

MEASURING AND ALLOCATING PORTFOLIO RISK CAPITAL IN THE REAL WORLD: PRACTICAL APPLICATION OF VALUE-AT-RISK AND EXPECTED SHORTFALL

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Abstract

Financial risk professionals are constantly interested in the risk capital allocation especially when dealing with management of portfolios under their control. This paper seeks to investigate two major risk measures namely the Value-at-Risk (VaR) and Expected Shortfall (ES) in dealing with the risk capital allocation problem. Data from the London Stock exchange was used for this study. Assuming no dividends payment, the Geometric Brownian motion (Black-Scholes Model) and a fair per-unit capital allocation principle were applied to ascertain the coherence of the two considered risk measures within a two year time horizon. It is evident from the results that stock with high mean rate of log returns and low volatility turns to have a lower fair per unit capital allocation of risk in any selected portfolio. Results of stocks with the least quantified risk (in pence) of all considered portfolios in this paper were Portfolio I (Mining: BLT - 926), Portfolio II (Media: PSON - 175), Portfolio III (Financial services: SDRC - 459), Portfolio IV (Bank: STAN - 739) and Portfolio V (FTSE 100 top 10 Companies: BATS - 1021) respectively.

Keywords: Coherent Risk Capital Allocation Value-at-Risk Expected Shortfall

1. Introduction

Financial risk professionals in their quest to boost investor and other market participant's confidence in the financial industry are always concerned with the future trends in market stocks. The problem of risk measurement and risk capital allocation is not only important but also critical to industry players.

Capital allocation is the process of distributing capital to individual functional units of a business in which capital is placed at risk for an expected return above the risk-free rate. The aim includes but not limited to profitability test of business units and the determination of business units that can be improved in order to add value to the business. Artzner *et al.*, (1999) proposed some fundamental properties that a risk measure should exhibit and called them "coherent". Risk is associated with the term "uncertainty" which is considered as an uncertain measurable random variable. The theory views risk as uncertainty regarding the future net worth of an investment portfolio or company at a specified point in time (Artzner *et al.*, 1999). Generally, financial theory suggests coherent risk measure as the one and only theoretically best way to measure risk. It measures risk as the additional amount of money needed to ensure the future net worth will fall within a predefined set of acceptable outcomes, called the acceptance set (Artzner *et al.*, 1999). Artzner (1999), Meyers (2002), and Wirch and Hardy (2002), are among several authors who have recommended the use of coherent risk measures in determining an appropriate capital requirement. In practice, there are different approaches in measuring risk. Common among them are variance, standard deviation, Value at Risk (VaR), the Proportional Hazard Transform (PHT) and Tail Value at Risk (TVaR) also referred to as Expected Shortfall (ES). Apart from the Expected Shortfall (ES) and Proportional Hazard Transform (PHT) which have proved to be coherent, the other risk measures violate one or more of the fundamental properties/axioms of coherent risk measures.

This paper examines the application of risk measures to data from the London Stock Exchange (LSE) in the determination of a coherent risk capital requirement. The London Stock Exchange (LSE) is one of the largest international financial markets. The exchange trades in equities of over 2,500 companies including some of the world's most profitable companies across 50 different countries. The exchange has traded over 300 years and currently has two markets, namely the Main Market and the Alternative Investment Market (AIM) with market capitalisations of £3.5 billion and £59 billion respectively. The companies listed on the exchange can be grouped into over 46 business sectors. In this paper we analyse and present results for a selection of The Financial Times Stock Exchange (FTSE) 100 listed companies of the LSE that are categorised in the mining, media, financial services and the banking sectors. The FTSE 100 consists of the 100 largest companies listed on the LSE in respect of market capitalisation. The mining and media services sectors comprise of about 184 and 99 companies contributing 7% and 3% to the total LSE market capitalisation respectively. A total of 47 and 179 listed companies fall under the banking and financial services sector respectively. These account for £457,048 and £39,869 respectively of the LSE market

capitalisation. By analysing data from the LSE, we hope to focus and capture those characteristics that are common in large exchanges around the world.

This paper seeks to investigate the coherence or otherwise of Value-at-Risk and Expected Shortfall and the practical application of the theory of risk measures to the allocation of risk capital to a portfolio of equities.

The remaining part of the paper is organized as follows: Section 2 describes the concept of methods employed in the research. The empirical analysis, results and discussion are presented in Section 3. Section 4 provides the concluding remarks.

