# Mathematical Modeling of Life Insurance Policies 

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#### Abstract

The Life Insurance Company calculates the policy price with intent to recover claims to be paid and administrative cost and to make profit. The cost of insurance is determined using Mortality Table calculated by Actuaries. The insurance companies receive premiums from the policy owner and invest them to create a pool of money from which to pay claims and finance the insurance company's operations. Rates charged for life insurance increase with insured's age because statistically people are more likely to die as they get older. In this paper, we discussed about different insurance policies including expenses. We have also discussed about the annual premium rates of both endowment plan and three-payment plans. We have used mathematical programming to calculate the premium rates.


Keywords: Insurance, Premium, Endowment Assurance, Life Annuities.

## 1. Introduction

Actuarial Science applies mathematical and statistical methods to finance, insurance particularly to risk assessment. Actuarial Mathematics deals with the mathematics of uncertainty and risk (Bowers, Gerber, Hickman, Jones and Nesbitt 1986). Some key areas where actuarial mathematics is principally applied are mortality study, financial risk, risk and ruin theory, credibility etc. Basic ideas from calculus, linear algebra, numerical analysis, statistics, mathematical programming and economics will appear as a building block in models of insurance system (Dixit, Modi and Joshi 2002).

An insurance system is a mechanism for reducing the adverse financial impact of random events that prevents the fulfillment of reasonable expectations, i.e. Insurance is designed to protect against serious financial reversals that may result from random events intruding on the plans of individuals. The face amount of the policy is normally the amount paid when the policy matures, although policies can provide greater or lesser amounts. The policy matures when the insured dies or reaches a specified age. The most common reason to buy a life insurance policy is to protect the financial interest of the owner of the policy in the event of the insured's demise (Uddin 1999). Life insurance is must for all and sundry who have family to look after. Notwithstanding this, the need is not equal for all. For rich people it may be a luxury but for low income community it is a must. Yet the later sections of the society are not convinced that they need it. The Government and Non Government Organizations need.

The Life Insurance Company calculates the policy price with intent to recover claims to be paid and administrative cost and to make profit (Ali 2004). The cost of insurance is determined using Mortality Table calculated by Actuaries. The insurance companies receive the premiums from the policy owner and invest them to create a pool of money from which to pay claims, and finance the insurance company's operations (Alam 1993). Rates charged for life insurance increases with insured's age because statistically people are more likely to die as they get older. Since adverse selection can have a negative impact on the financial results of the insurer, the insurer investigates each proposed insured beginning with the application which becomes a part of the policy (Ziam and Brown 2005).

Life insurance is must for all and sundry who have family to look after. Notwithstanding this, the need is not equal for all. For rich people it may be a luxury but for low income community it is a must. Yet the later sections of the society are not convinced that they need it. The Government and Non Government Organizations need. Lack of insurance can contribute to inequality in the society as whole and which has a direct effect on the economic growth of any country. Life insurance companies, selling agents, NGOs and the Government should take this issue together as a challenge equally. However considering the fact that number of people with low income far exceeds those with higher incomes, therefore low premium life insurance policies with no frills can also be remunerative to the selling agents and the Life Insurance companies, as low margins of profit would be more that offset by the high volumes of policies sold (Chaudhury 1994) . Need in such awareness should be created in both sides, life insurance companies, their selling agents and the concerned strata of people with low incomes. Here is where the regulatory bodies, Government and Non Government Organization can play important roles.

In this paper, we discussed about different insurance policies including expenses. We discussed about the annual premium rates of both endowment plan and three-payment plans. We have used mathematical programming to calculate the premium rates.

## 2. Mathematical Formulations

## Whole Life Assurance

The essence of Whole Life Assurance is that it provides for the payment of the face amount upon the death of the insured regardless of when the death occurs. This is one of the simplest forms of life assurance (Hafiz, Islam, and Chowdury 1995). The value of whole life assurance of 1 payable to a person aged $x$ i.e. the present value of the assurance is denoted by $A_{x}$, and is given by

$$
\begin{aligned}
& A_{x}=V \frac{d_{x}}{l_{x}}+V^{2} \frac{l_{x+1}}{l_{x}} \frac{d_{x+1}}{l_{x+1}}+V^{3} \frac{l_{x+1}}{l_{x}} \frac{l_{x+2}}{l_{x+1}} \frac{d_{x+2}}{l_{x+2}}+\cdots \\
& A_{x}=\frac{V d_{x}+V^{2} d_{x+1}+V^{2} d_{x+2}+\cdots}{l_{x}}
\end{aligned}
$$

