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Quantitative Literacy and High School Mathematics : The Evolution of a Collaboratively Constructed Course and Its Impact on Students' Attitudes and Numeracy

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QUANTITATIVE LITERACY AND HIGH SCHOOL MATHEMATICS: THE EVOLUTION OF A COLLABORATIVELY CONSTRUCTED COURSE AND ITS IMPACT ON STUDENTS' ATTITUDES AND NUMERACY

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ABSTRACT

QUANTITATIVE LITERACY AND HIGH SCHOOL MATHEMATICS: THE EVOLUTION OF A COLLABORATIVELY CONSTRUCTED COURSE AND ITS IMPACT ON STUDENTS' ATTITUDES AND NUMERACY

by Mark Francis Russo

This study describes a practitioner action research project in which I co-constructed a high school mathematics elective course with my students. The focus of the course was on developing students' quantitative literacy. I examined the impact of the co-construction process on the evolution of the course, analyzed how the course influenced students' quantitative literacy and attitudes about mathematics, and reflected on some of the lessons I learned about making mathematics more relevant for my students. This study fills a gap in the literature by describing the impact of a quantitative literacy course at the high school level and by documenting the effect of co-construction on an entire course. In order to answer my research questions, I relied on qualitative data gathered from surveys, questionnaires, classroom assessments, transcriptions of classroom discussions, field notes, and my own research journal. The findings of this study highlight some of the complexities involved in the co-construction process, the impact of co-construction on students' interest, and some key themes related to teaching for quantitative literacy.

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Introduction

In his seminal work *Innumeracy: Mathematical Illiteracy and its Consequences*, John Allen Paulos (1988) sounded an alarm on the widespread mathematical ignorance that he claimed had gripped our nation. Paulos noted various inadequacies in the mathematical knowledge of the general population, including a lack of numerical perspective, an acceptance of pseudoscience, a fascination with "meaningless coincidences," an inability to recognize trade-offs, and a dangerous disconnect between scientists' assessment of risks and public perception of those same risks. Niss (1994) made similar observations, for even as the prevalence of mathematics was increasing, it remained essentially unnoticed to politicians and the general public. Niss described this as a "relevance paradox," in which mathematics held "objective relevance" in terms of its utility for the world, but "subjective irrelevance" for individual citizens (p. 371). This innumeracy was not only apparent in a lack of mathematical knowledge, but also in the public's inability to understand the basic quantitative information they would find in a newspaper (Kolata, 1997; Paulos, 1996). Paulos (1988) blamed this problem partially on psychology, but he also blamed it on poor mathematics education.

As a high school mathematics teacher, I have taught many students who loved mathematics and achieved at a high level, but I have also worked with students who did not learn much in my classes, who achieved far below their potential, and who boasted a strong dislike, or even hatred, for mathematics. It was extremely challenging to work with these students, because I would not only struggle to help them understand the material, but I would struggle even more to justify why they had to learn these concepts at all. It is difficult to teach a lower performing high school senior polynomial division and Descartes' rule of signs, but it is far more difficult to explain to them why these things are worth learning in the first place. Year after year I tried different ways to teach these courses, but I began to consider that the problem might not be my pedagogy; rather, the problem might be the content. These students were not struggling solely because of a lack of understanding, though that was definitely part of it; rather, they might have been underperforming because of a lack of interest or a lack of relevance. What if I stopped trying to prepare students for a calculus course that they might never take, and instead, I focused on material that would capture their interests and relate to their lives or their future professions? Perhaps this change could help students enjoy mathematics and achieve at a higher level, while also equipping them with the knowledge and skills they would need to be educated consumers of quantitative information. I decided to implement this change in two of my senior elective classes, by designing courses with an emphasis on content and skills that would develop students' quantitative literacy (QL).

Problem Statement

In my experience, many students in senior elective classes do not consider mathematics class to be useful for their lives, nor do they learn much while they are enrolled in the course. One purpose of this study, then, is to learn whether a QL course can influence students' attitudes towards and understanding of mathematics. In order to do this I decided to construct a course around students' interests and future ambitions, since QL focuses on skills that are necessary to "engage effectively in quantitative situations arising in life and work" (Alsina, 2002, pp. 2-3). This proved to be difficult, though, because, as Appleton and Lawrenz (2011) suggested, "mathematics teachers' ideas of what is real world or practical are not the same as their students'" (p. 150). In order to better understand where students are coming from, the authors suggested that "mathematics teachers could productively spend more class time discussing with students what their ideas of real world or practical issues are and how these relate to classroom mathematics" (p. 151). Fredricks, Blumenfeld, and Paris (2004) also argued for the importance of student voices, as they described a positive relationship between student engagement in high school and "voluntary choice, …student participation in school policy and management, opportunities for staff and students to be involved in cooperative endeavors, and academic work that allows for the development of products" (p. 73).

One way to increase students' participation in the decision-making process is through the practice of *co-construction*. Unlike more traditional techniques where teachers incorporate students' feedback into their lesson planning, co-construction invites students to work alongside teachers as they together plan units, lessons, and assessments. This technique can address the problem presented by Appleton and Lawrenz (2011), because students are the best judges of what is important and relevant to them. Consequently, co-construction could be a valuable tool in a QL classroom, because it increases the likelihood that course content will be pertinent and meaningful to students' lives.

In order to make mathematics class more meaningful for my students, then, I invited them to co-construct the course with me. As a result, this study will not only analyze how a QL course can impact students' attitudes and understandings of mathematics, but it will also investigate how the process of co-construction impacts the development of the course. In this study, I attempt to answer the following research questions:

1. How does the ongoing co-construction of a QL course between my students and me affect the evolution and development of the course?
2. How does participating in this course affect students' QL and attitudes about mathematics?
3. Through these experiences, what do I as a teacher learn about teaching for QL and making mathematics more relevant to my students?

This study is significant for two main reasons. First, I have encountered a real problem in upper-level mathematics electives, where many students display negative attitudes towards mathematics and hardly advance in their knowledge and appreciation of mathematics. Through teaching for QL and co-construction, this study will seek to help this group of students by providing them with an experience that may not only be more interesting and engaging, but also more relevant to their lives. My hope is that this study will also generate some ideas that might be useful for other teachers who work with similar groups of students. Secondly, this study is significant because it will add to two distinct segments of the literature. First, there is a dearth of research examining the impact of shared decision-making on classroom experiences, particularly when it comes to whole-class co-construction. Second, this study would seek to fill a gap in the QL literature, because as I demonstrate in Chapter 2, there has been very little research on QL in high school.

Definition of Terms

In this study, I utilize the definition of quantitative literacy (QL) that comes from the International Life Skills Survey (2000). The ILSS has defined QL as "an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work" (Alsina, 2002, pp. 2-3). In order to better understand QL, it may be helpful to look at some of the characteristics of a quantitatively literate individual. Along with a serviceable knowledge of mathematics and a positive disposition towards it, a quantitatively literate person reasons mathematically, recognizes the value of mathematics in society, and understands the history of mathematics (Wilkins, 2000). A quantitatively literate individual can gather useful information from a problem, perform the mathematics, estimate whether solutions are reasonable and generalizable, and reflect on the results (Madison & Steen, 2009). One of the key characteristics of a quantitatively literate person is his or her ability to understand the quantitative elements of everyday life, and that includes the ability to read and understand the daily newspaper (Trefil, 2008; Watson, 2004). The National Research Council (2012) emphasized the importance of knowledge that can be transferred to novel situations, and in particular, they listed several 21st century competencies that directly relate to QL, including critical thinking, problem solving, decision making, communication, information literacy, and media literacy. This idea of transferable knowledge is a key concept, because quantitatively literate individuals, by definition, have the ability to face authentic quantitative situations with confidence and skill. The ability to deal with the quantitative elements of everyday life, or QL, is also referred to as numeracy, mathematical literacy and quantitative reasoning (Quantitative Literacy Design Team, 2001). Though some authors use each phrase differently, others use them interchangeably. Throughout this paper, I will primarily use the phrase "QL."

Throughout this study, I will also make reference to students' attitudes towards mathematics. Haladyna, Shaughnessy, and Shaughnessy (1983) defined "attitude toward mathematics" as "a general emotional disposition toward the school subject of mathematics," and they distinguished this definition from "attitude towards the field of mathematics, toward one's ability to perform in the field of mathematics, or toward some specific area within mathematics" (p. 20). The authors stated that positive attitudes towards mathematics are important not only for their own sake, but also because of the slight positive relationship between attitudes and achievement, and because of the tendency for students with positive attitudes to choose additional mathematics courses or enter mathematical careers. Additionally, the authors argued that "changes in instruction probably have a more pervasive effect on a class of students than they do on the individuals who compose the class," so an in-depth analysis of students' attitudes could go hand-in-hand with my analysis of the impact of QL on two upper-level mathematics classes (p. 19).

Theoretical Framework

Two theoretical frameworks that have impacted my understanding of this course, my students, and the data collection and analysis process are the constructivist theory of learning and the principles of democratic mathematics education. I believe that constructivism best describes how students learn, and I view the teaching and learning that takes place in my classroom from a constructivist perspective. Constructivist theory is based on the work of Piaget, whose theory of radical constructivism revolves around the idea that "knowledge arises from the active subject's activity, either physical or mental, and that it is goal-directed activity that gives knowledge its organization" (von Glasersfeld, 1995, p. 56). A constructivist approach to learning sees teachers and students as "active meaning-makers who continually give contextually based meanings to each other's words and actions as they interact" (Grady, Watkins, & Montalvo, 2012, p. 39). Simon, Tzur, Heinz, and Kinzel (2004) outlined three key principles of radical constructivism that relate to learning mathematics:

> 1. Mathematics is created through human activity. Humans have no access to a mathematics that is independent of their ways of knowing. 2. What individuals currently know (i.e., current conceptions) affords and constrains what they can assimilate – perceive, understand. 3. Learning mathematics is a process of transforming one's ways of knowing (conceptions) and acting (p. 306).

