European Journal of Business and Management
ISSN 2222-1905 (Paper) ISSN 2222-2839 (Online)
www.iiste.org

The differential game theory of RMB exchange rate under Marshall-Lerner Conditions and Constraints

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The research is financed by China Wenzhou Municipal Sci-tech Bureau, Project No.R20100117

Abstract

The Marshall-Lerner condition is a theory to study how the exchange rate changes influence the payment balance in international trade. However, in the international trade business, using the Marshall - Lerner condition for decision-making model always leads to conclusions that is contrary to expectations and it lacks of satisfactory technical demonstration effect. Therefore, this deficit tends to make the Marshall - Lerner Condition trapped in the scientific confusion. In this paper, differential game theory is applied to the Marshall - Lerner condition. The Marshall - Lerner condition correction model is derived, then the old Marshall - Lerner conditions are expanded. The new model meets the scope of control of the international trade balance and provides a new technical analysis tools. This preliminary research created precedent study of the Marshall-Lerner condition dynamics.

Keywords: differential game theory; Marshall - Lerner condition; import and export demand elasticity; dynamic constraints

1. Introduction

Marshall-Lerner condition (the Marshall-Lerner condition) is an important tool for the study of the exchange rate movements to control a country's balance of payments position in the international trade. It has been used in the developed countries to shape international trade, revenue and expenditure policies. It is a major decision-making theory of exchange rate changes. Marshall-Lerner condition is developed in the 30s of 20th century based on Marshall's partial equilibrium analysis. The core theory is: In international trade, when there are perfectly elastic supply of trade goods, no capital mobility, full employment and unchanged stable income, and the relative price elasticity of import demand is greater than 1 and the initial balance of zero-recurring items, the actual currency depreciation can increase export earnings and trade surplus will increase. The actual currency appreciation can reduce export earnings and import spending increase, the trade balance deficit will be increased .The the economic meaning of Marshall - Lerner condition is : if the basic constraints are unchanged (perfectly elastic supply of goods, no capital mobility, social achievement of full employment, income constant), import and export is flexible enough to the real exchange rate movements and the exchange rate depreciation will help to improve the balance sheet which increases international trade balance surplus. Real exchange rate appreciation would not help improve the balance sheet and it may reduce the international trade balance surplus.But there must also be noted that the basic premise of using the Marshall - Lerner condition is: foreign demand are in a sensitive situation of the price of their exports, and domestic demand are in a similar position of the price of foreign imported goods.

The current deepening process of marketization of China's economy, Market mechanisms, international trade, the exchange rate effect of mediation are becoming more and more important for the economic

growth. Therefore, the Marshall - Lerner condition is of great importance for the developing countries' sustainable international trade, especially the of China's international trade.

2. Differential game theory under the Marshall - Lerner condition

2.1 differential game theory

Differential game theory is also called differential theory. Differential game theory was started in the early fifties of 20th century, mostly used in military countermeasures. Now, the differential games has been widely used in economic control and market competition and other fields. Especially in recent years, the differential games are actively applied in many economic activities. And cooperation in multilateral competition can be explained with differential game theory. It helps the policy makers find reliable operation strategies. In general, differential game is to study more than two (more) decision-making program control, while adding in a dynamic system described by a differential equation, so that the parties can achieve an optimal goal. Its essence is a multi-equation optimal control problem. In addition, the differential game theory can also be extended to the discrete-time dynamic system, so it is also known as dynamic decision-making. In general, in foreign economic and trade policy formulation, the exchange rate changes and other issues are always regarded the dynamic decision problem. Therefore, the study Marshall-Lerner meaning of dynamic constraints, modified Marshall-Lerner Condition theoretical model, using the differential game theory can achieve a breakthrough.

