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Teaching Mathematical Modeling in Mathematics Education

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Abstract
In the present paper we have discussed about mathematical modeling in mathematics education.

2. Introduction and Preliminary work already done

Mathematics is not only a subject but it is the language with some different symbols and relations. Mathematics simplifies all the things easily but in different manner therefore mathematics taught as a compulsory subject up to 10th class. After that students choose independently mathematics as a main subject or other subjects. The basic aim or purpose is how to make mathematics education interesting and students enjoy doing mathematics, not only for their academic progress but discovers new tricks, methods and mainly they can be able to relate all the math problems or content of text book to real life problems.

It is seen that student's performance in mathematics is not at the desired level, they don't solve mathematical problems interestingly. Sometimes they feel difficulty in solving the problems and learn formulas, proofs of theorems. Although lots of effort and researches are going on in this field but mostly work has done on elementary level. But difficulty level become high on secondary level and higher level because we have lots of content in 11th and 12th class mathematics also so many things are new for students lots of formulas, relations and theorems and little part of their application. There is lots of efforts should be needed on secondary level by which we make mathematics interesting and students have fun to take challenges and relate math to real world. Because when students taking interest in study of mathematics they become able to achieve their goal at higher level. Also if we improve quality of mathematics education at secondary level we get better population of youth at higher math education and also improve quality work in research programmes. Our system of education needs more efforts to improve ability to relate problems with daily life and use of mathematical modeling work as a tool which linked mathematics with real world. Students feel mathematics with their own experiences and use their concepts at higher mathematics education. Especially in engineering field. Because engineers are the backbone of the development in infrastructure and their innovative ideas make them creative. Students face the problem when some tasks given to them they had been taught in the past, they unable to apply concepts and procedure to solve problems also they unable to relate concepts with real world problems. They always confused why they learn all these formulas, concepts? Where it can be used? Sometimes some students change mathematics subject in 12th class they take another subject. By this discussion we tried to improve the quality of school mathematics. It is not only concerned with practice or solving sums but application of all the mathematics in real life problems and mathematical modeling play very important role in this. Mathematical modeling is just like a bridge which associates mathematics with real world.

Mathematics teaching concern with some important aspect as follows:
Student
Teacher
Curriculum
Strategies and techniques

In mathematical modeling we study how to deal real world problems mathematically. There are so many real life problems based on pollution, population, environment, diseases, traffic control, and education. Advance technologies include in math teaching also new software's developed by mathematician which makes complicated and time consuming calculations easy in few seconds. Mathematical modeling can be done through Algebra, trigonometry, geometry, differential equation.

According to Steen (1990),"Changes in society, in technology, in schools-among others-will have great impact on what will be possible in school mathematics. All of these changes will affect the fundamentals of school mathematics". Here we established some models on high school, secondary and higher mathematics education.

Model 1

In this model we used the word "mathematics literacy" which means knowledge of basic concepts and having proficiency with basic skills and procedure. Also we emphasis on provide mathematics education with better techniques and strategies. The question arises what is taught? And how it is taught? The goal is to make students mathematically literate. In this model we have discussed purpose of mathematics education, also used some techniques to solve sums. By these techniques we tried to motivate students also parents for math education. Basically the main purposes of this model clear the main concept of problems and connect with real world.

NCTM (National Council of Teachers of Mathematics) recommends all the mathematics instructions and improvement in curriculum. According to mathematical literacy we conclude that

- Understand mathematical concepts, facts, and operations.
- Calculate problems using different methods and techniques.
- Identify, relate and solve the problem effectively.
- Capable in logical thinking and verify results.