These principles helped to guide my work as I relied heavily on a constructivist perspective to inform the creation of the course, my collaboration with students, and the data collection and analysis process.

In addition to a constructivist theory of learning, my research study has also been influenced by the democratic mathematics education literature. Ellis and Malloy (2007) described democratic education in the following way:

> Democratic education is a process where teachers and students work collaboratively to reconstruct curriculum to be inclusive of diversity. Each classroom will differ in its attributes because the interactions of democratic classrooms are based on student experiences and community and educational context…Democratic education is accessible to all students, rests on the assumption that all students can learn given the right circumstances, provides students with an avenue through which they can learn substantial mathematics, and helps students develop the tools to become productive and active citizens…Within the democratic mathematics classroom, students should see themselves in the curriculum and link mathematics to their everyday lives; they should see that mathematics is connected to social needs of the community; and that mathematics can expand and deepen their own democratic possibilities (pp. 160-161).

The democratic mathematics education literature played an important role in my decision to co-construct the course with my students, but it also impacted the way I planned the course, interacted with students, and collected and analyzed the data. Along with a constructivist theory of learning, the democratic mathematics education tradition has helped to shape the way I see myself as an educator, and similarly, it has provided a framework through which I have conducted this research study.

Literature Review

The purpose of this literature review is to examine previous research on QL and co-construction by determining the extent to which certain mathematics curricula have failed to combat innumeracy, analyzing the benefits and challenges of a pedagogy that focuses on QL, and discussing the relationship between QL and co-construction. In order to do so, I will outline some problems with the current educational system, present the case for QL as a curricular reform, position QL within broader theory on mathematics education, describe the relationship between QL and other segments of the literature, including the literature on co-construction, and describe previous research studies that implemented various QL initiatives.

Problems with Current Educational System

In order to frame the subsequent discussion, it may be helpful to look at editorials that were written by curricular experts. Welsh (2012) wrote an article entitled “Why our Kids Hate Math,” in which he argued that schools and parents instill in their students a dislike for mathematics by pushing them into advanced classes at an early age. Hacker (2012) contended that algebra serves as a gatekeeper that ends up filtering too many students, thus depleting our workforce of potential brainpower. He argued that mathematics can be used as “a hoop, a badge, a totem to impress outsiders and elevate a profession’s status,” rather than as a useful element of someone’s career training (p. SR1). Garfunkel and Mumford (2001) argued that our current abstract curriculum is not ideal, because “different sets of math skills are useful for different careers, and our math education should be changed to reflect this fact" (p. 27). Since many citizens do not grapple with concepts from algebra, geometry and calculus in their day-to-day lives, the authors recommended a shift in focus to more relevant problems with a contextual approach. What these authors all have in common is their concern about the relevance and effectiveness of our mathematics curricula. In order to better understand these concerns, it may be helpful to understand how mathematics education has responded to changing societal needs in the past.

Lack of Response to Changing Societal Needs

Cohen (2001, 2003) has stated that our current mathematics courses closely resemble the curriculum that has been in place since the 1820s. The fact that the curriculum has remained largely static is problematic, especially as the demands for active participation in society have changed so dramatically over the last two centuries, and in particular, the last two decades. With so much information available at the click of a mouse, students need to use quantitative thinking to understand issues as diverse as business decisions, finances, politics and environmental monitoring (Steen, 2001b). Disciplines like statistics have become much more important over the past century, yet our mathematics curriculum has barely changed (Cohen, 2003). Without an appropriate change in mathematics education, the "growing sophistication of numerical argument" could continue to hold people back from "full participation in this new style of thinking" (Cohen, 2001, p. 24). In a world where politicians, businesses and newspapers make regular use of numerical arguments, mathematics curricula must adapt to ensure that students are developing the skills that they need to reason quantitatively. The National Research Council (1989) anticipated this sluggishness more than twenty years ago: “we have inherited a mathematics curriculum conforming to the past, blind to the future, and bound by a tradition of minimum expectations” (p. 1). In an increasingly quantitative world, mathematics education has an important role to play, but if it cannot adapt, then many individuals will be left without the skills they need to actively participate in society.

Calculus Pipeline

One of the main criticisms of the current mathematics curriculum is that each course does little more than prepare students to take the next course. Steen (2001b) observed that school mathematics is often justified by its utility for future courses, thus making it a “hollow regime that many students unfortunately reject” (p. 15). Rather than choosing concepts “as prerequisites for something to follow that most students will never see,” Steen (2003) recommended that teachers choose topics to make connections within mathematics and between mathematics and the world (p. 65). Packer (2003b) argued that each year of mathematics should be important for its own sake, and not just for the sake of next year’s course. This temptation to justify mathematics courses merely as prerequisite knowledge for the next course is the danger behind the so-called “calculus pipeline” (Kennedy, 2001).

Kennedy (2001) argued that students are being prepared for college calculus and nothing else. Madison (2003) described this “calculus pipeline” as GATC, or geometry, algebra, trigonometry and calculus. GATC is firmly entrenched in policy, standardized assessments and college entrance requirements, so it is unlikely to change any time soon (Madison, 2004). Madison (2003) argued that GATC dominates not only the high school curriculum, but due to acceleration and tracking, it reaches down into middle school as well. Calculus is an important goal for many students, but when one course dominates the entire school curriculum, then we need to reconsider whether there is too much emphasis in too specialized an area. This "calculus pipeline" is so entrenched in our educational system that even reform efforts have not fundamentally changed the GATC sequence. Steen (2012) argued that even the most recent attempt at reform, the Common Core State Standards, continues to privilege pre-calculus skills over other types of mathematical abilities. As long as calculus is the primary goal of high school mathematics, then students who do not plan on taking calculus will not receive the necessary justification for the importance of their studies.

This "isolated trajectory of increasing difficulty and abstraction" is useful for some careers (Carnevale & Desrochers, 2003, p. 21), particularly in academia, but it should not be forced on every student (Orrill, 2001). Madison (2004) contended that GATC exists only because of the perceived needs of students who are seeking STEM careers, though they only account for one-fourth of the total population. Furthermore, many of the skills that are developed through these calculus-driven courses can be carried out much more efficiently with technology (Madison & Steen, 2009; Packer, 2003b; Trefil, 2008), so the traditional curriculum is not only inappropriate for the majority of the population, but it is also somewhat antiquated for the students that it was designed to serve. This leads one to question whether the "calculus pipeline" should continue to dominate our school curriculum, or whether there might be an alternative curriculum that better meets the needs of our student population.

Packer (2003b) went even further when he questioned the value of calculus, since mathematicians themselves often replace calculus with finite mathematics so that computers can perform required calculations. Much like Madison and Steen (2009), Packer wondered whether advances in technology should make us reconsider our traditional curriculum. Carnevale and Desrochers (2003) admitted that the abstract progression of high school mathematics is important, but they argued that the requirement for all students to experience the calculus course progression does not mesh well with society's need for citizens with basic numeracy and reasoning skills. Furthermore, the mathematics sequence has been appropriated as a sorting mechanism for many elite institutions and careers, so students experience increased abstraction, rather than relevance and utility. In sum,

> Most Americans seem to have taken too little, too much, or the wrong kind of math. Too many people do not have enough basic mathematical literacy to make a decent living even while many more people take courses in high school such as geometry, algebra, and calculus than ever will actually use the mathematical procedures taught in these courses (Carnevale & Desrochers, 2003, p. 25).

The problems with high school mathematics are definitely alarming, but the problems become even more complicated when one considers the influence of colleges. Colleges and universities have a large influence on high school curricula as a result of college admissions requirements. Unfortunately, this articulation is not clearly defined, and in some cases, it may even send conflicting messages. The SAT-I and ACT barely touch on intermediate algebra, trigonometry and pre-calculus, but many college placement tests require them (Kirst, 2003). High school students, therefore, receive mixed signals about what mathematics they need to learn to be successful in college. Furthermore, many high schools encourage the use of technology and graphing calculators as tools for the classroom, but some colleges prohibit the use of calculators on their admissions tests. What sort of messages are colleges sending, and why do they seem to run contrary to the messages that students receive in high school? Kirst (2003) suggested that the AP program is the only serious effort to provide clear articulation for grades 10-14, but even this program emphasizes standardized tests, rather than numeracy or quantitative reasoning. This march towards calculus has unified the mathematics curriculum towards a common goal; unfortunately, this common goal has little value for the majority of students who will not utilize calculus in their day-to-day lives.

Inadequate Pedagogy

The "calculus pipeline" is not solely to blame for the deficiencies in the educational system, as inadequate pedagogy can also hamper students' development of QL. Hughes-Hallett (2003) contended that traditional teaching may actually impede students' ability to develop QL. Poor teaching can train students to seek nothing more than the algorithm, and teachers often acquiesce to students' demands rather than helping them to construct deeper meanings. The result is that students memorize problem types, which is good in the short term for high-stakes tests, but not very beneficial in the long term. Textbooks exacerbate this situation, as they contain worked out problems for each problem type, so students are rarely faced with novel situations. This reliance on bad word problems is actually dangerous (Allen, 2001), since these problems are presented as real-life applications, when in reality they are meaningless (Schoenfeld, 2001; Usiskin, 2001). Since students may only see algebra in "sanitized template exercises" (Madison & Steen, 2009, p. 5), they can actually lose their good judgment when dealing with real world problems. Alsina (2002) cited the work of educational psychologist Lieven Verschaffel, who found that students experienced "suspension of sense making in mathematical modeling and problem solving" (p. 4). In other words, students lost the ability to use common sense when thinking about real life problems in the classroom. Unambiguous template problems allow for little interpretation or reflection, so they cannot effectively prepare students for reasoning in the real world (Madison, 2006). Schoenfeld (1990) lamented this development, as learning to solve these types of problems may actually teach students that mathematics problems aren't realistic at all (p. 324). Schoenfeld claimed that there is significant "nonreason" in school mathematics, as reasoning in school does not relate to reasoning necessary for real situations (p. 324). These deficiencies are extremely unsettling, particularly when one considers their impact on students' attitudes and achievement.

