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## Optimal Pricing and Ordering Policy under Permissible Delay in Payments

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### Abstract

This study develops an inventory model to determine an optimal cycle time and optimal total annual profit for non-deteriorating items under permissible delay in payments. Mathematical models have been derived for obtaining the optimal cycle time and optimal price, so that the annual total profit is maximized. This paper also develops the model by considering particular cases (A) and (B) respectively. We obtain price and lot size simultaneously when supplier offers a permissible delay in payments. The demand rate is assumed to be a function of price and time. Finally, a numerical example is given to illustrate the proposed model.

**Key words:** Pricing, Inventory, Permissible delay, Non- deterioration, Finance, Quantity

### 1. Introduction

The traditional economic order quantity (EOQ) model assumes that the retailer must be paid for the items as soon as the items were received. But it may not be true in general. In practice the supplier offers the retailer a period (called delay period or trade credit period) for settling the account. Before the end of this period, the retailer can sell the goods and accumulate revenue and earn interest. An interest is charged if the retailer unable to settle the account by the end of the credit period. Therefore, it makes economic sense for the retailer to delay the settlement of the replenishment account up to the end of the delay period allowed by the supplier. During the past few years, many articles dealing with various inventory models under permissible delay have appeared in various research journals.

In past decade, mathematical ideas have been used in different area for controlling inventory. The important concerns of the management are to decide when and how much to order or to manufacture, so that total cost associated with the inventory system should be minimum. Deterioration cannot be ignored in business management. Deterioration refers to damage, change, decay, spoilage obsolescence and loss of original value in the item those results in the decreasing usefulness from the original one. The certain products such as medicine, vegetable, blood, gasoline and radioactive chemicals decrease under deterioration during their normal storage period. As a result, the loss due to deterioration cannot be ignored for determining optimal inventory policy. To accumulate more practical features of the real inventory system, the deteriorating inventory models have been continuously modified. Number of researchers has been discussed inventory models for non- deteriorating items. However, there are certain substances in which deterioration play the main role and commodities cannot be stored for a long time. Non deteriorating items like, wheat, rice, some types of dry fruits, etc.

Teng et al. (2004) developed a model on optimal pricing and ordering policy under permissible delay in payments, in which deterioration rate is constant and demand rate is a function of price. In this paper Tenj et al.(2004) obtained optimal cycle time and optimal total annual profit. This paper is the extension of Teng et al. (2004) in which deterioration rate is zero and demand rate is a function of price and time. Teng (2002)

in his paper discussed on the economic order quantity under condition of permissible delay in payments for non-deteriorating items. Goyal (1985) developed an EOQ model under conditions of permissible delay in payments. He ignored the difference between the selling price and the purchase cost, and concluded that the economic replenishment interval and order quantity increases marginally under permissible delay in payments. Dave (1985) corrected Goyal's model by assuming the fact that the selling price is necessarily higher than its purchase price. Aggarwal and Jaggi (1995) then extended Goyal's model for deteriorating items. Jamal *et al.* (1997) further generalized the model to allow for shortages and deterioration. Liao *et al.* (2002) developed an inventory model for stock- dependent demand rate when a delay in payment is permissible.

Huang (2003) implicitly assume that the inventory level is depleted by customer's demand only. This assumption is valid for non- deteriorating or non- perishable inventory items. Mahata and Mahata (2009) modified Huang (2003) model by developing an inventory model for deteriorating items under condition of permissible delay in payments. Chung (1998) presented the discounted cash flow (DCF) approach for the analysis of the optimal inventory policy in the presence of the trade credit. Hwang and Shinn (1997) extended Goyal's (1985) model to consider the deterministic inventory model with a constant deterioration rate. Manisha Pal and S.K. Ghosh (2006) developed an inventory model with shortage and quantity dependent permissible delay in payment for non- deteriorating items.

In this paper we establish an appropriate model for a retailer to determine its optimal price and lot size simultaneously when the supplier offer a permissible delay in payments. In this paper the deterministic inventory model with time –dependent demand pattern is developed for non- deteriorating items in which inventory is depleted only by demand. The paper is organized as follows: In section 2 assumptions and notations are mentioned. In section 3, the mathematical model is formulated. In section 4 the optimal replenishment time for given price is mentioned in which we considered two particular cases viz; case (A) and case (B) respectively. In section 5 optimal prices is obtained. In next section numerical example is cited to validate the proposed model followed by concluding remark and future research is detailed in the last section.