Introducing Commutation Functions

$$
\begin{aligned}
& A_{x}=\frac{V^{x+1} d_{x}+V^{x+2} d_{x+1}+V^{x+3} d_{x+2}+\cdots}{l_{x} V^{x}} \\
& A_{x}=\frac{C_{x}+C_{x+1}+C_{x+2}+\cdots}{D_{x}}
\end{aligned}
$$

$$
A_{x}=\frac{M_{x}}{D_{x}}
$$

## Temporary or Term Assurance

Term life insurance furnishes life insurance protection for a limited number of years, the face amount of the policy being payable only if death occurs during the stipulated term and nothing being paid in case of survival. The value of $n$ year term assurance of 1 on the life of a person aged $x$ is denoted by $\mathbf{A}_{\mathbf{x}: \overline{\mathrm{n}} \mid}^{1}$. The number " 1 " over " $x$ " indicates that in order for the sum assured becoming payable the status ( $x$ ) must come to an end before the status $n$. Thus the expression for the present value of this assurance of 1 payable on death during $n$-year term is given by

$$
\mathbf{A}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}=\frac{V d_{x}+V^{2} d_{x+1}+V^{3} d_{x+2}+\cdots+V^{n} d_{x+n-1}}{l_{x}} ; \mathbf{A}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}=\frac{V^{x+1} d_{x}+V^{x+2} d_{x+1}+\cdots+V^{x+n} d_{x+n-1}}{l_{x} V^{x}}
$$

Introducing Community Function

$$
\begin{aligned}
& \mathrm{A}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}=\frac{C_{x}+C_{x+1}+C_{x+2}+\cdots+C_{x+n-1}}{D_{x}} \\
& \mathrm{~A}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}=\frac{\left\{C_{x}+C_{x+1}+\cdots\right\}-\left\{C_{x+n}+C_{x+n+1}+\cdots\right\}}{D_{x}} \\
& =\frac{\sum_{t=0}^{\infty} C_{x+t}-\sum_{t=n}^{\infty} C_{x+t}}{D_{x}}=\frac{M_{x}-M_{x+n}}{D_{x}}
\end{aligned}
$$

## Pure Endowment Assurance

An $n$-year pure endowment provides for payment at the end of the nth year if and only if the insured survives at least $n$ years from the time of policy issue The value of $n$-year pure endowment assurance of 1 on the life of a person aged $x$, is denoted by $\quad \begin{gathered}1 \\ x: \bar{n} \mid\end{gathered}$ is given by

$$
\mathrm{A}_{x: \bar{n} \mid}^{1}=\frac{V^{n} l_{x+n}}{l_{x}}=\frac{V^{x+n} l_{x+n}}{V^{x} l_{x}}=\frac{D_{x+n}}{D_{x}}
$$

## Endowment Assurance

An n - year endowment insurance provides for an amount to be payable either following the death of insured or upon the survival of the insured to the end of the $n$-year term, whichever occur first. This is a combination of pure endowment and temporary assurance. The present value of the assurance of 1 under this plan is denoted by $\mathrm{A}_{x: \bar{n} \mid}$.

$$
\begin{aligned}
& \mathrm{A}_{x: \bar{n} \mid}=\mathrm{A}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}+\mathrm{A}_{x: \bar{n} \mid}^{1} \\
&=\frac{\left\{V d_{x}+V^{2} d_{x+1}+\cdots+V^{n} d_{x+n-1}\right\}+V^{n} l_{x+n}}{l_{x}} \\
&=\frac{\left(C_{x}+C_{x+1}+\cdots\right)-C_{x+n-1}}{D_{x}}+\frac{D_{x+n}}{D_{x}}
\end{aligned}
$$

$$
\mathrm{A}_{x: \bar{n} \mid}=\frac{M_{x}-M_{x+n}+D_{x+n}}{D_{x}}
$$

## Life Annuities

A life annuity is a series of payments made continuously or at equal intervals while a given life survives. It may be temporary, that is, limited to a given term of years, or it may be payable for the whole of life. The payment intervals may commence immediately or, alternatively, the annuity may be deferred.

