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On common Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces

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Abstract
In this paper, we prove some common fixed point theorems for weakly compatible maps in intuitionistic fuzzy metric space.

Keywords: Intuitionistic fuzzy metric space, weakly compatible mappings.

1.Introduction:

It proved a turning point in the development of fuzzy mathematics when the notion of fuzzy set was introduced by Zadeh [Zadeh, 1965]. Atanassov [Atanassov,1986] introduced and studied the concept of intuitionistic fuzzy sets. Coker [Coker, 1997] introduced the concept of intuitionistic fuzzy topological spaces. Alaca et al. [Alaca, C. et. Al.2006] proved the well-known fixed point theorems of Banach [Banach 1932] in the setting of intuitionistic fuzzy metric spaces. Later on, Turkoglu et al. [Turkoglu et al. ,2006] proved Jungck's [Jungck, 1998] common fixed point theorem in the setting of intuitionistic fuzzy metric space.Turkoglu et al. [Turkoglu et al. ,2006] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [Pant, 1994]. Gregori et al. [Gregori et al. ,2006], Saadati and Park [Sadati et. al. 2006] studied the concept of intuitionistic fuzzy metric space and its applications.No wonder that intuitionistic fuzzy fixed point theory has become an area of interest for specialists in fixed point theory as intuitionistic fuzzy mathematics has covered new possibilities for fixed point theorists. Recently, many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric spaces (Dimri et.al. 2010, Grabiec1988,Ibdad et. al. 2006).

2.Preliminaries:

We begin by briefly recalling some definitions and notions from fixed point theory literature that we will use in the sequel.

Definition 2.1 [Schweizer st. al.1960] - A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norms if $*$ satisfying conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a*1=a$ for all $a\in[0,1]$;

(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.

Examples of the t-norms are $a * b = \min\{a,b\}$ and $a * b = a.b$.

Definition 2.2[Schweizer st. al.1960**]** A binary operation $\lozenge : [0,1] \times [0,1] \to [0,1]$ is continuous t-conorm if $\lozenge$ is satisfying the following conditions:
(i) $\lozenge$ is commutative and associative;
(ii) $\lozenge$ is continuous;
(iii) a ◊ 0 = a for all $a \in [0,1]$;
(iv) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d for all $a,b,c,d \in [0,1]$.

Examples of the t-norms are $a \lozenge b = \max\{a,b\}$ and $a \lozenge b = \min\{1, a+b\}$.

Definition 2.3[Alaca, C. et. Al. 2006**]** A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions:
(i) $M(x,y,t) + N(x,y,t) \le 1$ for all $x,y \in X$ and $t > 0$;
(ii) $M(x,y,0) = 0$ for all $x,y \in X$;
(iii) $M(x,y,t) = 1$ for all $x,y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x,y,t) = M(y,x,t)$ for all $x,y \in X$ and $t > 0$;
(v) $M(x,y,t) * M(y,z,s) \le M(x,z,t+s)$ for all $x,y,z \in X$ and s, $t > 0$;
(vi) for all $x,y \in X$, $M(x,y,.) : [0,\infty) \to [0,1]$ is left continuous;
(vii) $\lim_{t\to\infty} M(x,y,t) = 1$ for all $x,y \in X$ and $t > 0$;
(viii) $N(x,y,0) = 1$ for all $x,y \in X$;
(ix) $N(x,y,t) = 0$ for all $x,y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x,y,t) = N(y,x,t)$ for all $x,y \in X$ and $t > 0$;
(xi) $N(x,y,t) \lozenge N(y,z,s) \ge N(x,z,t+s)$ for all $x,y,z \in X$ and s, $t > 0$;
(xii) for all, $x,y \in X, N(x,y,.) : [0,\infty) \to [0,1]$ is right continuous;
(xiii) $\lim_{t\to\infty} N(x,y,t) = 0$ for all $x,y \in X$;
(M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4[Alaca, C. et. Al. 2006**].** An intuitionistic fuzzy metric spaces with continuous t-norm $*$ and Continuous t -conorm $\lozenge$ defined by $a * a \ge a, a \in [0,1]$ and $(1-a) \lozenge (1-a) \le (1-a)$ for all $a \in [0,1]$, Then for all $x,y \in X, M(x,y,*)$ is non-decreasing and, $N(x,y,\lozenge)$ is non-increasing**.**

Remark 2.5[Park, 2004**].** Let (X,d) be a metric space .Define t-norm $a*b=\min\{a,b\}$ and t-conorm $a\Diamond b=\max\{a,b\}$ and for all $x,y\in X$ and $t>0$

$$M_d(x,y,t)=\frac{t}{t+d(x,y)}\ ,\ N_d(x,y,t)=\frac{d(x,y)}{t+d(x,y)}$$

Then $(X,M,N,*,\Diamond)$ is an intuitionistic fuzzy metric space induced by the metric . It is obvious that $N(x,y,t)=1-M(x,y,t)$.

