8-2019

Examining Students’ Covariational Reasoning Through Mathematical Modeling Activities Embedded in the Context of the Greenhouse Effect

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EXAMINING STUDENTS' COVARIATIONAL REASONING THROUGH MATHEMATICAL MODELING ACTIVITIES EMBEDDED IN THE CONTEXT OF THE GREENHOUSE EFFECT

A DISSERTATION

Submitted to the Faculty of

Montclair State University in partial fulfillment

of the requirements

for the degree of Doctor of Philosophy

by

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Upper Montclair, NJ

August 2019

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MONTCLAIR STATE UNIVERSITY

THE GRADUATE SCHOOL

DISSERTATION APPROVAL

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EXAMINING STUDENTS' COVARIATIONAL REASONING THROUGH

MATHEMATICAL MODELING ACTIVITIES EMBEDDED IN THE CONTEXT

OF THE GREENHOUSE EFFECT

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Abstract

EXAMINING STUDENTS' COVARIATIONAL REASONING THROUGH MATHEMATICAL MODELING ACTIVITIES EMBEDDED IN THE CONTEXT OF THE GREENHOUSE EFFECT

by Debasmita Basu

The greenhouse effect is one of the most pressing environmental as well as social issues of the present age. In news media and weather reports, most of the essential information about the phenomenon is expressed in forms of graphs and pictures. However, the interpretation of such graphs is challenging for students; they often focus on the shape of the graphs, overlooking the covariational relationships between the concerned quantities. Building on the framework of critical mathematics literacy and social justice mathematics, in this study I aimed to explore the power of dynamic mathematical modeling activities for engaging students in covariational reasoning and developing their understanding about the greenhouse effect. More specifically, this study aimed to explore a) the extent to which students' understanding of the greenhouse effect and covariational reasoning changed as a result of their engagement with the mathematical modeling activities, and b) the ways in which students may reason covariationally as they engage with mathematical modeling activities in the context of the greenhouse effect.

To engage students in covariational reasoning in the context of the greenhouse effect, three NetLogo dynamic simulations and accompanied activities were developed and implemented in two sixth-grade classrooms in the form of a whole class design experiment. Both quantitative and qualitative data were collected in the form of pre- and post-assessments and video recordings of whole class discussions and small group interactions. The analysis of the quantitative data shows a significant improvement in post-assessment scores of the treatment group students compared to their peers in a control group. The qualitative analysis that followed helped me understand the meaning of the improved post-assessment scores by studying students' reasoning as they interacted with the simulations. The analysis of the qualitative data indicates that the design of the three simulations and activities as well as the targeted questioning provided a productive space for students to engage in different levels of covariational reasoning according to Carlson et al.'s mental action framework and helped them identify the causes and the consequences of the greenhouse effect.

Keywords: Greenhouse effect, covariational reasoning, mathematical modeling.

Acknowledgement

First, I would like to thank my advisor, Dr. Nicole Panorkou, without whose guidance, this dissertation would not have been possible. Dr. Panorkou, thank you for all the research and teaching opportunities you offered me during my time at Montclair State University. I am fortunate to have you as my mentor. I am grateful that you have always believed in my research and continually supported me to conduct it. Everything you taught me; every advice you gave has helped me grow as the researcher I am today.

I would also thank my committee members, Dr. Steven Greenstein and Dr. Eileen Murray for their support and guidance throughout my doctoral education. Dr. Greenstein, you have always encouraged me to think about research critically and get deep into any issue through questioning and inquisitiveness. Also, I appreciate you for supporting me during my stressful time and making me feel confident when I doubted my career as a mathematics educator. Dr. Murray, you have always given your valuable advice and helped me shape this study. I thank you for motivating me to ask questions and think deeply about my research. I would like to thank Dr. Teo Paoletti for introducing me to covariational reasoning and sharing his insight with me whenever I asked. I would also thank other faculty members and graduate students of mathematics education at Montclair State University.

I would extend my gratitude to the ACMES team members for offering me this fantastic learning and research opportunity and helping me design this study. Thank you, Dr. Pankaj Lal and Dr. Michelle Zhu, for providing insight and sharing your expertise with me. A special thanks to Sowmith and Corey for helping me develop the NetLogo simulations and Jay Prakash for assisting me in the quantitative data analysis. I would also express my gratitude towards the

Kearny middle school teachers and students for being a part of this study. You all have been wonderful.

I would thank my friends who have been most supportive through this process during the last five years. Madhavi, thank you for being my critical friend and for being there whenever I needed you. This journey would have been incomplete without you. Thank you Erell, Zareen, Atreyee, Matthew, Justin and all my other friends for sharing your wisdom and for your support and encouragement.

I would like to thank my parents, Amit Kumar Basu and Mita Basu, for all their love and support, and the sacrifices they have made to help me reach where I am today. You have stood firm by me in my thick and thin, and always believed in me when I doubted my decisions. A big thank you to my brother Anirban and sister-in-law Deepanwita for continuously motivating me and supporting me through the process. My nephew Vighnesh, you are my rock star, my inspiration. You cheer me up every day and energize me to work.

Lastly, I would like to thank my husband, Saikat, for all the love and support he has given me in the last five years. Saikat, you have always prioritized my work over yours and made changes in your schedule and life to make my life easy. Maybe I do not say this often, but you inspire me every single day and motivate me to get going.

Dedication

I dedicate this dissertation to my family: Amit Kumar Basu, Mita Basu, Anirban, Deepanwita, Vighnesh, and Saikat.

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1. Introduction

Climate change is one of the most pressing issues of the present age for our society. At the Earth Summit at Rio de Janeiro in 1992, the United Nations Framework Convention on Climate Change (UNFCCC) defined climate change as the "change of climate which is attributed directly or indirectly to human activity" (Kolbert, 2006, p. 153). Indeed, with the increasing population and expanding technological advancement, human activities have maintained a dominant influence on the natural climatic condition of the earth (Heng, Karpudewan, & Chandrakesan, 2017; Karl & Trenberth, 2003; Shepardson, Niyogi, Choi, & Charusombat, 2009; Vitousek, Mooney, Lubchenco, & Melillo, 1997). Over the past few decades, human activities have altered the "earth's ecosystems more rapidly and extensively than in any comparable period of human history" (MEA, 2005, p. 4). For instance, fossil fuel combustion in the economically developed regions adds 5.5 ± 0.5 billion metric tons of CO_2 to the atmosphere annually (Vitousek, Mooney, Lubchenco, & Melillo, 1997). Due to the large-scale burning of fossil fuels, the emission of greenhouse gases such as carbon dioxide, methane and nitrous oxide has increased, thus disrupting the normal atmospheric composition of the earth (Dolman, Verhagen, & Rovers, 2003).

Apart from the industrial emission of greenhouse gases, research shows that an average US citizen, through his/ her daily course of action, contributes more than sixteen metric tons of CO_2 into the atmosphere during their lifetime (Murn, 2017). We often remain unaware, but our simple act of choosing household goods, such as detergents, toilet papers, floor mops, and house paints, also impacts our environmental conditions (Black & Cherrier, 2010). Several of these household items often contain hazardous substances such as benzene, lead, asbestos and chlorine, which are added to nature due to the improper disposal of these items with common household wastes (Malandrakis, 2008). Additionally, our choice of transportation and household energy use, such as driving big-size cars instead of carpooling or taking public transports, excessive use of highly efficient electronics, game systems, and uninterrupted service of heating and cooling systems constitute anywhere from 32 to 41 percent of the total carbon dioxide emission (Barkenbus, 2010; Murn, 2017). Due to all the human activities, the amount of carbon dioxide gas in our atmosphere, which was stable to 280 ppm for thousands of years has increased exponentially since 1800 (Vitousek, Mooney, Lubchenco, & Melillo, 1997). In 1993, Boyes, Chuckran, and Stanisstreet (1993) asserted that if the carbon emission level of the world remains unchecked, we might see a global rise of temperature between 2-degree and 5-degree Celsius in future, which in turn would melt polar ice caps and raise the sea level (Shepardson, Niyogi, Choi, & Charusombat, 2009).

While a considerable section of the human population is concerned about the human-induced climate change and its impact on the natural habitats of the earth, many others deny the potential role of the human activities responsible for climatic disruption and often question the climate change authenticity (Abtahi, Gotze, Steffensen, Hauge, & Barwell, 2017). Hulme (2009) suggested that the complexities associated with the issue of climate change and the wide range of information and misinformation available to the general public about the climate might have resulted in such confusion. However, considering the gravity of the issue, if the general public, and especially the students, are not educated about climate change and its consequences, then that might worsen the current climatic situation. In other words, due to lack of knowledge and awareness, if human activities continue to put such a strain on the natural functions of the earth then "the ability of the planet's ecosystems to sustain future generations can no longer be taken for granted" (MEA, 2005, p. 5). Unless we become more sensitive towards climate and alter our attitudes and actions, the stress on the earth's ecosystems will increase in future.

1.1. Is Climate Change Affecting Everybody Equally?

Though climate change is considered to be a potential threat to the entire humanity, not everybody has an equal contribution to the causes of climate change and not every individual incurs the consequences of climate change equally. In other words, climate change is not only an environmental issue but also an issue of inequity and social injustice (Agyeman, Bullard, and Evans, 2002). According to many researchers, climatic and environmental problems bear down disproportionately upon the poor though they are not the primary consumers of the available resources (Agyeman, Bullard, & Evans, 2002; Costello et al., 2009; Pettit, 2004). The poor people are more likely to endure the harmful effects of climate change compared to the affluent section of society (Corvalan, Hales, McMichael, Butler, & McMichael, 2005). As Kemp (2011) suggested, "where people live profoundly influences how they live, with important implications for equity and social justice" (p. 1200). A study conducted by The Dallas Morning News and the University of Texas in 2000 reported that 870,000 out of 1.9 million (46%) houses of the poor people are located within the one-mile radius of factories that have been reported to emit toxic chemicals according to Environmental Protection Agency. Indeed, big multinational companies often adopt unsustainable forms of production and waste disposal to maximize their profit (Agyeman, Bullard, & Evans, 2002), and most of these factories are located in urban areas, which are heavily resided by poor and minority groups of people. In Atlanta, about 83% of the African American people live near waste site compared to 60.2% of the whites, and in Los Angeles, 60% of the Hispanic people reside in areas heavily contaminated by industrial wastes compared to 35.3% of the whites (Bullard, 2000).

Another example that describes how environmental health threat is connected to people's economy was given by Bullard (2000) who discussed the influence of lead poisoning on the life of American children arguing that a child belonging to the low socio-economic strata is eight times more susceptible to the lead poisoning compared to a rich child. For instance, in West Dallas, which is resided by 85% black people and most of the families live below the poverty line, a local lead smelter pumps more than 269 tons of lead particles in the air every year. Another example is the case of Institute, a small community in the Kanawha County in West Virginia, where 90% of whose population is black and their health is compromised daily as a result of polluted air and odors emission from a local Union Carbide chemical plant (Bullard, 2000). While the wealthy people can ensure their children healthier lives with clean air and non-polluted water supplies, the people belonging to the lower economic strata of the society are less able to avoid the environmental hazards such as lead emission, motor vehicle exhausts, polluting industry, and power generation (Agyeman, Bullard, & Evans, 2002). As a result, the poor population of our society is more vulnerable to health issues such as heart diseases, breathing problems, diarrhoea, malaria, and malnutrition (McMichael, 2013).

Apart from health hazards, flooding, which can be caused by climate change also impacts the different strata of society disproportionally. According to Walker and Burningham (2011), poor people are more vulnerable to be affected by floods over others because they lack preparedness and financial resources to make their houses more water resilient. Considering the above, climate change can rightfully be claimed as an issue of social injustice as it has "its greatest effect on those who have the least access to the world's resources and who have contributed least to its cause" (Costello et al., 2009, p. 1694).

1.2. What Can We Do About It?

To restrain the pace of the existing climatic disruption, several initiatives have been taken by the governments and different organizations worldwide. To mention a few, in 2015, all the United Nations Member States adopted the 2030 Agenda for Sustainable Development, which identified Climate Change as one of the 17 sustainable development goals that contribute towards a better and sustainable future for all. In November 2016, 175 countries in the world signed the Paris Agreement, with the unified intention to limit global temperature rise to 2 degrees Celsius for the next hundred years. They aimed to enhance the adaptability of the countries towards climate change, minimize the loss and damage associated with climatic disruption, mitigate the pace of climate change by providing financial support to the countries to reduce greenhouse gas emission, and educating, training, and making public aware of issues related to climate change. As of 2018, 175 countries ratified the Paris Agreement, and 168 countries have communicated their contributions to the United Nations Framework Convention on Climate Change Secretariat.

While governments and organizations create initiatives to fight climate change, research shows that students often have a wide range of misconceptions about the climatic condition of the earth. Some of those misconceptions include students believing that climate change is caused by ozone depletion (Shepardson, Niyogi, Choi, & Charusombat, 2009), excessive solar radiation (Boyes & Stanisstreet, 1993) and proximity of earth to the sun (Pruneau, Gravel, Bourque, & Langis, 2003). Consequently, many students often do not consider climate change as an immediate or future threat to the society or humans (Pruneau et al. 2001, 2003; Shepardson, Niyogi, Choi, & Charusombat, 2009). Hence, it is essential that we as educators acknowledge our own share of responsibility towards the planet and task ourselves to educate our students so that the future citizens develop an awareness about the issue of climate change and cultivate sensitivity towards climate (Bostrom, Morgan, Fischhoff, & Read, 1994; Shepardson, Niyogi, Choi, & Charusombat, 2009; UNESCO, 2013). As educators, our goal should also be to engage our students in a study of this scientific phenomenon using equity and social justice lens.

1.3. Climate Change and The Role of Mathematics Education

Climate change and mathematics are closely related. Governments and the policymakers develop laws and policies around climatic conservation, primarily based on the predictions made by mathematical models on climate (Barwell, 2013). Mathematics literacy is not only necessary to identify the different traits that indicate climatic disruptions, but it also helps us predict the future impacts of climate change. Abtahi et al. (2017) acknowledged the role of mathematics for understanding, predicting, and communicating issues related to climate change, and questioned the "ethical and moral responsibilities" (p. 2) of the mathematics educators and teachers to educate students about this complex yet pressing issue. They argued that, if teachers assume their ethical responsibilities to educate their students about the rapidly changing climate, then that would prepare the future decision makers to effect change for the betterment of the climate.

In line with this call, in this study, I aimed to explore the power of mathematical reasoning for developing students' understanding of the greenhouse effect, which is a significant cause behind climate change. Acknowledging the complexity of climate change, my goal was to engage students in targeted dynamic mathematical modeling activities that would not only help them develop their mathematical reasoning but would also provide them with a platform to identify the causes and consequences of the greenhouse effect and discover the role of human activities as a cause behind this phenomenon. To address the social impact of the greenhouse effect, this study utilizes the frameworks of critical mathematics education (Frankenstein, 1994) and teaching mathematics for social justice (Gutstein, 2003) to advocate the power of mathematics for questioning the existing social injustice and use mathematics as a potential tool to critique it.

1.4. Structure of The Thesis

This dissertation is organized into five chapters. In Chapter Two, I present my review of the literature around the following five major themes: critical education theory, critical mathematics education, social justice mathematics, mathematical modeling, and covariational reasoning. First, I discuss the essential idea of critical education theory as introduced by Freire (1973). Building on the framework of critical education theory, next I describe the concept of critical mathematics education (Frankenstein, 1994) and social justice mathematics (Gutstein, 2003) to illustrate how mathematics education can help students to read the world, generate within the learners a sense of agency to change the world, and question the hegemonies and the established norms of the society. Then I review the literature on mathematics modeling and discuss how mathematics modeling helps students to engage in meaningful learning of different real-life phenomena. I end chapter two focusing on the mathematical aspect of the greenhouse effect and reviewed pertinent literature illustrating what is covariational reasoning, and how this particular form of cognitive activity might help students to read graphs and understand the causes and the consequences of the greenhouse effect.

In Chapter Three, I describe the methodology of my study, the whole class design experiment. I explain why the particular methodology is best suited for this study. Then, I review the general design features of the tasks followed by design features my research. First, I discuss the role of technology in students' learning; then I provide a detailed description of all the three NetLogo simulations I designed, and the five Investigations to illustrate how the simulations and the activities could conjointly engage students in different stages of covariational reasoning and help them understand the greenhouse effect. Next, I described the participants of the study, the research setting, as well as my data sources and collection methods. Lastly, I end the chapter describing the methods of data analysis and the framework I used to analyze students' covariational reasoning.

In Chapter Four, I describe my findings. First, I present the results of the pre- and the post-assessments of the treatment and the control groups and compare them to illustrate the difference in the performance of the two groups of students in the pre- and the post-assessment. Next, to get an insight into the quantitative data and identify the possible reasons that might have resulted in the difference between the scores, I analyze the qualitative data. I look into the students' experiences interacting with the three NetLogo simulations and discuss how the simulations and the activities might have enabled students to reason covariationally and helped them identify the causes and the consequences of the greenhouse effect.

In Chapter five, first I summarize the findings of my study in connection to existing research literature on mathematical modeling and covariational reasoning and discuss the implication the study in the field of mathematics education. Then I discuss the limitations of the study and end the chapter by discussing implications for future research.

2. Literature Review

For my dissertation, I examine the impact of dynamic mathematical modeling activities on students' understanding of the greenhouse effect. More specifically, I focus on how the dynamic mathematical modeling activities helped students to engage in different levels of covariational reasoning and helped them shape their opinions about the issue of the greenhouse effect through data and graphs.

The literature review is organized into five sections. In section 2.1, I explore what is critical education theory and its role to generate within students a sense of critical awareness about different socio-political issues. Developing on the role of critical education theory, in section 2.2 and section 2.3, I discuss how critical mathematics education and social justice mathematics can help students to become familiar with deep-rooted social injustice and explore how an environmental issue such as the greenhouse effect can be studied as a social justice issue. I envision to use mathematical modeling activities to make students aware of the impact of greenhouse effect on their daily lives and propose strategies to curb the pace of the changing climate. Therefore, in section 2.6, I provide a brief overview about how mathematical modeling has been defined by educators, describe the role of modeling for integrating different subjects, such as mathematics, science, and social science, and present the current place of mathematical modeling in school education. Finally, section 2.7 describes how modeling activities can help students to interpret and analyze graphs and data that they encounter outside of the school context and focuses on how covariation reasoning may assist students in this interpretation and analysis.

2.1. Critical Education Theory

Historian Howard Zinn (1990) once wrote that "in a world where justice is maldistributed, there is no such thing as a neutral or representative recapitulation of the facts" (p. 24). Echoing the opinion of Zinn about non-neutrality in education, Gutstein (2005) stated that teaching is not a neutral activity; it is a political act. Indeed, in a society, which is highly diverse socially, culturally, and economically, anything that teachers do from selecting a mathematical activity for the students to deciding on the mode of instruction in a mathematics classroom, sends a message to the students about what is essential and valued in school mathematics.

A similar argument was also made by Setati (2008), who stated that the language of instruction in a multicultural classroom is never just a "vehicle to express ideas (a cultural tool), but also a political tool that we use to enact (i.e. to be recognized as) a particular 'who' (identity) engaged in a particular 'what' (situated activity)" (p. 105). Under such circumstances, it is essential for educators to reconsider the role of mathematics. We can either treat mathematics as it has been treated traditionally and pose nonsensical problems to the students, such as calculating the speed of two trains moving on a single track or finding the total amount of time a jet plane will take to catch a passenger plane, or we can use topics, such as corruption, child labor, impact of a war, and climate change to make students familiar with socially pressing issues (Cirillo, Bartell, & Wager, 2016; Koestler, 2010).

In his book "Pedagogy of the Oppressed," Freire (1973) argued that our education system has turned into an act of deposition, where the teachers and the students play the role of depositors and depositories respectively. Freire used the analogy of a banking system to describe that knowledge is treated as a gift, which is imparted by somebody knowledgeable to those whom they consider being less intellectual and less competent. Freire claimed that this banking education system positions the learners in passive roles and does not develop within them a sense of inquiry about their own lives or the society they live in. Eventually, the banking system of education serves the purpose of the oppressors by refraining the oppressed from asking questions, thus keeping the later under the impression that "causality is a static, established fact" (Freire, 1973, p. 44) and, therefore, not susceptible to change through their actions.

Opposing the banking system of education, Freire (1973) proposed that our education system should establish within students a sense of conscientizacao or a critical sociopolitical awareness to "understand what one reads and to write what one understands" (Freire, 2014, p. 284). Originally, the concept of critical consciousness was introduced by Freire (1973) as an approach to develop Brazilian peasants' ability to identify the different socio-political factors that contribute to their inequitable social conditions, and empower them to lift themselves up from the pitfall of social injustice and biases that became an eternal part of their lives. With a similar goal in mind, Freire emphasized the development of students' critical consciousness through dialogue and problem posing. He stated, "authentic thinking, thinking that is concerned about reality, does not take place in ivory tower isolation, but only in communication" (Freire, 1973, p. 77). Problem-posing education regards dialogue as indispensable to the act of cognition that encourages students to be critical thinkers and fosters within them deep consciousness towards apprehending the world and take actions to transform it. Comprehending the world, or in Freire's words reading the world and transforming it, that is writing the world, are the two fundamental components of critical education.

Henry Giroux (cited in Frankenstein, 1983) collectively defines the concept of reading and writing the world as "a critical mode of reasoning and behavior…[that] functions so as to help people analyze the world in which they live, to become aware of the constraints that prevent them from changing that world, and, finally, to help them collectively struggle to transform that world." (pp.114, 116). Building on Freire's framework of critical consciousness, recent studies (Diemer et al., 2006; Godfrey & Grayman, 2014; Watts et al., 1999) identified three components of critical consciousness: (i) critical reflection, (ii) socio-political efficacy, and (iii) critical action. Critical reflection, as described by Godfrey and Grayman (2014) is the "youth's ability to analyze current social realities critically, and recognize how social, economic, and political conditions limit access to opportunity and perpetuate injustice" (p. 1802). Critical literacy motivates an individual to critically reflect on accepted ways of living and thinking, discerning the established norms and hidden interests of a particular section of the society (Hopper, 1999). The Socio-political efficacy defines "one's perceived ability to act to change social and political conditions" (Godfrey & Grayman, 2014, p. 1802), and critical action describes the extent to which an individual takes active roles to condemn social biases and work towards abolition of unfair practices that slipped into and became an integral part of our lives. Though Godfrey and Grayman (2014) recognized critical reflection, socio-political efficacy, and critical action as three distinct components of critical literacy, these three components collectively highlight the goals of reading and writing the world. The three goals suggest that an individual's critical consciousness develops through their ability to understanding the world deeply and their perceived and active capability to work towards rewriting the world in their own terms.

2.2. Critical Mathematics Education

Developing on Paolo Freire's critical education theory in the context of mathematics, Frankenstein (1983, 1990, 1994) established what we refer to as critical mathematics literacy. She stated that mathematical numbers and representations would "explode the myths about the institutional structure of our society and understanding the limitations of the knowledge we gain from mathematical analyses of our world" (Frankenstein, 1994, p. 25). Similar to Freire, Frankenstein (1983) emphasized the role of dialogue in mathematics classrooms, as a tool for bridging the existing critical consciousness of the students with their developing critical approach towards knowledge. She argued that current school mathematics establishes and reinforces hegemonic ideologies; however, critical mathematics education can develop within students a deeper understanding of race, gender, and class along with learning mathematical concepts. Consequently, Frankenstein (1990) defined critical mathematics literacy as "the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues and the ability to present data to change people's perception of those issues" (p. 336). She suggested that when students start identifying mathematics embedded in social or political situations, they can then develop an in-depth understanding of the situation and question "sentences and myths that are slipped into their lives in so many ways" (p. 336). Frankenstein (1994; 2001) identified four goals of critical mathematics literacy: a) understanding the mathematics, b) understanding the mathematics of political knowledge, c) understanding the politics of mathematical knowledge, and d) understanding the politics of knowledge. In the following paragraphs, I provide some examples to explain each of those four goals.

For calculating the demographic percentage of unemployed people, students would require the knowledge of calculating the percentage. This is what Frankenstein refers to the understanding of mathematics (Goal 1). On the contrary, understanding the mathematics of political knowledge (Goal 2) refers to how students' mathematical knowledge may help them to understand the complexities of a real-life situation. For instance, if students are provided with unemployment data, then their data analysis ability would enable them to apprehend the unemployment situation of the working-class people of the country. According to Frankenstein, students' mathematical competencies would successfully organize their thoughts around the data and help them ask relevant questions to unmask other factors hidden in the data. Though researchers have emphasized on the power of numeracy to develop critical consciousness among students, numbers can also be misused to politicize an issue or to make a situation favorable for the dominant section of the society. Frankenstein regarded such misuse of numbers as understanding the politics of mathematical knowledge (Goal 3). For instance, in the United States, the unemployment rate is usually calculated based on the number of people applying for unemployment insurance. However, the current procedure of calculation understates the percentage of unemployed people in the US. Many people, who cannot find a full-time job and work part-time, underpaid people, and workers who ran out of their unemployment benefits, are omitted from unemployment insurance. As a result, they are not considered as unemployed. This example shows how certain policies are built to mellow or misrepresent relevant issues such as unemployment by using seemingly neutral mathematical data.

The fourth goal of critical mathematics literacy, the politics of knowledge (Goal 4) involves "what counts as mathematical knowledge and why" and "how mathematical knowledge is learned in schools" (Frankenstein, 1994, p. 36). Frankenstein said, in schools, students do not encounter difficulties in learning mathematics due to the discipline's difficult abstraction, but due to the cultural form in which mathematics is presented (Frankenstein, 1994).

Considering the above, critical mathematics literacy "ranges from demystifying the structure of mathematics to using numerical data for demystifying the structure of society" (Frankenstein, 1994, p. 24). Developing on Freire's theory of critical education, the goals of critical mathematics literacy is to use the power of mathematics to help students develop a critical understanding of the society, which in turn leads to critical action (Frankenstein, 1983). Critical mathematics literacy demands students to analyze the patterns in a given set of data and mathematical graphs, examine the connections between them and reflect on how certain people use numbers to serve their own purposes while pushing others towards a life filled with oppression and injustice. As Frankenstein (1983) states, "reflection which is not ultimately accompanied by action to transform the world is meaningless" (p. 3); as a result, critical mathematics literacy further motivates students to challenge the biases and injustice in the society and develop a plan of actions to transform them.

2.3. Social Justice Mathematics

Building on the framework of critical education theory by Freire (1973) and critical mathematics literacy by Frankenstein (1990, 1994), Gutstein (2006) developed a model for teaching mathematics for social justice. He acknowledged Freire's idea of liberation from oppression as the fundamental objective of teaching mathematics for social justice. More specifically, he defined the purpose for teaching mathematics for social justice to involve helping students learn mathematics more meaningfully through issues of social concern and enabling students "to investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts" (Gutstein, 2006, p. 4). In other words, social justice mathematics (SJM) develops within students an awareness against deep-rooted social inequities and unfair practices while developing a mathematical identity.

Gutstein (2006) identified three goals of teaching mathematics for social justice: a) a mathematics pedagogical goal, suggesting that we can help students learn mathematics through the study of social issues b) a social justice pedagogical goal, suggesting that we can use mathematics as a medium to teach students about the issues of equity, diversity, and social justice and c) developing positive cultural and social identities, that emphasizes on grounding mathematics instruction in students' languages, cultures, and communities, while providing them with the mathematical knowledge they need to survive and thrive in the dominant culture. This study is developed at the intersection of the first two goals and explores how mathematical activities embedded in the context of the greenhouse effect help students to understand the environmental and social aspects of the phenomenon through data and graphs. As identified by Gutstein (2006), the two goals of mathematics pedagogical goal and social justice pedagogical goal are dialectically connected and complement each other enabling students to learn mathematical concepts through socially relevant issues and develop socio-political awareness during their interaction with mathematical tasks.

In terms of the mathematics pedagogical goal, Gutstein emphasized students' academic success in the traditional sense. He argued that unless students gain mathematical competencies, their limited understanding of mathematics would keep them ignorant about socio-political ideas around them. In relation to the social justice pedagogical goal, Gutstein (2003) utilized Freire's literacy scholarship to introduce the concepts of "reading and writing the world with mathematics" (p. 24) as the two main objectives of this goal. The following paragraphs describe each of those two objectives in more depth.

2.3.1 Reading the world with mathematics

Building on Freire's concept of reading the world, Gutstein (2003), defined reading the world with mathematics as using:

> mathematics to understand relation of power, resource inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (p. 45)

In other words, reading the world with mathematics involves students seeing themselves and their future in a given set of numbers and as a result developing a sociopolitical consciousness of their own lives.

Gutstein (2003) developed a series of real-world mathematics projects on his students' lived experience as urban Latino immigrants belonging to working-class families. The purpose of his study was to explore how students may move beyond simple mathematical computation and use mathematics as an analytical tool to learn about their own positionality in the society. In one of the projects, Gutstein provided students with a data set containing the 1997 SAT and ACT scores of students and asked them to analyze the data based on race, gender, and social class. Interestingly, Gutstein found that during their interaction with the ACT and SAT scores, students moved beyond mathematics and started asking questions to find the truth embedded in the data and examine how a student's ACT/SAT score depends on his family income, race, and ethnicity. Some of those questions include, "How come whites and Asians get higher scores, yet everyone else lower scores?", "How does race affect your scores?", and "All of these [low-income] people want to become doctors, lawyers, sports players, etc. So why are 'rich' smarter?" (Gutstein, 2003, p. 52). Gutstein (2003) wrote, "their questions were genuine and from their experiences as educationally disadvantaged and marginalized students, who saw themselves in the data" (p. 52).

Likewise, Peterson (2005) suggested that teachers and educators integrate mathematics across curricula including history and social science and provide students with instances where mathematics has generated major controversies around social and political events. He asserted that an in-depth knowledge of mathematics and statistics may enable students to have better clarification about social structures and policies. Peterson (2005) also argued that students' ability to read numbers would make them competent enough to successfully participate in debates on social issues such as soaring unemployment rate, wage inequality and the federal budget.

In an attempt to make his students aware of the unequal wealth distribution in the world, Peterson (2005) provided them with data on the distribution of population and wealth in the six continents and engaged them in the discussion on the disparity of wealth in the US and other continents. He divided his twenty-four students into groups, as per the world's population distribution and used twenty-four chocolate chip cookies as representative of the total world wealth. When the cookies were divided among students following the proportional distribution of world wealth, students found that some of them received more cookies than others. One student who represented the USA and Canada received eight cookies, whereas three students representing the population of Asia and fifteen students representing the population Africa received .5 and six cookies respectively. By the end of the activity the participating students realized that similar to the result of this activity, in reality as well their "classmates in North American and European sections of the map get so many more cookies even though they have so many fewer people" (Peterson, 2005, p. 13). In this particular task, instead of learning about the issue from their teacher, students modeled the world-wealth disparity and discovered the disproportionate distribution of world wealth among its residents, concluding that the total amount of wealth possessed by a continent does not provide the true picture of its general population (Gutstein, 2003).

Similar to Gutstein (2003) and Peterson (2005), many other researchers also attempted to introduce students with real-life data to familiarize learners with socially relevant issues. For instance, Stinson, Bidwell and Powell (2012) conducted a study to explore how pedagogical practices of teachers evolve when they teach mathematics around issues related to social injustice. Stinson et al. (2012) described the classroom scenarios of two mathematics teachers and illustrated how students' attitude towards racial profiling changed when they were provided with an actual dataset on the particular issue. More specifically, during instruction one of the teachers provided her diverse group of urban/suburban high school students with a dataset on the racial composition of people pulled over for traffic violations in the neighboring state and asked her students to analyze the data, calculate its mean, mode, and standard deviation, and represent the data in graphical form. Following this activity, the teacher asked the students to reflect on their own experience regarding racial profiling. Initially, many students showed skepticism about the relevance of the topic in the mathematics classroom, but as they worked through the mathematics their attitude about the project changed, and they shared their own experience about racial profiling and discrimination. As the students got involved in the project, they got engaged in an interesting and provocative conversation about racial profiling, and they went a step ahead to find instances of racial discrimination in their communities.

2.3.2 Writing the world with mathematics

Writing the world with mathematics is defined as "developing a sense of social agency" (Gutstein, 2006, p. 27) and using mathematics as a tool to change the world. An excellent example of writing the world with mathematics has been described by Tate (1995), where a teacher organized her mathematics lessons around issues relevant to her students and their community. She used tasks that encouraged the students to engage in a series of mathematical activities to model their own society and asked her students to pose a problem that is important for their community and develop strategies to resolve that problem. Students discussed the issue of the inappropriate number of liquor stores in their community and studied in detail the codes, regulations, tax-advantage, and fiscal incentives applied to the situation. Following the data analysis, the students collected, interpreted and presented a data-based argument to the city council complaining about the excess number of liquor stores around their school. Following this movement, about fifteen per cent of the total number of liquor stores were closed, and the city council passed a bill making liquor consumption illegal within 600 feet of the school. This is certainly an example of students moving beyond "percentages, decimals, and fractions" (Tate, 1995, p. 170) and utilizing the power of mathematics to bring changes in their society.
Gutstein (2005) argued that Tate's story is a "relatively rare example" (p. 27) where younger students have used mathematics to bring some impact in the society. According to Gutstein, writing the world with mathematics is a developmental process, where students start to see themselves capable of fighting against social evils and bringing social changes. They see themselves as reformists in the movement and feel obligated to respond to social injustice, even though they are not physically or mentally ready to speak out for themselves.

2.4. Bridging Critical Education theory, Critical Mathematics Literacy and Social Justice Mathematics

Critical education theory, which was once introduced by Freire (1973) as a means to develop within an individual critical socio-political awareness, served as a platform to the development of critical mathematics literacy (Frankenstein, 1994) and social justice mathematics (Gutstein, 2003). All three concepts emphasize on the ability of an individual to critically reflect on the society and take critical actions to transform it. Freire (1973) established his conception of critical literacy on the fundamentals of reading and writing the world. Reading enables an individual to understand the structure of the society and perceive how their "lives are shaped by and in turn can shape the world" (Frankenstein, 1983, p. 7). Writing the world, on the other hand "concentrates on the role of human consciousness in changing the world" (Frankenstein, 1983, p. 23). Building on Frankenstein, Gutstein (2006) talked about the power of mathematics to help students interpret and "make sense of social reality" (p. 70) and support them to "develop a sense of agency, that they would stand up and speak out for that which they believe(d)" (p. 100).

The two factors that I identify to run across all the three concepts are critical reflection (through mathematics) and critical action (through mathematics). Critical reflection, as already been discussed focuses on an individual's ability to critically reflect on injustices and biases across the society, develop awareness about the socio-political inequality, and critical action calls for an "individual's ability to take action to change the socio-political situations around them" (Godfrey & Grayman, 2014, p. 1802). This study is motivated by critical mathematics literacy and teaching mathematics for social justice and is developed on the belief that like any other sociopolitical issues, mathematics literacy can make individuals aware of the greenhouse effect and motivate them to ask questions such as "What is the greenhouse effect?", "Why is it happening?", "Who is responsible for the greenhouse effect?", "Who is most victimized by the phenomenon? Why?", and "What are our responsibilities towards addressing the issue?" Like Freire (1973), Frankenstein (1990, 1994), and Gutstein (2003, 2005), in this study I argue that mathematics is a powerful tool that would provide students with the opportunity to understand the issue of the greenhouse effect and prepare them to critically reflect on the different aspects of the issue and take critical action to mitigate it.

2.5. Social Justice Mathematics and Climatic Issues

Though extensive research has been conducted to promote teaching mathematics as a catalyst to make learners aware of social and political issues (e.g. Frankenstein, 1994; Gutstein, 2005; Tate, 1995), there are only a few studies (e.g., Abtahi et al., 2017; Barwell, 2013; Karrow, Khan, & Fleener, 2017) that focused on the role of mathematics for addressing issues related to climate change or the greenhouse effect. As aforementioned in the introduction, the greenhouse effect is an issue of social justice, which brings down its effect on the poorer people disproportionately, who instead of being the minor consumers of the available resources (Agyeman, Bullard, & Evans, 2002; Costello et. al., 2009; Pettit, 2004), are more likely to endure its harmful effects. Hence, I anticipate that, like any other social justice issues, the greenhouse effect can be better interpreted and addressed through mathematics.

Acknowledging the complexities of the issues related to the climate, Abtahi et al. (2017) questioned the ethical and moral responsibilities of mathematics teachers and educators to educate students about this complex yet pressing issue. They argued that, if teachers assume their ethical responsibilities and incorporate climate change in their mathematics instruction, then that would help students identify the role of mathematics in climate change and prepare the future decision makers to effect change for the betterment of the climate. As Karrow, Khan, and

Fleener (2017) argued, mathematics education should "concern itself with the development of the individual, in relation with our Planetary Ecosystem" (p. 9).

In their study, Abtahi et al. (2017) reported results from a survey to explore Norwegians and Canadian mathematics teachers' opinion about incorporating climate change in mathematics classrooms and found that although overall teachers' action towards introducing climate change was guided by their personal awareness and ethical responsibility towards the environment, there were concerns about the effective implementation of such lessons. Common concerns that echoed through the teachers' responses regarding the implementation of climate change in mathematics instruction include the complexity of climatic issues, the lack of students' mathematical and technical knowledge, and the lack of resources and time. In order to enable mathematics teachers introduce climate-related issues in their classrooms, Gonzalez Martinez (2017) suggested including scientific concepts in mathematics teacher education courses and help them model real-world situations, such as global warming. He further said that such initiative would not only provide students with the opportunity to learn concepts in both mathematics and science more meaningfully but would help both teachers and students to make informed decisions in their lives about the impact of changing climate.

One of the many ways to enable students to recognize social injustice is through the use of mathematical modeling. Research has shown that social justice mathematics and mathematical modeling share some common goals and practices, that make them powerful tools for deepening students' understanding about issues related to their own life and their society (Cirillo et al., 2016). Determining a specific answer to a given problem is not the focus of either of them; rather both the systems encourage learners to engage in discussions with their peers and teachers, investigate a situation from different lenses, and develop an insight about the situation that was presented to them (Johnson, 2007). Similar to social justice mathematics, mathematical modeling provides students with the scope to examine any issue with reference to one's everyday experience via mathematics (Barbosa, 2006). In the following sections, I present how mathematical modeling has been defined by researchers over the last two decades, discuss the benefits of mathematical modeling for school education and for our daily life, and finally describe the role of mathematical modeling in the current study.

2.6. Mathematical Modeling

Mathematical modeling is a cognitive activity of conceptualizing a real-world situation through mathematics (Dym, 2004; Blum & Niss, 1991; Lesh, Amit, & Schorr, 1997; Lesh, 2006; Lesh & Caylor, 2007). It allows a problem solver to create a conceptual system, to understand how a real-world phenomenon occurs, why it occurs, and extend the understanding of the present phenomenon to anticipate a set of similar events in the future (Dym, 2004). This conceptual system, which is purely conceived within the mind of an individual (Lesh & Caylor, 2007), can be represented in the form of mathematical equations, algorithms, diagrams, graphs, computer programs, or any other representations (Abrams, 2001; Lesh, Carmona, & Post, 2002) and can be used to "describe, explain or predict the behavior of some other familiar system" (Doerr & English, 2003, p. 112) is called a mathematical model.

In a traditional problem-solving situation, students learn mathematical ideas and skills to interpret and solve real-life problems in future, whereas, the process of mathematical modeling starts by exploring a real-life situation (Lesh, Doerr, Carmona, & Hjalmarson, 2003), which students might experience in their everyday lives, in and outside the work place (Lesh, Amit, & Schorr, 1997; CCSSO, 2010). Doerr and Pratt (2008) considered mathematical modeling as an activity of mapping, where elements of the real world are selected, organized, and modified to map into the model world. When exposed to a real-life situation, the modeler asks questions, gathers information, and carefully observes the minute details of the situation aiming to develop a better understanding about the situation (Zbiek & Conner, 2006) and construct a model of the situation. For instance, consider the following situation:

To increase the cost efficiency of the school transport system, the finance department of the APJ university is considering reducing the frequency of shuttle buses from the university train station to campus on Fridays. Before making the decision final the authority asked the graduate student association to measure the effectiveness of this decision. To develop a better knowledge about the situation, the graduate student association might request or seek information about different factors involved in the situation. Based on the gathered information they would conduct their further analysis.

Mathematical modeling can be a complex process as a real-life situation might have several extraneous factors. However, to diminish the complexities of the situation and to reduce the situation to a structured and ideal one, a problem solver often excludes some of the insignificant factors (Blum, 2002) and creates a model with the dominant ones and their governing relationship (Voskoglou, 2015). The modeler's consideration about which factors are significant, and which are not dependent on the underlying assumption about the situation is the purpose of the modeling activity (Blum & Niss, 1991; Zbiek & Conner, 2006).

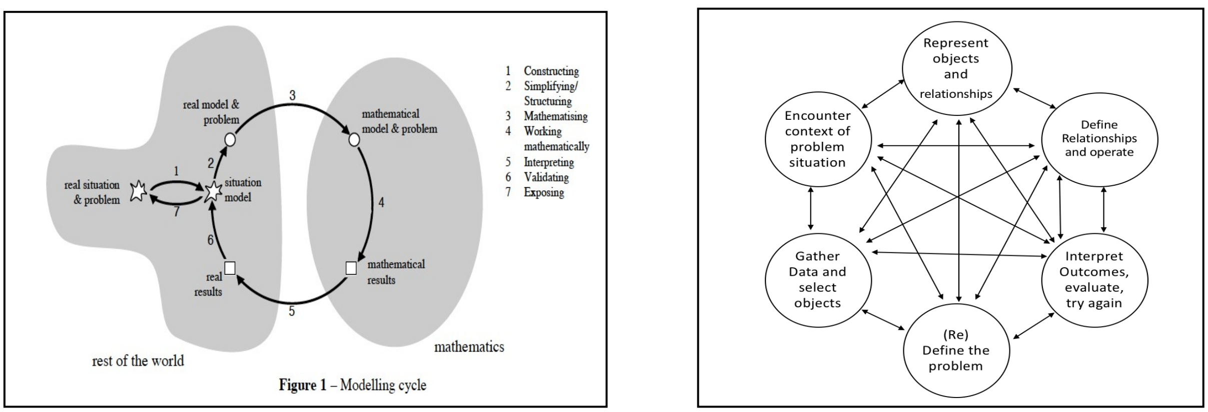
For instance, if we consider the previous example, the graduate student association might recognize that there are several factors involved in the situation, such as the number of classes on Fridays, the number of trains on Fridays, the services available on campus (e.g. cafeteria, library, and gym), other transport options available, the salaries of the shuttle drivers and the cost of fuel. If the graduate student association considers all the factors, then the situation would be extremely complex to be modeled. Instead, they can combine two or three factors and find if there is a correlation between the chosen factors and the number shuttle buses. Since in this example, the purpose of modeling is to determine the cost efficiency of school transport system, then the graduate students' association may choose to examine if there is a correlation between the number of Friday classes and the frequency of the shuttle bus from the train station to campus.

It is also worth mentioning that although mathematical modeling has a close association with problem-solving, there is a significant difference between the two processes. Problem-solving is a single cycle process (Wessels, 2014) where students are required to provide a specific response to a given problem (Lesh, Amit, & Schorr, 1997). However, development of a model seldom follows a unique developmental path (Lesh, 2006). It undergoes a series of iterative testing and revision cycles (Doerr & Pratt, 2008; Lehrer & Schauble, 2000; Lesh & Lehrer, 2003) to develop the initial fuzzy model to a more predictable, generalizable, re-usable and sharable one (Lesh, Amit, & Schorr, 1997; Blum & Niss, 1991; Lesh, Doerr, Carmona, & Hjalmarson, 2003).

Further, unlike mathematical modeling, problem-solving does not refer to an unedited real-life situation. It either includes non-contextual mathematical tasks or problems containing idealized real-life situations (Hirsch & McDuffie, 2016). I will refer to the previous example to explain this difference: Imagine that the graduate students' association found that there is a strong correlation between the number of Friday classes and the frequency of the shuttle bus from the train station to campus. Based on this result they might conclude that the decision of the university authority to reduce the frequency of shuttle buses might not be effective. However, on further analysis the association might find that only a few classes are scheduled on Fridays after

2 PM, therefore the association might revise their initial model to introduce time as a second factor and thus create a new model with a different outcome.

Although mathematical modeling, as described by many researchers seems to be a linear cyclic process (see Figure 1a), some researchers (e.g. Barbosa (2006); Doerr & Pratt (2008)) oppose to the idea of linearity of steps in the process of mathematical modeling. Instead, they argue that modeling involves multiple stages and several cycles of revision where learners move from one stage to another without aligning to a strict succession order. Doerr and Pratt (2008) expressed the non-linear nature of modeling process through nodes and interconnectedness in the following diagram (Figure 1b).



(a) (b)

Figure 1: a) Seven-stage framework of mathematical modeling by Blum and Ferri (2009); b) Non-linear process of modeling proposed by Doerr and Pratt (2008)

In summary, I used the example with the graduate student association to illustrate that mathematical modeling is a non-linear cyclic process, which uses mathematics to explain a reality-based phenomenon, analyze that phenomenon, modify and manipulate the different factors involved, and predict similar phenomena in future that are yet to be measured (Lesh et al., 1997). The process is repeated in a loop of continuous development (Lesh et al., 2003) until a generalizable and sharable mathematical model is developed that can be used for future references to parallel events.

Mathematical modeling activities require higher order thinking skills to reason between the different factors involved in a situation, recognize the patterns between them, synthesize the factors and analyze the situation to enable students to build a simple yet meaningful model (Maiorca & Stohlmann, 2016). Hence, the process of modeling contributes towards students' development of thinking and analyzing skills. In the following three sub-sections, I describe the role of modeling for mathematical sense-making and extend the role of modeling beyond mathematics to explore its connection with different other subjects. Next, I focus on how modeling activities can enable students to become aware of different social and political issues and investigated the position of modeling in current mathematics school curricula.

2.6.1 Mathematical modeling for mathematics sense-making

A students' experience with school mathematics is often limited to textbook problems, where a mathematical problem is "preprocessed and detached" (Doerr & English, 2003, p. 111) from its context and students are expected to apply rules and formulas to solve those problems (Lesh et al., 1997). However, as Mokros and Russell (1995) stated, in mathematics and statistics, "an average makes no sense until data sets make sense as real entities" (p. 35). Indeed, I would argue that unless students are provided with mathematical problems that are embedded in some relevant and meaningful contexts, the mathematical calculations that students perform remain meaningless to them.

Unlike traditional mathematics, mathematical modeling encourages students to focus on a meaningful problem situation, analyze the different factors involved in the situation, develop their thinking around it and finally build an explicit mathematical interpretation of the problem (Lesh, 2006). The cycle of mathematical modeling initiates from an experientially real situation that provides students with a rich platform for the independent development of their mathematical ideas and enables them “to use mathematics as a generative resource in life beyond the classroom" (Doerr & English, 2003, p. 112).

