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A Novel Approach to Construct Portfolio that Addresses Variance of Utility Function

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Abstract
Many studies in the area of portfolio selection have done based on trade-off among various moments especially between mean and risk of sample returns. Merton (1980) argued that the instability of portfolio weights and sampling errors are due more to estimate the amount of mean. Indeed, it is difficult to estimate the expected return from time series of realized expected return. By this paper, we first answer to this question how to eliminate the wrong effect of mean sample returns instead of ignoring it from calculating portfolio weights and how would be the relation of other moments after this consideration. Second, the substantial evidence from experiments shows that hide information exposes ambiguity aversion to investor's behavior, and hence decision making under ambiguity for portfolio choice has led to improving tractability of the main features of asset returns. By considering the volatility of utility preferences as ambiguity then individuals prefer to stabilize their utility preferences to maximize their expected utility.
Keywords: utility function, portfolio selection, risk aversion, ambiguity aversion

1. Introduction

So far, the idea of portfolio selection and diversification has been exhausting in the explaining and understanding of risk and return for making a decision by maximizing the expected utility function. tfolio theory (MPT) or mean–variance optimization problem (MVO). He provided a suggestion that inveMarkowitz (1952) provided the theory of portfolio selection which it is popularly called as a modern por stors should consider jointly return and risk on the basis of a trade-off between their security returns and risk to determine how to allocate their funds among investment choices. He assumed that utility of terminal wealth is well approximated by a two Taylor expansion. Hence, applying the expected operator (E[X]), one can approximate the expected utility which is derived from an investment in risky assets by the first two moments of portfolio return distribution. As a result, the portfolio selection is a trade-off between expected return and risk.

Many researchers tried to extend the classical mean-variance framework to higher moments. Scott & Horvath (1980), Jean (1971) and Harvey et al. (2004) considered the impact of a higher-order moment of asset returns on portfolio selection. They reported that the investors have preferences for higher odd and lower even moments. In particular, Harvey & Siddique (2000) found that the investors are willing to accept lower expected return and higher volatility in exchange for higher skewness and lower kurtosis.

A considerable effort has been devoted to the solving portfolio problems and estimating the parameter values of them. While these parameter values are estimated from time series sample of past returns, extreme various portfolio weights will be obtained over time then this provides unstable portfolios and performs poorly out of sample. Although a vast literature is played to handle estimation error of moments, minimum-variance portfolio surprisingly outperforms other portfolios and it has a highest sharp ratio (Jorion 1986; Merton 1980; Jagannathan & Ma 2003). They explore the estimation error in sample mean is so larger than variance which ignoring the mean improves the portfolio performance.

The classical portfolio optimization model hypothesizes that the investors are perfectly aware of their preferences by a utility function, therefore, they maximize expected utility function. However, some studies show that this is Incompatible with actual choices. One in particular surveys on decision-making under ambiguity aversion because of poorly performance in actual choices and dissatisfaction of expected theory framework introduced by Von Neumann & Morgenstern (1944) and earlier made by Denial Bernoulli in 1738, which individual welfare can be measured by computing the expected utility. The linearity (affine transformation) of expected utility function with respect to probability and risk preferences implies that the expected theory is neutral with any uncertainty about probability and risk preferences effect. By looking historical data, the investor may become confident about forecasting returns but there is some hidden information that would affect the quality of judgment, therefore, the investors will consider it as ambiguous which makes different ambiguous permia from risk premia. The more recent general model of expected theory under ambiguity introduced by (Epstein & Schneider (2008), Epstein & Schneider (2010), Ju & Miao (2012) and Gilboa & Marinacci (2011)). Under the expected utility theory all idiosyncratic shocks will wash away by well diversification and in this framework investors like expected portfolio returns and dislike variance of portfolio returns which the effect of imposing this model often perform poorly out of sample but in the general and more real preferences model, this no longer holds and well diversified portfolio may collapse (see, Klibanoff et al. (2005), Easley & O'Hara (2009) and Maccheroni et al. (2013)). As a result, ambiguity does matter then this paper wants to capture ambiguity aversion through smoothing the preferences utility function.