Alaca, Turkoglu and Yildiz [Alaca, C. et. Al. 2006] introduced the following notions:

Definition 2.6. Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space. Then

(i) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t>0$ and $p>0$

$$\lim_{n\to\infty}M(x_{n+p,}x_{n,}t)=1,\lim_{n\to\infty}N(x_{n+p,}x_{n,}t)=0$$

(ii) a sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ if, for all $t>0$,

$$\lim_{n\to\infty}M(x_{n,}x,t)=1,\lim_{n\to\infty}N(x_{n,}x,t)=0$$

Since $*$ and $\Diamond$ are continuous, the limit is uniquely determined from (v) and (xi) of respectively

Definition 2.7. An intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.8. A pair of self-mappings (f,g) of an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to be compatible if $\lim_{n\to\infty}M(fgx_{n,}gfx_{n,}t)=1$ and $\lim_{n\to\infty}N(fgx_{n,}gfx_{n,}t)=0$ for every $t>0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}fx_n=\lim_{n\to\infty}gx_n=z$ for some $z\in X$.

Definition 2.9 A pair of self-mappings (f,g) of an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to be non-compatible if $\lim_{n\to\infty}M(fgx_{n,}gfx_{n,}t)\neq 1$ or nonexistent and $\lim_{n\to\infty}N(fgx_{n,}gfx_{n,}t)\neq 0$ or non-existent for every $t>0$, whenever $\{x_n\}$ is a sequence in -such that $\lim_{n\to\infty}fx_n=\lim_{n\to\infty}gx_n=z$ for some $z\in X$.

In 1998, Jungck and Rhoades [Jungck et. al. 1998] introduced the concept of weakly compatible maps as follows:

Definition 2.10. Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Definition 2.10[Alaca, C. et. Al. 2006**].** Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space then $f,g:X\to X$ are said to be weakly compatible if they commute at coincidence points.

Lemma 2.10 [Alaca, C. et. Al. 2006**]:** Let $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X if there exists a number $k\in(0,1)$ such that:

i $M(y_{n+2,}y_{n+1,}kt)\geq M(y_{n+1,}y_{n,}t)$;

ii $N(y_{n+2,}y_{n+1,}kt)\leq N(y_{n+1,}y_{n,}t)$;

for all $t > 0$ and $n = 1, 2, 3....$ then $\{y_n\}$ is a Cauchy sequence in X .

3.Main Results:

Theorem3.1: Let $(X, M, N, *, \Diamond)$ be an intuitionstic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $t * t \geq t$ and $(1-t)\Diamond(1-t) \leq (1-t)$ $\forall t \in [0,1]$. Let A, B, S and T be self mappings in X s.t.

a) $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$.

b) $M(Ax, By, t) \geq \phi\left\{\min\{M(Tx, Sy, t), M(Tx, Ax, t)\} * \max\{M(Ax, Sy, t), M(Sy, Tx, t)\}\right\}$
$N(Ax, By, t) \leq \psi\left\{\min\{N(Tx, Sy, t), N(Tx, Ax, t)\} \Diamond \max\{N(Ax, Sy, t), N(Sy, Tx, t)\}\right\}$
$\forall x, y \in X$ and $t > 0$ where $\phi, \psi : [0,1] \to [0,1]$ is a continuous function s.t. $\varphi(s) > s$ and $\psi(s) < s$ for each $0 < s < 1$ and $\phi(1) = 1$ and $\psi(0) = 0$ with $M(x, y, t) > 0$.

c) If one of the $A(X), B(X), S(X)$ and $T(X)$ is a complete subspace of X then $\{A, T\}$ and $\{B, S\}$ have a coincidence point.

Moreover, if the pairs $\{A, T\}$ and $\{B, S\}$ are weakly compatible , then A, B, S and T have a unique common fixed point.

Proof: Let $x_0 \in X$ be any arbitrary point since $A(X) \subseteq S(X)$, there is a point $x_1 \in X$ s.t. $Ax_0 = Sx_1$. Again since $B(X) \subseteq T(X)$ for this x_1 there is an $x_2 \in X$ s.t. $Bx_1 = Tx_2$ and so on. Inductively we get a sequence $\{y_n\}$ s.t.