Based on the result of their exploratory study on data modeling with middle-grade students, Doerr and English (2003) stated that mathematical modeling allows students to take diverse approaches towards a mathematization of a real-life situation. It also provides the learners with the opportunity to see and understand the alternate approaches and explanations shared by other students solving the same situation. Doerr and English (2003) conducted a study across two countries, Australia and the USA, where they provided five modeling activities to the participants and asked them to evaluate an object, such as a pair of sneaker or a restaurant, by considering the different factors of the sneaker or the restaurants. For example, in the first activity, students brainstormed the different factors they consider important for buying a pair of sneakers and ordered those factors based on their weight in influencing the purchase decision. Some groups laid emphasis on the style of sneakers, while for some other groups comfort or the brand of the product was more important. During the process, the students engaged in discussion with each other and through negotiation, they modified and remodified their initial models until all the group members came to a consensus.

Lesh and Lehrer (2003) argued that when students are engaged in such meaningful activities of model building or sense-making of a system, then even the most under-privilege students “often produce high quality results that are far more impressive than anything that would have been predicted based on results from their prior work in traditional textbooks and tests” (p. 117). This kind of model-eliciting activities encourage students to spend a quality amount of time thinking about mathematics that they encounter in and outside the classroom (Lesh, 2006). For example, in an exploratory study, Lesh et al. (1997) provided seventh-grade students with a set of data representing money earned by vendors in an amusement park during the summer and asked them to evaluate the performance of each vendor and build a model to re-employ them for maximizing the profit. Lesh and his colleagues observed that the children, who were considered to be of average ability by their teachers emerged as most productive in constructing powerful mathematical models. These students went beyond the simple analysis of the data and investigated the strengths, weaknesses, and assumptions associated with the model they developed to reach their conclusion about re-employment Lesh et al. (1997).

In addition to making mathematics more meaningful, mathematical modeling assists in showing the connections between mathematics and other disciplines, such as science, engineering, and technology. This is because a mathematical model of a real-life situation usually requires a blend of more than one discipline in order to understand it. In the following sub-section, I describe how mathematical modeling may contribute towards the integration of the STEM subjects and explain the importance of this blend for students' understanding of the world.

2.6.2 Mathematical modeling for making the connection between STEM subjects

The process of modeling helps students not only to learn mathematics more meaningfully but also to develop a sense of interconnectedness between different subjects, topics and ideas that they learn in schools (Abrams, 2001; Zbiek & Conner, 2006). Smith (1996) for instance, stated that most of the scientific theories that students learn in schools are developed as mathematical models of "descriptive or predictive nature" (p. 38). In other words, scientific theories which involve multiple factors are expressed through mathematical equations, relations or diagrams, explaining an existing situation or predicting similar situations. For instance, the equation $M = \rho \times V$, which shows how the mass (M) of a liquid depends on the volume (V) and density (ρ) of the liquid, is an expressive form of mathematical model. Similarly, Figure 2 is another model expressed in a diagrammatic form, which shows the chemical structures of Glucose.

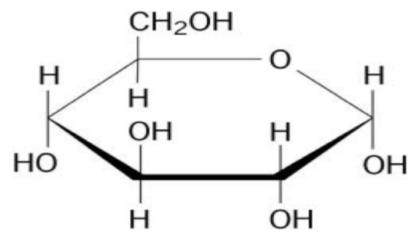


Figure 2: Chemical Structure of Glucose

Scientific theories and concepts presented under the umbrella of mathematical modeling not only help students to develop a deeper understanding of mathematics and science but also make the two curricula more relevant and purposeful (Lesh & Lehrer, 2003; Roehrig, Moore, Wang, & Park, 2012). In fact, Stewart and Golubitsky (2010) criticized the idea of presenting mathematical and scientific concepts as individual bodies of finished knowledge. They argued that our education system should adopt an activity-centered approach, which would engage learners in an ongoing production of knowledge through their active engagement with meaningful tasks. For instance, consider an engineering project of designing the model of a cargo ship. In order to create an optimal design of the ship, students would experiment with the shape of the ship and materials required to make the different parts of the ship. Following their research and multiple trials, students may determine which shape is best for what purpose, and understand different mathematical and scientific concepts, such as buoyancy and water displacement behind the engineering design of the ship. Further, during their engagement in the project students would also perform mathematical calculations to determine the optimal size or weights of different parts of the ship which would maximize its capacity to carry cargos. Thus, modeling activities focus on real-life situations, such as engineering problems, which in turn provide students a natural platform to develop mathematical competencies and scientific understanding.

Considering the above, mathematical modeling can help integrate the STEM subjects for understanding various phenomena of the world. Students often find the content matter of the individual subjects irrelevant, difficult and unrelated to other disciplines and their regular lives (Christensen, Knezek, & Tyler-Wood, 2014; Stohlmann, Moore, & Roehrig, 2012). An integrated STEM curriculum may assist students in understanding and address complex issues of their society, such as issues of social justice. In the following paragraph, I will discuss how modeling activities can also integrate social science and help students identify different socio-political issues.

2.6.3 Mathematical modeling for addressing social issues

As mentioned earlier, teaching mathematics for social justice and mathematical modeling share some common goals and practices that nurture within students a deep-rooted understanding of the society (Wessels, 2014). Cirillo et al. (2016) identified three overlapping features of modeling and social justice that allows learners to engage in real-life issues more meaningfully: a) engage students in ill-defined problems, b) leverage students' real-world knowledge, and c) raise students' awareness. The three features aim to emphasize students' engagement with mathematical problems, developed around learners' experiences or some issue of social and political relevance. These contexts can be used as "anchoring points for the reinvention of mathematics by the students themselves" (Gravemeijer & Doorman, 1999, p. 111) and vice-versa. Modeling activities emerge within students a sense of integrity and enable them to "see and judge independently, to recognize, understand, analyze, and assess representative examples of actual uses of mathematics, including (suggested) solutions to socially significant problems"

(Blum & Niss, 1991, p. 43). When students explore or develop a mathematical model of a realistic situation, they analyze the different factors involved in the situation, understand how the different factors influence each other and the situation, and evaluate the situation in an unbiased manner.

Barbosa (2006) argued that mathematical models are not "neutral descriptions about an independent reality" (p. 294) rather they encompass several socio-political factors, which are not comprehensible to general people. She argued that most policies and decisions in society are formed based on some form of mathematical models, hence it is important for students to be able to understand these models and engage in discussion around those models.

To illustrate how the modeling experience enables students to understand governmental policies, Barbosa (2006) described a classroom situation where she provided her seventh-grade students with newspaper excerpts on the government's decision about distributing bean and corn seeds among farmers. Since many of the students were the direct beneficiary of this program, the mathematical problems that Barbosa (2006) created around this news were meaningful to students. After a brief exploration, students pointed out some discrepancies in the government's announcement, such as discovering that if the government awards 5kg of seeds to each farmer, then 37.5 tons of seeds would not be sufficient for 8000 farmers. This disparity in data led students to investigate the factors based on which the government decided to distribute the seeds and also discussed what factors should be ideal in the distribution process. Through discussion with their peers and constant negotiation, the students discarded the government's criterion and developed new models containing new criteria. During this modeling process, not only students explored different mathematical concepts and ideas, but also explored and made sense of a social situation.

Cirillo et al. (2016) asserted that, when students are presented with a real-life situation, then the students may engage in the continuous cyclic process of developing an optimal model, or they may reframe the task, and brainstorm questions that would provoke them to engage in critical discussions about the concerned issues with their peers. For instance, a modeling task built around the topic of different food habit and obesity can be restructured by leveraging on students' real-life knowledge about the geographical location of fast food shops in different neighborhoods. The task may encourage the students to investigate how the consumption of fast food items is related to more calories and how the low pricing of fast foods and its easy availability is pushing the people of the certain section of the society towards health hazards. Hence, mathematical modeling tasks not only provide students with a meaningful context to learn mathematics, but it imbibes within students a socio-political awareness about the world they live in. However, "genuine modeling activities are still rather rare in mathematics lessons" (Blum, 2002, p. 150). In the following section, I will provide a brief account about the state of mathematical modeling in the current mathematics curriculum and in classrooms.

2.6.4 Current state of mathematical modeling in the school curriculum

Since the last few decades, a lot of work has been done in the field of mathematics education to promote modeling in school and university mathematics (Blum, 1993). Some countries, such as Germany, Netherlands, and Australia have included compulsory modeling components throughout their secondary mathematics curricula (Blum & Niss, 1991). In the United States, the Common Core State Standard Mathematics (CCSSO, 2010) has included modeling both as one of their eight mathematical practices (i.e. "*Model with mathematics"*) and as a content strand of standards in the high school curriculum. This dual nature of modeling in the math curriculum shows that the significant role of modeling is acknowledged. However, this action evokes some concerns that I discuss in the following paragraphs.

Though CCSSM has laid a substantial emphasis on the role of mathematical modeling in the content of high school, we find a limited mention of the term in elementary and middle school mathematics content standards. This comes in contrast with research showing that even young "children are natural modelers" (Lehrer & Schauble 2000, p. 40). Elementary school students have the capacity to solve non-routine mathematical problems which are personally meaningful to them even before classroom instruction (Hirsch & McDuffie, 2016). During her study with fourth and sixth-grade students, English (2016) found that when elementary school students were provided with model-eliciting activities, they engaged in the process of modeling and built various sophisticated models by completing multiple iterations of the modeling cycles. Further, during their involvement in the modeling process, students also developed their thinking about advanced mathematical topics, such as weighted means.

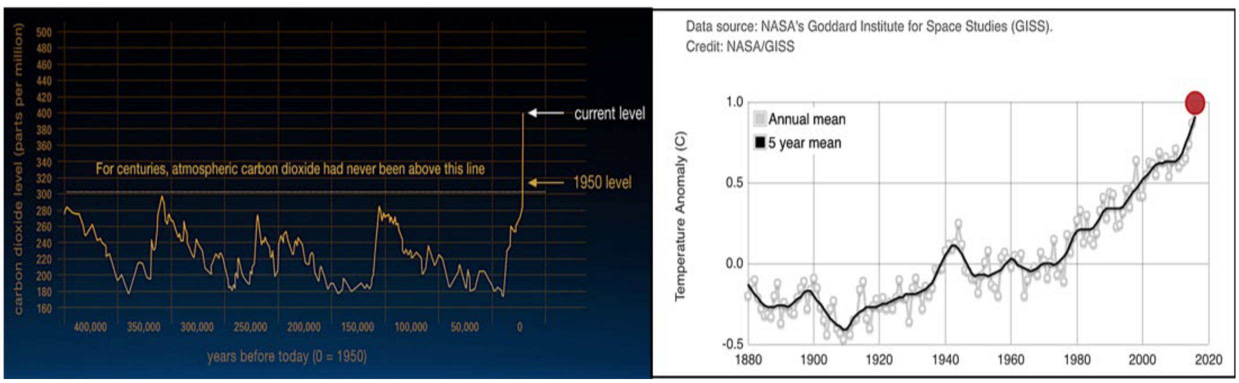
Envisioning the introduction of mathematical modeling in elementary school classrooms, Carlson, Wickstrom, Burroughs, and Fulton (2016) proposed a framework for teaching modeling to young kids that consist of three phases: a) developing and anticipating, b) enacting and c) revising. More specifically, teachers first develop modeling tasks and tailor questions to anticipate students' working strategies and possible misconceptions during their interaction with the tasks. Then they engage students in the process of modeling, pose relevant questions to help students translate a real-life situation to the mathematical world and finally revise their tasks and questions based on their reflection about students' response during the modeling cycles. Similar to Carlson et al. (2016), Lehrer and Schauble (2000) also suggested that modeling should be a central concept of early childhood mathematics education and teachers should design their instruction in order to stretch children's early competencies to long-term "that are complex, multifaceted, and subject to development over years, rather than weeks or months" (p. 40).

For decades, researchers such as Lehrer and Schauble (2000) and Carlson et al. (2016) have appealed for the inclusion of modeling activities in early childhood mathematics education. However, very few classrooms around the world organize their discourse around modeling activities and take initiatives to implement model-eliciting tasks in practice (Blum, 2002). Rather than nurturing within students a genuine disposition for realistic mathematical modeling, most of the classrooms implement "word problems" as an alternative to modeling tasks. Unlike modeling tasks, which encourage students to use common sense to tackle a real-life situation, word problems are pre-structured and are "nothing more than a dressing up of a purely mathematical problem in the words of a segment of the real world" (Blum, 2002, p. 153). In word problems, all the data that are necessary for finding the answer(s) are included. So, such problems provoke students to apply known mathematical rules and perform calculations to find a solution rather than fostering in students a new mathematical knowledge (Schukajlow et al., 2012; Zbiek & Conner, 2006).

Additionally, in the United States, although the CCSSM emphasized students' development of modeling skills to better analyze a mathematical situation within and outside school curriculum (CCSSO, 2010), it does not include mathematical modeling as an explicit topic in K-12 mathematics. According to CCSSM, students proficient in mathematical modeling are better able to analyze a practical situation through identification of different factors involved in that situation and map their relationships through diagrams, tables, graphs, formulas and other representations. However, in school education, mathematical modeling has remained an alternative and engaging setting, which assists students to develop a deeper understanding of curricular mathematics rather than gaining recognition as an instructional goal on its own (Zbiek & Conner, 2006). One of the reasons of why this might be the case is that mathematics content is not always explicit in those modeling activities. Consequently, the next two sections focus on the mathematics content in mathematical modeling aiming to illustrate the importance of modeling not only for developing mathematical practices but also mathematical concepts.

2.7. The Mathematics in Mathematical Modeling

In this age of technological advancement, the internet is a crucial data source. It is flooded with tons of information generated every day by people around the world and is used by researchers, government and multi-national institutions to understand different phenomenon connected to the data. However, considering the vastness of the data it is essential for modelers to extract relevant data from the data pool. Hence, data management and interpretation are essential parts of the 21st Century literacy. Data management, which includes data analysis, is one of the many concepts that students need to use outside of their classroom to better understand the information that they encounter in their daily lives through news media, internet, weather report, advertisement, sports and stock market (Friel, Curcio, & Bright, 2001; Glazer, 2011). Among all forms of data representations, graphs and tables are the most commonly practiced and effective way to visually represent numerical data (Tufte, 1983). Glazer (2011) suggested that in today's information age, an extreme use of visual demonstration to represent any social, political, or cultural issue, is based on the underlying assumption that the viewers would be well able to read and interpret graphs that they encounter in their daily lives. In other words, we assume that readers would gain a transparent and proper understanding of the pictures that they come across in newspapers, magazines, and educational journals without being engaged in an investigation of that situation. For instance, the following graphs (Figure 3a and b) are published in the website of NASA (https://climate.nasa.gov), illustrating how the level of carbon dioxide in the atmosphere has increased steeply over last few decades and how the global surface temperature has changed over the period of 136 years. If an individual is able to understand how the two factors plotted in the graph are changing in relation to each other, then he/she can become familiar with the reality of the greenhouse effect. Further, the reader could use the graphs to predict future global, regional or local effects of the greenhouse effect on human life based on the current trend.



(a) *(b)*

Figure 3a) Graph showing the level of carbon dioxide in the atmosphere has increased over last few years. b) The graph shows how global surface temperature has changed over last 136 years

However, understanding graphical representation is not an inherent skill (Roth & McGinn, 1997). Reading graphs and interpreting them can be challenging for many students and adults too (Glazer, 2011; Monk & Nemirovsky, 1994). Research shows that, when students are asked to interpret a graph, they often correctly respond to the mathematical questions embedded within the graph and fail to provide a correct explanation of the underlying situation. In an exploratory study conducted by Friel and Bright (1996), the researchers provided seventy-six middle school students with four graphs representing everyday life situations coupled with some questions to guide students to read the data and develop an in-depth understanding of the data.

Results of the study suggested that most of the students provided correct answers to the mathematical questions asked to them. However, when they were asked to provide an explanation for their responses, they failed to provide correct reasoning for their answers. Friel and Bright (1996) suggested that our usual emphasis on getting an answer of a mathematical problem without asking follow-up questions to learn about students' logical explanation behind the solution, often make our students unable to develop a deeper understanding of the situation.

Another common difficulty in reading graphs relates to viewing graphs as a mere picture of a particular situation and overlooking the underlying relationship between the factors represented there. According to Monk and Nemirovsky (1994), when students interact with graphs, they often make overly simple connections between the visual features of the graphs with the physical events. Students often get captivated by the pictorial representations of the graphs, such as steepness of a line segment, wavy curves or one graph is more spread apart than the other, rather than focusing the mathematical relationships represented there. During his work with advance level students from Calculus class, Kaput (1992) found that students often get distracted by the shape of a graph and overlook the covariational relationships between two variables involved in the situation. Consequently, many researchers in mathematics education have conducted research focusing on the relationships between quantities in a situation rather than working on decontextualized situations. In the following section, I discuss how covariational relationships have been defined in the literature and the role of such covariational relationships for developing students' understanding of reading and interpreting data.

2.7.1 Covariational reasoning and mathematical modeling

Covariational reasoning is an explicit form of thinking and reasoning in the field of mathematics; however, rarely it has been considered as an explicit mathematical concept

(Thompson & Carlson, 2017). Confrey and Smith (1994) defined covariational reasoning as a coordination between two sets of variables as the values of those variables change. They stated that two variables *x* and *y* are connected covariationally if the variable *y* moves operationally from y_m to y_{m+1} when the variable *x* moves from x_m to x_{m+1}. For instance, if the two sets of variables are written in a tabular form, covariational reasoning "involves the coordination of the variation in two or more columns as one moves down (or up) the table" (Confrey & Smith, 1994, p. 33). In the following table (Figure 4a), as the value of year (x) is increased by +1, the value of air temperature (y) is increased by +3, whereas in the second table the value of temperature (y) is increased multiplicatively by .2, as the value of year (x) is incremented by +1 (Figure 4b).

	Year (x)	Temperature (y)	
+1	1	2	+3
+1	2	5	+3
+1	3	8	+3
+1	…	…	+3
	10	29	

(a)

	Year (x)	Temperature (y)	
+1	1	2	x2
+1	2	0.4	x2
+1	3	.08	x2
+1	…	…	x2
	10	.000001024	

(b)

Figure 4: Students' covariational reasoning as described by Confrey and Smith (1994)

Thompson (1993) studied covariation from a quantitative reasoning perspective. His interpretation of covariation reasoning stemmed from students' understanding of quantities and the relationship between two changing quantities. According to him, a quantity is a measurable attribute of an object, such as length, width, and height of an object (Thompson, 1993, 2011) and quantification is the process of assigning numerical values to the quantities. According to Thompson's theory of quantitative reasoning, a person reasons covariationally when "she envisions two quantities' values varying and envisions them varying simultaneously" (Thompson

& Carlson, 2017). For instance, when the height of sea water rises due to the increase of atmospheric temperature, then the two quantities, the height of water level in the sea and the amount of habitable land, co-vary. Engaging in this type of reasoning is what constitutes covariational reasoning.

In an attempt to study high performing students' conceptions of functions, Carlson (1998) conducted an investigation with students enrolled in college algebra, calculus or those who started graduate study in math. She provided them with tasks on the concept of function that prompted them to construct a graph to represent how the values of two quantities change together dynamically. Some of the tasks given to the students including filling a bottle with water and dragging the base of a ladder leaning along a wall and reasoning about the change in the quantities involved. Based on students' work and excerpts collected during the follow-up interviews, Carlson reported that very few of the high performing Calculus students were able to represent continuously changing events graphically. Further, students' tendency to view a dynamic situation in a static way hindered them from expressing one quantity as the function of another. Aiming to classify students' reasoning of these relationships, Carlson, Jacobs, Coe, Larsen, and Hsu (2002) elaborated Saldanha and Thompson's (1998) conjecture that covariational reasoning is developmental and proposed a framework that describes five mental actions that an individual may go through when involved in covariational reasoning (see Table 1).

Table 1: Mental Action of Covariational Framework by Carlson et al. (2002)

Mental action	Description of mental action	Behaviors
Mental Action 1 (MA1)	Coordinating the value of one variable with changes in the other	• Labeling the axes with verbal indications of coordinating the two variables (e.g., y changes with changes in x)
Mental Action 2 (MA2)	Coordinating the direction of change of one variable with changes in the other variable	• Constructing an increasing straight line • Verbalizing an awareness of the direction of change of the output while considering changes in the input
Mental Action 3 (MA3)	Coordinating the amount of change of one variable with changes in the other variable	• Plotting points/constructing secant lines • Verbalizing an awareness of the amount of change of the output while considering changes in the input
Mental Action 4 (MA4)	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable.	• Constructing contiguous secant lines for the domain • Verbalizing an awareness of the rate of change of the output (with respect to the input) while considering uniform increments of the input
Mental Action 5 (MA5)	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function	• Constructing a smooth curve with clear indications of concavity changes • Verbalizing an awareness of the instantaneous changes in the rate of change for the entire domain of the function (direction of concavities and inflection points are correct)

Table 1 consists of three columns: the first column contains the different levels of mental actions, the second includes a description of each mental action and the third presents the behaviors that students exhibit as they reason covariationally. The five mental actions are hierarchical in the sense that, students reach a certain level of reasoning only when they are able to reason using the mental actions associated with that level and the levels lower to it (Moore, 2010). In the following paragraph, I explain Carlson's mental action of framework with an example to demonstrate how students' covariational reasoning may progress when they are provided with a graph similar to the one shown in Figure 5.

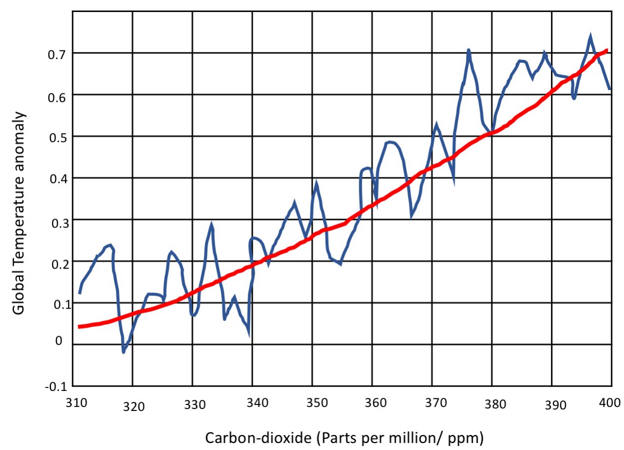


Figure 5: Rate of change of temperature for a change in the amount of carbon dioxide

The graph in Figure 5 represents how the temperature of the earth changes due to change in the amount of carbon dioxide. According to Carlson et al., (2002), students exhibit MA1 when they focus on coordination of quantities. For instance, if students recognize that the value of temperature changes as the value of carbo-dioxide changes, then they would exhibit a behavior supported by Mental Action 1 (MA1).

In Mental Action 2 (MA2), students turn their attention from the values of the two variables to the direction of change in one variable due to the change in the other variable. During this stage, they focus on the shape of the straight line representing the relationship between the two variables (increasing/ decreasing). In Figure 5, if students can reason that the value of temperature increases, as the value of carbon dioxide increases, then their behavior is guided by Mental Action 2 (MA2).

Mental Action 3 (MA3) involves the coordination between the amount of change in one variable due to change in the other variable. During this stage, a student may partition the x-axis into small intervals of equal length, say x_1, x_2, x_3, …, x_n, and observe the amount of change in the value of y, for every interval of x. In the temperature-carbon dioxide graph, if students partition the axis containing carbon dioxide and observe that the value of temperature increases by Y as the value of carbon dioxide increases by X (310 to 320 or 320 to 330), then students exhibit Mental Action 3 (MA3).

Students exhibit Mental Action 4 and 5 (MA4 and MA5) if they can coordinate the average and instantaneous rate of change of one variable with respect to change in the other variable. If students consider how air temperature increases with a uniform change of carbon dioxide amount respectively, then they exhibit behavior guided by MA4. For example, if students plot the amount of carbon dioxide along x-axis and air temperature along y-axis, then in Mental Action 4 (MA4) students would be able to verbalize that the average rate of change of temperature due to change in carbon dioxide for successive intervals (say [x_1= 310, x_2= 320]) is $\frac{yy_2 - yy_1}{xx_2 - xx_1}$ (Figure 5). If they identify the instantaneous rate of change of temperature over an interval of the domain, that is, if students identify that the rate of change of temperature for the change of carbon dioxide (say [x_1= 310, x_9= 400]) is $\frac{yy_9 - yy_1}{xx_9 - xx_1}$, then their behavior is guided by Mental Action 5 (MA5).

Although many studies have been conducted to investigate how covariational reasoning enables students to understand specific math topics, such as exponential growth (Confrey & Smith, 1994, 1995), rate of change (Saldanha & Thompson, 1998), and functions (Carlson, 1998), but none of these studies focused on students' understanding of covariational reasoning for reading and interpreting real-life data neither for interpreting and addressing environmental issues and social justice issues. Specifically, I want to employ Carlson's Mental Action framework as a lens to understand how students reason covariationally when they are provided with mathematical tasks embedded in the context of the greenhouse effect. The next section gathers all these gaps I found in the literature and describes how these gaps helped me formulate my dissertation research topic.

2.8. Conclusion

In this chapter, I reviewed the pertinent literature concerning the role of critical mathematics and social justice mathematics for developing within students a sense of critical consciousness about their life and about the society they live in. Some common social issues that were chosen by mathematics educators to study the impact of mathematics literacy on developing students' social awareness are unemployment rate (Frankenstein, 1990), racial profiling (Stinson et. al., 2012), and gentrification (Gutstein, 2003); however, there are very few studies where researchers focused their attention towards the social aspect of environmental issues, such as the greenhouse effect. Barwell (2013), Abtahi et al. (2017), and Karrow, Khan, and Fleener (2017) are some of the names that emerged in last few years in the field of mathematics education who identified and urged the inclusion of climatic issues in school mathematics curriculum in order to prepare the future generation to understand, predict, and mitigate climatic disruption through mathematics literacy. Following their path, in this study I attempt to engage students to understanding the environmental as well as social aspects of the greenhouse effect through dynamic mathematical modeling activities.

Mathematical modeling is a very powerful approach for generating within students an awareness about the society they live in. Consequently, I aim to engage students in mathematical modeling activities to help interpret and address social issues related to the greenhouse effect and climate change. In order to identify the impact of their daily actions on the greenhouse effect and understand how this can influence their lives in future, students would engage in covariational tasks in which they will explore quantities that co-vary, such as the amount of carbon dioxide, air temperature, and height of sea level rise. Apart from generating within students an awareness about the greenhouse effect, I hope that students will also develop their covariational reasoning through as a result of their interaction with the modeling activities, which in turn would help them understand the causes and consequences of the greenhouse effect.

3. Methodology

The primary goal of my research is to design dynamic mathematical modeling activities that engage students in covariational reasoning and help them identify the traits and consequences of the greenhouse effect. Considering the above, in this study I designed three mathematical modeling activities that I anticipated would support students' covariational reasoning and the greenhouse effect. To be specific, in this research, I investigated:

i) To what extent do the students' understanding of the greenhouse effect and covariational reasoning change as a result of their engagement with the mathematical modeling activities?

ii) How may students reason covariationally as they engage with mathematical modeling activities in the context of the greenhouse effect?

This chapter begins with an overview of my research methodology, the design experiment, and then describes the three phases of my research: the development phase, the implementation phase, and the data collection and analysis phase. The development phase provides a detailed description of the mathematical tasks I created using technological tools and describes how each of the task is expected to help students reach different levels of covariational reasoning based on Carlson et al.'s (2002) Mental Action of Covariation framework. A description of each task along with its intended goal helped me establish a transparency between the goal of this study and the principles of the design experiment methodology. Next, the implementation phase includes a detailed account of the research design, participants, and context. Subsequently, the data collection and analysis phase describes the methods I used to collect data and analyze them to study the above research questions.

3.1. Design Experiment

The primary methodology for this study is the design experiment. According to Cobb et. al. (2003) a design experiment is a highly interventionist method (involving some sort of design) where a researcher engineers particular forms of learning in naturalistic settings and studies the impact of those forms of learning in the given context with the implicit goal of supporting them. An individual might argue that to test the effectiveness of a given intervention, researchers can conduct the study in a controlled laboratory setting. However, as Schoenfeld (2006) states, "design experiments are set in the messy situations that characterizes real life learning, in order to avoid the distortions of laboratory experiment" (p. 9). Hence, a design experiment is a test-bed for innovation (Schoenfeld, 2006) where a researcher identifies a problem in the context of education, such as some unquestioned institutionalized instructional goals (Cobb & Gravemeijer, 2008), generates a hypothetical solution to that problem, conducts an anticipatory thought experiment regarding how the anticipated solution might support students' learning (Cobb, Stephan, McClain, & Gravemeijer, 2001), and tests the hypothesis with students in classroom settings with the aspiration to improve the students' learning as a result of the given intervention.

The hypothetical solution is usually in the form of testable conjectures formulated by the researcher as a means to support students' learning and these hypothetical solutions often include instructional activities, associated resources such as computers and other manipulatives, and the norms and classroom discourse (Cobb & Gravemeijer, 2008). As mentioned during the literature review, we know very little about how mathematical modeling activities might help middle school students to develop an awareness about the causes and consequences of climatic issues such as the greenhouse effect. Hence, in this study, I conjectured that when students are introduced to dynamic mathematical modeling activities, embedded in the context of some relevant environmental and social issue, such as the greenhouse effect, then the dynamic nature of the tasks would help students to reason covariationally, which in turn would help them identify the different traits of the greenhouse effect.

My initial conjecture, which was also the broader goal of my study, was based on the framework of critical mathematics literacy (Frankenstein, 1994) and reading and writing the world with mathematics (Gutstein, 2003). Both these frameworks advocate the positive role of mathematics to develop within students a sense of critical awareness towards the world they live in and question different socio-political and environmental phenomena. Any methodology, as observed by Saxe (1994) offers a two-fold goal to a research study. It informs the researchers about the techniques of gathering and analyzing data and also contributes towards framing of research questions by relating the methodological approach to central epistemological assumptions. Building on Saxe's argument, this study is also informed by design experiment methodology both in terms of selecting theoretical frameworks and design techniques, which is thoroughly described in the following sections.

An integral property of a design experiment is the development of theories regarding both the process of learning and the means that are designed to support that learning (Cobb et. al., 2003; Gravemeijer, & Cobb, 2006). As mentioned above, a design experiment is conducted with the goal to generate a plausible solution to a problem in a given setting. The solution might refer to the entire problem-solving process or it might indicate an actual design, such as a set of tasks, tools, materials, program or instruction that a researcher anticipates might address the concerned teaching-learning problem. In either case, the design is not an "ostensible product" (Crompton, 2015, p. 4), rather, it is a set of conjectures that are tentative and provisional and are subject to a regular modification based on the ongoing analysis of classroom events (Cobb, Stephan, McClain, & Gravemeijer, 2001). Such designs, which are constantly generated and refined during a design experiment, are tools that help reveal important information and develop theories and test those theories to understand the learning ecology (Cobb et. al., 2003). These theories are considered "humble" (Cobb et. al., 2003, p. 9) because they focus on the specific learning processes and are accountable to the activity of the designs.

Similarly, in this study, I designed a set of dynamic mathematical modeling activities on the greenhouse effect. I theorized that as students would engage with the tasks and reason between two or more dynamically changing quantities, such as the amount of carbon dioxide, global air temperature, and height of future sea level, they would engage in covariational reasoning as per Carlson et al.'s (2002) Mental Action Framework. Further, I conjecture that students' conception of covariation would help them discover the causes and consequences of the greenhouse effect, such as recognizing, the more the amount of carbon dioxide in the atmosphere, the higher the global air temperature and the higher the height of future sea level. The theory I developed about students' developing conception of covariation is "humble" (Cobb et. al., 2003, p. 9). The theory is best justifiable when similar dynamic mathematical modeling activities are administered in middle school classrooms and might not be applicable in other settings.

Though design-based research develops theories in local contexts, its goal is "not just to meet local needs, but to advance a theoretical agenda to uncover, explore and confirm theoretical relationships" (Barab & Squire, 2004, p. 5). In other words, researchers do not conduct design experiments just to provide warrants regarding the effectiveness of a proposed strategy in a local context, but also to treat the changes made in one context, as evidence towards forming a theory in the broader educational context. Similarly, in this study I developed artifacts and practices that

I hoped would not only help middle school students in understanding covariational reasoning and the greenhouse effect but, when revised and refined, would also be applicable in other settings. I hoped that the results of the study would lay a promising path for other similar studies in the field of mathematics education, which would intend to use the power of mathematics to address environmental and social issues such as the greenhouse effect.

Considering the above, I used the design experiment methodology to undertake the following four primary objectives:

a) to develop dynamic mathematical modeling tasks for the middle school students;

b) to study students' thinking as they engage with the tasks and observe the progression of their covariational reasoning; and,

c) to examine the role of covariational reasoning in students' identification of the different traits and consequences of the greenhouse effect;

3.1.1 Development Phase

The primary goal of a design-based research is the development of instructional sequences guided by a domain-specific theory (Cobb, Stephan, McClain, & Gravemeijer, 2001). Likewise, in this study, I developed a series of dynamic mathematical modeling activities, which consisted of three NetLogo simulations, five investigations (APPENDIX II to APPENDIX VI) containing tasks in order to prompt students to focus on specific features of the simulations and reason about dynamic events, and discussion questions in order to guide students' reasoning through particular forms of covariational reasoning. All the simulations, tasks, and the questions were guided by Carlson et al.'s (2002) Mental Action framework of covariation. In this section, first I will describe the framework and explain how the five mental actions described in the framework have helped me to anticipate the probable levels of students' covariational reasoning in light of my study. Next, I present the rationale of using technology in the task design and describe the three simulations I developed together with the tasks that accompany each of the simulations.

3.1.1.1 Task design framework

To develop mathematical modeling activities that would engage students in covariational reasoning, I employed Carlson et al.'s (2002) Mental Action framework of covariation. As aforementioned in the literature review, this constructivist framework captures five mental actions that an individual may go through when involved in covariation reasoning through graphical activities. Every simulation of this study was accompanied with a set of tasks, some of which explicitly asked students to express the relationships between two or more concerned quantities in words, while others were activities involving graphical representations, which I anticipated might create a platform for students to visualize how two quantities covary with respect to each other and help them construct a relationship between them.

For instance, a particular simulation connecting 'global air temperature' and 'the height of future sea level' includes questions, such as "How does the height of future sea level change as the global air temperature rises by 2 degrees?" It was my hope that such a question would prompt students to focus on the two quantities and coordinate the amount of change of the output variable (height of future sea level) with the change in the input variable (rise in global air temperature), thus helping them engage in Level 3 covariational reasoning. The same simulation includes a graphical activity that asked students to collect the values of future sea level for different rises in global air temperature in a table and then plot those ordered pairs on a graph. I conjectured that this graphing activity would provide students with the scope to think deeply about the relationship between the two quantities and investigate the amount of change (MA3) and the rate of change (MA4) of one quantity with respect to the change in the other quantity.

The following table (*Table 2*) is an adaptation of the Mental Action of Covariational Framework table developed by Carlson et al. (2002), where in the third column I included questions that I created to complement the simulations and prompt the participating students to attain different levels of mental actions.

Table 2: Mental Action of Covariational Framework by Carlson et al. (2002)

Mental Action	Description of mental action	Probing Questions
Mental Action 1 (MA 1)	Coordinating the value of one variable with changes in the other.	What happens to the value of air temperature as you change the value of carbon dioxide?
Mental Action 2 (MA 2)	Coordinating the direction of change of one variable with changes in the other variable.	How does the value of air temperature change as we increase the value of carbon dioxide?
Mental Action 3 (MA 3)	Coordinating the amount of change of one variable with changes in the other variable.	How does the value of air temperature change when the amount of carbon dioxide increases from 200 to 300 and 300 to 400?
Mental Action 4 (MA 4)	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable.	What can you say about the change in the value of air temperature for each interval of carbon dioxide?
Mental Action 5 (MA 5)	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function	How did the value of air temperature change for the entire change in carbon dioxide?

The five mental actions are hierarchical, in the sense that if a student exhibits a Level 3 reasoning (MA3), then the student has already reached the lower levels of understandings (Carlson et al., 2002). The objective behind developing such mathematical modeling activities was to provide students with an exploratory space to engage in different levels of covariational reasoning that closely aligns to the practice standards laid by Common Core State Standard of Mathematics (CCSSO, 2010). CCSSO encourages students a) to use mathematics to identify important quantities in a practical situation, b) map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas, and c) analyze the relationships between the concerned quantities to draw conclusions of the given situation. According to CCSSO, these practices help students connect classroom mathematics to everyday life, work, and decision-making. Consistent with the practice standards of CCSSO, this study utilized the power of technology to develop dynamic mathematical modeling activities and created an opportunity for students to engage in covariational reasoning between two or more quantities and build their understanding about the causes and consequences of the greenhouse effect. To discuss, how the dynamic nature of the activities created an interactive environment for the students to explore the relationships between covarying quantities, in the next section, I describe the role that technology played in the task design procedure.

3.1.1.2 The role of technology in task design

Jonassen, Carr, and Yueh (1998) emphasized the importance of technology in the modern education system and stated that technology should not only be used to provide students an engaging ambience and supporting students' learning, but it should also be used as a knowledge construction tool. They stated that technology has the potential to provide students with a discovery space where they could explore different real-life phenomena, experiment with them and engage in critical thinking about the context and the underlying content that they are studying. Technology-enhanced learning environment offers students a scope to learn new mathematical and scientific concepts through visualizations and helps them develop a sound mental representation (Varma & Linn, 2012). For instance, when students learn certain mathematical or scientific concepts using software, such as Geometer sketchpad, GeoGebra, or using programing languages such as NetLogo or Scratch, then these programming languages provides students an interactive space to explore and create various mathematical models dynamically along with satisfying the cognitive demand of the curricular mathematics (Johnson, 2007). Instead of receiving direct instructions about any algorithm, technology provides students with an interactive mathematical interface where they can explore relationships between different quantities based on the increasing or decreasing values of the quantities (Bos, 2009). In a self-exploratory and dynamic interface, students get the opportunity to tinker and manipulate the values of different quantities and observe patterns between them (Resnick, 2014). Such mathematical patterns that emerge intuitively, help students construct their own algorithms and form conjectures based on their own observations and interpretations (Bos, 2009; Pratt, 2012).

In this study, I used NetLogo (Wilensky, 1999) to design a set of three simulations aiming to provide students with a research bed (Pratt, 2012) to self-explore different quantities included in the simulation and develop a deeper conception about covariational reasoning. NetLogo is an agent-based modeling tool developed by Uri Wilensky in 1999, which uses Java and a version of the Logo programming language (Papert & Harel, 1991). According to Wilensky and Reisman (2006), there is a sharp contrast between the ways a subject matter is learned by students in a usual school setting and in NetLogo. They argued that though in both approaches the object of study might remain the same, the process involved in understanding the intended conception is quite different. Unlike traditional classroom instruction, NetLogo treats its participants as "active theorizers" (Wilensky & Reisman, 2006, p. 172), who are involved in the process of constructing and testing theories in light of new evidence students gather through observing the simulations. Similarly, in my study, on the onset of exploring each simulation, I encourage students to hypothesize the outcome of the simulation and then engage with the tasks "searching for confirming-disconfirming evidence" (Wilensky & Reisman, 2006, p. 172) to either accept their initial hypothesis or to refine it. For instance, in a simulation involving the two quantities of carbon dioxide and air temperature, students may create a conjecture regarding the relationship between the two quantities, such as arguing that either the two quantities change in the same or in the opposite direction, and then explore the simulation to gather evidence to either accept or refine their theory.

3.1.1.3 Simulations and tasks

The design experiment is an iterative ongoing process where the researcher designs an initial task and based on students’ responses, it undergoes continuous cycles of refinement and enactment (Middleton, Gorard, Taylor, & Ritland, 2006; Schoefeld, 2006). During each cycle, the researcher engages in a daily cycle of classroom analysis, reform modification and implementation to nurture within students a deep mathematical understanding (Brown, 1992; Cobb, 2000; Schoenfeld, 2006). Following the same principle, I attempted to design and refine mathematical activities that went through two rounds of iterations based on the participants’ responses. It is worth mentioning that, I do not claim that the latest version of the tasks that I have at present is beyond further revision.

This study is a part of a bigger STEM project developed by an interdisciplinary team of mathematics educators, computer scientists and environment scientists. I was fortunate to be a part of the STEM project and work with the interdisciplinary team who assisted me in the task design process. The computer scientists helped me design the simulations in NetLogo while I consulted the environmental scientists about the greenhouse effect content.

For the first iteration of my study, except the first simulation, the Climate Change, I could not find any pre-existing mathematical tasks that I could use. This might have happened due to a lack of substantial research in the field of mathematics education exploring the role of covariational reasoning in middle school students' understanding of the greenhouse effect. The only relevant tasks that I found were primarily designed for high school and college undergraduates and could not be used in a middle school setting. Therefore, I created an original sequence of tasks using NetLogo simulations to provide middle school students with an opportunity to engage in dynamic experiences of exploring the greenhouse effect. All the tasks had two-fold goals, mathematical and environmental, and consisted of a series of questions, which prompted students to reach the target understandings. The following sub-sections describe the three simulations I used and the "investigations" (set of tasks) I designed to accompany each simulation.

3.1.1.4 Description of NetLogo interface

NetLogo provides students with a naturalistic environment to engage in mathematical modeling activities (Johnson, 2007). Its design is based on an "embodied modeling approach [that] connects more directly to students' experience, enables extended investigations as well as deeper understanding" (Wilensky & Reisman, 2006, p. 171). In this section, I will provide some description of the NetLogo environment, with reference to one of the simulations I used in this study to provide the readers an intuitive understanding of the interface.

The interface of NetLogo simulation is divided into two sections. On the right is the graphics window section and on the left is the control section. The graphic window section, as shown in Figure 6, contains the visuals that makes the "world" of the model visible. The Control section comprises of (i) the buttons that controls the model, (ii) the sliders that regulates all the parameters, and (iii) plotting windows that contains graphs expressing the relationship between two or more quantities. The Setup button, on the top left of the control section prepares the model, resets it to the initial interface, and the Go button runs the model. In this particular

Climate Change simulation, every time a user clicks the Setup button, the graphics returns to its initial display mode and restarts the simulation from the beginning. The sliders, below the Setup and Go buttons in Figure 6, enable the users to control the values of different parameters within given ranges and observe the impact of the change on the graphs and the graphics. For example, in this particular simulation when users move the albedo slider to the left and right, thus decreasing and increasing the value of albedo between zero and one, the thin rectangular patch on the earth's surface (green in the picture) changes its color from black to white, thus representing the reflective surface of the earth. Apart from the visual on the right side, the movement of the albedo slider also leads to change of the Global temperature vs. Time graph, thus expressing the impact of albedo on the Global air temperature.

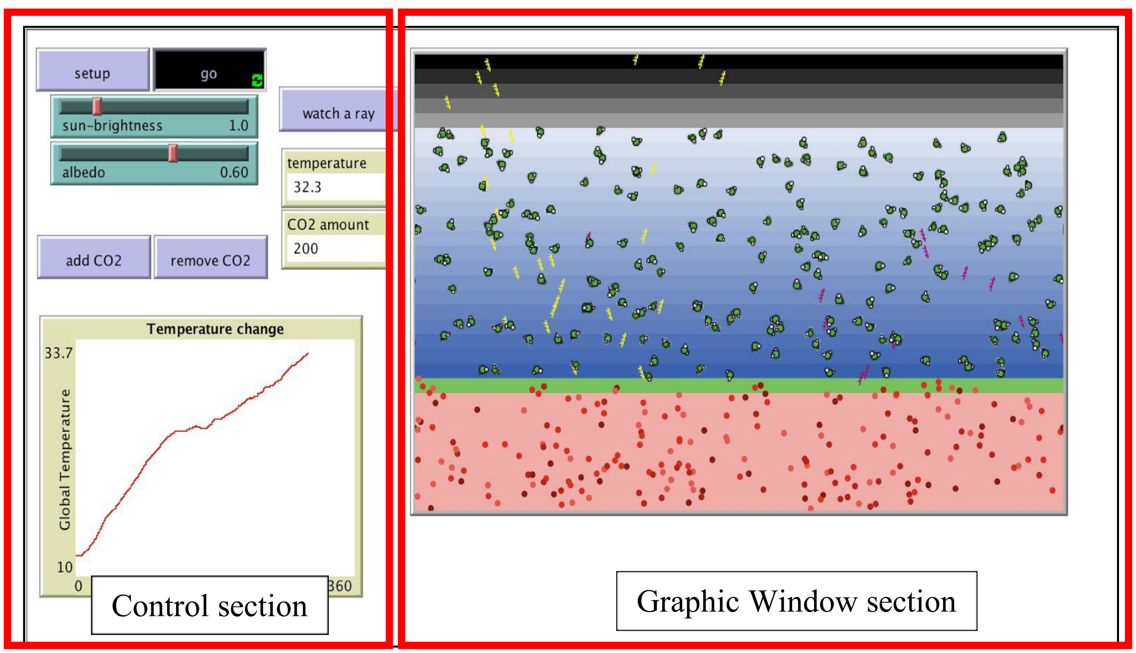


Figure 6: Description of the NetLogo Interface

3.1.1.5 Simulation 1: Climate Change

The first simulation that I use in this study is the Climate Change simulation (*Figure 7*). The simulation has been adopted from NetLogo (https://ccl.northwestern.edu/NetLogo/models/ClimateChange). The Climate Change simulation represents a model of the heat energy flow in the earth. When sunlight falls on the earth's surface it either gets reflected in the atmosphere or is absorbed by the earth. The absorbed particles are infrared rays and they are represented as red-dots in the simulation. The red dots randomly move around the earth and the simulation shows that as the amount of infrared rays absorbed by the earth increases, the air temperature also increases.

