Research Journal of Finance and Accounting
ISSN 2222-1697 (Paper) ISSN 2222-2847 (Online)
Vol.6, No.3, 2015
www.iiste.org

Heuristic Approach to Satisfaction Level of Demand Maker in Three Stage Scheduling with Fuzzy Due Time

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Abstract
This paper discusses three stage flow shop scheduling including transportation time in frame work of fuzzy due date with bi-objective criteria. On one side to minimize the tardiness of jobs while on other way to determine the satisfaction level of demand maker with the help of fuzzy due date to each job. The analysis has been made through heuristic algorithm based on mathematical theorem regarding tardiness of the jobs.
The objective of the paper is to find due time to each job and optimal or near optimal sequence of jobs in order to minimize the tardiness of jobs so that they can be performed within time. The paper ends with a numerical illustration and analyse graphically the satisfaction level of demand maker.
Keywords: Fuzzy processing time, trapezoidal, tardiness, <AHR>, linguistic variable, fuzzy quantifier.

1. Introduction

The research in the field of scheduling has drawn a great attention in the last two decades with the objective to increase the effectiveness of industrial production. In flow shop scheduling, various jobs are to be done on two or more facilities (called machines) available in the sequential manner of facilities for this purpose. We want to know "when each job is to begin and when its completion due date is?" We also want the order in which these jobs are processed. If a penalty is imposed for all tardy jobs such as the quantum of penalty varies with magnitude of tardiness (greater penalty for greater tardiness), then obviously our attempt will be to minimize the mean tardiness.

The concepts of fuzzy processing times, fuzzy due date, fuzzy precedence relation etc. have been studied by various researchers in the field of scheduling. Conway etal (1965), Ikram (1986) discussed about earliness and tardiness of jobs in flow shop scheduling. Mc Cahon & Lee (1992) studied the job sequencing problem in which processing times were given in fuzzy numbers. Shibuchi et al (1996) and Ishii (1995) have studied fuzzified scheduling models taking in account fuzzy due date. T.P. Singh, Sunita (2008, 2010) investigated the demand maker satisfaction level under triangular fuzzy rule in 2 and 3 stage scheduling problems. Recently Meenu, Singh T.P. & Deepak Gupta (2014) investigated a bi- objective criteria in nx2 flow shop scheduling to minimize the tardiness of jobs as well as to find the satisfaction level of demand maker with the help of fuzzy due date to each job.

This paper is the extension of our earlier work. In this Paper we discuss 3 stage flow shop scheduling models which includes transportation time in the frame work of fuzzy due time. The membership function of fuzzy due time assign to each job shows the satisfaction level of demand maker. The processing times of jobs are given whose elements are fuzzy quantifiers characterized.

The linguistic variables are replaced by fuzzy trapezoidal numbers. We find that if a job completes very early, it has no significance due to expenditure on inventory or storage or in much advanced financial investment. On the other hand if job is done beyond due date, it will be again either creates dissatisfaction among the customers or market demand has already been expired or some penalty as per condition may be imposed. In fact the lateness in job is neither beneficial for the company nor for customer as it increase the level of dissatisfaction.

The Organization of this paper is as follows:

In section 2 we provide a brief introduction about fuzzy set and Graded Mean Integration Representation Method. In section 3 we define the satisfaction level of demand maker which includes assumptions and notations. Section 4 provides a proof of the theorem on tardiness of jobs. In section 5 an algorithm is given which is applied to solve our problem. Section 6 provides a numerical illustration with linguistic variables. Concluding remarks are given in section 7.

2. Fuzzy Concept and Graded Mean Integration Representation Method

The significance of fuzzy variables is that they facilitate gradual transitions between states and consequently process a natural capability to express and deal with observation and measurement uncertainties. Since fuzzy variables capture measurement uncertainties as part of experimental data they are closer to reality than crisp variables. Mathematics based on fuzzy sets has far greater expressive power than classical mathematics based on crisp sets, its usefulness depends critically on our capability to construct appropriate membership functions for given concept in various contexts.

Chen S. H & Hsieh C.H. (1999) introduced graded mean integration representation method based on the integral value of graded mean h level of generalized fuzzy number for defuzzifying generalized fuzzy member. The generalized fuzzy number is defined as follows:

Suppose $\tilde{A}$ is a generalized fuzzy number. It is described as any fuzzy subset of the real line R, whose membership function $\mu_{\tilde{A}}$ satisfies the following conditions.