$y_{2n} = Ax_{2n} = Sx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Tx_{2n+2}$, $n = 0, 1, 2...$

Putting $x = x_{2n}, y = x_{2n+1}$ in (b) we have ,

$$M(Ax_{2n}, Bx_{2n+1}, t) \geq \phi\left\{\begin{matrix}\min\{M(Tx_{2n}, Sx_{2n+1}, t), M(Tx_{2n}, Ax_{2n}, t)\} \\ * \max\{M(Ax_{2n}, Sx_{2n+1}, t), M(Sx_{2n+1}, Tx_{2n}, t)\}\end{matrix}\right\}$$

$$M(y_{2n}, y_{2n+1}, t) \geq \phi\left\{\begin{matrix}\min\{M(y_{2n-1}, y_{2n}, t), M(y_{2n-1}, y_{2n}, t)\} \\ * \max\{M(y_{2n}, y_{2n}, t), M(y_{2n}, y_{2n}, t)\}\end{matrix}\right\}$$

$$= \phi\{M(y_{2n-1}, y_{2n}, t) * 1\}$$

i.e. $M(y_{2n}, y_{2n+1}, t) \geq \phi\{M(y_{2n-1}, y_{2n}, t)\} > M(y_{2n-1}, y_{2n}, t)$, …(1)

as $\varphi(s) > s$ for each $0 < s < 1$.

and

$$N(Ax_{2n}, Bx_{2n+1}, t) \leq \psi\left\{\begin{matrix}\min\{N(Tx_{2n}, Sx_{2n+1}, t), N(Tx_{2n}, Ax_{2n}, t)\} \\ \Diamond \max\{N(Ax_{2n}, Sx_{2n+1}, t), N(Sx_{2n+1}, Tx_{2n}, t)\}\end{matrix}\right\}$$

$$N(y_{2n}, y_{2n+1}, t) \leq \psi\left\{\begin{matrix}\min\{N(y_{2n-1}, y_{2n}, t), N(y_{2n-1}, y_{2n}, t)\} \\ \Diamond \max\{N(y_{2n}, y_{2n}, t), N(y_{2n}, y_{2n}, t)\}\end{matrix}\right\}$$

$$= \psi\{N(y_{2n-1}, y_{2n}, t) \Diamond 0\}$$

i.e. $N(y_{2n}, y_{2n+1}, t) \leq \psi\{N(y_{2n-1}, y_{2n}, t)\} < N(y_{2n-1}, y_{2n}, t)$, …(2)

as $\psi(s) < s$ for each $0 < s < 1$.

Thus $\{M(y_{2n}, y_{2n+1}, t), n \geq 0\}$ is an increasing sequence of positive real numbers in $[0,1]$ which tends to a limit $l \leq 1$.We assert that $l = 1$.If not $l < 1$, which on letting $n \to \infty$ in (1) one gets $l \geq \phi(l) > l$,a contradiction yielding thereby $l = 1$. Therefore for every $n \in I^+$ using analogous argument one can show that $\{M(y_{2n+1}, y_{2n+2}, t), n \geq 0\}$ is an increasing sequence of positive real numbers in $[0,1]$ which tends to a limit $l = 1$. Also $\{N_{2n}, n \geq 0\}$ is a monotonic decreasing sequence of positive real numbers in $[0,1]$ and therefore tends to a limit $k \geq 0$ .We assert that $k = 0$ .If not , $k > 0$, which on letting $n \to \infty$ in (2) one gets $k \leq \psi(k) < k$,a contradiction yielding thereby $k = 0$ .Therefore for every $n \in I^+$ using analogous argument one can show that $\{N(y_{2n+1}, y_{2n+2}, t), n \geq 0\}$ is a decreasing sequence of positive real numbers in $[0,1]$ which tends to a limit $k = 0$. Therefore foe every $n \in I^+$

$$M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n, t) \text{ and } \lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$$

And $N(y_n, y_{n+1}, t) < N(y_{n-1}, y_n, t)$ and $\lim_{n\to\infty} N(y_n, y_{n+1}, t) = 0$.