2. Methodology

Real market data of different stocks prices in different sectors of the London Stock Exchange is used to measure the underlying risk of the stock and the portfolio respectively within a given time horizon (Δ). The data was downloaded from yahoo finance website (<http://uk.finance.yahoo.com>) consisting of 759 daily (2nd Dec 2008 – 2nd Dec 2011) adjusted closing stock price of companies grouped under four (4) sectors namely Mining, Media, Financial Services and Banks. A portfolio consisting of the FTSE 100 top 10 companies is also constructed. The underlying risks of investing in these stocks are calculated using Value-at-Risk and Expected Shortfall at $\alpha = 0.05$ for an equally weighted portfolio with portfolio value of £100,000. One thousand scenarios of future stock prices of each of the respective company stocks in the respective sectors are simulated two years ahead (i.e. time horizon ($\Delta = 2$)) from latest stock prices $S_j(0)$. The two widely used risk measures (Value-at-Risk and Expected Shortfall) are applied to quantify the risk associated with the portfolios. A fair risk capital allocation principle is then implemented to allocate a per-unit risk contribution (a_j) with respect to individual stocks in each of the portfolios. The portfolios are organized as follows:

- a) Portfolio I: Mining
- b) Portfolio II: Media
- c) Portfolio III: Financial Services
- d) Portfolio IV: Banks
- e) Portfolio V: FTSE 100 top 10 companies

The following chronological steps were followed in calculating the fair per unit risk capital allocations and the portfolio risk.

2.1. Coherent Risk Measures

Let M be a set of real-valued random variables representing portfolio payoffs over some fixed time interval. We define a risk measure ρ , as a real-valued function $\rho : M \rightarrow R$ mapping the payoffs to a set R of realisations. The realisation $\rho(X)$ may be interpreted as the extra minimum amount of capital that should be added to a position with payoff X in order that the position becomes acceptable to the investor. In other words, $\rho(X)$ is the amount of capital required as a cushion/buffer against the payoff X . A position with $\rho(X) \leq 0$ indicates an acceptable payoff which requires no further capital injection. Artzner *et al* (1999) defined a risk measure ρ to be "coherent" if it satisfies all the following properties:

1. **Monotonicity:** If X and Y belongs to M , and $X \geq Y$, then $\rho(X) \geq \rho(Y)$.
Thus, given two portfolios with payoffs X and Y , the required risk capital $\rho(X)$ should be more than the required risk capital $\rho(Y)$, implying that positions that lead to higher payoffs require less risk capital.
2. **Sub-Additivity:** If X, Y and $X + Y \in M$, then $\rho(X + Y) \leq \rho(X) + \rho(Y)$.
Sub-additivity reflects the fact that due to diversification effects, risk inherent in the union of two portfolios should be at most the same as the sum of the risk of the two portfolios considered separately.
3. **Positively- Homogeneous:** If $X \in M$, $\lambda > 0$, then $\rho(\lambda X) = \lambda \rho(X)$.
This means that the risk capital required does not depend on any scale change in the unit of the risk being measured. For example currency change will not affect the risk.
4. **Translation invariance:** If $X \in M$, and $\phi \in R$, then $\rho(X + \phi) = \rho(X) - \phi$.
This axiom states that by adding or subtracting a risk free portfolio (a deterministic quantity ϕ) to a portfolio with payoff X creates no change in risk. That is we only alter our capital requirement by exactly that amount (ϕ).

2.2. Definitions of Risk Measures

2.2.1. Value-at-Risk (VaR)

VaR is a statistical measure of downside risk that is easily and intuitively understood by non-specialists. It is generally defined as the risk capital sufficient to cover losses from a portfolio over a fixed holding period. The VaR can also be interpreted as a forecast of a given quantile, usually in the lower tail of the payoff distribution of a portfolio for a fixed holding period. Formally the VaR of a portfolio, at a given confidence level $100*(1-\alpha)$ where $\alpha \in (0,1)$ is defined as

$$VaR_{\alpha}(X) = -\inf\{x : P(X \leq x) \geq \alpha\} \quad (1)$$

Even though VaR is very popular in practice, it is not a coherent measure of risk as it violates the sub-additive property of coherent risk measures.

2.2.2. Expected Shortfall (ES)

Expected Shortfall (ES) or the Tail Conditional Expectation (TCE) is another informative measure of risk which estimates the potential size of the loss X^- exceeding VaR. The Expected shortfall has been proposed as an alternative measure of risk to VaR since it satisfies all the properties of coherent risk measure. It measures the expected value of a portfolio returns given that a specified threshold (usually the Value-at-Risk) has been exceeded. Basically, the expected shortfall does not depend on the definition of the quantile (which are generally not sub-additive) but rather on the distribution of payoffs and the confidence level. Given a portfolio with payoff X and expectation $E(X)$, if we assume that $E(X) < 1$ and $0 < \alpha < 1$, we define expected shortfall as