Our intention with this paper is not to explain and develop the mean-variance or higher moments framework and improve the estimation of moments neither. We think it is reasonable for assessing the impact of asset returns on portfolio selection, decrease the effect of expected return restriction from models and do consider other information. We thus consider that one can approximate the utility of terminal wealth through a two Taylor expansion, by applying the variance operator (Var[x]) to both sides of the equation, we can approximate the expected variance of utility. We see that risky assets may be approximated by the derivatives of the variance of the utility function. Hence, portfolio optimization will be a minimization variance of the utility function.

This paper studies the question to what is the trade-off between moments when we use the variance of utility instead of expected utility. First, we answer theoretically to this question. Then, we compare them to a possible explanation for using sample moments. To better understand why the aversion coefficient of portfolio selection computed from expected utility is different among moments, we will describe mean-variance portfolio through Taylor expansion. If the fluctuation in portfolio weight is due almost to estimation error in the first sample moment, then one should employ the model which does not corporate it. Then perhaps investors should consider more confidently the values of no mean framework such as the approach that we explain.

The paper is categorized as follows. Section 2 describes the concept of portfolio selection in different models and derives the model from the variance of utility. Section 3 discusses the differences among these approaches.

2. The Models Description

In this section, we briefly discuss a set of portfolios framework including Mean-Variance portfolio, Global Minimum-Variance and then present our model which we call it “variance-skewness portfolio”. Therefore, we answer the question raised in the introduction.

2.1. Mean-variance portfolio

Markowitz (1952) gives a model that the investor obtains efficient frontier which is the efficient trade-off between return and the risk of diversified portfolios. The investor can reduce only unsystematic risk through diversification, but systematic risk cannot be moderated in this approach because it is unpredictable. In Markowitz’s seminal paperwork, he minimizes the amount of risk portfolio for a given portfolio expected return, which is called as the mean-variance framework. The following formulation can express this:

$$\begin{aligned} &\min_{w}\ w'\Sigma w \\ &\text{s.t: } w'\mu \geq \mu_0 ;\ w'\iota=1,\ \iota'=[1,1,...,1] \end{aligned} \qquad (1)$$

here, $w=(w_1,w_2,...,w_n)'$ is the weight vector of N risky assets, Σ is an $N \times N$ covariance matrix of returns between N risky assets, $\mu=(\mu_1,\mu_2,...,\mu_N)'$ is the vector of expected returns, μ_0 is the target expected return. For the single-period framework, a rational investor with U a utility function and W_0 initial wealth chooses his portfolio to maximize his expected utility. At the end of period, his wealth becomes:

$$W=W_0(1+w'\mu) \qquad (2).$$

Let λ denote an investor risk-aversion coefficient. Under the assumption that an investor’s utility function is given by quadratic utility function (that is, asset returns are fully described by mean and variance), the expected utility of terminal wealth can be approximated through a second-order Taylor expansion such that the following equation holds:

$$E[U(W)] \approx U(E[W])+\frac{U^{(2)}(E[W])}{2}(E[(W-E(W)^2)]) \qquad (3)$$

here, $U^{(i)}(E[W])$ denote ith-order derivative of the utility function, where W is the terminal wealth of investor. Then, by considering CRRA[1] investors, define $\lambda=(-W*U^{(2)}(E[W]))/U^{(1)}(E[W]);\ \Sigma=E[(W-E(W)^2)]$. Finally, it can be shown that the Markowitz’s model can be written as following:

$$\begin{aligned} &\max_{w}\ (w'\mu-\frac{\lambda}{2}w'\Sigma w) \\ &\text{s.t: } w'\iota=1,\ \iota'=[1,1,...,1] \end{aligned} \qquad (4)$$

for a general utility function, the above problem will no longer be expressed by the Markowitz framework which is the trade-off between risk and return.

[1] Constant Relative Risk Aversion

2.2. Global Minimum-variance Portfolio

In this model, it only considers variance of historical past return of assets to maximize the expected utility. As we have noted this model ignores the mean of sample return and chooses a set of assets which minimize the variance of returns.

$$\min_{w} \ (w'\Sigma w) \quad \text{s.t: } w'\iota=1,\ \iota'=[1,1,...,1] \tag{5}$$

2.3. Variance-Skewness Portfolio Model

Suppose that the price changes in excess of the risk-free rate are independently and identically distributed with mean vector μ and define Σ as the matrix of covariance of asset returns. We will construct the expected volatility utility of terminal wealth by:

$$\mathrm{Var}(U(W))=E[(U(W)-E[U(W)])^2]=E[U^2(W)]-E[U(W)]^2 \tag{6}$$

We then minimize this variance of utility to better diversify efficient portfolios from sample moments. This problem can become even more well-diversified portfolios because the extreme behavior of the weights is more due to the estimation of the sample first moment which obviously disappear from our analysis in the following calculations.