## Annuity Due

Consider $l_{x}$ lives. Since the payments are to be made at the beginning of each year, $l_{x}$ lives will receive first payment at the present time.

$$
\ddot{a}_{x}=1+V \frac{l_{x+1}}{l_{x}}+V^{2} \frac{l_{x+2}}{l_{x}}+\cdots
$$

Introducing the commutation functions, we have

$$
\ddot{a}_{x}=\frac{D_{x}+D_{x+1}+D_{x+2}+\cdots}{D_{x}}=\frac{N_{x}}{D_{x}}
$$

## Temporary Annuities

A temporary life annuity is a series of payments made at regular intervals to a person during his life time for a specified period, each payment being made at the end of each year of life during $n$ years. The present value of such annuity is denoted by $a_{x: n}$. Thus

$$
\begin{aligned}
a_{x: n} 7 & =V \frac{l_{x+1}}{l_{x}}+V^{2} \frac{l_{x+2}}{l_{x}}+\cdots+V^{n} \frac{l_{x+n}}{l_{x}} \\
& =\frac{D_{x+1}}{D_{x}}+\frac{D_{x+2}}{D_{x}}+\cdots+\frac{D_{x+n}}{D_{x}}+\frac{D_{x+n+1}}{D_{x}}+\cdots-\frac{D_{x+n+1}}{D_{x}}-\frac{D_{x+n+2}}{D_{x}}-\cdots \\
a_{x: n}= & =\frac{\sum_{t=1}^{\infty} D_{x+t}-\sum_{t=n+1}^{\infty} D_{x+t}}{D_{x}}=\frac{N_{x+1}-N_{x+n+1}}{D_{x}}
\end{aligned}
$$

## Temporary Life Annuities Due

If instead at the end of the year, the n payments are made at the beginning of each year, the series of payments are known as temporary life annuity due for n years. The present value of temporary life annuity due of 1 to a person aged $X$ is denoted by $\ddot{a}_{x: n\rceil}$, and its value is given by

$$
\ddot{a}_{x: n}=\left(\frac{D_{x}}{D_{x}}+\frac{D_{x+1}}{D_{x}}+\cdots+\frac{D_{x+n-1}}{D_{x}}+\frac{D_{x+n}}{D_{x}}+\cdots\right)-\left(\frac{D_{x+n}}{D_{x}}+\frac{D_{x+n+1}}{D_{x}}+\cdots\right)
$$

$$
\ddot{a}_{x: n}=\frac{\sum_{t=0}^{\infty} D_{x+t}-\sum_{t=n}^{\infty} D_{x+t}}{D_{x}}=\frac{N_{x}-N_{x+n}}{D_{x}}
$$

## 3. Mathematics of Premiums

## Net premiums for Assurance Plans

The net premiums are obtained by dividing the present value of benefits by the present value of premiums. Present value of various assurance plans also represents the single premium to be paid at the beginning of a contract to secure the benefits under the assurance plan.

## Whole Life Assurance

Let $P_{x}$ be the annual premium for a whole life assurance of 1 on the life aged $x$. Under this plan the premium is payable throughout the life time of the assured. The value of the of the premium would therefore, be equal to $P_{x}$ $\ddot{a}_{x}$. We also know that the value of the whole life sum assured of 1 is $A_{x}$. Therefore, we get $P_{x} \ddot{a}_{x}=A_{x}$

$$
P_{x}=M_{x} / N_{x}
$$

## Temporary Assurance

Under this plan life assured aged $\boldsymbol{x}$ will pay the level annual premium $\mathbf{P}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}$ at the beginning of each policy year for $\mathbf{n}$ years.

