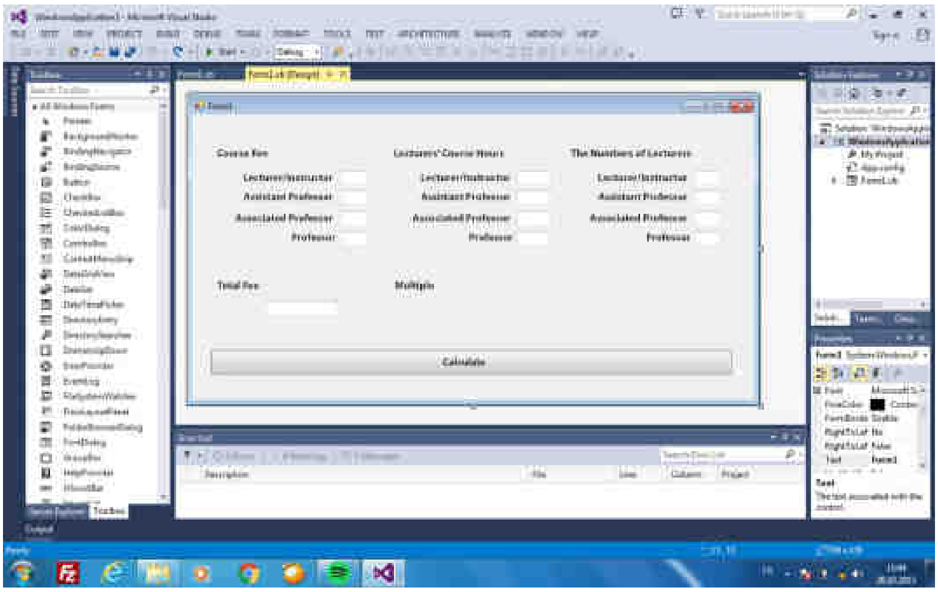

Figure 7. Running the model in a computer programme

And below there is an example of the programme running in Figure 8.

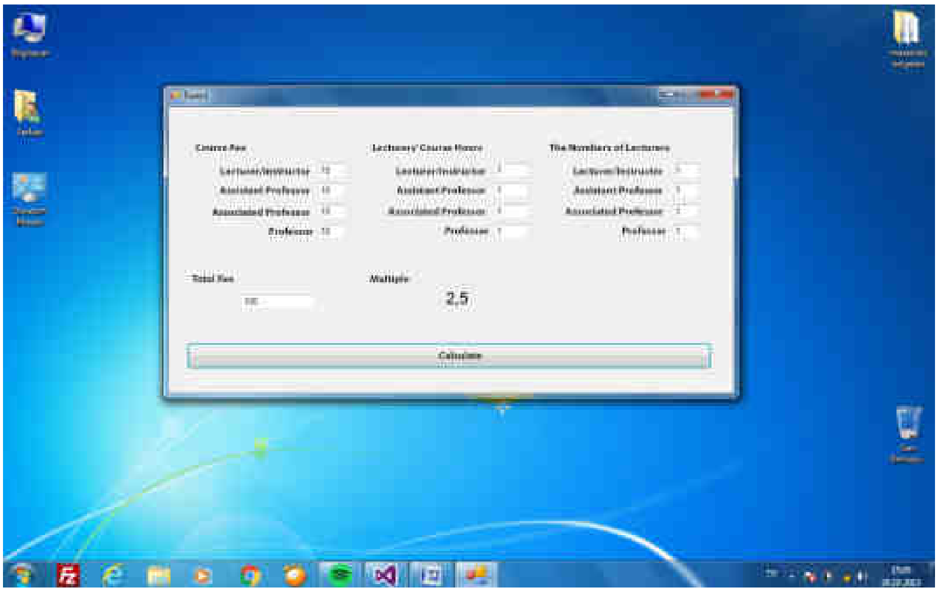

Figure 8. Modelling Example

5.2. Generalizing the Problem Status (The Example of Total Additional Fees in an Organization)

An organization pays a fee for all employees that put in work overtime. This payment comes from the money these workers bring into the organization. In this organization, the employees are sectionalized into certain categories. Naturally, there is certain number of people employed in each category and the fee of one hour overtime for each category is different (the additional fee is the same for all employees in the same category). A part of the money earned through overtime will be left to the organization, and the rest will be distributed among the employees in various categories. The proprietor wishes to distribute the money fairly. Let's design the mathematical model to calculate the fee to be received by each employee.

Solution: Let's assume the employees have been sectionalized into n number of categories. Let's show the number of employees in each category with x_1, x_2, x_3, …,x_n. T= The total money to be paid to all employees for overtime. a_i = the fee for one additional hour for the employee x_i (i=1,2,3,…,n), = the total additional hours for employee x_i (i=1,2,3,…,n). According to all of these, the share to be payed of for i (i=1,2,3,…,n), will be $K.a_i.b_i$ (i=1,2,3,…,n). As a result, our formula (mathematical model) for distributing payment would be as follows:

$$\sum_{i=1}^{n}(a_i.b_i.x_i).K = T$$

Frome here our mathematical model for K would be,

$$K = \frac{T}{a_1.b_1.x_1 + a_2.b_2.x_2 + a_3.b_3.x_3 + ... + a_n.b_n.x_n}$$

or in short,

$$K = \frac{T}{\sum_{i=1}^{n}(a_i.b_i.x_i)}$$

This generalized problem status can also be run in the computer programme.

5.3. Concluding Comments

The importance of content knowledge in modelling cannot be ignored. Content knowledge (especially mathematical analysis) needs to be comprehensive for mathematical modelling. Because if the content knowledge is not sufficient, it cannot be related efficiently to real-life situations. In this case neither the (mathematics $\rightarrow$ real-life), nor (real-life $\rightarrow$ mathematics approach) can be properly addressed.

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