$$ES_{\alpha}(X) = -E[X | X \leq -VaR_{\alpha}(X)] \quad (2)$$

2.3. Risk Capital Allocation Principle

Let's consider an investment in a portfolio consisting of a fixed number of shares (d) with payoffs represented by the random variables X_1, \dots, X_d . Estimation of contributory risk capital which is indeed a unique fair per-unit allocation of considered investment options given a portfolio risk capital $\rho(X)$. It is shown by Tasche (2000), Aubin (1979) and Denault (2001) that, the most suitable way of finding a unique fair per-unit risk allocation is by the gradient method. The gradient method looks at the partial derivative of the underlying risk measure with respect to asset weights of the portfolio under consideration. However, not all risk measures are appropriate in allocating risk capital. Hence for any feasible risk capital allocation, the differentiability of the risk measure (ρ) must be guaranteed (i.e. the risk measure must be "sufficiently smooth" for ensuring that the respective derivative exists). The VaR and ES are considered quantile-based risk measures and not differentiable in general. Likewise the portfolio base ρ_B , will not be differentiable on R^n since the considered risk measures are all quantile-based. The persistent of this condition call for finding a suitable fair per-unit allocation of a given risk capital, else it will be meaningless since the differentiability of the portfolio base must be guaranteed. Even though the ES is not differentiable, it is an appropriate risk measure to be used for allocating risk capital as compared to VaR which is also not differentiable and also lack the sub-additivity property of a coherent risk measure. For risk measure differentiability to be achieved, at least one payoff has to possess a continuous density (Tasche, 2000). This condition puts a restriction on discrete spaces, since not all portfolio payoff distributions are differentiable in nature. (e.g. insurance claims and credit portfolios). Benfield, (2005) indicated that if the underlying payoff distribution is "sufficiently smooth", then ES is partially differentiable with respect to the exposure weights (i.e. weights of the sub-portfolio u_j).

Assuming sufficient properties, results from Tasche (2000) show that

$$\frac{\partial ES_{\alpha}}{\partial u_j}(u_1, \dots, u_n) = E[X_j | X \leq -VaR_{\alpha}(X)] \quad (3)$$

Given a portfolio base with u_B and a risk measure ρ on R^n , a per-unit allocation in $u \in R^n$ is a vector

$\{a_j \rho(u_B); u\}$ such that

$$\sum_{j=1}^n u_j a_j = \rho(u_B) \quad (4)$$

Aubin (1979), Tasche (2000) and Denault (2001), also attest the fact that, in the case of a sub-additive, 1-homogenous and coherent risk measure, differentiable at a portfolio $u \in R^n$; the gradient is the $\frac{\partial \rho(u)}{\partial u_j}$ unique fair per - unit allocation. According to Fischer et al [2003] the total risk of a portfolio (u) with respect to risk contribution per unit sub-portfolio $(u_j * e_j)_{1 \leq j \leq d}$ is given by the Euler theorem;

$$\rho(u_B) = \sum_{j=1}^n u_j * \frac{\partial}{\partial u_j} \rho(u_B) \quad (5)$$

This gives the unique fair per - unit allocation to each of the sub-portfolio $(u_j * e_j)_{1 \leq j \leq d}$.

Finding a fair per-unit capital allocation of the d stocks in a portfolio, the Brownian motion (i.e. Black Scholes model) is used in forecasting future stock prices given historical data of the d -stocks. The mean rate (μ_j) of log-returns is estimated from the historical data as well as the volatility (σ_j). The two estimates are the main parameters of the geometric Brownian motion. Simulated outcomes of the d -stocks will be the foundation of our quest of measuring the underlying risk (i.e. using VaR and ES) associated with these stocks and the allocation of risk capital in a very fair manner to the respective stocks.

3. Empirical Estimation and Forecasting

The mean rate of the log-returns and the volatility are estimated empirically from historical stock price data as follows:

$$\hat{\mu}_j = \frac{1}{M\delta t} \sum_{m=1-M}^0 \ln \left(\frac{S_{j,m}}{S_{j,m-1}} \right) \quad (6)$$

$$\hat{\sigma}_j = \sqrt{\frac{1}{M\delta t} \sum_{m=1-M}^0 \left\{ \ln \left(\frac{S_{j,m}}{S_{j,m-1}} \right) - \delta t \hat{\mu}_j \right\}^2} \quad (7)$$

where $S_j(j = 1, \dots, d; m = -M, \dots, 0)$ is the historical price of stock j at the end of the m^{th} time interval. The price process of stock j , according to the Black Scholes model is given as

$$S_j(t) = S_j(0) * \exp(\hat{\mu}_j t + \hat{\sigma}_j W_j) \quad (8)$$

where $(t = \Delta)$ is the time horizon, $\mu_j \in R$ is the drift (mean rate of log-return), $\sigma_j \in R^+$ the diffusion coefficient of the Brownian motion in the exponent (volatility of stock prices), W_j is the j^{th} Brownian motion at time (t) respectively.