The value of temporary assurance of 1 on a life aged $x$, is $A_{x: \bar{n} \mid}^{1}$. The present value of the premium is

$$
\mathrm{P}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1}=\frac{M_{x}-M_{x+n}}{N_{x}-N_{x+n}}
$$

## n - Year Endowment Assurance

The value of an $n$-year endowment assurance on 1 of the life aged $x$ is ${ }^{\mathrm{A}} x: \bar{n} \mid$. The present value of the premiums
is $_{x: \bar{n} \mid}{ }^{\ddot{a}_{x: n}}$. Hence

$$
\begin{aligned}
& \mathrm{P}_{x: \bar{n} \mid}^{\ddot{u}_{x: n}}=\mathrm{A} \\
& x: \bar{n} \mid \\
& \mathrm{P}_{x: \bar{n} \mid}=\frac{M_{x}-M_{x+n}+D_{x+n}}{N_{x}-N_{x+n}}
\end{aligned}
$$

## Insurance Models Including Expenses

A more realistic view of the insurance business includes provision for expenses. The profit for the company can also be included here as an expense. The common method used for the determination of the expenses loaded premium is a modification of the equivalence principle. According to the modified equivalence principle the gross premium P is set to that on the policy issue date the actuarial present value of the benefit plus expenses is equal to the actuarial present value of the premium income. The premium is usually considered to be constant. Under these assumptions it is fairly easy to write a formula to determine P . Three elements which is to be taken into consideration while designing a product and pricing the product, i.e to calculate the premium are:

1) Rate of mortality
2) Expenses incurred by life insurance business
3) Rate of return on investment.

## Product Design

As per art. 39 of the insurance rule 1958, the limitation of expenses of management (including commission and any other remuneration for procreation of business) in any calendar year is an amount not exceeding $90 \%$ of the $1^{\text {st }}$ year premium and $15 \%$ of renewal premium for a life insurance company whose year of operation are 10 years or more and terms of the insurance policy not less than 12 years.

## Annual premium of an endowment plan

We calculate the annual Premium of a product which provides benefit of Tk. 1000 on survival up to maturity and Tk. 1000 on death before maturity. This type of plan is called endowment plan.

If we consider the term of the policy to be $n$ years and we want to calculate the annual premium for a person aged $(x)$, if $P$ is the annual premium then,

$$
\begin{aligned}
& \text { Value of death benefit is } 1000 \mathrm{~A}_{\mathrm{x}: \overline{\mathrm{n}} \mid}^{1} \\
& \text { Value of survival benefit is } 1000 \mathrm{~A}^{1} \begin{array}{l}
1 \\
\\
x: \bar{n} \mid
\end{array}
\end{aligned}
$$

Hence the present value of the premium is $\left.P \ddot{a}_{x: n}\right\rceil$. Considering the expenses following the rule of insurance act we have

$$
\begin{align*}
\left.P \ddot{x}_{x: n}\right\urcorner & =1000 \mathrm{~A}_{\mathrm{x}: \bar{n} \mid+1000}^{1} \mathrm{~A}_{x:\left.\bar{n}\right|^{+.75 \mathrm{P}+.15 \mathrm{Pä} \mathrm{x}: n 7}}^{1} \\
P & =\frac{1000\left(M_{x}-M_{x+n}+D_{x+n}\right)}{0.85\left(N_{x}-N_{x+n}\right)-0.75 D_{x}} \ldots \ldots \ldots \ldots \tag{1}
\end{align*}
$$

We use Mathematical Program for equation (1). We obtain a polynomial for all the commutation function using Newton's Forward Interpolation method (Burden and Faires 2003).

The annual premium table per Tk. 1000 for an insurance policy of term 15 years using mathematical program is given below.

Table for 15 years plan

| Age | Premiums | Age | Premiums |
| :---: | :---: | :---: | :---: |
| 20 | 64.419 | 41 | 66.260 |
| 21 | 64.628 | 42 | 66.605 |

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| 22 | 64.508 | 43 | 66.927 |
| :---: | :---: | :---: | :---: |
| 23 | 64.436 | 44 | 67.398 |
| 24 | 64.429 | 45 | 67.871 |
| 25 | 64.449 | 46 | 68.465 |
| 26 | 64.471 | 47 | 68.989 |
| 27 | 64.493 | 48 | 69.670 |
| 28 | 64.518 | 49 | 70.344 |
| 29 | 64.549 | 50 | 71.442 |
| 30 | 64.588 | 51 | 72.014 |
| 31 | 64.641 | 52 | 72.851 |
| 32 | 64.703 | 53 | 74.255 |
| 33 | 64.783 | 54 | 75.366 |
| 34 | 64.878 | 55 | 76.727 |
| 35 | 65.001 | 56 | 78.372 |
| 36 | 65.127 | 57 | 80.885 |
| 37 | 65.300 | 58 | 81.671 |
| 38 | 65.494 | 59 | 84.532 |
| 39 | 65.696 | 60 | 86.239 |
| 40 | 65.966 |  |  |

The annual premium Table per Tk. 1000 of a life insurance policy of term 20 is given below which is also obtained by using mathematical programming.