## 2. Assumptions and Notations

The following assumptions are being made to develop the mathematical model

- The demand for the item is a downward sloping function of the price and variable time  $t$ .
- Shortage is not allowed.
- Time horizon is infinite.

In addition the following notations are also used throughout the manuscript

H: The unit holding cost per year excluding interest charges

c: The unit purchasing cost, with  $c < p$

p: The selling price per unit

$i_d$ : The interest earned per dollar per year

$i_c$ : The interest charged per rupee in stocks per year by the supplier

m: The period of permissible delay in setting account; that is, the trade credit period

s: The ordering cost per order

Q: The order quantity

$I(t)$ : The level of inventory at time  $t$ ,  $0 \leq t \leq T$

T: The replenishment time interval

D: The annual demand, as a decreasing function of price and time, we set  $D(p, t) = \alpha p^{-\beta} t$ , where

$\alpha > 0$  and  $\beta > 1$ , ( $a = \alpha p^{-\beta}$ )

$Z(T, p)$ : The total annual profit

The total annual profit consists of (a) the sales revenue, (b) cost of placing orders, (c) cost of purchasing, (d) cost of carrying inventory (excluding interest charges), (e) cost of interest payable for items unsold after the permissible delay  $m$  (note that this cost occurs only if  $T > m$ ), and (f) interest earned from sales revenue during the permissible period.

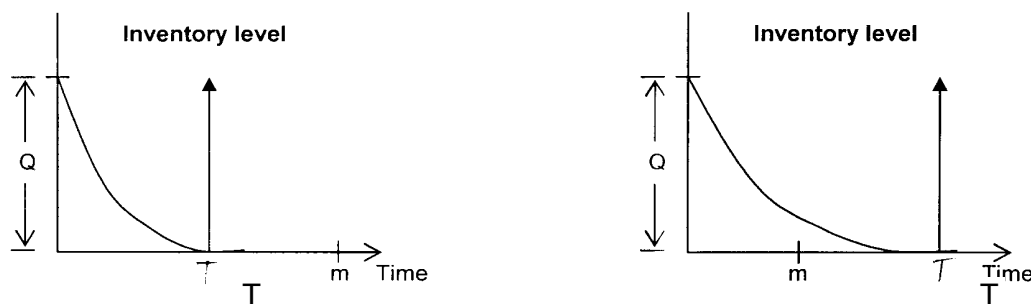
### 3. Mathematical Formulation

The level of inventory  $I(t)$  gradually decreases mainly to meet demands. Hence the variation of inventory with respect to time can be determined by the following differential equations:

$$\frac{dI(t)}{dt} = -D(p, t), \quad 0 \leq t \leq T \quad (1)$$

$$\frac{dI(t)}{dt} = -at, \quad 0 \leq t \leq T, \quad [\text{where, } a = \alpha p^{-\beta}] \quad (2)$$

With boundary condition  $I(T) = 0$ . We have the following two possible cases based on the values of  $T$  and  $m$ . These two cases are given graphically in Fig. 1.



Case 1:  $T \leq m$

Case 2:  $T \geq m$

Fig. 1: Graphical representation of two inventory systems

#### Case 1: $T \leq m$

In this case, the customer sells  $\frac{aT^2}{2}$  units in total by the end of the replenishment cycle time  $T$ , and has

$\frac{caT^2}{2}$  to pay the supplier in full by the end of the credit period  $m$ . Consequently, there is no interest payable. However, the interest earned per year is

$$\frac{pl_d}{T} \left[ \int_0^T at^2 dt + (m - T) \int_0^T at dt \right] = \frac{pl_d a T}{2} \left( m - \frac{T}{3} \right) \quad (3)$$

The total annual profit  $Z_1(T, p)$  is

$Z_1(T, p) =$  Sales revenue – Cost of placing order– Cost of purchasing – Cost of carrying inventory + interest earned per year.

$$Z_1(T, p) = \frac{paT}{2} - \frac{s}{T} - \frac{caT}{2} - \frac{haT^2}{3} + \frac{pl_d a T}{2} \left( m - \frac{T}{3} \right) \quad (4)$$

**Case 2:  $T \geq m$**

The buyer sells  $\frac{am^2}{2}$  unit in total by the end of the permissible delay  $m$  and has  $\frac{caT^2}{2}$  pay the supplier.