[] Author has work on Canadian schools. Some principles and standards are given for mathematics. In which author focus on curriculum (full of activities), teaching (need to learn), learning (learning with experiences), technology. Some standards are explain for school mathematics-

Content standard: students should know

- Number and operations
- Algebra
- Geometry
- Measurement
- Data analysis and probability

Process standard

- Problem solving
- Reasoning and proof
- Communication
- Connection
- Representation

These grades are applied to all grades from kindergarten to 12^{th} grade. In primary grade, emphasis on numbers, in middle emphasis on geometry and algebra. All these grades are essential for higher education to make the students confident for their work. it necessary to make them capable as they use their knowledge of mathematics which they learn up to secondary level. They understand the importance of mathematics in real world and improve conceptual understanding and fluency on certain points in curriculum. All the standards have skill to develop mental math, reasoning power, and make capable to communicate mathematically.

Author also give ideas for best practicing in math education by developing mathematical thinking through "do-talk-record", attitude of teacher and student, grouping. Assessment in mathematics is also very important aspect in math education it should be small scale or large scale. In schools formative and summative assessments are the main process of evaluation of student.

Author present comparative study of math education of China and United State. This study conducted by "Liping Ma". According to her, teacher must be effective. Teacher able to connect students with math positively. Give them to chance to solve the problem by their own efforts. Also she mentioned some differences between China and US mathematics education.

At last author include technology in mathematics education. Use of calculator and give computer projects, use of internet.

3. Proposed Model & Methodology

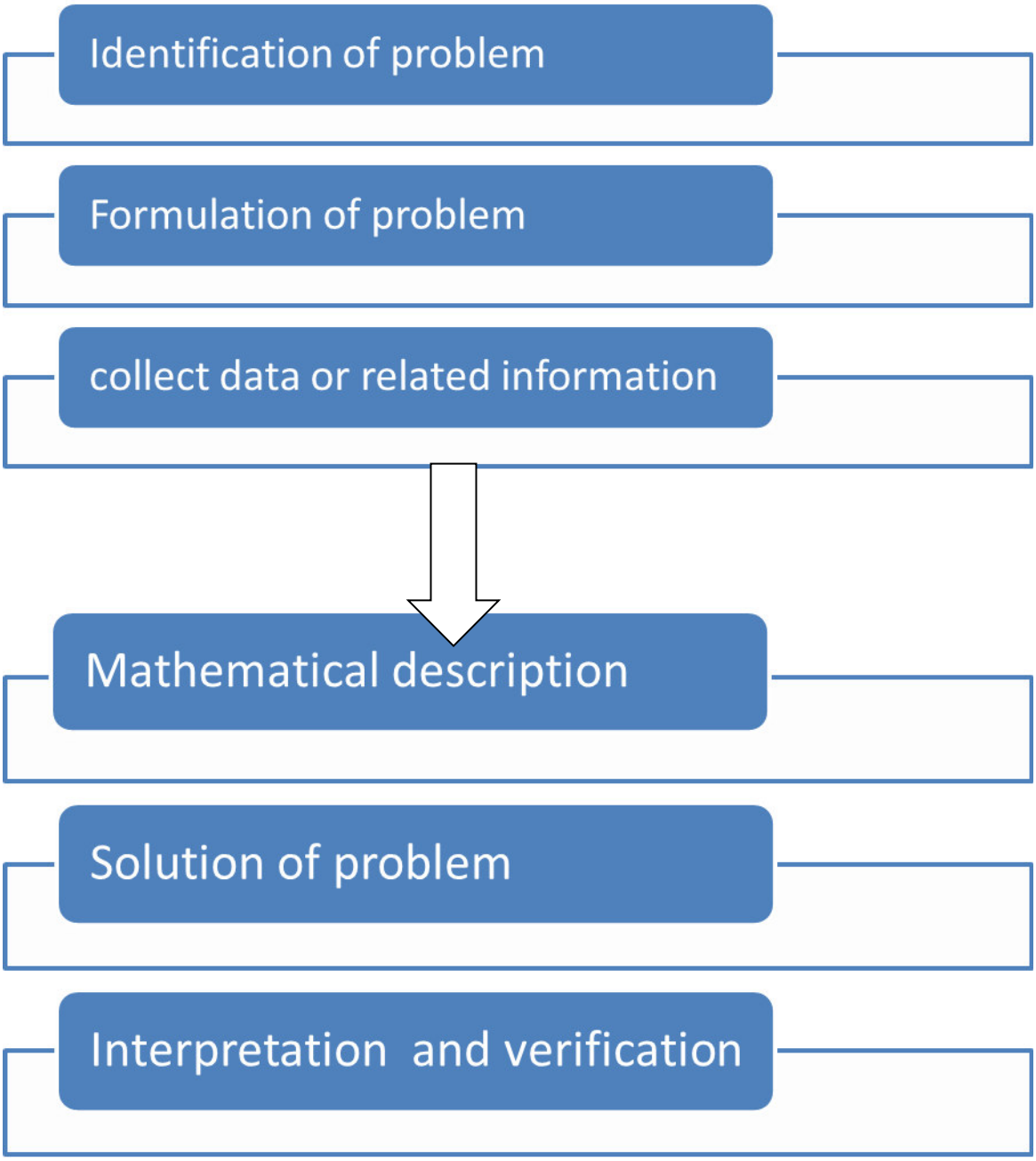
In this model we explain modeling plays a vital role in daily life problem by the help of mathematical modeling, discuss benefits and challenges in teaching learning also improve traditional way of teaching learning process, introduce new authentic activities in teaching learning process.