Table for 20 years plan

| Age | Premiums | Age | Premiums |
| :---: | :---: | :---: | :---: |
| 20 | 43.605 | 41 | 46.297 |
| 21 | 43.783 | 42 | 46.685 |
| 22 | 43.691 | 43 | 47.192 |
| 23 | 43.640 | 44 | 47.698 |
| 24 | 43.646 | 45 | 48.500 |
| 25 | 43.676 | 46 | 48.966 |
| 26 | 43.714 | 47 | 49.620 |
| 27 | 43.752 | 48 | 50.655 |
| 28 | 43.799 | 49 | 51.483 |
| 29 | 43.855 | 50 | 52.496 |
| 30 | 43.929 | 51 | 53.724 |
| 31 | 44.007 | 52 | 55.527 |
| 32 | 44.116 | 53 | 56.238 |
| 33 | 44.239 | 54 | 58.235 |
| 34 | 44.369 | 55 | 59.697 |
| 35 | 44.546 | 56 | 61.211 |
| 36 | 44.742 | 57 | 63.401 |
| 37 | 44.976 | 58 | 67.054 |
| 38 | 45.203 | 59 | 75.998 |
| 39 | 45.531 | 60 | 74.447 |
| 40 | 45.867 |  |  |

The following curve (Fig. 1) shows the variation of premiums with respect to the age for a 15 years and 20 years insurance policy.


Figure 1. Variation of premiums with respect to the age for 15 years and 20 years insurance policy

## Annual premium for a three-payment plan

In three payments plan survival benefit is given at 3 stages of the total term of the policy. If the term of the policy is 12 years then we may consider that $25 \%$ of the sum assured is provided after the expiry of 4 years, $25 \%$ of the sum assured is provided after the expiry 8 years and finally $50 \%$ of the sum assured is provided at the end of the term i.e. after 12 years as survival benefit. So the Mathematical formulation for a three payment plan, where the basic sum assured is Tk 1000 using the commutation function $M_{x}, D_{x}, N_{x}$ for $n$ years and for a person aged $x$ is:

$$
\text { Value of survival benefit is } 1000 \mathrm{~A} \begin{gathered}
1 \\
x: \bar{n} \mid
\end{gathered}
$$

The present value of the premium is $\left.P \ddot{a}_{x: n}\right\rceil$. Considering the expenses following the rule of insurance act we have,

$$
P=\frac{1000\left(M_{x}-M_{x+n}\right)+250 D_{x+n / 3}+250 D_{x+2 n / 3}+500 D_{x+n}}{0.85\left(N_{x}-N_{x+n}\right)-0.75 D_{x}}
$$

The annual premium table per Tk. 1000 of a three-payment plan for a term of 12 years using mathematical program is given below.

Table for 12 years three-payment plan

| Age | Premiums | Age | Premiums |
| :---: | :---: | :---: | :---: |
| 20 | 96.361 | 41 | 98.982 |
| 21 | 96.605 | 42 | 99.416 |
| 22 | 96.473 | 43 | 99.929 |
| 23 | 96.393 | 44 | 100.501 |
| 24 | 96.387 | 45 | 101.158 |
| 25 | 96.413 | 46 | 101.822 |
| 26 | 96.443 | 47 | 102.611 |
| 27 | 96.474 | 48 | 103.480 |
| 28 | 96.510 | 49 | 104.479 |
| 29 | 96.555 | 50 | 105.382 |
| 30 | 96.615 | 51 | 106.613 |
| 31 | 96.689 | 52 | 107.808 |
| 32 | 96.783 | 53 | 109.474 |


| 33 | 96.896 | 54 | 110.751 |
| :---: | :---: | :---: | :---: |
| 34 | 97.035 | 55 | 112.320 |
| 35 | 97.199 | 56 | 114.560 |
| 36 | 97.398 | 57 | 116.386 |
| 37 | 97.629 | 58 | 118.516 |
| 38 | 97.905 | 59 | 121.045 |
| 39 | 98.208 | 60 | 124.952 |
| 40 | 98.573 |  |  |

