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The Effect of Current Expected Variance of Return on Future Trading Optimal Strategy

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Abstract
Due to the presence of transaction cost, most investors do not keep changing their portfolio to an optimal portfolio over time. This paper adopts a new approach to investigate the linkages between current optimal portfolio variance and the expected future portfolios variances. It is given a closed-form solution for optimal dynamic portfolio selection with trading cost; considering the minimum variance of the utility function as an optimal or selected portfolio by an investor for any period of time based on Gârleanu & Pedersen (2013) framework. Finally, we introduce the multi-period portfolio model based on CRRA preference utility function.
Keywords: multiperiod portfolio selection, higher-order moments, CRRA utility function, optimal wealth changes

1. Introduction

The main focus of modern portfolio theory has been on mean-variance framework (Markowitz 1952's seminal paperwork) with the assumption that the investor is *myopic* and the utility function is *quadratic*. Many researchers have been exhausting it on: First, they considered the investor chooses her/his portfolio based on only the single-period framework as a myopic behavior, for example, see Merton (1969) and Campbell & Viceira (2002), but in general most investors take a longer period and make a decision based on multi-period framework (such as, Grinold 2006; Engle & Ferstenberg 2007). Second, the mean-variance problem only represents an expected utility maximization for the special case of quadratic utility because it can be interpreted as a second order utility function without considering transaction cost, whereas in reality trading is costly in financial, see Constantinides (1979). Also, several studies have shown trading cost may be economically no negligible.

In this research, we study multi-period portfolio in the presence of trading cost. The goal is to describe, discuss, and illustrate a portfolio choice model with the best portfolio which is based on Markowitz portfolio, naïve rule and minimizing variance of the utility function in the multiperiod framework. We make use of the Gârleanu & Pedersen (2013) formulation to describe analytically the optimal multi-period portfolio under the optimal portfolio of minimum variance of utility's investor. The difference between our model and the G&P model is that: we consider an investor who maximizes her CRRA utility of terminal wealth by minimizing the variance of utility function while the G&P model is only based on maximizing the expected utility. The same as the G&P model, we assume that the price changes are predictable in excess of the risk-free rate.

Our work is related to Biglari Kami & Ouyang (2016), who consider that the investors are willing to minimize the variance of the utility function in portfolio selection. We shall build up the portfolio problem based on this assumption and the multiperiod framework.

2. Literature Review

2.1. Mean-variance portfolio

Under the theory of portfolio selection, the investor obtains efficient frontier which is the efficient trade-off between return and the risk of diversified portfolios. The investor can reduce only unsystematic risk through diversification, but systematic risk cannot be moderated in this way because it is unpredictable. In Markowitz's seminal paperwork, he minimizes the amount of risk portfolio for a given portfolio expected return, which is called as the mean-variance framework. The following formulation can express this:

$$\begin{aligned} &\min_{w}\ w'\Sigma w \\ &s.t:\ w'\mu \geq \mu_0 \\ &\qquad w'\iota = 1,\ \iota' = [1,1,...,1] \end{aligned} \tag{1}$$

Here, $w = (w_1, w_2, ..., w_n)'$ is the vector of the weight of N risky assets, Σ is an $N \times N$ covariance matrix of returns between N risky assets, $\mu = (\mu_1, \mu_2, ..., \mu_N)'$ is the vector of expected returns, μ_0 is the target expected return.

If we assume that an investor's utility function is given by quadratic utility (that is, asset returns are fully described by mean and variance), this problem can be formulated in maximization of the objective function using an investor risk-aversion coefficient λ :

$$\max_{w} \ (w'\mu - \frac{\lambda}{2} w'\Sigma w)$$
$$s.t.: \tag{2}$$
$$w'\iota = 1, \ \iota' = [1,1,...,1]$$

For a general utility function, the above problem will no longer be expressed by the Markowitz framework which is the trade-off between risk and return.