2.2 Differential game theory under the Marshall-Lerner condition

2.2.1 Analysis of Marshall-Lerner constraints

Constraint is the basic of differential game theory and the premise to construct a differential equation. In 1937, Marshall studied the exchange rate movements in the control of a country's international trade, starting from the partial equilibrium analysis. Marshall assume that the supply of import and export commodities have full flexibility, so exchange rate change depends on demand elasticity of imports and exports. Especially in an economic and financial crisis, a period of oversupply of goods, the assumption is valid. In addition, because the international capital flow is still relatively small comparing to the trade flows, the trade almost equals to the all the balance of payments When there is a currency devaluation or appreciation, there would an immediate changes in the import and export commodity prices and quantities. Elasticity theory is valid here. On this basis, Lerner conducted in-depth research and proposed Marshall-Lerner condition, which uses exchange rate changes to control a country's trade balance

Marshall-Lerner constraints are:

1) To use the exchange rate movements to control a country's international trade balance, require other conditions (B) remain unchanged. B = {commodity supply remain completely flexible, there is no capital mobility, social achievement of full employment, constant income} = CON. Only consider the import and export commodity prices, exchange rate changes (P) and quantity (Q) effects. Where e is the nominal exchange rate (direct quotation), Δe is the nominal exchange rate changes. q is the real exchange rate, Δq is the actual exchange rate movements;

2) The supply of import and export commodities have full flexibility, Sx (elasticity of supply of export commodities) = Sm (elasticity of supply of imported goods) = ∞;

3) No capital flows, that is, Al = 0 .NV (the amount of payments) = TV (trade balance amount);

4) the balance of initial trade , that is, Xx (initial exports) = Mm (initial imports.)

Clearly, according to the above constraints, time is ignored At best, this kind of elasticity analysis is only a comparative static.

2.2.2 Differential theory model and the Marshall-Lerner conditions

When the constraints are determined, we constructed the Marshall-Lerner condition model based on differential theory.

$$TV = X_{x\text{-}M\,m} = P_x^{*} Q - P_m^{*} M \quad (1)$$

TV is the formal trade balance, X_x is exports, $_{M\,m,}$ is imports, Q is the quantity of commodity exports, P_x^{*} is the export prices in foreign currency, M indicated quantity of the foreign physical goods imports, P_m^{*} is prices of the imported goods in a foreign currency.

Take differentiation on both sides:

$$dTV = Q\,dP_x^{*} + P_x^{*}\,d\,Q - {}_{(M}\,d\,P_m^{*} + P_m^{*}\,d\,M) \quad (2)$$

(2) both sides can be divided by $QP_x^{*} MP_m^{*,}$ at the same time, we have:

$$dTV / QP_x^{*} MP_m^{*} = DP_x^{*} / MP_x^{*} P_m^{*} + d\,Q / QMP_m^{*} - (dP_m^{*} / QP_x^{*} P_m^{*} + d_M / QP_x^{*}{}_{M)} \quad (3)$$

Substituting $X_x = P_x^{*} Q$, and $_{M\,m} = P_m^{*} M$, we have:

$$dTV / X_{x\,M\,m} = dP_x^{*} / M_m P_x^{*} + dQ / QM_m - (d\,P_m^{*} / X_x P_m^{*} + dM / X_x M) \quad (4)$$

(4) with both sides multiple $X_{x,}$ we have:

$$dTV / {}_{M\,m} = X_x / M_m\,(dP_x^{*} / P_x^{*} + dQ / \mathrm{Q}) - (dP_m^{*} / P_m^{*} + dM / M) \quad (5)$$

Ex, *Em*, are the elasticity of exports and imports demand ,respectively. *Sx*, *Sm*, are the elasticity of exports goods and the imported goods supply, respectively. We can have:

$$Ex = -\frac{DQ / Q}{dP_x^{*} / P_x^{*}} \quad (6)$$

$$Em = -\frac{dM / M}{dP_m / P_m} \quad (7)$$

$$Sx, = \frac{dQ / Q}{dP_x / P_x} \quad (8)$$

$$Sm = \frac{dM / M}{dP^{*}{}_m / P^{*}{}_{\mathrm{m}}} \quad (9)$$

Here introduce the nominal exchange rate *e*, the domestic and foreign trade prices are as follows:

$$P_m = P_m^{*} e,\ P_x = P_x^{*} e.$$

Take the full-differentiation, we get:

$$dP_m / P_m = dP^{*}{}_m / P^{*}{}_m + de / e \quad (10)$$

$$dP_x / P_x = DP_x^{*} / P_x^{*} + de / e \quad (11)$$

Plug (6) into (11) to find the solution of differential equations,

$$dP_x^{*} / P_x^{*} = -\,Sx / (Ex + Sx)\,de / e \quad (12)$$

$$dP^{*}{}_m / P^{*}{}_{\mathrm{m}} = -\,Sm / (E_{\mathrm{m}} + S_{m)}\,de / e \quad (13)$$

$$dQ / Q = Ex\,Sx / (Ex + Sx)\,de / e \quad (14)$$

$$dM / M = E\,m\,S\,m / (E\,m + S\,m)\,de / e \quad (15)$$

Substituting the above 4 equations into (5), we obtain:

$$dTV / {}_{M\,\mathrm{m}} = [X_x / {}_{M\,m} \cdot (Ex - 1) / (Ex / Sx + 1) + E_m\,(1 + 1 / S_{m)} / (E_m / S_m + 1)]\,de / e \quad (16)$$

Taking the Marshall-Lerner conditions into account :

1) $Sx = \mathrm{Sm} = \infty$, $Ex / \mathrm{Sx} = 1 / S\,m = E\,m / S\,m = 0$;

2) $X_x = {}_{M\,m,}\ X_x / M_m = 1$.

Then (16) can be simplified as:

$$dTV / de = {}_{M\,m}\,(Ex + E_m - 1) / e \quad (17)$$

From (17), we can derive the following conclusions:

1) to consider Marshall-Lerner constraint 1) B = CON, have $TV = f\,(e)$, dTV / *de* is the rate of change in the name of *e*.
2) Generally speaking, in $TV = f\,(e)$ in, $_{M\,m>}$ 0, $e>$ 0.Only when $Ex + E\,m - 1 > 0$, dTV / $de>$ 0, indicating that exchange rates changes can impact on international trade balance.

3) when $Ex + E\ m - 1 > 0$, changing the exchange rate e to control a country's international trade balance is Marshall-Lerner conditions model.
4) Since $TV = f(e)$, has no time variable t, so, Marshall-Lerner conditions model is an only static model.

3. The Marshall-Lerner conditions and amended constraints

3.1 Marshall-Lerner conditions and constraints after introducing the real exchange rate

1) the Marshall-Lerner constraints after introduction of the actual currency exchange rate between the two countries

From the trade exchange rate equation:

$$P_x = EP^*_x,\ P^*_x = P_x / e \quad (18)$$

Plug (18) into (1) and get,

$$e\ \mathrm{TV} = \mathrm{Q}\ P_x - e\ P_m{}^*_{\mathrm{M}} \quad (19)$$

(19) divided by $P_{x \text{ on both sides,}}$ and use the initial balance of trade condition $Q = qM$, we can find:

$$(E / P_{x)}\ TV = Q - (E\ P^*_m / \mathrm{P}_{x)}\ M \quad (20)$$

In (20), let $e\ P_m{}^* / \mathrm{P}_x = {}^{q,}$ $(e / \mathrm{P}_{x)}\ \mathrm{TV} = \mathrm{TV}^{*,}$ the trade equation under the constraints of the real exchange rate is established.

$$TV^* = Q - q\ M \quad (21)$$

Where, $e\ P_m{}^* / \mathrm{P}_x = q$ is the Marshall-Lerner constraints between two countries.

2) The introduction of the currency exchange rate of the Marshall-Lerner conditions

Now, we introduce the real exchange rate constraints q to the Marshall-Lerner conditions.

In (21), take differentiation of q , we have:

$$d\ TV^{*\prime} dq = d\ Q^{\prime} dq - qd\ M^{\prime} dq - M\ dq^{\prime} dq.$$

In the above formula, consider the trade balance and the initial constraints, we can make some simplification, then:

$$d\ TV^{*\prime} dq = Q^{\prime} q\ (d\ Q^{\prime} dq \cdot q^{\prime} Q - d\ M^{\prime} dq \cdot q^{\prime} M - 1). \quad (22)$$

Introduce demand elasticity of the real exchange rate between the two countries $\eta^{*,}$ the elasticity of demand of imported goods η.Where, $\eta^* = d\ Q / dq \cdot q / Q;$ $\eta = - d\ M / dq \cdot q / M.$ (22) can be expressed as:

$$d\ TV^* / dq = Q / \mathrm{q}\ (\eta^* + \eta - 1). \quad (23)$$

So we constructed the real exchange rate based on the Marshall-Lerner conditions.