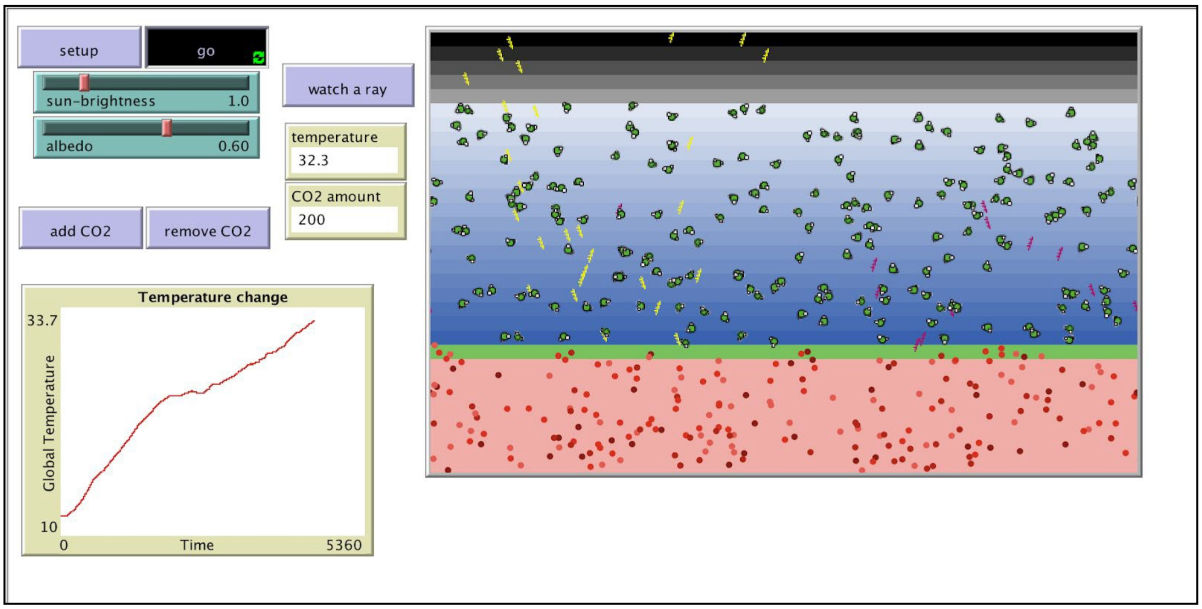


Figure 7: Change of global temperature with change in time

The simulation includes two more factors of the environment, the albedo of the earth and the amount of carbon dioxide. The albedo of the earth, otherwise known as terrestrial albedo, is the measure of the reflective nature of the earth's surface. It is the proportion of the sun's radiation reflected by the surface of the earth. Various elements of the earth, such as the clouds, oceans, deserts, and forests absorb solar radiation and contribute to the global temperature of the planet. Since regions like forests and oceans are darker in color, they have lower albedo and absorb more amounts of sun's energy. On the contrary, ice and white clouds have a high albedo and absorb less amount of the sun's energy. All these different albedos at different sections of the earth are averaged out to give the albedo of the planet. In this simulation, the user can manipulate the value of albedo of the earth (from 0 to 1 and back) and observe the effect of albedo on the graphics of the simulation and on the atmospheric temperature.

When the value of albedo was zero, the graphic of the simulation exhibited a black rectangular patch on the earth's surface, and the number of reflected sun rays was significantly low. When the slider was dragged to the right, halfway through, the black patch turned green, and the reflected rays increased simultaneously. Finally, when the albedo reached its maximum value, the green patch was replaced by a white patch, indicating the higher reflective surfaces such as ice or snow and the number of reflected rays became maximum (Figure 8).

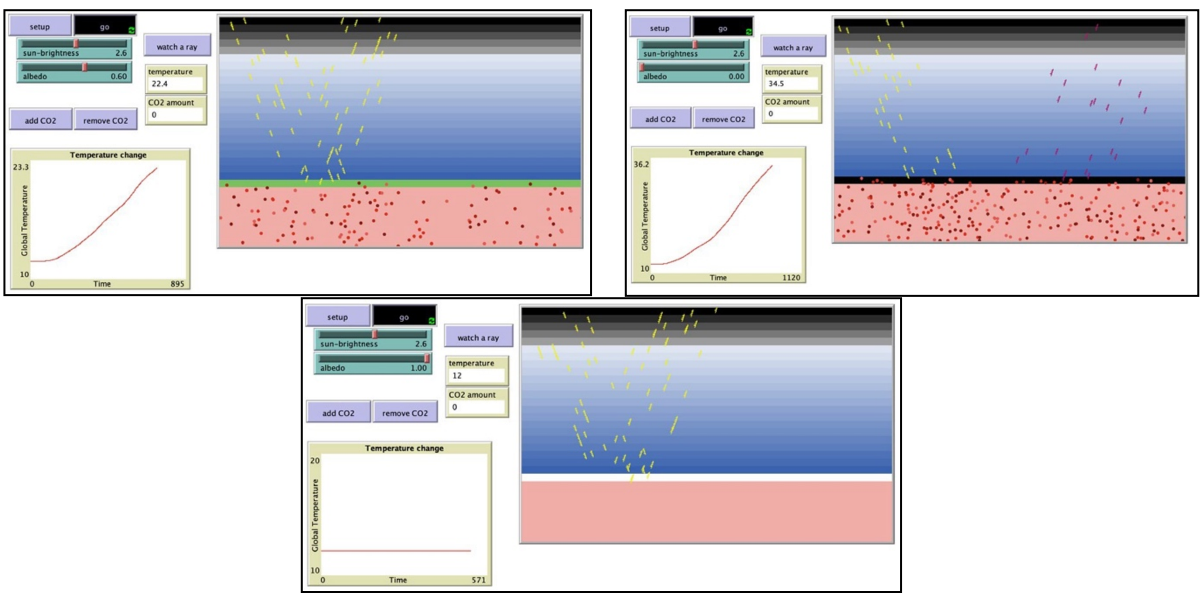


Figure 8: Graphics showing the reflective natures of the earth's surface for different values of albedo

The original 'Climate Change' model also included clouds, where the users were allowed to add and remove clouds, thus manipulating the value of albedo of the earth and influencing the atmospheric temperature. For simplifying the simulation for middle school students, I removed the cloud from the modified version to reduce the number of input variables that might affect the air temperature.

The simulation also allows the users to increase and decrease the amount of carbon dioxide molecules and investigate how the value of atmospheric temperature changes along with it. The climate change simulation includes a time-series graph representing the change in the value of global air temperature with respect to time. In the graph, time is plotted along the horizontal axis and global temperature along the vertical axis. Though the graph represents time along the x-axis, the users do not have the capability to manipulate the value of time. Rather they can change the value of carbon dioxide or albedo and observe the graph to understand how these changes influence the value of global temperature with increasing time.

The simulation is accompanied by two investigations (see APPENDIX II and APPENDIX III). In the first investigation, students were asked to freely and independently explore the simulation and respond to a series of statements pointing to the non-numeric covariational relationships between carbon dioxide, albedo, and air temperature. The goal of the second investigation is to prompt students to engage in numeric covariation reasoning and develop an in-depth understanding of how the global air temperature changes with the enhanced concentration of carbon dioxide and the value of albedo. Both investigations have questions that engage students in the first, second, and third levels of covariational reasoning as per Carlson et. al.'s (2002) Mental action framework. For instance, to prompt students to identify the direction of change of the input and output variables, the first investigation includes questions such as, "If I increase the amount of carbon dioxide, the air temperature <u>increases/ decreases"</u>. To answer this question, students might focus on the values of the two variables or observe the graph to see if the curve representing air temperature moves upward or downward as the value of carbon dioxide increases (MA2).

Similarly, in the second investigation, instead of providing students with any specific formula connecting carbon dioxide, air temperature and albedo, I ask the students to collect the values of air temperature for different values of carbon dioxide in a table and coordinate the values of the two variables as they change with respect to each other (MA3, MA4). To ensure that students equitably focus on the change of both the variables, in this investigation I included questions such as "find the air temperature when carbon dioxide is 300" and "find the carbon dioxide when the air temperature is 33". Further, students are asked to plot the ordered pairs in a graph paper and focus on the nature of the graph (linear, curved, increasing from left-to-right, increasing from right-to-left) to investigate the relationship between the concerned quantities.

3.1.1.6 Simulation 2: Sea level rise

The second simulation, the Sea Level Rise (*Figure 9*) was developed to model the phenomenon of sea level rise. A sea level rise is an increase in the global mean sea level as a result of an increase in the volume of ocean water. Sea level rise is usually attributed to the rise in atmospheric air temperature, which causes melting of ice sheets and glaciers in the land. To make students feel more connected to the problem, I deliberately included the names of four places in New Jersey-New York City area. I selected the places according to their elevations from sea level starting with downtown Manhattan (10 feet) being at the lowest elevation followed by East Newark (20 feet), Newark (32 feet) and Kearny (108 feet). Students are able to increase and decrease the value of temperature rise from zero to five degrees Celsius and observe how the change in global temperature rise influences the height of sea level.

Similar to the previous simulation, the Sea Level Rise simulation is accompanied by a series of non-numeric and numeric covariational reasoning questions encouraging students to recognize a connection between the increased global temperature, height of sea level and habitable land area. Questions such as "As the global temperature is increasing by 0.5, the height of the future sea level is increasing by____feet" were included with the anticipation that they would prompt students to coordinate the values of the two variables and extend that understanding to construct a relationship between the input and output variables. If students can reason about the relative change in the value of future sea level for a unit change of air temperature, then that would indicate their level 3 understanding, as per Carlson et al.'s (2002) Mental Action framework of Covariational reasoning.

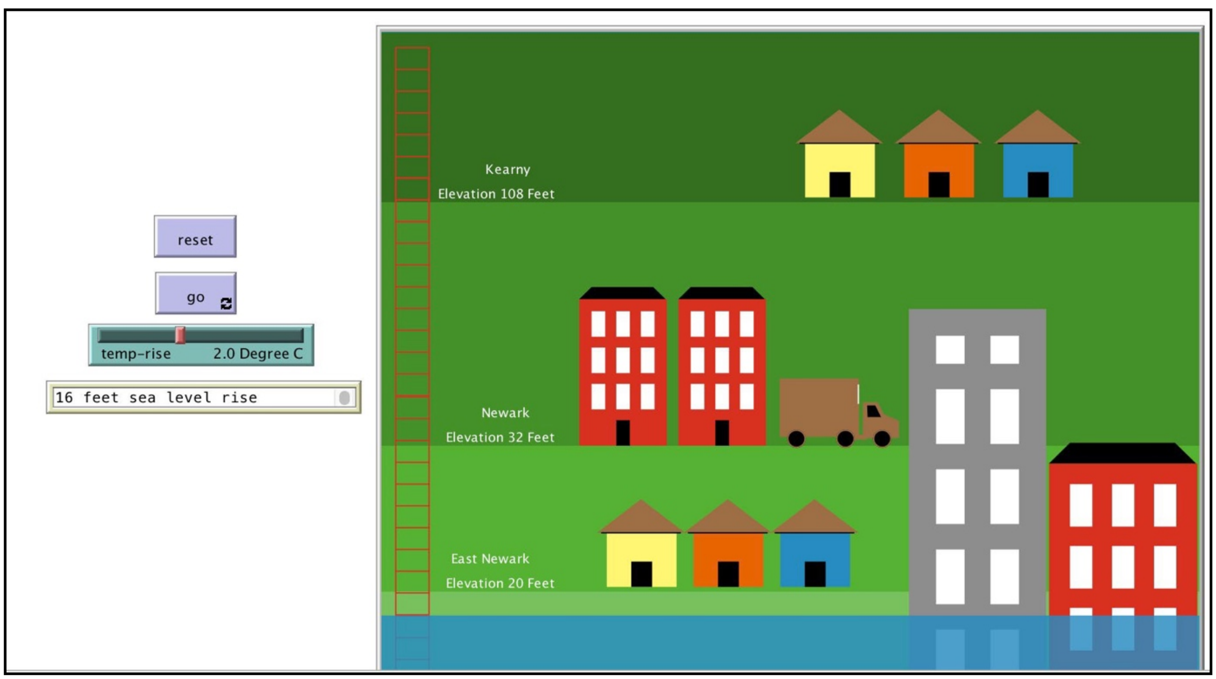


Figure 9: Future Sea Level Rise

Another major goal of this investigation is for students to recognize that if the current trend of temperature rise persists as a result of the increasing concentration of greenhouse gases, then the sea level would rise, and shorelines would move inland by hundreds of meters displacing millions of people from their places. To help students identify the risk factor associated with sea level rise, the investigation also asks students questions such as the one presented in Figure 10.

The elevation of East Newark (20 feet) is double the elevation of downtown Manhattan (10 feet) from the sea level. Which of the statements best describes the impact of sea-level rise on the two places? a) The risk of going under sea water of downtown Manhattan is the same as East Newark. b) The risk of going under sea water of downtown Manhattan is double than East Newark. c) The risk of going under sea water of downtown Manhattan is half than East Newark.

Figure 10: Sample Question included with Simulation 2

3.1.1.7 Simulation 3: The carbon calculator

The first two simulations were developed with the intention to help students identify the different traits and consequences of the greenhouse effect through mathematical tasks. This is what Gutstein (2003) referred to as "reading the world with mathematics." However, to develop an equitable society, it is important to develop within students a sense of agency, that is, a belief in themselves as people who can make a difference in the world, as ones who are makers of history. Educators working toward an equitable and unbiassed society can help students develop not only a sophisticated understanding of power relations in society but also the belief in themselves as conscious actors in the world. Helping young people develop a sense of personal and social agency is the goal of this next simulation.

As the climate around us is changing alarmingly, the term carbon-footprint is gaining enhanced attention among people of different sectors such as environmentalists, researchers, educators, and politicians. According to Wiedmann and Minx (2008), "the carbon footprint is a measure of the exclusive total amount of carbon dioxide emissions that is directly and indirectly caused by an activity or is accumulated over the life stages of a product" (p. 4). In other words, the carbon footprint is a sophisticated term used as a generic synonym of carbon dioxide emission over a given period of time.

According to Padgett, Steinemann, Clarke, and Vandenbergh, (2008), to restrict the emission of carbon dioxide and preserve the natural consistency of our climate, it is essential to estimate the amount of CO_2 an individual emits over a given period of time. With growing awareness about the elevated atmospheric CO_2 amount, at present, numerous websites contain a tool named carbon calculator or carbon emission calculator that helps an individual calculate their carbon footprint or the total amount of released carbon dioxide as a result of their daily activities. These calculators, developed by government agencies, non-government organizations, and private companies are usually very powerful and interpretable tools that help people understand the impact of their personal behavior on climate change. For instance, when an individual drives a car, the engine burns fuel which releases a certain amount of CO_2, depending on its fuel consumption and the driving distance. Similarly, when we heat up our houses with oil, gas, coal or electricity, tons of carbon dioxide are emitted, which enhance the proportion of greenhouse gases in the atmosphere.

Discussing the reliability of the different carbon calculators available to common people, Padgett, Steinemann, Clarke, and Vandenbergh, (2008) argued that the carbon calculators produced by different bodies might give different estimation of carbon footprints that can vary by several metric tons per annum per individual. Different calculators include different factors and follow different methodologies for calculation of carbon dioxide, which are not often clearly described to the users. As a consequence, these different calculators provide variable results. Padgett, Steinemann, Clarke, and Vandenbergh, (2008) argued that although the variability observed across different calculators invokes further research, it does not necessarily imply that the results are invalid. Building on the argument that if an individual wants to minimize their share of carbon emission, the calculation and constant monitoring of personal carbon footprint is essential, in this study I created a carbon calculator for the participating students. The Carbon calculator simulation (*Figure 11*) aims for students to reflect on their own lives and inspect the amount of carbon dioxide they contribute to the environment annually.

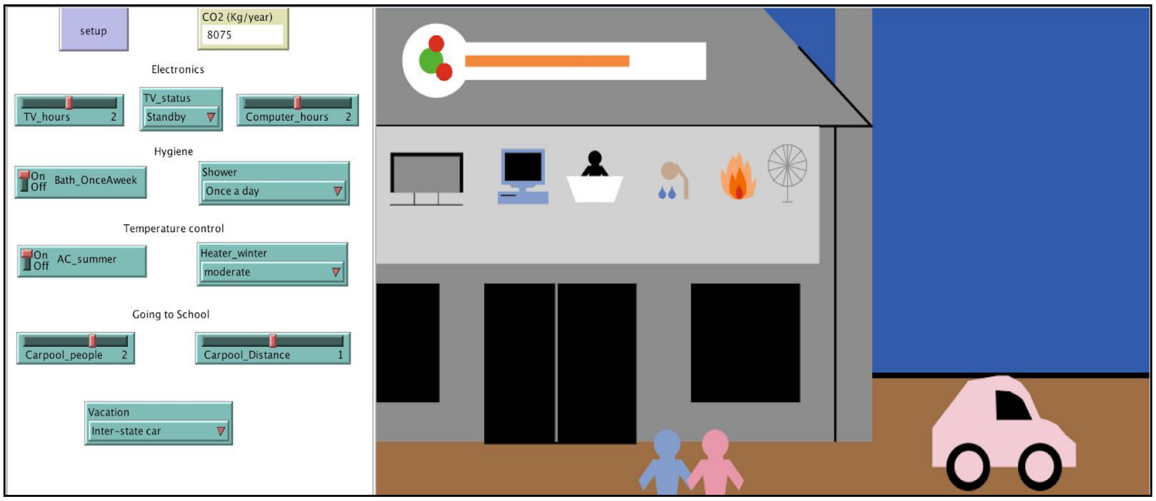


Figure 11: Carbon calculator simulation to calculate the amount of CO_2 we add as individuals through our daily activities

I adopted this simulation from the carbon calculator developed by "The Nature Conservancy" (https://www.nature.org/en-us/get-involved/how-to-help/consider-your-impact/carbon-calculator/) and developed a much simpler version suitable to the middle school students. I assumed that the participating students might not be familiar with their household's primary heating sources or average monthly electricity bills, so I overlooked several such factors and developed a carbon calculator that focuses on activities that might be more familiar to students, such as watching TV, playing video games or using an air conditioner. For each of these factors, students can manipulate the number of hours and calculate the total amount of carbon dioxide they emit in a year. For instance, if students watch TV for one hour and switch it off after watching, then the total amount of carbon dioxide added in the atmosphere in one year is 82Kg. The calculator contains a slider for the TV hours, which the students can manipulate and increase the time watching to up to four hours and calculate the total amount of carbon dioxide added in the atmosphere. To facilitate students to engage in different forms of covariational reasoning, both linear and non-linear, I deliberately chose the values of carbon dioxide which would prompt students to plot and identify such relationships between two given quantities.

This simulation is accompanied by two investigations. In the first investigation, students were asked to reflect on some of their daily activities and calculate the amount of carbon dioxide that each of their activities adds in the atmosphere. Further, through their engagement with carbon calculator, students were prompted to make generalizations, such the double/triple the distance travelled by a car, the double/triple the amount of carbon dioxide released, or the more the number of people carpools, the less the amount of carbon dioxide that is released in the atmosphere. The investigation also asked the students to plot graphs and discuss the two relationships depicted in the graph. Students focused on the shapes of the two graphs, 'number of TV hours - amount of released CO_2' and 'number of friends carpooling - amount of released CO_2' to identify the contrasting covariational relationship between the variables, that is the amount of carbon dioxide increases as the number of TV hours increases and the amount of carbon dioxide decreases as the number friends carpooling increases.

The last part of the investigation asks students to select some of their daily life activities, such as number of hours they spend watching TV and playing video-games, or the number of hours they use AC or heater, and calculate the total amount of carbon dioxide they produce annually as a result of their current lifestyle (*Table 3*). Students are not required to fill all the rows of the table, rather, they are asked to choose those activities that are most closely related to their own lives.

Table 3: Carbon Calculator table given to students to record their daily activities

Consumption	Value chosen	Amount of Carbon dioxide	Total Carbon dioxide
TV-hours_Stand-by			
TV-hours_Turn off			
Video_Game_hour_Standby			
Video_Game_hour_Turn off			
Battery_charger_plugged_ unused			
Computer-hours			
Bath once a week			
Shower			
Number of AC			
Heater			
Carpool			
Carpool distance			
Vacation (within state car, inter-state car, inter-state bus, inter-state plane, international plane)			
Total Carbon dioxide			

As mentioned earlier, the goal of this activity is to also motivate the students to take initiative and build strategies to change their own life-style and reduce carbon emission. Therefore, in the last investigation of the study I ask them to find ways to minimize the carbon emission by reconsidering some of the activities they mentioned in Table 3, such as video games hours or turning televisions off after watching. This activity aims to instill within students a sense of responsibility towards the environment they live in and to motivate them to be cognizant about their role in disrupting the natural climatic condition of the earth.

3.1.2 Implementation Phase

In the implementation phase, I describe the research setting and the research participants. I also present a pictorial representation of the entire implementation phase of my study to provide the readers with an organized structure of the implementation process.

3.1.2.1 Research setting

As mentioned earlier, the primary methodology for my study is the design experiment, and more specifically the whole class design experiment. In a whole class design experiment, the research team collaborates with the teachers, and the teachers assume the responsibility of the instruction (Cobb, et. al., 2003). Likewise, in this study I collaborated with two teachers, Mr. Doug (Pseudonym) and Ms. Chelsea (Pseudonym) from two schools located in the town of Kearny. Both Doug, the science teacher and Chelsea, the mathematics teacher were participants of the bigger STEM project. During Summer 2018 both the teachers expressed their interest about implementing the greenhouse effect module in their middle school science and mathematics classrooms and for the purpose of my dissertation data collection, I visited their classes and conducted whole class design experiments. As mentioned above, both the schools were located in Kearny, which is a culturally-diverse community in western Hudson County, New Jersey. The Kearny school district has a total enrollment of 4,300 students, and nearly 59% of them are Hispanic or African Americans and 59% of them are classified as economically disadvantaged. According to Partnership for Assessment of Readiness for College and Careers (PARCC), the two schools I considered for this study are low performing schools where a very low percentage of students either met or exceeded school-wide expectations in PARCC. These performance reports indicate an alarming signal that these underrepresented and economically disadvantaged students desperately need an education intervention to be prepared for the higher grades.

3.1.2.2 Macro-cycles

I used two classrooms because I conducted two cycles of design experiments, which I refer to as macro-cycles. Each macro-cycle of this design-based study consisted of two components: (a) classroom-based instruction and b) small group interaction with the students. Both macro cycles lasted for a week (5 days). Each macro cycle consisted of a series of mini-cycles which are the daily teaching-learning episodes. The ongoing analysis during those mini cycles helped me develop and refine a local instruction theory. Figure 12 illustrates the macro- and mini-cycles of this study.

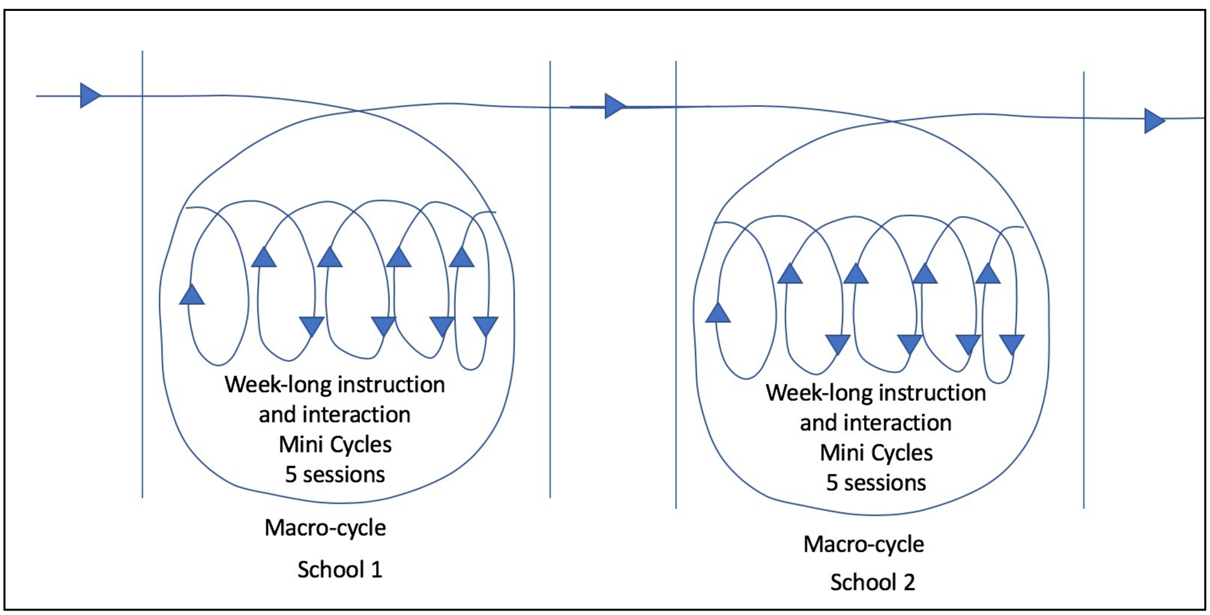


Figure 12: Macro-cycles and mini-cycles of this study

The first macro cycle of design experiment was conducted in late Spring 2018. It took place in a sixth-grade science classroom containing 27 students. I collaborated with Mr. Doug

(Pseudonym), a science teacher, who conducted the whole-class instruction and I assumed the role of a facilitator. The second macro-cycle took place a week after the conclusion of the first. It took place in a sixth-grade mathematics classroom and 17 students participated in the study. Similar to the first macro-cycle, the second cycle also consisted of a series of five mini-cycles of discussion sessions. I collaborated with Ms. Chelsea (Pseudonym), a math teacher, who assumed responsibility for instruction. Additionally, one of these teachers taught the greenhouse effect as he traditionally does without our STEM module in a third classroom. This third classroom acted as the control group and consisted of 31 students. The assignment of which classroom was the treatment or control was determined based on convenience as the research team could video-record the treatment classroom in the morning.

Before each mini-cycle session, I sat with the teacher to discuss the goals of the study and go over the lesson. After each mini-cycle session, I met the teacher again to discuss any modifications in simulations, tasks and questions that might be required before the next session. All the instruction and interactions were audio- and video-recorded.

During each design experiment, I was not only responsible for recording the classroom discourse and take notes on the classroom activities, but when students worked individually on the tasks on their computer, I sat with a small group of students and facilitated a discussion among them regarding covariational reasoning and the greenhouse effect. The students belonging to the small groups were selected by the teachers based on their articulateness and expressiveness. That is, for the small group discussion, the teachers chose those students who they identified as expressive and eloquent. My aim behind engaging in one-to-one interaction with the small group of students were to "create a small-scale version of learning ecology so that it can be studied in depth and detail" (Cobb et. al., 2003, p. 9). To explore how students' reasoning about covarying quantities progress as they engage with the tasks, I used open-ended questions that aimed to prompt the students to talk about the covarying quantities and their interpretations of the graphs involved. Starting from general questions, such as "Can you explain what you are doing here?" and "What are some relationships you found?" I then moved to more specific ones such as, "What do you think the graph will look like?", or "How do you recognize the relationship between amount of carbon dioxide and air temperature in the graph?"

3.1.3 Data Collection and Analysis Phase

In this study, to receive a comprehensive view of students' covariational reasoning and development of their awareness regarding the causes and consequences of the greenhouse effect I collected both quantitative and qualitative data. I anticipated that analysis of the quantitative data would help me identify if there has been any change in the participants' covariational reasoning and their understanding of the greenhouse effect. On the other hand, I hoped that the analysis of the qualitative data would give me an insight about the quantitative data. It would walk me through the students' experience as they engaged with the dynamic activities and help me understand the probable causes behind the shift in students' covariational reasoning and their understanding of the greenhouse effect. In this study, to collect the two types of data I drew upon a variety of data sources including a) whole class and small group video recordings, b) students' written artifacts, and c) pre- and post-assessments. Given this wealth of data, I provide an outline of the data collection and analysis (Figure 13) method to provide an organization of the process that captures and analyzes students' reasoning about covariation and understanding of the greenhouse effect.

<table>
<tr><th>Data</th><th>Data Collection</th><th colspan="2">Data Analysis</th></tr>
<tr><td rowspan="2">Qualitative Data</td><td>Whole class and small group videos</td><td rowspan="2">Ongoing</td><td rowspan="2">Retrospective
Carlson's Mental Action Framework</td></tr>
<tr><td>Students' written artifacts</td></tr>
<tr><td>Quantitative Data</td><td>Pre- and Post-Assessment: Multiple Choice Questions</td><td colspan="2">Linear mixed effect model</td></tr>
</table>

Figure 13: Outline of Data Collection and Analysis

The rich collection of data ensures the rigor of the study. I anticipate that the quantitative and qualitative data that I collected would complement each other and strengthen the claim of the study. While students' responses in the pre- and post-assessments would inform me if the dynamic mathematical modeling activities have been effective to deepen students' understanding of covariational reasoning and the greenhouse effect, the qualitative data would provide me with detailed information about students' development of covariational reasoning and identification of the traits of the greenhouse effect through "observation, description and interpretation of the features of interactions" (Anderson & Shattuck, 2012, p. 19). Further, the data collected through the whole class instruction and small group interaction would inform me whether the conjectures I proposed at the beginning of the study have been supported by the design intervention or if I need to further revise my tasks for an additional task reimplementation.

A distinct characteristic of the design experiment methodology is that the researcher or the research team develops an in-depth understanding of the phenomenon under investigation while the research is in progress (Cobb et. al., 2003). Therefore, it is crucial for the research team to generate and maintain a comprehensive record of the ongoing design process, which later on would aid the retrospective analysis (Figure 14) of the experiment (Cobb et. al., 2003). In the following section, I will provide a brief account of the data I collected from the two classrooms along with the method I employed to analyze those data.

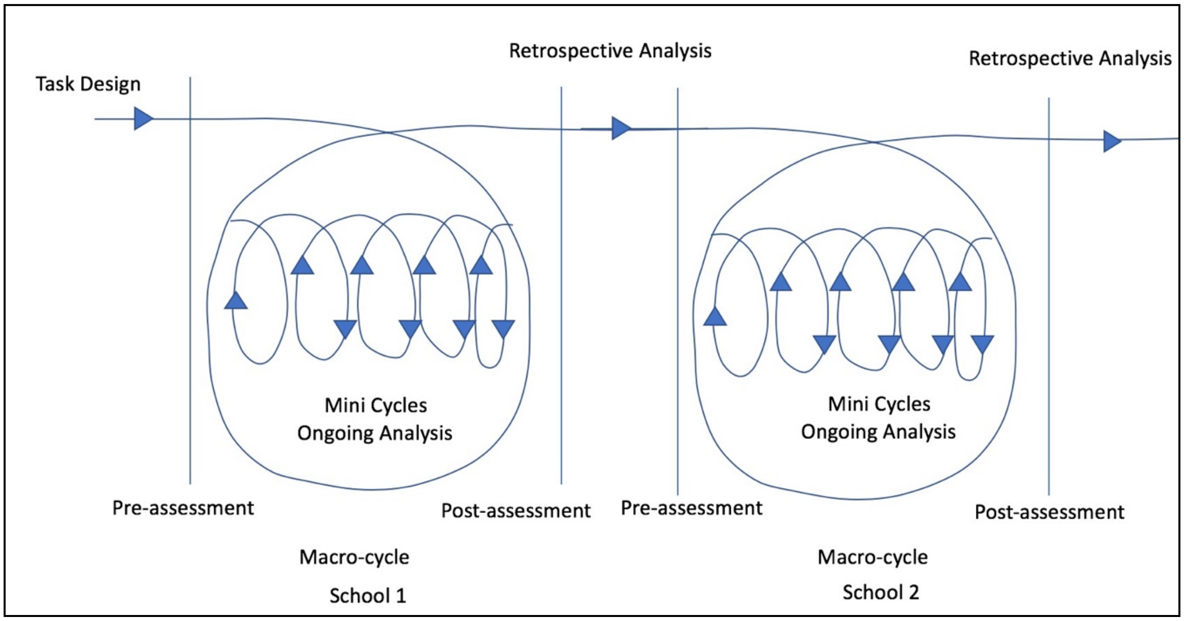


Figure 14: Data Collection and Analysis

3.1.3.1 Whole class and small group video recordings and students' artifacts

During the whole class instruction and the small group discussion, the conversations between teacher, students, and researcher were audio- and video-recorded. Two cameras, set on two tripods, were used to record the teaching sessions. During the whole class instruction, one camera constantly focused on the instructor and the other camera followed the students' responses. When the students worked individually or in small groups, then each of the camera focused on one chosen group of students and recorded their activities on the computer screen and the worksheets. Apart from video recording the classroom sessions, I also collected students' written artifacts in order to complement their verbal utterances. For each of the five investigations students received a worksheet that contained questions and prompts, and spaces to enter their responses. These worksheets were collected with the intent to support my investigation of understanding students' covariational reasoning

3.1.3.2 Ongoing analysis during mini cycles

The primary goal of a design experiment, as described by Cobb et. al., (2003) is "to improve the initial design by testing and revising conjectures as informed by ongoing analysis" (p. 11). During each micro cycle, based on the constant analysis of individual students' activity and discourses, researchers perform anticipatory thought experiments, revise the instructional activities, and modify the learning goals (Cobb, Stephan, McClain, & Gravemeijer, 2001). Consistent with this principle, during each teaching episode I took field notes to record my interpretation of the students' responses to the tasks and wrote down the modifications that I might require to make in the simulations or the tasks before the next cycle of design experiment. For instance, in the Carbon-Calculator task, initially I did not ask the students to calculate the annual amount of carbon dioxide added in the atmosphere as a result of playing video games or charging phones. When students from the first macro-cycle identified these activities as parts of their daily lives, I modified the initial version of the carbon calculator and added consumption caused as a result of 'Video_Game_hour_Standby', 'Video_Game_hour_Turn off', and 'Battery_charger_plugged_unused'.

Apart from taking field notes, every day I watched parts of the recorded videos to observe how the other students of the class responded to the implemented tasks and how my conversation with the selected group of students went, so that I could plan my questions before next session. The videos also helped me to notice if students encountered any difficulty as they engaged with the tasks. My aim was not only to help the participating students to reach the intended mathematical and environmental goal, but also to refine and develop a set of instructional tools that might best fit similar settings.

3.1.3.3 Retrospective analysis at the end of each macro cycle

As the ongoing analysis helped me to construct and de-construct local instructional theories, the data collected from the two macro-cycles will be used to create a more robust instructional theory that might be applicable to a larger instructional setting. This type of analysis is called retrospective analysis. During each retrospective analysis, all the classroom instruction and interaction videos were transcribed and analyzed. At this stage, the analysis was more fine-grained and included students' and teachers/researchers' conversations in order to capture the students' development of covariational reasoning that might have formed during their interaction with the simulations and tasks. To investigate how students reasoned covariationally when they interacted with the dynamic mathematical modeling activities, I used Carlson et al.'s (2002) Mental action framework of covariation.

After transcribing the videos, first I read the data and then I coded them using the software program Quirkos. Quirkos is a CAQDAS (Computer-assisted qualitative data analysis software) software package for the qualitative data analysis. After uploading the textual product in Quirkos, I went through the data and extracted phrases and sentences that might be identified under any one of the five mental levels or indicated students' identification of the traits and consequences of the greenhouse effect and showed their developing critical consciousness about the issue. Some of my codes, as shown in Figure 15 are albedo-air temperature, consequences of the greenhouse effect, and agency. Quirkos provides a thematic framework represented with a series of circles. When any new code was generated, I created a circle as a representative of the code and dragged all the phrases or sentences on the circle that falls under the specific code. As the number of phrases or sentences under each code increased, the size of the circle became bigger. Hence, the size of each circle in the figure indicates the amount of data coded to them. For example, looking at the Quirkos interface in Figure 15 it can be concluded that there are more students' excerpts under the code CO_2 and Temperature compared to Carpool.

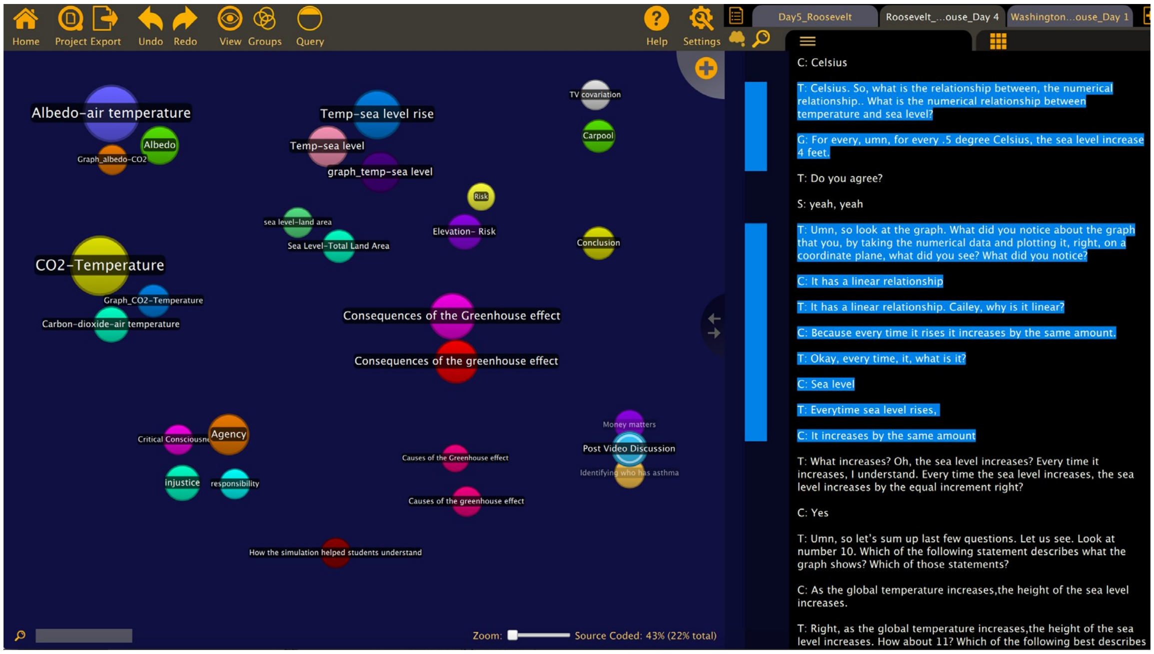


Figure 15: Quirkos for Qualitative Data Analysis

Once all the codes are created, a user can click on any code and see all the students' excerpts under that code on the right side of the interface. Further, Quirkos allows its users to create multiple files and merge them to have a convenient look into the data. In this study, initially I created two separate files for the two schools and later merged them to put all the codes together and have a better interpretation of the data.

3.1.3.4 Carlson et al.'s (2002) mental action framework of covariation

As aforementioned, to examine students' covariational reasoning, I used Carlson et al.'s Mental Action of Covariational Framework. To provide a detailed explanation of my analysis plan, I present a modified version of Carlson et al. (2002)'s Mental Action of Covariational Framework (Table 4), where in the third column I include some probable students' excerpts that I used as identifiers to determine the students' level of reasoning.

Table 4: Mental Action of Covariational Framework by Carlson et al. (2002)

Mental Action	Description of mental action	Probable responses
Mental Action 1 (MA 1)	Coordinating the value of one variable with changes in the other.	• As the amount of carbon dioxide changes from 25 to 50, and the value temperature also changes. • The global temperature is changed from 1 degree Celsius to 2 degree Celsius, the height of sea level is increased by 8 feet.
Mental Action 2 (MA 2)	Coordinating the direction of change of one variable with changes in the other variable.	• As the value of carbon dioxide increases, the value of global air temperature also increases. • The greater number of hours I watch TV, the more the amount of carbon dioxide I add in the atmosphere.
Mental Action 3 (MA 3)	Coordinating the amount of change of one variable with changes in the other variable.	• When the number of computer hours is increased by 1, the amount of carbon dioxide is increased by 36 Kg/ year. • The relation between air-temperature and carbon dioxide in interval [x_1-x_2] is different from the relation between air-temperature and carbon dioxide in interval [x_2-x_3].
Mental Action 4 (MA 4)	Coordinating the average rate-of-change of the function with uniform increments of change in the input variable.	• The rise in the height of future sea level with respect to increasing global air temperature is 8 feet. • The average rate of change of temperature due to change in carbon dioxide for intervals [x_1= 100, x_2= 200] is $\frac{yy_2-yy_1}{xx_2-xx_1}$
Mental Action 5 (MA 5)	Coordinating the instantaneous rate of change of the function with continuous changes in the independent variable for the entire domain of the function.	• The rise in the height of future sea level with respect to increasing global air temperature is linear.

Since in each macro cycle I interacted with a small group of students working together, I did not segregate individual students' reasoning regarding covariation; rather I focused on how students' covariational reasoning progressed as a group when they engaged with the dynamic mathematical modeling activities. During this stage I also reviewed students' written artifacts and used them as reference to justify their verbal utterances. For instance, during the coding when I identified any students' excerpts that suggested their level 2 understanding, I returned to the students' written artifacts and reviewed the corresponding mathematical work complementing the generalization.

3.1.3.5 Pre- and post-assessments

To measure the effectiveness of the dynamic mathematical modeling activities in developing students' covariational reasoning and their understanding of the causes and the consequences of the greenhouse effect, a pre-assessment and a post-assessment were administered to the participating students one day prior to the commencement of the macro-cycles and after the completion of the task implementation respectively. The assessment consists of nineteen multiple choice questions (see APPENDIX VII for the complete assessment). All the pre- and post-assessment questions were created by me with sufficient help from my advisor and expertise of the earth and environmental science fellows. Almost all of the pre- and post-assessment questions were created by me with sufficient help from my advisor and expertise of the earth and environmental science fellows, but a few questions were either taken or adapted from other researchers' questionnaires (Varma & Linn, 2012).

Out of the nineteen questions included in the pre- and post-assessments, nine focused on the several factors responsible for the greenhouse effect and ten questions addressed the consequences of the greenhouse effect. For example, questions such as "If I use my computer for 1 hour every day, I release 36 kg of CO_2 in the atmosphere in one year. How many kgs of CO_2 will I release in the atmosphere if I use my computer for 3 hours? (#8)" or "What will happen if you go to school every day by carpooling with your two friends? (#2)" were developed to encourage students to think about the causes of the greenhouse effect. Whereas the intention behind developing questions such as "Which of the following statements is correct for the global temperature and the height of future sea level? (#6)" and "Which of the statements is true about the height of sea level and elevation of a place? (#7)" were to help students identify the consequences of the greenhouse effect. Additionally, all the questions required students to engage in covariational reasoning between two or more quantities. For example, to answer question #4 (Figure 16), students were required to coordinate the direction of change of carbon dioxide and air temperature. Likewise, question #5, #6, #7 (see APPENDIX VII, p. 231) demanded students to engage in MA2 reasoning.

4. Which of the following statements is true about atmospheric carbon-dioxide and air temperature?

a) **If the atmospheric carbon-dioxide increases, the air temperature increases.**
b) **If the atmospheric carbon-dioxide increases, the air temperature decreases.**
c) **If the atmospheric carbon-dioxide increases, the air temperature stays the same.**

Figure 16: Question #4 from the Pre- and Post-Assessment asking students to coordinate the direction of change of carbon dioxide and air temperature

To investigate if students could coordinate the amount of change of one quantity with respect to the change in the other quantity, in the assessments I included questions such as,

#9: If I use my computer for 1 hour every day, I release 36 kg of CO_2 *in the atmosphere in one year. This year I released 540 kg of* CO_2*. For how many hours did I use my computer every day?*

a) 8 hours
b) 10 hours
c) 15 hours

and

#11f: What is the Increase in Height of Sea Level, when the Global temperature rise is 8 degree Celsius? (Refer the given graph)

i. 40 feet
ii. 80 feet

To answer question #11, first students were required to coordinate the amount of change of the height of future sea level for 1-degree Celsius temperature rise, followed by using the value to calculate the height of sea level for 8-degree Celsius temperature rise (MA3 reasoning). Therefore, considering the nature of the questions included in the pre- and the post-assessment, I hypothesized that the students' correct responses for any of the questions not only indicate their understanding of the greenhouse effect, but also proclaims their reasoning ability between two or more covarying quantities.

3.1.3.6 Analysis of multiple-choice questions

The linear mixed effect model within the repeated measures framework was selected as an analysis technique for the quantitative data, which consist of the pre- and post-assessment scores of the students in the treatment and control groups. The repeated measures framework was chosen as an appropriate tool to repeatedly measure the pre- and post-assessment scores of the subjects (student in this case) to examine if there has been any significant change in the treatment students' understanding of the greenhouse effect compared to their peers from the control group. Further, I used the R programing language (R core team, 2014) to perform the linear mixed effect model analysis using the package nlme (Pinheiro et al., 2018) and utilized the tidyverse package (Wickham, 2017) to generate a visual figure of this analysis.

3.2. Conclusion

This chapter outlines the methodology for investigating the role of mathematical modeling activities in developing students' covariational reasoning, which in turn I hope would facilitate students to identify the different traits and consequences of the greenhouse effect and develop within them an awareness towards the rapidly changing climate. I followed the design experiment methodology aiming to design, implement, and revise tasks that I anticipate would help in developing students' covariation reasoning.

I divided my methods into three phases: the development phase, implementation phase, and the data collection and analysis phase. In the developmental phase, first I explained how I used Carlson et al.'s (2002) Mental Action of Covariation framework to design my tasks. Next, I provided a detailed description of the three NetLogo simulations, Climate Change simulation, Sea Level Rise simulation, and Carbon Calculator simulation and explained how I anticipate these simulations may help students develop understanding about covariational reasoning and the greenhouse effect.

The implementation phase described the macro and micro-cycles of my study to provide readers with an overview of the whole process I conducted to implement the tasks in two middle school classrooms and collect quantitative and qualitative data. The last phase of the method section described the methods I followed to collect and analyze data. In this study I collected both quantitative and qualitative data to receive a comprehensive view of students' covariational reasoning as a result of their interaction with the dynamic mathematical modeling tasks. I collected three types of data, namely a) whole class and small group video recordings, b) students' written artifacts, and c) pre- and post-assessments. I used Carlson et al.'s (2002) Mental Action of Covariation framework to analyze students' covariational reasoning. In the following chapter I discuss the findings of this study.

4. Findings

The primary purpose of this chapter is to provide the results of the study which explored the power of mathematical reasoning for developing students' understanding of the greenhouse effect, a significant cause behind climate change. Students engaged with dynamic mathematical modeling activities, which I anticipated might help them to reason covariationally between two or more quantities and provide them with a platform to identify the causes and consequences of the greenhouse effect.

This study sought to answer two research questions around students' covariational reasoning and the greenhouse effect. To answer the first research question, I begin the chapter describing how students belonging to both the treatment and control group performed in the pre- and post-assessments. Next, to get an insight of the quantitative data analysis, I focus on the analysis of the qualitative data, which were collected in the form of whole class and small group video recordings. My goal was to observe the video recordings and investigate the possible causes behind the shift in students' performance from the pre- to the post-assessment. Through the video data I explored students' reasoning about covarying quantities in connection with the three mathematical modeling activities. I used Carlson et al.'s (2002) Mental Action Framework to identify how students navigated through the different levels of covariational reasoning during their interaction with the activities. Through the discussion of students' covariational reasoning, I sought to answer my second research question in which I investigated how students may navigate through different levels of covariational reasoning as they engage with mathematical modeling activities in the context of the greenhouse effect.

As mentioned earlier in the Method section, during the small group discussions I interacted with six students from the first macro cycle and five students from the second macro cycle. The names of the six students from the first macro cycle are Ani, Elen, Nia, Gina, Amber, and Paula and the names of the five students from the second macro cycle are Gio, Celine, Myra, Jake, and Simi. In this chapter, I present the students' excerpts from the whole class discussions and small group interactions to tell stories of these students' experiences interacting with the simulations. Also, students' written works, graphs, tables, and pictures are included along with students' verbal utterances to provide the readers a window into the students' thought processes. It is worth mentioning that, since the methodology of this study is whole class design experiment, the small group interactions occurred only for brief periods between whole class discussions. As a result, considering the time constraint, on several occasions the voices of one or two students were more prominent than others.