First, for calculating $E[U(W)]^2$ let $\mu^{(i)}$ denote *i*th central moment. The following equation holds if we approximate the expectation of utility wealth by a second-order Taylor expansion at $\mu=E(W)$:

$$E[U(W)] \approx U(E[W])+\frac{U^{(2)}(E[W])}{2}\mu^{(2)} \tag{7}$$

multiply above equation by itself to get:

$$E[U(W)]^2 \approx (U(E[W]))^2+U^{(2)}(E[W])\mu^{(2)}U(E[W])+\frac{(U^{(2)}(E[W]))^2}{4}(\mu^{(2)})^2 \tag{8}$$

Then, similarly we take the first term $E[U^2(W)]$ by implying a second-order Taylor expansion for utility function at $\mu=E(W)$ gives:

$$U(W) \approx \frac{(W-E(W))^0}{0!}U(E(W))+\frac{(W-E(W))^1}{1!}U^{(1)}(E(W))+\frac{(W-E(W))^2}{2!}U^{(2)}(E(W)) \tag{9}$$

multiply above by itself to get

$$U(W)^2 \approx \left[\frac{(W-E(W))^0}{0!}U(E(W))+\frac{(W-E(W))^1}{1!}U^{(1)}(E(W))+\frac{(W-E(W))^2}{2!}U^{(2)}(E(W))\right]^2 \tag{10}$$

applying both sides by expected operation to get

$$\begin{aligned} E[U(W)^2] \approx\ & E[U(\mu)^2+2U(\mu)(W-\mu)U^{(1)}(\mu)+(W-\mu)^2(U^{(1)}(\mu))^2 \\ & +(U(\mu)+(W-\mu)U^{(1)}(\mu))(W-\mu)^2U^{(2)}(\mu)+\frac{1}{4}(W-\mu)^4(U^{(2)}(\mu))^2] \end{aligned} \tag{11}$$

Finally, we can use equation (8) and (11) to construct equation (6):

$$\begin{aligned} \mathrm{Var}(U(W)) &= E[(U(W)-E[U(W)])^2]=E[U(W)^2]-E[U(W)]^2 \\ &= [(U^{(1)}(\mu))^2]\mu^{(2)}+U^{(1)}(\mu)\,U^{(2)}(\mu)\mu^{(3)}+\frac{1}{4}(U^{(2)}(\mu))^2\mu^{(4)}-\frac{1}{4}(U^{(2)}(\mu))^2(\mu^{(2)})^2 \end{aligned} \tag{12}$$

if suppose the investors have CRRA preferences with risk aversion parameter λ , for example, let define $U=\frac{W^{1-\lambda}}{\lambda}$ be utility function for CRRA investor. Then higher-order moment tensors can easily parametrize portfolio moments as:

$$\begin{aligned} \mu^{(2)} &= w'\Sigma w \\ \mu^{(3)} &= w'M_3(w\otimes w) \\ \mu^{(4)} &= w'M_4(w\otimes w\otimes w) \end{aligned} \tag{13}$$

note that

$$\begin{aligned} M_3 &= E[R-E[R]]\otimes E[R-E[R]]\otimes E[R-E[R]]\ , \\ M_4 &= E[R-E[R]]\otimes E[R-E[R]]\otimes E[R-E[R]]\otimes E[R-E[R]] \end{aligned} \tag{14}$$

by taking the initial wealth as a numeraire, the following explanation can be suggested by our analysis:

$$U^{(1)}(W)=1$$
$$U^{(2)}(W)=-\lambda$$
$$U^{(3)}(W)=\lambda(\lambda+1)$$
$$U^{(3)}(W)=-\lambda(\lambda+1)(\lambda+2) \quad (15)$$

therefore, we can rewrite the investor optimization's problem as a minimizing the following portfolio moments:

$$\mu^{(2)}-\frac{(\lambda)^2}{4}(\mu^{(2)})^2+\lambda\mu^{(3)}+\frac{(\lambda)^2}{4}(\mu^{(4)}) \quad (16)$$

which clearly removes the first moment impact of sample return in trade-off among moments.