3.2 The initial trade imbalance and Marshall-Lernerconditions and constraints

3.2.1 Marshall-Lerner under the initial trade imbalance constraint

Under the initial trade imbalance, $X_x \neq M_{m,}$ but the constraints $Sx = Sm = \infty$ exists.The nominal trade balance,

Derivative equation (16),

$$dTV / de = [X_x / M_m \cdot (Ex - 1) / (Ex / Sx + 1) + E_m\ (1 + 1 / S_{m)} / (E_m / S_m + 1)]\ M_m / e$$

Substitute the constraints $Sx = Sm = \infty$, we can get,

$$dTV / de = {}_{M\,\mathrm{m}} / e\ [X_x / M_{\mathrm{m}} \cdot (Ex - 1) + E_{\mathrm{m}]}\ . \quad (24)$$

3.2.2 Marshall-Lerner conditions under initial trade imbalance constraints

The initial trade imbalance in the term of derivative trade balance equation (24)

$$dTV / de = {}_{M\,\mathrm{m}} / e\ [X_x / M_{\mathrm{m}} \cdot (Ex - 1) + E_{\mathrm{m}]}$$

In general, ${}_{M\,\mathrm{m}} > 0$, $e > 0$,the constraint makes $dTV / de > 0$ is

$$X_x / {}_{M\,m} \cdot (Ex - 1) + E_{m} > 0.25$$

Inequality (25), is the Marshall-Lerner conditions under initial trade imbalance constraints.

3.3 Marshall-Lerner conditions and constraints under the initial trade imbalance, and exports and

imports of non-perfectly elastic goods supply

The Marshall-Lerner constraints under these conditions are: $X_x \neq M_m$; $Sx \neq \infty$, $Sm \neq \infty$. That is to give up the import and export commodity supply full-elasticity constraints, and the new elasticity of supply is added to the Marshall-Lerner conditions. The nominal trade balance equation is

$$dTV / de = (M_m / e) \cdot [X_x / M_m \cdot (Ex - 1) / (Ex / Sx + 1) + E_m (1 + 1 / S_m) / (E_m / S_m + 1)], \quad (25)$$

The sufficient condition of $dTV / de > 0$ is

$X_x / M_m \cdot (Ex - 1) / (Ex / Sx + 1) + E_m (1 + 1 / S_m) / (E_m / S_m + 1) > 0$, that is the condition to improve a country's trade balance using currency exchange rate depreciation is:

$$X_x / M_m \cdot (Ex - 1) / (Ex / Sx + 1) + E_m (1 + 1 / S_m) / (E_m / S_m + 1) > 0.26 \quad (26)$$

This condition is given by the Robinson-Metzler, called the Robinson-Metzler Condition, however this is beyond the scope of this study.

3.4 Marshall-Lerner conditions after the introduction of time variable t

Introducing the time variable *t*, the trade balance function is $TV = f(e, t)$, the trade balance derivative equation becomes:

$$TV' = f'(e, t) = \partial TV / \partial e + \partial TV / \partial t \quad (27)$$