4.1. Overall impact of the module on students' reasoning

To examine the extent to which students' scores shifted from the pre- to the post-assessment, I used a linear mixed effect model within the repeated measures framework to compare the treatment and control groups. First, I compared the pre- and post-assessment scores of the students participating in both the treatment and the control groups. The analysis shows that students belonging to both the groups exhibited significant improvement ($p < 0.005$) in their performance from the pre- to the post-assessment (see Testpost_test on Table 5).

Table 5: Linear mixed effect model

	Value	Standard Error	DF	t-value	p-value
Intercept	11.741935	0.5540278	73	21.193767	0.0000
Testpost_test	1.483871	0.5402645	73	2.746564	0.0076
Module_Treatment	-0.219208	0.7233289	73	-0.303055	0.7627
Testpost_test:Module_Treatment	1.447947	0.7053599	73	2.052778	0.0437

To examine if the shift in the treatment students' scores from the pre- to the post-assessment can be attributed to their engagement with modeling activities, I compared the post-assessment scores of the treatment and control groups. From the analysis of the results I found a significant difference ($p < 0.05$) between the post-assessment scores of the students belonging to the treatment and the control groups (see Testpost_test: Module_Treatment in *Table 5*). As Figure 17 visually illustrates, the difference between the medians of the pre- and post-assessment of the treatment group is greater than the difference in the medians of the pre- and post-assessment of the control group.

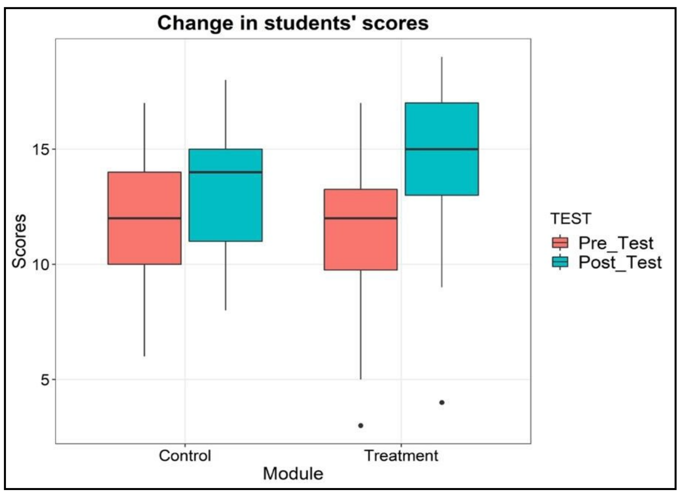


Figure 17: Change in students' scores (control and treatment) from pre- to post assessment

The analysis of the pre- and post-assessment scores using the linear mixed effect model suggests that the treatment group students who worked with the greenhouse module showed significant improvement in their post-assessment scores compared to their peers in the control group, who also learned about the phenomenon of the greenhouse effect without this particular module. To get an insight into the quantitative data and identify the possible reasons that might have resulted in the increase of the post-assessment scores, in the following sections I looked into the students' experiences interacting with the three NetLogo simulations. I anticipated that the analysis of the qualitative data would shed some light into the quantitative data and help me understand the meaning of the increase in the post-assessment scores.

As mentioned earlier in the Method section, out of the nineteen questions included in the pre- and post-assessments, nine focused on the several factors responsible for the greenhouse effect and ten questions addressed the consequences of the greenhouse effect. Additionally, all the questions as explained earlier required students to engage in covariational reasoning between two or more quantities. To give the reader a comprehensive view of how the three NetLogo simulations provided the students a space to engage in covariational reasoning and thus helped them develop their understanding of the causes and consequences of the greenhouse effect, the following sections are divided into two sub-sections: students' reasoning about the consequences of the greenhouse effect and students' reasoning about the causes of the greenhouse effect. Under each of the sub-section, first I present the analysis of the treatment-group students' scores in the two sets of questions - consequences of the greenhouse effect and causes of the greenhouse effect - and discuss how their performance shifted from the pre- to the post-assessment. Next, to understand the possible reasons behind the shift in the post-assessment scores, I delve deep into the qualitative data. I present the stories of the students, illustrating their reasoning about two or more covarying quantities as they interacted with the simulations and the accompanying activities. Each sub-section ends with a discussion about how the design of the simulations might have provided a space for students to think critically about the causes and consequences of the greenhouse effect.

4.2. Students' reasoning about the consequences of the greenhouse effect

As aforementioned, ten out of nineteen questions in the pre- and post-assessment were developed concentrating on the consequences of the greenhouse effect. To investigate how the treatment group's responses to the ten items about covariation and the consequences of the greenhouse effect shifted from the pre- to the post-assessment, I calculated the total scores of the students for all ten items in both assessments (maximum possible score 10 and minimum possible score 0) and determined the frequency of the students belonging to each scoring category (each score formed a category). The results from a comparison of the pre- and the post-assessment (Table 6) indicate that during the pre-assessment, only 29.5% of the students' scores were 8 or above, whereas during the post-assessment the percentage of the students scoring equal to or above eight rose to 68.2%.

Table 6: Frequency of the students scoring 0 through 10 in the pre- and post-assessment (covariation and consequences of the greenhouse effect)

Consequences of the greenhouse effect				
Percentage	Pre-Assessment	Score	Post- Assessment	Percentage
29.50%	1	10	7	68.2%%
	4	9	10	
	8	8	13	
	8	7	7	
	8	6	3	
	5	5	2	
	7	4	1	
	3	3	1	
0%	0	2	0	0%
	0	1	0	
	0	0	0	

Figure 18 further illustrates how the frequency of the students scoring eight or above increased during the post-assessment, while the number of students scoring between seven and three declined. The figure indicates a positive shift in students' understanding of covariational reasoning and the consequences of the greenhouse effect from the pre- to the post-assessment.

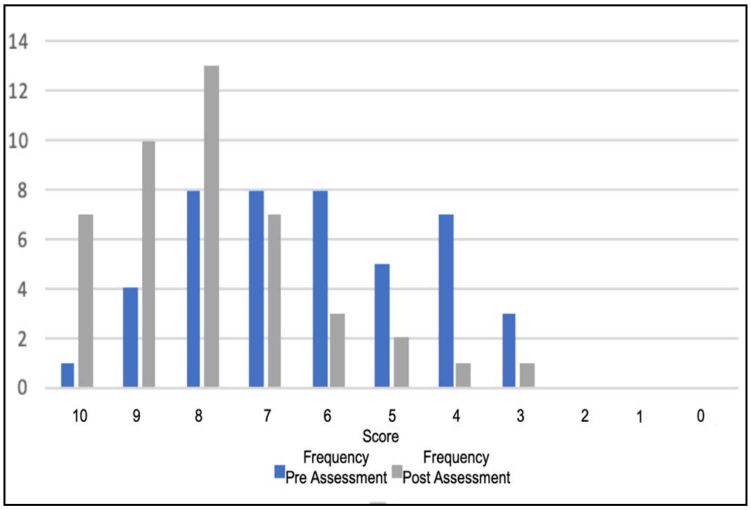


Figure 18: Bar Graph showing frequency of students in each scoring category (0 through 10) in the pre- and post-assessment (covariation and consequences of the greenhouse effect)

I delve deeper into the analysis of some these ten questions to provide examples of these changes. One of the major consequences of the greenhouse effect, as addressed in this study, is the rise of sea level. In two items, #6 and #11, students were asked to identify the relationship between air temperature and height of future sea level. During the pre-assessment, 54.5% of the students identified that as global temperature increases, height of sea level increases. However, the percentage of the students reasoning between the two quantities increased to 68.2% during the post-assessment. Since one of the three NetLogo simulations was developed around the issue of Sea Level Rise and the participating students engaged considerably in reasoning between air temperature, height of future sea level, and total land area, I would argue that the shift in students' responses during the post-assessment might have occurred as a result of their engagement with the intervention.

One of the ten questions in the pre- and post-assessment also asked students to reason covariationally about the elevation of a place and its associated risk of going under water. In the pre-assessment, 59.1% (n=26) of the students identified that the higher the elevation of a place, the lower the risk being affected by sea level rise, while in the post-assessment 82% (n=36) of the students recognized this relationship between the two quantities. During their engagement with the Sea Level Rise simulation, students were prompted to think about the covariational relationship between elevation of a place and risk of going under water. As a result, the simulation might have helped the students understanding of the risks associated with the elevations of places.

Overall, the quantitative data analysis of the treatment group students' mean scores in the pre- and post-assessment indicate that a higher percentage of treatment group students identified the consequences of the greenhouse effect during the post-assessment. I conjecture that the treatment group students' engagement with the Climate Change simulation and the Sea Level Rise simulation might have helped them identify the consequences of the greenhouse effect. However, it may be too early to make this claim. To get insight into how the students' engagement with the simulations and their covariational reasoning might have impacted their performances in the post-assessment, in the following paragraphs I describe the design principles of each of the simulations and activities and discuss stories of the students' experiences interacting with the simulations. By doing so I will present substantiated claims about what knowledge they abstracted from those interactions.

4.2.1 Exploring the relationships of the Climate Change simulation

The first NetLogo simulation that I used in this study is the Climate Change simulation (Figure 19). This simulation was designed with the intention of supporting students in reasoning about two sets of covarying quantities, albedo and air temperature and carbon dioxide and air temperature, helping them identify the consequences of the greenhouse effect. Students were asked to explore the simulation, change the values of albedo by dragging the albedo slider left and right, increase and decrease the amount of carbon dioxide by clicking the add and remove

CO_2 buttons, and observe the impact of the changes on the value of air temperature. I conjectured that the exploration of the simulation would provide a space for the students to identify the direction of change of two covarying quantities, for example, as albedo increases, the air temperature decreases or as carbon dioxide increases, the air temperature increases, and thus provide students an opportunity engage in Level 2 covariational reasoning as per Carlson et al.'s (2002) Mental Action Framework. In the following paragraphs I present how students reasoned about those two relationships by discussing four cases from the two macro-cycles.

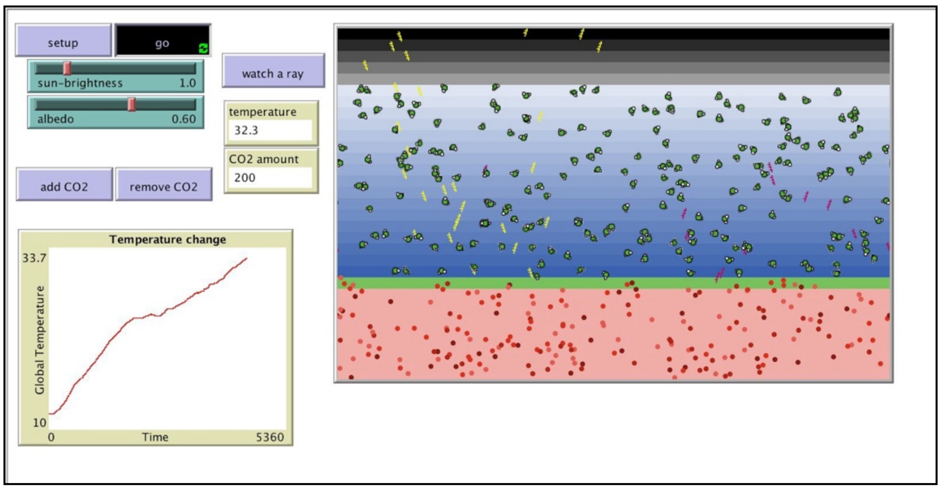


Figure 19: The Climate Change Simulation

4.2.1.1 Relationship between albedo and temperature: The case of Ani and Gina from MC1

For the first simulation, Doug (teacher) asked the students to explore the simulation and identify the different factors that might impact the value of air temperature. Students identified carbon dioxide and albedo as two factors that influence air temperature. During the small group discussion, when I asked the students if they identified any relationship between albedo and air temperature, Ani moved the albedo slider to the left and right, thus decreasing and increasing the value of albedo, and stated that when value of albedo increases, air temperature decreases. Further, he and Gina identified the reflection of sunlight as a third quantity to bridge the relationship between albedo and air temperature. The following excerpt illustrates Ani and Gina's reasoning:

Interviewer: What do you think, if I increase the value of albedo?

Ani: It (earth) will reflect more.

Interviewer: It will reflect more.

Gina: It will reflect more sunlight.

Ani: Which means no heat.

Interviewer: Which means?

Ani: It will decrease.

Interviewer: It will decrease? What will decrease?

Ani: Well the temperature (pointing the time graph on the screen).

[Excerpt 1]

Excerpt 1 shows that Ani and Gina were able to illustrate MA2 reasoning and coordinate three quantities: albedo, reflection of sunlight, and air temperature. Ani pointed to the time graph to justify his response. Because the graphics of the Climate Change simulation (*Figure 20*) explicitly expresses the connection between the three quantities, I conjecture that the graphics of the simulation might have helped Ani and Gina see the connection between the three quantities (MA2). More specifically, in the simulation, when the value of albedo is set to zero, the graphics of the simulation exhibit a black rectangular patch on the earth's surface and the number of sun rays reflected back from the earth's surface is zero (see Figure 20a). During this time, all the incident sunlight is absorbed by the earth in the form of infrared rays (red dots) and air temperature, expressed through time graph and temperature monitor, is maximum. When the albedo slider is dragged to the right and stopped halfway through, the black patch turns green and part of the reflected sun rays are absorbed by the earth as infrared rays while the rest are reflected back (see Figure 20b). Finally, when the albedo reaches its maximum value of one, the green patch is replaced by a white patch to indicate the higher reflective surfaces such as ice or snow and the number of reflected rays become maximum, with no sun rays being absorbed by the earth (see Figure 20c). At this time, the value of air temperature remains the lowest. Because Ani and Gina stated that when albedo increases, the earth reflects maximum sunlight, which in turn decreases heat and reduces air temperature, I conjecture that the graphics of the Climate Change simulation (Figure 20c) might have had a significant impact on both Ani and Gina's reasoning. The students' responses indicate that the graphics of the simulation not only helped them see the connection between the different quantities, but also enabled them to model the covariational relationship between them.

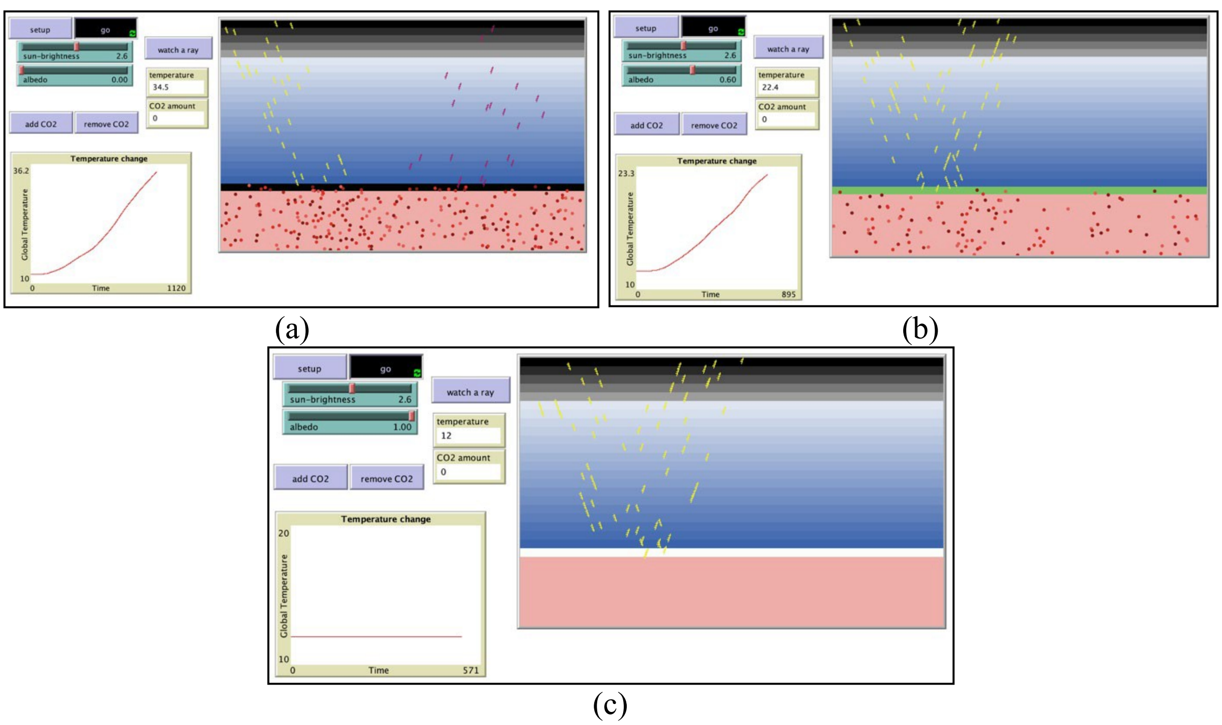


Figure 20: (a) When albedo is zero, the earth's surface is represented by a black patch and no sunlight is reflected back; (b) When albedo is 0.5, the earth's surface is represented by a green patch and part of sunlight is reflected back and part of sunlight is absorbed; (c) When albedo is one, the earth's surface is represented by a white patch and no sunlight is reflected back

4.2.1.2 Relationship between albedo and temperature: The case of Gio from MC2

Similar to the first macro cycle, the second macro cycle also started with students exploring the first simulation. Following the exploration, during the whole class discussion when Chelsea (teacher) asked the students if they noticed any relationship between albedo and air temperature, Gio moved the albedo slider to the right and then left and replied, "as the albedo goes down, temperature goes up." This statement shows that by manipulating the albedo slider and observing the change in temperature, Gio was able to identify the direction of change of the two quantities, albedo and air temperature, which is what Carlson et al. (2002) distinguished as MA2 reasoning.

In order to have the students further identify the covariational relationship between albedo and air temperature, they were given four graphs and were asked to explore the Climate Change simulation, observe how air temperature changes with increasing and decreasing values of albedo, and accordingly choose the graph that correctly represents the relationship between earth's albedo and air temperature (Figure 21).

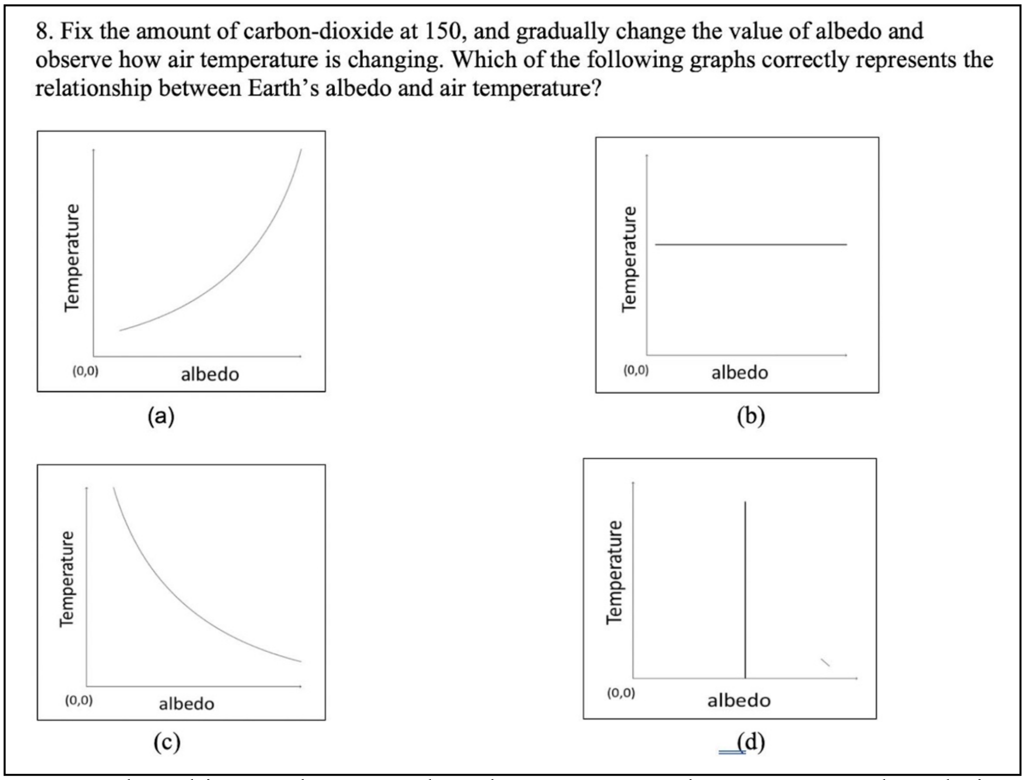
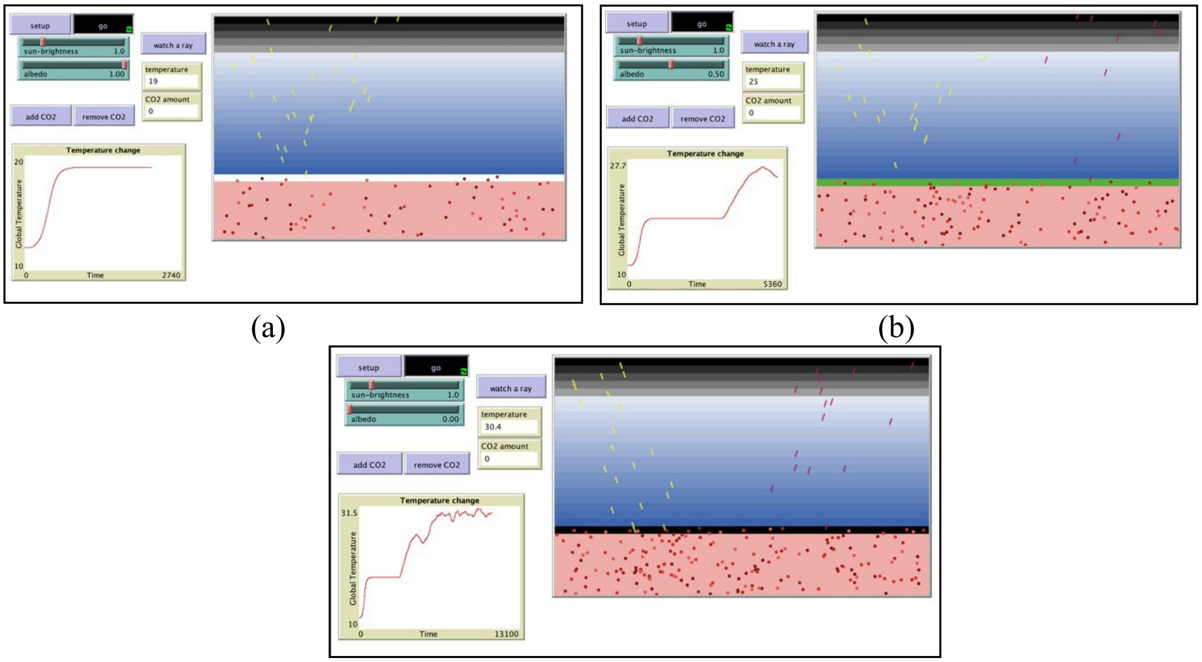


Figure 21: Four graphs asking students to select the correct one that represents the relationship between albedo and air temperature

Gio selected graph C as a representative of the albedo-temperature relationship. Aiming to examine why Gio selected graph C, I asked him, "How did you identify that the graph C represents that (relationship)?" In response he said, "as albedo was higher, this means the albedo was going higher, then the temperature is dipping down in a curved way because as we saw before it is not everything is on the same straight line, it is in a curved line." At this point Gio not only focused on the direction of change of temperature with direction of change of albedo ("albedo was going higher, then the temperature is dipping down"), a type of reasoning that aligns to MA2 as per Carlson et al.'s (2002) Mental Action Framework, but also tried to reason about the curvilinear nature of the albedo and air temperature graph. When he said "because as we saw before" it seems that he referred back to the simulation and focused on the curved nature of the time graph being developed as a result of changing albedo (Figure 22).



(a) (b)

(c)

Figure 22: (a)Time-Graph showing low values of temperature (19 degrees Celsius) when albedo is maximum, that is one; (b) Time-Graph showing increased values of temperature (around 25 degrees Celsius) when albedo is decreased to 0.50; (c) Time-Graph showing maximum air temperature

In order to prompt Gio to think about the nature of the graph, I asked him, "Why it is not a straight line and curved?" I anticipated that this question might encourage Gio to focus on the amount of change of temperature for different values of albedo. In response to my question, Gio took some time and said, "it is not a straight line because it is different every time in range of temperature, because if we do it with a certain amount of carbon dioxide it doesn't just stay the same but actually dips up and dips down." From Gio's response, it appears that he not only identified that temperature would be different every time albedo is changed, but also referred to the simulation and identified how temperature "dips up and dips down" for a specific value of albedo. Because Gio talked about the dipping up and dipping down of the temperature graph, it seems that the time graph in the Climate Change simulation (Figure 22) provided space for the students to notice the fluctuating values of air temperature (MA3). Hence, based on Gio's response I infer that the time graph of the simulation provided Gio with the space to reason about the direction of change of air temperature for direction of change of albedo (MA2) and allowed him to see how air temperature fluctuates at and between different values of albedo (MA3).

4.2.1.3 Relationship between CO_2 and temperature: the case of Nia and Elen from MC1

After students identified the covariational relationship between albedo and air temperature, next, students explored the relationship between the amount of carbon dioxide and air temperature. During the small group discussion, when I asked the students, "what will happen if I increase carbon dioxide?", Nia responded, "it increases the temperature." Aiming to examine how Nia identified the increase of temperature, I asked her, "how do you know it is increasing?" In response to the question, both Nia and Elen pointed to the time graph (Figure 23) in the simulation and identified that if carbon dioxide increases, temperature gets higher. Excerpt 2 illustrates Nia and Elen's reasoning.

Interviewer: How do you know it is increasing?

Elen: It's going up.

Interviewer: It is going up? Okay, say it goes up, then what happens? If it goes up, what happens?

Nia: The temperature gets higher

[Excerpt 2]

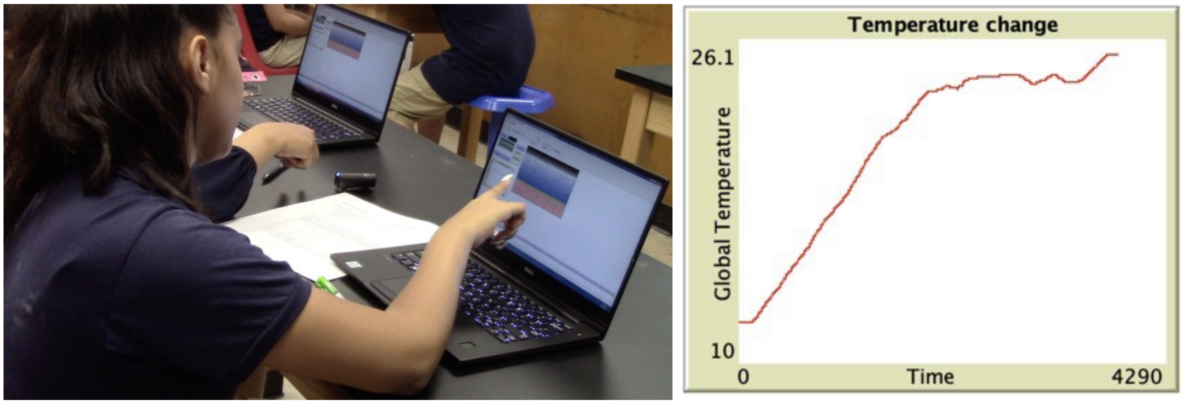


Figure 23: (a) Nia pointing the time graph to show how air temperature increases with carbon dioxide (b) Time graph showing how air temperature increases with carbon dioxide

I interpret that both the students engaged in reasoning aligned with Carlson et al.'s (2002) MA2 as they focused on the direction of the air temperature curve (increasing curve line going from left to right) in the graph and coordinated the direction of change of air temperature (going up from left to right) with changing values of carbon dioxide.

Next, I was interested to know more about the impact of the simulation on the students' understanding of the phenomenon underlying covariational relationship between carbon dioxide and air temperature. So, I asked, "you are adding more carbon dioxide? Okay. But why is it increasing? Why is the temperature increasing?" While Elen was thinking of the answer, Nia replied, "it is gaining more energy." Being unsure of what Nia referred to as energy, I prompted her to describe the term energy. After a brief thought, Nia said, "The red dots. Some are going up. Some are going up here, but some are going down. That's increasing the temperature." When I asked her if she remembers the name of the red dots, she said, "infrared rays." Nia's response indicates that during the exploration of the carbon dioxide and air temperature relationship, she not only focused on the numerical values of the two quantities or the graph, but also observed how the changes in the quantities impact the simulation interface. She changed the value of carbon dioxide and noticed that when the amount of the gas increases, then the red dots representing the infrared rays also increase (Figure 24). She said a part of the red dots "are going down" and "some are going up" however, the increased amount of carbon dioxide (represented by small green dots floating in the atmosphere) (Figure 24) "will not allow the infrared to escape," concluding that "the more carbon dioxide, the more temperature." From her response it appears that Nia focused on the graphics of the simulation interface to observe and understand the phenomenon underlying the relationships. By engaging with the simulation, Nia observed that when she added the amount of carbon dioxide, both the number of carbon dioxide molecules and infrared rays (represented by green and red dots) increased (Figure 24) and as a result, the value of air temperature on the earth also increased. Nia showed evidence that she coordinated the direction of change of carbon dioxide and air temperature, thus engaging in a type of covariational reasoning that aligns to MA2 according to Carlson et al.'s (2002) framework.

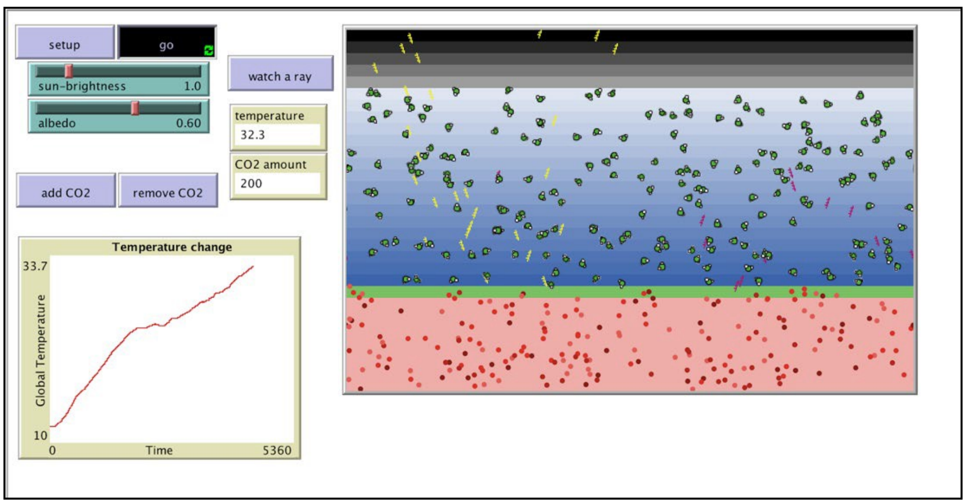


Figure 24: Climate Change Simulation showing what happens to the simulation interface when amount of carbon dioxide increases

4.2.1.4 Relationship between CO_2 and temperature: The case of Gio from MC2

Similar to the first macro cycle, in the second macro cycle also students explored the Climate Change simulation to identify the relationship between carbon dioxide and air temperature. They clicked the add CO_2 and remove CO_2 buttons and observed the impact of the change on the simulation interface and value of air temperature. In order to encourage students to articulate their reasoning about carbon dioxide and temperature, during the small group discussion, I asked them "can you explain what is happening to carbon dioxide and temperature?" It is worth noting that from the first to the second macro cycle, I modified my questioning style. Unlike the first macro cycle, I did not ask the students to think about the value of air temperature when carbon dioxide increases or decreases. Rather, I tried to encourage the students to think about and articulate the relationship between the two quantities without any prompt. I hypothesized that if the students are supported in thinking about the relation between carbon dioxide and air temperature, without providing them any external cues, then that might help them to engage in reasoning between covarying quantities. The following excerpt describes my interaction with Gio.

Interviewer: can you explain what is happening to carbon dioxide and temperature?

Gio: It is increasing.

Interviewer: It is increasing? Like how do you know it is increasing?

Gio: It is going higher. [showing the time graph on the simulation]

Interviewer: Okay. What is going higher?

Gio: Carbon dioxide.

Interviewer: Carbon dioxide?

Gio: The air temperature

Interviewer: The air temperature?

Gio: As the carbon dioxide increases.

[Excerpt 3]

As Excerpt 3 suggests, when Gio was asked to explain the relationship between carbon dioxide and air temperature, he did not articulate the relationship between both the quantities. Rather he said, "it is increasing." As a result, when I asked him "how do you know it is increasing?", Gio referred to the time graph in the simulation and pointed at the direction of the air temperature curve to justify his response (*Figure 25*). Similar to Nia and Elen from the first macro cycle, Gio also focused on the direction of the air temperature curve and recognized that as the value of carbon dioxide increases, the air temperature graph goes higher (going up from left to right). Such reasoning indicates that Gio coordinated the change of direction of the two quantities, carbon dioxide and air temperature, and engaged in MA2 reasoning as per Carlson et al.'s (2002) Mental Action Framework.

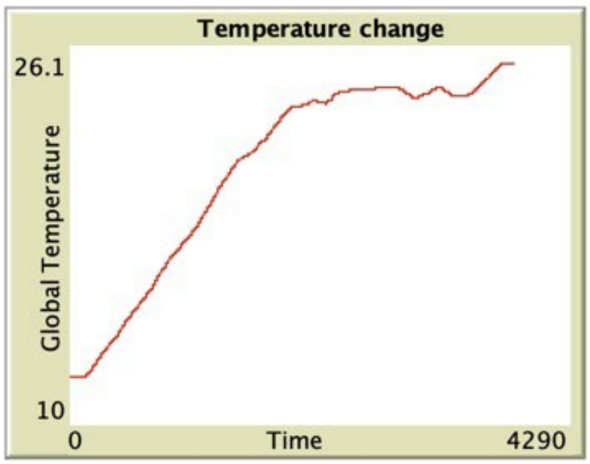


Figure 25: Time graph showing how air temperature changes with carbon dioxide

4.2.2 Graphing the relationship between CO_2 and temperature

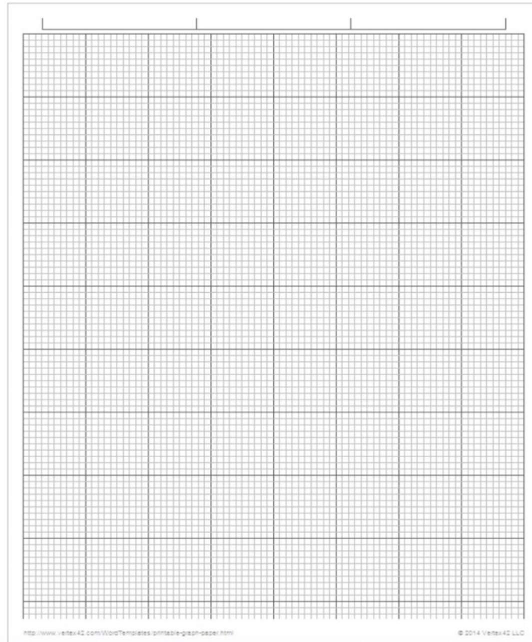
After students explored the Climate Change simulation and coordinated the direction of change of albedo and air temperature and carbon dioxide and air temperature (MA2), they engaged in a graphing activity. First, students were provided with a carbon dioxide and air temperature table (Figure 26a) and were asked to change the value of carbon dioxide in the simulation as indicated in the table, observe the values of air temperature for corresponding values of carbon dioxide, and record the values of air temperature in the table.

2. Understanding how the model works can be a complex process if we have many variables. That's why in this task we will just change one variable only and see what happens. Change the carbon-dioxide as shown in the table below and record the air temperature accordingly. Remember to give some time after each change and let the temperature stabilize and take the nearest whole number (e.g. if the temperature is 22.3, 22.2, 21.9 then temperature is around 22).

Carbon-dioxide	Air Temperature
0	
100	
200	
300	
400	

(a)

4. Use the values you found in the table above to plot a graph showing the relationship between the carbon-dioxide and air temperature. Label your axes as follows:
x-axis : Amount of carbon-dioxide
y-axis: Air temperature



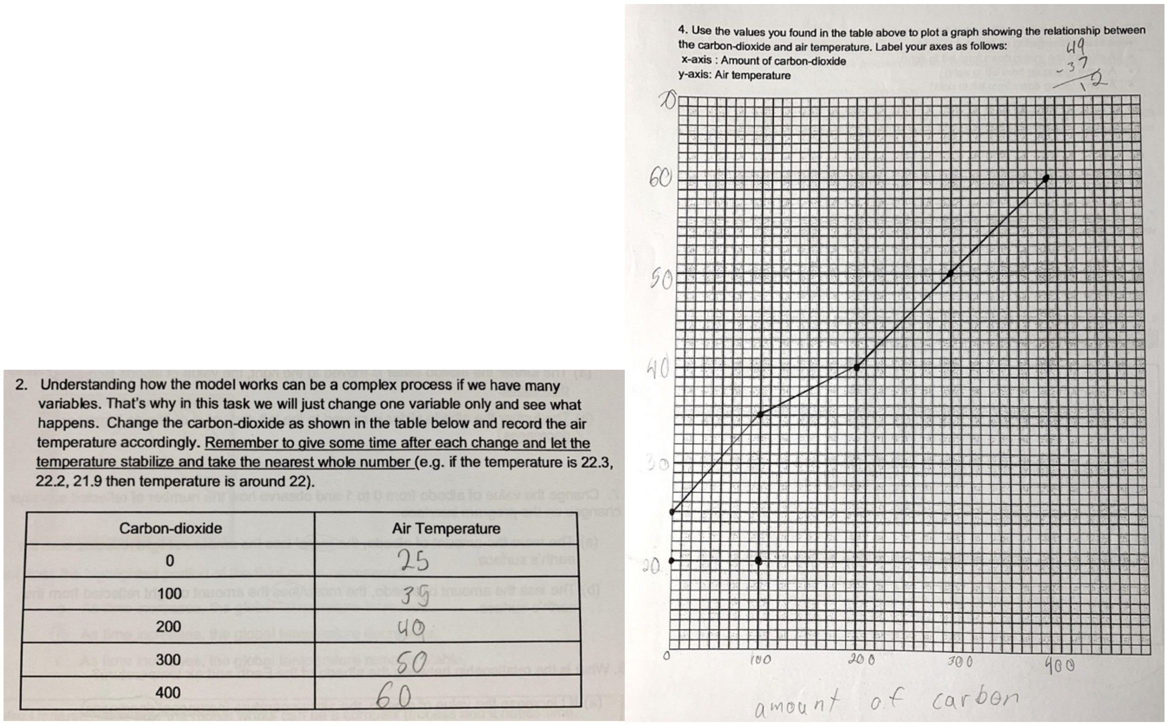
(b)

Figure 26: (a) Graphical Activity 1 where students collected the values of air temperature for given values of carbon dioxide, and (b) Graphical Activity 2 where students were asked to plot the carbon dioxide and air temperature ordered pairs

Next, students were asked to use the values of carbon dioxide and air temperature recorded in the table to plot the ordered pairs on a graph (Figure 26b). I hypothesized that the graphing activity might allow the students to recognize visually the amount and rate of change of air temperature with change in the value of carbon dioxide and express covariational reasoning aligned to Carlson et al.'s (2002) MA3 and MA4. In the following paragraphs, I describe two cases of students interacting with the activity to illustrate the forms of reasoning that they developed.

4.2.2.1 Relationship between CO_2 and temperature: the case of Ani from MC1

When Ani engaged with the graphing activity, first he recorded the values of air temperature for given values of carbon dioxide in the table (Figure 27a) and then he plotted the carbon dioxide versus air temperature ordered pairs on a graph (Figure 27b).


2. Understanding how the model works can be a complex process if we have many variables. That's why in this task we will just change one variable only and see what happens. Change the carbon-dioxide as shown in the table below and record the air temperature accordingly. Remember to give some time after each change and let the temperature stabilize and take the nearest whole number (e.g. if the temperature is 22.3, 22.2, 21.9 then temperature is around 22).

Carbon-dioxide	Air Temperature
0	25
100	35
200	40
300	50
400	60

Figure 27: Ani's (a) table showing the carbon dioxide air temperature ordered pairs and (b) graph representing the relationship between carbon dioxide and air temperature

During the small group discussion when Ani was asked to explain the graph, he measured the 'space' between two consecutive values of air temperature, arguing (Excerpt 4):

Ani: This one from here has more space than this one from here, and from this one to here. This one has more space in between of them. [Showing two different intervals on the graphs]

Interviewer: It has more space between them? Like what do you mean by more space? Can you show me with fingers?

Ani: Yes, like here. Here from here, like 3 fingers and from here to here like 4 fingers. So, it has more space here than here.

[Excerpt 4]

By measuring the space between the various intervals, Ani seemed to focus on the change of value of air temperature when the amount of carbon dioxide changed from 0 to 100 units, 100 to

200 units, and so on. He used his fingers to measure the space between two consecutive values of air temperature (that space between air temperature at 100 and 200 units of carbon dioxide versus space between air temperature at 200 and 300 units of carbon dioxide) and determined the relationships between carbon dioxide and air temperature in corresponding intervals of carbon dioxide. Ani's response indicates that the graphing activity provided him a space to coordinate the amount of change of air temperature with change in the value of carbon dioxide, a type of reasoning aligned to Carlson et al.'s (2002) MA3.

4.2.2.2 Relationship between CO_2 and temperature: the case of Myra and Celine from MC2

Similar to students of the first macro cycle, students of the second macro cycle also collected the values of air temperature for different values of carbon dioxide and plotted the different ordered pairs to graph the relationship between the two quantities. Students displayed MA2 reasoning during their engagement with this particular activity and identified the covariational relationship between carbon dioxide and air temperature. During this activity, Myra looked at the table she created (Figure 28) and stated that "as the carbon dioxide gets higher, the temperature rises." By observing that every time she increased the value of carbon dioxide, the value of air temperature also increased, she was able to coordinate the direction of change of air temperature with the direction of change of carbon dioxide and identify carbon dioxide as a major contributor to the increased air temperature.

Carbon-dioxide	Air Temperature
0	25
100	34
200	44
300	48
400	56

Figure 28: Myra's carbon dioxide vs. air temperature table

When I asked Myra if she observed any pattern in the values of air temperature for different values of carbon dioxide, she argued that every time the carbon dioxide is changing, "the number (air temperature) is getting 10 degrees higher." This shows that she was able to reason about the change of the value of air temperature for every 100-unit change in carbon dioxide and illustrate evidence of MA3 reasoning. Next, I asked students to use the graph (Figure 29) and estimate the value of air temperature for 400 units carbon dioxide:

Interviewer: How much would be the temperature when carbon dioxide is 400?

Myra: Maybe like, 50-56.

Interviewer: 50-56? How do you know that?

Myra: Because the numbers like, keep going on like that, umm, as they get higher, they get closer intervals? Like the range of number gets smaller.

Interviewer: Can you explain, what do you mean by closer interval?

Myra: Like over here 44 to 48 and carbon dioxide went up to 100, and this one went up 4. When it is 100 to 200, it went up 10. So, every time like it goes up, the carbon dioxide goes up, the air temperature as they go up like, the range between them would be smaller. So, it be around 50.

Interviewer: Around 50?

Myra: 50 or 53 something like that.

[Excerpt 5]

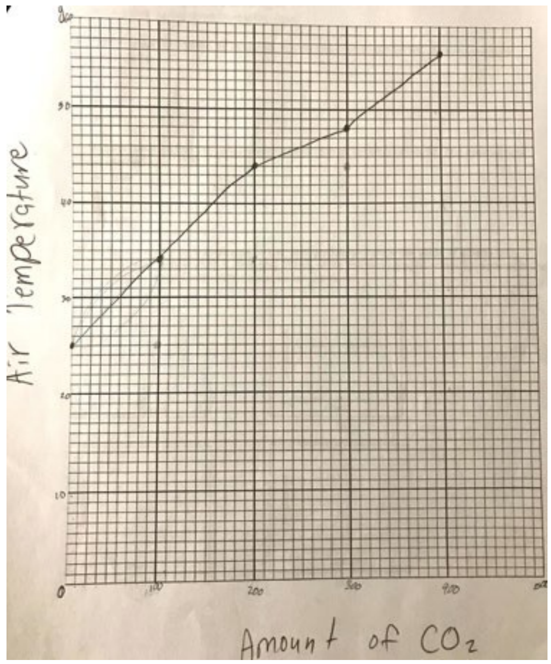


Figure 29: Myra's graph representing relationship between carbon dioxide and air temperature

Excerpt 5 shows that Myra observed the change of air temperature for each interval change of carbon dioxide (100 units) and concluded that as the value of carbon dioxide went up by 100 units, the value of air temperature increased but the amount of the amount of that increase became smaller. For instance, when the amount of carbon dioxide rose from 0 to 100 units, the air temperature went up by 10 degrees, while as the value of carbon dioxide increased from 100 to 200 units, the value of air temperature increased by only 4 degrees. Following this pattern, Myra stated that the probable value of air temperature for 400 units of carbon dioxides would be around 50 to 53 degrees Celsius.

A similar argument was also made by Celine when I asked her to use the graph (*Figure* 30) to find the probable value of air temperature for 300 units of carbon dioxide (Excerpt 6):

Interviewer: Can you say for sure what the temperature would be when carbon dioxide is 300?

Celine: I can make an estimate.

Interviewer: You can make an estimate? How are you making the estimate?

Celine: Each of them increases more than at least 5. So then next temperature would be 46 or higher.

Interviewer: 46 or higher? Why 46 or higher? Like how are you finding out?

Celine: Because if it is more than a 5, you would add 5 to 41 it would be 46.

[Excerpt 6]

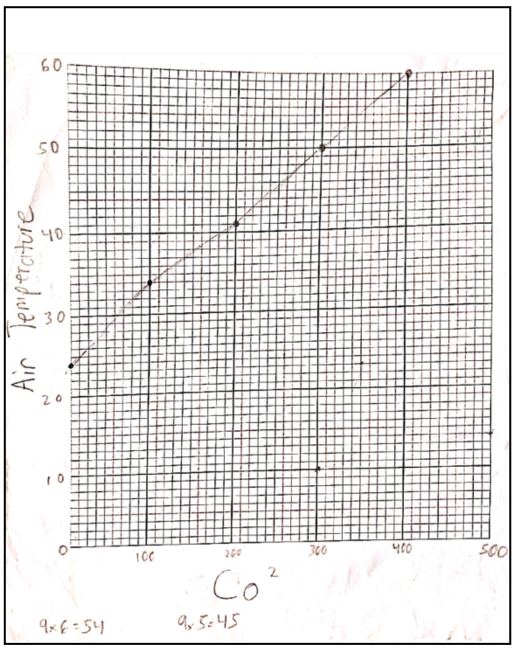


Figure 30: Graph representing relationship between carbon dioxide and air temperature (Celine)

Excerpt 6 shows that Celine also observed the amount of increase of air temperature for consecutive intervals of carbon dioxide and identified that for every 100 unit increase in the value of carbon dioxide, the value of air temperature "increases more than at least 5." Leveraging on that pattern of increase of air temperature, Celine estimated that when the value of carbon dioxide would increase from 200 to 300 units, air temperature would increase by at least 5 degrees. She added 5 degrees to 41 degrees Celsius (the temperature for 200 units of carbon dioxide) and found 46 degrees Celsius to be the value of air temperature corresponding to 300 units of carbon dioxides.