Consequences of the Shortcomings of the Current Mathematics Curriculum

The literature focuses on three major consequences of supposed shortcomings in the current mathematics curriculum: poor overall performance and skills, poor habits and attitudes, and an inability to transfer knowledge to new situations.

Using a TIMSS literacy study, Wilkins (2007) found that American students enjoy mathematics and find it easier than students in other nations, but their performance is still among the lowest. Wilkins wondered whether this discrepancy was a result of low standards, because students felt successful in their classes even though they performed at a low level. Others have cited specific examples of this poor performance, such as students' inability to work with fractions (Packer, 2003b; Schield, 2008). Packer (2003b) blamed this particular deficiency on middle school teachers, who, according to Packer, teach fractions in a way that is unrelated to the way fractions are actually used in the real world. Students also retain very little of what they learned in high school (Madison, 2003; Steen, 2012). Steen (2012) argued that students only remember a "pale shadow" of what they were taught and what the standards recommend (p. 3). In addition, he compared today's students with students in the 1980s, when half of them left high school without knowing mathematics because they were not required to take the courses. The unsettling difference, though, is that in today's schools, students take the courses, but they retain "little or nothing of the mathematics they have been taught" (p. 6). Madison (2003) argued that remedial mathematics courses in college can be even more depressing, since they are often just rehashed versions of arithmetic or high school algebra.

Along with poor performance and skills, students also develop poor habits and attitudes about mathematics. For many, it is culturally acceptable to hate mathematics, and this societal norm exerts a large influence on students (Dewdney, 1993). Some students and teachers approach mathematics courses only for the credit, because they do not believe that the subject has any relevance for their lives (Madison, 2006). This discrepancy between mathematics class and real life has a debilitating effect on students' perceptions, as they tend to see the two as being incongruent. Madison (2006) maintained that "some of the habits learned and attitudes formed in mathematics classes are actually obstacles to achieving the [numerate] habit of mind" (p. 2323), and some students can even develop views of mathematics that impede their development of QL (Hughes-Hallett, 2003). This separation of mathematics from real life has a negative impact on the utility of the material, and it hampers students' ability to apply knowledge in unfamiliar situations.

An inability to transfer knowledge is one of the most harmful consequences of a curriculum that separates mathematics from real life (Hughes-Hallett, 2001). If students are unable to transfer knowledge into new settings, then the knowledge they acquire and the skills that they gain will not help them in life after school. This is particularly troubling when one considers the amount of quantitative information that individuals face in the 21st century. In order to understand this information and reason through it in an educated, thoughtful way, students need transferable knowledge, rather than general skills (Steen, 2012). When discussing the prerequisite knowledge of business students, Albers (2002) stated that students' mathematics education is "either insufficient or difficult to apply to the situations they face in professional settings" (p. 14). This is a recurring theme, as students are not learning the skills they need, forgetting the skills they once knew, or learning things that they cannot utilize in their lives after graduation.

How then, are we to address this predicament? Is there any way for the mathematics curriculum to adjust its content and pedagogy to better prepare citizens for the quantitative demands of everyday life? The Quantitative Literacy Design Team (2001) attempted to answer this question with the publication of their seminal work, "The Case for Quantitative Literacy."

The Case for Quantitative Literacy

The National Council on Education and the Disciplines (NCED) encouraged a group of teachers, professors and leaders in education to inquire into the meaning of numeracy in the 21st century (Orrill, 2001). Led by Lynn Steen, this Design Team developed "The Case for Quantitative Literacy." The Quantitative Literacy Design Team (2001) lamented the fact that in a world "awash in numbers," most students are not quantitatively literate (p. 1). QL is a necessary component of a person's overall literacy, and government agencies codify this point by subdividing literacy into prose, document and QL. QL is a "direct analog" of verbal literacy in terms of what is needed for "active and alert participation in contemporary society" (p. 9). Unfortunately, most students are not quantitatively literate, and while this problem often applies to students who do poorly in mathematics, even students with advanced mathematical backgrounds are often "unable to comprehend (much less to articulate) the nuances of quantitative inferences" (p. 2). As stated earlier, the high school mathematics curriculum prepares students primarily for college mathematics, rather than for the situations they will face in their everyday lives. The inability for high school mathematics to prepare students for QL is understandable, as mathematics focuses on abstractions while QL focuses on context. Connecting mathematics to authentic contexts is a balancing act, as context hides the broad patterns that define mathematics. On the other hand, those same contexts provide the relationships and the motivation that are critical for life-long learning. The Design Team highlighted several important aspects of QL, with the goal of helping educators better understand how to restructure their classrooms.

The Quantitative Literacy Design Team (2001) listed several elements of QL: confidence with mathematics, cultural appreciation, interpretation of data, logical thinking, quantitative decision-making, contextual mathematics, number sense, practical skills, prerequisite knowledge and symbol sense. These elements overlap with elements of the traditional curriculum, but they place particular emphasis on reasoning and context. Some of the key skills of QL include arithmetic, data analysis, comfort with computers, modeling, knowledge of statistics and chance, and general mathematical reasoning. The Design Team emphasized that all of these skills must be taught and learned in context, rather than in unrelated classroom scenarios. Some of these contexts are commonplace, such as splitting a bill three ways or understanding interest, but there are many other valuable expressions of QL. Some of these expressions include voting, sampling, analyzing economic and demographic data, personal finance, personal health, management, work, and applications in a good number of college majors, including biology, medicine, social sciences, psychology, art history and language. The key difference between QL and the traditional mathematics curriculum is that QL is "driven by issues that are important to people in their lives and work, not by future needs of the few who may make professional use of mathematics or statistics" (p. 18). The Design Team argued that teaching for QL, much like other student-centered approaches, will require a very different way of approaching education, and content, pedagogy, context, culture and interdisciplinary work will all have to be a part of this change. The case for QL as one potential solution to the problem of innumeracy is a provocative one, and in order to better understand what a QL curriculum looks like, it may be helpful to go more in depth into some of its primary characteristics.

Characteristics of a QL Curriculum

Teaching for QL requires progressive pedagogy that seeks out real-world applications, emphasizes understanding over memorization, and prefers depth over breadth (Cuban, 2001). Burkhardt (2008) stated that teachers must embrace the world beyond mathematics, and they should empower students by providing strategic guidance and supplementary questions that are unique to each student, with the goal of giving students ownership over their work. Technology must also be an integral part of teaching for QL (Catalano, 2010; Edwards, 2008; Madison, 2006; SCANS, 1991; Steen, 2003; Taylor, 2008), and it should be used organically throughout the curriculum. While these attributes can be found in many mathematics curricula, the authors have stated that they are nonnegotiable in a QL curriculum. Along with technology, the literature focuses on two additional elements of pedagogy that can effectively develop QL skills: applications and interdisciplinary work.

Applications and modeling skills are essential for teaching QL (Alsina, 2002), and the implications of these applications and models should also be considered (Davis, 1993). Specifically, teachers must focus on “real applications, based on real data, on objects and instruments, on everyday situations, on frequent or recent events" (Alsina, 2002). This real-world problem solving not only requires mathematical accuracy, but practicality as well, since all problems are contextual in nature (Pollak, 1997). The fact that QL relies on context is a strength (Madison & Steen, 2009; Steen, 2001b), as context lends itself to motivation and student learning (Steen, 2004). Teaching for QL is different from traditional pedagogy because instruction moves from complicated, real-life situations to generalizable abstractions, and not the other way around (Dewdney, 1993). De Lange (2003) stated that "applications should not be reserved for consideration only after learning has occurred; they can and should be used as a context within which the learning of mathematical concepts takes place" (p. 87). The focus then shifts to using mathematics as a tool, as teachers and students become concerned primarily with solving real-world problems.

Since quantitatively literate students must be able to understand numerical aspects of any context, teaching for QL must be interdisciplinary (Cohen, 2001; Ellis, Jr., 2001; Hughes-Hallett, 2001; Madison, 2004; Orrill, 2001; Richards, 2001; Steen, 2001a). Steen (1999) argued that numeracy feeds the entire curriculum, and Malcom (1997) contended that "mathematics needs to make explicit connections with other subject areas, with the world of work, and with people's everyday lives" (p. 73). Authors from several disciplines made the case for why QL should be an integral part of their curriculum as well. In social studies, Crowe (2010) claimed that traditional mathematics courses do not help students "make reasonable judgments of and inferences from information presented to them in the media, by the government, or by other citizens" (p. 105). In order to rectify this situation, Crowe suggested interdisciplinary work in which history students learn to analyze raw numeric data, understand percentages and averages in context, and interpret and question graphs and charts. Miller (2010) made a compelling case for the intersection of English and QL, because while students need to "read, understand, solve, and write about word problems," they also need to write the answer "in prose in ways that place it back in its original substantive context, thus bringing the word problem full circle" (p. 336). Lutsky (2008) claimed that QL should not only be modeled after the writing across the curriculum initiative, but he went further to say that it should be "intertwined with teaching writing" (p. 60). Lutsky continued:

> We need to show others that numbers can contribute to precision in our thinking, facilitate the public discussion and evaluation of claims, help us grasp the attributes of large and complex phenomena, organize vast domains of information, and help us discover patterns of relationships not readily available to human perception (p. 61).

Authors also made the case for the intersection of QL with business (Albers, 2002; McClure & Sircar, 2008; Taylor, 2008), economics (Schuhmann, McGoldrock, & Burrus, 2005), and sociology (Atkinson, Czaja, & Brewster, 2006; Howery & Rodriguez, 2006; Lindner, 2012; Sweet & Strand, 2006). Many disciplines see value in the improvement of QL, despite the fact that QL is a relatively new movement in mathematics education. One potential reason for the attractiveness of QL is the fact that it applies some of the key theories in mathematics education over the past century.