The value of $S_j(\Delta)$ during the time horizon can be regarded as the sum of increases in $S_j(\Delta)$ in n intervals of δt (where $n = \Delta/\delta t$). The value of the stock price at the end of the time horizon (Δ) which cuts short of all the intermediate price processes.....

Equation (8) above can be represented by a stochastic differential equation

$$d(\ln S_j) = \hat{\mu}_j dt + \hat{\sigma}_j d(W_j) \quad (9)$$

Solving Equation (9) we obtain

$$W_j(t) = W_{j,t-0} = \frac{\log \left[\frac{S_j(t)}{S_j(0)} \right] - \hat{\mu}_j t}{\hat{\sigma}_j} \quad (10)$$

where $W_{j,t-0}$ are un-correlated normal random variables such that $W_j(0) = 0$ with $E(W_{j,t-0}) = 0$ and covariance matrix $\tilde{\Sigma}$. For square-integrable random variables $X, Y \in L^2$, the covariance is defined as the measure of dependence between two random variables X and Y whereas correlation is the standardised measure of dependence between random variables the two variables. We consider a d -dimensional Brownian motion $W_{j=1,\dots,d} = (W_1, \dots, W_d)$ where the 1-dimensional Brownian motion W_j is assumed to be standard (i.e. $W_{j,t-s} \approx N(\mu = 0, \sigma^2 = t-s)$) and correlated with covariance matrix $\Sigma = \sigma_{ij}$. Then for $t > s$ such that $W_{j,t-s} = W_j(t) - W_j(s)$, the covariance and correlation are

$$\text{Cov}(W_{i,t-s}, W_{j,t-s}) = (t-s)\sigma_{ij} \quad (11)$$

and

$$\text{Cor}(W_{i,t-s}, W_{j,t-s}) = \frac{(t-s)\sigma_{ij}}{\sqrt{(t-s)^2}} = \sigma_{ij} \quad (12)$$

respectively.

For the purposes of the simulation in this paper, we use the empirical unbiased variance estimator

$$\hat{\sigma}_{ij} = \frac{1}{M} \sum_{m=-M+1}^0 (N_{i,m} - \bar{N}_i)(N_{j,m} - \bar{N}_j) \quad (13)$$

where $\bar{N}_i = \sum_{k=1}^m N_{i,k}$ and $\bar{N}_j = \sum_{k=1}^m N_{j,k}$. When $i = j$, we have the empirical unbiased covariance estimator. We

introduce the notation $W_{j,m} = N_{j,m} * \sqrt{t}$ where $N_{j,m}$ are correlated standard normal random variables. From (10), we get the Euler-approximation

$$\ln S_{j,m} - \ln S_{j,m-1} = \hat{\mu}_j t + \hat{\sigma}_j N_j \sqrt{t} \quad (14)$$

$N_{j,m}$ can easily be deduced from (14) given historical market data and a given time horizon ($t = \Delta$). To do that we proceed by solving Equation (10) for the $M \times d$ historical uncorrelated normally distributed random variables $W_{j,m}$

$$W_{j,m} = \begin{bmatrix} W_{1,1} & W_{2,1} & \dots & W_{1,d} \\ W_{2,1} & W_{2,2} & \dots & W_{2,d} \\ \vdots & \vdots & \ddots & \vdots \\ W_{M,1} & W_{M,2} & \dots & W_{M,d} \end{bmatrix}$$

where M is the number of historical stock prices. In order to make the assets in a portfolio correlated with each other, we need to make the $N_{j,m}$ in Equation (14) correlated. The Cholesky decomposition of the covariance matrix $\tilde{\Sigma} = \text{Cov}(W_{j,m})$ is computed as follows

$\Sigma = C * C^T$
 where

$$C = \begin{bmatrix} C_{1,1} & 0 & \cdots & 0 \\ C_{1,2} & C_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{1,d} & C_{2,d} & \cdots & C_{d,d} \end{bmatrix} \quad (15)$$

is a lower triangular matrix and d is the number of stocks (assets) within each portfolio. The Cholesky decomposition to simulate correlated standard normal random variables $N_{j,m}$ is computed by simulating $k \times d$ independent identically distributed (i.i.d) standard normal random variables $Z_{j,m}$.