Table for 15 years three-payment plan

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Age $\quad \frac{\text { www.iiste.org }}{\text { Premiums }}$

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| Vol 3, No. ${ }_{2}{ }^{2} 011$ | 74.431 | 42 | 78.031 |
| :---: | :---: | :---: | :---: |
| 22 | 74.322 | 43 | 78.596 |
| 23 | 74.259 | 44 | 79.250 |
| 24 | 74.266 | 45 | 79.993 |
| 25 | 74.301 | 46 | 80.827 |
| 26 | 74.344 | 47 | 81.637 |
| 27 | 74.392 | 48 | 82.617 |
| 28 | 74.447 | 49 | 83.678 |
| 29 | 74.517 | 50 | 85.047 |
| 30 | 74.602 | 51 | 86.179 |
| 31 | 74.708 | 52 | 87.477 |
| 32 | 74.835 | 53 | 89.313 |
| 33 | 74.989 | 54 | 90.827 |
| 34 | 75.169 | 55 | 92.816 |
| 35 | 75.386 | 56 | 94.816 |
| 36 | 75.626 | 57 | 97.964 |
| 37 | 75.916 | 58 | 99.906 |
| 38 | 76.243 | 59 | 103.277 |
| 39 | 76.599 | 60 | 106.092 |
| 40 | 77.022 |  |  |

The following curve (Fig. 2) shows the variation of premiums with respect to the age for a 12 years and 15 years three-payment plan.


Figure 2. Variation of premiums with respect to the age for 12 years and 15 years insurance policy

## Premium Table of American Life Insurance Company (Alico)

| Age | 12 years <br> 3PPP | 15 years 3PPP | Age | 12 years 3PPP | 15 years 3PPP |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 100.80 | 80.70 | 35 | 102.20 | 82.60 |
| 21 | 100.80 | 80.80 | 36 | 102.50 | 82.90 |
| 22 | 100.80 | 80.80 | 37 | 102.80 | 83.20 |

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| 23 | 100.90 | 80.90 | 38 | 103.10 | 83.60 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | 100.90 | 80.90 | 39 | 103.40 | 84.00 |
| 25 | 100.90 | 81.00 | 40 | 103.80 | 84.40 |
| 26 | 101.00 | 81.10 | 41 | 104.30 | 85.00 |
| 27 | 101.10 | 81.10 | 42 | 104.90 | 85.60 |
| 28 | 101.10 | 81.30 | 43 | 105.50 | 86.30 |
| 29 | 101.20 | 81.40 | 44 | 106.20 | 87.10 |
| 30 | 101.30 | 81.50 | 45 | 106.90 | 87.90 |
| 31 | 101.50 | 81.70 | 46 | 107.60 | 88.80 |
| 32 | 101.60 | 81.90 | 47 | 108.50 | 89.70 |
| 33 | 101.80 | 82.10 | 48 | 109.40 | 90.70 |
| 34 | 102.00 | 82.30 | 49 | 110.40 | 91.80 |

We have seen that the premium rate charged by ALICO, and different life insurance companies like Delta Life Insurance, National Life Insurance etc. are higher than what we have calculated using mathematical program. This may be due to the following reasons:

1) Since we haven't got any information about the calculation of the premium rates from different existing company.
2) The rate of interest assumed by me in the premium rates calculation is higher than what have been assumed by