Theoretical Underpinnings of QL

Theories of how students learn mathematics have changed quite considerably over the last century. The traditional belief was that the teacher had the knowledge, and learning only took place when he or she filled students' heads with that knowledge. Over the past century, however, this belief has been refuted by prominent thinkers such as Piaget, Vygotsky, and Lave and Wenger. In his theory of radical constructivism, Piaget argued that children construct knowledge for themselves, and therefore, learning only takes place when a child incorporates something new into his or her own preexisting knowledge structure (von Glasersfeld, 1995). Vygotsky agreed with Piaget, but he went a step further to argue that the social milieus surrounding the child are key elements in the child's construction of his or her own knowledge (Van Oers, 1996). Lave and Wenger extended the work of Vygotsky to describe the phenomenon of situated learning, in which learning can only be understood within a particular context or environment. Each of these theories has significant ramifications for QL, and in a sense, QL can be seen as a practical application of these important theories.

Piaget's theory of radical constructivism revolves around the idea that "knowledge arises from the active subject's activity, either physical or mental, and that it is goal-directed activity that gives knowledge its organization" (von Glasersfeld, 1995, p. 56). For Piaget, knowledge was not a collection of information or facts; rather, it was something that is continuously created and recreated by each individual. Radical constructivism revolutionized the way that educators thought about education. For the first time, educators had to take into account the individuals themselves, since learning could only take place if an individual could incorporate new results into their preexisting knowledge structure. New theories about how children learn would not cease with Piaget, however. Where Piaget did a great service by focusing the discussion on the individual, Vygotsky enriched the discourse by stressing the importance of society and culture:

> Constructivism often seems to stick to the view that children build and develop their own mental structures through interaction with the (social) environment. Cognitive apprenticeship from a Vygotskian perspective, on the other hand, implies that the qualities of mental development are derived from the distinctive properties of the sociocultural organization of the activity (Van Oers, 1996, pp. 107-108).

Vygotsky, then, built on Piaget by incorporating the way in which culture influences how an individual constructs his or her own knowledge.

Like Piaget, Vygotsky believed that education takes place when students develop their creative potentials through personal activity (Van Oers, 1996). Unlike Piaget, though, Vygotsky's cultural-historical theory states that the development of human personality "has a specifically historical character, content and form" (Davydov, 1995, p. 15). For Vygotsky, "education…was basically a process of enculturation" in which members of the community came alongside a student to help them "reconstruct (reinvent) valuable cultural elements in a meaningful way" and "grow into the intellectual life of those around them" (Van Oers, 1996, p. 93). Engaging in a sociocultural activity with the help of an adult lies at the heart of Vygotsky's *zone of proximal development* (ZPD). The ZPD "is constructed in the cooperation between the child and the adult on the basis of what the child wants and the actions the child actually can carry out, as well as the help the child gets from the adult" (Van Oers, 1996, p. 97). The construction of knowledge is at its heart Piagetian, but the assistance from the adult is distinctly Vygotskian. A learning model based on the ZPD relies on the child's activities and interests and the expertise and guidance provided by the adult.

Taking Vygotsky's model a step further, Lave and Wenger described a theory of situated learning, in which "learning, thinking, and knowing are relations among people engaged in activity *in, with, and arising from the socially and culturally structured world*" (Lave, 1991, p. 67). Greeno (1989) described situated cognition in the following way: "Thinking is situated in physical and social contexts. Cognition, including thinking, knowing, and learning, can be considered as a relation involving an agent in a situation, rather than as an activity in an individual's mind" (p. 135). As an example, Greeno referenced the work of Scribner (1984), who observed the behaviors of workers for a dairy company. The workers were required to fill containers of various sizes, and in more than 90% of the cases, they chose the optimal packing method. What is interesting, though, is that the workers' performance was not based on the knowledge they would have acquired in school; rather, they developed the ability to work effectively through their day-to-day experiences. Lave (1991) described these groups as communities of practice, where "common, shared, knowledgeable skill gets organized" through "ongoing everyday activities," rather than more formal approaches to education (p. 71). Lave and Wenger's theory of situated learning extends the work of Vygotsky, because they emphasized the importance of context as well as culture. Unfortunately, Lave (1991) noted that "genuine participation, membership, and legitimate access to ongoing practice" are rare within schools (pp. 78-79). One way to apply the principles of situated learning, while also taking into account the work of Piaget and Vygotsky, would be to focus the mathematics curriculum on teaching for QL.

QL builds on the work of Piaget because it affords students an opportunity to study what they care about. QL activities build on students' prior knowledge, and they create situations where students can expand on their previous understandings. QL also takes into account Vygotsky's ZPD, because adults come alongside students to help them develop new understandings. Effective QL pedagogy scaffolds instruction to help students extend their own knowledge to new and complex situations. Finally, teaching for QL is a practical application of Lave and Wenger's situated learning, since it attempts to simulate authentic experiences in the classroom. In sum, teaching for QL attempts to implement the theories of Piaget, Vygotsky, and Lave and Wenger, as it creates an environment that helps students extend their own knowledge to understand complex, real-life phenomena.

Benefits of Teaching for QL

The literature discussed many benefits associated with teaching for QL, but five main themes emerged: QL as necessary for democracy, QL as a requirement for a democratic mathematics classroom, QL as an impetus for co-construction, QL as a force for equity, and QL as a stimulus for higher achievement.

Democracy

In the late 1980s, the National Research Council (1989) declared that "mathematical illiteracy is both a personal loss and a national debt" (p. 18). Like the work of Paulos (1988), the NRC sounded an alarm that innumeracy was both a personal and a national problem. Since quantitative reasoning is such a necessary skill for US citizens (Madison & Steen, 2009), QL becomes essential for the preparation of future citizens and the functioning of our democracy (Alsina, 2002; Cuban, 2001). As Steen (2003) stated, "numeracy lies at the intersection of statistics, mathematics, and democracy" (p. 62). "Just as verbal literacy gives students the tools to think for themselves, to question experts, and to make civic decisions, quantitative literacy does exactly the same in a world increasingly drenched in charts, graphs, and data" (Cuban, 2001, p. 87). Quantitatively literate individuals have greater potential to influence the world around them, so they are better prepared to improve their own position and the positions of others (Wiest, Higgins, & Hart Frost, 2007). If students learn to plan, challenge, negotiate, and evaluate the work they do in mathematics class, then they will be better prepared for civic life (Noddings, 1993), and better equipped to improve their position in life. If students learn to apply the analytical tools of mathematics to examine inequalities and study societal problems (Ball, Goffney, & Bass, 2005), then they will also be better prepared to improve the positions of others.

Democratic Mathematics Classroom

Teaching for QL can also be compatible with the principles of a democratic mathematics classroom. Ellis and Malloy (2007) have argued that a democratic mathematics classroom must include the following: a problem solving curriculum, inclusivity and rights, equal participation in decisions that affect students' lives, and equal encouragement for success. Most QL curricula are designed to satisfy the first of these criteria, and certain techniques that are related to teaching for QL, some of which I incorporated in this research study, may satisfy the other three.

A QL curriculum satisfies Ellis and Malloy's (2007) problem solving requirement because it gives students opportunities "to draw on their accumulated knowledge to solve problems important to their lives and society" (p. 161). These types of important problems are common in a QL classroom, since the very definition of QL highlights the skills "that people need in order to engage effectively in quantitative situations arising in life and work" (Alsina, 2002, pp. 2-3). In addition, teaching for QL can promote inclusivity and rights by "affirming the worth of diverse experiences" (Ellis & Malloy, 2007, p. 161), equal participation in classroom decision-making by allowing students to design their own experiences (Allen, 2011), and equal encouragement for success by providing students with "access to materials that engage them actively in the learning of mathematics" (Ellis & Malloy, 2007, p. 161). One way that QL classrooms can do this is through the process of co-construction, where students and the teacher come together to plan content and lesson types that will best develop students' QL.

Co-construction

Co-construction is one of many approaches to structuring a QL classroom, but because it directly impacts inclusivity and rights and equal participation in decisions that affect students' lives, and may indirectly lead to a problem solving curriculum and equal encouragement for success, then it may be the one that most closely aligns with Ellis and Malloy's (2007) characteristics of a democratic mathematics classroom. A small number of research studies from other disciplines have considered aspects of co-construction, but I was only able to find two that investigated a style of co-construction that was similar to the one employed by this study. The first study examined how a responsive teaching environment impacted scientific inquiry in one teacher's fifth-grade classroom over two consecutive years (Maskiewicz & Winters, 2012). As opposed to a classroom with a pre-scripted curriculum, the daily activities in a responsive classroom depend on the ideas and interests of the students themselves. Practically speaking, "the teacher listens carefully to students' ideas and brainstorms—either in-the-moment or between class periods, either alone or with others—what possible next moves might be warranted by the ideas in play" (p. 433). Students' ideas "become the terrain for discussions and investigations," and the "teacher's and students' expectations for how to pursue a scientific understanding are negotiated over time and vary depending on communal resources" (p. 433). The authors found that different forms of inquiry became normative in each of the classes, and they attributed these differences to the students' unique intellects and interests, to the teacher's flexibility in how she ran her courses, and to professional development that promoted responsiveness to student thinking.

A second study examined the co-construction of midterm exams at the undergraduate level (Ahn & Class, 2011). In this study, the instructor gave students some criteria regarding the types of questions that could be used, but students were given the freedom to create the actual exam questions in small groups. The authors found that students approached this task very differently, as one group had each person write the entire exam and then compare, a second group assigned each member one question to complete, and a third group assigned some members to be writers and others to be researchers. A student reflected on her experience, and she found that this exercise "provided an entry point for every student, regardless of ability level, and enabled their active and successful participation in the activity, and, thereby, their learning" (p. 274). The instructor offered a slightly different perspective, as she described feeling "nervous and uneasy when I decided to shift the power of creating midterm exam questions to my students and treat them as partners" (p. 274). The authors concluded that "by allowing each individual to bring his or her own unique contributions to a particular task, as well as creating a climate of open dialogue between students at all academic levels," this activity fostered an inclusive classroom environment that may have benefited and empowered all students, while excluding none (p. 277). It is exactly this type of outcome that makes co-construction a valuable tool for a democratic classroom, because co-construction can promote inclusivity, reduce exclusion, and provide equal encouragement for success.