To prompt students to think about the rate of change of air temperature in each interval of carbon dioxide, I asked them to explain why they joined the plotted points by straight lines and if they could draw curves between the consecutive points. In response to this question, Celine said that "the straight line probably means there is a relationship, like a solid relationship and then, these curved lines probably mean that sometimes it goes down, sometimes it goes up." Being intrigued by the term "solid relationship", I asked Myra to elaborate her answer. Myra then added, "So it would be straight line up. It is like one number, like between the air temperature and the amount of carbon dioxide." Through the discussion of the amount of change of air temperature for consecutive values of carbon dioxide, both Myra and Celine's responses were consistent to the third mental action (MA3) of Carlson et al.'s (2002) framework of covariational reasoning. However, I was not convinced if the students were trying to express the idea of the constant rate of change through terms such as "solid relationship" or "one number." Such an argument, in that case, would establish students' engagement in level four covariational reasoning (MA4) as per Carlson et al.'s (2002) Mental Action Framework of covariational reasoning. Therefore, I asked students for further explanation (Excerpt 6):

Celine: [Pointing at Myra's amount of CO_2 and air temperature graph; Figure 31a] Like right here, this, this relationship [A; Figure 31b] is different from this relationship [B; Figure 31b].

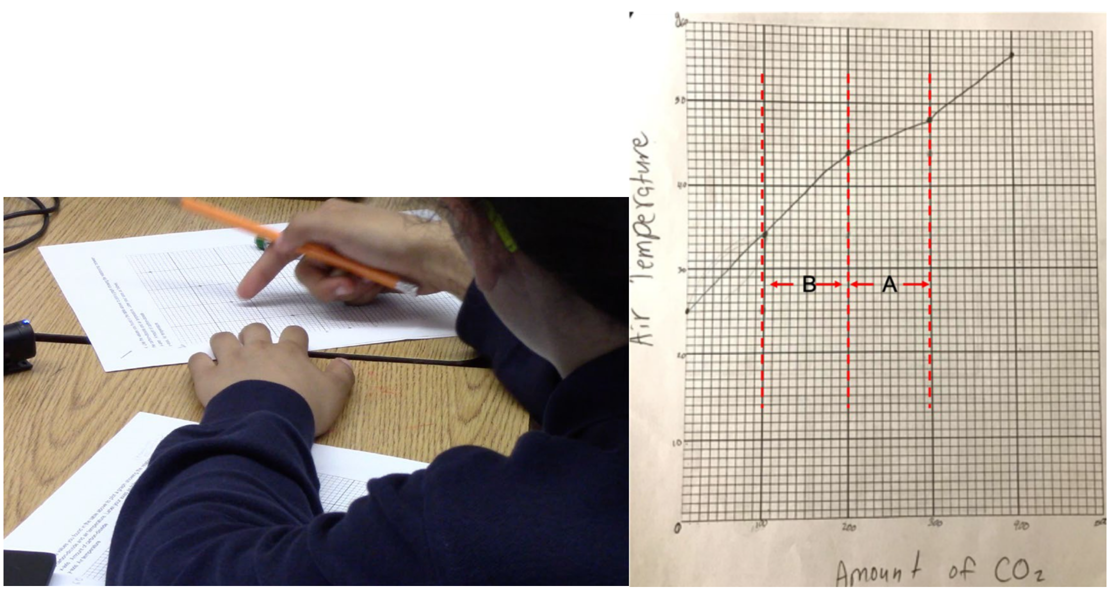
Interviewer: Oh, so the relationship is different for different points?

Myra: Yes.

Interviewer: Can you explain the relationships? Like, if I have to, if you have to compare the relationship here vs here, what is the difference?

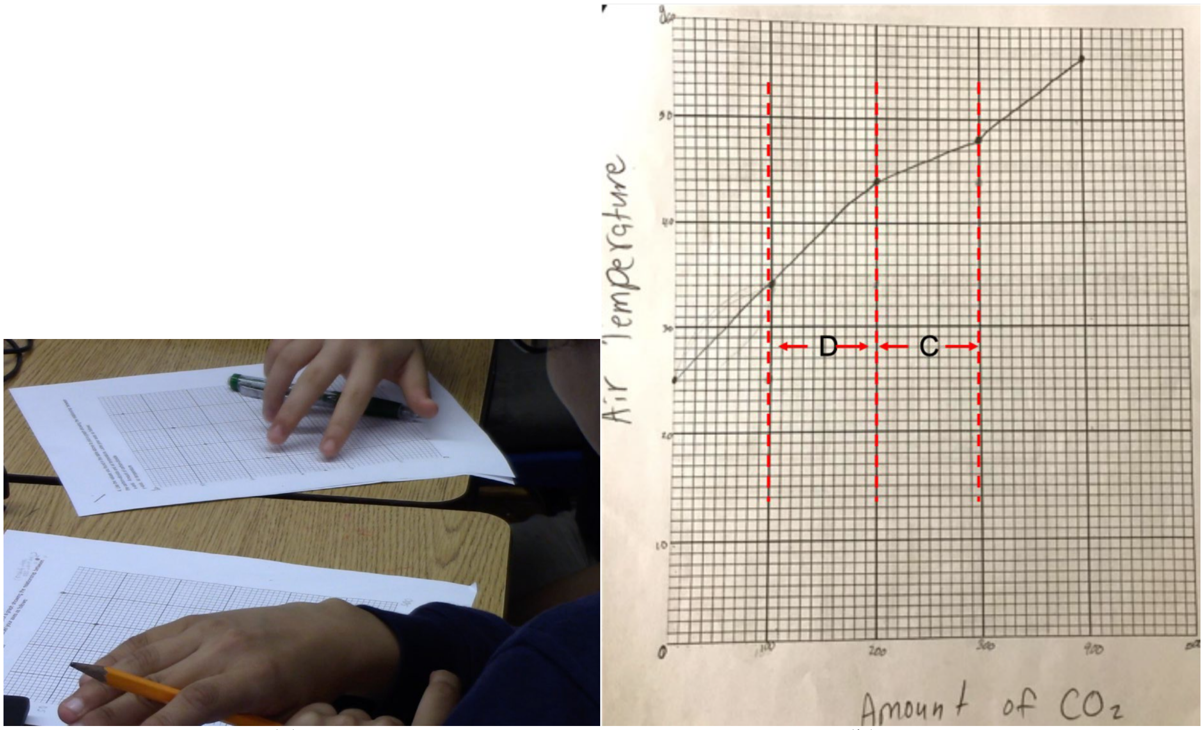
Myra: [Pointing to the amount of CO_2 and air temperature graph; Figure 32a] That, at 300, and 200, the relationship between them [C; Figure 32b] was different because the numbers were closer together, and over here [D; Figure 32b] amount of CO_2 and air temperature relationship between 200 and 100 units of carbon dioxide) the numbers are further apart. So, one will be higher, one will be lower (Figure 31b).

[Excerpt 7]



(a) (b)

Figure 31: (a) Celine pointing at Myra's carbon dioxide and air temperature graph to explain how the relationship between the two quantities is different in different intervals, and (b) Myra's graph, in which Celine pointed the amount of CO_2 and air temperature curve to show that the relationship between 200 and 300 carbon dioxide (A) is different from the relationship between 200 and 100 carbon dioxide (B)



(a) (b)

Figure 32: (a) Myra pointing at her amount of CO_2 and air temperature graph to explain the relationship between air temperature and carbon dioxide; (b) An illustration of her explanation (red dotted lines) that the value of air temperature between 300 and 200 carbon dioxide is closer (C), while the value of air temperature between 200 and 100 carbon dioxide is farther apart (D)

Both Celine and Myra used the graph and identified that the relationship between carbon dioxide and air temperature is different in different intervals of carbon dioxide. Continuing the discussion, when I asked the students to explain how they would find the difference between two corresponding values of air temperature, Celine replied, "you would subtract from your table." Seeing students fluctuating their attention between the numerical values of air temperature, as expressed in the table, and the carbon dioxide and air temperature graph, I wanted to ensure that the students do not overly rely on numbers to build their arguments. I anticipated that, instead of concentrating on the numbers, if students are encouraged to focus on the graphical relationship between the two quantities then that would prompt them to move beyond the amount of change of one quantity with respect to the other (MA3) and direct their attention towards the rate of change of one quantity with respect to the change in the other quantity (MA4). Hence, I asked them:

Interviewer: Suppose I do not have the table, just looking at the graph can we find out which one is growing more and which one is growing less?

Celine: Yes.

Interviewer: How?

Celine: [Pointing to the carbon dioxide and air temperature graph; Figure 33a] Because these you can see, these points are higher like they are more steep [A; Figure 33b]. So, that means there is a higher increase.

[Excerpt 8]

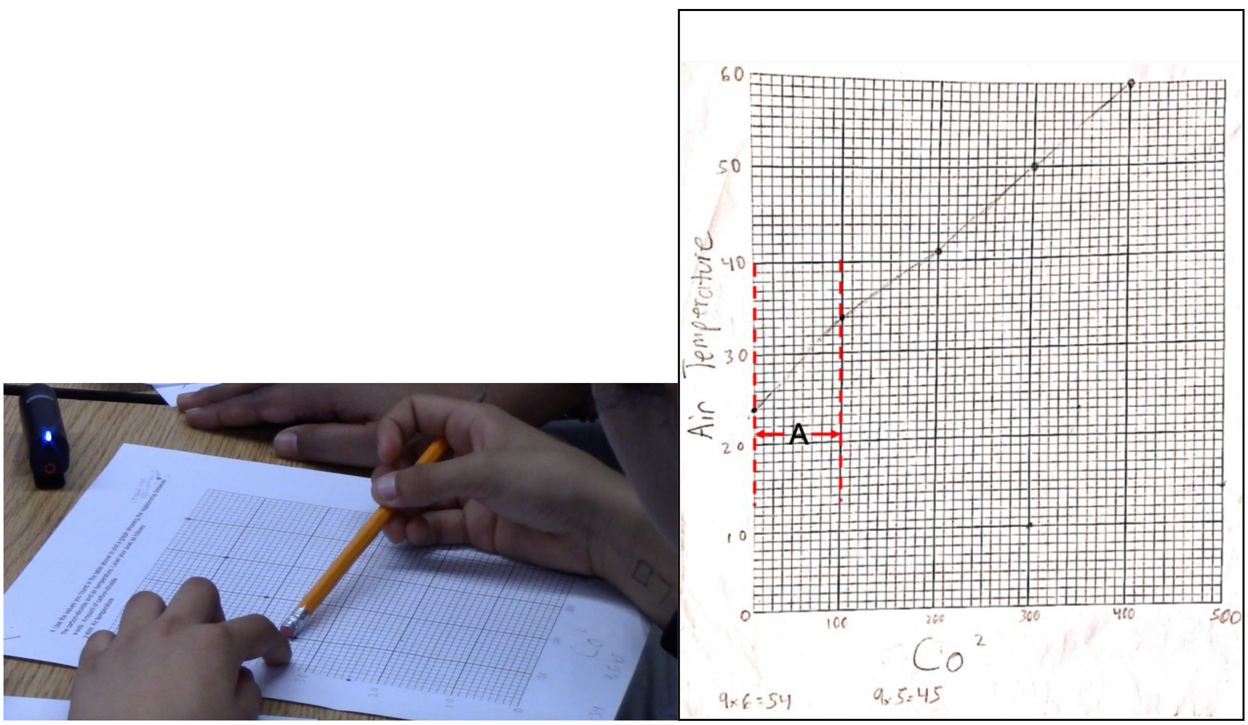
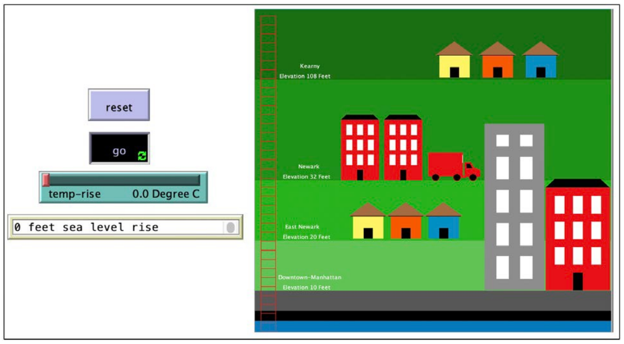


Figure 33: (a) Celine pointing the carbon dioxide and air temperature graph to show that a steeper air temperature curve indicates more increase in value of air temperature; (b) The carbon dioxide and air temperature graph where the air temperature curve (A) between 0 and 100 units of carbon dioxide is more steep. An illustration of her explanation is presented with red dotted lines.

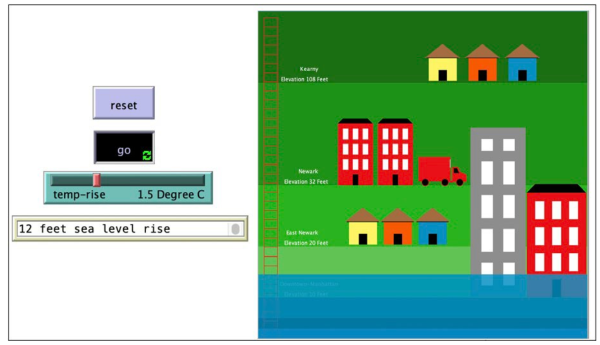
The above excerpt shows that to compare the slopes of air temperature curves in corresponding intervals of carbon dioxide, Celine focused on the carbon dioxide and air temperature graph and examined the slope of the line segments drawn in each interval of carbon dioxide. Though the students did not use the word 'rate' explicitly in their explanation, their focus on the steepness of the line segments joining the corresponding values of air temperature indicates students' attention towards slope of line segment, thus arguably establishing their conception of level 4 covariational reasoning (MA4), which as per Carlson et. al.'s Mental Action Framework prompt students to coordinate the average rate-of-change of the function with uniform increments of change in the input variable.

4.2.3 Exploring the relationships of the Sea Level Rise simulation

After students explored the Climate Change simulation, next I engaged the students with the Sea Level Rise simulation. The Sea Level Rise simulation provides the users with a dynamic environment where the users can play with the temperature rise slider to increase or decrease the value of temperature rise and observe the impact of this change on the simulation interface and the value of height of future sea level (Figure 34). This simulation was designed with the intention to support students' covariational reasoning about the quantities involved and help them identify sea level rise as a consequence of the greenhouse effect.



(a)



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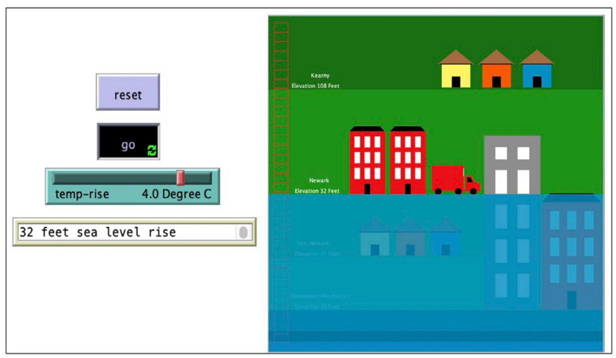
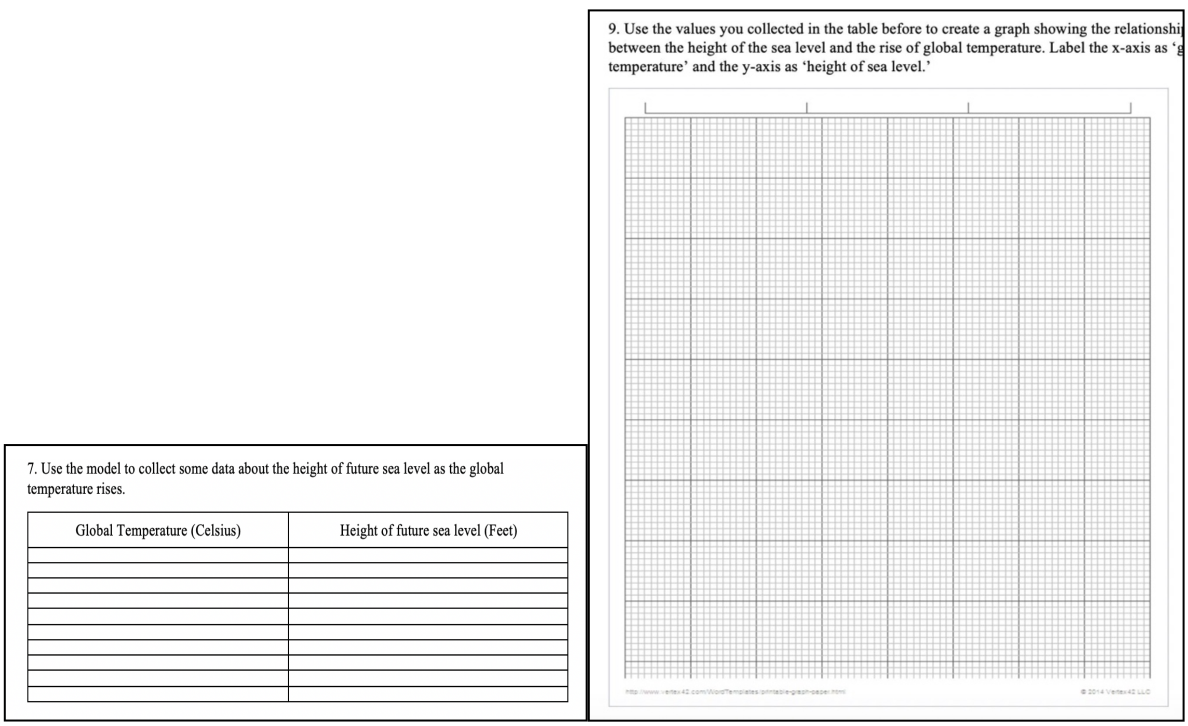


(c)

Figure 34: (a) When height of sea level is low, total land area is more (b) When height of sea level increases, total land area decreases (c) When height of sea level is maximum, total land area is minimum

Apart from temperature rise and height of future sea level, the Sea Level Rise simulation also contains a third quantity called the total land area. This quantity was included with the intention to enable students to explore the impact of sea level rise on their own lives. The directional relationship between height of future sea level and total land area varies according to the value of temperature rise. That is, when temperature rise is minimum, height of future sea level is minimum also, and the total land area is maximum. However, when the temperature rise is maximum, height of future sea level is maximum, and the total land area is minimum. I conjectured that the Sea Level Rise simulation would provide the students a platform to coordinate the direction of change of these two sets of quantities, and thus provide students an opportunity to engage in level 2 covariational reasoning as per Carlson et al.'s (2002) Mental Action Framework.

Further, students engaged in graphing activity in which they modified the values of global temperature rise in the simulation, collected the values of height of future sea level for corresponding values of the temperature rise, and plotted the ordered pairs on a graph to express the relationship between the height of future sea level and the rise of global temperature (Figure 35). I anticipated that the graphing activity might allow the students to identify the amounts of change and rate of change of height of future sea level for change in the value of global temperature and exhibit covariational reasoning aligned to Carlson et al.'s (2002) MA3 and MA4. In the following paragraphs I present two cases from the two macro cycles to discuss how students reasoned as they interacted with the simulation and the activities.


9. Use the values you collected in the table before to create a graph showing the relationshi
between the height of the sea level and the rise of global temperature. Label the x-axis as 'g
temperature' and the y-axis as 'height of sea level.'

7. Use the model to collect some data about the height of future sea level as the global temperature rises.

Global Temperature (Celsius)	Height of future sea level (Feet)

Figure 35: (a) Graphical Activity 1 where students collected the values of Height of future sea level for different values of Global Temperature; (b) Graphical Activity 2 where students were asked to plot the Height of future sea level and Global Temperature ordered pairs.

4.2.3.1 The case of Ani from MC1

After being given some time to explore the Sea Level Rise simulation, students were asked to change the value of temperature rise and observed its impact on the height of future sea level. The simulation allowed the students to drag the slider and change the value of temperature rise from 0 degrees Celsius to 4 degrees Celsius at an increment of 0.5 degrees Celsius. During the small group discussion when I asked the students to explain their activity, Ani said that he was changing temperature rise to increase the height of future sea level. The following excerpt illustrates Ani's reasoning during this activity:

Interviewer: So, what are you doing here?

Ani: I am raising the sea level.

Interviewer: You are rising the sea level? How are you rising the sea level?

Ani: The temperature rises.

Interviewer: So how are you rising the sea level?

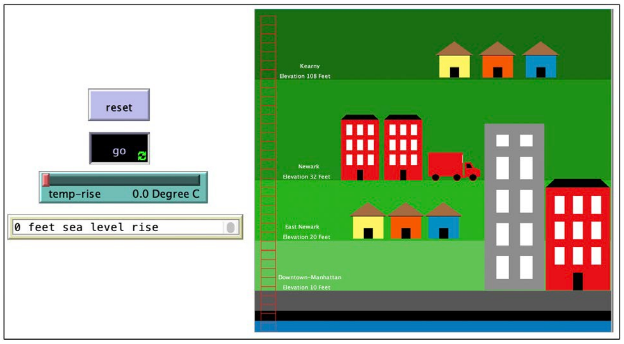
Ani: With the temperature.

[Excerpt 9]

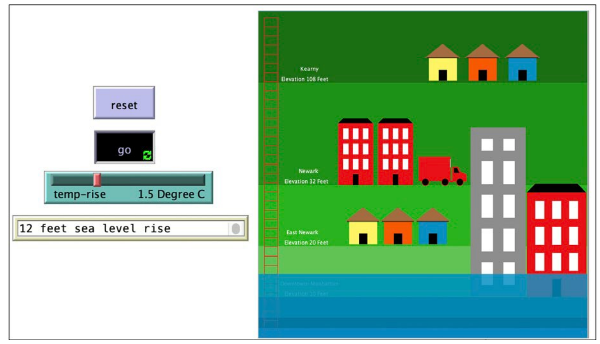
Excerpt 9 suggests that Ani focused on the directional relationship between the two quantities. He identified that the more he increased the air temperature, the higher the height of future sea level would be, a type of reasoning that aligns to MA2 as per Carlson et al.'s (2002) Mental Action Framework.

Next, to help the students identify the consequences of sea level rise for their own lives, I encouraged them to think about the impact of sea level rise on total land area. When I asked Ani, what would happen to total land area if the sea level rises, he responded that "the less land, the total land area is going to be less." When I asked him to justify his answer, he stated, "because the more higher the sea level is, it takes over land. So, instead of land over water, it will be under water." Through his reasoning, Ani coordinated the direction of change of the height of sea level with the change on the total land area, in other words coordinating the direction of change of the two quantities (MA2).

Further, from Ani's reasoning it appears that the that the graphics of the simulation helped him notice the connection between higher sea level and total land area. In the Sea Level Rise simulation, as demonstrated in *Figure 36*, as the value of temperature rise increases, blue rectangular strips representing sea water overlap the green rectangular strips of land area and icons of buildings and houses. The simulation shows possible flooding in three locations at different elevations: downtown Manhattan, East Newark, and Newark. So, considering the design of the simulation and Ani's articulation "instead of land over water, it will be under water" I infer that the graphics of the overlaying sea water helped Ani to observe the relationship between the increased sea level and decreased land area.



(a)



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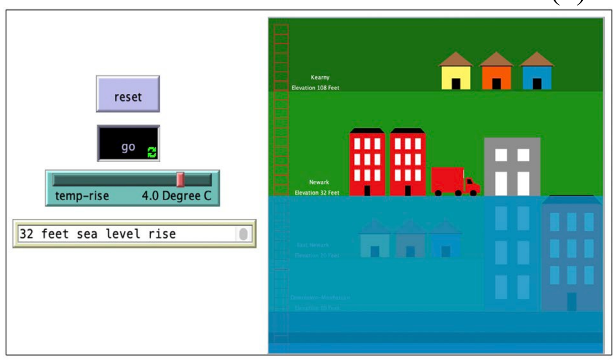


(c)

Figure 36: (a) When height of sea level is low, total land area is more; (b) When height of sea level increases, total land area decreases; (c) When height of sea level is maximum, total land area is minimum

Next, to prompt students to focus on the amount of change of the height of future sea level with respect to change in temperature rise, I asked them to graph the relationship between the temperature rise and the height of future sea level. Students used the simulation to find the height of future sea level for different values of temperature rise and plotted the ordered pairs on a graph. During the small group discussion when I asked the students to explain the graph, Ani identified that the graph represents the relationship between the temperature rise and height of future sea level (Figure 37).

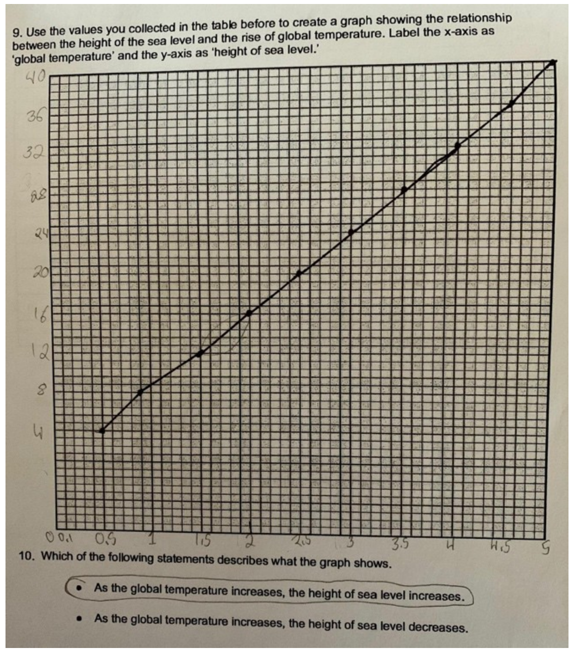


Figure 37: Graph showing the relationship between Global Temperature and Height of future sea level

Ani stated that the temperature rise versus height of future sea level graph was "rising like super straight line" because "when temperature rises, 0.5, it rises by 4 feet every time." From his response it seems that he attributed the "straight" shape of the graph to the constant increase of height of future sea level for an equal change of temperature rise. I interpret that Ani's coordination of the amount of change of height of future sea level for uniform change in temperature rise indicates an MA3 type of covariational reasoning (Carlson et al., 2002).

In order to have Ani further consider the amount of change of height of future sea level with changes in global air temperature (MA3), I asked him to explore the temperature rise versus height of future sea level graph and the carbon dioxide versus air temperature graph and compare them (Figure 38). I anticipated that comparing the two graphs would encourage Ani to examine corresponding values of the two sets of quantities and allow him to focus on the rate of change of height of future sea level with respect to the global air temperature rise (MA4-MA5, Carlson et al., 2002):

Interviewer: Remember, yesterday also we did a graph. Was it also super straight like this?

Ani: No, it was curvy.

Interviewer: It was curvy? Why do you think this is straight and that was curvy?

Ani: This one is straight because when temperature rises, 0.5, it rises by 4 feet every time. Unlike the other graph, it was all mixed up. And each time it rises it was a different height. Height it goes up to….

Interviewer: Can you show me what do you mean by height?

Ani: So, the sea level rises by 4 feet every time, but this one, like here, here it is all mixed up [showing the graph in *Figure 38*a]. Some of them are super straight line, some are super close, some are higher than others. In this [showing the graph in *Figure 38*b] they are all evenly spaced out.

[Excerpt 10]

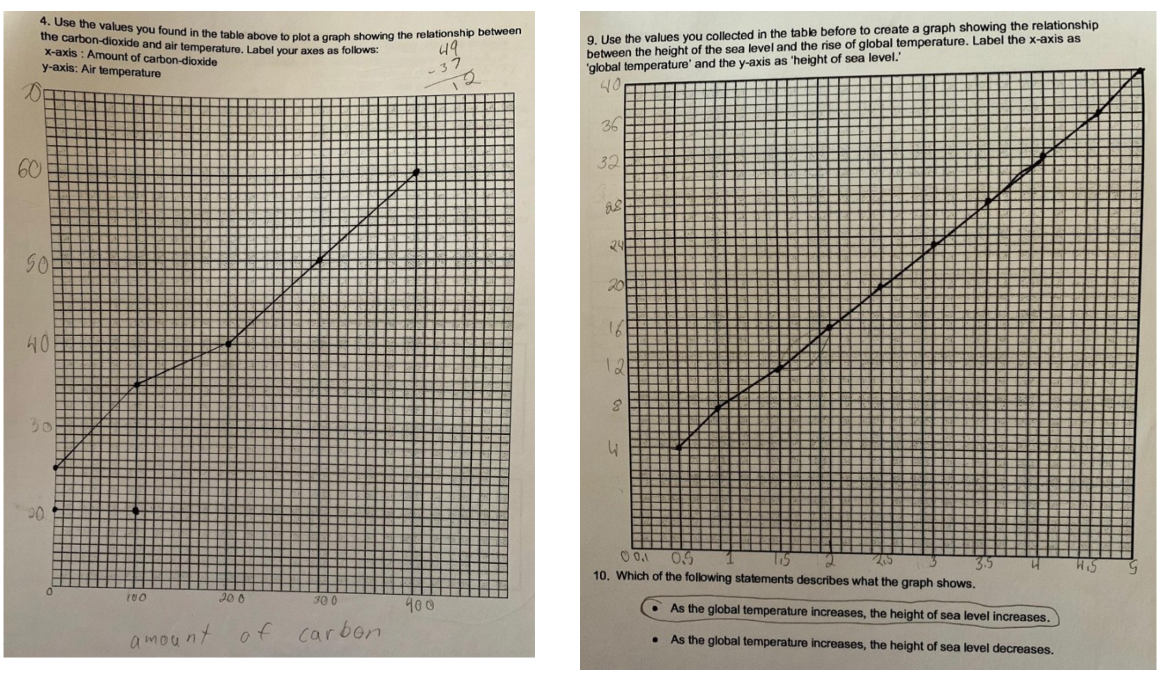


Figure 38: (a) The carbon dioxide and air temperature graph; (b) The rise in air temperature and height of future sea level graph

As Excerpt 10 shows, Ani first distinguished the two graphs (Figure 38) based on their "super straight" and "curvy" shapes. When probed to explain his answer, he reasoned about the constant increase of the height of future sea level for every 0.5 degrees Celsius increase of temperature rise as the cause behind getting a straight line representing the relationship between the two quantities. He also explained his curvy graph by reasoning that in the carbon dioxide versus air temperature graph, for every equal interval of carbon dioxide, air temperature increased randomly. For an interval of carbon dioxide, the air temperature rise is a "super straight line," while for others it is "super close" and "higher than others." By coordinating the amount of change between the various quantities in each graph, Ani showed a consistent pattern of behaviors supported by MA3.

4.2.3.2 The cases of Gio and Jake and Myra and Celine from MC2

Similar to the first macro cycle, in the second macro cycle students also explored the Sea Level Rise simulation to determine if there is a relationship between temperature rise and height of future sea level. During the whole class discussion, when Chelsea (teacher) asked the students what would happen to future sea level when global temperature would increase or decrease, Gio and Jake stated that when global temperature increases, future sea level increases and vice versa. The following excerpts illustrates their reasoning.

Teacher: The higher the temperature, what happens to the future sea level?

Gio: Higher.

Teacher: It goes higher right? It increases. The lower the global temperature and you saw this when you were exploring, the lower the global temperature…

Jake: The lower the sea level.

[Excerpt 11]

The above excerpt suggests that by manipulating the slider of temperature rise and observing the change of the sea level in the graphic of the simulation, both Gio and Jake coordinated the direction of change of global temperature and future sea level, and thus engaged in MA2 reasoning as per Carlson et al.'s (2002) Mental Action Framework.

The dynamic nature of the Sea Level Rise simulation, as we saw before, allows the users to modify the values of temperature rise and observe the change in height of future sea level and total land are simultaneously. After students explored the relationship between temperature rise and height of future sea level, next they engaged with the simulation and identified a third quantity embedded in the Sea Level Rise simulation, that is the total land area. As mentioned in the last section, my goal behind including total land area in the simulation was to have students explore the impact of sea level rise on their own lives. During the small group discussion when I asked the students if they identified any relationship between the quantities global temperature, height of future sea level, and total land area, Myra stated, "The higher the global temperature, the higher the height of the future sea level, and the less the total land area." In her response, Myra reasoned about the direction of change of the three covarying quantities (MA2), which appeared to be attributable to the design of the simulation.

Following students' reasoning about the impact of sea level rise on total land area, next I prompted them to think about the risk associated with places at lower elevations. Places located in lower elevations endure a higher risk of going under water in events of sea level rise. Consequently, I asked students to reflect on the relationship between the two quantities, elevation of a place and risk of going under water. In response Celine (MC2) argued, "the higher the elevation, the lower the risk of going under sea water." Celine's focus on the directional relationship between the elevation of a place and risk of going under water indicates her MA2 reasoning as per Carlson et al.'s (2002) Mental Action Framework.

Further, to engage students in numerical covariational reasoning about reason about elevation of a place and risk of going under water, I provided them with three statements asking whether the risk of downtown Manhattan going under water is the same, double, or half that of East Newark, given that the elevation of East Newark is double that of Manhattan (Figure 39).

12. The elevation of East Newark (20 feet) is double the elevation of downtown Manhattan (10 feet) from the sea level. Which of the statement best describes the impact of sea-level rise on the two places?

- The risk of going under sea water of downtown Manhattan is the same as East Newark.
- The risk of going under sea water of downtown Manhattan is double than East Newark.
- The risk of going under sea water of downtown Manhattan is half than East Newark.

Figure 39: Question on covariational relationship between elevation of a place and its risk of going under water

In response to these statements Celine said "I think it is the second one (The risk of going under sea water of downtown Manhattan is double than East Newark) because it says that [reading question], that is because East Newark has, is more elevated, and because of that it would be doubled because like this is double, double the amount of downtown Manhattan (Figure 40).

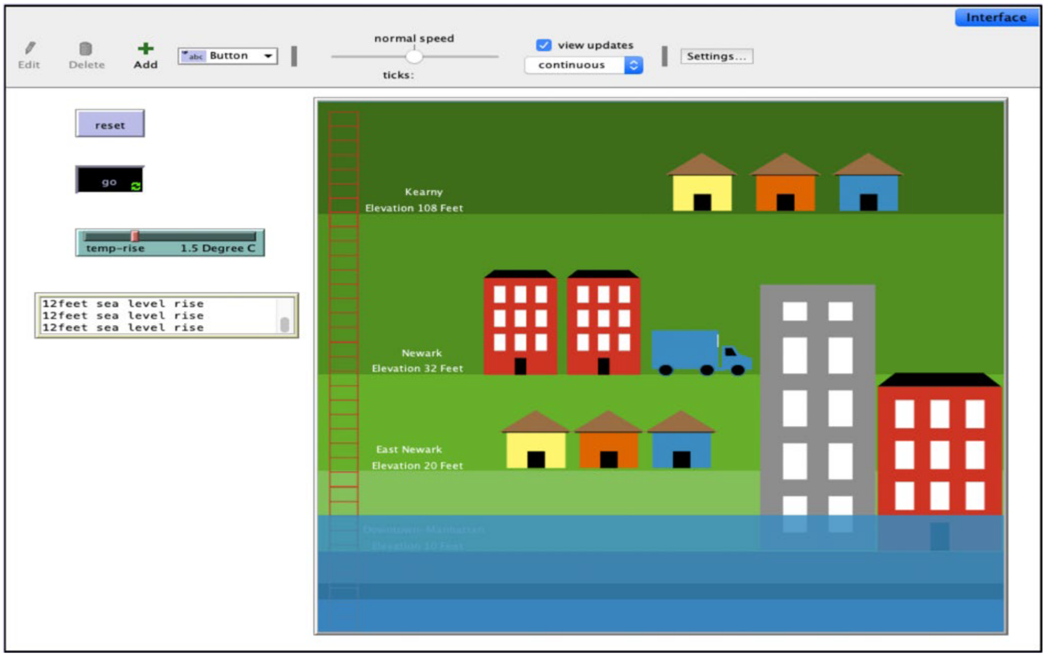


Figure 40: The Sea Level Rise simulation illustrating the elevations of four different places

From Celine's response it appears that she argued that since the elevation of East Newark is double the elevation of downtown Manhattan, the risk of downtown Manhattan going under water would be doubled. However, being unsure of what Celine referred to by saying "it would be doubled because like this is double, double the amount of downtown Manhattan", I asked her "what is double the amount?" In response Celine replied, "the elevation." Though Celine's response indicated MA2 reasoning, I still asked her to explain why the risk of downtown Manhattan going under water would be double. In response to this question, Myra added, "the higher the elevation, the lower, the less is the risk it will go under the water. So, if the water is 15 feet, downtown Manhattan will be flooded." To explain her answer, Myra chose a particular value of sea level at 15 feet however she did not say anything indicating the double risk factor. Hence, I asked her "why double?" and she argued that "it is half the elevation of East Newark, so, I feel like the double the chance to get flooded." This question not only helped Myra to identify the covariational relationship between elevation and risk by coordinating the direction of change of the two quantities (MA2), but also enabled her to identify the specific relationship between the two quantities.

Next, to prompt students to focus on the amount of change of the height of future sea level with respect to change in temperature rise, I asked them to graph the relationship between the temperature rise and the height of future sea level. Students used the simulation to find the height of future sea level for different values of temperature rise and plotted the ordered pairs on a graph. Before plotting, the teacher asked the students to predict the shape of the graph representing the relationship between temperature rise and height of future sea level. Celine (MC2) stated "I know what it is, because I felt a relationship already on line." Celine's answer intrigued me since she made the statement about the "relationship already on line" even before she plotted the ordered pairs. After plotting the temperature rise and the height of future sea level graph (Figure 41), she argued that "every time you increase by 0.5 degrees, the sea level rises 4 feet." Celine's reasoning shows that she coordinated the amount of change of the two quantities and identified that for every identical change of the air temperature by 0.5 degrees Celsius, the height of sea level increases by an equal amount of 4 feet, a type of reasoning that aligns to level 3 covariational reasoning according to Carlson et al.'s Mental Action Framework.

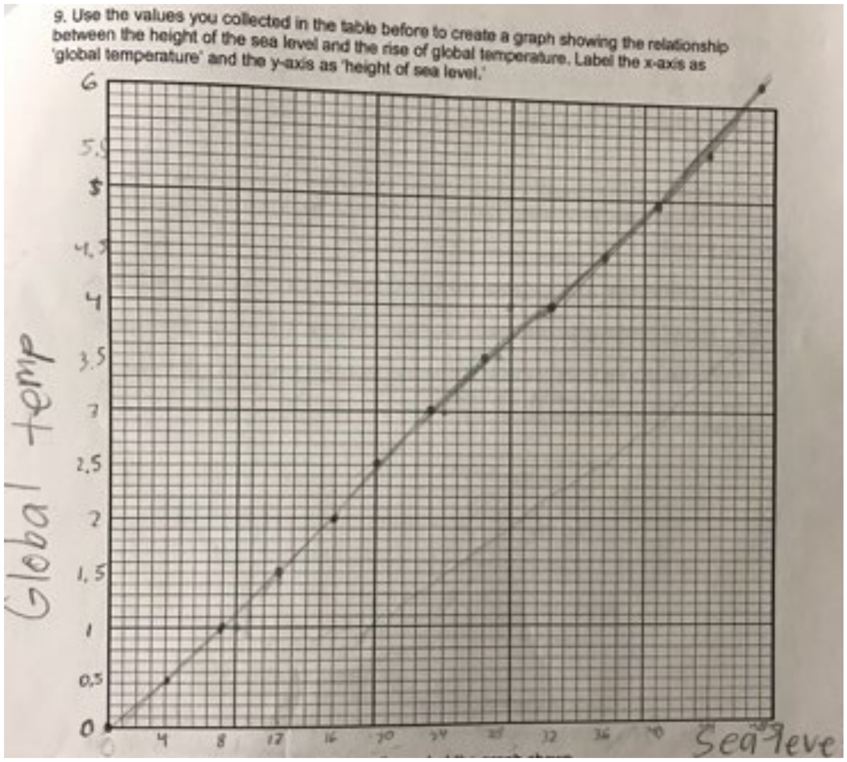


Figure 41: Celine's graph representing the relationship between temperature rise and height of future sea level

Though Celine's initial response indicated that she coordinated the amount of change of the height of future sea level with changes in temperature rise, I wanted to know more about her conception about linearity of a relationship between two or more quantities. I conjectured that if she could move beyond the specific values of temperature and height and focus on reasoning about a linear relationship in a more general sense, then that would help her attend to a higher level of covariational reasoning (Excerpt 12):

Teacher: What did you notice?

Celine: It has a linear relationship.

Teacher: It has a linear relationship. Celine, why is it linear?

Celine: Because every time it rises it increases by the same amount.

Teacher: Okay, every time, it, what is it?

Celine: The sea level.

Teacher: Every time sea level rises,

Celine: It increases by the same amount.

[Excerpt 12]

Celine predominantly used MA3 reasoning (Carlson et al., 2002) and identified that when one quantity changes by a constant amount (height of future sea level) with the uniform change in the other quantity (temperature rise), then the two quantities are linearly related to each other (Figure 41). Even though Excerpt 13 does not establish that the graphing activity engaged Celine in covariational reasoning beyond MA3, it suggests Celine's understanding of the linear relationship.

4.2.4 Bridging quantitative and qualitative data

The analysis of the pre- and post-assessment scores of the treatment and control group students indicated that overall students belonging to both the groups exhibited a significant improvement ($p < 0.005$) in their performance from the pre- to the post-assessment. Further, comparison of the post-assessment scores of the students belonging to the treatment and the control groups illustrated that the difference between the medians of the pre- and post-assessment scores of the treatment group is greater than the difference in the medians of the pre- and post-assessment of the control group. That is, the improvement of scores from the pre- to the post-assessment is higher for treatment group students compared to their peers in the control group. Further, analysis of the 10 out of 19 pre- and post-assessments questions that addressed the consequences of the greenhouse effect shows that a higher percentage of treatment group students identified different consequences of the greenhouse effect during the post-assessment. During the pre-assessment only 29.5% of the treatment group students scored 80% or more in the

10 consequences questions, whereas during the post-assessment, the percentage of students scoring 80% or more was more than doubled (68.2%). To get an insight into the quantitative data and understand what the increase in the post-assessment scores mean and what are some possible reasons that might have increased the post-assessment scores, I looked into the qualitative data collected in the form of video recordings during whole class discussion and small group interactions.

Overall, the qualitative data suggest that the dynamic simulations provided students with a space to engage in different levels of covariational reasoning as per Carlson et al.'s Mental Action Framework. Consistent with Carlson et al.'s (2002) description of students' covariational reasoning as an emergent concept, this study also found that the participating students' reasoning about covarying quantities shifted based on their interaction with the different activities. Students' exploration of the dynamic simulations allowed them to coordinate the direction of change of two or more quantities; however, the graphing activities provided them with the platform to gradually refine their directional reasoning and coordinate the amount of change and rate of change of one quantity with respect to the change in the other quantity. For example, during their exploration of the Sea Level Rise simulation, students modified the value of temperature rise and observed the impact of the change on the height of future sea level, thus coordinating the direction of change of the two covarying quantities (MA2). However, when students engaged in graphing activities, they focused on the amount of change of height of sea level with respect to change in temperature rise and identified that for every 0.5-degrees Celsius increase in temperature rise, the height of sea level increases by 4 feet. The graphing activities encouraged the students not only to focus on in what direction the two quantities were covarying but helped them refine their directional understanding to reason about how the two quantities were covarying (MA3-MA4).

Consequently, I consider students' engagement with the dynamic simulations and accompanying investigations as a significant reason behind their shift in performance from the pre- to the post-assessment. For example, question #4 (Figure 42) asks students to identify the relationship between carbon dioxide and air temperature and question #7 (Figure 43) requires students to focus on the covariational relationship between elevation of a place and risk of being affected by sea level rise. During the design experiment, students explored the relationship between the concerned sets of quantities in the Climate Change and Sea level rise simulations respectively. As a result, during the post-assessment, students might find the phenomena and the involved quantities familiar and performed better after their interaction with the simulations. Secondly, the ten pre- and post-assessment questions on the consequences of the greenhouse effect, as shown in Figure 42 and Figure 43, require students to have a conception of covariation.

4. Which of the following statements is true about atmospheric carbon-dioxide and air temperature?
   a) If the atmospheric carbon-dioxide increases, the air temperature increases.
   b) If the atmospheric carbon-dioxide increases, the air temperature decreases.
   c) If the atmospheric carbon-dioxide increases, the air temperature stays the same.

Figure 42: Pre- and Post-assessment question #4 illustrating requirement of students' covariational reasoning to be able to answer the question

7. Which of the statements is true about height of sea level and elevation of a place:
   a) The higher the elevation of a place, the lower the risk being affected by sea level rise.
   b) The higher the elevation of a place, the greater the risk of being affected by sea level rise.

Figure 43: Pre- and Post-assessment question #7 illustrating requirement of students' covariational reasoning to be able to answer the question

For example, to answer question # 4 (Figure 42) students need to reason how air temperature covaries with respect to increases and decreases of carbon dioxide. Likewise, to answer question

7 (Figure 43), students need to coordinate the direction of change of elevation of a place and the risk of being affected by sea level rise. As a result, because the activities of the simulations encouraged the students to reason about the different covarying quantities, it is arguable that the engagement of the students with the module facilitated their performance during the post-assessment.

4.3. Students' reasoning about the causes of the greenhouse effect

The nine remaining items of the assessment attended to the different contributing factors of the greenhouse effect including carbon dioxide emission due to watching televisions and carpooling. Consequently, this section focuses on the pre- and post-assessment responses of the students from the treatment group to identify how the students' perception about the causes of the greenhouse effect shifted over the course of five days of design experiment. To examine how the treatment group's understanding about covariation and the causes of the greenhouse effect shifted from the pre- to the post-assessment, I executed the following procedure. First, I calculated the amount of correct responses of each individual student in the pre- and post-assessment for those nine items. The maximum possible score of correct responses is 9 if they responded to all the nine questions correctly and the minimum possible score is 0 if they responded to none of the questions correctly (see third column of Table 7 for possible scores). Then I calculated the number of students that got each score in the pre- and post-assessment, presented in the second and fourth columns in Table 7 respectively. The results of the analysis show that in the pre-assessment 29.5% of the students scored correctly in 7 or more responses, while during the post-assessment, the percentage of students scoring 7 or higher changed to 52.3%. Also, during the pre-assessment 11.4% of the students' score was 2 or below, which was reduced to 4.5% during the post-assessment.