Now for any positive integer p ,we obtain

$$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * \ldots\ldots * M(y_{n+p-1}, y_{n+p}, t/p)$$

And $N(y_n, y_{n+p}, t) \leq N(y_n, y_{n+1}, t/p) \lozenge \ldots\ldots \lozenge N(y_{n+p-1}, y_{n+p}, t/p)$

Since $\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$ and $\lim_{n\to\infty} N(y_n, y_{n+1}, t) = 0$ for $t > 0$, it follows that

$$\lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1*1*1*\ldots*1 = 1 \text{ and}$$

$$\lim_{n\to\infty} N(y_n, y_{n+p}, t) \leq 0\lozenge 0\lozenge 0\lozenge \ldots \lozenge 0 = 0$$, which shows that $\{y_n\}$ is a Cauchy sequence in X .

Now suppose that $S(X)$ is a complete subspace of X .Note that the sequence $\{y_{2n}\}$ is contained in $S(X)$ and has a limit in $S(X)$ call it u .Let $w \in S^{-1}u$ then $Sw = u$. We shall use the fact that subsequence $\{y_{2n+1}\}$ also converges to u . Now by putting $x = x_{2n}, y = w$ in (b) and taking $n \to \infty$

$$M(Ax_{2n}, Bw, t) \geq \phi\left\{\begin{matrix} \min\{M(Tx_{2n}, Sw, t), M(Tx_{2n}, Ax_{2n}, t)\} \\ * \max\{M(Ax_{2n}, Sw, t), M(Sw, Tx_{2n}, t)\} \end{matrix}\right\}$$

$$M(u, Bw, t) \geq \phi\left\{\begin{matrix} \min\{M(u,u,t), M(u,u,t)\} \\ * \max\{M(u,u,t), M(u,u,t)\} \end{matrix}\right\}$$

$$= \phi(1) = 1$$

i.e. $M(u, Bw, t) \geq 1$ …(3)

Also,

$$N(Ax_{2n}, Bw, t) \le \psi \begin{Bmatrix} \min\{N(Tx_{2n}, Sw, t), N(Tx_{2n}, Ax_{2n}, t)\} \\ \Diamond \max\{N(Ax_{2n}, Sw, t), N(Sw, Tx_{2n}, t)\} \end{Bmatrix}$$

$$N(u, Bw, t) \le \psi \begin{Bmatrix} \min\{N(u,u,t), N(u,u,t)\} \\ \Diamond \max\{N(u,u,t), N(u,u,t)\} \end{Bmatrix}$$

$$= \psi(0) = 0 \qquad \ldots(4)$$

i.e. $N(u, Bw, t) \le 0$

from (3) and (4) $u = Bw$.Since $Sw = u$ we have $Sw = Bw = u$ i.e. w is the coincidence point of B and S.

As $B(X) \subseteq T(X)$, $u = Bw \Rightarrow u \in T(X)$. Let $v \in T^{-1}u$ then $Tv = u$.
By putting $x = v, y = x_{2n+1}$ in (b), we get

$$M(Av, Bx_{2n+1}, t) \ge \phi \begin{Bmatrix} \min\{M(Tv, Sx_{2n+1}, t), M(Tv, Av, t)\} \\ * \max\{M(Av, Sx_{2n+1}, t), M(Sx_{2n+1}, Tv, t)\} \end{Bmatrix}$$

as $n \to \infty$

$$M(Av, u, t) \ge \phi \begin{Bmatrix} \min\{M(u,u,t), M(u, Av, t)\} \\ * \max\{M(Av, u, t), M(u,u,t)\} \end{Bmatrix}$$

$$= \phi\{M(u, Av, t) * 1\}$$

$$= \phi\{M(u, Av, t)\} > M(u, Av, t) \qquad \ldots(5)$$

And

$$N(Av, Bx_{2n+1}, t) \le \psi \begin{Bmatrix} \min\{N(Tv, Sx_{2n+1}, t), N(Tv, Av, t)\} \\ \Diamond \max\{N(Av, Sx_{2n+1}, t), N(Sx_{2n+1}, Tv, t)\} \end{Bmatrix}$$

as $n \to \infty$

$$N(Av, u, t) \le \psi \begin{Bmatrix} \min\{N(u,u,t), N(u, Av, t)\} \\ \Diamond \max\{N(Av, u, t), N(u,u,t)\} \end{Bmatrix}$$

$$= \psi\{N(u, Av, t) \Diamond 0\}$$

$$= \psi\{N(u, Av, t) \Diamond 0\} < N(u, Av, t) \qquad \ldots(6)$$

From (5) & (6) we get $Av = u$.