Methodology In first model we discussed the work already done. Now we discuss some problems related with school mathematics and higher mathematics. Firstly we discussed Problems related with application of trigonometry. Actually students face difficulties to apply problems in daily life. We tried to make interesting and full of enjoy these problems. Suppose if we want to measure height of any object we use some measuring tools like scale, measuring taps but in previous time people used hand span or other traditional tools for measurement. But measuring object become easy with the help of mathematics because it gives us better methods for this. And with the help of mathematics now we not only measure height of tree, building even height of satellites, planet from earth and many more. Now it is time to learn how to students take interest with these kinds of problems. Firstly we take a problem of height and distance.

Problem: To find the height of Tajul Masjid in Bhopal.

First of all we ask to students how they find height of Tajul Masjid without actually measure them. They replied by trigonometry. They used Pythagoras theorem to find out the perpendicular distance of Masjid. But to find out the height of Masjid we need some information like base (distance between observer and Masjid), also hypotaneous. But how we find hypotaneous. With the help of angle from the observer eye to the top of the Masjid. Now students raised the question how we measure the angle. Then we explain them that for this we used an instrument "Theodolite", which is used to measuring angles with a rotating telescope. After that we check

their previous knowledge of trigonometric ratios and their solutions.
Mathematical modeling has some stages such that:
1. Formulation
2. Solution
3. Interpretation
4. Validation



By following these steps we relate real world situations with mathematical modeling. We give some problems related with height and distance, mensuration for class 9^{th} and 10^{th} class students. And tell them to solve those problems by their methods. And note the speed how much time they have taken. Also this survey is done in two schools Model higher secondary school T.T. Nagar Bhopal and Sanskar Bharti School Bagsevania Katara hills Bhopal.

2. Modeling in high school

Problem 1: A man swims downstream 30km and upstream 18 km taking 3 hrs. each time. What is velocity of current?

Traditional methods used by students they use formulas of distance and speed. And try to solve the problem.

By mathematical modeling we explained them some techniques or steps for solving above problem. Actually boat and stream problems having three basic concepts as follows time, distance and speed. Now before solution we explain them some important formulas which make solution easy.