2.2. Portfolio of Minimizing the Variance of Utility Function

In this section, we describe a portfolio of variance utility based framework proposed by Biglari Kami & Ouyang (2016) that minimize the variance of the utility function for investors. Suppose that the price changes in excess of the risk-free rate are independently and identically distributed with mean vector μ and define Σ as the matrix of covariance of asset returns. We will construct the expected volatility utility of terminal wealth by:

$$\mathrm{Var(U(W))=E[(U(W)-E[U(W)])^2]=E[U^2(W)]-E[U(W)]^2} \tag{3}$$

We then minimize this variance of utility to better diversify efficient portfolios from sample moments. This problem can become even more well-diversified portfolios because the extreme behavior of the weights is more due to the estimation of the sample first moment which obviously disappear from our analysis in the following calculations.

First, for calculating $\mathrm{E[U(W)]^2}$ let $\mu^{(i)}$ denote ith central moment. The following equation holds if we approximate the expectation of utility wealth by a second-order Taylor expansion at μ=E(W) :

$$\mathrm{E[U(W)] \approx U(E[W]) + \frac{U^{(2)}(E[W])}{2}\mu^{(2)}} \tag{4}$$

multiply above equation by itself to get:

$$\mathrm{E[U(W)]^2 \approx (U(E[W]))^2 + U^{(2)}(E[W])\mu^{(2)}U(E[W]) + \frac{(U^{(2)}(E[W]))^2}{4}(\mu^{(2)})^2} \tag{5}$$

Then, similarly we take the first term $\mathrm{E[U^2(W)]}$ by implying a second-order Taylor expansion for utility function at μ=E(W) gives:

$$\mathrm{U(W) \approx \frac{(W-E(W))^0}{0!}U(E(W)) + \frac{(W-E(W))^1}{1!}U^{(1)}(E(W)) + \frac{(W-E(W))^2}{2!}U^{(2)}(E(W))} \tag{6}$$

multiply above by itself to get

$$\mathrm{U(W)^2 \approx \left[\frac{(W-E(W))^0}{0!}U(E(W)) + \frac{(W-E(W))^1}{1!}U^{(1)}(E(W)) + \frac{(W-E(W))^2}{2!}U^{(2)}(E(W))\right]^2} \tag{7}$$

applying both sides by expected operation to get

$$\begin{aligned}\mathrm{E[U(W)^2]} &\approx \mathrm{E[U(\mu)^2 + 2U(\mu)(W-\mu)U^{(1)}(\mu) + (W-\mu)^2(U^{(1)}(\mu))^2} \\ &\quad \mathrm{+ (U(\mu)+(W-\mu)U^{(1)}(\mu))(W-\mu)^2U^{(2)}(\mu) + \frac{1}{4}(W-\mu)^4(U^{(2)}(\mu))^2]}\end{aligned} \tag{8}$$

Finally, we can use equation (5) and (8) to construct equation (3):

$$\begin{aligned}\mathrm{Var(U(W))} &= \mathrm{E[(U(W)-E[U(W)])^2]=E[U(W)^2]-E[U(W)]^2} \\ &= \mathrm{[(U^{(1)}(\mu))^2]\mu^{(2)} + U^{(1)}(\mu)\,U^{(2)}(\mu)\mu^{(3)} + \frac{1}{4}(U^{(2)}(\mu))^2\mu^{(4)} - \frac{1}{4}(U^{(2)}(\mu))^2(\mu^{(2)})^2}\end{aligned} \tag{9}$$

if suppose the investors have CRRA preferences with risk aversion parameter λ, for example, let define $U=\frac{W^{1-\lambda}}{\lambda}$ be utility function for CRRA investor. Then higher-order moment tensors can easily parametrize portfolio moments as:

$$\begin{aligned}\mu^{(2)} &= w'\Sigma w \\ \mu^{(3)} &= w'M_3(w \otimes w) \\ \mu^{(4)} &= w'M_4(w \otimes w \otimes w)\end{aligned} \tag{10}$$