$\because Tv = u$ we have $Av = Tv = u$. Thus v is a coincidence point of A and T.If one assumes $T(X)$ to be complete ,then an analogous argument establishes this claim.
The remaining two cases pertain essentially to the previous cases. Indeed if $B(X)$ is complete then $u \in B(X) \subset T(X)$ and if $A(X)$ is complete then $u \in A(X) \subset S(X)$. Thus (c) is completely established.
Since the pairs $\{A, T\}$ and $\{B, S\}$ are weakly compatible i.e. $B(Sw) = S(Bw) \Rightarrow Bu = Su$ and $A(Tv) = T(Av) \Rightarrow Au = Tu$.

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Putting $x=u$, $y=x_{2n+1}$ in (b) ,we get

$$M(Au,Bx_{2n+1},t)\geq\phi\left\{\min\left\{M(Tu,Sx_{2n+1},t),M(Tu,Au,t)\right\}*\max\left\{M(Au,Sx_{2n+1},t),M(Sx_{2n+1},Tu,t)\right\}\right\}$$

taking $n\to\infty$

$$M(Au,u,t)\geq\phi\left\{\min\left\{M(Au,u,t),M(Au,Au,t)\right\}*\max\left\{M(Au,u,t),M(u,Au,t)\right\}\right\}$$

$$M(Au,u,t)\geq\phi\left\{\left\{M(Au,u,t)\right\}*\left\{M(Au,u,t)\right\}\right\}$$

$$M(Au,u,t)\geq\phi\left\{M(Au,u,t)\right\}>M\left(Au,u,t\right) \quad \ldots(7)$$

And

$$N(Au,Bx_{2n+1},t)\leq\psi\left\{\min\left\{N(Tu,Sx_{2n+1},t),N(Tu,Au,t)\right\}\Diamond\max\left\{N(Au,Sx_{2n+1},t),N(Sx_{2n+1},Tu,t)\right\}\right\}$$

taking $n\to\infty$

$$N(Au,u,t)\leq\psi\left\{\min\left\{N(Au,u,t),N(Au,Au,t)\right\}\Diamond\max\left\{N(Au,u,t),N(u,Au,t)\right\}\right\}$$

$$N(Au,u,t)\leq\psi\left\{\min\left\{N(Au,u,t),0\right\}\Diamond\max\left\{N(Au,u,t),N(u,Au,t)\right\}\right\}$$

$$N(Au,u,t)\leq\psi\left\{0\Diamond N(Au,u,t)\right\}$$

$$N(Au,u,t)\leq\psi\left\{N(Au,u,t)\right\}<N(Au,u,t) \quad \ldots(8)$$

(7) & (8) implies that $Au=u$, so $\Rightarrow Au=Tu=u$

Similarly by putting $x=x_{2n}$, $y=u$ in (b) and as $n\to\infty$, we have $u=Bu=Su$.

Thus $Au=Bu=Su=Tu=u$ i.e. u is a common fixed point of A,B,S and T .

Uniqueness: Let $w(w\neq u)$ be another common fixed point of A,B,S and T .then

By putting $x=u,y=w$ in (b), we have

$$M(Au,Bw,t)\geq\phi\left\{\min\left\{M(Tu,Sw,t),M(Tu,Au,t)\right\}*\max\left\{M(Au,Sw,t),M(Sw,Tu,t)\right\}\right\}$$

$$M(u,w,t)\geq\phi\left\{\min\left\{M(u,w,t),M(u,u,t)\right\}*\max\left\{M(u,w,t),M(w,u,t)\right\}\right\}$$

$$M(u,w,t)\geq\phi\left\{M(u,w,t)\right\}>M(u,w,t)$$

And

$$N(Au,Bw,t)\leq\psi\left\{\min\left\{N(Tu,Sw,t),N(Tu,Au,t)\right\}\Diamond\max\left\{N(Au,Sw,t),N(Sw,Tu,t)\right\}\right\}$$

$$N(u,w,t)\leq\psi\left\{\min\left\{N(u,w,t),N(u,u,t)\right\}\Diamond\max\left\{N(u,w,t),N(w,u,t)\right\}\right\}$$

$$N(u,w,t)\leq\psi\left\{0\Diamond N(u,w,t)\right\}<N(u,w,t)$$

Hence $u=w$ for all $x,y\in X$ and $t>0$. Therefore u is the unique common fixed point of $A,B,S\ \&\ T$ .This completes the proof.