Interestingly, it shows that if only the sample second moment is between zero and $(\lambda)^2/4$, the investor are willing to accept higher variance in exchange of lower skewness and higher kurtosis, otherwise vice versa. As compared to consequences portfolio constructed using mean-variance framework, only when the sample second moment is bounded between zero and $(\lambda)^2/4$, we find a consistent result. This behavior unlikely can explain why optimized portfolios are not optimum.

3. Methodology

Our methodology, which is motivated by the article from Merton (1980), is on three important dimensions. First, we describe how to measure the outperformance of proposed model. This model has some significant advantages. Specifically, it avoids estimation of the first moment of past sample returns, it provides a way to evaluate the investor portfolio selection problem which assumes the sample mean of returns estimation fluctuate a lot when we rebalance the portfolio according to some investigates as the same as Merton (1980) and Jagannathan & Ma (2003). Although there has been considerable effort to improve the estimation of the expected return such as using Bayesian estimation, robust optimization and option-implied information, the estimation of the expected retunes in empirical and simulation-based analysis is poorly behaved and needs very long time series data.

Second, we conclude this section with a discussion on the relation between expected utility and variance of expected utility objective function. We focus on minimizing the expected variance utility function as it avoids the expected return than maximizing the mean-variance models because the estimation error would result in extreme rebalancing portfolios even run the model by robust estimation.

Finally, we are tremendously interested to discover the behavior of portfolio asset when we bound variance in the posited area of the variance-skewness portfolio with empirical data and compare the result with mean-variance portfolio. We collect monthly returns stocks listed on the Center for Research in Securities Prices (CRSP), from the period January 1971 to December 2010. This gave us over 200-month return series which from this estimated sample mean, variance and higher order moments.

3.1. Performance evaluation

We consider an economy with the R returns vector of N different risky assets. Let M_i denote ith higher order moment tensor for the assets which is introduced by Jondeau & Rockinger (2003) using Kronecker product as equation (17). The investor's terminal wealth can be defined such that equation (2) and considered the initial wealth as a numeraire. Then, the central moments of portfolio returns can satisfy equation (18).

$$M_i=E[R-E[R]]^{\otimes i} \;;\; i>1 \quad (17)$$
$$\mu^{(i)}=w'M_i w^{\otimes(i-1)};\; i>1 \quad (18)$$

we can rewrite the investor optimization's problem as a function portfolio of weight vector with two first moment tensors (only trade-off between variance and skewness):

$$\min_{w}\; w'\Sigma(w)-\frac{(\lambda)^2}{4}(w'\Sigma(w))^2+\lambda w'M_3(w\otimes w) \quad (19)$$

which define the trade-off between variance and skewness. To analyze the out-performance of constructed portfolio, we compare certainty equivalents for an investment in different competing portfolios. Differentiating the above objective function with respect to (w) gives optimum weight values of portfolio:

$$2\Sigma w-(\lambda)^2\Sigma w(w'\Sigma w)+3\lambda M_3(w\otimes w)=0 \quad (20)$$

write the third order moment tensor for n assets:

$$M_3=[S_1|S_2|\cdots|S_i|\cdots|S_n] \quad (21)$$

then the above equation is equivalent to:

$$[2\Sigma-(\lambda)^2\Sigma(w'\Sigma w)+3\lambda(\sum_{i=1}^{n} w_i S_i)]w=0$$

or (22)

$$[2\Sigma-(\lambda)^2\Sigma(w'\Sigma w)+3\lambda w' S]w=0$$

so, the explicit solutions can be written as following:

$$(\lambda^2 w'\Sigma w\text{-}2)=\frac{3\lambda M_3(w\otimes w)}{\Sigma w}$$

$$w=\{w|(\lambda^2 \text{var-}2)=\frac{3\lambda \text{ skew}}{\text{var}};\ \text{var}=w'\Sigma w,\ \text{skew}=w'M_3(w\otimes w)\ \} \quad (23)$$

$$w=\frac{(\text{-}3M_3 \pm \sqrt{(3M_3)^2+8\Sigma^3})\Sigma^{-2}}{\text{-}2\lambda} \text{ or } w=0$$

the second optimum solution can help to answer the question of DeMiguel et al. (2009) "How Inefficient is the 1/N Portfolio Strategy?" because zero matrix gives a kind of naïve diversification, then maybe we can say naïve diversification is so close to an optimum solution.