| Age | 15 years <br> Endowment <br> plan | 20 years <br> Endowment <br> plan | Age | 15 years <br> Endowment plan | 20 years <br> Endowment <br> plan |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 70.99 | 53.85 | 38 | 74.63 | 58.56 |
| 21 | 71.07 | 53.95 | 39 | 75.15 | 59.29 |
| 22 | 71.16 | 54.07 | 40 | 75.73 | 60.12 |
| 23 | 71.26 | 54.26 | 41 | 76.38 | 61.04 |
| 24 | 71.37 | 54.32 | 42 | 77.11 | 62.05 |
| 25 | 71.48 | 54.47 | 43 | 77.93 | 63.15 |
| 26 | 71.60 | 54.79 | 44 | 78.85 | 64.34 |
| 27 | 71.71 | 54.79 | 45 | 79.90 | 65.65 |
| 28 | 71.83 | 54.98 | 46 | 81.10 | 67.12 |
| 29 | 71.97 | 55.19 | 47 | 82.46 | 68.80 |
| 30 | 72.13 | 55.41 | 48 | 83.98 | 70.74 |
| 31 | 72.31 | 55.65 | 49 | 85.66 | 72.98 |
| 32 | 72.52 | 55.93 | 50 | 87.51 | 75.57 |
| 33 | 72.76 | 56.24 | 51 | 89.55 | 78.47 |
| 34 | 73.04 | 56.58 | 52 | 91.81 | 81.67 |
| 35 | 73.37 | 56.96 | 53 | 94.29 | 85.17 |
| 36 | 73.75 | 57.40 | 54 | 97.00 | 88.98 |
| 37 | 74.17 | 57.93 | 55 | 100.00 | 93.10 |

these companies.
3) Expenses loaded in the premium determination formula are higher than what have been allowed in the insurance rule.
4) A combination of both the above reasons.

## 4. Conclusion

In this paper, we have presented how one can apply mathematical programs to calculate the annual premiums of various insurance policies. It is very difficult to get the age specific premium rates but by coding mathematical
program we can easily get the premium rates for different insurance policies for a person aged $(x)$ and for a term of the policy of $n$ years in a customized way. It is found that life expectancy of the insured population is more than the actual population, that means insurance companies are charging more premium rates than what they should charge. In this paper, we have discussed how to evaluate the premium of different assurance plans such as whole life assurance, temporary assurance, endowment assurance etc. We have calculated the premium for different life insurance policies like endowment assurance plan, three payment plan, six payment plan, twelve payment plan, and micro life insurance policy using Mathematical Program. At the beginning different commutation function has been evaluated which are further used to calculate the premium of a person aged ( $x$ ) for an insurance policy of term n years, using Newton's Forward interpolation method. Then these functions are used to evaluate the premium of a person using Mathematical Program.

We have calculated the annual premium for an n- year endowment assurance of the life aged $(x)$, where the basic sum assured is Tk.1000. We also calculated the annual premium for a three payment plan. The basic sum assured for all these policy is TK.1000. Then we have given a tabular form of premium rates for these policies and we have also compared it with the premium rates of some existing companies like American Life Insurance Company, Delta Life Insurance Company and Popular Life Insurance Company etc. We have found that the premium rates of these companies are higher than that we have computed, and we have came to a conclusion that these variation in the premium rates might occur because of the following reasons.

1) The rate of interest assumed by us in the premium rates calculation is higher than what have been assumed by these companies.
2) Expenses loaded in the premium determination formula are higher than what have been allowed in the insurance rule.
3) Or a combination of both the above reasons.

Three-payment plan is a very popular life insurance plan. In a three-payment Life Insurance Plan of term 12 years the insurer pays premium after every 4 years. On the continuation of three payment plan we have proposed six-payment plan and twelve-payment plan. We have seen that customer will be more interested to buy a sixpayment plan rather than buying a three-payment, on the other hand customer will be more interested in buying a twelve-payment plan rather than buying a six-payment plan. This is because if the term of the policy is 12 years then in a six-payment plan customer will get some part of his sum assured at the end of every $2^{\text {nd }}$ year while on the other hand in a three-payment plan the customer will get some part of his sum assured after every four years. Similarly in a twelve-payment plan for a policy of term 12 years a customer will get some part of his sum assured after every one year. Hence the customer will be more attracted towards a twelve-payment plan. At the same it will be easier for the company to convince people to buy a six-payment plan rather than to buy a three-payment plan and to buy a twelve-payment plan rather than buying a six-payment.

We have also calculated premium rates for micro insurance policies for low class population of the country. Here we have considered the basic sum assured to be 6000 and we have lower the expenses. Again since it is easier for the poor people to give premium monthly hence we have calculated the premium rates monthly rather than annually as we have calculated for other policies. We have come into a conclusion that these companies are charging more premium rates than what they should actually charge i.e. the insurance companies are earning more profits than usual.

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