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Evaluation of Simultaneous Equation Techniques in the Presence of Misspecification Error: A Monte Carlo Approach

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Abstract
One of the assumptions of Classical Linear Regression Model (CLRMA), is that the regression model be 'correctly' specified. If the model is not 'correctly' specified, the problem of model misspecification error arises. The objective of the study is to know the performances of the estimator and also the estimator that is greatly affected by misspecification error due to omission of relevant explanatory variable. Four simultaneous equation techniques (OLS, 2SLS, 3SLS, LIML) were applied to a two-equation model and investigated on their performances when plagued with the problem of misspecification error. A Monte Carlo method simulation method was employed to investigate the effect of these estimators due to misspecification of the model. The findings revealed that the estimates obtained by 2SLS and 3SLS are similar and variances by all the estimates reduced consistently as the sample size increases. The study had revealed that 2 3 SLS performed best using average of parameter criterion while OLS generated the least variances. LIML is mostly affected by misspecification.

Keywords: Monte Carlo, Misspecification error, Simultaneous equation.

1. INTRODUCTION

The first step in any econometric research, is the specification of the model with which, one will attempt the measurement of the phenomenon being analysed that is, to study the relationship between economic variables.

Most econometric relationships are subject to misspecification. Misspecification errors occur when the true model is not known, and these can arise as are result of omission of relevant explanatory variable(s), inclusion of redundant variable(s), errors of measurement or adoption of wrong functional form.

Consequently, misspecification of models is difficult to avoid, it can lead to biased coefficients and error terms, which in turn can lead to incorrect inference and models. Swamy and George (2008).

OLS is mostly inefficient since its estimates are inconsistent but is used here to facilitate comparison. 2SLS and LIML methods use the same amount of information; they have the same degree of efficiency since they use all the predetermined variables of the method.

3SLS and FIML methods are complete systems techniques and use more information than any of the above methods, because apart from all the variables of the system, they also use information concerning the mathematical form of the equations, that is, they take into account the structure of all the equations of the system). In general, these two methods have the same efficiency. It should be noted that if all assumptions based on each method are satisfied including no specification error, FIML and 3SLS seem the best methods since they are the most efficient of all the others. However, these methods are generally more sensitive than the others to specification errors, because an error of specification anywhere in the system affects all the parameter estimates.

Given therefore our uncertainty about the correctness of the specification of our model, as well as the errors in variables and the extremely complicated computations of FIML and 3SLS and in particular FIML, it seems that these methods are the least attractive for economic research. For this reason, only 3SLS method is used in this study while FIML is completely dropped for its complexities, (Adepoju and Olaomi (2009))

Small sample properties of various econometrics techniques have been studied by many authors including Bryon (1972), Light (2010), Shoukwiller (2010). Wagnar (1958) in his work employed Monte Carlo to compare small properties of LIML, Single Equation Least Squares (LISE) and concluded that least squares generally gives more biases, but less variable estimates than LISE method, while Nagar A.L (1960) using Monte Carlo approach concluded that 2 3SLS showed the smallest bias.

The estimation problem posed by the existence of misspecification error due to omission of relevant explanatory variable in the model is examined in this study. The estimation techniques are judged for their performances using average of estimates and variance of parameter estimates.

2. PRESENTATION OF THE MODEL

Consider the following model;

$$y_{1t}= \beta_{12}y_{2t}+\gamma_{11}x_{1t}+u_{1t} \qquad (1)$$

$$y_{2t}= \beta_{21}y_{1t}+\gamma_{22}x_{2t}+\gamma_{23}x_{3t}+u_{2t}$$

Where y_{1t} , y_{2t} and x_{1t}, x_2 x_{3t} are the endogenous and exogenous variables respectively. u_{1t} and u_{2t} denote stochastic disturbance terms which are assumed to be independently and identically normally distributed with zero mean and finite variance-covariance matrix Σ i.e. $u \sim$ NID (0, Σ). β_{12}, β_{21}, γ_{11}, γ_{22} and γ_{23} are the coefficients of predetermined variables.

3. MATERIALS AND METHOD

The method employed in this study is a Monte Carlo approach. The main task of Monte Carlo approach is the generation of stochastic dependent (endogenous) variables i.e y_{1t} and y_{2t} which are subsequently used in estimating the parameters of the model, to achieve this, the following have to be assumed.

(i) Values of the parameters of the model which are:

$$\beta_{12} = 1.5, \beta_{21} = 1.8, \gamma_{11} = 1.5, \gamma_{22} = 0.5, \gamma_{23} = 2.0$$

(ii) Values of elements Ω

$$\text{i.e} \quad \Omega = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \begin{pmatrix} 5.0 & 2.5 \\ 2.5 & 3.0 \end{pmatrix}$$

(iii) Values of predetermined variables x_{1t} x_2 and x_{3t} (t=1... T)

These values were obtained from uniform distribution with mean 0 and standard deviation 1 (Kmenta (1971)). Samples sizes were set at N=10, 20 and 30each replicated 250 times.