There exists a time lag effect in the previous formula $\partial TV / \partial t$. The U.S. and Japan's $Ex + E m > 1$ in the 80s of 20th century. According to Marshall-Lerner conditions, the U.S. decided to depreciate U.S. dollar against the yen. The U.S. expected to control the international trade deficit. However, though there's a significant depreciation of the dollar against the yen, the U.S. international trade deficit is still expanding, US-Japan trade friction continues to emerge. In the late 90s of 20th century, China's $Ex + E m > 1$,satisfy the Marshall-Lerner conditions, but using the Social Accounting Matrix in 2000 estimated impact analysis results show that the depreciation did not improve the Chinese trade balance. These empirical analysis shows that Marshall-Lerner conditions have serious shortcomings. The time delay effect produced by *t* can not be ignored. This is because that the production volume and currency exchange rate factors are time dependent. Generally speaking, the delay effect made depreciation fail to improve of trade balance, but may cause it to deteriorate. Only after a period of time, when the export supply and import demand were adjusted accordingly, the trade balance was likely to improve gradually. There is no ideal Marshall-Lerner model with time variable *t*, .However, this process can be expressed by the J-curve effect. J-curve effect is shown in Figure -1.

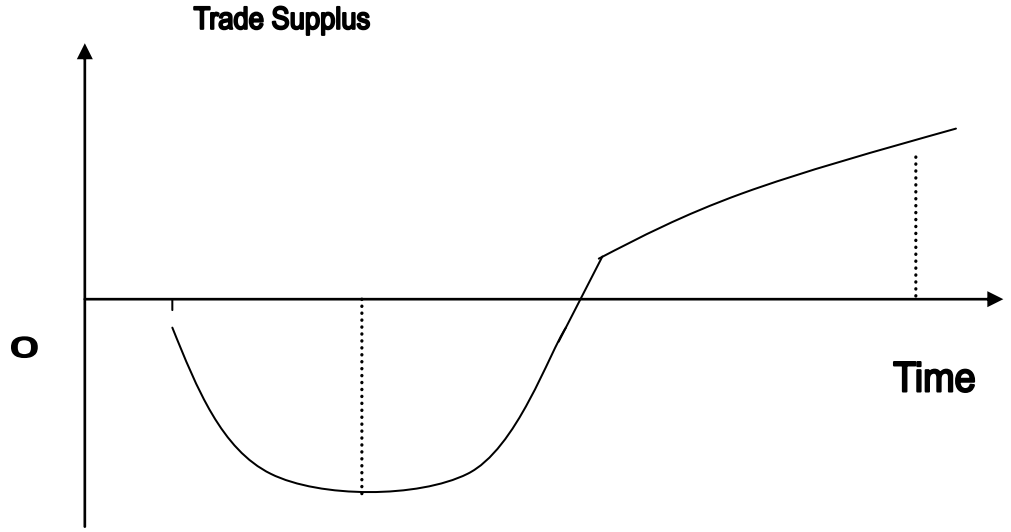

Figure -1 J-curve effect

According to Figure -1 , between t_0 to t_1 , due to the impact of currency devaluation, the import price in foreign currency will increase; the export price in local currency, converted into foreign currency earnings, will be reduced. That is, the increase of export volume in foreign currency can not offset the decline of foreign currency prices, and decrease of import can not offset the increment of import prices. Therefore, the trade balance did not improve, or even got worse. Empirical analysis shows that in the late 20th century, the RMB exchange rate movements has a lag time $t_0 t_1 \approx\approx$ 6-12 months. Only in the time between t_1 to t 2, the trade balance was improved.

4. The revised Marshall-Lerner Conditions and constraints

After a series of amendments, though the Marshall-Lerner Conditionis further close to the real economy, it must be pointed out that the Marshall-Lerner condition is only the sufficient condition (but not the necessary) to improve the trade balance. Changing a country's trade balance is a complex, interrelated process.Here we explore the applications of the revised Marshall-Lerner conditions and constraints.