Equity

The benefits of QL for democracy carry over to issues of equity as well. A democratic society should give all citizens the opportunity to advance their position in life, but mathematics education's role as gatekeeper has traditionally acted as a filter, rather than as a potential equalizer. Madison (2003) stated that the role of mathematics as a filter "misuses mathematics and abuses students;" even worse, "many mathematics faculty accept the long tradition of their discipline as a filter and expect a large number of students to fail" (p. 160). With this misuse of mathematics, the United States is at risk of becoming divided "both economically and racially by knowledge of mathematics," as poor mathematics preparation has a disproportionate effect on minorities (NRC, 1989, p. 13). QL, on the other hand, is about "the democratization of mathematics," rather than mathematics education's traditional task of separation (Steen, 2002). QL offers the opportunity for a more equitable society because it equips more students with the skills they need to participate in civic life (Wiest, Higgins, & Hart Frost, 2007). QL curricula could decrease the perception that some students are better at mathematics than others, as more students become engaged and ultimately successful in mathematics (Stith, 2001). By teaching students to reason quantitatively, mathematics could not only become "accessible to all students," but it could also provide them with the tools they need to become successful and participatory citizens (Ellis & Malloy, 2007, p. 161). QL then, is one possible way to transform mathematics education from a sorting mechanism to a mechanism that can uplift and empower.

Moses and Cobb (2001) have argued that "math literacy and economic access are how we are going to give hope to the young generation" of Blacks in the United States (p. 13). They described algebra as a gatekeeper not only for higher mathematics, but also as a barrier for citizenship. Consequently, Moses and Cobb championed the successful completion of algebra as a civil right (Ellis & Malloy, 2007). In order to help students gain mathematics literacy, Moses and Cobb (2001) have recommended a version of experiential learning in which students reflect on their common culture, form abstract conceptualizations, and apply those abstractions back on their experiences. In practice, the Algebra Project involves physical trips, modeling, intuitive language, structured language, and symbolic representation. Much like QL, the Algebra Project capitalizes on student interests and experiences to engage students in big ideas and complex problems. In so doing, Moses and Cobb hope to give Black students the opportunity to develop the skills and credentials to access more lucrative employment opportunities.

Rivera-Batiz (1992) contended that QL has an independent effect on the probability of an individual obtaining employment. Carnevale and Desrochers (2003) went further to say that individuals with quantitative skills earn more than others. Specifically, they found Algebra II to be the "threshold mathematics course taken by people who eventually get good jobs in the top half of the earnings distributions" (p. 26). Further, they found that the number of mathematics courses students take beyond Algebra II was highly correlated with future employment in the top quarter of the earnings distribution. QL can serve two purposes, then, because it can provide students with skills to help them improve their own position in life, but Gutstein (2006) argued that students can also develop skills that would help them combat injustices on a much broader scale.

In *Reading and Writing the World with Mathematics: Towards a Pedagogy for Social Justice*, Gutstein (2006) argued that many of the elements of QL could help to empower students and challenge inequalities. Gutstein described a mathematics curriculum that utilized “real and potentially controversial issues,” where mathematics became a tool to investigate, understand, and possibly act on social issues (p. 3). He argued that equity in mathematics education is not just about what students learn in the classroom, but it is also about what they can do with the mathematics they learn. Gutstein distinguished two types of literacies that can be developed in the classroom: functional and critical. Functional literacy, he contended, reproduces the social purposes of schooling, while critical literacy develops critical and skeptical abilities. Unfortunately, the traditional curriculum develops functional literacy, since “schooling tends to reproduce dominant social relations” (p. 7). Gutstein advocated for critical literacy, and in order to achieve this goal, he utilized many of the practices that promote QL. Gutstein described how he devoted 15% to 20% of his class time to real-world projects that empowered students to read and write the world with mathematics. Gutstein’s is the type of pedagogy that uses mathematics in context, and it is exactly the type of approach that makes QL such a powerful tool in the drive for a more equitable system of mathematics education.

Achievement

While QL can equip students with skills to fight inequality and provide them with greater economic opportunities, it can also serve the more general role of improving student achievement. If mathematics classrooms took a QL approach and offered relevant and functional contexts (Steen, 2001b) that students actually cared about (Allen, 2001; Malcom, 1997), then maybe fewer students would fail (Stith, 2001). Several studies provide evidence for this. For example, Schiefele and Csikszentmihalyi (1995) found a positive relation between interest and achievement, Ma (1997) noted a reciprocal relationship between attitudes in mathematics (specifically enjoyment) and achievement, and Koller, Baumert, and Schnabel (2001) found a correlation between interest and achievement at the upper secondary level, especially when instruction doesn't involve written examinations with positive and negative consequences. Each of these examples underscores the importance of developing students' interests, which along with self-efficacy and motivation, is one of the primary benefits of teaching for QL.

Since one of the major benefits of teaching for QL is the promotion of students' interests, it may be beneficial to look more closely at a theoretical model of how interest develops. Hidi and Renninger (2006) use a four-phase model to describe a theory of how students' interests are triggered, and they provide a variety of implications for education. The basic premise of their model is that situational interest, or "focused attention and the affective reaction that is triggered in the moment by environmental stimuli," can develop into individual interest, or "a person's relatively enduring predisposition to reengage particular content over time" (p. 113). More specifically, the first phase is known as “triggered situational interest”, which can be generated by “environmental or text features such as incongruous, surprising information; character identification or personal relevance; and intensity” (p. 114). Pedagogical techniques such as group work, puzzles, and the use of technology have been found to promote situational interest. The second phase is known as “maintained situational interest.” Pedagogical techniques that foster its development include “meaningful and personally involving activities, such as project-based learning, cooperative group work, and one-on-one tutoring” (p. 114). It is not surprising that many of the techniques that support these two phases are commonly found in QL classrooms, since QL focuses on content and skills that are relevant to students’ daily lives. In order for students to move to the next phase, though, the burden shifts from the teacher to the learner.

The third phase is known as the “emerging individual interest” phase, and Hidi and Renninger (2006) have described some indicators that a student has moved to this phase:

> The student values the opportunity to reengage tasks related to his or her emerging individual interest and will opt to do these if given a choice; the student begins to regularly generate his or her own “curiosity” questions about the content of an emerging individual interest; as an outcome of such curiosity questions or self-set challenges, students may redefine and exceed task demands in their work with an emerging individual interest; [and] the student is likely to be resourceful when conditions do not immediately allow a question about content of emerging individual interest to be answered (p. 115).

Though emerging individual interest is often self-generated, there are some techniques that can create an environment that is conducive to its development, such as the use of models or support and encouragement from experts or peers. The fourth and highest phase of interest, known as “well-developed individual interest,” is similar to the third in many ways, but in this phase learners will "persevere to work, or address a question, even in the face of frustration" (p. 115). Though this level of interest is "typically but not exclusively self-generated," classroom structures that foster interaction and introduce appropriate challenges may support its growth (p. 115). This four-phase model has major implications for education, and consequently, for the development of students' QL.

Research has shown that teachers can promote student interest in many ways, but they can be particularly influential during the first and second stages of interest development (Hidi & Renninger, 2006). The primary way to promote interest is to help students develop positive attitudes about their ability to engage with classroom content. Positive feelings can be supported in the following ways:

> Offering choice in tasks, promoting a sense of autonomy, innovative task organization, support for developing the knowledge that is needed for successful task completion, …building a sense of competence[,] …project-based learning that includes students' work with peers or other social situations, computer environments that are attractive, and word problems or passages that have contexts specifically addressing students' individual interests (p. 122).

These characteristics are definitely illustrative of a QL classroom, particularly one that gives students the time and the resources they need to explore their developing interests. Teaching for QL often requires that teachers have the flexibility to respond to students in this way, and by supporting the development of emerging or well-developed individual interest, teachers can hope to parlay student interest into higher levels of achievement.

A QL classroom is not only well-suited to encourage student interest, but it also has the potential to promote self-efficacy. Self-efficacy is defined as the belief in "one's capabilities to organize and execute the courses of action required to produce given attainments" (Carmichael, Callingham, Hay, & Watson, 2010, p. 85). Carmichael et al.

found that students' self-efficacy for statistical literacy is highly correlated with their interest, so if a QL classroom has more potential to pique students' interest, then it would follow that teaching for QL could develop students' self-efficacy as well. It is beneficial to develop students' self-efficacy because "of all the psychosocial factors, self-efficacy is reported to be the best predictor of achievement in an educational context" (p. 85). The relationship between self-efficacy and interest gets more complicated, though, as research has shown that neither low self-efficacy nor high self-efficacy promotes interest, because in both situations, the outcome is certain (Silvia, 2003). Rather, "students with mid-levels of self-efficacy, those with sufficient uncertainty regarding their mastery of the task, are expected to report the most interest" (Carmichael, et al., 2010, p. 85).

While it would appear that this diminishes the link between teaching for QL and self-efficacy, there are some attributes of a QL classroom that might still lend themselves well to this relationship. On the one hand, a QL classroom is very accessible to students, since it draws on topics that are relevant to students' lives and requires very little in terms of advanced mathematics. On the other hand, QL classrooms address problem situations that are sophisticated and complex, and they require students to understand the nuances that can be involved in real-life problem solving. Perhaps teaching for QL strikes the right balance between giving students an entryway into the mathematics while still challenging them with problems that do not always have clearly defined answers. By finding this middle ground and choosing tasks that are in each student's ZPD, QL classrooms can attend to students' self-efficacy, and thereby elevate their interest, and hopefully achievement, in mathematics.