3.2. The role of avoiding the mean on diversification

Based on our strategy, we can now formally say that variance-skewness portfolio optimization (VSO) is well diversified at skewness, therefore, we can make portfolio diversification based on considering jointly securities skewness and their co-movements. While it is difficult to make an accurate estimation of return due to the direct impact of idiosyncratic volatility on the first moment of individual security rather than other moments, the estimation error of mean significantly moves portfolio weights from optimum one. We conclude from our obtained objective function (equation 18) that if we decrease the variance of the portfolio, in fact, we increase proportionally the variance of expected utility it means that we make worse our utility portfolio.

This portfolio is the Global Minimum Variance of Utility portfolio (GMVU) which can be formulated by the optimization portfolio

$$\min_{w} \text{VarE(U)}=w'\Sigma(w)\text{-}\frac{(\lambda)^2}{4}(w'\Sigma(w))^2+\lambda w'M_3(w\otimes w)$$

S.T: (24)

$$w'i=1;\ i=[1,\cdots,1,1]$$

refer to GMVU we can obtain the efficient frontier of skewness and variance which is totally different with the efficient portfolio of risk-return. Another efficient portfolio that we can introduce is an efficient portfolio of expected variance and return of utility which we call it EVU. We can formulate it as following

$$\min_{w} \text{VarE(U)}$$

S.T:

$$E(U) \geq M$$

$$w'i=1;\ i=[1,\cdots,1,1]$$

(25)

3.3. Estimation of moments

Our goal is to analysis the performance of our model compared to a benchmark portfolio on the asset allocation of the data set. In order to improve the result of our model, we need to mitigate the impact of estimation error in portfolio optimization which it increases exponentially with the number of risky assets. Following the literature on improved estimation method, the shrinkage estimators are the most effective approach suggested by Ledoit & Wolf (2004) and Martellini & Ziemann (2010) for covariance, skewness and kurtosis respectively, gives better performance than original sample estimator and easy to implement which gives us more motivation to consider these estimators. They define the posterior misspecification function of convex linear combination estimator as:

$$L(\delta)=\|\delta F+(1\text{-}\delta)S\text{-}\Omega\|^2 \quad (26)$$

here, δ is the shrinkage intensity which is between 0 and 1; F the shrinkage target which we estimate by the sample constant correlation approach, S the sample estimator and Ω is the true moment tensor matrix. Note that Frobenius norm of a matrix is defined as

$$\|s\|^2=\sum_i s_i^2 \quad (27)$$

by finding the optimum shrinkage intensity, the expected value of loss will be minimized and asymptotically behave like a constant over time period T . This optimal value can be written

$$\delta^* = \frac{1}{T}\frac{\pi-\rho}{\gamma} \quad (28)$$

where, π denotes an asymptotic tensor moment of the sample estimator, ρ represents the asymptotic tensor moment between the sample and structured estimator and γ represents misspecification of the structured estimator. Then, the shrinkage estimators are calculated by:

$$\delta^* F + (1-\delta^*) S \quad (29)$$

We now compare the out-of-sample performance of our portfolio with benchmark portfolio (Markowitz portfolio model) using “rolling sample-horizon” procedure by the historical market dataset. We chose window estimation (M) less than a total number of observations (T) then compute the portfolio's return over the estimation window(t=1,2,...,M). To calculate the out-of-sample portfolio return ($\mu_{t+1}=w_t' R_{t+1}$) of each model, we should estimate the portfolio weight (w_t) by considering the approach of any strategy. We repeat it by including next period and dropping the beginning period until the end of the observed dataset. Finally, in order to assess performance of each models following DeMiguel et al. (2009) we measure the out-of-sample sharp ratio, certainty-equivalent (CEQ) return, turnover and wealth for each model which is defined by:

A: Sharp Ratio

$$SR = \frac{\hat{\mu}}{\hat{\sigma}} \quad (30)$$

which,

$$\hat{\mu} = \frac{1}{T-M}\sum_{t=M}^{T-1} (w_t' R_{t+1})$$
$$\hat{\sigma}^2 = \frac{1}{T-M-1}\sum_{t=M}^{T-1} (w_t' R_{t+1} - \hat{\mu})^2 \quad (31)$$