Table 7: Frequency of students scoring zero through 9 in the pre- and post-assessment (covariation and the causes of the greenhouse effect)

Factors responsible for the greenhouse effect				
Percentage	Pre-Assessment	Score	Post-Assessment	Percentage
29.50%	2	9	9	52.30%
	6	8	8	
	5	7	6	
	7	6	9	
	8	5	5	
	8	4	4	
	3	3	1	
11.40%	3	2	1	4.50%
	1	1	1	
	1	0	0	

Figure 44 shows a bar graph illustrating the frequency of the students belonging to each scoring category (0 to 9) during the pre- and the post-assessment. The figure indicates a shift in students' understanding of covariation and the causes of the greenhouse effect as a result of the intervention.

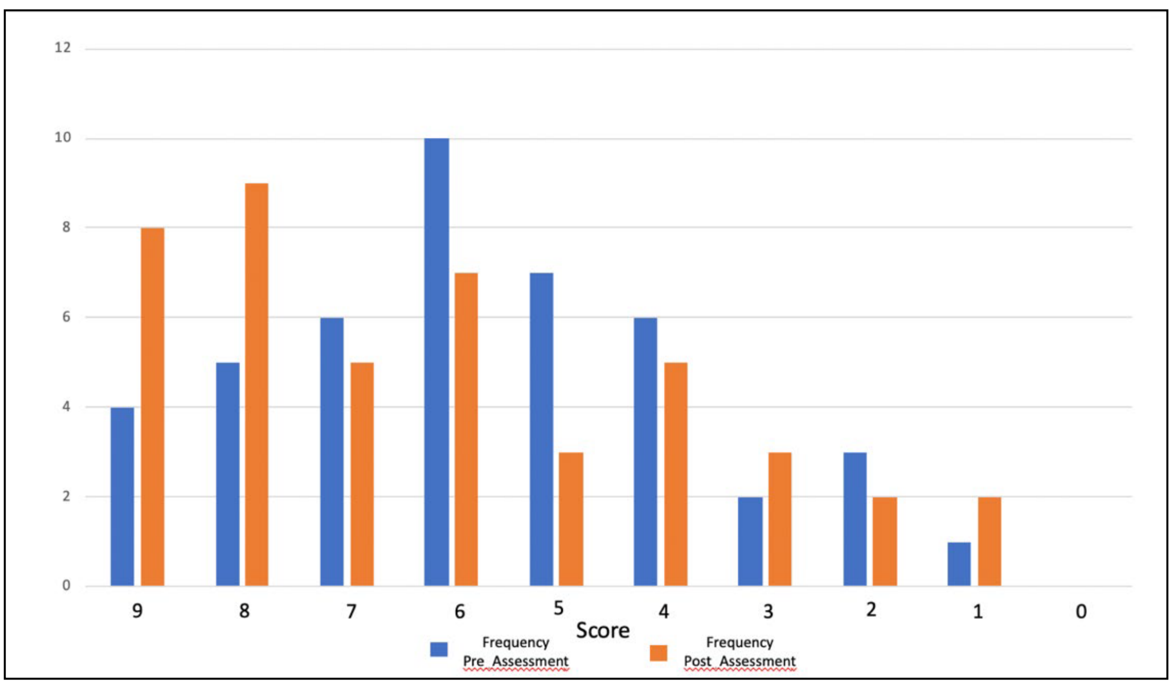


Figure 44: Bar Graph showing frequency of students in each scoring category (0 through 9) in the pre- and post-assessment (covariation and the causes of the greenhouse effect)

Delving deeper into the analysis of some those nine questions, five of the nine items, #8, #9, #13a, #13b, #13c (see APPENDIX VII), in the assessments entailed different daily-life practices that result in carbon dioxide emission in the atmosphere. In these five items, students were asked to calculate the total amount of carbon dioxide emitted over the period of one year as a result of watching TV or playing video games. During the pre-assessment, slightly more than half of the students (56.4%) belonging to the treatment group responded correctly to these five questions. During the post-assessment the percentage of the participants responding correctly to the questions increased to 67.8%.

Carpooling or sharing of cars by two or more people cuts down the number of cars on roads, thus resulting in less emission of carbon dioxide in the atmosphere. One of the nine items, #2, in this particular category asked students about the relationship between carpooling and carbon dioxide emission (see APPENDIX VII). During the pre-assessment, 22 students (50%) indicated that carpooling with friends would add less carbon dioxide in the atmosphere. However, during the post-assessment, the number of students identifying the correct relationship between the two quantities increased significantly (n=38, 86.4%).

Although the treatment group students' engagement with the Carbon Calculator simulation and the accompanying activities might have helped them identify the causes of the greenhouse effect and be the reason for the above increase, the quantitative data do not provide enough information about students' development of reasoning to support this claim. To get an insight into how the students' engagement with the simulation and their covariational reasoning might have impacted their performances in the post-assessment, in the following paragraphs I describe the design principles of the Carbon Calculator simulation and activities and discuss stories of the students' experiences interacting with the simulations to present substantiated claims about what knowledge they abstracted from those interactions.

4.3.1 Exploring the relationships of the Carbon Calculator simulation

The first two simulations, Climate Change and Sea Level Rise, were designed to provide students a platform to engage in covariational reasoning between different relevant quantities and thus identify the causes and consequences of the greenhouse effect. The third simulation, the Carbon Calculator, was developed with the goal of helping students to model the covariational relationships between different quantities such as watching TV carpooling or amount of carbon dioxide and foster within them an awareness towards human impact on the climate.

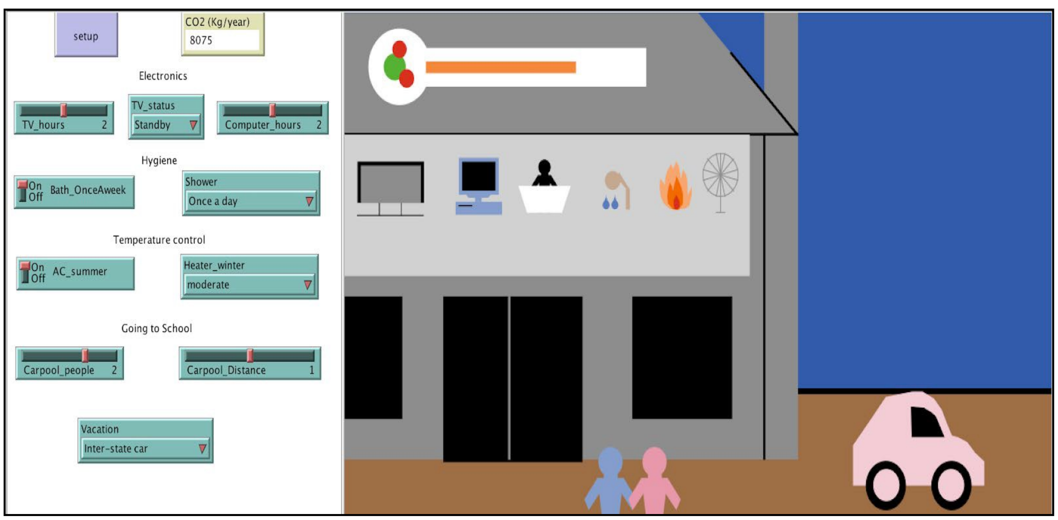


Figure 45: The Carbon Calculator Simulation

The Carbon Calculator simulation (Figure 45) provided the students with a dynamic environment where they could modify their daily practices and calculate the total amount of carbon dioxide being released in the atmosphere as a result of those practices. I conjectured that the simulation would provide students with a space to engage in covariational reasoning between different quantities, for example, as TV hours increases the carbon dioxide emission increases, thus providing students an opportunity to engage in Level 2 covariational reasoning as per Carlson et al.'s (2002) Mental Action Framework. Further, students engaged in graphing activities where they collected the carbon dioxide emission amount for different practices such as number of TV hours, carpool distance, and number of people carpooling and plotted the ordered pairs on a graph, expressing the covariational relationship between the different sets of quantities. I anticipated that the graphing activity might allow the students to identify the amounts of change and rate of change of carbon dioxide emission for different practices and exhibit covariational reasoning aligned to Carlson et al.'s (2002) MA3 and MA4 levels. In the following paragraph first, I describe the how the students belonging to the two macro cycles reasoned covariationally during their interaction with the simulation. Next, I discuss in what ways the activities might have generated within students a sense of critical consciousness about the impact of human activities on the natural climatic condition of the earth.

4.3.1.1 The case of Amber from MC1

At the beginning of the Carbon Calculator simulation, students were asked to manipulate different daily practices such as working on computers, taking showers, and using air conditioners, or heaters and observe the impact of these activities on the annual carbon dioxide emission. One of these practices included in the Carbon Calculator was the number of TV hours. Students were asked to modify the number of hours they watch TV and observe the change in the amount of carbon dioxide they add in the atmosphere annually. Students increased and decreased the TV hours between zero and four, calculated the amount of carbon dioxide for corresponding values of TV hours (Figure 46), and entered the ordered pairs in the TV hours and CO_2 amount table (Figure 47b).

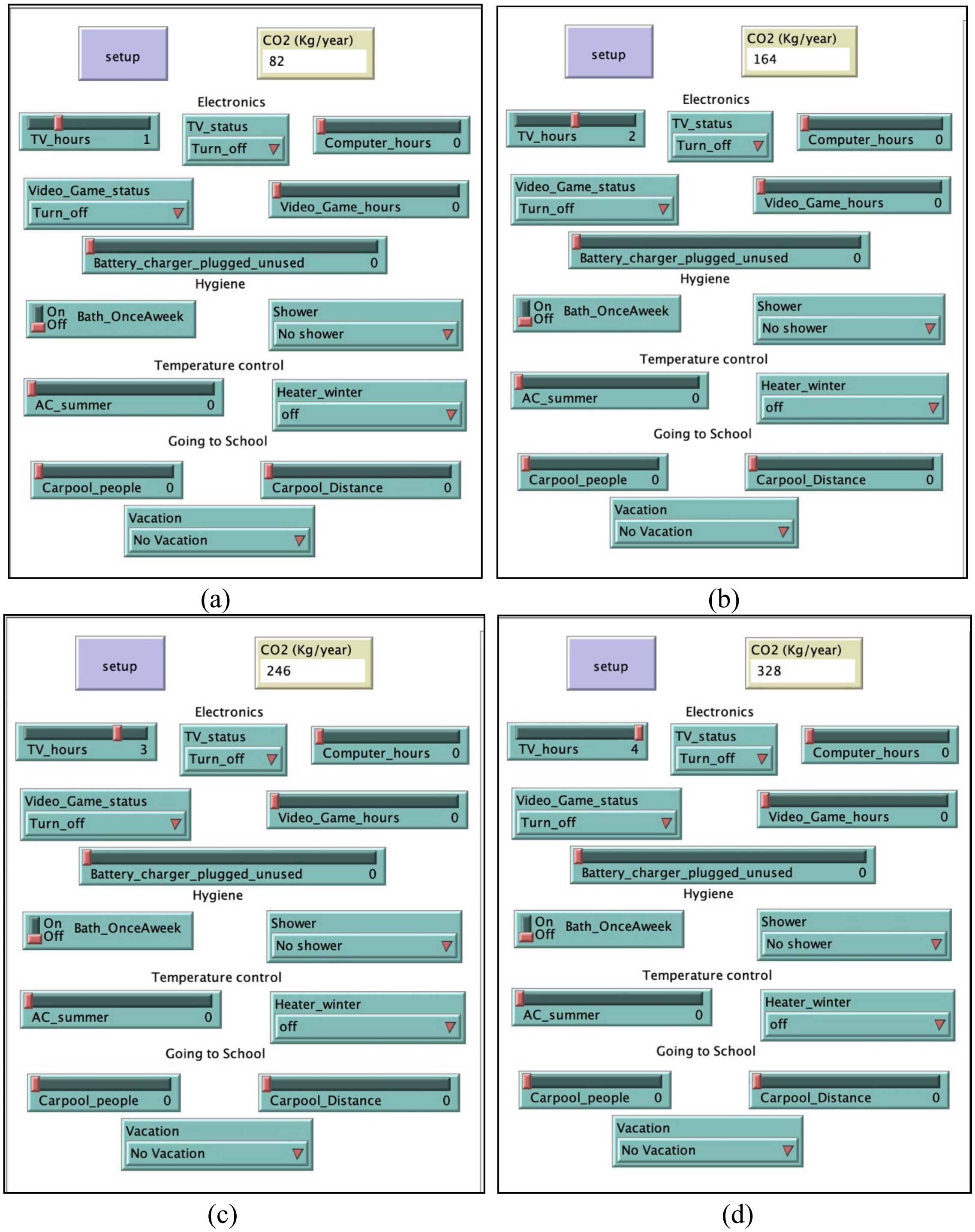
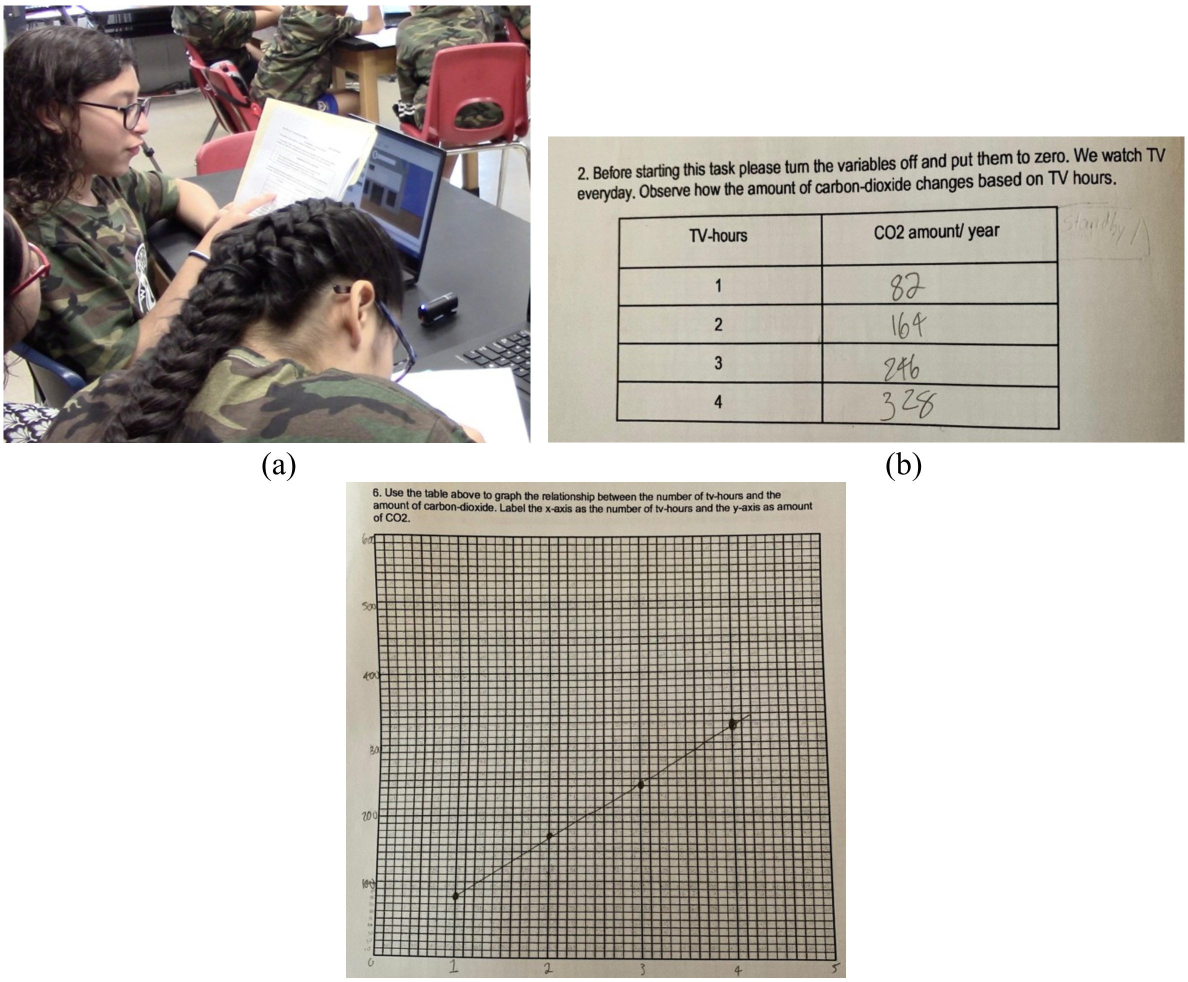


Figure 46 (a) Amount of carbon dioxide released for one TV hour; (b) amount of carbon dioxide released for two TV hour; (c) amount of carbon dioxide released for three TV hours; (d) amount of carbon dioxide released for four TV hours.

Next, students engaged in a graphing activity where they were asked to plot the TV hours versus amount of carbon dioxide ordered pairs and graph the covariational relationship between the two quantities. However, before students graphed the relationship, during the small group interaction I asked the students to reflect on the relationship between the two quantities. In response Amber referred to the TV hours and CO_2 amount/year table (Figure 47b) and stated that when TV hours doubles, the amount of carbon dioxide also doubles.



(a) (b)

(c)

Figure 47: (a) Pointing towards the TV hours and *CO_2* amount/ year table to show how *CO_2* amount changes with TV hours; (b) TV hours and *CO_2* amount/ year table; (c) Amber's table and graph showing the relationship between TV hours and Carbon dioxide.

When I asked Amber to explain her response of doubling carbon dioxide, she added, "You just keep adding depending on the hours of usage of TV." Being unable to understand if Amber was thinking across the two quantities or coordinating the change of one quantity with the change in another, I prompted Amber to explain her answer. Excerpt 13 demonstrates my conversation with Amber.

Amber: Per hour it is 82. The amount of….

Interviewer: Carbon dioxide?

Amber: Yeah. And if you multiply 82 times 2, the total is 164 which is 2 hours. So, for just every hour you just keep adding 82.

[Excerpt 13]

Amber's statement, "So, for just every hour you just keep adding 82" (*Figure 18*) indicates that by looking at the table she focused on the amount of change of carbon dioxide for every hour change of TV usage and incremented the carbon dioxide amount by 82 for each unit increment of TV hours. Excerpt 15 not only establishes Amber's covariational reasoning but is also indicative of MA3 (Carlson et al., 2002).

Another activity in the Carbon Calculator simulation asked the students to explore the covariational relationship between the number of people carpooling and the amount of carbon dioxide (Figure 48). First, they were asked to use the simulation to find the different values of carbon dioxide for corresponding number of people carpooling and fill the values in a table. Then they were asked to plot the ordered pairs to graph the relationship. However, before students could plot the carpooling versus carbon dioxide graph, I asked Amber to predict the nature of the graph. The following excerpt illustrates my conversation with Amber.

Interviewer: Before plotting can you give me some idea how the graph will look like?

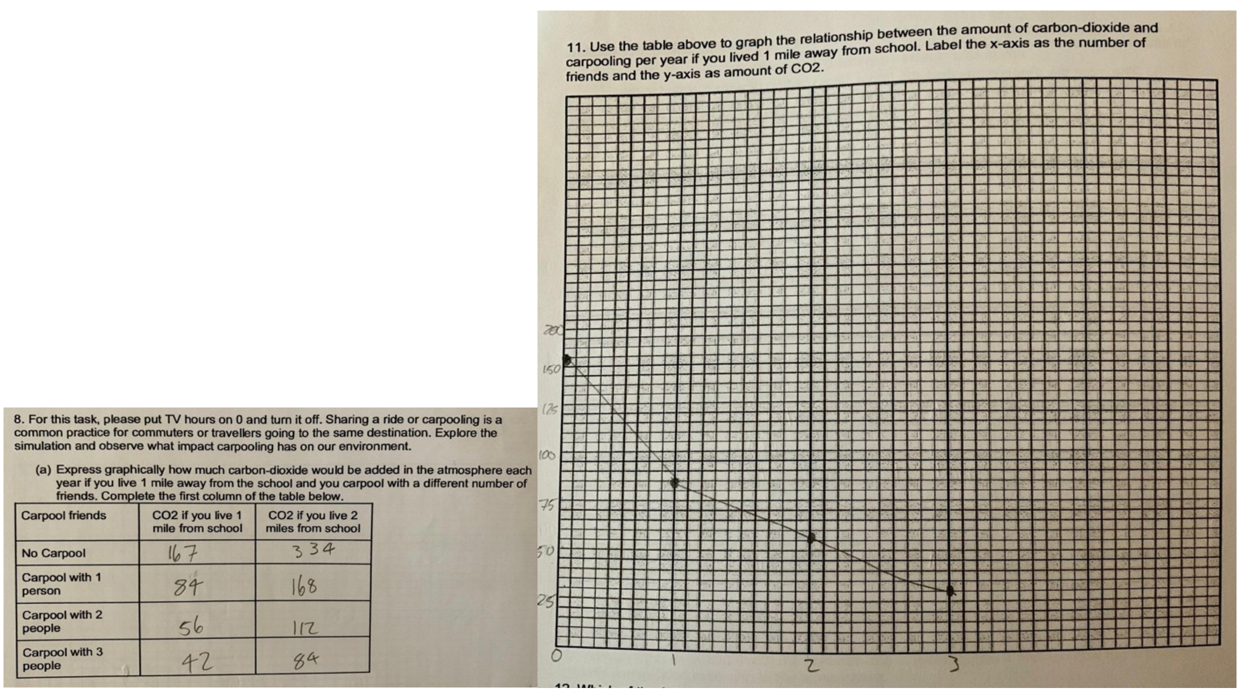
Amber: It will start going down, decreasing.

Interviewer: Why?

Amber: Because since you are carpooling, the more people you carpool, the less cars you use. So, that means the less carbon dioxide you are using.

[Excerpt 14]

To determine the nature of the graph between carpooling and carbon dioxide, Amber focused on the directional relationship between the two quantities and identified the more number of people carpool, the lesser would be the number of cars, and as a result, the reduced would be the amount of emitted carbon dioxide (MA2). Next, Amber plotted the collected values of carpooling and carbon dioxide and constructed a concave down graph expressing the relationship between the number of people carpooling and amount of carbon dioxide added in the atmosphere (Figure 48). When I asked Amber to explain the graph, she reasoned that "It started at 167 and ended at 42, which is big difference." Amber's reasoning shows that she coordinated the change of the carbon dioxide values from zero carpools to carpooling with three friends.



8. For this task, please put TV hours on 0 and turn it off. Sharing a ride or carpooling is a common practice for commuters or travellers going to the same destination. Explore the simulation and observe what impact carpooling has on our environment.

(a) Express graphically how much carbon-dioxide would be added in the atmosphere each year if you live 1 mile away from the school and you carpool with a different number of friends. Complete the first column of the table below.

Carpool friends	CO2 if you live 1 mile from school	CO2 if you live 2 miles from school
No Carpool	167	334
Carpool with 1 person	84	168
Carpool with 2 people	56	112
Carpool with 3 people	42	84

Figure 48: Amber's table and graph representing the relationship between number of people carpooling and carbon dioxide

When Amber was asked to explain why the TV hours versus carbon dioxide graph is a straight-line graph but the carpool people versus carbon dioxide graph is curved, she stated "Because here they were going by 82 (Figure 48). Going up every time by 82 and here (Figure 19) it just, gone down by don't know how much." As a result, the graph representing the relationship between TV hours and carbon dioxide is a "straight line." Through comparison of the two graphs, Amber observed the change of values of carbon dioxide for corresponding changes in the TV hours and number of people carpooling and identified that for the first set of quantities (TV hours versus carbon dioxide), carbon dioxide changed by the same amount. As a result, the graph representing the relationship between TV hours versus carbon dioxide is a "straight line." On the contrary, Amber did not recognize any pattern in the change of carbon dioxide for different values of carpool people. In her reasoning, the absence of a pattern in the relationship between number of people carpooling and carbon dioxide led to a representation of a curved line (Figure 48). Amber's responses illustrate that she reasoned about the amount of change between the two quantities, illustrating a behavior that was suggestive of MA3 reasoning (Carlson et al., 2002).

4.3.1.2 The case of Gio from MC2

Similar to the students of the first macro cycle, students of the second macro cycle also explored the Carbon Calculator simulation and calculated the total amount of carbon dioxide they add in the atmosphere annually. However, it is worth mentioning that when the Carbon Calculator was implemented in the first classroom, Doug asked the students to think of other daily practices that might also emit carbon dioxide in the atmosphere. In response, students suggested additional activities that were more familiar to them and that they practice regularly. Some of those activities were playing video games, video games kept in stand-by mode or turned off after use and keeping battery chargers plugged in when not in use. Hence, before implementing the Carbon Calculator simulation in the second classroom, I modified the simulation and added the suggested activities.

After exploration of the simulation, students engaged in graphing activities where they were asked to find the carpool friends and CO_2 amount/year ordered pairs and plot those points to graph the relationship between the two quantities. Before students could draw the graph, when I asked them to predict the shape of the graph, Gio (MC2) replied that the graph would be an increasing "straight line moving from left to right," but immediately changed his response to "a straight line decreasing from left to right." When Gio was asked to explain his answer, he stated, "If you have more people in the car, then you will have less [carbon dioxide]." He used his hand-gesture, from up to down and stated, "we have to start up here and go down." (MA2). Next, when Gio plotted the ordered pairs and joined the points, he noticed that the graph (Figure 49), which he had anticipated to be linear, was curved. Gio justified his decreasing curve construction by saying, "it is not the same every time. It is not the same amount of CO_2, there is no doubling or tripling. It is all different. It is not tripling or doubling or anything like that. Whole bunch of different numbers."

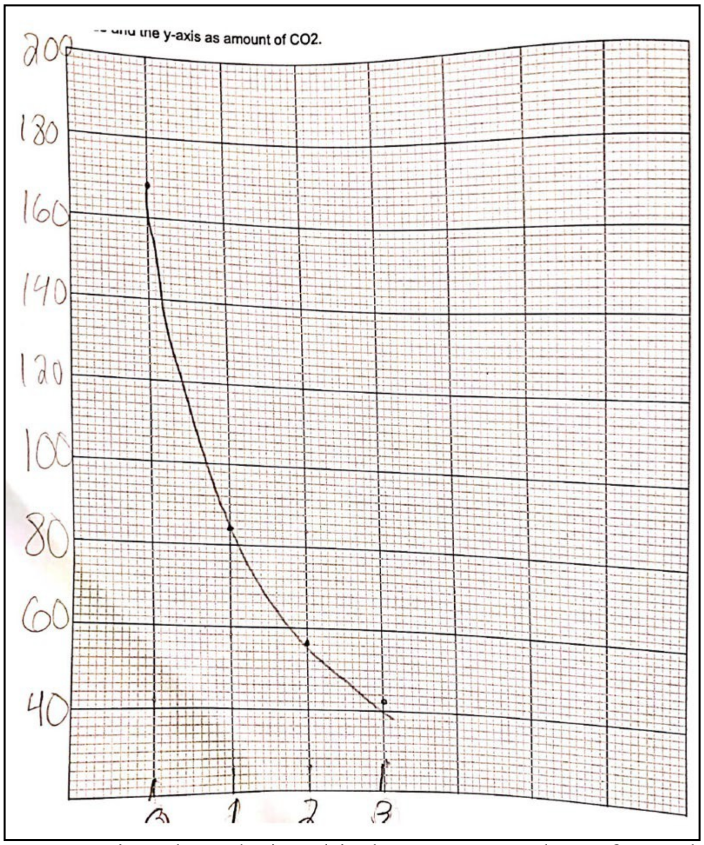


Figure 49: Gio's graph representing the relationship between number of people carpooling and carbon dioxide

Gio's reasoning shows that he compared the change of carbon dioxide value for different intervals of carpool people (i.e., [1-2], [2-3], [3-4]) and found that the change in the amount of carbon dioxide is different in consecutive intervals, and accordingly the graph representing the relationship between the two quantities "would be curved." This type of reasoning is aligned to Carlson et al.'s (2002) MA3.

4.3.2 Bridging quantitative and qualitative data

The analysis of the nine pre- and post-assessments questions that addressed the causes of the greenhouse effect showed that during the post-assessment a higher percentage of treatment group students identified different factors responsible for the greenhouse effect. Out of the total of nine questions, during the pre-assessment only 29.5% of the treatment group students scored seven or above, while during the post assessment the percentage of students scoring equal to or above seven increased to 52.3%. To get an insight into the analysis of the quantitative data and understand the possible reasons that might have led to the increase in the post-assessment scores, I looked into the qualitative data collected during students' interaction with the Carbon Calculator simulation. Because the Carbon Calculator was designed with the intention to make students aware of their own contribution towards carbon dioxide emission, I anticipated that a window into students' experience interacting with the Carbon Calculator simulation might help me find the possible causes that enhanced treatment group students' mean scores during the post-assessment.

The qualitative analysis showed that the Carbon Calculator simulation provided students with an interactive space where they modified their daily practices such as watching televisions, playing video games, and taking showers, and observed the impact of these modification on their annual carbon dioxide emission. By exploring the dynamic sliders of the simulation, students showed evidence that they coordinated the direction of change of two or more quantities, such as TV hours and carbon dioxide amount and number of people carpooling and carbon dioxide amount, thus engaging in Level 2 covariational reasoning as per Carlson et al.'s (2002) Mental Action Framework. The graphing activity provided them with the platform to refine their directional reasoning and coordinate the amount of change of one quantity with respect to the change in the other quantity (MA3). For example, students plotted both TV hours and CO_2 amount/year and carpool friends and CO_2 amount/year ordered pairs and compared the two graphs to identify that in the first scenario (TV hours and carbon dioxide amount), amount of carbon dioxide increased uniformly with uniform increase in TV hours, thus producing a linear increasing graph. On the contrary, the carpool friends and CO_2 amount/year graph would be "a straight line decreasing from left to right" because "If you have more people in the car, then you will have less [carbon dioxide]" (Gio, MC2). On further discussion, Gio identified that unlike the TV hours and CO_2 amount/year graph, the carpool friends and CO_2 amount/year graph would be curved downward since the change in CO_2 amount/year "is not same every time." Hence, the graphing activities encouraged the students not only to focus on in what direction the two quantities were covarying but helped them refine their directional understanding to reason about how the two quantities were covarying (MA3-MA4).

Hence, considering students' interactions with the Carbon Calculator simulation, it seems that the simulation itself and the students' engagement with covariational reasoning collectively helped them perform better during the post-assessment. The questions on the causes of the greenhouse effect in the assessment closely aligned with the quantities included in the Carbon Calculator simulation. For instance, in the pre- and the post-assessment, to measure students' understanding of the causes of the greenhouse effect, students were asked "What will happen if you go to school every day by carpooling with your two friends?" and "If I use my computer for 1 hour every day, I release 36 kg of CO_2 in the atmosphere in one year. How many kg of CO_2 will I release in the atmosphere if I use my computer for 3 hours?" As it can be seen, both the practices of carpooling and using computers are included in the Carbon Calculator simulation, so students' familiarity these practices might have influenced their performance in the post assessment.

Further, as discussed earlier, all the questions included in the pre- and the post-assessment required students to reason between covarying quantities. For example, to answer question #2 (Figure 50) students need to reason about how carbon dioxide emission covaries with number of friends carpooling to school.

2. What will happen if you go to school every day by carpooling with your two friends?
   a) You will add less carbon-dioxide in the atmosphere.
   b) You will not add any amount of carbon-dioxide in the atmosphere.
   c) You will add more carbon-dioxide in the atmosphere.

Figure 50: Pre- and Post-assessment question #2 illustrating requirement of students' covariational reasoning to be able to answer the question

Hence, I conjecture, because the Carbon Calculator simulation engaged students in covariational reasoning between different quantities and the pre- and post-assessment questions also necessitate students to engage in reasoning between different covarying quantities, there is a possibility that the engagement of the students with the Carbon Calculator simulation might facilitate the improvement in their performance during the post-assessment.

4.4. Conclusion: Providing a space for students' covariational reasoning

The findings of the study suggest that the three NetLogo simulations, the five investigations, and the discussion questions conjointly helped the students to navigate through the different levels of covariational reasoning as per Carlson et al.'s Mental Action Framework and developed their understanding of the greenhouse effect. Table 8 is an outline of the different models of covariational relationships that students expressed as a result of their engagement with the three dynamic activities. Under each mental action, in the first column of the table I discuss the models of covariational relationships that the students articulated, and in the second column I describe the activities that provided students a space to engage in the particular forms of reasoning.

Table 8: Models of Covariational Reasoning developed by students and activities that helped them to build the models.

MA2: **Coordinating the direction of change of one variable with changes in the other variable.**	
Students' Articulations	**Activities**
Albedo versus air temperature The higher the albedo, the lower the temperature; the lower the albedo, higher the temperature.	Dragging the albedo slider of the Climate Change simulation to the left and right, thus decreasing and increasing the value of albedo, and studying the impact of the change on the graphics of the simulation and the value of air temperature. Increasing and decreasing the amount of carbon dioxide by clicking the Add CO_2 and Remove CO_2 buttons on the Climate Change simulation and studying the impact of the change on the graphics of the simulation and value of air temperature.
Carbon dioxide versus air temperature The higher the amount of carbon dioxide, the higher the temperature; the lower the amount of carbon dioxide, the lower the temperature.	
Temperature Rise versus Height of Future Sea Level The higher the temperature rise, the higher the height of future sea level; the lower the temperature rise, the lower the height of future sea level.	Dragging the temperature-rise slider to the left and right on the Sea Level Rise simulation, thus decreasing and increasing the value of temperature rise, and studying the impact of the change on the graphics of the simulation and value of height of future sea level and total land area.
Height of Future Sea Level versus Total Land Area The higher the height of future sea level, the lesser the total land area; the lower the height of future sea level, the more the total land area.	
TV hours versus Total Carbon dioxide The more the number of TV hours, the higher the amount of carbon dioxide released in the atmosphere; the less the number of TV hours, the lower the amount of carbon dioxide released in the atmosphere.	Changing the value of TV hours between 0 and 4 on the Carbon Calculator simulation and studying the impact of the change on amount of CO_2/year.
Carpooling versus Total Carbon dioxide The more the number of people carpooling, the lower the amount of carbon dioxide released in the atmosphere; the less the number of people carpooling, the higher the amount of carbon dioxide released in the atmosphere.	Changing the number of friends carpooling between 0 and 3 on the Carbon Calculator simulation and studying the impact of the change on amount of CO_2/year.

MA3: Coordinating the amount of change of one variable with changes in the other variable.	
Carbon dioxide versus air temperature Every time the carbon dioxide is increased or decreased by 100 units, air temperature is increased or decreased by 10 degrees or higher. The change in air temperature in one interval of carbon dioxide (say X) is more/less than the change in air temperature in another interval of carbon dioxide (say Y).	Using the Climate Change simulation to record the value of air temperature for given values of carbon dioxide, plotting the ordered pairs to graph the relationship between the two quantities, and studying the relationship in the graph.
Temperature Rise versus Height of Future Sea Level Every time temperature rises by 0.5 degrees Celsius, height of future sea level increases by 4 feet. The graph representing the relationship between temperature rise and height of future sea level is straight/linear because every time you increase by 0.5 degrees, the sea level rises 4 feet/ every time you increase by 0.5 degrees, the sea level rises by the same height.	Using the Sea Level Rise simulation to record the value of height of future sea level for different values of temperature rise, plotting the ordered pairs to graph the relationship between the two quantities, and studying the relationship in the graph.
Carbon dioxide versus air temperature and Temperature Rise versus Height of Future Sea Level Since air temperature changes randomly in every equal interval of carbon dioxide, the carbon dioxide versus air temperature graph is curvy. On the contrary, since the increase of the height of future sea level for every 0.5 increase of temperature-rise is constant, the graph is straight.	Compare the carbon dioxide versus air temperature graph with the temperature rise- height of future sea level graph. What do you notice?
TV hours versus Total Carbon dioxide Every time the TV hours increase by an hour, the amount of carbon dioxide increases by 82 units. The graph representing the relationship between TV hours and amount of carbon dioxide is straight because every time TV	Using the Carbon Calculator simulation to record the amount of CO_2/year for different TV hours, plotting the ordered pairs to graph the relationship between the two quantities, and studying the relationship in the graph.

hours are increased by 1 hour, amount of CO_2/year is increased by 82 units.	
Carpooling versus Total Carbon dioxide The graph representing the relationship between carpooling and amount of carbon dioxide decreases from left to right, because every time the number of people carpooling increases, the amount of carbon dioxide decreases. The graph representing the relationship between carpooling and amount of carbon dioxide is not straight/linear because every time the number of people carpooling is increased by 1, the amount of carbon dioxide is not increased by the same amount.	Using the Carbon Calculator simulation to record the amount of CO_2/year for different number of friends carpooling, plotting the ordered pairs to graph the relationship between the two quantities, and studying the relationship in the graph.
MA4: Coordinating the average rate-of-change of the function with uniform increments of change in the input variable.	
Carbon dioxide versus air temperature The air temperature between 0 and 100 units of carbon dioxide is growing more because the air temperature graph is steeper between these two points, that means higher increase of air temperature.	Using the Climate Change simulation to record the value of air temperature for given values of carbon dioxide, plotting the ordered pairs to graph the relationship between the two quantities, and studying the relationship.

As Table 8 indicates, the modeling activities provided students a space to explore the relationships between different sets of quantities such as carbon dioxide versus air temperature and temperature rise versus height of future sea level and conceptualize phenomena such as the greenhouse effect and sea level rise. More specifically, the three NetLogo simulations, accompanied non-graphing activities, and the interview questions during the small group interactions prompted students to engage in level 2 covariational reasoning, as per Carlson et al.'s mental action framework. Students explored the simulations, observed how the different quantities covaried with respect to each other, and coordinated the direction of change of one quantity with respect to the change in the other quantity. For example, in the first NetLogo simulation, Climate Change, students increased and decreased the amount of carbon dioxide by clicking the Add CO_2 and Remove CO_2 buttons and reasoned about the direction of change of air temperature with the direction of change of carbon dioxide amount, that is as the carbon dioxide increases, the air temperature increases and vice versa. Further, students dragged the albedo slider to the left and right and identified that as they decrease or increase the value of albedo, the surface of the earth changes and so does the reflection of the sun rays, thus impacting the air temperature. Through their interaction with the simulation, students modeled the dynamic event as a covariational relationship between albedo and air temperature. They stated that as albedo increases air temperature decreases and as albedo decreases air temperature increases, a type of reasoning that aligns with level 2 covariational reasoning as per Carlson et al.'s mental action framework of covariation.

Likewise, in the second NetLogo simulation, Sea Level Rise, students focused on and analyzed the factors responsible for increased height of sea level. The simulation allowed the students to increase and decrease the value of temperature rise, observe the impact of the change on the height of future sea level, and develop cognitive models of the covariational relationship between the two quantities. Students modified the values of temperature rise and identified that as the value of temperature rise increases, height of future sea level increases, a type of reasoning that aligns to MA2 as per Carlson Mental Action (2002). Further, through their identification of the direction of change of the two quantities, students built an explicit mathematical interpretation of the situation, which as Lesh (2006) identified is an important goal behind any modeling activity. Consistent with the principle of mathematical modeling, students' engagement with the Sea Level Rise simulation initiated with them exploring the impact of increased height of sea level on four places located at different elevations and gradually developed their mathematical understanding, thus enabling them to "to use mathematics as a generative resource in life beyond the classroom" (Doerr & English, 2003, p. 112).

Students also observed and modeled the relationship between the height of future sea level and total land area and recognized that as the height of future sea level increases, total land area decreases, a type of reasoning that aligns to MA2 as per Carlson Mental Action framework. As discussed earlier, mathematical modeling is a cognitive activity that helps students to conceptualize a real-world situation (Blum & Niss, 1991; Lesh, Amit, & Schorr, 1997). It further provides an opportunity to the learners to understand how a real-world phenomenon occurs, why it occurs, and extend the present understanding of the learners to anticipate similar events in the future (Dym, 2004). Likewise, the findings of the study suggest that the graphics of the simulation helped the students to conceptualize the impact of sea level rise on total land area as one student Ani said, "because the more higher the sea level is, it takes over land. So, instead of land over water, it will be under water." It seems that the graphics of blue rectangular patch overlapping the green rectangular patch helped Ani to identify that increased height of sea water would replace and hence decrease total land area. Further, the vertical arrangement of the four places according to their elevations from sea level also allowed the students to identify the impact of sea level rise on those places and reason about their associated risk of going under water.

The last NetLogo simulation, Carbon Calculator, was developed to encourage the students to examine the amount of carbon dioxide emissions that are directly and indirectly caused by their modern lifestyles. Students reflected on their daily practices such as watching television or using an air conditioner and calculated the total amount of carbon dioxide they add to the atmosphere annually. The activities helped students to develop cognitive models of the covariational relationships between different sets of quantities and identify the activities that might seem trivial but add significant amounts of carbon dioxide gas into the atmosphere, and activities that help resist the carbon dioxide emission. For example, students calculated the amount of carbon dioxide added in the atmosphere as a result of watching television and carpooling and identified the direction of change of one quantity with respect to the other, illustrating their second level (MA2) covariational reasoning as per Carlson et al.'s mental action framework.

Apart from the design of the simulations, this study also showed the power of the interview questions during the small group interactions for prompting students to reason covariationally according to the different levels of mental actions. In fact, the teachers' and interviewer's questions during the exploration of the simulation provided a wide window into students' MA2 reasoning. Some of those questions were, "What is the relationship between albedo and air temperature?" or "What happens to total land area when temperature is increased? Why?" Such questions acted as external cues to facilitate students' engagement in different forms of covariational reasoning.

As Table 8 illustrates, the graphing activities allowed students to move beyond MA2 and engaged them in higher levels of covariational reasoning, as per Carlson et al.'s mental action framework (MA3-MA4). Students plotted the ordered pairs of different quantities (for example, carbon dioxide and air temperature, temperature rise and height of future sea level), coordinated the amount of change of one quantity with respect to the change in the amount of the other quantity, and compared the rate of change of a quantity in consecutive intervals of the second quantity. For example, in the carbon dioxide and air temperature graph, students identified that the air temperature does not increase by any specific values for a constant increase in the value of carbon dioxide, therefore they described the graph representing the relationship between the two quantities as not linear. They focused on each interval of carbon dioxide (0-100, 100-200, and so on) and recognized that for some interval the growth of the air temperature graph is more compared to other intervals. In other words, the graphing activities provided students with a space to develop more sophisticated and meaningful models through exploration of the pattern of change of the two quantities. In the first simulation, after students coordinated the direction of change of albedo and air temperature and carbon dioxide and air temperature (MA2), they explored the simulation to record the values of air temperature for different values of carbon dioxide, plotted those ordered pairs, and mapped the relationship between the two covarying quantities. The graphing activity allowed students to focus on specific values of air temperature for values of carbon dioxide and coordinate the amount of change of one quantity with the amount of change of the other quantity (MA3). Students observed the change of air temperature in different intervals of carbon dioxide (100) and identified that the relationship between carbon dioxide and air temperature is different in different intervals of carbon dioxide. Further, students stated that as the value of carbon dioxide increases by 100, the value of air temperature also increases, but the amount of the increase becomes smaller.

While exploring the relationships of the second NetLogo simulation, students were also asked to graph the temperature rise and height of future sea level ordered pairs and examine how the height of future sea level changes for every 0.5 degree increase of temperature rise. Students identified that the temperature rise versus height of future sea level graph was linear and increasing because every time temperature rise increased by 0.5 degrees

Celsius, height of future sea level increased by 4 feet, a reasoning aligned to MA3 as per Carlson et al.'s mental action framework. Students were also further encouraged to compare the carbon dioxide versus air temperature and temperature rise versus height of future sea level graphs. They distinguished the two graphs based on the "super straight" and "curvy" shapes of the graphs and reasoned that since air temperature changed differently in every equal interval of carbon dioxide, the carbon dioxide versus air temperature graph is curvy. On the contrary, the constant increase of the height of future sea level for every 0.5 degree increase of temperature-rise produced a straight line. Such reasoning that aligns to MA3 as per Carlson Mental Action framework indicates that the graphing activity not only exposed students to the real life situation of the greenhouse effect and sea level rise, but also encouraged students to carefully observe the minute details of the situation and develop a better understanding about the situation (Zbiek & Conner, 2006).

Students also engaged in graphing activities during the third NetLogo simulation, the Carbon Calculator. They modeled the relationship between the two sets of quantities graphically and analyzed how and why the TV hour and carbon dioxide ordered pairs led to a linear increasing graph, while the carpooling and carbon dioxide ordered pairs generated a non-linear decreasing graph. To do the analysis, students focused on the change in carbon dioxide amount for each interval of TV hours and carpooling and identified that in the former case the change in carbon dioxide amount is uniform for each TV hours interval but does not maintain any pattern for carpooling. Such reasoning indicates students' engagement in MA3 as per Carlson et al.'s mental action framework.

Though the three NetLogo simulations were designed with the goal to engage students in different levels of covariational reasoning through MA1 and MA4, during the five days of the design experiment, on limited occasions students reasoned about the rate of change of the quantities, thus illustrating their level 4 covariational reasoning. Most of the students, as seen in the Findings chapter, reached MA3 as the highest level of their covariational reasoning. The only exception was Celine. Celine engaged with the graphing activity and identified that the relationship between carbon dioxide and air temperature is different for different intervals of carbon dioxide. When I asked her "just looking at the graph can we find out which one is growing more, and which one is growing less?" Celine focused on the carbon dioxide versus air temperature graph and examined the steepness (slope) of the line segments drawn in each interval of carbon dioxide. Though Celine did not explicitly used the term 'rate' to explain her answer, the graphing activity enabled Celine to focus on the steepness of the line segments in each interval of carbon dioxide. This illustrates her attention towards slope of line segment, thus arguably establishing a conception of level 4 covariational reasoning (MA4).

5. Discussion and Conclusion

The purpose of this study was to examine the role of mathematical modeling activities in engaging students in covariational reasoning and helping them identify the causes and the consequences of the greenhouse effect. Consequently, three dynamic mathematical modeling activities embedded in the context of the greenhouse effect were developed and implemented in two middle school classrooms to examine the following research questions:

1. To what extent do the students' understanding of the greenhouse effect and their use of covariational reasoning change as a result of their engagement with the mathematical modeling activities?
2. How may students reason covariationally as they engage with mathematical modeling activities in the context of the greenhouse effect?

This study collected pre- and post-assessment data from two treatment groups and one control group and qualitative data in the form of video recordings of the whole class and small group discussions from the two treatment groups. By examining middle school students' covariational reasoning through mathematical modeling activities embedded in the context of the greenhouse effect, my dissertation makes three fundamental contributions to mathematics education research. First, it provides empirical evidence regarding the development of dynamic mathematical modeling activities that provided students with an exploratory space to engage in reasoning between different covarying quantities. Secondly, this study illustrates how research in mathematics and science education can be integrated to develop integrated STEM activities that provide learners with an opportunity to engage in learning of mathematical and scientific concepts in a more meaningful way. These STEM activities can help students understand the complex yet pressing issues such as the greenhouse effect and sea level rise. Lastly, this study shows the potential of bridging students' mathematical and scientific understanding through covariational reasoning.