4.1 The Marshall-Lerner model after introducing the real currency exchange rate.

By introducing the real exchange rate q, the elasticity of demand for exports is expressed as $\eta^* = dQ/dq \cdot q/Q$; elasticity of demand for imported goods is expressed as $\eta = -dM/dq \cdot q/M$.These conditions made Marshall-Lerner to be more stringent. The real exchange rate indicates the relative price between domestic goods and foreign imported goods, after deducting the price factors.In general, the decrease of the q value indicates that the relative price of foreign goods are more expensive, which means the domestic products are more competitive in the international market. If q values have increased and the competitiveness of the domestic goods in the international market will decline.Therefore, the derivative of trade balance $dTV^*/dq = Q/\text{q}\,(\eta^* + \eta - 1) > 0$ has a necessary condition: $\eta^* + \eta > 1$.The condition $\eta^* + \eta - 1 > 0$, is the Marshall-Lerner conditions after introducing the real currency exchange rate.The real exchange rate $q = e(P_m^* / P_{x)} = \beta e$, where $\beta = (P_m^* / P_{x)}$ is the commodity export price ratio coefficient.Since the introduction of β coefficient does not only consider the bilateral exchange rate movements in international trade, but also takes into account the inflation of bilateral trade. It basically can reflect the relative purchasing power a currency. Therefore, Marshall-Lerner is more practical after introducing the real exchange rate.

4.2 Marshall-Lerner Condition and constraints under the conditions of the initial trade imbalance,

The Marshall-Lerner conditions under the initial trade imbalance constraints are:

$$X_x / M_m \cdot (Ex - 1) + E_m > 0.$$

Now,when $E_x < 1$,

1) When $X_{x} < M_m$, There was a trade deficit before the depreciation.

$$X_x / M_m \cdot (Ex - 1) > Ex - 1$$

So, Marshall-Lerner Condition become less constrained, and the effect of currency depreciation on the trade balance improvement is even more apparent.

2) When $X_x > M_m$, , there was a surplus before the currency depreciation, then

$$X_x / M_m \cdot (Ex - 1) > Ex - 1$$

So, M arshall-Lerner Condition become more stringent, that is, control import and export situation using exchange rate movements alone is even more difficult.

4.3 Marshall-Lerner Condition and dynamic analysis of constraints after the introduction of time

variable *t*

After the introduction of time variable *t*, $TV' = f'(e, t) = \partial TV / \partial e + \partial TV / \partial t$.As $\partial TV / \partial t$, existence of time delay effect is significant, therefore, the following dynamic factors should be considered when the policy make use Marshall-Lerner Condition to improve the import and export trade balance,

1) The domestic inflation caused by the national currency devaluations. As a long-term delay effects,the inflation between t_0 to t_1 can not be ignored. The devaluation increases the cost of imports, then increases domestic market prices of imported goods. Thereby promoting the domestic price index rise in the t_0 to t_1. Resulting in the actual benefits of currency depreciation swallowed by the inflation.

2) Tariffs and trade subsidies between t_0 to t_1 .Changes in tariffs and the role of trade subsidies in the period between t_0 to t_1 can not be ignored. Changes in domestic tariffs and trade subsidies often lead to increased cost of imports and exports. In general, the change of commodity price caused by the nominal exchange rate movements will be offset by the decrease or increase of the import costs. The impacts of domestic tariffs and trade subsidies changes in the time lag period devalues the Marshall-Lerner Conditions.

3) structural adjustment of domestic import and export commodities between t_0 to t_1.Generally speaking, a country's international trade structure is not stable and always ready to change If the raw materials import is too big, the devaluation will increase the production cost of imported goods, then decreases the price of export goods. Only when the increase of commodity price caused by the increase of import prices of raw materials is smaller then value of currency depreciation, export commodity prices will fall.At this point, Marshall-Lerner Condition can help to improve the import and export trade balance situation.

5. Conclusions

After the above analysis, we can reach the following conclusions;

1) the Marshall-Lerner Condition is the basis of elastic theory. Although it has some shortcomings, but it can help to use exchange rate movements to control a country's international trade balance.
2) the Marshall-Lerner Condition is based on neoclassical economic theory and take the price mechanism and complete information as constraints. Therefore, Marshall-Lerner conditions can not be fully applicable to the balance analysis of international trade.
3) The revised Marshall - Lerner condition based on differential game theory is further study, which not only shows a new Marshall-Lerner Condition theoretical model, but also expand the Marshall - Lerner condition applications.
4) After the introduction of time variable *t* and the value analysis. The new model is particularly useful to control the Chinese international trade balance.
5) Different constraints for the Marshall-Lerner Condition are critical. The Marshall - Lerner condition based on differential game theory created a precedent step in the area of Marshall-Lerner theory.

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