[1] If speed of boat= x km/h and
Speed of stream =y km/h then

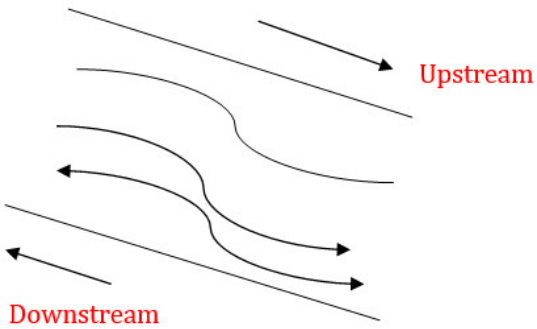
1. Boat speed in downstream=x+y
2. Boat speed in upstream = x-y

[2] If the speed of boat is x km/h in downstream and y km/h in upstream then

1. Speed of boat in still water =(x+y)/2
2. Speed of stream = (x-y)/2

[3] If the speed of boat or person in still water is "x" and speed of stream is "y" and the boat has to cover a distance 'd" km then

Time taken in downstream T_1= d/(x+y)
Time taken in upstream T_2 = d/(x-y)
Total time taken in going downstream and upstream $T=T_1+T_2$



Step 1

Formulation of the problem: In this process we check that what is given and what has to find out. So in above problem man's rate in upstream and downstream are given also time is given and we have to find out speed of boat in still water?

Mathematical description: Now we know distance in upstream is 30 km/h in 3 hours and distance in downstream is 18 km/h in 3 hours. We know velocity is change in distance with respect to time. So we have

Velocity "V"=distance/time

Also we can write man's rate downstream V_1=30/3km/h

And man's rate upstream V_2=18/3km/h

Velocity of current V= (V_1-V_2)/2km/h

Step 2

Finding the solution: Man's rate downstream V_1=30/3 =10km/h

Man's rate upstream v_2 =18/3=6km/h

Velocity of current V= $(V_1-V_2)/2$

V= (10-6)/2

V=2km/h

Step 3

Interpretation: In above problem we have seen that when man swim with the stream it takes time 3 hours also when he swim in opposite direction it takes same time 3 hours. And if we have to find velocity of current it means we have to find velocity of flow of river is obtained by direct formula $(V_1-V_2)/2$. Actually the total velocity is find out by the total distance covered by the man in same time [same direction of stream + (-opposite direction of stream)] opposite direction having negative sign. Divided by total round which is 2.

Name of students	class	topic	Speed of students in traditional method	Speed of students using mathematical modeling
Surbhi Gurjar	9th	Boat / Stream problems	9min.	7min
Poonam Raikwar	9th		9min.	6min
Ashi Rajak	9th		11min.	8min
Tarun Niware	9th		10min.	7min.
Aakash Darokar	9th		10min.	6min.
Shakeb Khan	9th		10min.	5min.
Sanjay Tiwari	9th		9min.	7min.
Ganga Satoday	9th		11min.	8min.
Pooja Rajak	9th		10min.	8min.

Problem 2: How many square tiles of sides 20 cm are needed to cover the floor area of room. Room has length 8m and breadth 6m.

When we give above problem to the students they use formula of areas of square and rectangle. And try to find out the solution according to their capacity.

Using Mathematical Modeling

Step 1

Formulation of problem:

For this firstly we check that what is given and what has to be found? In above problem we used area of room and area of tile for solution of the problem. We choose variable for length "l" and for breadth "b"

Now talk about square tile with side 20 cm. but we have to change side into m because length and breadth are given into meter.

Given side of tile is "a" = 20 cm.

Or we can write this "a" = 0.2 m.

Now given length of room "l" = 7m

So we have 8/0.2=40, therefore 40 tiles fit in one row in length.

Again given breadth of room "b" = 6m

So we get 6/0.2 =30, therefore 30 tiles fit in one column.

Step 2

Mathematical description:

Given length "l" =8m

And breadth "b"=5m

Side of tile "a"=0.2m

Then if length is 8m we can fit 8/0.2=40 tiles in one row lengthwise

Again breadth is 5 m so we can fit 5/0.2 = 25 tiles in one column.

Now we used the formula to find out the number of tiles needed

Total number of tiles= number of tiles along the length* number of tiles along the breadth

1200 = 40*30

Step 3

Solution:

Now we have

Total no of tiles required = number of tiles lengthwise * number of tiles breadth wise =(40*25)

Total tiles= 1000.