$$Z_{j,m} = \begin{bmatrix} Z_{1,1} & Z_{2,1} & \cdots & Z_{d,1} \\ Z_{1,2} & Z_{2,2} & \cdots & Z_{d,2} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{1,k} & Z_{2,k} & \cdots & Z_{d,k} \end{bmatrix}$$

such that $Cov(C * Z_{j,m}) = \tilde{\Sigma}$ and $E(N_{j,m}) = 0$. However such decomposition only exists if the covariance matrix $\tilde{\Sigma}$ is a symmetric definite matrix. Using $W_{j,m}$ and C we get a simulated $k \times d$ matrix of correlated standard normal random variables $N_{j,m}$ for the d stocks as

$$\begin{aligned} N_{j,m} &= [C * (Z_{j,m})^T]^T \\ &= \begin{bmatrix} N_{1,1} & N_{2,1} & \cdots & N_{d,1} \\ N_{1,2} & N_{2,2} & \cdots & N_{d,2} \\ \vdots & \vdots & \ddots & \vdots \\ N_{1,k} & N_{2,k} & \cdots & N_{d,k} \end{bmatrix} \end{aligned} \quad (16)$$

where k is the number of simulations.

Substituting $N_{j,m} \sqrt{t}$ for $W_{j,m}$ in Equation (8) we have 2-year horizon k simulated prices

$$S_{j,m}(t) = S_{j,m}(0) * \exp(\hat{\mu}_j t + \hat{\sigma}_j N_{j,m} \sqrt{t}) \quad (17)$$

for the stock j where $j = 1, \dots, d$.

The payoff distribution $X_{j,m}$ for the stock (asset) j can also be calculated as

$$X_{j,m} = S_{j,m}(t) - S_{j,m}(0) \quad (18)$$

3.1. Results and Discussion

The results for a hundred thousand pounds (i.e. £100,000) investment in each of the five (5) portfolios are summarised in tables 2 to 6 respectively. As part of the table, the mean rate of log returns and volatility of historical stock prices of the assets are also presented. The required number of investment units for each asset, a fair per-unit risk allocation and the corresponding risk capital is also presented. The Value –at-Risk and Expected Shortfall for each portfolio are also shown in each table.

The respective total risk capital allocation known as portfolio risk for the constituent assets of each portfolio is also shown. A numerical ranking (with 1 indicating a less risky asset) based on the quantified risk of each stock in each portfolio is presented. Figures 1 to 5 show unitised stock price movements for each stock. It is the plot of the ratio

$\frac{S_{j,m}}{S_{j,-M}}$ ($j = 1, \dots, d : m = -M + 1, \dots, 0$) of historical stock prices to the initial stock price value. It shows the

deviation of the stock price process from its initial value. For each of tables 2 to 6 we make the following definitions:

Mean rate (drift) of log returns for stock j , μ_j , volatility of stock j , σ_j , last stock price of stock j , $S_j(0)$, number of units allocated to stock j , u_j , per unit risk allocation of stock j , a_j , risk capital allocation for stock j , $u_j * a_j$, risk ranking for stock j , RR , portfolio risk, $\rho(u_B)$.

Intuitively, if it is assumed that all assets in a portfolio have the same volatility, then an asset with higher returns would require a lower risk capital allocation. Theoretically Fund managers expect to have higher returns on their investments with a lower volatility in order to improve portfolio payoff. However in practice this assumption does not hold as the volatility of a stock in a portfolio cannot remain constant irrespective of its price history. In all the five portfolios considered, the portfolio risks were consistent with the Expected Shortfall estimate unlike the Value-at-Risk which underestimates the true risk of the entire portfolios. This is seen through tables 1 to 5.

From table 6, which constitutes the top ten (10) FTSE 100 companies, the results are not only intuitive but very simple and definite. BATS in the portfolio of the top 10 FTSE 100 companies had the highest mean rate of log returns of 0.7275 and 0.5812 as its volatility. Even though the volatility was higher, it is compensated by its apparently higher mean rate of log returns. HBSA on the other hand has a mean rate of log returns -0.0403 and 0.3921 as its volatility. Among the stocks considered in portfolio V, BATS ranked first as a stock with lowest risk with HBSA ranking last, being the worse stock in the portfolio.

Also in table 5 (i.e. Portfolio IV), STAN and LLOY were the stocks with the lowest and highest risk in the portfolio. STAN had the highest mean rate of log returns of 0.2555 whereas LLOY was -0.6218. The volatility of LLOY even though not the highest in the portfolio has a correspondingly lower mean rate of log return making it the most risky ranked asset. The results so far show STAN as the best stock to invest in as compared to the considered stocks in the portfolio.

It is also evident in table 4 (i.e. Portfolio III); that SDRC is the best asset as IAP is the worse asset in the portfolio. The mean rate of log return of SDRC is 0.2112 with 0.3661 as the volatility. On the other hand, IAP, the worse asset in the portfolio has a mean rate of log return of 0.1027 and 0.454 respectively.