The items in stock are charged at interest rate  $I_c$  by the supplier starting at time  $m$ . Therefore the buyer gradually reduces the amount of financed loan from the supplier due to constant sales and revenue received. As a result, the interest payable per year is

$$\frac{cl_c \int_m^T l(t) dt}{T} = \frac{cl_c}{T} \int_m^T \frac{a(T^2 - t^2)}{2} dt = \frac{cl_c a}{6T} (2T^3 - 3mT^2 + m^3) \quad (5)$$

During the permissible delay period, the buyer sells product and deposits the revenue into an account that earns  $I_d$  per dollar per year. Therefore, the interest earned per year is

$$\frac{pl_d \int_0^m at^2 dt}{T} = \frac{pl_d am^3}{3T} \quad (6)$$

Hence the total annual profit  $Z_2(T,p)$  is

$$Z_2(T, p) = \frac{paT}{2} - \frac{s}{T} - \frac{caT}{2} - \frac{haT^2}{3} + \frac{cl_c a}{6} \left( 2T^2 - 3mT + \frac{m^3}{T} \right) + \frac{pl_d am^3}{3T} \quad (7)$$

Note that there are many different ways to calculate the interest payable as well as interest earned. For simplicity, we use Goyal's approach throughout this paper.

Hence the total annual profit  $Z(T,p)$  is written as

$$\begin{cases} Z_1(T, p) & \text{for } T \leq m \\ Z(T, p) = Z_2(T, p) & \text{for } T \geq m \end{cases}$$

Although  $Z_1(m,p) = Z_2(m,p)$ ,  $Z(T,p)$  is a continuous function of  $T$  either in  $(0, m)$  or in  $(m, \infty)$ , but not in both.

**4. Determination of the optimal replenishment time for given price**

Differentiating (10) partially with respect to  $T$ , we get

$$\frac{\partial Z_1(T, p)}{\partial T} = \frac{ap}{2} + \frac{s}{T^2} - \frac{ca}{2} - \frac{2haT}{3} + \frac{pl_d am}{2} - \frac{pl_d aT}{3} \quad (8)$$

and 
$$\frac{\partial^2 Z_1(T, p)}{\partial T^2} = - \left( \frac{2s}{T^3} + \frac{2ah}{3} + \frac{pl_d a}{3} \right) < 0 \quad (9)$$

Again differentiating (13) partially with respect to  $T$ , we get

$$\frac{\partial Z_2(T, p)}{\partial T} = \frac{ap}{2} + \frac{s}{T^2} - \frac{ca}{2} - \frac{2haT}{3} - \frac{2cl_c aT}{3} + \frac{cl_c am}{2} + \frac{(cl_c - 2pl_d)am^3}{6T^2} \quad (10)$$

$$\text{and } \frac{\partial^2 Z_2(T, p)}{\partial T^2} = - \left[ \frac{2s}{T^3} + \frac{a}{3} \{ 2h + 2cl_c + (cl_c - 2pl_d)m^3 \} \right] < 0 \quad (11)$$

For a fixed p,  $Z_1(T, p)$  is strictly concave function of T. Thus there exists a unique value of T, which maximizes  $Z_1(T, p)$ . Also for a fixed p,  $Z_2(T, p)$  is a concave function of T. Thus there exists a unique value of  $T_2$  which maximizes  $Z_2(T, p)$ .  $T^* = T_1$ , is obtained by solving  $\frac{\partial Z_1(T, p)}{\partial T} = 0$ , i.e.

$$2(2h + pl_d) aT^3 - 3a(p - c + pl_d m) T^2 - 6s = 0 \quad (12)$$

For example, let  $h = 0.65/\text{unit}/\text{year}$ ,  $I_c = 0.09/\$/\text{year}$ ,  $I_d = 0.06/\$/\text{year}$ ,  $c = \$9.0$  per unit,  $p = \$10$  per unit,  $m = 2.0$  year,  $s = 50$ ,  $\alpha = 10^5$ ,  $\beta = 2$ . Equation (18) becomes  $38T^3 - 66T^2 - 3 = 0$ , by trial, we get  $T_1 = 1.76226$  year (approximately). At  $T^* = T_1$ ,  $Z_1(T, p)$  gives the optimal value (maximum value). And optimal (maximum) value of  $Z_1(T, p) = \$926.6859114$  (approximately).

Similarly  $T_2$  is obtained by solving  $\frac{\partial Z_2(T, p)}{\partial T} = 0$ , we get

$$4a(h + cl_c) T^3 - 3a(p - c + cl_c m) T^2 - \{6s + (cl_c - 2pl_d)am^3\} = 0 \quad (13)$$

For example, let  $h = \$0.60 / \text{unit} / \text{year}$ ,  $I_c = 0.09/\$/\text{year}$ ,  $I_d = 0.03/\$/\text{year}$ ,  $c = \$8.0$  per unit,  $p = \$10$  per unit,  $m = 2.0$  year,  $s = 200$ ,  $\alpha = 10^5$ ,  $\beta = 2$ . From (21), we get,  $66T^3 - 129T^2 - 27 = 0$ , by trial we get  $T^* = T_2 = 2.05173$  year (approximately). And optimal (maximum) value of  $Z_2(T, p) = \$1503.202202$  (approximately).