Step 4

Interpretation:

By solving us get 1200 tiles are required to cover the floor.

Step 4

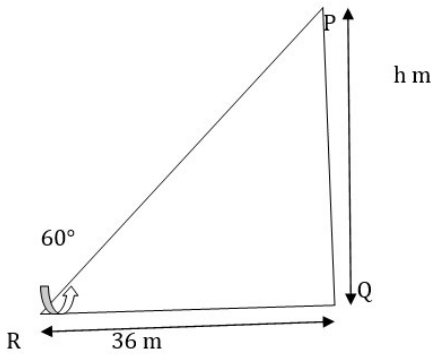
Validation:

In this model according to situations and ability of workers are the main factors as how they work without any mistakes and damage of tiles. obviously in real life situations it is not confirm that number of tiles are sufficient sometimes some tiles get damaged also defective so we in this case we need more tiles. So mathematical modeling gives us a rough idea to understand the problem. Also it provides some changes according to conditions.

So mathematical modeling gives us the last step validation by which we check our model efficiency and applicability. First of all we illustrate this problem by picture.

Name of students	class	topic	Speed of students in traditional methods	Speed of students using mathematical modeling
Sanjeevni Salve	9th	Mensuration Problems	11min	10min
Vishesh Mishra	9th		10min	9min
Auras Dwivedi	9th		15min	10min
Varsha vishwakarma	9th		17min	11min
Varun Chaturvedi	9th		11min	9min
Pallavi Gupta	9th		15min	10min
Kalpna Rajak	9th		14min	10min
Khushboo Jain	9th		13min	9min
Prathviraj Purohit	9th		10min	10min

Firstly we have given to the students
1. How to measure your box?
2. How they measure their height?
3. Find the height of tree in their school?
1. They started with their ruler and measure length of box within seconds.
2. Measuring the height students need measuring tape. It is not convenient with ruler.
Then they measure their height in 5 to 10 minutes.
3. For measuring the height of tree they face difficulty like they don't know how to exactly measure height of tree.
But some students tried to apply Pythagoras theorem, and used trigonometric ratios. After these problems we have given them problem such that:
Problem 3: We have to find the height of Tajul Masjid at Bhopal. Observer was standing 36 m away from Masjid. Angle of elevation is 60°.
Step 1
Formulation
Suppose "P" is the top of Masjid and "Q" is foot of the Masjid also "R" is the point of observer. And angle of elevation is 60 degree. So we have distance, angle of elevation and we have to find height of Masjid. See figure of Tajul Masjid at last page.
Step 2
Mathematical description
Suppose height of Masjid is "h", distance from foot of Masjid to observer be QR = 36m, angle of elevation angle R =60°.
Now we know
tan θ= perpendicular / base
tan θ = PQ/QR
tan 60° =h/36m
We know tan 60°=$\sqrt{3}$
Solution- $\sqrt{3}$ = h/36
h= $\sqrt{3}$ * 36m
h= 36 * 1.732m
h = 62.35m
Step 3



Interpretation

Therefore we get height of Tajul Masjid h= 62.35m

Actual height of Tajul Masjid is 62 m. so we apply this model for finding height of any object.

Name of students	class	topic	Speed of Students in traditional method	speed of students using mathematical modeling
Chahat Kushwaha	10^{th}	Height & Distance problem	15 min	
Ankit Sahu	10^{th}		20min	14min
Vishwash Mishra	10^{th}		14min	9min
Ashish Mishra	10^{th}		16min	11min
Amit Dhurvey	10^{th}		15min	10min
Deepanjali Mishra	10^{th}		17min	12min
Gudia shukla	10^{th}		18min	13min
Teena Rajput	10^{th}		20min	15min
Gracy Yadav	10^{th}		17min	11min
Muskan Yadav	10^{th}		16min	11min
Prateek Singh	10^{th}		14min	10min
Anjali Sharma	10^{th}		19min	9min
Kamal Sharma	10^{th}		16min	10min
Tarun vanshkar	10^{th}		15min	11min
Khushi Dhakad	10^{th}		20min	10min
Golu Yadav	10^{th}		14min	9min
Vaishnavi Singh	10^{th}		15min	10min
Varsha Jogi	10^{th}		13min	10min
Akash Jha	10^{th}		15min	11min
Sumit Ahirwar	10^{th}		18min	12min

We should emphasis on variety of experiences with applied mathematics, enhance mathematical concepts by describe the fact, explain uses of theories, and relate problems with real life situations. Educationists also consider NCTM policies for mathematical curriculum.