Last but not the least, PSON and ITV were the good and worse assets respectively in Portfolio II as shown in table 3. The mean rate of log returns of PSON and ITV were 0.2088 and 0.2165 respectively. However, the volatility of PSON is 0.2454 whereas ITV is 0.5184 which is significantly higher. PSON parameter looks more stable than ITV.

Lastly, in the case of the first portfolio as in table II, BLT, the safest asset in the portfolio has a mean rate of log returns of 0.3100 and the worse asset, XTA as 0.3094. Volatility of BLT is lower as compared to XTA. BLT volatility is 0.4268 which is lower than that of XTA which is 0.6111.

4. Conclusion

The mean rate of log returns and volatility of an asset remains the two main parameters of the Black Scholes model. Their variations therefore have different effect on the type of risk measure being used to estimate the portfolio risk. Consistently, it is evident throughout tables 2 to 6, that the Value-at-Risk underestimates the true portfolio risk unlike the Expected Shortfall whose estimate of the risk is the same as that of the portfolio risk. The fair per unit risk capital allocation estimate associated with the respective stocks in all the five portfolios considered in this paper adds up to the portfolio risk. This principle is also supported by the sub-additivity property of coherent risk measures. Expected Shortfall is a coherent risk measure unlike Value-at-Risk which violates the sub-additivity property. It is therefore evident that Value-at-Risk violates the sub-additivity property as shown consistently in tables 2 to 6. Hence, the usage of Value-at-Risk as a risk measure by practitioners should serve as a rough estimate in taking decision rather than as a true risk estimate. The Expected Shortfall as a risk measure apart from being coherent also estimates the true portfolio risk as seen in tables 2 to 6.

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Table 1: List of Companies

Sector	Company Name/Ticker	Sector	Company Name/Ticker
Mining	Anglo American plc (AAL)	Banks	HSBC Holding plc (HSBA)
	Rio Tinto plc (RIO)		Barclays plc (BARC)
	BHP Billiton Group plc (BLT)		Lloyds Banking Group plc (LLOY)
	XSTRATA plc (XTA)		Royal Bank of Scotland Group plc (RBS)
	Vedanta Resources plc (VED)		Standard Chartered plc (STAN)
Media	WPP plc (WPP)	FTSE100 Top 10 companies	British American Tobacco plc (BATS)
	Reed Elsevier plc (REL)		HSBC Holding plc (HSBA)
	Pearson plc (PSON)		BG Group plc (BG)
	ITV plc (ITV)		BHP Billiton Group plc (BLT)
	British Sky Broadcasting Group plc (BSY)		BP Group plc (BP)
Financial Services	ICAP plc (IAP)		GlaxoSmithKline plc (GSK)
	Investec plc (INVP)		Royal Dutch Shell plc 'A' (RDSA)
	Schroders plc (SDR)		Unilever plc (ULVR)
	Schroders plc (SDRe)		Rio Tinto plc (RIO)

Table 2: Parameter Estimate and Risk Measure for Portfolio I

STOCK	β_i	α_i	$S_i(0)$	u_i	a_i	$u_i * a_i$	RR_i
AAL	0.2432	0.4912	2,487.00	8	1576.73	12,680	3
RIO	0.3221	0.5363	3,346.50	6	2112.56	12,625	2
BLT	0.3100	0.4268	2,000.50	10	925.91	9,257	1*
XTA	0.3094	0.6111	1,036.00	19	733.69	14,164	5**
VED	0.2307	0.5723	1,086.00	18	768.86	14,159	4
$B(u)$	62,885						
VaR	53,112						
ES	62,885						

* - (Portfolio Stock with lowest risk) ** - (Portfolio stock with highest risk)

Table 3: Parameter Estimate and Risk Measure for Portfolio II

STOCK	β_i	α_i	$S_i(0)$	u_i	a_i	$u_i * a_i$	RR_i
WPP	0.2236	0.3198	673.50	30	219.65	6,523	3
REL	-0.0016	0.2527	524.00	38	219.29	8,370	4
PSON	0.2088	0.2456	1,143.00	17	174.69	3,057	1*
ITV	0.2165	0.5184	65.20	307	35.28	10,823	5**
BSY	0.1958	0.2657	764.50	26	136.25	3,564	2
$B(u)$	32,337						
VaR	23,420						
ES	32,337						

* - (Portfolio Stock with lowest risk) ** - (Portfolio stock with highest risk)

Table 4: Parameter Estimate and Risk Measure for Portfolio III

STOCK	β_i	α_i	$S_i(0)$	u_i	a_i	$u_i * a_i$	RR_i
IAP	0.1027	0.4543	369.00	68	216.73	14,684	4**
INVP	0.6120	0.7200	372.50	67	194.49	13,053	3
SDR	0.1898	0.3718	1,383.00	18	617.22	11,157	2
SDRC	0.2112	0.3661	1,142.00	22	458.95	10,047	1*
$B(u)$	48,941						
VaR	37,460						
ES	48,941						