**(i) Particular case (A).** If  $c = p(1 + I_d m)$ , from equation (8) we obtain

$$T = T_1 = \left\{ \frac{3s}{a(2h + pl_d)} \right\}^{1/3} \quad (14)$$

To ensure  $T_1 \leq m$ , we substitute (14) into inequality  $T_1 \leq m$  and obtain that if only if,

$$3s \leq a(2h + pl_d)m^3, T_1 \leq m \text{ for } c = p(1 + I_d m) \quad (15)$$

**(ii) Particular case (B).** If  $p = c(1 - I_c m)$ , from equation (13), we obtain,

$$T = T_2 = \left\{ \frac{6s + (cl_c - 2pl_d)am^3}{4a(h + cl_c)} \right\}^{1/3} \quad (16)$$

To ensure  $T_2 \geq m$ , we substitute (16) into inequality  $T_2 \geq m$  and obtain that if and only if,

$$3s \geq a(2h + pl_d + \frac{3}{2} cl_c)m^3, T_2 \geq m, \text{ for } p = c(1 - I_c m) \quad (17)$$

In classical EOQ model, the supplier must be paid for the items as soon as the customer receives them. It is a special case of (2) with  $m = 0$ , as a result,

$$T^* = \left\{ \frac{3s}{2a(h + cl_c)} \right\}^{1/3} \quad (18)$$

$Z(T, p)$  is a continuous function of T either in  $(0, m)$  or in  $(m, \infty)$  but not in  $(0, \infty)$ . We know from Theorem 1 below that  $Z(T, p)$  is not continuous in  $(0, \infty)$ , but continuous in  $(0, m)$  and  $(m, \infty)$ . For example choose c, p and  $I_d$  such that  $c = p(1 + I_d m)$ , for this let  $c = \$6$  per unit,  $p = \$5$  per unit,  $I_d = 0.06/\$/\text{year}$ ,  $m = \frac{5}{3}$  year,  $s = 200$ ,  $\alpha = 10^6$ ,  $\beta = 4.0$  and  $h = \$0.065/\text{unit}/\text{year}$ . We obtain Theorem 1 below that  $3s \leq a(2h + pl_d)m^3 = 740.741$  i.e.  $Z(T, p) = Z_1(T, p)$  and optimal  $T^* = 1.55362 < m$  as shown in Fig. 2. For an example of case 2 (i.e.  $Z(T, p) = Z_2(T, p)$ ). Choose c, p and  $I_c$  such that  $p = c(1 - I_c m)$ , let  $p = \$5$  per unit,  $c = \$6$  per unit,  $I_d = 0.06/\$/\text{year}$ ,  $I_c = 0.1/\$/\text{year}$ ,  $\alpha = 10^6$ ,  $\beta = 4.0$ ,  $s = 400$ ,  $h = \$0.65/\text{unit}/\text{year}$  and  $m = \frac{5}{3}$  year. Then we obtain from Theorem 1 that  $3s \geq a(2h + pl_d + \frac{3}{2} cl_c)m^3 = 1157.74$ ,  $Z(T, p) = Z_2(T, p)$  and the optimal  $T^* = 1.686865 > m$ , as shown in Figure 3.