Theorem3.2: Let : Let $(X,M,N,*,\Diamond)$ be an intuitionstic fuzzy metric space with continuous t-norm $*$ and continuous t-conorm $\Diamond$ defined by $t*t\geq t$ and $(1-t)\Diamond(1-t)\leq(1-t)$ $\forall t\in[0,1]$. Let A,B,S,T,P and Q be self mappings in X s.t.

a) $P(X)\subseteq ST(X)$ and $Q(X)\subseteq AB(X)$,

b) There exists a constant $k\in\left(0,1\right)$ s.t.

$$(\alpha+\beta)M(Px,Qy,kt)-\gamma M(Qy,ABx,2kt)$$
$$\geq\alpha M(Px,ABx,t)+\beta M(STy,ABx,t)-\gamma\left\{M(STy,Qy,kt)*M(STy,ABx,kt)\right\}$$

And
$(\alpha+\beta)N(Px,Qy,kt)-\gamma N(Qy,ABx,2kt)$
$\leq \alpha N(Px,ABx,t)+\beta N(STy,ABx,t)-\gamma\{N(STy,Qy,kt)\Diamond N(STy,ABx,kt)\}$
$\forall x,y\in X$ and $t>0$ where $\alpha+\beta>0$.

c) If one of the $P(X),Q(X),ST(X)$ and $AB(X)$ is a complete subspace of X then $\{AB,P\}$ and $\{Q,ST\}$ have a coincidence point.
d) $AB=BA, ST=TS, PB=BP \,\&\, QT=TQ$

Moreover, if the pairs $\{AB,P\}$ and $\{Q,ST\}$ are weakly compatible , then A,B,S,T,P and Q have a unique common fixed point.

Proof: Let $x_0\in X$ be an arbitrary point. since $P(X)\subseteq ST(X)$, there exist $x_1\in X$ s.t. $Px_0=STx_1=y_0$. Again since $Q(X)\subseteq AB(X)$ for this x_1 there is $x_2\in X$ an s.t. $Qx_1=ABx_2=y_1$ and so on. Inductively we get a sequence $\{x_n\}$ and $\{y_n\}$ in X s.t. $y_{2n}=Px_{2n}=STx_{2n+1}$ and $y_{2n+1}=Qx_{2n+1}=ABx_{2n+2}$, $n=0,1,2...$ Putting $x=x_{2n}, y=x_{2n+1}$ in (b) we have ,

$$(\alpha+\beta)M(Px_{2n},Qx_{2n+1},kt)-\gamma M(Qx_{2n+1},ABx_{2n},2kt)$$
$$\geq \alpha M(Px_{2n},ABx_{2n},t)+\beta M(STx_{2n+1},ABx_{2n},t)-\gamma\{M(STx_{2n+1},Qx_{2n+1},kt)*M(STx_{2n+1},ABx_{2n},kt)\}$$

i.e.
$$(\alpha+\beta)M(y_{2n},y_{2n+1},kt)-\gamma M(y_{2n+1},y_{2n-1},2kt)$$
$$\geq \alpha M(y_{2n},y_{2n-1},t)+\beta M(y_{2n},y_{2n-1},t)-\gamma\{M(y_{2n},y_{2n+1},kt)*M(y_{2n},y_{2n-1},kt)\}$$
$$(\alpha+\beta)M(y_{2n},y_{2n+1},kt)-\gamma M(y_{2n+1},y_{2n-1},2kt)\geq(\alpha+\beta)M(y_{2n},y_{2n-1},t)-\gamma\{M(y_{2n-1},y_{2n+1},2kt)\}$$
i.e. $(\alpha+\beta)M(y_{2n},y_{2n+1},kt)\geq(\alpha+\beta)M(y_{2n},y_{2n-1},t)$
i.e. $M(y_{2n},y_{2n+1},kt)\geq M(y_{2n},y_{2n-1},t)$
and

$$(\alpha+\beta)N(Px_{2n},Qx_{2n+1},kt)-\gamma N(Qx_{2n+1},ABx_{2n},2kt)$$
$$\leq \alpha N(Px_{2n},ABx_{2n},t)+\beta N(STx_{2n+1},ABx_{2n},t)-\gamma\{N(STx_{2n+1},Qx_{2n+1},kt)\Diamond N(STx_{2n+1},ABx_{2n},kt)\}$$
$$(\alpha+\beta)N(y_{2n},y_{2n+1},kt)-\gamma N(y_{2n+1},y_{2n-1},2kt)$$
$$\leq \alpha N(y_{2n},y_{2n-1},t)+\beta N(y_{2n},y_{2n-1},t)-\gamma\{N(y_{2n},y_{2n+1},kt)\Diamond N(y_{2n},y_{2n-1},kt)\}$$
$$(\alpha+\beta)N(y_{2n},y_{2n+1},kt)-\gamma N(y_{2n+1},y_{2n-1},2kt)\leq(\alpha+\beta)N(y_{2n},y_{2n-1},t)-\gamma\{N(y_{2n+1},y_{2n-1},2kt)\}$$
$$(\alpha+\beta)N(y_{2n},y_{2n+1},kt)\leq(\alpha+\beta)N(y_{2n},y_{2n-1},t)$$
$$N(y_{2n},y_{2n+1},kt)\leq N(y_{2n},y_{2n-1},t)$$