$$\Sigma = PP' \Sigma = PP'$$

$$\text{Let } P = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix}$$

$$\text{Then} \quad \Sigma = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} S_{11} & 0 \\ S_{12} & S_{22} \end{pmatrix} = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\Rightarrow S_{11}^2 + S_{12}^2 = 5.0$$

$$S_{12} S_{22} = 2.5$$

$$S_{22} S_{12} = 2.5$$

$$S_{22}^2 = 3.0$$

$$S_{22} = \sqrt{3.0} = 1.732050808$$

$$S_{12} = \frac{2.5}{\sqrt{S_{22}}} = \frac{2.5}{\sqrt{3.0}} = 1.443375673$$

$$S_{11} = \sqrt{5.0 - \frac{2.5}{3.0}} = 1.707825128$$

Then the random disturbance terms will be given as:

$$u_t = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = P\varepsilon_t = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$$u_{1t} = S_{11}\varepsilon_{1t} + S_{12}\varepsilon_{2t}$$

$$u_{2t} = S_{22}\varepsilon_{2t}$$

Re-arranging model (1), we have:

$$y_{1t} - \beta_{12} y_{2t} = \gamma_{11} x_{1t} + 0x_{2t} + 0x_{3t} + u_{1t}$$

$$-\beta_{21} y_{1t} + y_{2t} = 0x_{1t} + \gamma_{22} x_{2t} + \gamma_{23} x_{3t} + u_{2t}$$

Which can be written compactly as;

$$\begin{bmatrix} 1 & -\beta_{12} \\ -\beta_{21} & 1 \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \gamma_{11} & 0 & 0 \\ 0 & \gamma_{22} & \gamma_{23} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

Using the reduced form of equations which gives;

$$y_{1t} = \pi_{11}\, x_{1t} + \pi_{12}\, x_{2t} + \pi_{13}\, x_{3t} + v_{1t}$$

$$y_{2t} = \pi_{21}\, x_{1t} + \pi_{22}\, x_{2t} + \pi_{23}\, x_{3t} + v_{2t}$$

$$\text{Where: } \pi = \begin{bmatrix} \gamma_{11}\beta^* & \beta_{12}\gamma_{22}\beta^* & \beta_{12}\gamma_{23}\beta^* \\ \beta_{21}\gamma_{11}\beta^* & \gamma_{22}\beta^* & \gamma_{23}\beta^* \end{bmatrix}$$

$$\beta^* = (1 - \beta_{12}\beta_{21})^{-1}$$

$$v_{1t} = \beta^* (\, u_{1t} + \beta_{12}\, u_{2t})$$

$$v_{2t} = \beta^* (\beta_{21} u_{1t} + u_{2t}$$

4. RESULTS AND DISCUSSION

Samples were set at size N=10, 20 and 30 and each of this were replicated 250 times for an upper triangular matrix in the presence of misspecification errors due to omission of relevant explanatory variable.

Ordinary Least Squares (OLS), 2 Stage Least Squares (2SLS), 3 Stage Least Squares (3SLS) and Limited Information Maximum Likelihood (LIML) were used to obtain parameter estimates. The performances of these estimators are examined using average and variance of parameter estimates.

Hence, the table 1-2 shown below the true parameters values, average estimated values and variance.

TABLE 1
PERFORMANCE EVALUATION OF ESTIMATORS TO CHANGES IN SAMPLE SIZE INCREASES USING AVERAGE OF PARAMETER ESTIMATES

		EQUATION 1						EQUATION 2								
Estimators		N=10		N=20		N=30		N=10			N=20			N=30		
		β_{12}=1.5	γ_{11}=1.5	β_{12}=1.5	γ_{11}=1.5	β_{12}=1.5	γ_{11}=1.5	β_{21}=1.8	γ_{22}=0.5	γ_{23}=2.0	β_{21}=1.8	γ_{22}=0.5	γ_{23}=2.0	β_{21}=1.8	γ_{22}=0.5	γ_{23}=2.0
OLS	P1	0.0017	0.1021	0.0006	0.0343	0.0003	0.0194	0.0026	0.6259	0.5694	0.0006	0.1880	0.2234	0.0003	0.1160	0.1168
23SLS	P1	0.1650	1.5882	1.6189	12.3722	1.6312	1.6312	5.6361	41938.58	5213.185	69.1098	19.6	599.7174	461.699	1528.84	1200.242
LIML	P1	136.464	741.215	32.4162	265.5259	103.64	103.6358	5.6361	41938.58	5213.185	69.1098	19.6	599.7174	461.699	1528.84	1200.242

TABLE 2:PERFORMANCE EVALUATION OF ESTIMATORS TO CHANGES IN SAMPLE SIZE INCREASES USING VARIANCE OF PARAMETER ESTIMATES

		EQUATION 1						EQUATION 2								
Estimators		N=10		N=20		N=30		N=10			N=20			N=30		
		β_{12}=1.5	γ_{11}=1.5	β_{12}=1.5	γ_{11}=1.5	β_{12}=1.5	γ_{11}=1.5	β_{21}=1.8	γ_{22}=0.5	γ_{23}=2.0	β_{21}=1.8	γ_{22}=0.5	γ_{23}=2.0	β_{21}=1.8	γ_{22}=0.5	γ_{23}=2.0
OLS	P1	0.0017	0.1021	0.0006	0.0343	0.0003	0.0194	0.0026	0.6259	0.5694	0.0006	0.1880	0.2234	0.0003	0.1160	0.1168
23SLS	P1	0.1650	1.5882	1.6189	12.3722	1.6312	1.6312	5.6361	41938.58	5213.185	69.1098	19.6	599.7174	461.699	1528.84	1200.242
LIML	P1	136.464	741.215	32.4162	265.5259	103.64	103.6358	5.6361	41938.58	5213.185	69.1098	19.6	599.7174	461.699	1528.84	1200.242

5. CONCLUSION

This investigated the performances of four estimators in the presence of misspecification errors due to omission of relevant explanatory. A two equation model with one just identified and other over identified was used for the Monte Carlo simulation analysis. The sample size was set at N=10, 20 and 30 each replicated 250 times.

The performances were judged by using average and variance of estimates. The results shown that 2 3 SLS performed best using average of parameter estimate criterion. OLS generated the least variances while other estimators generated reduced variances as the sample size increases confirming the consistency property of good estimators.

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