note that

$$\begin{aligned}M_3 &= E[R-E[R]] \otimes E[R-E[R]] \otimes E[R-E[R]], \\ M_4 &= E[R-E[R]] \otimes E[R-E[R]] \otimes E[R-E[R]] \otimes E[R-E[R]]\end{aligned} \tag{11}$$

by taking the initial wealth as a numeraire, the following explanation can be suggested by our analysis:

$$U^{(1)}(W)=1$$
$$U^{(2)}(W)=-\lambda$$
$$U^{(3)}(W)=\lambda(\lambda+1)$$
$$U^{(3)}(W)=-\lambda(\lambda+1)(\lambda+2) \quad (12)$$

therefore, we can rewrite the investor optimization's problem as a minimizing the following portfolio moments by taking into account only two moments:

$$\mu^{(2)}-\frac{(\lambda)^2}{4}(\mu^{(2)})^2+\lambda\mu^{(3)} \quad (13)$$

which clearly removes the first moment impact of sample return in trade-off among moments.

2.3. Multi-period portfolio Based on Markowitz Portfolio as an Optimal Portfolio

In the classical mean-variance framework, an investor attempts to maximize only tomorrow's payoff and is not concerned about future payoffs. But in a realistic situation, one may maximize his expected utility of wealth along some future date. The investor's terminal wealth can be defined:

$$W_{t+1}=W_t(x_t' r_{t+1}+R_t^f) \quad (14)$$

Which (W_{t+1}) is the investor terminal wealth at the time (t+1), x_t is the portfolio weights over of the period t , R_t^f is the return of risk-free rate at the period of t , and r_{t+1} is the portfolio return at the end of the period t . Then the investor's expected utility function for multi-period portfolio problem along future date $t+\tau$ by trading in N risky assets at times $t, t+1, ..., t+\tau-1$ can be formulated as (Aït_Sahalia & Hansen 2009):

$$\begin{aligned} V(\tau,W_t,z_t) &= \max_{\{x_s\}_{s=t}^{s=t+\tau-1}} E_t[U(W_{t+\tau})] \\ &= \max_{x_t} E_t\left[\max_{\{x_s\}_{s=t}^{s=t+\tau-1}} E_{t+1}[U(W_{t+\tau})]\right] \\ &= \max_{x_t} E_t[V(\tau-1, W_t(x_t' r_{t+1}+R_t^f), z_t)] \end{aligned} \quad (15)$$

Where τ is the investment horizon, $V(\tau,W_t,z_t)$ denotes the investor's expectation at the time t , conditional on the information z_t, of the utility of terminal wealth $W_{t+\tau}$ over the next τ period. The last equation is called Bellman equation. This problem can be solved numerically using First Order Condition (FOCs).

(Gârleanu & Pedersen 2013) G&P consider N securities traded at each time $t\in\{?\ \ -1,0,1,?\}$. The securities' price changes in excess of the risk-free return, $p_{t+1}-(1+R_f)p_t$ between times t and $t+1$, are predictable. In their model, trading is costly with the quadratic transaction costs (TC) are proportional to the covariance matrix Σ is obtained by:

$$TC(\Delta x_t)=\frac{1}{2}\Delta x_t'\Lambda\Delta x_t \quad (16)$$

Here, $\Lambda=\gamma\Sigma$, γ is the transaction cost parameter, and $\Delta x_t = x_t - x_{t-1}$ denotes trading shares at a time t . G&P provide the investor's dynamic trading objective to maximize the present value of all future expected excess returns, the investor's objective is:

$$\max_{x_0,x_1,\ldots} E_0\left[\sum_t (1-\rho)^{t+1}(x_t' r_{t+1}-\frac{\lambda}{2}x_t'\Sigma x_t)-\frac{(1-\rho)^t}{2}\Delta x_t'\Lambda\Delta x_t\right] \quad (17)$$

Which $\rho\in(0,1)$ is a discount factor and λ is risk-aversion parameter.