- $\mu_{\tilde{A}}$ (x) is a continuous mapping from R to [0,1],
- $\mu_{\tilde{A}}(x) = 0, -\infty < x \leq\cong a_1$,
- $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,
- $\mu_{\tilde{A}}(x) = w_A, a_2 \leq x \leq a_3$,
- $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,
- $\mu_{\tilde{A}}(x) = 0, a_4 \leq x < \infty$,

Where $0< w_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers.

Generalized fuzzy numbers are denoted as $\tilde{A}= (a_1, a_2, a_3, a_4 ; w_A)_{LR}$. The graded mean h-level value of generalized fuzzy number $\tilde{A}= (a_1, a_2, a_3, a_4 ; w_A)_{LR}$ is given by :

$(h)/2\{L^{(-1)}(h)+R^{(-1)}(h)\}$

The Graded Mean Integration Representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{w_A} \frac{h}{2}\{L^{-1}(h)+R^{-1}(h)\}\, dh}{\int_0^{w_A} h\, dh},$$

Where $0 < h \leq w_A$ and $0< w_A \leq 1$.

Let $\tilde{A}$ be a trapezoidal fuzzy number and denoted as $\tilde{A} = (a_1, a_2, a_3, a_4)$. Then we can get the Graded Mean Integration Representation of $\tilde{A}$ by the formula as:

$$P(\tilde{A}) = \frac{\int_0^1 \frac{h}{2}\{(a_1+a_4)+h(a_2-a_1-a_4+a_3)\}dh}{\int_0^1 h\, dh} = \frac{a_1+2a_2+2a_3+a_4}{6}$$

3. Satisfaction Level

As demand maker wants that the job should be completed within time but in real situation, it has been observed that the output generally falls before or after the fixed time, which creates dissatisfaction to the demand maker. The satisfaction level depends on the time how much early or later it has been reached. The satisfaction level is defined as follows:

$$\mu_i = \begin{cases} 0 & ; \text{if } x<d_{i1} \\ (x-d_{i1})/(d_{i2}-d_{i1}) & ; \text{if } d_{i1}<x<d_{i2} \\ 1 & ; \text{if } d_{i2}<x<d_{i3} \\ (d_{i4}-x)/(d_{i4}-d_{i3}) & ; \text{if } d_{i3}<x<d_{i4} \\ 0 & ; \text{if } x>d_{i4} \end{cases}$$

4. Assumptions and Notations

4.1. Assumptions:

- No job pre- emption is allowed.
- The machine can process one job at a time.
- All jobs are available at time zero.
- The machine set up time is negligible.
- Due date are in linguistic fuzzy variable.
- Fixed transportation time is taken.

4.2 Notations:

- A-**<AHR>** Average high ranking of fuzzy numbers (a, b, c, d) = (a +2b+2c+d)/6
- A_{i1} - Average high ranking of the processing time of i^{th} job on 1^{st} machine.
- A_{i2} - Average high ranking of the processing time of i^{th} job on 2^{nd} machine.
- A_{i2} - Average high ranking of the processing time of i^{th} job on 3^{rd} machine.
- P_i – Processing time of i^{th} job.

- L – Total tardiness of all the jobs.
- K- Constant factor for which total tardiness is optimized.
- d_{ij} - Due time for i^{th} job and j^{th} fuzzy time
- C_i – Completion time of i^{th} job.
- t_i – Transportation time from first machine to second machine.
- g_i – Transportation time from second machine to third machine
- μ_i – Degree of satisfaction level of i^{th} job for the demand maker.

5. Theorem On Tardiness Of The Jobs

Let Pi be the process time of i^{th} job on both machines and KA_{i1} is the due time of the job. Then tardiness of the job is $L^2 = \sum(P_i - KA_{i1})^2$ where K is the factor depending on P_i & A_i is minimum for $K = \sum \frac{P_i A_{i1}}{A_{i1}^2}$

Proof

The result can be proved by simple calculus of Maxima and Minima. Total tardiness of the jobs is the time in which the jobs are idle i.e.

$$L^2 = \sum (P_i - KA_{i1})^2$$

where i =1 to n.