B: Certainty-Equivalent return

$$CEQ = \hat{\mu} - \frac{\lambda}{2}\hat{\sigma}^2 \quad (32)$$

C: Turnover

$$TRO = \frac{1}{T\text{-}M}\sum_{t=1}^{T\text{-}M}\sum_{j=1}^{N} \left(\left|w_{j,t+1} - w_{j,t}\right|\right) \quad (33)$$

D: Wealth-Loss

$$WLR = R-(1+R)*CT$$

$$\text{cost of trade(CT)} = c*\sum_{j=1}^{N}\left(\left|w_{j,t+1} - w_{j,t}\right|\right)$$

Which, c denotes proportional cost of trade.

E: Utility ratio

$$UR = \frac{E[U(W)]}{V(E[U(W)])}$$

$$E[U(W)] = U(E[W]) + \frac{U^{(2)}(E[W])}{2}\mu^{(2)} = w'\mu - \frac{\lambda}{2} w'\Sigma w$$

$$V(E[U(W)]) = \mu^{(2)} - \frac{(\lambda)^2}{4}(\mu^{(2)})^2 + \lambda\mu^{(3)} + \frac{(\lambda)^2}{4}(\mu^{(4)}) = w'\Sigma(w) - \frac{(\lambda)^2}{4}(w'\Sigma(w))^2 + \lambda w' M_3 (w \otimes w)$$

4. Data and Empirical results

In this section, we examine the performance of our constructed portfolio using returns from Center for Research in Security Prices (CRSP) monthly returns data.

A. Data

We consider the sample period January 2004 to October 2013. We select top 7 different firms from CRSP database, as seen in table *, and then we collect monthly returns for these stocks from January 2004 to October 2013. As a result, we obtain valid monthly returns of 7 socks for 118 periods. We first set window estimation M=60 and then to measure the stability of each portfolio we estimate moment and comoments parameter which obviously are not known by using (Martellini & Ziemann 2010; Ledoit & Wolf 2004) shrinkage estimation method. The relative risk aversion coefficient is taken equal to different cases of λ=1,3,5,15 .

B. Empirical result

Although this was presented naïve the rule by some new measurements such as Sharpe ratio has superior out-of-sample performance than the Markowitz model, we developed the Markowitz model to consider ambiguity and hidden information as a novel model. Table 1 describes the summarized results of empirical data for novel model and Markowitz mean-variance theory. We see the novel model almost has better Sharpe ratio, CER and Adjusted for Skewness Sharpe Ratio than mean-variance strategy. For example, the out-of-sample Sharpe ratio for Markowitz is 0.088, while the novel model has 0.091 monthly out-of-sample ratio. Similarly, the Certainty Equivalent Return (CER) for Markowitz is negative, while that for the novel approach is strongly positive in different coefficients of risk aversion. Moreover, both strategies almost have same Adjusted for Skewness Sharpe Ratio around 0.09. The comparison of different measurements typically enhances the improvement results of using the novel model at dealing with estimation error. Thus, considering portfolio under ambiguity as an optimal target is very successful and much more reasonable measuring with current gauges like Sharpe ratio and CER.

Table 6: how well is the novel model rather than Markowitz?

PANEL A: N=7	λ=1		λ=3	
Measurements\ Models	Novel	Markowitz	Novel	Markowitz
Sharp Ratio	0.091599491*	0.088835632	0.091599491*	0.088835632
Certainty Equivalent Return	0.003185329*	-0.179559716	0.001008434*	-0.662098514
Adjusted for Skewness Sharpe Ratio	0.092350955	0.090964793	0.092036462	0.097386299

5. Conclusion:

We display that constructing a minimum variance of utility function subject to the ambiguity aversion that portfolio manager should consider hidden information is better than to constructing a maximum expected utility function without any involving ambiguity and hidden information. This consideration has typically two important effects. On the one hand, to the extent a large estimation error is due to estimating the mean of sample data rather than other moments, variance of utility function leads to eliminating the effect of first moment of sample data and then finally more precise estimation and portfolio weights by covariance of sample data.