3. Methodology

3.1. Multi-Period Portfolio based on Minimum Variance Utility Framework

We consider the securities price changes are predictable collected in μ_{t+1} between time period $(t, t+1]$ which can be obtained by a vector of factors that predict return which could be the past return of securities over different horizons or other predicting variables:

$$\mu_{t+1} = Bf_t + u_{t+1}$$
$$\Delta f_{t+1} = -\Phi f_t + \varepsilon_{t+1} \tag{18}$$

Here, f_t is a $K \times 1$ vector of factors, B is a $N \times K$ matrix of factor loadings, and u_{t+1} is an unpredictable zero-mean term with the covariance matrix $\sum$.

The change of factors has a mean-reversion Φ which is a $K \times K$ matrix of mean-reversion coefficients, ε_{t+1} is the shock affecting predictors. Then, considering the minimum variance, the optimal strategy can be obtained by the following investor's objective function which is the present value of all future minimized for risk and trading cost. In our model, we assume that trading costs are proportional to the quadratic variance of trading in each period of time. So, the investor's objective is:

$$\min_{S_0, S_1, \ldots} E_0 \left[\sum_t (1-\rho)^{t+1} (S_t - (\frac{\lambda}{2})^2 S_t' S_t) + \frac{(1-\rho)^t}{2} \Delta S_t' \Lambda \Delta S_t \right] \tag{19}$$

where, $S_t = x_t' \Sigma x_t$

The above model is a dynamic programming problem, we solve the model by introducing a value function for the bellman equation. The bellmen equation can be written as following:

$$V(S_t) = \min_{S_t} \left[(1-\rho)(S_t - (\frac{\lambda}{2})^2 S_t' S_t + E_t[V(S_t)]) + \frac{\lambda}{2} \Delta S_t' \Sigma \Delta S_t \right] \tag{20}$$

This model has a unique solution which is given in Proposition 1.

Proposition 1: *The unique solution for the above bellmen equation is given by:*

$$V(S_t) = \left[S_t' (\frac{A}{2} - \frac{B}{4} S_t) + C \right] \tag{21}$$

The coefficient A, B, C *are coefficient matrices which are stated in the proof of the proposition1.*

Proof of Proposition 1: traders try to minimize the Bellman equation for any period:

$$V(S_{t-1}) = \min_{S_t} \left[(1-\rho)(S_t - (\frac{\lambda}{2})^2 S_t' S_t + E_t[V(S_t)]) + \frac{\lambda}{2} \Delta S_t' \Sigma \Delta S_t \right] \tag{22}$$

First, we guess the value function is the posited form, the expected future value function is calculated as:

$$E_t[V(S_t)] = \left[S_t' (\frac{A}{2} - \frac{B}{4} S_t) + C \right] \tag{23}$$

Therefore, the Bellman equation becomes:

$$\min_{S_t} \left[(1-\rho)(S_t - (\frac{\lambda}{2})^2 S_t' S_t + [\left[S_t' (\frac{A}{2} - \frac{B}{4} S_t) + C \right]]) + \frac{\lambda}{2} \Delta S_t' \Sigma \Delta S_t \right] \tag{24}$$

Now, the bellman equation can be simplified as bellow:

$$d_t + S_t' (\frac{1}{2} j_t - \frac{1}{4} h_t S_t) \tag{25}$$

Where,

$$S_t = x_t' \Sigma x_t$$
$$j_t = [(1-\rho)(1 + \frac{A}{2}) - \lambda \Sigma S_{t-1}]$$
$$h_t = [(1-\rho)(-\frac{\lambda^2}{4} - \frac{B}{4}) + \frac{\lambda}{2} \Sigma] \tag{26}$$
$$d_t = [(1-\rho)C + \frac{\lambda}{2} S_{t-1}' \Sigma S_{t-1}]$$

The optimal solution is attained by:

$$S_t^* = h_t^{-1} j_t$$
$$V^*(x_{t-1}) = \frac{1}{2} j_t' h_t^{-1} j_t + d_t \tag{27}$$

The above equation must be hold for all x_t, which implies the following limitations:

$$\frac{1}{2} j_t' J_t^{-1} j_t + d_t = \left[S_{t-1} (\frac{A}{2} - \frac{B}{4} S_{t-1}) + C \right] \tag{28}$$