Hence we have $M\left(y_{2n},y_{2n+1},kt\right)\geq M\left(y_{2n-1},y_{2n},t\right)$ and $N\left(y_{2n},y_{2n+1},kt\right)\leq N\left(y_{2n-1},y_{2n},t\right)$.Similarly, we also have $M\left(y_{2n+1},y_{2n+2},kt\right)\geq M\left(y_{2n+1},y_{2n},t\right)$ and $N\left(y_{2n+1},y_{2n+2},kt\right)\leq N\left(y_{2n+1},y_{2n},t\right)$.In general, for all n even or odd, we have

$M\left(y_{2n},y_{2n+1},kt\right)\geq M\left(y_{2n-1},y_{2n},t\right)$ and $M\left(y_{2n+1},y_{2n+2},kt\right)\geq M\left(y_{2n+1},y_{2n},t\right)$. Hence by Lemma 2.10, $\{y_n\}$ is a Cauchy sequence in X . Now suppose $AB(X)$ is complete . Note that the subsequence $\{y_{2n+1}\}$ is contained in $AB(X)$ and has a limit in $AB(X)$ call it z. Let $w\in AB^{-1}(z)$ .Then $ABw=z$ .We shall use the fact that subsequence $\{y_{2n}\}$ also converges to z.

By putting $x=w$, $y=x_{2n+1}$ in (b) and taking limit as $n\to\infty$,we have

$$(\alpha+\beta)M(Pw,Qx_{2n+1},kt)-\gamma M(Qx_{2n+1},ABw,2kt)$$
$$\geq\alpha M(Pw,ABw,t)+\beta M(STx_{2n+1},ABw,t)-\gamma\{M(STx_{2n+1},Qx_{2n+1},kt)*M(STx_{2n+1},ABw,kt)\}$$

as $n\to\infty$

$$(\alpha+\beta)M(Pw,z,kt)-\gamma M(z,z,2kt)\geq\alpha M(Pw,z,t)+\beta M(z,z,t)-\gamma\{M(z,z,2kt)\}$$
$$(\alpha+\beta)M(Pw,z,kt)\geq\alpha M(Pw,z,t)+\beta\geq\alpha M(Pw,z,kt)+\beta$$
$$\beta M(Pw,z,kt)\geq\beta$$
$$M(Pw,z,kt)\geq 1 \qquad \ldots(9)$$

And

$$(\alpha+\beta)N(Pw,Qx_{2n+1},kt)-\gamma N(Qx_{2n+1},ABw,2kt)$$
$$\leq\alpha N(Pw,ABw,t)+\beta N(STx_{2n+1},ABw,t)-\gamma\{N(STx_{2n+1},Qx_{2n+1},kt)*N(STx_{2n+1},ABw,kt)\}$$

As $n\to\infty$

$$(\alpha+\beta)N(Pw,z,kt)-\gamma N(z,z,2kt)\leq\alpha N(Pw,z,t)+\beta N(z,z,t)-\gamma\{N(z,z,kt)\lozenge N(z,z,kt)\}$$
$$(\alpha+\beta)N(Pw,z,kt)-\gamma N(z,z,2kt)\leq\alpha N(Pw,z,t)+\beta(0)-\gamma\{N(z,z,2kt)\}$$
$$(\alpha+\beta)N(Pw,z,kt)\leq\alpha N(Pw,z,kt)+\beta(0)$$
$$\beta N(Pw,z,kt)\leq 0$$
$$N(Pw,z,kt)\leq 0 \qquad \ldots(10)$$

from (9) and (10) $z=Pw$. Since $ABw=z$ thus we have $Pw=z=ABw$ that is w is coincidence point of P and ABw.Since $P(X)\subset ST(X)$, $Pw=z$ implies that $z\in ST(X)$.Let $v\in ST^{-1}z$.Then $STv=z$.