In addition to promoting interest and self-efficacy, QL classrooms have the potential to support student motivation as well. Middleton and Spanias (1999) defined motivations as "reasons individuals have for behaving in a given manner in a given situation" (p. 66). The authors differentiated between intrinsic and extrinsic motivation, and they argued that "providing opportunities for students to develop intrinsic motivation in mathematics is generally superior to providing extrinsic incentives for achievement" (p. 81). Citing the earlier work of Middleton, Littlefield and Lehrer (1992), the authors described a model of how intrinsic motivation develops in the classroom:

> They asserted that when one first encounters an academic activity, she will tend to evaluate the stimulation (challenge, curiosity, fantasy) it provides and the personal control (free choice, not too difficult) the activity affords. If her arousal and control requirements are met consistently, she may choose to include the activity among her interests (p. 75).

Therein lies the true power of the QL classroom: if students repeatedly engage with mathematics that is relevant, interesting, and accessible, then they may begin to develop intrinsic motivation for the subject. Teaching for QL might be the "radical and consistent change" that is necessary to overcome students' lack of motivation (p. 75), since it emphasizes skills that students need to understand the quantitative situations in their everyday lives.

Challenges of QL

While teaching for QL has many potential advantages, it is not without some formidable challenges. Two of the primary challenges relate to the construction of authentic situations in the classroom and the so-called planning paradox, but there are additional theoretical and practical challenges as well.

Difficulties in Creating Authentic Situations

One challenge faced by the QL classroom is the difficulty of trying to incorporate authentic situations in a classroom setting. Jurdak (2006) has argued that real-world problem solving and situated problem solving in schools are very different exercises:

> The situated problem solving in the school context is an activity within the school community, which results in a written solution using mostly mathematical tools and constrained by school rules, norms, and expectations; whereas, decision-making in real life is a complex activity that occurs within the larger social context and which results in a decision constrained by the acceptable social and personal rules and using all available mathematical and non-mathematical tools (p. 296).

Beswick (2011) added to this complexity, as she noted that each authentic problem must be filtered not only by the context of the classroom, but also by each individual's prior knowledge and experience. The result is a "nested pair of contexts, both of which include subjective aspects—the context evoked by the problem sitting within the context in which the problem is encountered" (pp. 383-384). While it might not be possible to bring truly authentic problem situations into the classroom, there are still benefits to simulating real life situations in the classroom. Real life situations may be able to make mathematics more meaningful for students, and they also "may provide an opportunity for appreciating the power and limitations of using mathematics in the real world" (Jurdak, 2006, p. 298). Unfortunately, lack of time, poor attendance, lack of materials and funding, and inflexible and traditionally trained teachers can often serve as major obstacles to this type of instruction (Dennis & O'Hair, 2010).

Planning Paradox

Ainley, Pratt, and Hansen (2006) described a slightly different challenge for the QL classroom with their idea of the planning paradox:

> If teachers plan from tightly focused learning objectives, the tasks they set are likely to be unrewarding for the pupils, and mathematically impoverished. If teaching is planned around engaging tasks the pupils' activity may be far richer, but it is likely to be less focused and learning may be difficult to assess (p. 24).

There is a great deal of tension between closely monitored, state-mandated curricula, and the freedom and professional preparation that may be necessary to engage in QL-related content. The authors provided two examples to highlight this paradox. On the one hand, a teacher who wants students to learn how to add two-digit numbers "will find it difficult to make the tasks interesting in more than a superficial way" (p. 24). At the other end of the spectrum, a teacher who asks students to design an ideal bedroom "may find it difficult to take advantage of such [mathematical] opportunities, or to monitor any mathematical thinking" (p. 24). As a result, attempts at authentic instruction will ultimately fall short of true authenticity, because "the more [mathematical activities] are shaped to have clear mathematical focus, the further removed they become from socially meaningful contexts" (p. 27). Consequently, the authors suggested that it may be easier to engage students and focus classroom instruction if teachers use contextual mathematics to emphasize the purpose and usefulness of mathematics. While this does not prevent teachers from teaching for QL, it does offer a slightly different perspective than an authentic or situated approach.

Additional Theoretical and Practical Challenges

There are some additional theoretical arguments against teaching for QL, and four are particularly prevalent: QL is too difficult, QL cannot actually be taught, major curricular changes are unrealistic, and any emphasis on QL will harm mathematics (Madison, 2004). Madison acknowledged that these concerns have been stubbornly persistent, but he attempted to refute them one by one. Firstly, Madison contended that QL does involve sophisticated applications and confusing terminology, but he claimed that the content itself is nothing more than elementary mathematics. Secondly, Madison noted that critical thinking and problem solving are emphasized in schools, and QL could not be more difficult to cultivate than any other habit of mind (but to be fair, it is not easy to cultivate critical thinking or problem solving skills either). Thirdly, major curricular changes may be rare, but they are possible, with writing across the curriculum as a recent example. Finally, Madison admitted that emphasizing QL might require some sacrifices in the traditional program of study, but he believed that QL is important enough that the curriculum should adjust to meet both needs. Steen (2012) also addressed the concern that a new approach could slight some traditional topics: "That's almost certainly true…but the current approach virtually guarantees that large numbers of students will never learn (or at least, not remember) these same rarely reinforced topics. So there may be little risk and much potential gain in trying a different approach" (p. 5).

Along with some theoretical concerns, there are also some very practical challenges in teaching for QL. Hughes-Hallett (2003) stated that teachers have no experience teaching for QL, and as a result, extensive professional development would be necessary (Ewell, 2001; Usiskin, 2001). Some studies have shown that professional development does have a positive impact on teaching for QL (Carbone, 1998; Edwards, 2008), but schools may be unlikely to make the requisite financial investment. Even with highly trained teachers, there are still some major obstacles to developing students' QL. Madison and Dingman (2010) documented some of the challenges they faced over a six-year period in the creation and implementation of a college-level QL course. Specifically, the authors found that QL was not an automatic consequence of mathematical or statistical fluency (implying that more mathematics might not necessarily lead to higher QL), that students tended to struggle transferring knowledge to new situations, and that students were reluctant to claim that they were good at mathematics, which served as an impediment to their development of QL. In terms of content, the authors found that students persistently used the wrong base for percents, they were unable to determine the reasonableness of answers, they confused magnitude with relative change, and they tended to think that bigger numbers, even death rates at a hospital, were always better. Finally, students had major problems with algebra, and they rarely understood it as an appropriate method to solve problems. Students' algebraic knowledge was organized around a "fragmented collection of methods" rather than "core concepts," and they had a limited ability to reflect back on their answer once they employed an appropriate model (p. 11). These practical challenges made teaching for QL even more complicated, and when combined with the pedagogical and political concerns, one can see why the traditional mathematics curriculum has remained dominant for so many years.

Research Projects in QL

Several studies have been conducted at the college level to explore the efficacy of teaching for QL. Many of these projects have looked at how colleges address QL in a comprehensive manner, while others have investigated the efficacy of classes that were designed to develop QL. No claims are made about whether these programs and courses meet the requirements for QL that were outlined in "The Case for Quantitative Literacy." Rather, since there is a paucity of research on QL initiatives, each study is reported here if it claims to support the recommendations to teach for QL.

How Colleges Address QL

In 1996, the Mathematical Association of America (MAA) published a report describing how colleges should address quantitative reasoning. In this report, the authors stated four main conclusions:

> 1) Colleges and universities should treat quantitative literacy as a thoroughly legitimate and even necessary goal for baccalaureate graduates; 2) Colleges and universities should expect every college graduate to be able to apply simple mathematical methods to the solution of real-world problems; 3) Colleges and universities should devise and establish quantitative literacy programs each consisting of a foundation experience and a continuation experience, and mathematics departments should provide leadership in the development of such programs; 4) Colleges and Universities should accept responsibility for overseeing their quantitative literacy programs through regular assessments (Sons, et al., 1996).

Based on the literature, it is clear that many colleges and universities have taken up this charge. Brakke and Carothers (2004) described how James Madison University utilizes multiple approaches to develop quantitative reasoning skills. The authors described this initiative as a balancing act, as liberal education goals for all, the need to develop specific skills for students in quantitative disciplines, and the desire to develop quantitative skills for students in other disciplines all compete for primacy. Add teacher preparation to the mix, and one can see how difficult it is to have a school-wide goal for QL at a university with specialized majors. In order to implement this program, James Madison has focused on freshman advising, student support, pre- and post-testing, and interdisciplinary work, while also instituting curricular changes in the mathematics department and supporting minors in quantitative subjects. Diefenderfer, Doan, and Salowey (2004) described the quantitative reasoning program at Hollins University, in which students have to satisfy a basic skills and an applied requirement. The basic skills component for QL requires a weekly computer lab, and the applied component requires students to complete at least two projects where they must use quantitative reasoning in real-world situations.

Much like the program at James Madison, Hollins University places a great deal of emphasis on professional development, which is understandable given the nontraditional pedagogy that is required to teach for QL. The authors found that students improved on a post-test as a result of completing their QL courses, particularly in regards to their applied skills, and they also reported a substantial improvement in their self-assessment. Richardson and McCallum (2003) echoed the two previous papers in their description of Wellesley College's QL requirements. Wellesley requires that students take a course that emphasizes literacy, authenticity, applicability, understanding, and practicality. These components closely mirror characteristics of QL from "The Case for Quantitative Literacy," so it would appear that many colleges are buying into the importance of QL for a liberal arts education. Additional studies have evaluated the efficacy of QL programs at the college level. Steel and Kilic-Bahi (2010) described an improvement in QL and a minimal improvement in basic skills for students at Colby-Sawyer College, while Jordan and Haines (2003) stated that students at Lawrence University report an increased appreciation for the utility of statistics. Many colleges have responded to the call for improved programming on QL, and while the previous authors addressed school-wide initiatives, the following authors described the efficacy of specific college-level classes.