Then,

$$\frac{1}{2}j_t'J_t^{-1}j_t + d_t = \left[S_{t-1}(\frac{A}{2} - \frac{B}{4}S_{t-1}) + C\right]$$
$$-\frac{B}{4} = \frac{1}{2}\left[\lambda^2\Sigma'[(1-\rho)(-\frac{\lambda^2}{4} - \frac{B}{4}) + \frac{\lambda}{2}\Sigma]^{-1}\Sigma + \frac{\lambda}{2}\Sigma\right]$$
$$\frac{A}{2} = \frac{1}{2}\left[-2\lambda(1-\rho)*(1+\frac{A}{2})*[(1-\rho)(-\frac{\lambda^2}{4} - \frac{B}{4}) + \frac{\lambda}{2}\Sigma]^{-1}\Sigma\right] \quad (29)$$
$$C = \frac{1}{2}\left[(1-\rho)(1+\frac{A}{2})*[(1-\rho)(-\frac{\lambda^2}{4} - \frac{B}{4}) + \frac{\lambda}{2}\Sigma]^{-1}*(1-\rho)(1+\frac{A}{2})\right] + (1-\rho)C$$

A, B and C can obtain easily by above equations. Finally, differentiating the Bellman equation with respect to S_{t-1}, we can obtain the optimal portfolio trading rate which is expressed in the proposition 2.

Proposition2: *the optimal portfolio change in any period is*

$$\Delta S_t = \lambda\Sigma^{-1}(1-\rho)^{-1}(\frac{B}{2}S_{t-1} - \frac{A}{2}) \quad (30)$$

The properties of optimal portfolio change ΔS_t for any period are as follows:

a. ΔS_t *is decreasing in coefficient of factors which predict securities return (B).*
b. Δx_t *is decreasing in transaction cost parameter* Λ .
c. Δx_t *is increasing in discounting factor* ρ .

Proof of proposition 2: Differentiating the bellman equation with respect to S_{t-1} gives:

$$\frac{d}{dS_{t-1}} = 0$$
$$\Delta S_t = \lambda\Sigma^{-1}(1-\rho)^{-1}(\frac{B}{2}S_{t-1} - \frac{A}{2}) \quad (31)$$

Which shows that the optimal portfolio change for any period is decreasing in B, Λ , $\rho > 0$.

3.2. The investor's utility function based framework:

In this section, we consider an economy with the investors maximizing their wealth base on the utility function framework on CRRA preferences by making decision on how much to invest, not only mean, variance and skewness of utility, which can be written as follow:

$$\max_W E_0\left[\sum_t (1-\rho)^t (U(W))\right] \quad (32)$$

Here, W is the wealth of investor in any period of time.

Then, we define ($\theta_{:}$) as the rate of investment as n-state Markov process with transition matrix P_{ij} :

$$\begin{array}{lll} period & Wealth & \\ 0 & W_0 & \\ 1 & W_1 \rightarrow \frac{W_1}{W_0} = \theta_1 & \\ 2 & W_2 & \\ \vdots & \vdots & \vdots \rightarrow \theta_{\vdots} \\ t & W_t & \\ t+1 & W_{t+1} \rightarrow \frac{W_t}{W_{t+1}} = \theta_{t+1} & \\ \vdots & \vdots & \vdots \end{array} \quad (33)$$

The mentioned objective function is a recursive dynamic programming problem. We solve this problem by introducing value function for Bellman equation, therefore:

$$V(W_0,\theta_0)=\max_{\theta_0\theta_1\cdots} E_0\left[(1-\rho)^t U(W_t)\right]$$

$$CRRA\ preferences: \qquad U(W)=\frac{W^{1-\lambda}}{1-\lambda} \tag{34}$$

$$V(W_t,\theta_t)=U(W_t)+(1-\rho)E_t V(W_{t+1},\theta_{t+1})$$

In the following, we state the unique value function for the Bellmen equation.