Also get information about recently developed US' mathematics curriculum standards, the common core state standards (2010) modeling is about habits of mind and productive way of thinking. It links classroom mathematics to daily life and develop decision making among students. The research indicates that most schools do not engage students in creating, modifying or meaningful representation of problem situations. Also face difficulties in solving word problems of mathematics given in text books. Explanation in Syllabus is not interesting also less detail. There is no relation with everyday problems especially in higher classes. We also described problems on different levels of education process to remove the complexities faced by students in mathematics. Consider some mathematical models and techniques of teaching of mathematical modeling. Problems based on fractions, shapes, measurement, probability and statistics focus on searching patterns and developing problem solving skills. Also focus on teachers role which help students to engage in activities and challenging tasks. Teacher developed mathematical attitude among students, improve habit of discover new ideas and facts of mathematics. So teacher should be more effective. All the higher education is depending upon secondary education interest develop about the subject at this age. If here we do less efforts then the subject become more complicated.

3. Modeling on secondary school

In this section we suggest some example to engage students in mathematical activity. Also explain those problems by some diagram which shows relation between problem and real world. These problems involved some following steps – constructing, mathematically described, simplifying. We tried that students worked mathematically, interpreting, comparing with similar situations, also verify with their own efforts. In this section we also survey in Model higher secondary T.T. Nagar Bhopal and Sanskar Bharti school Bagsevania Katara hills Bhopal.

We have explained them what is matrix and how it is related with real life situations. We have asked is there library in your school? They replied yes. Suppose there are 15 mathematics books in one column and 10 physics books in second column and 10 chemistry books in third column. Then we represent arrangement of above books such that: [15 10 10]. We have given them different types of matrices and ask about those. Questions based on matrix operations. Students used traditional methods for solving equations like Cross multiplication method, Elimination method, and Substitution method.

Problem 1

How to solve given equations? $x + y + z = 5, 2x - y + 3z = 0, x + 3y + 4z = 2$

Formulation of problem

We have three equations and we have to solve these equations i.e. find the value of x, y and z.

Mathematical description

First of all we write above equation as

$$X = A^{-1}B$$

Where A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

B= $\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$

X= $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now we find a inverse by formula

$A^{-1} = Adjoint\ of A/|A|$ For adjoint we have to find cofactors

$cij = (-1)^n Mij$ Where Mij is minor.

Minor M11 = $\begin{vmatrix} -1 & 3 \\ 3 & 4 \end{vmatrix}$ =-13

Similarly we have M12 = 5, M13 =7, M21 = -1, M22 = 3, M23 = 2, M31 = 4, M32 =1, M33= -3

Now we have to find cofactors C11 =-13, C12 =-5, C13= 7, C21 = -1, C22 = 3, C23 =-2, C31 =4, C32 = -1, C33 =-3.

Adjoint of A = $\begin{bmatrix} -13 & -1 & 4 \\ -5 & 3 & -1 \\ 7 & -2 & -3 \end{bmatrix}$

A-1 = Adjoint A/$|A|$

Where $|A|$ = -11

A-1 = -1/11$\begin{bmatrix} -13 & -1 & 4 \\ -5 & 3 & -1 \\ 7 & -2 & -3 \end{bmatrix}$

Now we have to find value of x, y, z

So we solve equation

X=A-1B

$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = -1/11$\begin{bmatrix} -13 & -1 & 4 \\ -5 & 3 & -1 \\ 7 & -2 & -3 \end{bmatrix} * \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$

Then we get x= 57/11, y= 27/11, z= -29/11.