On equating to zero the first derivative, we have

$K = \sum \frac{P_i A_{i1}}{A_{i1}^2}$. For the minimum value, second derivative should be positive.

$\frac{d^2L}{dK^2} = 2\sum A_{i1}^2 > 0$,being square .

L^2 is minimum for $K = \sum \frac{P_i A_{i1}}{A_{i1}^2}$

As due time depends upon the processing time of jobs on the first machine, so we desire to get the job early whose due time is early . Smaller the processing time early is the due time. Hence, the sequence of jobs can be made in non decreasing order. We summarise our work in the following Algorithm.

6.Algorithm

Step 1**:** Take fuzzy number in place of linguistic variable

Step 2: Find <AHR> of the processing time of each job in fuzzy environment

(a, b, c, d) = (a +2b+2c+d)/6 on the basis of grade mean integration representation.

Step 3: Add transportation time as follows: $A'_{i1} = A_{i1} + t_i + g_i$ & $A'_{i2} = A_{i2} + t_i + g_i$.

Step 4: Arrange the sequence in non decreasing order of A'_{i1}.

Step 5: Find the processing time P_i of each job i

Step 6: Find the value of K by using the formula

$K = \sum \frac{P_i A_{i1}}{A_{i1}^2}$

Step 7: Using the value of K, find due times of each job in fuzzy environment d_{ij} =K*processing time of i^{th} job in j^{th} fuzzy time.

Step 8: Find the completion time C_i of each job in fuzzy environment. We find the completion time C_i by using the formula $C_i = (C_{i1}+2C_{i2} + 2C_{i3} +C_{i4})/6$ on the basis of grade mean integration representation.

Step 9: On comparing completion time and due time of the jobs, the satisfaction of demand maker is determined.

7.Numerical Example:

Consider there are 5 jobs & 3 machines whose processing time is given in linguistic variables which converted into trapezoidal fuzzy number and transportation time is given from first machine to second machine & second to third machine are given

Table 1:Processing time of each job in linguistic terms

JOB	M_1	t_i	M_2	g_i	M_3
1	High Processing Time	2	Average	3	Very Low
2	Extremely high	4	High	2	Medium
3	Medium	3	Extremely low	4	Low
4	Very High	2	Medium	1	Very Low
5	Extremely high	1	Low	2	Extremely Low

Table 2: List of Fuzzy Number with respect to Linguistic Variables

S.No.	Linguistic Variable	Fuzzy Number
1	Extremely Low Processing Time	(2,4,6,8)
2	Very Low processing time	(3,5,6,7)
3	Low processing time	(5,6,8,10)
4	Average processing time	(6,7,8,9)
5	Medium processing time	(7,8,10,12)
6	High processing time	(8,10,11,12)
7	Very high processing time	(10,12,14,18)
8	Extremely high processing time	(12,13,14,15)

Our objective is to find the completion time as well as due time of each job also to find the degree of satisfaction level of each job for the demand maker.

8.Solution

Step 1**:** Substituting the value of linguistic variables in terms of trapezoidal fuzzy number as per step 1 of algorithm

Job	M_1	t_i	M_2	g_i	M_3
1	(8,10,11,12)	2	(6,7,8,9)	3	(3,5,6,7)
2	(12,13,14,15)	4	(8,10,11,12)	2	(7,8,10,12)
3	(7,8,10,12)	3	(2,4,6,8)	4	(5,6,8,10)
4	(10,12,14,18))	2	(7,8,10,12)	1	(3,5,6,7)
5	(12,13,14,15)	1	(5,8,9,10)	2	(2,4,6,8)

Step2: Find <AHR> of the processing time of each job as per in step 2

Job	1	2	3	4	5
A_1	62/6	81/6	55/6	80/6	81/6
A_2	45/6	62/6	30/6	55/6	43/6
A_3	32/6	55/6	43/6	32/6	30/6

Step 3: Add transportation time as defined in step 3

Job	1	2	3	4	5
A_1’	137/6	179/6	127/6	153/6	142/6
A_2’	107/6	153/6	115/6	105/6	91/6

Step 4: The Non decreasing sequence in which jobs are processed is 3-1-5-4-2.