Proposition 3: *the unique solution for the Bellman equation is given by:*

$$V(W_t,\theta_t)=U(W_t)\psi(\theta_t)$$

$$which,$$

$$\psi=[1-(1-\rho)P\,diag(\theta_1^{1-\lambda},\theta_2^{1-\lambda},\ldots,\theta_N^{1-\lambda})]^{-1}1,1'=[1,1,\ldots,1] \tag{35}$$

$$P\ is\ transition\ matrix\ \ P_{ij}=prob(\theta period\ j=\theta_j\mid\theta period\ i=\theta_i)$$

Proof of Proposition 3: first, we guess that the value function is the posited form, so:

$$V(W_t,\theta_t)=U(W_t)+(1-\rho)E_t V(W_{t+1},\theta_{t+1}),$$

$$guess: V(W_t,\theta_t)=U(W_t)\psi(\theta_t)$$

$$U(W_t)\psi(\theta_t)=U(W_t)+(1-\rho)E_t[U(W_{t+1})\psi(\theta_{t+1})]$$

$$\psi(\theta_t)=1+(1-\rho)E_t[\frac{U(W_{t+1})}{U(W_t)}\psi(\theta_{t+1})];$$

$$\frac{U(W_{t+1})}{U(W_t)}=(\frac{W_{t+1}}{W_t})^{1-\lambda}=\theta_{t+1}{}^{1-\lambda}$$

$$\psi(\theta_t)=1+(1-\rho)E_t[\theta_{t+1}{}^{1-\lambda}\psi(\theta_{t+1})] \tag{36}$$

$$E_t[\theta_{t+1}{}^{1-\lambda}\psi(\theta_{t+1})]=\sum_i P_{i,t+1}\theta_{t+1}{}^{1-\lambda}\psi(\theta_{t+1})$$

$$P\ is\ matrix\ N\times N\ with\ P_{ij}=prob(\theta period\ j=\theta_j\mid\theta period\ i=\theta_i)$$

$$\psi(\theta_t)=1+(1-\rho)\sum_{i=1}^{N}P_{i,t+1}\theta_{t+1}{}^{1-\lambda}\psi(\theta_{t+1})\ for\ t=1,2,\ldots,N$$

$$\psi=[1-(1-\rho)P\,diag(\theta_1^{1-\lambda},\theta_2^{1-\lambda},\ldots,\theta_N^{1-\lambda})]^{-1}1,1'=[1,1,\ldots,1]$$

We next obtain the closed-form expressions and properties of the optimal investment rate policy in our setting which are stated in the following.

proposition4: *the optimal investment rate policy in our setting is given by:*

$$W_{t+1}=(\frac{P^{-1}}{2(1-\rho)})^{\frac{1}{1-\lambda}}W_t$$

$$\theta^*=(\frac{P^{-1}}{2(1-\rho)})^{\frac{1}{1-\lambda}} \tag{37}$$

The properties of optimal investment rate θ^* for any period are as follows:

a. θ^* *is increasing in coefficient of risk aversion parameter (λ).*
b. θ^* *is decreasing in the transition matrix P.*
c. θ^* *is decreasing in discounting factor ρ.*

Proof of the proposition4: differentiating the mentioned bellmen equation with respect to W_t:

$$V(W_t,\theta_t)=U(W_t)\psi(\theta_t)$$

$$V(W_t,\theta_t)=\frac{W_t^{1-\lambda}}{1-\lambda}[1-(1-\rho)P\,diag(\frac{W_{t+1}}{W_t})^{1-\lambda}]^{-1}1,1'=[1,1,\ldots,1]$$

$$\frac{dV}{dW_t}=0$$

$$W_t^{-\lambda}[1-(1-\rho)P\,diag(\frac{W_{t+1}}{W_t})^{1-\lambda}]-[(1-\rho)P\,diag(\frac{(1-\lambda)W_{t+1}^{1-\lambda}}{W_t^{2-\lambda}})][\frac{W_t^{1-\lambda}}{1-\lambda}]=0$$