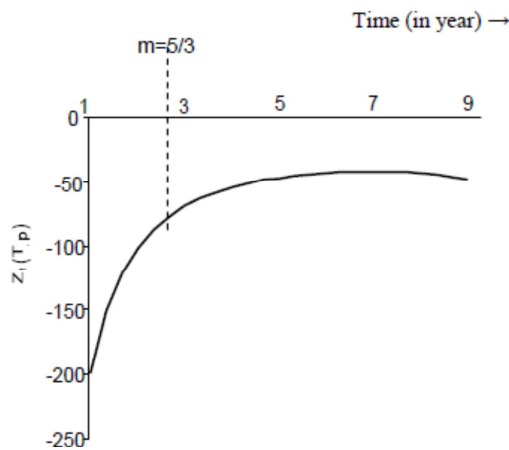


Fig.2. Case1. Particular case(A) when,  
 $c = p(1 + I_d m)$ ,  $T \leq m$

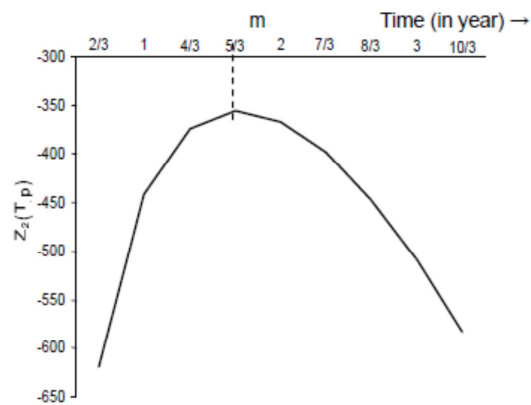


Fig.3. Case 2. Particular case (B), when  
 $p = c(1 - I_c m)$  &  $T \geq m$

From (16), the optimal EOQ for case 1 (i.e.  $T_1 \leq m$ ) for  $c = p(1 + I_d m)$

$$Q^*(T_1) = \frac{a}{2} \left\{ \frac{3s}{a(2h + pI_d)} \right\}^{2/3}, \text{ for } c = p(1 + I_d m) \quad (19)$$

From (16) into (1), we obtain

$$Z_1(p) = -\frac{1}{2} \{9as^2(2h + pI_d)\}^{1/3} \quad (20)$$

Again, the optimal EOQ for case 2 (i.e.  $T_2 \geq m$ ) for  $p = c(1 - I_c m)$

$$Q^*(T_2) = \frac{a}{2} \left\{ \frac{6s + (cI_c - 2pI_d)am^3}{4a(h + cI_c)} \right\}^{2/3}, \text{ for } p = c(1 - I_c m) \quad (21)$$

Substituting (16) into (7), we obtain

$$Z_2(p) = -\frac{1}{4} \{4a(h + cI_c)\}^{1/3} \{6s + (cI_c - 2pI_d)am^3\}^{2/3} \quad (22)$$

From (18), the classical optimal EOQ is

$$Q^* = \frac{aT^{*2}}{2} = \frac{a}{2} \left\{ \frac{3s}{2a(h + cI_c)} \right\}^{2/3} \quad (23)$$

By comparing (15) and (17), we have the following results:

**Theorem 1:** If

- (i)  $3s \leq a(2h + pI_d)m^3$ , for  $c = p(1 + I_d m)$ , then  $T^* = T_1$
- (ii)  $3s \geq a(2h + pI_d + \frac{3}{2}cI_c)m^3$ , for  $p = c(1 - I_c m)$ , then  $T^* = T_2$
- (iii)  $3s = a(2h + pI_d + \frac{3}{2}cI_c)m^3$ , for  $p = c(1 - I_c m)$ , then  $T^* = m$ .

**Proof:** It immediately follows from (15) and (17).

Similarly, from (19), (21) and (23), we have the following theorem :

**Theorem 2:** If

- (i)  $cl_c > 2pI_d$ , for  $c = p(1 + I_d m)$ , then  $Q^*(T_2)$  and  $Q^*(T_1) > Q^*$
- (ii)  $cl_c < 2pI_d$ , for  $p = c(1 - I_c m)$ , then  $Q^*(T_2)$  and  $Q^*(T_1) < Q^*$
- (iii)  $cl_c = 2pI_d$ , for  $p = c(1 - I_c m)$ , then  $Q^*(T_2) = Q^*$  and  $Q^*(T_1) > Q^*$

**Proof:** It is obvious from (19), (21) and (23).

**Note:** Theorem 1 and 2 given above are obtained by particular cases (A) and (B).

### 5. Determination of the Optimal Price

Taking the first derivative of  $(2h + pI_d + \frac{3}{2}cl_c)a(p)m^3$  with respect to  $p$ , we obtain

$$I_d a(p)m^3 + (2h + pI_d + \frac{3}{2}cl_c) \left( -\frac{\beta}{p} a(p) \right) m^3$$

$$= m^3 \{ (2h - \frac{3}{2}cl_c) a'(p) - I_d(\beta - 1) a(p) \} < 0 \quad (24)$$

Hence  $(2h + pI_d + \frac{3}{2}cl_c)a(p)m^3$  is a strictly decreasing function of  $p$ .