Courses Designed to Develop QL

Briggs, Sullivan, and Handelsman (2004) described a QL course at CU-Denver. This course sought to strengthen and broaden QL skills, restore confidence to students, and demonstrate the relevance of mathematics. In order to improve attitudes and increase student engagement, classes contained a "number patrol" activity, in which students were asked to bring in articles that possessed a quantitative element. Teachers always presented applications first, and they emphasized group problem solving, process rather than solution, discussion and effort. This course sought to develop critical thinking, number sense and statistical reasoning by looking at financial problems, probability, and exponential growth, as well as voting, apportionment, mathematics and the arts, graph theory, and energy and environmental problems. The authors distributed pre- and post-course questionnaires, and students reported decreased anxiety, an increase in confidence, and an increase in comfort as a result of working in groups. The authors also found that unmotivated students reported higher levels of motivation, and student performance was highly correlated with diligence and study.

Madison (2006) described a course designed to analyze and criticize newspaper articles using mathematical and statistical reasoning. Madison highlighted several important features of the course, including the fact that all materials were required to be fresh and authentic. Since all articles were genuine, Madison found that students did not dispute the fact that they should understand them, even if they were not interested. Madison wanted students to be more engaged than they would be in a traditional mathematics or statistics course, so the course awarded extra credit if students brought in an interesting article with a quantitative component. Madison's course had nine lessons that focused on numbers, percent, linear and exponential functions, indices, graphs, counting, probability, weights, and maps. Teachers in this course attempted to situate the mathematics in context, and while fewer topics were covered, each was covered in much greater detail. The mathematics itself was often elementary in nature, but the contexts and reasoning were rather "sophisticated" (p. 2325). Further, technology was an integral part of the classroom, and interdisciplinary endeavors often arose unpredictably. Dingman and Madison (2010) offered additional thoughts on this course, as they evaluated its efficacy after six years. Using pre- and post-course tests, attitude surveys, and think-aloud sessions, the authors found that there was a modest shift in students' regard for the relevance of mathematics. Compared to a group using a more traditional text, the students' attitude scores increased, but only slightly, and their confidence with quantitative reasoning also increased. Student retention and grades were also better in this course, but the authors admitted that this result may have been due in part to the grading scale. Dingman and Madison also described several challenges of this course, such as managing bad habits that students had developed in previous mathematics courses, determining scope of content, finding authentic tasks for assessments, and struggling with various pedagogical issues.

Catalano (2010) described a college algebra course with a contextual focus, and Van Peursem, Keller, Pietrzak, Wagner, and Bennett (2012) compared a college algebra and QL class. Catalano (2010) described a constructivist classroom that was learner-centered, inquiry-based, data-driven and activity-oriented. The author noticed that traditional college algebra courses leave pre-service elementary teachers with negative attitudes about the utility of mathematics, so his goal was to improve their perceptions of mathematics and to increase their QL. Catalano argued that college algebra used to serve as a bridge to calculus, but since it has transformed into a terminal mathematics course for many students, educators must rethink its purpose. The author reorganized the typical college algebra content, and he inserted topics in probability and statistics while removing polynomial, rational and radical functions, as well as systems of equations. The course focused on modeling, and it utilized pertinent social issues, such as income inequality, the 2008 election, homelessness, and life expectancy. Catalano utilized the Student Assessment of Learning Gains (SALG) survey, along with interviews, student data, and final exams to evaluate the course. He found that the course was more effective at helping students learn mathematics, and students reported higher levels of confidence. Furthermore, despite a slight degradation in skills, fewer students withdrew, and student success rates were much better.

Van Peursem, et al. (2012) hoped to compare a college algebra and QL course with an exam modeled after the Collegiate Assessment of Academic Proficiency (CAAP) exam, with problems from pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, college algebra and trigonometry. The authors found no significant differences in scores on the exams, despite the fact that students with weaker backgrounds self-selected into QL (57.8% met college readiness benchmark in college algebra, while 26.9% met benchmark in QL). At the same time, students in QL reported higher gains in feelings about mathematics, and they reported that mathematics had more value and utility for their lives. QL students felt that they could better apply knowledge, and they scored higher on the application problems. A further analysis of the students who did not meet the college readiness benchmark showed that this subset of students performed much better in QL than college algebra.

Finally, Boersma and Kylve (2013) looked at a course that taught QL through media articles, using the book from Madison, Boersma, Diefenderfer, and Dingman (2010). The authors wondered whether students needed a basic set of skills in order to learn QL, and they found that students who lacked basic mathematics skills, and possibly basic critical reading and study skills as well, "may not be able to overcome these deficiencies in a fast-paced demanding course without some form of supplemental instruction or remedial reinforcements" (p. 9). The authors suggested that instructors need to dedicate time to study habits and basic skills if students are to be successful learning QL in authentic, contextual situations.

In sum, research on QL has focused primarily on students at the undergraduate level. Some researchers looked at school-wide initiatives, while others analyzed specific courses. Research on school-wide initiatives took a broader approach, as authors described the efficacy of programs at James Madison University, Hollins University, Wellesley College, Colby-Sawyer College, and Lawrence University. The research on specific courses went into greater detail not only on curricular changes, but also on the assessment techniques that the authors used to determine the efficacy of the courses. Briggs, Sullivan, and Handelsman (2004) used pre- and post-course questionnaires to determine that students in a QL course displayed an increase in confidence and a decrease in anxiety. Madison (2006) and Dingman and Madison (2010) used pre- and post-course tests, attitude surveys and think-aloud sessions to discover that students in a QL course showed higher confidence and improved attitudes towards mathematics. Catalano (2010) utilized a survey, focus group interviews, student data and final exams to show that students in a contextual college algebra course reported higher confidence and achieved higher success rates than students in a traditional college algebra course. Van Peursem, et al. (2012) compared students in a QL course with their peers in a college algebra course using the CAAP exam, and they found that the QL students not only performed better, but they also discovered more value in the mathematics. Finally, Boersma and Kylve (2013) used an assessment to conclude that students had basic skill deficiencies that were preventing them from fully developing their QL. Each of these studies adds to the research on QL in a unique way, but they all focus on undergraduate students and programs, and they mostly use quantitative methods to perform their analysis.

QL in High School

While a good amount of literature has been written on QL in college (Madison & Steen, 2007; Steen, 2001a; Wiest, Higgins, & Hart Frost, 2007), there has been very little done on QL in high school (Madison & Steen, 2007; Steen, 2001a). Steen (2001a) attempted to offer a rationale for why: “Numeracy in secondary schools is harder to detect and describe, primarily because, as an interdisciplinary enterprise, it must live in an atmosphere dominated by pressure for disciplinary standards and, recently, by vigorous arguments about competing mathematics curricula” (p. 115). At least two authors described QL efforts before college. Gutstein (2006) described a pedagogy that was infused with QL, but it took place in a middle school, and his focus was on social justice, not QL. Packer (2003b) referred to inner-city students in Baltimore that outperformed traditional students by a wide margin when they were taught algebra using QL, but he did not elaborate any further. Since there is so little information on QL in high school, this next section seeks to describe a potential model for QL in high school, and the literature gives guidance in two areas: programming suggestions, and the relationship between mathematics and QL.

Programming Suggestions

Several authors recommended certain types of content that should be addressed in a QL classroom. Steen (2012) submitted that QL can be implemented in high school if the curriculum is reorganized to emphasize knowledge and skills that are transferable to out-of-classroom situations. Lusardi and Wallace (2013) suggested that financial literacy should be included in the high school curriculum, while Hoachlander (1997) proposed organizing education around work, with different contexts for different students, based on their interests. In Madison and Steen (2008), NCTM President-Elect Henry S. Kepner, Jr. recommended that QL tasks be highlighted in a senior elective course for all students, particularly for those who do not plan on taking calculus. Gillman (2010) advocated for a radical restructuring of the curricula for grades 8-12, because “secondary students may be frequently pushed too fast into, or through, material that they may not need and may not be prepared to take” (p. 1). His alternative model would have students take two years of Algebra I, a year of Discrete Math and Fundamentals of Geometry, a year of AP Statistics and Analytic Geometry, and a terminal year for Functions and Modeling. He argued that this model would not only prepare STEM and non-STEM students, but it would also accommodate acceleration for students to take Calculus. Furthermore, this alternative model would give teachers time to focus on algebra and problem solving. Though this two-year model of algebra is unconventional, Gillman contended that the “one-year model for teaching algebra [creates] an artificial conflict between the need of students to master the technical skills of algebra and the need of the same students to internalize the ability to utilize algebra as a modeling system” (p. 10). Gillman also felt that an early focus on problem solving, reasoning, and statistics would translate well into upper-level science and social science courses.

In addition to curricular changes, some authors have recommended certain activities that could best support students' developing QL. For example, Madison (2006) offered a blueprint for how teachers and students might approach QL situations. According to Madison, students and teachers should find a challenging contextual circumstance, interpret it, glean critical information, model that information, and reflect their results back onto the original circumstance. Although this model may have been developed for a college setting, it can potentially be applied to high schools as well. Lesh, Middleton, Caylor, and Gupta (2008) have argued that data modeling should become a major priority in future curricula, particularly if the objective is to develop students' QL for a technology-based world. The authors made this suggestion not only because many data modeling concepts require nothing more than basic mathematics, but also because "many data modeling situations that once required the use of calculus (e.g., to minimize functions) can now be solved computationally (e.g., with easily manipulable tables and graphs)" (p. 116). Thus, they recommended the use of model-eliciting activities, which can be recognized by the following characteristics:

> (a) Participants (students, teachers, or researchers) are functioning in familiar situations where the chances are high that their actions will be governed by realistic judgments about sensibility rather than by school-based notions of correctness, (b) the design specs that participants are given call for the development of an artifact or a conceptual tool for accomplishing some well recognized goal, and (c) the development and testing of alternative artifacts or tools inherently involves developing and testing underlying conceptual systems (p. 118).

While I was unable to find research that employed these techniques at the secondary level, both Madison's (2006) blueprint and the model-eliciting activities of Lesh, et al. (2008) have the potential to support students in a high school QL classroom.

Relationship between School Mathematics and QL

If QL is to become a successful part of the high school experience, then mathematics teachers will likely have to take the lead in rethinking the current curriculum. The literature describes the relationship between school mathematics and QL through three lenses: the differences between school mathematics and QL, the possibility for both to coexist, and the potential that they cannot coexist.