Step 5: Processing time for each job is as follows:

Job			H_i	
	Time In	Time Out	Time In	Time Out
3	0	127/6	127/6	242/6
1	127/6	264/6	264/6	371/6
5	264/6	406/6	406/6	497/6
4	406/6	559/6	559/6	664/6
2	559/6	738/6	738/6	891/6

Hence P_1=371/6, P_2=891/6, P_3=242/6, P_4=664/6, P_5=497/6.

Step 6: Using the formula as per step 6, we get K=3.74.

Step 7: The list of due time in fuzzy environment as step 7 of algorithm is as follows:

d_{11}=29.9	d_{12}=37.4	d_{13} =41.14	d_{14}=44.88
d_{21}= 44.88	d_{22}=48.62	d_{23}=52.36	d_{24}=56.10
d_{31}=26.18	d_{32}=29.9	d_{33}=37.4	d_{34}=44.88
d_{41} =37.4	d_{42} =44.88	d_{43}=52.36	d_{44}=62.86
d_{51}=44.9	d_{52}=53.88	d_{53}=58.37	d_{54}=67.3

Step 8: The completion time of jobs as per step 8 of algorithm are as follows:
C_3=28.0,
C_1=37.33,
C_5=49.16,
C_4=63.8,
C_2=85.33

step 9: **Analytical Study**

(a) C_3=28 lies between d_{31}=26.18 & d_{32}=29.9, μ=.49, satisfaction level is 49%

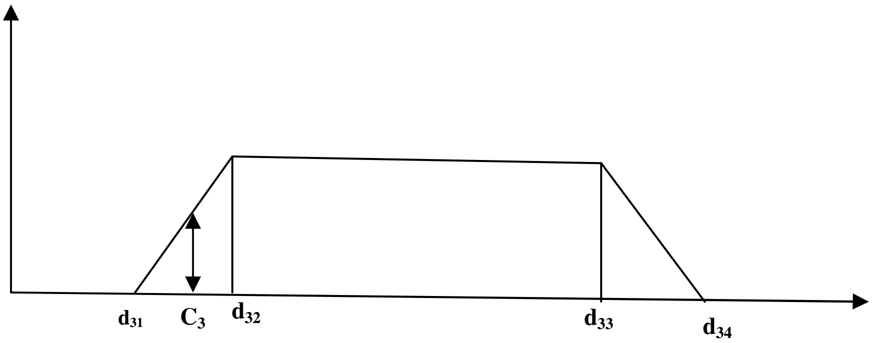

Figure 1:Analytical study of 3rd job

(b)C_1=37.33 lies between d_{11}=29.9 and d_{12}=37.4,μ= .99,Satisfaction level is 99%

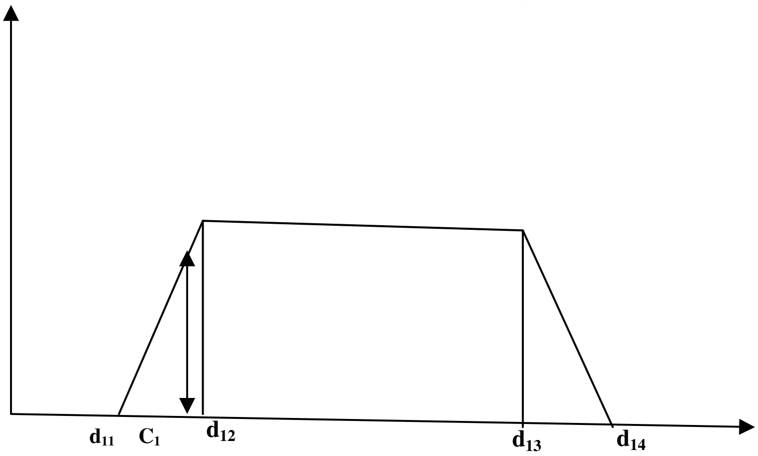

Figure 2: Analytical study of 1st job

(c) C_5 =49.16 lies between d_{52}=48.62 & d_{53}=52.36,μ=1, satisfaction level is 100%.