By putting $x=x_{2n}$ and $y=v$ in (b) ,we have

$$(\alpha+\beta)M(Px_{2n},Qv,kt)-\gamma M(Qv,ABx_{2n},2kt)$$
$$\geq\alpha M(Px_{2n},ABx_{2n},t)+\beta M(STv,ABx_{2n},t)-\gamma\{M(STv,Qv,kt)*M(STv,ABx_{2n},kt)\}$$

as $n\to\infty$,

$$(\alpha+\beta)M(z,Qv,kt)-\gamma M(Qv,z,2kt)$$
$$\geq\alpha M(z,z,t)+\beta M(z,z,t)-\gamma\{M(z,Qv,kt)*M(z,z,kt)\}$$
$$(\alpha+\beta)M(z,Qv,kt)-\gamma M(Qv,z,2kt)\geq(\alpha+\beta)-\gamma\{M(z,Qv,2kt)\}$$
$$(\alpha+\beta)M(z,Qv,kt)\geq(\alpha+\beta)$$
$$M(z,Qv,kt)\geq 1 \qquad \ldots(11)$$

And

$$(\alpha+\beta)N(Px_{2n},Qv,kt)-\gamma N(Qv,ABx_{2n},2kt)$$
$$\leq\alpha N(Px_{2n},ABx_{2n},t)+\beta N(STv,ABx_{2n},t)-\gamma\{N(STv,Qv,kt)\lozenge N(STv,ABx_{2n},kt)\}$$

as $n\to\infty$,

$$(\alpha+\beta)N(z,Qv,kt)-\gamma N(Qv,z,2kt)$$
$$\le \alpha N(z,z,t)+\beta N(z,z,t)-\gamma\{N(z,Qv,kt)\Diamond N(z,z,kt)\}$$
$$(\alpha+\beta)N(z,Qv,kt)-\gamma N(Qv,z,2kt)\le(\alpha+\beta)(0)-\gamma\{N(z,Qv,2kt)\}$$
$$N(z,Qv,kt)\le 0 \quad \ldots(12)$$

From (11) & (12), $z=Qv$. Putting $x=z$, $y=x_{2n+1}$ in (b) and as $n\to\infty$

$$(\alpha+\beta)M(Pz,z,kt)-\gamma M(z,Pz,2kt)$$
$$\ge \alpha M(Pz,Pz,t)+\beta M(z,Pz,t)-\gamma\{M(z,z,kt)*M(z,Pz,kt)\}$$
$$(\alpha+\beta)M(Pz,z,kt)\ge \alpha+\beta M(z,Pz,t)$$
$$(\alpha+\beta)M(Pz,z,kt)\ge \alpha+\beta M(z,Pz,kt)$$
$$\alpha M(Pz,z,kt)\ge\alpha$$
$$M(Pz,z,kt)\ge 1 \quad \ldots(13)$$

and

$$(\alpha+\beta)N(Pz,z,kt)-\gamma N(z,Pz,2kt)\le \alpha N(Pz,Pz,t)+\beta N(z,Pz,t)-\gamma\{N(z,z,kt)*N(z,Pz,kt)\}$$
$$(\alpha+\beta)N(Pz,z,kt)\le\beta N(z,Pz,t)$$
$$N(Pz,z,kt)\le 0 \quad \ldots(14)$$

From (13) & (14) $z=Pz$. So $Pz=ABz=z$.

By putting $x=x_{2n}, y=z$ in (b) and taking limit as $n\to\infty$ we have $M(z,Qz,kt)\ge 1$ and $N(z,Qz,kt)\le 0$.Thus, $z=Qz$. and $Qz=STz=z$.

By putting $x=z, y=Tz$ in (b) and using (d), we have $M(z,Tz,kt)\ge 1$ and $N(z,Tz,kt)\le 0$. Thus, $z=Tz$. Since $STz=z$ therefore $Sz=z$. To prove, $Bz=z$ we put $x=Bz, y=z$ in (b) and using (d), we have $M(z,Bz,kt)\ge 1$ and $N(z,Bz,kt)\le 0$. Thus, $z=Bz$. Since $ABz=z$ Therefore $Az=z$. By combining the above results we have $Az=Bz=Sz=Tz=Pz=Qz=z$. That is z is a common fixed point of A,B,S,T,P and Q.

Uniqueness: Let $w(w\ne z)$ be another common fixed point of A,B,S,T,P and Q then $Aw=Bw=Sw=Tw=Pw=Qw=w$.

By putting $x=z, y=w$ in (b), we have $M(z,w,kt)\ge 1$ and $N(z,w,kt)\le 0$. Hence $z=w$ for all $x,y\in X$ and $t>0$. Therefore z is the unique common fixed point of A,B,S,T,P and Q.This completes the proof.

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