$$[1-(1-\rho)P\,diag(\frac{W_{t+1}}{W_t})^{1-\lambda}]=(1-\rho)P\,diag(\frac{W_{t+1}^{1-\lambda}}{W_t^{1-\lambda}}) \quad (38)$$

$$2(1-\rho)P\,diag(\frac{W_{t+1}^{1-\lambda}}{W_t^{1-\lambda}})=1$$

$$diag(\frac{W_{t+1}}{W_t})=(\frac{P^{-1}}{2(1-\rho)})^{\frac{1}{1-\lambda}}$$

$$W_{t+1}=(\frac{P^{-1}}{2(1-\rho)})^{\frac{1}{1-\lambda}}W_t$$

As to the above, it is clear that the optimal investment rate for any period is decreasing in B, Λ, $\rho>0$.

4. Empirical application:

In this section, we compare the performance of our optimal dynamic portfolio with static Markowitz portfolio using data from Center for Research in Security Prices (CRSP) monthly returns data.

We consider the sample period January 2004 to October 2013. We select top 7 different firms from CRSP database, and then we collect monthly returns for these stocks from January 2004 to October 2013. As a result, we obtain valid monthly returns of 7 socks for 118 periods. We first set window estimation M=60 and then to measure the stability of each portfolio we estimate moment and comoments parameter which obviously are not known by using (Martellini & Ziemann 2010; Ledoit & Wolf 2004) shrinkage estimation method. The relative risk aversion coefficient is taken equal to different cases of λ=3 .

Table 1 describes the summarized results of empirical data for our dynamic model and Markowitz mean-variance theory. We see our optimal dynamic model almost has better Sharpe ratio than mean-variance strategy. For example, the out-of-sample Sharpe ratio for Markowitz is 0.088, while the in our model has 0.091 monthly out-of-sample ratios. Thus, considering portfolio under dynamic strategies as an optimal target is very successful and much more reasonable measuring with static strategies.

Table 6: how well is the dynamic model rather than a static model?

PANEL A:	λ=1		λ=3	
Measurements\ Models	dynamic	Markowitz	dynamic	Markowitz
Sharp Ratio	0.091599491*	0.088835632	0.091599491*	0.088835632
Certainty Equivalent Return	0.003185329*	-0.179559716	0.001008434*	-0.662098514

5. Conclusion

In this research work, we studied the optimal portfolio policy for multi-period mean-variance-skewness framework and investor utility function to consider the impact of skewness and rate of investment. We provided a closed-form expression for two different multi-period frameworks. We showed that the investor's optimal multi-period portfolio considering skewness decreases in trading cost and the optimal portfolio change involves two terms, the predictable price change term and the independent and identically distributed price changes . Moreover, we proposed a utility function based framework for multiperiod portfolio selection in which we found out the effects of investment parameter on this framework.

References

Aït_Sahalia, Y. & Hansen, L.P., 2009. Handbook of Financial Econometrics, Vol 1: Tools and Techniques, Elsevier.

Biglari Kami, M. & Ouyang, H., 2016. Optimization makes estimation much worse. Working paper, Huazhong

University of Science and Technology.
Campbell, J.Y. & Viceira, L.M., 2002. Strategic asset allocation: portfolio choice for long-term investors, Oxford University Press.
Constantinides, G.M., 1979. Multiperiod consumption and investment behavior with convex transactions costs. Manage. Sci., 25(11), pp.1127–1137.
Engle, R.F. & Ferstenberg, R., 2007. Execution Risk. The Journal of Portfolio Management, 33(2), pp.34–44.
Gârleanu, N. & Pedersen, L.H., 2013. Dynamic trading with predictable returns and transaction costs. J. Finance, 68(6), pp.2309–2340.
Grinold, R., 2006. A dynamic model of portfolio management. Journal of Investment Management, 4(2), p.5.
Markowitz, H., 1952. Portfolio selection. J. Finance, 7(1), pp.77–91.
Merton, R.C., 1969. Lifetime portfolio selection under uncertainty: The continuous-time case. The review of Economics and Statistics, pp.247–257.