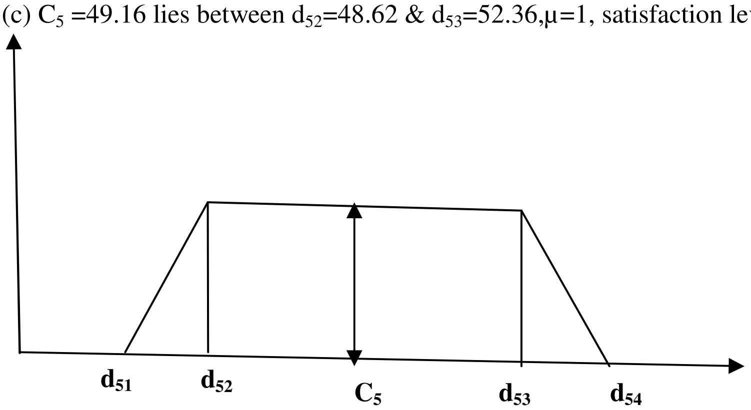

Figure 3: Analytical study of 5th job

(d)C_4=63.8 lies between d_{43} =52.36 and d_{44} =67.3, μ= .23,Satisfaction level is 23%

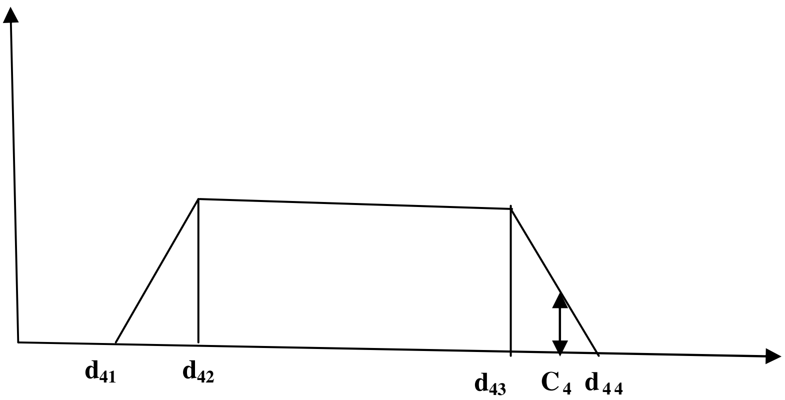

Figure 4: Analytical study of 4th job

(e)C_2=85.33 d_{24}=56.10, C_2>d_{24}, μ=0, Satisfaction level is 0%

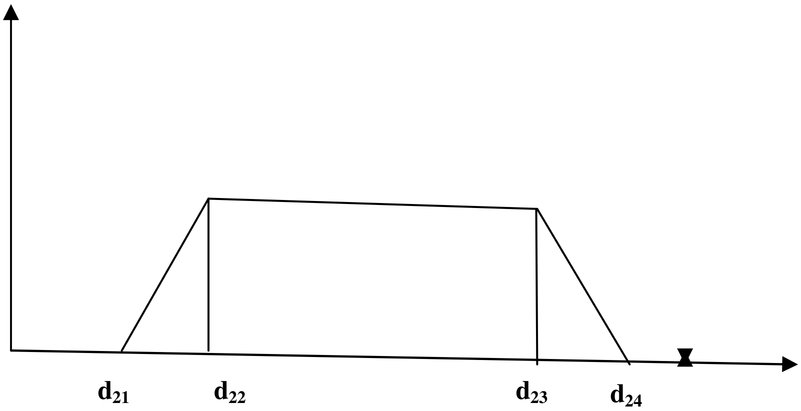

Figure 5:Analytical study of 2nd job

Concluding Remarks and Further Research Area:

In this study, a comparative analysis of completion time and due times of jobs has been calculated quantitatively and are presented graphically.

It has been shown that the satisfaction level of demand maker depends upon due date. As in our study the completion time of third job is 28 which lies between 1st and 2nd due dates of third job where satisfaction level of demand maker is 49 % and customer feels 51% dissatisfaction while in case of 5th job, whose completion time is 49.16 lying between 2nd and 3rd due dates of fifth job completely satisfies satisfaction level being 100%. Similar other cases for each jobs has been discussed. The main focus in the study is given for lateness of jobs as lateness factor is supposed to be the sign of dissatisfaction among the customers and in some organizations a penalty is imposed after crossing due date. Since flexibility or uncertainty is hidden in time factor hence fuzzy due date has been taken in account. The study can be extended to general n jobs x m machines problem.

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