In this chapter, first I discuss the task design principles emerged from the study that helped students engage in different forms of covariational reasoning within the context of the greenhouse effect. Subsequently, I discuss the insights that this study provided for developing integrating curriculum and the role of covariational reasoning for integrating the math and science disciplines. Then I reflect on the findings from a critical lens and discuss instances where students showed evidence of developing critical consciousness about the greenhouse effect as a social issue. I end this chapter by discussing the limitations of this study as well as implications that this study has for research and practice.

5.1. Design Principles for Engaging Students in Covariational Reasoning in the context of the Greenhouse Effect

Thompson, Carlson, and Silverman (2007) stated that "educational tasks are designed for a purpose and with an intended effect" (p. 416). Indeed, in this study the three mathematical modeling activities were designed with two main purposes: firstly, to provide students with a space to engage in covariational reasoning, and secondly, to identify the different causes and consequences of the greenhouse effect. The modeling activities contained three dynamic NetLogo simulations, five investigations containing questions to assist students to focus their attention to particular features of the simulations and reason about dynamic events, and discussion questions that prompted students to engage in particular forms of covariational reasoning. The findings of the study suggest three design principles of the modeling activities that can make a contribution to the field of mathematics education: a) the power of dynamic simulations in engaging students in covariational reasoning, b) using meaningful contexts to design the dynamic simulations, and c) the power of questioning. In the following paragraphs, I will reflect on each of the design principles and discuss how they provided students with the opportunity to engage in covariational reasoning and helped them examine the causes and consequences of the greenhouse effect.

5.1.1 The Power of Dynamic Simulations in Engaging Students in Covariational Reasoning

Prior research on students' covariational reasoning showed the power of technology for helping students envision the change in the quantities as well as to reverse change, which is not always practical with physical manipulations (Castillo-Garsow, Johnson, & Moore, 2013). In this study, the dynamic NetLogo simulations allowed the students to engage in an exploration of covariational relationships between different sets of quantities. As Jonassen, Carr, and Yueh (1998) stated, technology provides students with a discovery space to explore various real-life phenomena, experiment with them, and engage in critical thinking about the content and the underlying context. Further, it allows students to engage in an interactive interface where students can explore the relationships between different quantities based on their increasing or decreasing values (Bos, 2009). Consistent with these statements, in this study, the design of the three NetLogo simulations contained several features, that provided the participants with an exploratory space to reason both non-numerically and numerically about covarying quantities and enabled them to identify the mathematical and scientific aspects of the phenomenon in a more meaningful context. For example, in the first simulation, when the students were asked to identify the relationship between carbon dioxide and air temperature, students increased and decreased the value of carbon dioxide by clicking the Add CO_2 and Remove CO_2 buttons, observed the impact of the change on air temperature, and identified the directional relationship between the two quantities.

Further, the graphics of the simulation helped the students to understand the scientific aspect of the phenomenon in more depth. Students observed that, as the value of carbon dioxide was increased, the green dots representing carbon dioxide and the red dots, representing infra-red rays also increased. When the red infra-red rays tried to escape through the atmosphere, the green carbon dioxide molecules reflected them back, thus trapping the heat and increasing the air temperature. To identify the relationship between albedo and air temperature, students dragged the albedo slider to the left and right and identified that as they decrease or increase the value of albedo, the surface of the earth changes and so does the reflection of the sun rays, thus impacting the air temperature. They stated that as albedo increases air temperature decreases and as albedo decreases air temperature increases, a type of reasoning that aligns with level 2 covariational reasoning as per Carlson et al.'s mental action framework of covariation.

During students' engagement with the Sea Level Rise simulation, they observed and modeled the relationship between temperature rise, the height of future sea level, and total land area. Students recognized that as temperature rise increases, the height of future sea level increases, and total land area decreases, a type of reasoning that aligns to MA2 as per Carlson Mental Action framework. Further, the graphics of the simulation helped the students to conceptualize the impact of sea level rise on the total land area as one student Ani said, "because the more higher the sea level is, it takes over land. So, instead of land over water, it will be under water." The graphics of the blue rectangular patch (representing water) overlapping the green rectangular patch (representing land) provided an opportunity to Ani to understand why the real-world phenomenon of sea level rise occurs and how it displaces people living in places at lower elevations. Further, the vertical arrangement of the four places according to their elevations from sea level also allowed the students to identify the impact of sea level rise on those places and reason about their associated risk of going under water.

The last simulation, the Carbon Calculator helped students to develop cognitive models of the covariational relationships between different daily life practices and the total amount of carbon dioxide emitted in the atmosphere. The simulations provided students with a dynamic exploratory space for students to explore and discover different practices that might seem trivial but add significant amounts of carbon dioxide gas into the atmosphere. For example, students calculated the amount of carbon dioxide added to the atmosphere as a result of watching television, taking showers, and carpooling and identified the direction of change of one quantity with respect to the other, illustrating their second level (MA2) covariational reasoning as per Carlson et al.'s mental action framework. Further, students were surprised to identify the amount of carbon dioxide they add in the atmosphere through simple acts of taking showers or brushing the teeth.

5.1.2 Using meaningful Contexts to Design the Dynamic Simulations

To provide students with a meaningful mathematical experience, Gutstein (2005) said that mathematics should originate from the students' real lives. Likewise, Thompson, Carlson, and Silverman (2007) suggested that "one must design tasks with the learner in mind" (p. 417). With that goal in mind, at the onset of the module development, I searched for local issues relevant to the students which I used to introduce the lesson and then transition to the exploration of the first NetLogo simulation. The video of Asthma in Newark appealed to the students' consciences and triggered them to ask some questions around the causes behind the disparate environmental conditions in the two neighboring places.

Apart from the introductory video, the three simulations and the accompanying activities were designed with the intention to engage students with situations that were meaningful to them. For example, in the Sea Level Rise simulation, the inclusion of the names of four places familiar to the students helped them understand the significance of the sea level rise in connection to their own lives. Students were relieved to find themselves located at a higher elevation. Also, the students identified that if sea level rises, then that will "cause places like … low elevation like Newark will go under water (Gio, MC2)." Further, observing downtown Manhattan at a lower elevation, Gio said, "Oh that is why you can see New York through your window. You can look down on it. Oh, makes so much more sense." It seemed to me that the simulation allowed Gio to bridge their prior experience with the formal learning of the greenhouse effect and created a platform for him to understand the significance of sea level rise.

5.1.3 The Power of Questioning

Though the three NetLogo simulations provided students with a discovery space to explore the relationships between different covarying quantities, the simulations alone could not invoke students' covariation conception. According to Boaler and Brodie (2004), questioning in a mathematics classroom can be both critical and challenging. Good questioning, as the authors suggested, has a positive influence on students' classroom engagement, critical thinking, and has the potential to shape students' cognition. Consistent with Boaler and Brodie (2004), this study also identified the significance of efficient questioning and witnessed its potential through the two cycles of the design experiment. For instance, during the ongoing analysis of the first cycle of the design experiment, I found that most of the questions that I asked the students were leading. Some examples include "what would happen if I increase the amount of carbon dioxide?" or "how would the value of carbon dioxide change if the number of TV hours is increased by an hour?" These leading questions left students with limited opportunities to express the relationships between the two covarying quantities in their own words. Instead, they were responding to the questions using one or two words only and as a result it was not evident whether they could reason covariationally. Consequently, during the second cycle of the design experiment, I modified my questioning to allow for a range of student responses, such as asking "What are some relationships you found?" "What is the relationship between air temperature and carbon dioxide?" and provide students with an opportunity to express the covariational relationships in their own words.

Additionally, I found that it was the careful questioning that prompted students to engage in different levels of covariational reasoning. For instance, to prompt students reason about MA3 and the amount of change of the covarying quantities I asked them questions such as "What would happen to air temperature when carbon dioxide changes from 100 to 200 units? " and "Why is the temperature rise versus height of future sea level graph linear?" Likewise, to prompt them to notice the rate of change between the quantities (MA4) I asked questions such as "How does the relationship between carbon dioxide and air temperature change in each interval of carbon dioxide?"

5.2. Integrated STEM activities

Wang, Moore, Roehrig, and Park (2011) stated that to provide students with a more meaningful STEM experience and retain them in STEM fields, STEM education should follow an interdisciplinary approach and "cut across subject areas and focus on interdisciplinary content and skills, rather than subject-based content and skill" (p. 3). With that goal in mind, this study provides some important insights for the design of integrated curriculum through a seamless of the concepts of the greenhouse effect and covariational reasoning. The study showed that these integrated activities could be implemented in both mathematics and science classrooms to help students to study both the concepts together, which addresses recent calls to focus on integrated STEM education (Baldwin, 2009; McCright, O'Shea, Sweeder, Urquhart, & Zeleke, 2013). Because the topics of the greenhouse effect and covariational reasoning were consistent with both science and mathematics education standards, teachers of both math and science found the module relevant to their curriculum. Specifically, in the Next Generation Science Standards (NGSS), the middle school standard MS-ESS3-5 focuses on the impact of human activities causes a rise in global temperature, thus changing the climate. In the Common Core State Standard for Mathematics (CCSSM) for middle school, though covariational reasoning does not explicitly belong to any specific content or practice standards, it aligns with multiple standards of the Expressions and Equations strand, such as CCSS-M 6.EE.C.9 (using variables to represent two quantities in a real-world problem and focusing on the analysis of the relationship between the depended and independent variables using graphs and tables), CCSS-M 8.EE.B.5 (graphing proportional relationships and comparing two different proportional relationships represented in different ways). Also, it aligns with standards in the Ratio and Proportion strand, such as CCSS-M 7.RP.A.2 (recognizing and representing direct proportional relationships and deciding whether two quantities are in a proportional relationship by graphing), and the Functions strand such as CCSS-M 8.F.B.5 (describing qualitatively functional relationships between two quantities in graphs). In addition to the content standards, the activities in this study address the MP4 practice standard (Model with Mathematics).

The activities were designed bridging the science and mathematics standards, which in turn fostered within students a sense of interconnectedness between the mathematical and scientific concepts of the greenhouse effect (Zbiek & Conner, 2006). As the findings suggest, the three NetLogo simulations helped students to develop both the mathematics and scientific reasoning as one unified construct. In their generalizations, such as that as carbon dioxide increases, air temperature increases, students did not distinguish between the "math reasoning" and "science reasoning" of the greenhouse effect, rather their integrated reasoning showed that they developed "interdisciplinary content and skills" (Wang et al., 2011), avoiding disconnected disciplinary learning that other studies have previously noted (Barnes, 2000; Honey et al., 2014; Tytler et al., 2019).

5.3. The power of covariational reasoning for integrating science and math

The third fundamental contribution of this research in the field of mathematics education is exploring the potential of covariational reasoning to bridge the mathematical and scientific concepts of covariational reasoning and the greenhouse effect. The review of pertinent literature illustrates that there is a scarcity of empirical research in mathematics education that focuses on the application of covariational reasoning to integrate mathematics and science conceptions. Most of the studies explored the role of covariational reasoning in developing students' mathematical reasoning. However, this study focused on students' covariational reasoning as they engaged with the mathematical modeling activities and investigated how covariational reasoning contributed towards students' understanding of the greenhouse effect. The effectiveness of the module, as evidenced by the findings of this study, was likely due to the integration of a strong covariational reasoning component into the module design of the greenhouse effect. Covariational reasoning, as illustrated by the students' excerpts, bridged the mathematical and scientific aspects of the greenhouse effect, and helped students develop an integrated understanding of the phenomenon. Possibly, that is why the treatment group students, who engaged with the greenhouse effect modules and engaged in covariational reasoning, showed significant improvement in post-assessment scores compared to the control group of students, who explored the same phenomena without being engaged with the module or covariational reasoning.

As already has been discussed, covariational reasoning does not explicitly belong to any specific Common Core State content or practice recommendations from the CCSS-M. However, in middle school, covariational reasoning aligns with multiple middle-school and high-school standards of CCSS-M. Additionally, research has also shown that reasoning covariationally can be the basis on which functional thinking can be developed and built in the later years of schooling (Confrey & Smith, 1994). As a result, this study suggests the development of future research on designing similar covariational situations that would seamlessly connect the scientific aspect of different phenomena, such as gravity and water cycle, with advanced mathematical concepts and contribute towards developing students' reasoning about both science and mathematics. Instead of directly giving students scientific formula such as Newton's law of gravity or teaching them factors responsible for evaporation or precipitation, if students are encouraged to think about the conditions under which the force of gravity increases or the rate of evaporation decreases, then that would not only help them learn about the factors that impact the phenomenon but would provide them with an opportunity to engage in covariational reasoning and seamlessly learn about the direct and inverse proportional relationships between these quantities.

5.4. Students' Critical Consciousness about the Greenhouse Effect

This study was motivated by the observation of an insufficient number of studies addressing the inclusion of climate justice issues in the school mathematics curriculum. An increasing body of literature is available on exploring different human-induced factors that are responsible for the rapidly changing climate and discussing the potential threats of climate change on the natural environmental condition of the earth and human vulnerabilities. However, there is a distinct lack of attention in the field of mathematics education with respect to studying how mathematics can be used as a potential tool to make students aware of the different climatic issues and injustice associated with them. According to Karrow, Khan, and Fleener (2017), "of the many environmental challenges that we face, climate change currently epitomizes great ecological complexity and ethical urgency" (p. 2). They suggested that mathematicians and mathematics educators should work towards addressing climate change "by modeling types of thinking necessary to make sense of the phenomenon itself" (Karrow, Khan, & Fleener 2017, p. 19). Moore, Paoletti, and Musgrave (2013) argued that covariational reasoning is essential for understanding graphs. Therefore, in this study, students' cognitive models of the covariational relationships between different quantities of the greenhouse effect phenomenon can be foundational for understanding graphs related to climate change.

Apart from examining students' covariational reasoning and their understanding of the greenhouse effect, this study also intended to encourage students to develop their consciousness about the social aspect of the greenhouse effect. More specifically, it aspired to help students identify the different human-induced factors that contribute towards the greenhouse effect and motivated them to recognize how the economic disparity might make some people more vulnerable towards the impact of changing climate over others. In the following paragraphs I describe how this study tried to develop students' consciousness and present some students' articulations that might contain seeds for this development.

On the first day of the design experiment, I showed the students a news report on asthma in Newark. I anticipated that the video would engage the students in discussion within an environmental context familiar to them and help me build the lesson on their thinking. The news report stated that one out of every four kids in Newark has asthma, which is three times the state average. They identified pollution from trucks, industries, and chemical sites also as fumes from airplanes as some of the major reasons behind the poor environmental condition in Newark. Further, the news report referred to a study conducted by the Environmental Protection Agency (EPA) and stated that there exists a strong correlation between areas in Newark with high pollution and low economic condition. I predicted that the video would allow the students to see how people living so close to their town are victims of asthma and think critically about why the environmental and health condition of Newark residents are so vulnerable despite the proximity of the city to the students' towns.

After showing the video, the teachers initiated a whole class discussion. They asked the students a set of pre-decided questions, such as, "What is the video about?", "Why do you think so many children in Newark suffer from asthma?", and "Since Kearny is so close to Newark, do you think that you should be worried about your health too? Why/Why not?" These questions were designed with the intention to encourage students to critically reflect on the factors that might have resulted in such unfavorable living conditions in Newark and discuss if, as a neighboring town to Newark, the students are also exposed to any potential environmental or health threat. Though the questions engaged students in a brief discussion around the differential environmental conditions in Newark and Kearny, there is a lack of evidence that can ensure the development of students' critical consciousness in terms of critical reflection, socio-political efficacy, and critical action. That is, the discussion students had after watching the video does not ensure if they developed the ability to analyze the social realities of climate change critically and identified how the economic condition might limit access to healthy environmental condition to some people. Further, there is not enough evidence to affirm if students recognized their perceived and active capability to work towards rewriting the world, which offers everyone equal access to resources. I explain this lack of evidence by describing the conversations below.

After students watched the asthma video, both Doug and Chelsea conducted a whole class discussion asking students to express their thoughts on the video. When Doug asked his students, "what is the video about?" Ani responded, "we are treating the earth like it is nothing." Adding to Ani's comment, another student Paula said, "People are kind of careless where they put the factories, where most people live." While Ani's voice expressed his displeasure towards lack of human awareness about their exploitation of the earth and its resources, Paula was more anxious about the location of factories in highly populated regions and its impact on human lives.

Paula's comment stirred a discussion in the classroom. Students discussed the correlation between the economic condition of a neighborhood and the number of factories that have been established there. When Doug asked the students "Why do you think there is a lot of factories or industries in that region?", Ani replied, "they do not have that much money." Adding to Ani's response, Paula said, "You don't find them in the rich people's places. You find them in poor people's places." Both Ani and Paula attributed lack of monetary affluence as a significant reason behind the establishment of factories in impoverished areas such as Newark. These articulations indeed indicate that students recognized how the economic condition might limit access to healthy environmental condition to some people over others. However, there is no clear evidence which can ensure if Ani or Paula could reflect critically to investigate why economic conditions create such differential treatment and who is responsible for such unjust condition.

When the Asthma in Newark video was shown in Chelsea's classroom, and students were asked about their thoughts on the video, Gio said, "it is very different to see actually someone so close to you being affected." Gio sounded both upset and sympathetic towards his peers in Newark, who in spite of being so close to his town, are suffering from higher rates of asthma. The above excerpts illustrate that while Doug's students focused on the role of people's economic status on their environmental and health conditions, Chelsea's students were more compassionate towards the victims who live so close to their town yet experience such different environmental conditions. However, in both the classrooms, the discussion never led the students to dig deeper into the issue and critically reflect on the possible factors that might result in such health and environmental conditions in Newark.

Following the whole class discussion, students explored the Climate Change simulation. Students identified that the increased amount of carbon dioxide enhances air temperature. When I asked them if we share any responsibility towards the increased concentration of carbon dioxide in the atmosphere, Ani (MC1) answered affirmatively. Agreeing with Ani, Gina (MC1) also identified cars and pollutions to be some factors contributing to the issue.

Interviewer: Do you think we also add carbon dioxide in the atmosphere? Are we responsible?

Ani: Yes.

Interviewer: How?

Gina: Cars.

Interviewer: Cars. Do you agree? [Turning to Ani]

Ani: Yes.

Gina: Pollution.

Interviewer: Pollution. […] Am I doing anything , which is also adding carbon dioxide?

Ani: Driving cars as she said. Using technology, like some time they give out carbon dioxide. Some factors mean technology, which gives more carbon dioxide. Since the world is evolving with technology, so then more technology would mean factories have to work more spreading out more carbon dioxide.

Ani identified, "since the world is evolving with technology" there would be more establishment of factories, which in turn would increase the concentration of carbon dioxide gas in the atmosphere. Both Ani and Gina emphasized the contribution of human activities and modern lifestyle on the climatic condition of the earth.

Later, during students' engagement with the Carbon Calculator simulation, they identified that many of their daily practices emit a profound amount of carbon dioxide in the atmosphere annually. Some of the daily practices that drew the students' attention and stimulated a discussion in the classroom were using AC, taking showers, and keeping chargers and other electronic devices plugged even after use. Simi (MC2) said, "I did not realize how much CO_2 is produced by taking a shower every day." Adding to Simi's comment, Gio (MC2) uttered, "It surprised me like everyday things, I am guessing even like brushing your teeth could be releasing CO_2 like doing everyday things you need to do." He further said, "I did not think even if the TV is plugged in or the video game or X-box is plugged in it still releases CO_2, I didn't think." Paula expressed a similar concern about carbon dioxide release without the consumers' knowledge. When Paula realized that if a TV is kept in standby mode, it still releases carbon dioxide, she said, "I never knew that that (TV) can use electricity as we go, no one told me that thing. No one told me." The Carbon Calculator activity was not only an eye-opener for the students, but it also encouraged Gio, Simi, and Paula to think about their daily practices, which with or without their knowledge releases carbon dioxide in the atmosphere.

According to Padgett, Steinemann, Clarke, and Vandenbergh (2008), to restrict carbon dioxide emission to preserve the natural consistency of the climate, it is essential for an individual to be aware of the different sources of carbon dioxide emission. If individuals identify various sources of carbon dioxide and can estimate their contributions to the issue, then that would lead them to change their own behavior and work towards mitigation of the problem. So, there are two steps of mitigating carbon dioxide emission: a) identification of the sources of carbon dioxide emission, and b) taking action to reduce carbon dioxide emission. In the first half of their exploration with the Carbon Calculator simulation, students identified their daily life practices that emit carbon dioxide in the atmosphere. I consider students' identification of the different CO_2 sources as a pre-cursor to their sense of agency, which enables them to perceive themselves as an individual capable of mitigating carbon dioxide emission by adjusting their daily practices. Hence, to persuade students to take the next step to reduce carbon dioxide emission, I then encouraged them to propose strategies to lower atmospheric carbon dioxide concentration. Students suggested reducing TV hours, turning TVs off after watching, lessening the number of daily showers, and modification of modes of transportation to decrease carbon dioxide concentration. For example, Ani suggested that all students need to "talk to their parents not to use cars so much." As a substitute for using cars, he recommended: "stop driving, more walking, using bicycles." A similar proposal was also made by Gio who suggested using "public transport" to reduce the concentration of carbon dioxide in the atmosphere. While Ani and Gio focused on the other alternatives to driving cars, Celine suggested "carpooling" as a potential solution towards the problem. The mutual concern that emerged through the conversation of Ani, Celine, and Gio was the excessive usage of cars. It was also interesting to see that one of the students mentioned the need to have a conversation with their parents to make other family members aware of the environmental crisis. Consequently, I interpret that the carbon calculator simulation allowed students to see themselves as individuals capable of bringing changes in the earth's environment through some simple changes in their daily practices. Although the simulation or the classroom discussions provided limited opportunities for the students to critically reflect on how their actions would impact the environment and health conditions of the society, it gave the students an outset to work towards developing their critical consciousness about climatic issues.

The Sea Level Rise simulation allowed the students to recognize how the elevation of a place determines its chance of going underwater, thus displacing the people living there. As Gio (MC2) said, "the higher the future sea level, which is gonna cause places like … low elevation like Newark will go under water." To persuade the students to think about the displaced people, Chelsea asked her students, "And now your house is underwater. What would you do?" Students suggested repositioning displaced people to other places at higher altitudes such as Kearny, but then the discussion shifted around the feasibility of the solution. Some of the anxieties that emerged among the students included big family size, space limitation, the inadequacy of transportation, and lack of money to meet the demands of the rising population. The following is an excerpt of these conversations:

Teacher: And now your house is underwater. What would you do?

Gio: You move to Kearny.

Celine: Well you can't move […] you can't really move if everything is flooded and Kearny is like a mile away. But that is not how it works. No.

Teacher: What are some things that would affect how it works?

Simi: If you have big family.

Teacher: If you have a big family. Okay.

Myra: Transportation.

Teacher: Transportation. […]

Student: Finding another place to stay.

Teacher: Finding another place to stay.

Celine: Also, if this entire thing is going to flooded, everyone is going to need a place. There might not be enough for everyone.

Teacher: Right. How about, would you have enough money to move?

Celine: Probably not since everything got destroyed.

Similar concerns around financial limitations also surfaced in Doug's classroom when he asked his students to reflect on the effects of sea level rise. Interestingly, students not only recognized that the increased sea level would cause flooding, thus displacing most of the people residing in lower elevations, but also expressed concerns regarding the shelter of the displaced people. I sensed an anxiousness among some of the students concerning the lack of economic affluence to endure the impact of displacement caused by flooding. The following excerpt illustrates an example of one such conversation that Doug had with his student Ani.

Teacher: What is gonna happen to our home?

Ani: It is gonna be destroyed, and we cannot rebuild it.

Teacher: What is gonna happen to their homes?

Ani: Destroyed, but they can rebuild it.

Elen: They have the chance to rebuild it.

Both Ani (MC1) and Elen (MC1) resonated the argument of Corvalan et al., (2005) and Agyeman, Bullard, and Evans (2002) that environmental threats, such as sea level rise and flooding, are issues of social justice since they bear down a disproportionate impact on the people belonging to different socioeconomic strata. Their articulations of "they can rebuild it" and "they have the chance to rebuild it" and "It (home) is gonna be destroyed, and we cannot rebuild it" indicate that students recognized how low socioeconomic conditions of certain people limit their *access to resources and opportunities* (Godfrey & Grayman, 2014) to fight the impact of climatic disruption. However, the conversation does not ensure that students critically reflected on the reasons why some people are better able to cope with climatic disruption, or the responsibilities they share towards ensuring a healthy and sustainable environment for all.

Overall, students' articulations show that to some extent they identified that people belonging to lower economic strata experience disparate environmental and climatic conditions compared to the affluent people. However, this study has inadequate evidence to claim that students developed a sense of critical consciousness following their engagement with the greenhouse effect module. Further, I did not follow up with the students to examine if there has been any change in their own daily practices that could ensure students socio-political efficacy and critical action. Consistent with Gutstein's (2003) definition of agency, I acknowledge that the development of agency within an individual is a developmental process. Students identifying themselves as agents of change, that is identifying themselves as capable of making changes (socio-political efficacy) and taking actions to actually make the changes (critical action), is an emergent process and therefore requires time and more work for development. Though this study aimed towards developing students' critical consciousness through critical reflection, socio-political efficacy, and critical action, it also identified that the design of the activities and the teacher/researcher questioning was limited to examine the development of students' reasoning of critical consciousness. Nevertheless, this study showed that the dynamic activities provided students with an opportunity to engage in critical thinking around the phenomenon of the greenhouse effect, and I hope that the limited evidence I have about students' emerging critical consciousness can act as seeds for developing critical reflection, socio-political efficacy, and critical action in the future through careful design and questioning.

5.5. Limitations

In this study, I discussed the potential of dynamic mathematical modeling activities in engaging students to reason covariationally and helping them develop an understanding and critical consciousness towards relevant environmental and social issues such as the greenhouse effect. A fine-grained analysis of students' interaction with the dynamic activities indicates that the NetLogo simulations provided space for students to explore the relationships between different covarying quantities and thus allowed them to identify the different causes and consequences of the greenhouse effect. Though I anticipate that this study would contribute significantly towards similar research connecting mathematics and climate, I recognize there were limitations to this study, many of which provide avenues for future research.

First, the findings of the study would have been more rigorous and generalizable if I had a larger number of students participating in the study. The purpose of this study was to examine the effectiveness of the dynamic mathematical modeling activities on students' covariational reasoning as well as their understanding of the greenhouse effect. Such integrated activities, if tested in more classrooms and proved effective, can be used by both science and mathematics teachers in their classrooms to provide students with a holistic meaningful learning experience. As it is now, for quantitative data I focused on the pre- and post-assessment scores of forty-four treatment group students and thirty-one control group students, and for qualitative data, I focused on the small group interaction of six students from the first macrocycle and five students from the second macrocycle. With such a small sample size, it would be difficult to claim the generalizability of the findings. However, I am hopeful that if these integrated activities are implemented in a higher number of mathematics and science classrooms, then that might produce some significant and generalizable findings.

The second limitation of the study is the inadequate time allotted for the small group conversations. As I discussed in the Methods section, the two teachers Doug and Chelsea assumed the responsibility of the whole class instruction and I sat with a small group of students, identified by the teachers, and facilitated discussions around the three simulations. The video recordings of the small group conversations contributed towards the qualitative data of this study, which provided me with a window to students' covariational reasoning and their understanding of the greenhouse effect. However, on several occasions, the small group interaction was interrupted by the whole class discussion, and the students' chain of thoughts were disturbed, leaving me to wonder about the data I missed. Additionally, I only interacted with the students in person during the time when they were asked to self-explore the simulations and this was the time when all other students were also working independently. As a result, the entire classroom was loud and did not offer the best ambience to capture students' mental models as they engaged with the mathematics modeling activities. A probable way to address this limitation would be to conduct one-to-one design experiments or design experiments with pairs of students, where I would receive the opportunity to interact with a small number of students and would be able to "create a small-scale version of a learning ecology so that it can be studied in depth and detail (Cobb et al., 2003, p. 9).

The third limitation of the study was the implementation of the greenhouse module in two different classrooms: science and mathematics. In fact, I would treat such implementation as both a strength and a weakness. Since the module was first implemented in a middle school science classroom, followed by implementing it in a middle school mathematics classroom, the enactment ensures the integrated nature of the activities included in the greenhouse module. As a result, such activities could be considered as empirically tested integrated activities and could be used as samples for designing similar tasks. Though the activities are effective, the limitation is the difference in the module implementation by the two subject teachers. For example, in the first macrocycle, Doug spent a lot of time introducing the topic of the greenhouse effect through the Asthma in Newark video. He encouraged his students to think about the causes of the greenhouse effect and how the issue might impact different people from different economic sectors disproportionately. Unlike Doug, Chelsea kept her introduction brief and spent more time discussing the graphs representing the covariational relationships between different quantities. Though the two teachers took different approaches to implement the greenhouse effect module in their science and mathematics classroom, in this study I did not focus on the difference in module implementation or examined the impact of the dynamic mathematical modeling activities on students' covariational reasoning and their understanding of the greenhouse effect. In future research, it would be interesting to explore the nature of module implementation by different subject teachers and its effect on students' forms of reasoning.

The fourth limitation of the study is the restrictive nature of the questioning that was used by the teachers and me to prompt students' reasoning. One common issue that I noticed in both the classrooms was that the teachers were too leading in facilitating the whole class discussions. In several instructional episodes, Doug and Chelsea would monopolize the conversation and provide limited opportunities for students to talk. Also, other instances students answered to teachers' questions in a word or two, which made me uncertain about their covariational reasoning ability or their own understanding of the greenhouse effect. For example, in the following excerpt it is seen that instead of providing students with enough time to reason covariationally between carbon dioxide and air temperature, the teacher decides to ask leading questions and students just "fill in" the statements made by the teacher:

Teacher: What trend are you finding between CO_2 and temperature? […] Which one is increasing?

Student 1: The temperature.

Teacher: The temperature is increasing as,

Student 1: The carbon dioxide.

Teacher: Carbon dioxide is increasing yes. Is it increasing a lot, or a little bit?

Student: 10 degrees Celsius.

Teacher: 10 degrees Celsius, it is quite significant.

A fifth limitation of the study that I realized during the retrospective analysis was my own questioning style. At several occasions, I felt that my questions were too leading and left students with limited opportunity to engage in in-depth thinking about the relationships between the different quantities. For example, instead of asking students to articulate the relationship between two covarying quantities, say albedo and air temperature, I asked: "What do you think [would happen], if I increase the value of albedo?" I recognize that this type of questions provided students with leading cues to increase the value of albedo and observe its impact on the air temperature. Instead, if students were encouraged to explore the simulation without such external cues but more open questions such as "What relationships do you see?" or "What is the relationship between albedo and air temperature?", then that might have helped me better understand students' covariational reasoning as this emerged.

5.6. Implications to Research and Practice

This exploratory study provided empirical evidence of how dynamic mathematical modeling activities can be designed to engage students in studying the greenhouse effect through covariational reasoning. In this section, I discuss some of the implications for research and practice that arise from this study.

Barwell (2013) argued that mathematics literacy is needed to interpret data and graphs on climate as available in the news and public media. Consistent with this argument, in this study I focused on students' covariational reasoning as a fundamental concept for reading data and graphs about climate. However, other mathematical topics that can be seamlessly integrated with the phenomena of the greenhouse effect or sea level rise. Barwell (2013) suggested several school mathematics topics, such as mathematical modeling, differential equations, non-linear systems, and stochastic processes that are intimately connected to several climatic issues and can be utilized to help students understand the current climatic condition of the earth and predict future global and regional climate. Future studies could build on the findings of this study and focus on other mathematical topics such as ratio-proportion and percentages to explore questions such as "How has the percentage or proportion of different greenhouse gases changed over the past few decades?" or "If 3 trillion tons of ice melted in the Atlantic region in last 25 years, then under the assumption that the rate of melting stays constant, how many years would it take to melt the current 27 million billion tons of ice?"

Additionally, as already discussed, though the rapidly changing climate is a potential threat to humanity, it bears down its effect disproportionately upon the impoverished and minority people (Agyeman, Bullard, & Evans, 2002; Costello et al., 2009). People belonging to lower economic strata often experience a higher chance of being affected by environmental hazards compared to the people belonging to the richer section of society. Following the path laid by Frankenstein and Gutstein, it is essential that we mathematics educators work towards educating the future generation about issues related to climate justice. In this study, we saw that the introductory video and the three NetLogo simulations initiated some discussions that could be fruitful in developing students' critical consciousness towards the climate. Although the design of this study lacked to explore these ideas in depth, it illustrated the potential of mathematics for developing students' critical consciousness about the climate change in the future. In a similar way, I believe that mathematics can be utilized to make students aware of other environmental and climate justice issues such as inappropriate waste disposal system or access to clean drinking water. As Cirillo, Bartell, and Wager (2016) suggested, instead of treating mathematics traditionally and posing nonsensical problems to students, mathematics educators and teachers should choose relevant topics that will make students familiar with socially pressing issues. Consistent with their argument, I propose re-developing mathematics tasks on environmental and climate justice issues, which I anticipate would not only provide students with a meaningful math and science learning experience but would also encourage them to question these social and environmental disparities.

Furthermore, as discussed in the Literature Review, research shows that there are many misconceptions among students about different climatic issues such as the greenhouse effect, global warming, and climate change (Bostrom, Morgan, Fischhoff, & Read, 1994; Shepardson, Niyogi, Choi, & Charusombat, 2011). For instance, students often consider ozone layer depletion as a consequence of climate change (Shepardson, Niyogi, Choi, & Charusombat, 2011), which in turn allows increased ultraviolet rays to enter the atmosphere and results in global warming (Bostrom, Morgan, Fischhoff, & Read, 1994). Another common misconception about climate prevalent amongst students is that climate change occurs as a result of seasonal variation, or due to proximity of the earth to the sun; alarmingly many students do not even see climate change as an immediate or future threat to society or humans (Shepardson, Niyogi, Choi, & Charusombat, 2011). In this study, I provided a space for students to explore the causes and the consequences of the greenhouse effect and to think about the immediate and future threat of climatic disruption on people belonging to different socioeconomic strata. However, this was an initial attempt towards helping students become aware of the climate issues. To address inconsistencies in ideas, misconceptions about climate, and lack of awareness about the rapidly changing climate, it is necessary to educate students about the various environmental and social aspects of climate. Mathematics provides a concrete platform to do so. In this particular study, I focused on the greenhouse effect and its consequences of sea level rise, but future studies can explore other climatic and environmental issues, such as global warming, waste disposal, and pollution.

5.7. Conclusion

This study was motivated by the urgency of issues related to climate (the greenhouse effect in this case), and the role that mathematics can potentially play to address them. Three mathematical modeling activities were developed and implemented in two middle school classrooms to help the future generation become aware of the causes and consequences of the greenhouse effect on the normal atmospheric condition of the earth and human lives. The overall findings of the study suggest that the treatment group students showed significant improvement in covariational reasoning as well as their understanding of the greenhouse effect from the pre- to the post-assessment compared to their peers in the control group. Since, the treatment group students engaged with the three dynamic mathematical modeling activities during the five-day long design experiment, this study concludes that the intervention has had a significant impact on the students' performance during the post-assessment. Further, the study showed that the dynamic modeling activities accompanied with careful questioning provided a space for students to reason covariationally and suggested some possible design principles that helped constructing that learning space. For instance, students engaged in MA2 covariational reasoning, as per Carlson et al.'s mental action framework, as a result of their exploration of the NetLogo simulations. The graphical activities that followed prompted students to focus on specific values of the quantities and coordinate the amount of change of the two quantities, a reasoning that aligns with MA3 reasoning.

The overall findings of the study conveyed hope in the current climatic crisis. Students identified several human-induced factors that are responsible for climatic disruptions and indicated how economic disparity could make a certain group of people more vulnerable to the change of climate compared to others. Over the five days of the design experiment, students expressed their concerns regarding the consequences of a rapidly changing climate and showed compassion towards the people who bear a disproportionate impact of climatic disruption because of their economic conditions. I see optimism in the students' attitudes about the climate and find the necessity of similar studies, because if we do not take the initiative to educate our next generation to conserve this planet then "we are the ones who are gonna live with the damaged planet." (Paula, MC1)

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6. APPENDIX I

Module: Greenhouse Effect Theme: Earth System Science

This module focuses on how greenhouse gases, especially carbon dioxide, impact the temperature of the earth and the height of sea level. The module also provides students with a space to explore and discover how their own everyday practices may increase the carbon dioxide in the atmosphere and find ways to limit this.

Objectives (S for Science, M for Math, CS for Computer Science)

Students will:

- (S) Define greenhouse effect.
- (S) Identify the impact of greenhouse gases, especially, carbon dioxide on the temperature of the earth.
- (S) Define albedo and identify its impact on the temperature of the earth.
- (S) Identify the impact of increased global temperature on the height of sea level.
- (M) Define a variable as a quantity that varies.
- (M) Express non-numeric and numeric covariation relationships between the amount of carbon dioxide and the temperature of the air.
- (M) Express non-numeric and numeric covariation relationships between the temperature of the earth and the height of sea level.
- (M) Express non-numeric and numeric covariation relationships between the number of hours students use resources and the total amount of carbon dioxide.
- (M) Identify independent and dependent variable and express the relationships between them.
- (CS) Explore a computer simulation model on greenhouse effect.
- (CS) Modify the parameters of the simulation model and show how different variables influence carbon dioxide, air temperature, and height of sea level.

Key Terms: Greenhouse Effect, carbon dioxide, temperature, albedo, height of sea level, Covariational reasoning, coordinate system, rate of change, simulation.

Content and Practices

NGSS Standards
ESS3.C: Human Impacts on Earth Systems: *Human activities have significantly altered the biosphere, sometimes damaging or destroying natural habitats and causing the extinction of other species. But changes to earth's environments can have different impacts (negative and positive) for different living things. (MS-ESS3-3)* **ESS3.D: Global Climate Change:** *Human activities, such as the release of greenhouse gases from burning fossil fuels, are major factors in the current rise in earth's mean surface temperature (global warming). Reducing the level of climate change and reducing human vulnerability to whatever climate changes do occur depend on the understanding of climate science, engineering capabilities, and other kinds of knowledge, such as understanding of human behavior and on applying that knowledge wisely in decisions and activities. (MS-ESS3-5)*

Computational Thinking
Troubleshooting *Comprehensive troubleshooting requires knowledge of how computing devices and components work and interact. A systematic process will identify the source of a problem, whether within a device or in a larger system of connected devices.* **Algorithms** *Different algorithms can achieve the same result. Some algorithms are more appropriate for a specific content than others.* **Variables.** *Programming languages provide variables, which are used to store and modify data. The data type determines the values and operations that can be performed on that data.* **Control.** *Control structures, including loops, event handlers, and conditionals, are used to specify the flow of execution. Conditionals selectively execute or skip instructions under different conditions.*
Mathematical Thinking
CCSS.MATH.CONTENT.5.G.A.1 *Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).* ***CCSS.MATH.CONTENT.5.G.A.2*** *Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.* ***CCSS.MATH.CONTENT.7.RP.A.2*** *Recognize and represent proportional relationships between quantities.* ***CCSS.MATH.CONTENT.6.RP.A.3*** *Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.*
STEM+C practices
M4. Model with Mathematics; **S7.** Engage in argument from evidence; **CS4.** Developing and using abstractions. **S5:** Use mathematics and computational thinking.

Prerequisite Knowledge

(S) Different sources of air pollution.
(S) Knowing that names of some greenhouse gases such carbon dioxide, ozone, methane
(M) Defining a variable as an unknown quantity.
(M) Basic knowledge of coordinate system

(M) Basic knowledge of plotting graphs.
(M) Basic knowledge of mathematical operations such as multiplication division, and addition.
(CS) Basic familiarity with computers (user interfaces). No programming experience required for students.

Teacher Preparation

We will be using 3 simulations in this module:

Simulation #1: Climate change simulation on NetLogo.

Simulation #2: Height of future sea level simulation on NetLogo.

Simulation #3: Carbon calculator simulation on NetLogo.

Prior to the lesson, please explore the NetLogo simulations and become comfortable with changing and using the elements provided and setting up different scenarios.

Materials

Required Materials	Media	Equipment
• Whiteboard and markers. • 1 computer for each student or pair of students. • Notebooks and pens for each student.	• NetLogo must be on each computer to be used for the class.	• Computer and a projector for the instructor.

Safety

No physical safety issues.

Lesson 1: Simulation #1 & Forming non-numeric relationships (45 min)

Introduction 15 minutes	**[Whole class discussion] Use Greenhouse-Introduction powerpoint** 1. Show the following "Asthma in Newark" video (It's on the same folder as this module plan) and ask students the following questions: • What's the video about? • How did the video make you feel? (e.g. upset, worried, etc) Why? • What do you think about the situation shown in the video? • Why do you think so many children in Newark suffer from asthma? • Do you know anyone who has asthma? If yes, where does this person live? • Since Kearny is so close to Newark, do you think that you should be worried about your health too? Why/Why not?

	Show the graph that shows the difference between the age adjusted asthma Emergency department visits per 100,000 population in Essex county and the state of New Jersey. (Slide 2 at the Greenhouse-Introduction powerpoint) • Why do you think this is happening in some towns such as Newark? • Why did the factories choose Newark as their base? • What can we do to help the community of Newark deal with this problem? Now the video shows that one reason behind asthma is pollution. Different gases emitted as a result of pollution from factories, trucks and other vehicles are ground level ozone, oxides of nitrogen etc. Have you heard about Ozone gas before? (Based on the students' responses) We know that ozone protects us from UV radiation. Now that is good ozone. However, when the same ozone comes in contact with us (i.e. when it is formed/found at ground level) it is considered bad for us. Again, we have bad ozone. Ground level ozone is bad ozone. Ozone is a greenhouse gas. The increase in the ground level ozone and fine particulate concentrations is primarily responsible for asthma. It can trigger a variety of reactions including chest pains, coughing, throat irritation, and congestion. Other greenhouse gases responsible for asthma are oxides of nitrogen which is result of automobile emission. Another reason which also enhances the asthma problem among people is pollen. The greenhouse gas that increases the production of pollens is carbon dioxide. Apart from health, carbon dioxide also has a major influence on our natural environment. Over last 10 years the concentration of atmospheric carbon dioxide has increased enormously. (Show the graph of CO_2 concentration) In this module we are going to delve deeper and learn about the different sources of carbon dioxide and consequences of the increased carbon dioxide concentration.
Exploration 20 minutes	**[Investigation 1, Whole Class Investigation]** <u>Focus:</u> Exploring the program interface and forming non-numeric relationships For this investigation, students will not have a computer, but the teacher will project everything on the board and initiate the discussion. Students will be provided with a handout (see Investigation 1_Student version) that they will complete as the discussion unfolds. The teacher follows the instructions in the Investigation 1_Teacher's copy.

	The purpose of this investigation is for the students to explore the tools, find the variables of the model and construct non-numeric relationships among the tools/variables. The non-numeric relationships we would like the students to identify are: 1. The greater/less the amount of carbon dioxide, the higher/lower the temperature. 2. The more/less the amount of albedo, the more/less the amount of light reflected from the earth's surface. 3. If I increase/decrease the value of albedo, the air temperature decreases/ increases. **Questions that you can use throughout the investigation to help students reach the generalizations:** 1. What do you notice in the simulation? 2. What are the model's variables? How do they work? 3. How does air temperature change if I add more carbon dioxide? 4. How does air temperature change if I remove carbon dioxide? 5. What change do you observe in air temperature if the albedo is increased from 0 to 1? 6. What change do you observe in air temperature if the albedo is decreased from 1 to 0? 7. What can you conclude about influence of carbon dioxide on air temperature? 8. How does the albedo of the earth influence the air temperature, if at all?
Reflection 5 minutes	**[Whole class discussion]** Students reflect on their model exploration. **Ask:** • What have you learned today? • What surprised you? • What phenomenon does the model represent? • How would you describe the model to someone who is not here? • How does the model work? • What are some relationships you found? • What are you wondering about after today's lesson?

Lesson 2: Simulation #1 & Forming numeric relationships (45 min)

Introduction 10 minutes	**[Whole class discussion]** In the previous lesson we explored a simulation model. • What was the model about? • What were its variables?

	What are some relationships we found? <u>Say the following:</u> The relationships of the type, "The more the amount of carbon dioxide, the higher the temperature of the air" or "The less the amount of carbon dioxide, the lower the temperature of the air are called **non-numeric relationships**, because they don't involve any numbers. Today we will explore some **numeric relationships** between albedo, amount of carbon dioxide and temperature. They are called numeric because they involve numbers, for example, "What will happen to temperature of the earth, if the amount of carbon dioxide is doubled?" Or "What will happen to temperature of the earth, if the amount of albedo is changed from 0 to 1?"
Exploration 30 minutes	**[Investigation 2, Students work individually or in pairs]** <u>Focus:</u> Forming numeric relationships The purpose of this investigation is for the students to: (a) record the air temperature for different concentrations of carbon dioxide and plot the values graphically (b) identify that "As the amount of carbon dioxide in the atmosphere increases, the air-temperature increases." (c) identify the graph that represents the relationship between the albedo of the earth and air-temperature (that is, if the albedo of the earth increases, the air temperature decreases.)
Reflection 5 minutes	**[Whole class discussion]** Students reflect on their model exploration. **Ask:** • What have you learned today? • What are some relationships you noticed in this simulation? • How would you describe the relationship between amount of carbon dioxide and air temperature to somebody who is not present here? • How would you describe the relationship between the albedo and air temperature to your friend who is not here? • What are you wondering after today's lesson?

Lesson 3: Simulation #2 & Forming numeric relationships (45 min)

Introduction 5 minutes	**[Whole class discussion]** In the previous lesson we explored a model showing the relationships between air temperature, carbon dioxide and albedo.

	Ask: • What are some relationships you found? • If the amount of carbon dioxide increases, what happens to air temperature? • If the value of albedo decreases, what happens to the air temperature? • What do you think it will happen if this trend persists and the amount of carbon dioxide in the air continues to increase in that way? (Hear what they have to say) Today we will work on a simulation which will show the effect of the increasing air temperature. We will see how the increased air temperature might affect our lives.
Exploration 30 minutes	**[Investigation 3, Students work individually or in pairs]** <u>Focus:</u> Constructing numeric relationships The purpose of this investigation is for the students to: • identify that if the amount of air temperature gets higher, the sea level also rises; • recognize that as the height of future sea level rises, the amount of land area decreases; • reason about the relationship between three quantities, stating that if air temperature gets higher, the height of future sea level rises, and the amount of land area decreases.
Reflection 10 minutes	**[Whole class discussion]:** Students reflect on their model exploration. **Ask:** • What have you learned today? • What are some relationships you found? • How would you describe the relationship between the air temperature and level of sea? • Have you thought about any of these relationships before? • How do you think the sea level rise might affect your life? • What do you think we should do to delay this rise of sea level? • What are you wondering after today's lesson?