On the other hand, because of poorly performance in actual choices and dissatisfaction of expected theory framework introduced by Von Neumann & Morgenstern (1944) and earlier made by Denial Bernoulli in 1738, which individual welfare can be measured by computing the expected utility, portfolio manager surveys on decision-making under ambiguity aversion. The linearity (affine transformation) of expected utility function with respect to probability and risk preferences implies that the expected theory is neutral with any uncertainty about probability and risk preferences effect. This raises the question of interaction between the property which is referred to as the “independent axiom” and “Ellsberg paradox” the most famous challenge has been proposed by Ellsberg (1961). As a result, ambiguity does matter then this paper presents a model which capture ambiguity aversion through smoothing the preferences utility function based on Gilboa & Schmeidler (1989) *(Maxmin theory)*. In addition, we find by this new proposed model, enhancing that the Sharpe ratio of this model outperforms better than Markowitz strategy.

References

DeMiguel, V., Garlappi, L. & Uppal, R., 2009. Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? *Review of Financial Studies*, 22(5), pp.1915–1953.

Easley, D. & O’Hara, M., 2009. Ambiguity and Nonparticipation: The Role of Regulation. *The Review of Financial Studies*, 22(5), pp.1817–1843.

Ellsberg, D., 1961. Risk, ambiguity, and the Savage axioms. *The quarterly journal of economics*, pp.643–669.

Epstein, L.G. & Schneider, M., 2010. Ambiguity and Asset Markets. *Annual Review of Financial Economics*, 2(1), pp.315–346.

Epstein, L.G. & Schneider, M., 2008. Ambiguity, information quality, and asset pricing. *J. Finance*, 63(1), pp.197–228.

Gilboa, I. & Marinacci, M., 2011. Ambiguity and the Bayesian paradigm. *Chapter*, 7, pp.179–242.

Gilboa, I. & Schmeidler, D., 1989. Maxmin expected utility with non-unique prior. *Journal of mathematical economics*, 18(2), pp.141–153.

Harvey, C.R. et al., 2004. Portfolio selection with higher moments. *Duke University FRPS04-123*.

Harvey, C.R. & Siddique, A., 2000. Conditional skewness in asset pricing tests. *J. Finance*, 55(3), pp.1263–1295.

Jagannathan, R. & Ma, T., 2003. Risk reduction in large portfolios: Why imposing the wrong constraints helps.

J. Finance, 58(4), pp.1651–1684.
Jean, W.H., 1971. The extension of portfolio analysis to three or more parameters. *Journal of Financial and Quantitative Analysis*, 6(01), pp.505–515.
Jondeau, E. & Rockinger, M., 2003. Conditional volatility, skewness, and kurtosis: existence, persistence, and comovements. *Journal of Economic Dynamics and Control*, 27(10), pp.1699–1737.
Jorion, P., 1986. Bayes-Stein estimation for portfolio analysis. *Journal of Financial and Quantitative Analysis*, 21(03), pp.279–292.
Ju, N. & Miao, J., 2012. Ambiguity, Learning, and Asset Returns. *Econometrica*, 80(2), pp.559–591.
Klibanoff, P., Marinacci, M. & Mukerji, S., 2005. A Smooth Model of Decision Making under Ambiguity. *Econometrica*, 73(6), pp.1849–1892.
Ledoit, O. & Wolf, M., 2004. Honey, I shrunk the sample covariance matrix. *The Journal of Portfolio Management*, 30(4), pp.110–119.
Maccheroni, F., Marinacci, M. & Ruffino, D., 2013. Alpha as Ambiguity: Robust Mean-Variance Portfolio Analysis. *Econometrica*, 81(3), pp.1075–1113.
Markowitz, H., 1952. Portfolio selection. *J. Finance*, 7(1), pp.77–91.
Martellini, L. & Ziemann, V., 2010. Improved estimates of higher-order comoments and implications for portfolio selection. *Review of Financial Studies*, 23(4), pp.1467–1502.
Merton, R.C., 1980. On estimating the expected return on the market: An exploratory investigation. *J. financ. econ.*, 8(4), pp.323–361.
Von Neumann, J. & Morgenstern, O., 1944. Theory of games and economic behavior.
Scott, R.C. & Horvath, P.A., 1980. On the direction of preference for moments of higher order than the variance. *J. Finance*, 35(4), pp.915–919.