Name of students	Class	Topic	Speed of students in traditional method	Speed of students using mathematical modeling
Vikas Dwivedi	11th	Matrix	12min.	9min.
Namrta Ubrani	11th		15min.	10min.
Dayasagar	11th		13min.	8min.
Amit Sharma	11th		12min.	8min.
Rohit Tiwari	11th		15min.	10min.
Kajal Jain	11th		13min.	12min.
Priyash Jain	11th		14min.	9min.
Yogesh Sahu	11th		12min.	11min.
Sakshi Agrawal	11th		14min.	8min.

CONCLUSION & Discussion

After study of above work we conclude that there is lots of scope in mathematics education and mathematical modeling is the best tool or we can say link for connecting mathematics with real world problems. But lots of

efforts are needed to improve math education for students and teachers. In second model we tried to distinguish between traditional methods and using mathematical modeling also tried to illustrate with examples that how speed is affected in both methods, also focus how to teach math? And what to teach? We tried to emphasis on teacher's role because teacher play very important role in students life sometimes he change the whole life of students and students very much depend on him for their studies. So teacher should have complete knowledge of content and relate mathematics with real world problems. Also we have discussed different country education system like Korea, US, Singapore. And then compare those from Indian system of education.

Mathematics education modeling: mathematical modeling is also very useful in higher education. We discuss some areas where it is used effectively.

Engineering as a branch of mathematics

1. Math education at higher level especially in engineering field, is really different from school learning. Engineering students used their knowledge analytically, graphically, and logically. Because they learn practical aspect of mathematics in terms of engineering. Engineering connect students with technology and enhance their innovative ideas also improve their skills to use mathematics in daily life.

2. How mathematical modeling related with engineering?

With the help of mathematics engineering students design or develop a system to solve the problem. Also they become able to shape their abstract thinking and imagination in a right way. Their creativity is enhanced.

Modeling give a proper system to solve the problems mathematically also with the help of modeling prediction of solution is also possible but it need proper strategies. So mathematical modeling used as an important tool in engineering. Basically modeling is a systematic process in which some steps we have to follow like:

(1) Identification of problem.

(2)Collect the related data and analyze them.

(3)Then design a model that fulfills all the needs related with problem. Also create some tools those verify the predictability of the model. And leave some space to modify the model according to the conditions.

3. Sometimes modeling becomes the only way to solve the problem because resource and choices are not enough. Engineers follow some way to find out the solution by they collect all the information's related with problem then correlate with involved content for development of working model and try to find out the solution on the basis of including variables like time ,cost ,size ,performance. According to Rodencker" modeling is a process in which we proceed from abstract to concrete". He has given eight rules to design a technical system. Such that

1. Clarify the task (the required relationship).
2. Establish the function structure (the logical relationship).
3. Choose the physical process (the physical relationship).
4. Determine the embodiment (the constructional relationship).
5. Check the logical, physical and constructional relationships by appropriate calculation.
6. Eliminating disturbing factors and errors.
7. Finalize the overall design.
8. Review the chosen design.

5. Mathematical modeling is increasing rapidly in every aspect of life. At elementary school to higher math education. It parts a main and important role in different fields of life. Also linked content to daily life problems. But it needs complete knowledge of content and information related to problems. Mathematical modeling create an environment to achieve solution of problems mathematically even it develop mathematical attitude among student. Increase interest among students which is really a good thing because of this students relate text book material in daily life and try to able to make models and solving them. It is also applicable in research field. Actually there is no progress without research, all the development depends upon it. but to improve quality of research, we some technology and system which makes research work easy and interesting and mathematical modeling gives a better system, innovative ideas and logically process.