Using the fact in (17), we set  $p_0$ , such that

$$3s = a(p_0)(2h + p_0I_d + \frac{3}{2}cl_c)m^3$$

Therefore

$$Z_1(p) = Z_1(T_1(p), p), \text{ for } p \leq p_0$$

$$Z(p) = \begin{cases} Z_1(p) & \text{for } p \leq p_0 \\ Z_2(p) & \text{for } p \geq p_0 \end{cases}$$

To obtain the optimal price taking the first derivative of (20) with respect to  $p$  and setting the result to be zero, we have

$$\frac{dZ_1(p)}{dp} = \frac{(3s)^{2/3}}{6} a^{1/3} \left( \frac{\beta g_1^{1/3}}{p} - g_1^{-2/3} I_d \right) = 0 \quad (25)$$

Where,  $g_1 = (2h + pI_d)$

Next, we need to check the second order condition for concavity. That is

$$\frac{d^2 Z_1(p)}{dp^2} = -\frac{(3s)^{2/3} a^{1/3} g_1^{-5/3}}{18} \left\{ \frac{\beta(\beta + 3)}{p^2} g_1^2 - 2\beta g_1 I_d - 2I_d^2 \right\} < 0 \quad (26)$$

From (22) we obtain the first order condition for  $Z_2(p)$  as

$$\frac{dZ_2(p)}{dp} = \frac{4^{-2/3} (h + cl_c)^{1/3} a^{1/3}}{3p} \left[ \beta g_2^{2/3} + 2am^3 g_2^{-1/3} \{-2(\beta - 1)pI_d + \beta cl_c\} \right] = 0 \quad (27)$$

Where,  $g_2 = cl_c - 2pI_d$ .

The second order condition for concavity is

$$\frac{d^2Z_2(p)}{dp^2} = \frac{-g(\beta + 3)a^{1/3}}{6p^2} \left\{ \beta g_2^{2/3} + 2am^3 g_2^{-1/3} (2\beta pI_d + 2pI_d + \beta cI_c) \right\}$$

$$\frac{-ga^{1/3}}{p^2} \left[ \frac{2}{3} \beta g_2^{-1/3} a(\beta cI_c - 2\beta pI_d + 2pI_d) + 2am^3 \left\{ \beta g_2^{-1/3} + \frac{1}{3} am^3 g_2^{-4/3} (-\beta cI_c + 2\beta pI_d - 2pI_d) \right\} \right. \\ \left. \left\{ -2\beta pI_d + 2pI_d + \beta cI_c \right\} + 4am^3 g_2^{-1/3} p(\beta - 1)I_d \right] < 0$$

Where,  $g = \frac{4^{-2/3} (h + c I_c)^{1/3}}{3}$ .

Based on the above discussion we develop the following algorithm:

**Algorithm**

Step 1. Determine  $p_0$  on solving equation (17).

Step 2. If there exist  $p_1$  such that  $p_1 < p_0$ , and  $p_1$  satisfies both the first order condition as in (25) and the second order condition for concavity as in (26), then we find  $T_1(p_1)$  by (14), and  $Z_1(T_1(p_1), p_1)$  by (20).

Step 3. If there exists a  $p_2$  such that  $p_2 > p_0$ , and  $p_2$  satisfies both the first order condition as in (27) and the second order condition for concavity as in (28), then calculate  $T_2(p_2)$  by (16), and  $Z_2(T_2(p_2), p_2)$  by (22).

Step 4. If  $Z_1(T_1(p_1), p_1) > Z_2(T_2(p_2), p_2)$ , then optimal total annual profit is  $Z^*(T(p^*), p^*) = Z_1(T_1(p_1), p_1)$  otherwise optimal total annual profit is  $Z^*(T(p^*), p^*) = Z_2(T_2(p_2), p_2)$ .

**6. Numerical Examples**

**Example 1.** For generality, we use the following example in which  $cI_c < 2p^*I_d$ . Given  $h = .5/\text{unit/year}$ ,  $I_c = 0.09/\text{\$/year}$ ,  $I_d = 0.06/\text{\$/year}$ ,  $c = \$ 4.5 \text{ year}$ ,  $s = \$ 200/\text{per order}$   $\alpha = 100000$ , and,  $\beta = 2$ . We obtain the computational results for various values of  $m$  as shown in Table 1.

Table 1.

Optimal solution for different Trade credit period ‘m’.