* - (Portfolio Stock with lowest risk) ** - (Portfolio stock with highest risk)

Table 5: Parameter Estimate and Risk Measure for Portfolio IV

STOCK	β_i	α_i	$S_i(0)$	u_i	a_i	$u_i * a_i$	RR_i
HBSA	-0.0403	0.3921	511.20	39	342.77	13,410	2
BARC	0.0742	0.7622	190.65	105	160.64	16,852	3
LLOY	-0.6218	0.8849	25.39	788	24.53	19,320	5**
RBS	-0.3614	0.9465	21.58	927	20.54	19,041	4

STAN	0.2555	0.4306	1,452.50	14	739.16	10,178	1*
$B(u)$	78,800						
VaR	73,246						
ES	78,800						

* - (Portfolio Stock with lowest risk) ** - (Portfolio stock with highest risk)

Table 6: Parameter Estimate and Risk Measures for Portfolio V

STOCK	β_i	α_i	$S_i(0)$	u_i	a_i	$u_i * a_i$	RR_i
HBSA	-0.0403	0.3921	511.20	20	272.67	5,334	10**
BATS	0.7275	0.5812	2,956.50	3	-1021.41	-3,455	1*
BG	0.1659	0.3475	1,364.00	7	460.22	3,374	7
BLT	0.3100	0.4268	2,000.50	5	624.36	3,121	6
BP	0.0100	0.3181	464.80	22	195.13	4,198	9
GSK	0.0791	0.2447	1,421.50	7	228.18	1,605	4
RDSA	0.1488	0.2527	2,239.00	4	571.98	2,555	5
ULVR	0.1193	0.2238	2,102.00	5	212.14	1,009	2
RIO	0.3221	0.5363	3,346.50	3	1,368.96	4,091	8
VOD	0.1128	0.2442	172.10	58	21.15	1,229	3
$B(u)$	23,061						
VaR	13,100						
ES	23,061						

* - (Portfolio Stock with lowest risk) ** - (Portfolio stock with highest risk)