Nowadays an environment is created for research programme. And we really have lots of talented people but there is lake of some factors like economic factors, lake of inspiration and motivation, lake of knowledge of technology. In India we need lot of work in the field of research but it is difficult to achieve this in shortage of education programme, seminar, and good quality of teaching learning process. Now in India mathematical modeling introduce at school level but it really needs more efforts because it relates us with daily life and gives assurance to successful planning for solution of any problems.

6. By the help of mathematical modeling people able to translate real situations into mathematical language. Using some variables related with situations and verifies result on actual problem. There is no actual procedure for mathematical modeling for getting solution. According to (Haines & crouch) modeling process is cyclic in nature. Author considered here are different approaches which may have similarity and differences also so author categories approaches as follows:

1. Realistic or applied modeling
2. Contextual modeling

3. Educational modeling
4. Sociocritical modeling
5. Epistemological or theoretical modeling
6. Cognitive modeling

Also we have given comparison between traditional methods of problem solving and solving problems by mathematical modeling. In modeling process student's work in group and in traditional way they work individually. The term model is not only mathematical representation of problem but it shows the conceptual framework of the students. In modeling process the learning environment motivate students to give better result.

References

1. Bhogoya, J. D. , Bhogoya, J.M. "An academic value-added mathematical model for higher education in Columbia" Ingenieria e Investigacion, vol 33, no. 2, (2013), pp. 76-81.
2. Ayhan Kursat Erbas, Mahmut Kertil "Mathematical modeling in mathematics education: basic concept and approaches" Educational sciences:Theory and practices, 14(4), (2014), 1621-1627.
3. Tanja Sekulic, Smiljana Mirkov, Marija Matotek "College students attitude towards engineering profession innovation in mathematics education and mathematical modeling" Journal of the technical university of Gabrovo, vol. 47,(2014), pp.83-87.
4. Ok Ki kang 12th International congress on Mathematical Education (2012).
5. Nelson,T, Mathematical education: "A summary of research theories, and practice.(2002, august)"
6. Reston,VA, "principles and standards for school mathematics" National council of teachers of mathematics (2001).
7. Ok-Ki Kang, "Teaching Mathematical Modeling In School Mathematics", 12th International Congress On Mathematical Education, (2012) Seoul, Korea
8. Kang O.K., "A study on modeling process for fitting mathematical modeling", Journal of the Korean Society of research in Mathematics,(2010),20(1), 73-84.
9. Cheng A.K.,"Teaching Mathematical Modeling in Singapore school.", The Mathematics Educator,(2001), 16(1), 63-75.
10. Brown S.& Walter, M. "The art problem posing", Mahwah, NJ: Lawrence Erlbaum Associate,(2005)
11. Ambrose R., "Initiating change in prospective elementary school teachers orientations to mathematics teaching by building on beliefs", Journal of the mathematics teacher education, (2004), 7(291-119).
12. Beswick K., "Teachers beliefs about school mathematics and mathematicians mathematics and their relationship to practice", Educational studies in Mathematics, (2012), 79(1), 127-147.
13. Ponte J.P."Teachers knowledge, practice, and Teacher's knowledge and identity: essential aspects of teachers learning", Journal of Mathematics Teacher Education, (2011), 14(6), 413-417.
14. Zazkis R. & Mamolo A., "Reconceptualizing knowledge at the mathematical horizon." For the Learning of Mathematics, (2011), 21(2), 8-13.
15. Mosvold R., & Fauskanger,J "Teachers beliefs about mathematical knowledge for teaching definitions.", International Electronic journal of Mathematics Education, (2013), 8(2-3), 43-61.
16. Lew, H. c. "Middle school mathematics" 2, Mirae & Culture Group. (2011), Seoul, Korea:
17. Reston, VA, "Principles and standards for school mathematics", NCTM. (2000).
18. Helping Children Learn Mathematics. Washington D.C.: National Research Council. National Academy of Science, (2001).
19. Dr.Rita Borromeo Ferri, "Mathematical Modeling in school and in teacher education-conceptions and examples", university of Kassel (Germany), (2013)

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