M (days)	$p_0$	$p^*$	$T^*$	Q	$Z^*$
10	7.048268	$p_1 = 4.554794$	$T_1 = 0.460667$	511.4542	- 651.2292
20	7.73048	$p_1 = 4.609589$	$T_1 = 0.463956$	506.5230	- 646.6136
30	8.042800	$p_1 = 4.664384$	$T_1 = 0.467224$	501.6857	-642.0898
40	8.217605	$p_1 = 4.719178$	$T_1 = 0.470474$	496.9452	-637.6551
50	8.329943	$p_1 = 4.773973$	$T_1 = 0.473704$	492.2936	-633.3064
60	8.408180	$p_1 = 4.828767$	$T_1 = 0.476916$	487.7320	-629.0413
70	8.465783	$p_1 = 4.883562$	$T_1 = 0.480110$	483.2569	-624.8572
80	8.509957	$p_1 = 4.938356$	$T_1 = 0.483285$	478.8636	-620.7517
90	8.544907	$p_1 = 4.993151$	$T_1 = 0.486442$	474.5508	-616.7224
100	8.573246	$p_1 = 5.047945$	$T_1 = 0.489582$	470.3181	-612.7672
110	8.596687	$p_1 = 5.102740$	$T_1 = 0.492705$	466.1622	-608.8839
120	8.616398	$p_1 = 5.157534$	$T_1 = 0.495810$	462.0791	-605.0705
130	8.633205	$p_1 = 5.212329$	$T_1 = 0.498898$	458.0697	-601.3249



140	8.647704	$p_1=5.267123$	$T_1=0.501970$	454.1283	-597.6453
150	8.660341	$p_1=5.321918$	$T_1=0.505025$	450.2559	-594.0298
160	8.671452	$p_1=5.376712$	$T_1=0.508064$	446.4505	-590.4767
170	8.681298	$p_1=5.431507$	$T_1=0.511087$	442.7096	-586.9843
180	8.690084	$p_1=5.486301$	$T_1=0.514094$	439.0316	-583.5510
190	8.69797	$p_1=5.541096$	$T_1=0.517085$	435.4141	-580.1751
200	8.70509	$p_1=5.595890$	$T_1=0.520061$	431.8573	-576.8552
210	8.711551	$p_1=5.650685$	$T_1=0.523022$	428.3588	-573.5899
220	8.717438	$p_1=5.705479$	$T_1=0.525967$	424.9157	-570.3775
230	8.722826	$p_1=5.760274$	$T_1=0.528898$	421.5291	-567.2169
240	8.727775	$p_1=5.815068$	$T_1=0.531814$	418.1961	-564.1068
250	8.732336	$p_1=5.869863$	$T_1=0.534716$	414.9162	-561.0458
260	8.736555	$p_1=5.924658$	$T_1=0.537603$	411.6867	-558.0327
270	8.740467	$p_1=5.979452$	$T_1=0.540476$	408.5075	-555.0664
280	8.744105	$p_1=6.034246$	$T_1=0.543335$	405.3772	-552.1457
290	8.747467	$p_1=6.089041$	$T_1=0.546180$	402.2942	-549.2694
300	8.750667	$p_1=6.143836$	$T_1=0.549012$	399.2588	-546.4366
310	8.753636	$p_1=6.198630$	$T_1=0.551829$	396.2667	-543.6462
320	8.756423	$p_1=6.253425$	$T_1=0.55463$	393.3210	-540.8972
330	8.759044	$p_1=6.308219$	$T_1=0.557425$	390.4176	-538.1886
340	8.761513	$p_1=6.363014$	$T_1=0.560204$	387.5580	-535.5195
350	8.763843	$p_1=6.417808$	$T_1=0.562969$	384.7384	-532.8890
360	8.766046	$p_1=6.472602$	$T_1=0.565722$	381.9604	-530.2962
380	8.770109	$p_1=6.582192$	$T_1=0.571189$	376.5205	-525.2202
400	8.774437	$p_1=6.691781$	$T_1=0.576607$	371.2328	-520.2852
420	8.777089	$p_1=6.801370$	$T_1=0.581976$	366.0896	-515.4848
440	8.780109	$p_1=6.910959$	$T_1=0.587298$	361.0858	-510.8135
460	8.782869	$p_1=7.020548$	$T_1=0.592574$	356.2158	-506.2658
480	8.785402	$p_1=7.130137$	$T_1=0.597804$	351.4729	-501.8365
500	8.787735	$p_1=7.239726$	$T_1=0.602990$	346.8534	-497.5207
520	8.789891	$p_1=7.349315$	$T_1=0.608132$	342.3513	-493.3140
540	8.791888	$p_1=7.458904$	$T_1=0.613231$	337.9622	-489.2120
560	8.797785	$p_1=7.568493$	$T_1=0.618288$	333.682	-485.2106
580	8.795473	$p_1=7.678082$	$T_1=0.623304$	329.5068	-481.3060
600	8.797088	$p_1=7.787671$	$T_1=0.628280$	325.4328	-477.4943
620	8.798600	$p_1=7.897260$	$T_1=0.633216$	321.4555	-473.772
640	8.800018	$p_1=8.006844$	$T_1=0.638113$	317.5718	-470.1364
660	8.801351	$p_1=8.116438$	$T_1=0.642972$	313.7785	-466.5836