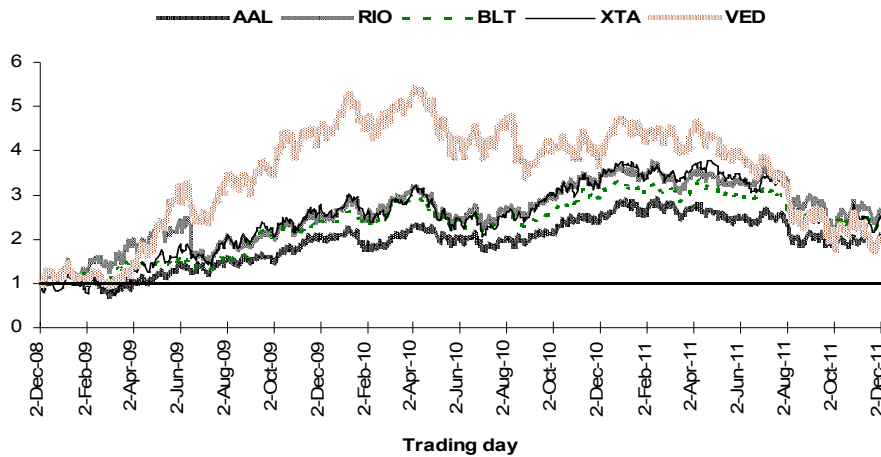


Figure 1: Unitised Chart for Portfolio I

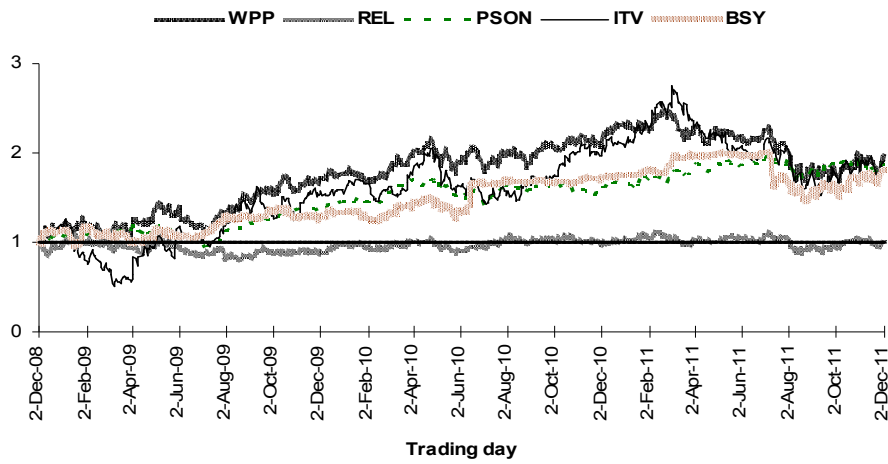


Figure 2: Unitised Chart for Portfolio II

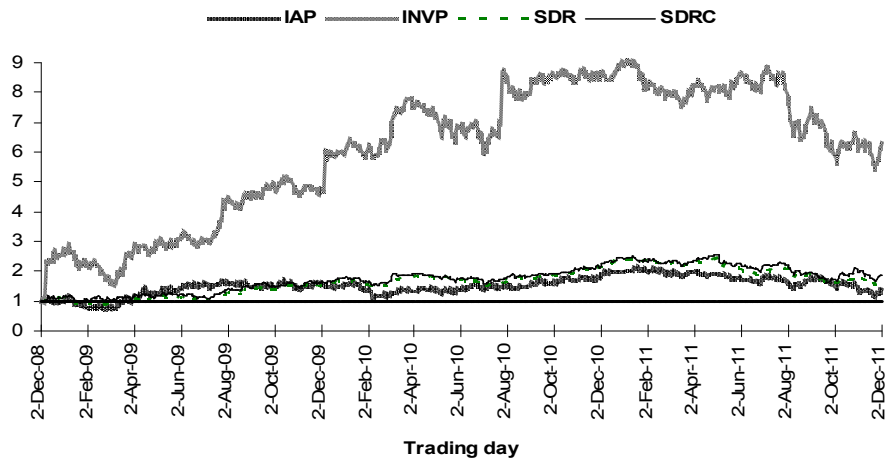


Figure 3: Unitised Chart for Portfolio III

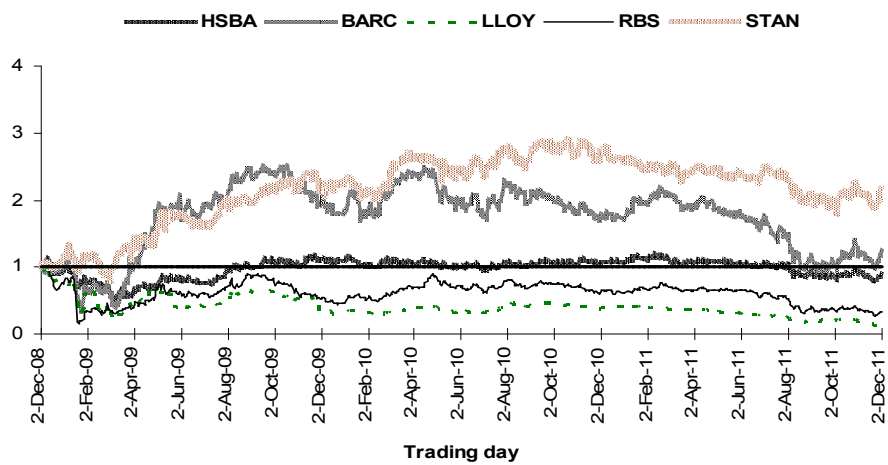


Figure 4: Unitised Chart for Portfolio IV

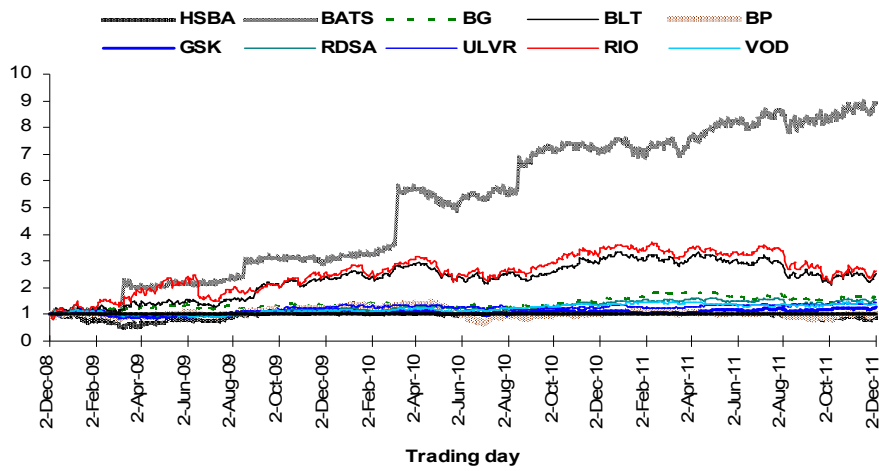


Figure 5: Unitised Chart for Portfolio V