Lesson 4: Simulation #3 & Forming numeric relationships (45 min)

Introduction 5 minutes	**Ask:** In the previous lesson we explored the relationships between the amount of carbon dioxide, air temperature and height of sea level. • Do you remember any of these relationships?

	• What happens to air temperature if the concentration of carbon dioxide increases? • What happens to the height of sea level, if the value of air temperature increases? • So, what happens to the height of sea level, if the concentration of carbon dioxide increases? • What are the different sources of carbon dioxide in the atmosphere? Today we will explore how we, as humans, also add carbon dioxide in the air. We will explore a simulation that will show us how we do that and also calculate how much carbon dioxide we are adding in the atmosphere every year.
Exploration 35 minutes	**[Investigation 4, Students work individually or in pairs]** <u>Focus:</u> Constructing numeric Relationships The purpose of this investigation is for students to: (a) calculate how much carbon is added to the atmosphere every year due to daily activities, such as watching TV or traveling to school by cars. (b) realize that the more resources they use, the more the amount of carbon dioxide they add to the atmosphere every year. **Ask:** • How much carbon dioxide are you adding each year, if you watch TV for 4 hours and keep it on standby? • How much carbon dioxide will you add in one year if you watch TV for 4 hours and turn it off after watching it? • How much carbon dioxide will you add each year if you travel to your school alone in a car? • What will happen, if you carpool with two other friends of yours?
Reflection 5 minutes	**[Whole class discussion]** Students reflect on their model exploration. **Ask:** 1. Today we explored how some of our daily activities add carbon dioxide in the atmosphere. What are some daily activities that do that? 2. Can you think of any other activities that you do every day not presented in the simulation, that might also add carbon dioxide in the atmosphere? 3. What are the possible effects of increased carbon dioxide in the air? 4. What might happen if the amount of carbon dioxide continues to increase? 5. What are some things that we can do to reduce the amount of carbon dioxide that is added in the atmosphere? 6. Is there anything in the simulation that surprised you? 7. What are you wondering after today's lesson?

Lesson 5: Simulation 3 & Building a model to take action (45 min)

Introduction 5 minutes	**[Whole class discussion]** Last few days we discussed the different sources that enhance the atmospheric carbon dioxide and also saw the impact of increasing amount of carbon dioxide in the atmosphere. Do you think our daily life activities also adds to the problem? How can you reduce the amount of atmospheric carbon dioxide you add every year? What are some other ways which you can adopt to reduce harmful carbon emissions and combat climate change?
Exploration 20 minutes	**[Investigation 4, Students work in groups of three.]** Focus: Comparing and creating a model of daily life activities to minimize the amount of carbon dioxide released in the atmosphere. The purpose of this investigation isis for the students to: • calculate how much carbon they add to the atmosphere every year due to their daily activities such as watching TV or traveling to school by cars; • build a model to optimize their actions for minimizing the amount of CO_2 they add to the atmosphere every year (e.g. by adjusting their different daily life actions such as hours they spend watching TV or number of people who carpool to school with the intention);
Presentations 10 minutes	**Each group presents their models to the whole class.** The purpose of this activity is for students to compare and contrast their model with the other students and think critically what are some other actions they can take to combat climate disruption.
Reflection 10 minutes	**[Whole class discussion]** Students reflect on the building of their model and their overall experience with the module. 1. What did you learn these past few days? 2. Has anything that we discussed surprised you? How? 3. Have you heard about this issue before? Where? 4. What are some relationships we discovered? 5. How will this module impact your life? 6. Will you continue taking actions to save our planet? How? 7. How will you discuss this issue with your friends and parents? 8. What are some of the issues that you will like to discuss more in future?

7. APPENDIX II
Investigation 1

Module 2.5: Greenhouse Effect **Theme: Human Impact**

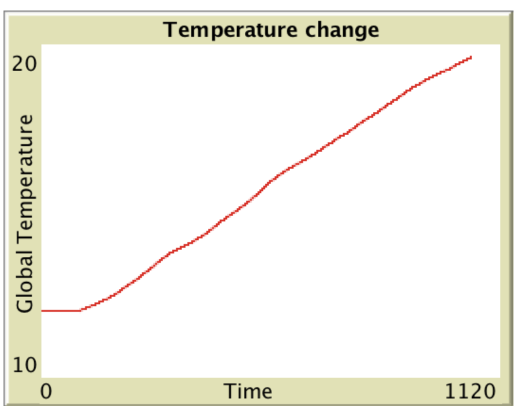
Investigation 1: Exploring the program interface and forming non-numeric relationships

Simulation: Simulation 1_Climate Change.nlogo

1. Let us explore the simulation together! A ***variable*** is any quantity that can be changed or controlled. What are the variables of the model? Check all that apply.
   - Add Carbon dioxide
   - Remove Carbon dioxide
   - Albedo
   - Temperature
   - Sun's brightness
   - Infrared Rays (Red dots)

2. Each time you add carbon dioxide, the value of carbon dioxide goes up by________.

3. This simulation includes an interactive graph. It's interactive because it changes as you manipulate the simulation. It can take many forms depending on your activity. Let's explore some of those forms.



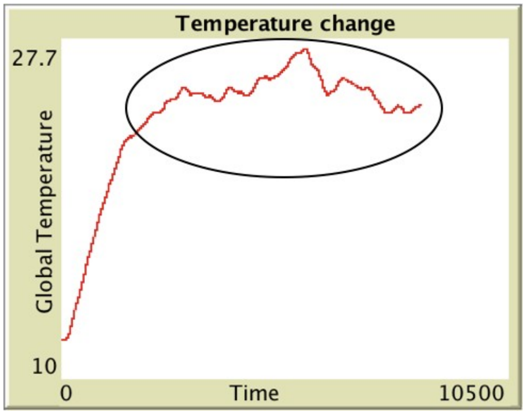
(A)

What does Graph A show?

a) As time increases, the global temperature increases.
b) As time increases, the global temperature decreases.

c) As time increases, the global temperature remains stable.

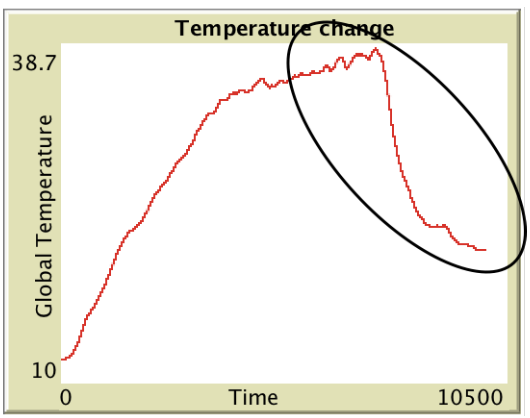
4. Graph B:



What does the highlighted portion of Graph B show?

a) As time increases, the global temperature increases.
b) As time increases, the global temperature decreases.
c) As time increases, the global temperature remains stable.

5. Graph C:

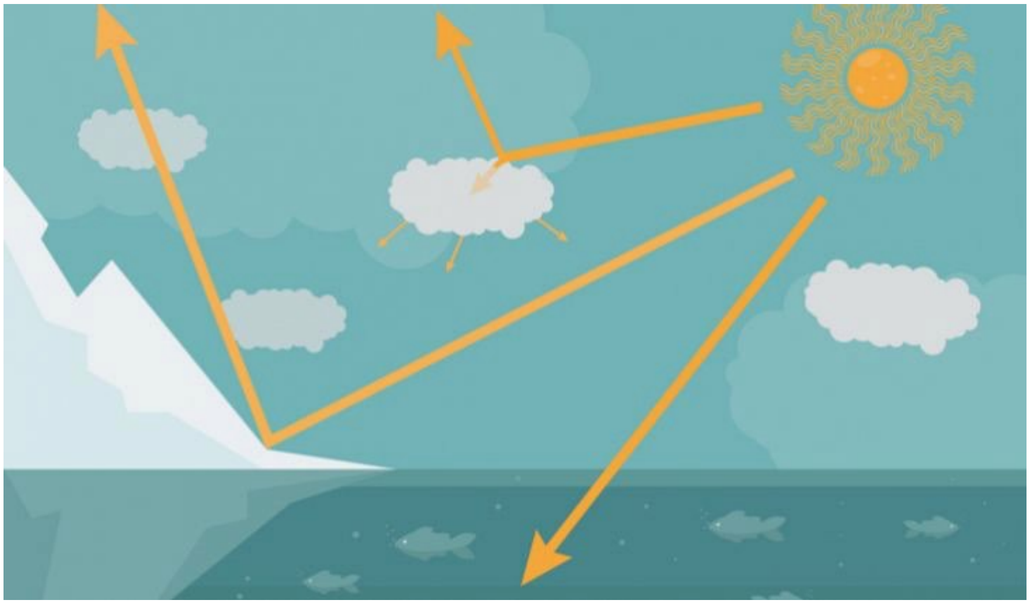


What does the highlighted portion of the third curve represent?

a) As time increases, the global temperature increases.
b) As time increases, the global temperature decreases.
c) As time increases, the global temperature remains stable.

Note: Understanding how the model works can be a complex process and it needs time. Remember, while working with any model whenever you are making any change in one variable, the other variables might get impacted. So, after making change, give some time to the model and let it stabilize.

6. The term albedo refers to the amount of solar energy that gets reflected off of the earth and lands back in space.



In this simulation, the albedo slider shows the value of albedo of the earth's surface.

a) The further the albedo slider is moved to the right, the value of albedo increases/ decreases.

b) The further the albedo slider is moved to the left, the value of albedo increases/ decreases.

7. Change the value of albedo from 0 to 1 and observe how the number of reflected sun-rays changes on the program interface:

a) The more the amount of albedo, the more/ less the amount of light reflected from the earth's surface.

b) The less the amount of albedo, the more/ less the amount of light reflected from the earth's surface.

8. What is the relationship between the albedo of the earth and air temperature?

a) If I increase the value of albedo, the air temperature increases/ decreases.

b) If I decrease the value of albedo, the air temperature increases/ decreases.

9. What is the relationship between amount of carbon dioxide and air temperature?

a) If I increase the amount of carbon dioxide, the air temperature increases/ decreases.

b) If I decrease the amount of carbon dioxide, the air temperature increases/ decreases.

8. APPENDIX III

Module 2.5: Greenhouse Effect **Theme: Human Impact**

Investigation 2: Explore numerical relationships

Simulation: Simulation 1_Climate Change.nlogo

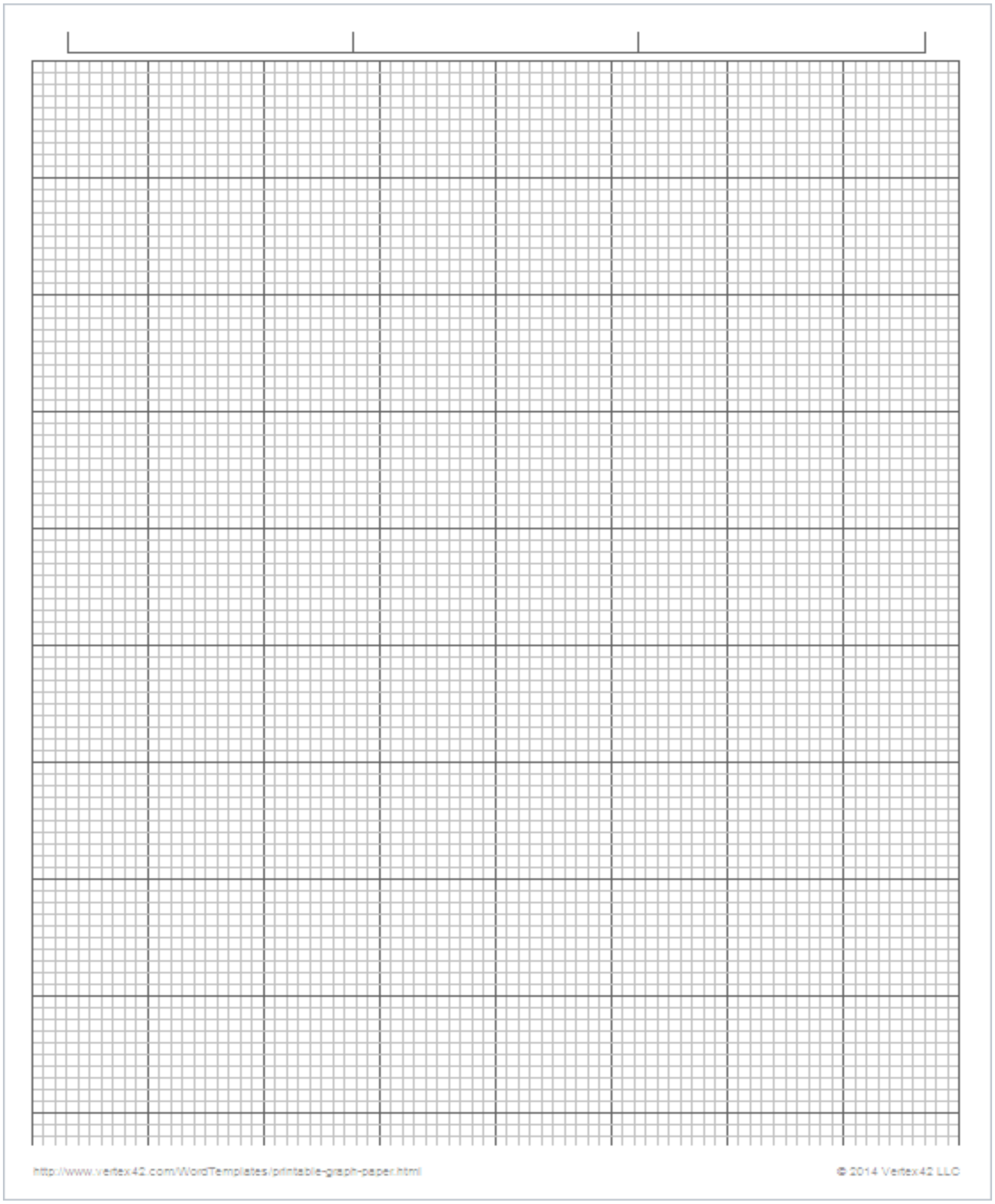
1. Which are the *two factors* that change the temperature of the earth? (Remember to give some time and let the model stabilize.).
a) Amount of carbon dioxide
b) Albedo
c) Infrared rays
d) Clouds
e) Time
f) Sun's rays

2. Understanding how the model works can be a complex process if we have many variables. That's why in this task we will just change one variable only and see what happens. Change the carbon dioxide as shown in the table below and record the air temperature accordingly. Remember to give some time after each change and let the temperature stabilize and take the nearest whole number (e.g. if the temperature is 22.3, 22.2, 21.9 then temperature is around 22).

Carbon dioxide	Air Temperature
0	
100	
200	
300	
400	

3. What patterns do you see in the table?
a) As the carbon dioxide is increasing by 100, the air temperature is also increasing.
b) As the carbon dioxide is decreasing by 100, the air temperature is decreasing.

4. Use the values you found in the table above to plot a graph showing the relationship between the carbon dioxide and air temperature. Label your axes as follows:
x-axis: Amount of carbon dioxide
y-axis: Air temperature

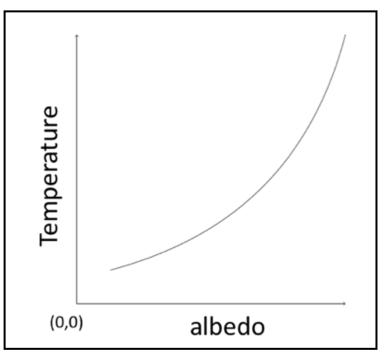


5. Which of the following describes what the graph shows.
- A straight line (going up from left to right)
- A straight line (going down from left to right)
- A curve (going up from left to right)
- A curve (going down from left to right)

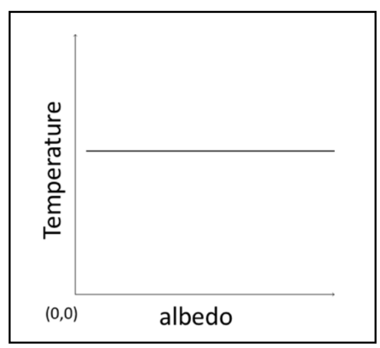
6. Use the graph to find the air temperature when carbon dioxide is 300. The approximate temperature is:
- 10
- 46
- 15
- 33

7. Use the graph to find the carbon dioxide when the air temperature is 33. The approximate value of carbon dioxide is:
- 300
- 400
- 0
- 100

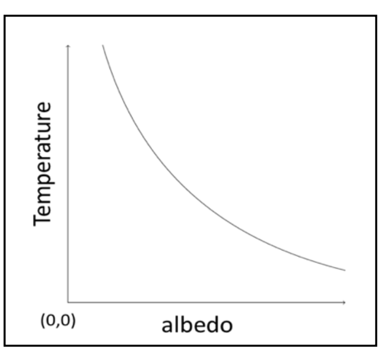
8. Fix the amount of carbon dioxide at 150, and gradually change the value of albedo and observe how air temperature is changing. Which of the following graphs correctly represents the relationship between earth's albedo and air temperature?



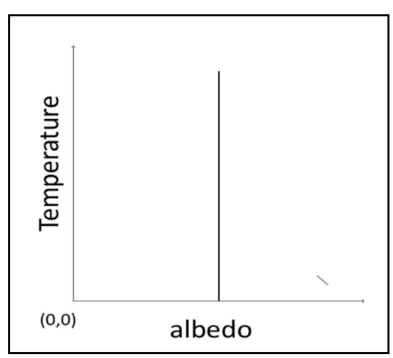
(a)



(b)



(c)



(d)

9. APPENDIX IV

Module 2.5: Greenhouse Effect **Theme: Human Impact**

Investigation 3: Explore numerical relationships

Simulation: Simulation 2_Sea level rise.nlogo

1. Free Play: Take 5 minutes to explore the simulation!

Terms we will be using in this investigation
An **independent variable** is a factor that is changed by the scientist. • What is tested • What is manipulated
A **dependent variable** is a factor that might be affected by the change in the independent variable. • What is observed • What is measured • The data collected

2. An ***independent variable*** is a factor that can be changed by the scientist. Which is the independent variable of the model?

- Global temperature
- Future sea level
- Total land area
- The height of the buildings

3. A ***dependent variable*** is a factor that might be affected by the change in the independent variable. Which are the independent variables of the model?

- Global temperature
- Future sea level
- Total land area
- The height of the buildings

4. The interactive graph shows how the sea level may change in the future as the global temperature changes. Manipulate the global temperature. What do you notice? Circle what applies.

a) The higher the global temperature, the (higher / lower) the height of future sea level.

b) The lower the global temperature, the (higher / lower) the height of future sea level.

4. Take a moment and imagine what will happen to the total land area if the height of future sea level increases.

   a) The higher the height of the future sea level, the (more / less) is the total land area.

   b) The lower the height of the future sea level, the (more / less) is the total land area.

5. Based on the above, what is the relationship between the three variables: global temperature rise, height of future sea level, and total land area? Choose the correct statement.

   a) The higher the global temperature, the lower is the height of future sea level, and the more is the total land area.
   b) The higher the global temperature, the higher is the height of future sea level, and the more is the total land area.

7. Use the model to collect some data about the height of future sea level as the global temperature rises.

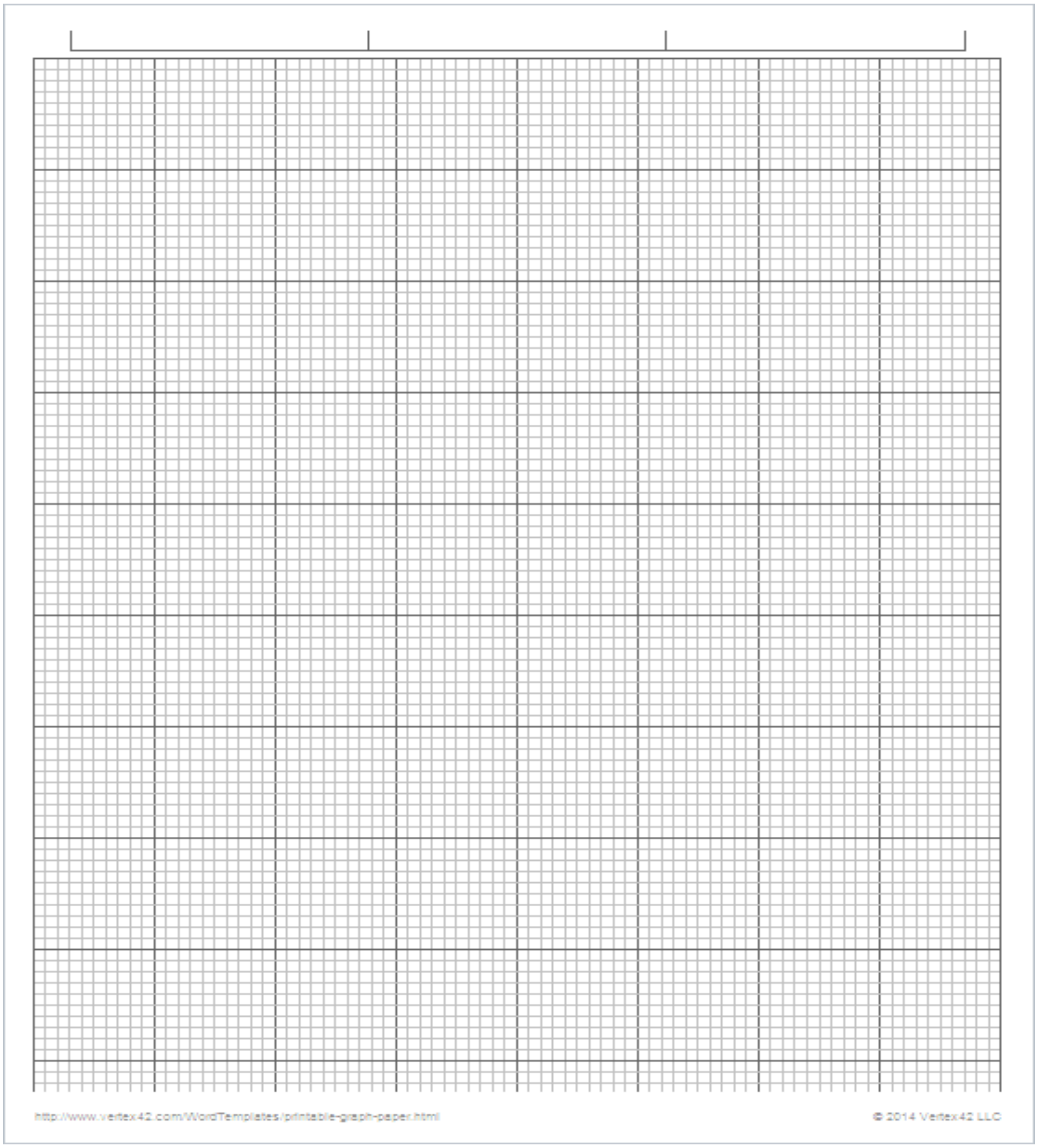
Global Temperature (Celsius)	Height of future sea level (Feet)

8. What is the relationship between the global temperature and the height of future sea level?

a) As the global temperature is increasing by 0.5, the height of the future sea level is increasing by____feet.

b) As the global temperature is decreasing by 1, the height of the future sea level is decreasing by____feet.

9. Use the values you collected in the table before to create a graph showing the relationship between the height of the sea level and the rise of global temperature. Label the x-axis as 'global temperature' and the y-axis as 'height of sea level.'



10. Which of the following statements describes what the graph shows.

- As the global temperature increases, the height of sea level increases.
- As the global temperature increases, the height of sea level decreases.

11. Which of the following relationships is best described in the simulation?

- The higher the elevation of a place, the lower the risk of going under sea water.
- The higher the elevation of a place, the greater the risk of going under sea water.

12. The elevation of East Newark (20 feet) is double the elevation of downtown Manhattan (10 feet) from the sea level. Which of the statement best describes the impact of sea-level rise on the two places?

- The risk of going under sea water of downtown Manhattan is the same as East Newark.
- The risk of going under sea water of downtown Manhattan is double than East Newark.
- The risk of going under sea water of downtown Manhattan is half than East Newark.

10. APPENDIX V

Module 2.5: Greenhouse Effect **Theme: Human Impact**

Investigation 4: Carbon-Calculator

Simulation: Simulation 3_Carbon Calculator.nlogo

1. Free Play: Take 5 minutes to explore the simulation! Click 'Setup' to start the simulation. Modify the different variables and observe the change in the value of carbon dioxide.

Explaining how the model works

- The model shows the amount of CO_2 in the atmosphere per year.
- Every time you change a variable please click on Setup.
- The amount shown is cumulative, meaning that it includes all the variables involved.

2. Before starting this task please turn the variables off and put them to zero. We watch TV every day. Observe how the amount of carbon dioxide changes based on TV hours, when you turn off the TV after watching it.

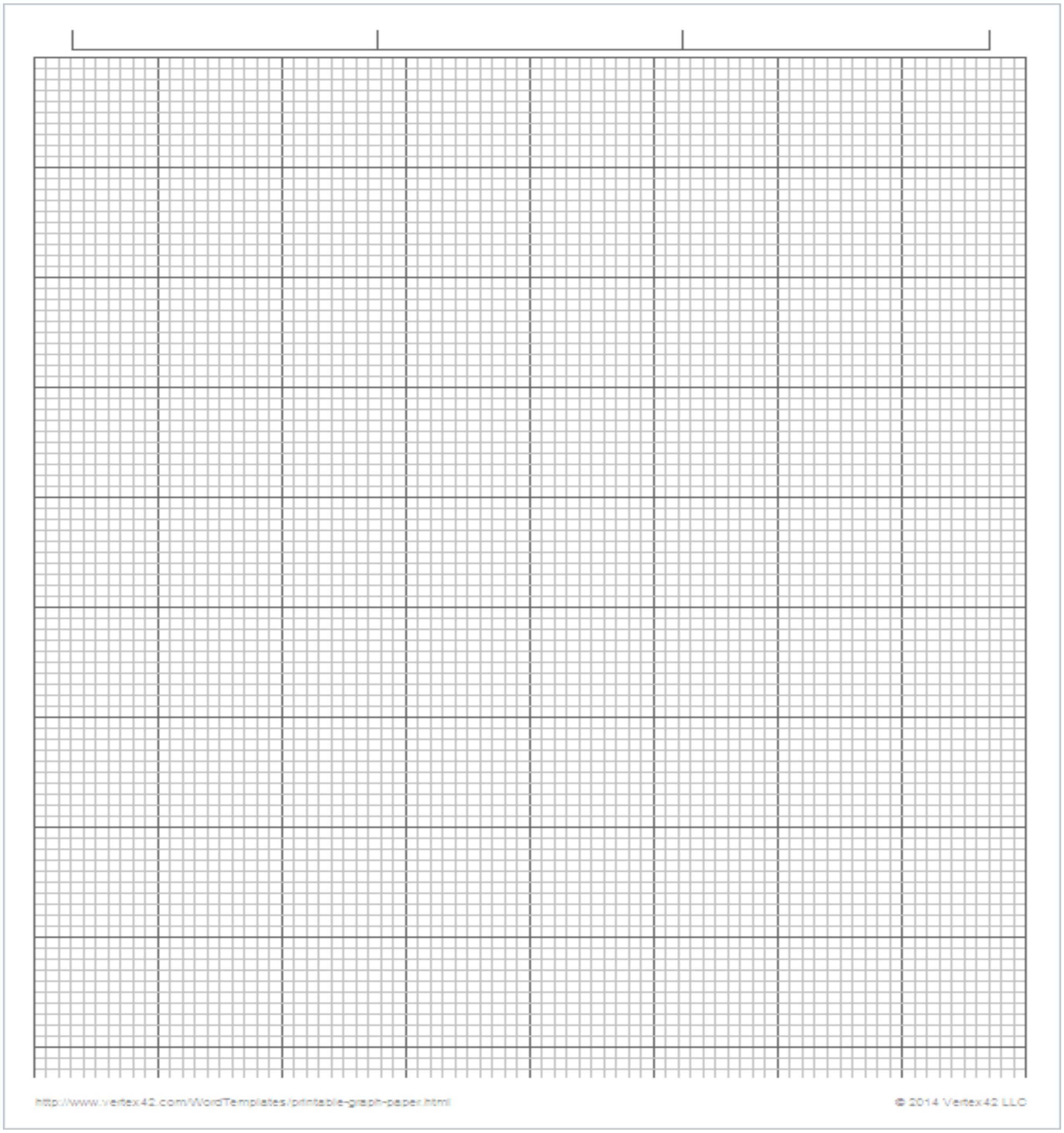
TV-hours	CO_2 amount/ year
1	
2	
3	
4	

3. Based on the above table, we can say that:
   a) As the tv-hours increase by 1, the CO_2 amount increases by 82 kg.
   b) As the tv-hours increase by 1, the CO_2 amount decreases by 82 kg.
   c) As the tv-hours increase by 1, the CO_2 amount doubles.

4. Based on the above table, we can say that:
   a) If the number of tv-hours becomes 10 times bigger, the amount of CO_2 becomes 10 times bigger.
   b) If the number of tv-hours becomes 10 times bigger, the amount of CO_2 becomes 10 times smaller.

5. What is the relationship between the number of TV-hours and the amount of CO_2 released in the atmosphere?
   a) The amount of CO_2 released is 82 times the number of TV-hours.
   b) The amount of CO_2 released is 164 times the number of TV-hours.

6. Use the table above to graph the relationship between the number of tv-hours and the amount of carbon dioxide. Label the x-axis as the number of tv-hours and the y-axis as amount of CO_2.



7. Which of the following describes the shape of the graph?

- A straight line (going up from left to right)
- A straight line (going down from left to right)
- A curve (going up from left to right)
- A curve (going down from left to right)

8. For this task, please put TV hours on 0 and turn it off. Sharing a ride or carpooling is a common practice for commuters or travelers going to the same destination. Explore the simulation and observe what impact carpooling has on our environment.

a. Express graphically how much carbon dioxide would be added in the atmosphere each year if you live 1 mile away from the school and you carpool with a different number of friends. Complete the first column of the table below.

Carpool friends	CO_2 if you live 1 mile from school	CO_2 if you live 2 miles from school
No Carpool		
Carpool with 1 person		
Carpool with 2 people		
Carpool with 3 people		

(b) What if you live 2 miles away from the school? Use the simulation to complete the second column of the table.

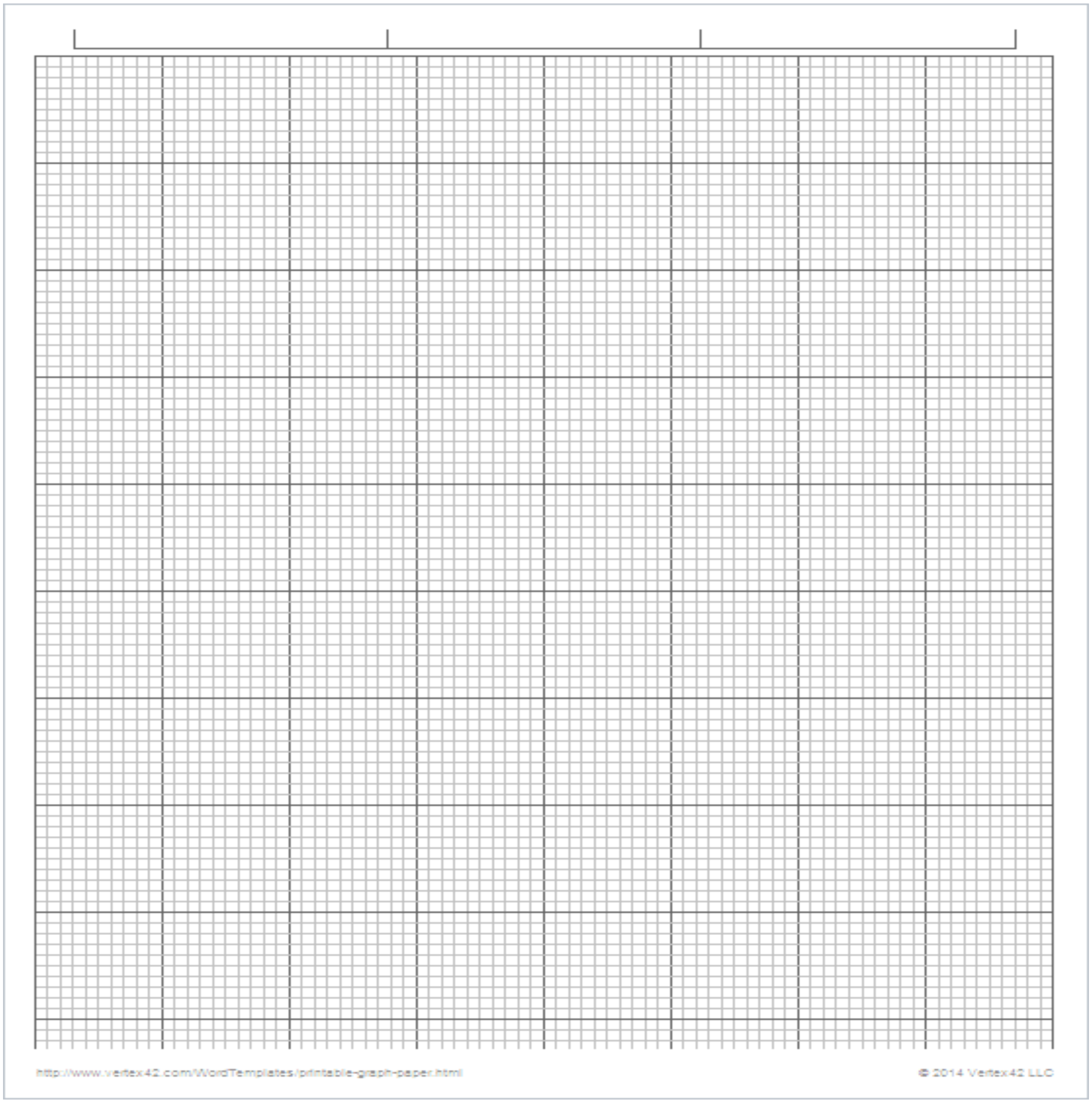
9. Based on the table created, we can say that:

a) As the number of friends carpooling increases, the amount of CO_2 released increases.
b) As the number of friends carpooling increases, the amount of CO_2 released decreases.

10. Based on the table created, we can say that:

a) If the distance between your school and house doubles, the amount of CO_2 released doubles.
b) If the distance between your school and house doubles, the amount of CO_2 released becomes half.

11. Use the table above to graph the relationship between the amount of carbon dioxide and carpooling per year if you lived 1 mile away from school. Label the x-axis as the number of friends and the y-axis as amount of CO_2.



12. Which of the following describes the shape of the graph?

- A straight line (going up from left to right)
- A straight line (going down from left to right)
- A curve (going up from left to right)
- A curve (going down from left to right)

13. Based on the graph, what are the relationships between the distance travelled, the amount of carbon dioxide, and carpooling?

a) The more the distance travelled, the (more / less) carbon dioxide is added to the atmosphere each year.
b) The more the number of people in carpool, the (more / less) carbon dioxide is added to the atmosphere each year.

14. Manipulate each factor, based on your own life, and calculate how much carbon dioxide you add to the atmosphere every year.

Consumption	Value chosen	Amount of Carbon dioxide	Total Carbon dioxide
TV-hours_Stand-by			
TV-hours_Turn off			
Video_Game_hour_Standby			
Video_Game_hour_Turn off			
Battery_charger_plugged_ unused			
Computer-hours			
Bath once a week			
Shower			
Number of AC			
Heater			
Carpool			
Carpool distance			
Vacation			
Total Carbon dioxide			

11. APPENDIX VI

Module 2.5: Greenhouse Effect **Theme: Human Impact**

Investigation 5: Building a model to take action

Simulation: Simulation 3_Carbon Calculator.nlogo

1. Modify the table you created in Investigation 4 to help reduce the amount of carbon dioxide you add every year. Manipulate some of the factors below to create a model that minimizes the total carbon dioxide that you add to the atmosphere. Note that some of these factors are important for you and you may decide not to change them.

Consumption	Value chosen	Amount of Carbon dioxide	Total Carbon dioxide
TV-hours_Stand-by			
TV-hours_Turn off			
Video_Game_hour_Standby			
Video_Game_hour_Turn off			
Battery_charger_plugged_ unused			
Computer-hours			
Bath once a week			
Shower			
Number of AC			
Heater			
Carpool			
Carpool distance			
Vacation			
Total Carbon dioxide			

2. Compare this new model with the previous one in Investigation 4 and see how much carbon dioxide you avoided releasing to the atmosphere.

a) Previous amount of carbon dioxide: ____________________________
b) New amount of carbon dioxide: _________________________
c) I saved: _________________________

3. What are some other ways which you can adopt to reduce the CO_2 release in the atmosphere?

a) Ride my bike
b) Carpool to school
c) Use night lamps
d) Practice eco driving
e) Take public transportation whenever possible
f) Increase use of plastic bags
g) Switch off lights
h) Plant trees
i) Print double-sided
j) Increase use of high-voltage light bulbs

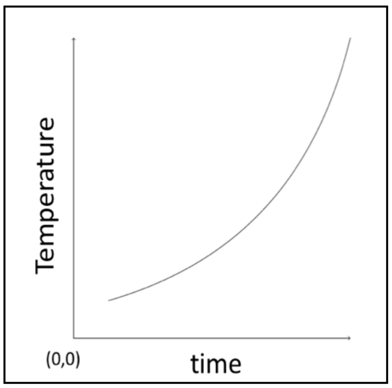
12. APPENDIX VII

Pre- and Post-Assessment

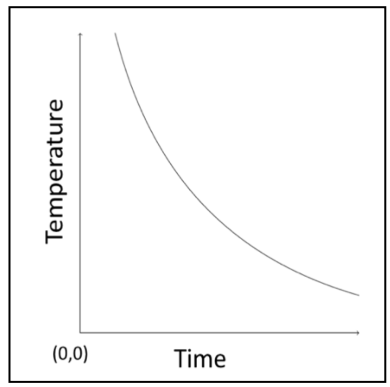
Full Name:____________________________________**Circle One: PRE / POST** (2.5 p1)

Greenhouse Effect Module
Summative Assessment

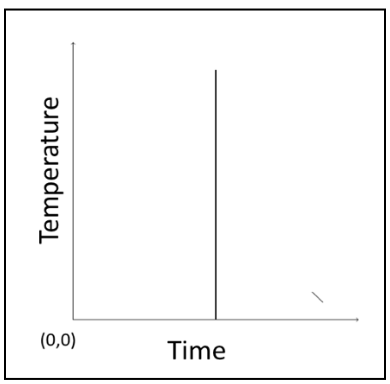
1. Which of the following graphs show that as the time increases, the air temperature increases?



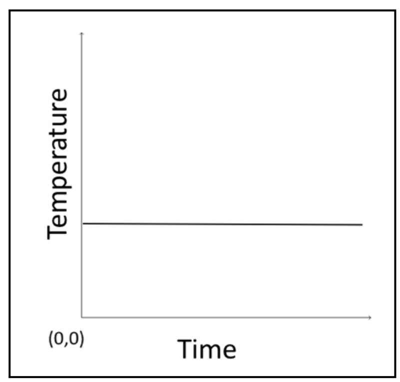
(a)



(b)



(c)



(d)

2. What will happen if you go to school every day by carpooling with your two friends?
   a) You will add less carbon dioxide in the atmosphere.
   b) You will not add any amount of carbon dioxide in the atmosphere.
   c) You will add more carbon dioxide in the atmosphere.

3. <u>Two</u> factors that influence the air temperature are:
   a) Time and the height of sea level.
   b) Sun rays and the height of sea level.
   c) The albedo of the earth and the greenhouse gas concentration in the atmosphere.
   d) Time and the elevation of the land.

4. Which of the following statements is true about atmospheric carbon dioxide and air temperature?
   a) If the atmospheric carbon dioxide increases, the air temperature increases.
   b) If the atmospheric carbon dioxide increases, the air temperature decreases.
   c) If the atmospheric carbon dioxide increases, the air temperature stays the same.

5. Which of the following statements is true about albedo and air temperature?
   a) If the albedo of the earth increases, the air temperature increases.
   b) If the albedo of the earth increases, the air temperature decreases.
   c) If the albedo of the earth increases, the air temperature stays the same.

6. Which of the following statements is correct for the global temperature and the height of future sea level:

   a) The higher the global temperature, the higher the height of future sea level.
   b) The higher the global temperature, the lower the height of future sea level.

7. Which of the statements is true about height of sea level and elevation of a place:
   a) The higher the elevation of a place, the lower the risk being affected by sea level rise.
   b) The higher the elevation of a place, the greater the risk of being affected by sea level rise.

8. If I use my computer for 1 hour every day, I release 36 kg of CO_2 in the atmosphere in one year. How many kg of CO_2 will I release in the atmosphere if I use my computer for 3 hours?
   a) 12
   b) 108
   c) 324

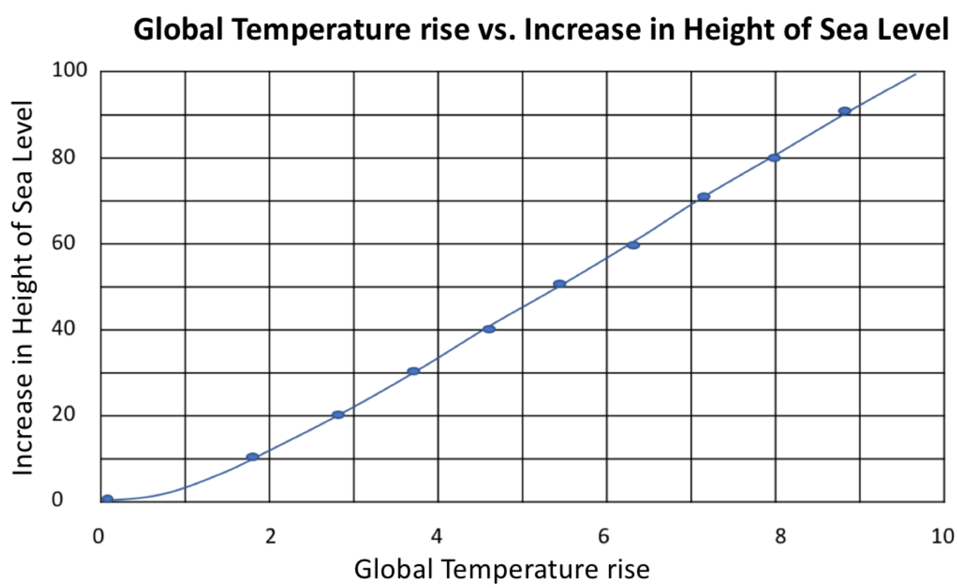
9. If I use my computer for 1 hour every day, I release 36 kg of CO_2 in the atmosphere in one year. This year I released 540 kg of CO_2. For how many hours did I use my computer every day?
   d) 8 hours
   e) 10 hours
   f) 15 hours

10. Consider the following statement:
"If I use my computer for 3 hours every day, I release 108 Kg of carbon dioxide in one year."

(i) The independent variable in the above statement is:
a) Number of years.
b) Number of computer hours.
c) Amount of carbon dioxide released in one year.

(ii) The dependent variable in the above statement is:
a) Number of years.
b) Number of computer hours.
c) Amount of carbon dioxide released in one year.

11. Explore the following graph:



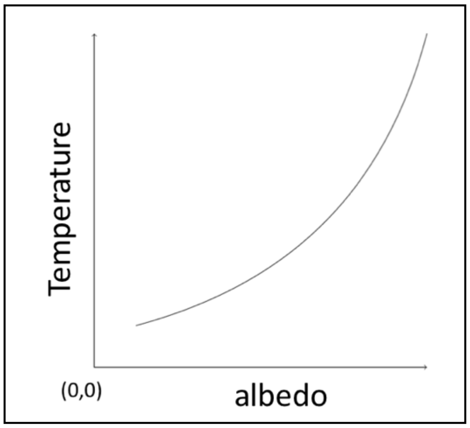
d. What relationship does the graph show?
   i. As the global temperature increases, the height of sea level increases.
   ii. As the global temperature increases, the height of sea level decreases.

e. What do you notice when you look at the graph?
   i. The graph is increasing from left to right.
   ii. The graph is decreasing from left to right.

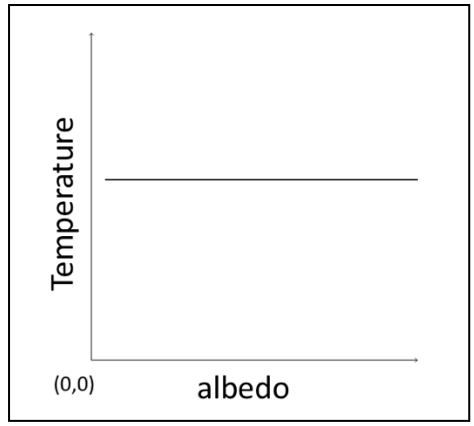
f. What is the Increase in Height of Sea Level, when the Global temperature rise is 8 degree Celsius?
   i. 40 feet
   ii. 80 feet

g. For what value of Global temperature rise, the increase in height of sea level is 70 feet?
   i. 7 degree Celsius.
   ii. 9 degree Celsius.

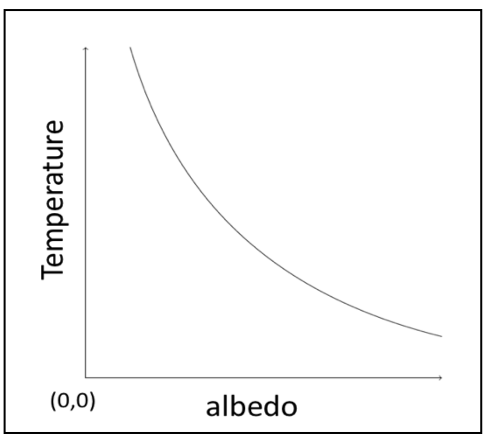
12. Which of the following graphs correctly represents the relationship between earth's albedo and air temperature?



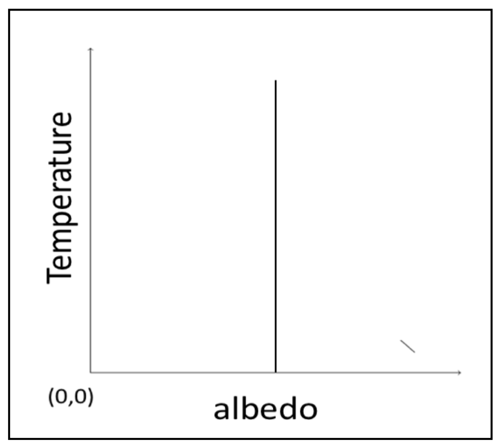
(a)



(b)



(c)



(d)

13. Complete the following table:

Activities	Number of hours	Amount of CO_2 released/ year	Total CO_2
TV-hours	_______hours	82 Kg per hour	246 Kg
Video Games	2 hours	40 Kg per hour	__________Kg
Music system	1/2 hour	_______Kg per hour	40 Kg