680	8.802606	$p_1=8.226027$	$T_1=0.647793$	310.0718	-463.1110
700	8.803790	$p_1=8.335616$	$T_1=0.652577$	306.4489	-459.7157
720	8.804908	$p_1=8.445205$	$T_1=0.657325$	302.9074	-456.3949
740	8.805966	$p_1=8.554794$	$T_1=0.662038$	299.4448	-453.1462
760	8.806970	$p_1=8.66438$	$T_1=0.666716$	296.0577	-449.9669
780	8.807922	$p_1=8.773973$	$T_1=0.671359$	292.7434	-446.8548
800	8.808827	$p_1=8.883562$	$T_1=0.675968$	289.4996	-443.8076
820	8.809688	$p_1=8.993151$	$T_1=0.680545$	286.325	-440.8233
860	8.811291	$p_1=9.212329$	$T_1=0.689600$	280.1726	-435.0349
900	8.812753	$p_1=9.431507$	$T_1=0.698528$	274.2682	-429.4744
940	8.814090	$p_1=9.650685$	$T_1=0.707334$	268.5980	-424.1276
980	8.815319	$p_1=9.869863$	$T_1=0.716021$	263.1475	-418.9818
1020	8.816453	$p_1=9.0890411$	$T_1=0.724594$	257.9050	-414.0252
1060	8.817501	$p_1=10.308219$	$T_1=0.733054$	252.8568	-409.2469
1100	8.818473	$p_1=10.527397$	$T_1=0.741406$	247.9934	-404.6367
1140	8.819377	$p_1=10.746575$	$T_1=0.749652$	243.3040	-400.1855
1180	8.820221	$p_1=10.965753$	$T_1=0.757797$	238.7805	-395.8846
1220	8.821009	$p_1=11.184932$	$T_1=0.765841$	234.4124	-391.7260
1260	8.821747	$p_1=11.404110$	$T_1=0.773789$	230.1931	-387.7020
1300	8.822441	$p_1=11.623288$	$T_1=0.781643$	226.1147	-383.8068

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$$Q^* = 407.8278$$

Table 1 reveals that (a) a higher value of trade credit period 'm' causes a higher value of  $Z^*$  and higher values of  $p^*$  and  $T^*$ . (b) a higher value of 'm' causes a lower value of  $Q^*(T)$ . From equation (23) the classical EOQ,  $Q^* = 407.8278$  which confirms the result in part (b) of Theorem 2 (i.e.  $Q^*(T_1) < Q^*$ , if  $cI_c < 2pI_d$ ), which is applicable only for credit period 280 days or more than 280 days. Less than 280 days credit period Theorem 2 contradicts the hypothesis. From the above example we are unable to obtain any value of  $p_2$  which is greater than or equal to  $p_0$ . Hence we consider only  $T_1^*$ ,  $p_1^*$ ,  $Q^*(T_1)$  and  $Z(T_1^*)$  only to compare the result. The special cases (A) and (B) are applicable for limited range, limited value of credit periods for managerial point of view.

## 7. Conclusion and Future Research

In this paper, we developed an appropriate pricing and lot sizing model for a retailer when the supplier provides permissible delay in payments. We establish the necessary and sufficient conditions for the unique optimal replenishment interval by taking particular cases i.e. case (A) and case (B). Next we derive the first and second order conditions for finding the optimal price. We establish Theorem 1, which provides us to obtain the optimal replenishment interval by taking particular case (A) and case (B). We also obtained Theorem 2 on these particular cases, we also verified case  $T_1 \leq m$ , for  $c = p(1 + I_d m)$  and  $T_2 \geq m$ , for  $p = (1 - I_c m)$ . On these particular cases (A) and (B), the total annual profit is negative which gives us contradictory results. On particular cases (A) and (B) we obtained total annual loss (due to negative sign of  $Z_1(T, p)$  and  $Z_2(T, p)$ ), while Fig 2 and Fig.3 proves the theoretical results (curve is concave in both the cases). Numerical example is given to illustrate the model.

The model proposed in this paper can be extended in several ways. For instance, we may extend the model by considering time dependent deterioration rate. Also we could consider the demand as a function of quantity. We could generalize the model to allow for shortage, quantity, discounts and inflation rates etc.

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