Madison and Steen (2009) argued that QL is different from remedial mathematics, because while the mathematics in QL may be elementary, the contexts are advanced. Steen (2001b) stated that the case for QL is not a case for more school mathematics, or even for more applied school mathematics; rather, it is a call for a different and more meaningful pedagogy in every subject. Orrill (2001) argued that "unlike mathematics, numeracy does not lead upward in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life's diverse contexts and situations" (p. xviii). Manaster (2001) went into great detail to describe the stark differences between QL and school mathematics: mathematics focuses on proof, while numeracy frequently makes inferences based on estimates or incomplete data; mathematics studies abstractions, while QL studies reality; mathematics studies numbers for their own sake, while QL uses numbers to describe situations. Furthermore, a school mathematics perspective says that as students advance in mathematics, they need to see applications, but a QL perspective says that as students study more complex applications, they can learn deeper mathematics.

These differences might lead one to believe that school mathematics and QL cannot coexist, and certain authors hint at this dilemma. Madison (2004) argued that the mathematics curriculum could use a good weeding, in order to make room for contextual teaching. Cobb (1997) stated that in mathematics, “context obscures structure,” while in QL, “contexts provides meaning” (p. 77). Davis (1993) argued that since computers can do the computations and manipulations that we traditionally learn in mathematics class, then the curriculum should change. Moses and Cobb (2001) made a similar point, as they claimed that “math labs in inner-city schools for the most part are used to remediate students about things the technology makes obsolete” (p. 117). The idea that certain elements of school mathematics are obsolete, and thus should be replaced by topics in QL, is a definite source of friction, but many other authors claim that students can and should study both subjects.

Steen (2001b) claimed that students need to develop QL and master traditional mathematics, and elsewhere, he stated that numeracy should be an “equal and supporting partner” with mathematics in helping students meet the quantitative demands of society (2001a, p. 115). Porter (1997) claimed that achievement in formal mathematics is a critical element for QL, and others stated that a balance between pure and practical mathematics is necessary (Ellis, Jr., 2001; Hughes-Hallett, 2001). Ellis, Jr. (2001) claimed that abstract thought and practical problem solving are “not mutually exclusive but mutually supportive" (p. 64), and Schoenfeld (2001) stated that a good amount of mathematics can be motivated by problems and learned while solving problems. Furthermore, since many of the prerequisite skills for school mathematics and QL are the same, one could argue that they can and should support one another. Finally, the MAA suggested that "mathematics departments should provide leadership in the development of [QL] programs," (Sons, et al., 1996) and while the MAA was referring to colleges and universities, one could argue that the same sort of collaboration should take place at the high school level.

Summary

In conclusion, there has been a great deal of research on innumeracy and the failures of the educational system to develop quantitatively literate citizens. The Quantitative Literacy Design Team (2001) made a compelling case for the importance of QL, and several authors described the characteristics of QL in great detail. The literature presented many benefits of QL, with a particular emphasis on democracy and equity, as well as the many challenges inherent in teaching for QL. QL as a mathematical practice grew out of the work of Piaget, Vygotsky, and Lave and Wenger, and it has already begun to take hold at the college level. In fact, several authors studied QL at the undergraduate level, both in terms of college-wide initiatives and specific QL courses. Unfortunately, there has been very little done on QL in high school, which is concerning, because high school may be the last opportunity to ensure that all students are prepared to meet the quantitative demands of society. In addition, there has been very little research on co-construction, and I was unable to find any studies that looked at whole-class co-construction or attempted to use co-construction in a QL classroom.

In summary, then, this literature review has suggested that there is a need for research, and particularly for qualitative research, on QL in high schools, and there is a need for research on co-construction at all levels. Therefore, a qualitative study on how co-construction can impact high school students' QL would fill a gap in the literature, but it would also benefit elementary and secondary educators who recognize the importance of shared decision-making, and who understand that students need to develop a basic level of QL to be thoughtful, well-educated, active citizens in a complex, quantitative world.

Methods

Once again, this study attempted to answer the following research questions:

1. How does the ongoing co-construction of a QL course between my students and me affect the evolution and development of the course?
2. How does participating in this course affect students' QL and attitudes about mathematics?
3. Through these experiences, what do I as a teacher learn about teaching for QL and making mathematics more relevant to my students?

In this chapter, I describe the study design, the structure of the course, and my classroom setting. I explain the procedures for collecting and analyzing data, and I discuss the validity and reliability of this methodology. In addition, I reflect on my own positionality and other ethical considerations that were involved in this project.

Research Design

I chose practitioner action research as the design of my study because it allowed me to intentionally and methodically study some of the major challenges that I faced in my classroom. Herr and Anderson (2005) defined action research as "inquiry that is done *by* or *with* insiders to an organization or community, but never *to* or *on* them" (p. 3). This type of research was appropriate for my study because I wanted to co-construct a course with my students and analyze its impact on my students and me in relation to mathematics and QL. Anderson, Herr and Nihlen (2007) argued that "action research is best done in collaboration with others who have a stake in the problem under investigation, such as other educational practitioners in the setting, students, parents, or other members of the community" (p. 3). In addition, action research is emergent, so I had the opportunity to adjust my instruction and planning as I learned more about my students, rather than having to wait until the end of the course to analyze my data. In particular, I engaged in action research using qualitative methods, which was appropriate for my study because I wanted to understand how the course evolved over time, as well as how my students and I were impacted by the course. Qualitative data gathering techniques focus on process, meaning, and understanding (Merriam, 2009), and since I wanted to make meaning of the students' experiences, as well as my own, qualitative methods were most appropriate for this study.

The Students

As is common with practitioner action research, I chose to utilize purposeful sampling. Purposeful sampling is based on the idea that "the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam, 2009, p. 77). The criteria I used to select my sample included the following: 1) select a course that I could develop and teach, because I wanted to learn more about a problem that I noticed in my own teaching; 2) select an upper-level elective course, since these are the courses where students had tended to underperform and demonstrate negative attitudes towards mathematics; 3) select a course without a predefined curriculum, where I would have the freedom to design and revise the course as I go. After discussing these criteria with my supervisor, we decided that I could teach two sections of a new, upper-level elective course, entitled Discrete Math.

Thirty-seven students enrolled in the course in the beginning of the year, but by the time I ended data collection, forty-five students were enrolled. My morning course consisted of nineteen students (it started with fifteen), and my afternoon course consisted of twenty-six students (it started with twenty-two). Of the forty-five students, twenty-eight were boys and seventeen were girls, forty-three were seniors and two were juniors, and I had taught ten of the students in previous years. Most of the students had come from our school's lower mathematics track, and not surprisingly, many did not consider mathematics to be their favorite or best class.

Setting

This study took place in a small high school in New Jersey, in a middle- to upper middle-class community. Of the approximately 600 students enrolled, 85% are White, 6% are Hispanic, 5% are Asian, and 2% are Black (State of New Jersey Department of Education, 2012). Approximately 95% of students graduate within four years, and the school has a postsecondary enrollment rate of 75%. I teach five mathematics classes each year, and each class has twenty to twenty-five students on average. My school operates with block scheduling, where classes meet for fifty-five minutes to an hour on a rotating four-day schedule, with the fourth day off.

My classroom is located in the back corner of the high school, furthest from the entrance. As opposed to most other classrooms, my room is tucked away at the end of a small hallway, so students walking the hallway do not walk by my room. This is beneficial because it eliminates some distractions, but it is a challenge because students have to walk farther to get to class on time. My classroom is well lit, with windows looking out onto the football field. The furniture is fairly new, the walls are painted white, and there are bulletin boards located on three of the four walls. Depending on the time of the year, I have student work displayed on the bulletin boards and the walls.

At both ends of my classroom, I have several white boards. I use both for writing, but since the projector shines onto the white board at the front of my room, I primarily use the white boards in the back of the room. My desk is located off to the side in the front of the room, and it is actually just a table with a computer on it, which makes it easier for students to sit and ask questions. Student desks are organized in groups of four moveable desks, facing each other. I have seven groups spread around the classroom, positioned in such a way that students can easily see either board.

The Original Vision of the Course

This new course was created with the hope that it would better meet the needs of the student population. In previous years, students could enroll in AP Calculus, AP Statistics, precalculus or college algebra, but there was no option for students who either had not been very successful in previous mathematics courses, or who weren't interested in abstract mathematics. In response, a new Discrete Math course was proposed, with the idea that topics like graph theory, probability and statistics, polling, game theory, and combinatorics could better motivate this specific group of students. After speaking with my supervisor, we agreed that I would teach this course, and while I was asked to consider some of the traditional topics covered in a Discrete Math course, I was given the flexibility to respond to the particular needs of my students and emphasize topics that would best develop students' QL. My goal for this course was not only to motivate and engage students, but also to equip them with the skills they would need to meet the quantitative demands of everyday life. Additionally, I wanted to give students an opportunity to co-construct the course with me, with the idea being that students should have input into the content they study, because they are the best judges of their skills, interests, and future aspirations.

The Course in Practice

Henningsen and Stein (1997) argued that five factors are associated with student engagement at the level of doing mathematics: tasks that build on students' prior knowledge, scaffolding, appropriate amount of time, modeling of high-level performance, and sustained pressure for explanation and meaning. I referred back to these factors as I planned lessons throughout the year, and while I often struggled to balance these principles with the realities in my classroom, I believe that they still played an important role in the creation of the course. My goal from the beginning of the course was to explore content that would build on students' prior knowledge, as this would allow students to develop their QL and construct new knowledge for themselves. At the same time, since students arrived with uneven experiences in previous mathematics classes, I needed to scaffold new concepts and allow a sufficient amount of time for students to grasp new ideas. Similarly, I attempted to demonstrate high-level work for my students, and I continually tried to get students to think more deeply and articulate themselves more clearly about our units of study. I was not always successful, but I did attempt to create a learning environment that would help students engage with mathematics at more than a superficial level.

In practice, Discrete Math functioned much like a project-based course, as students regularly worked together on assignments that built towards an end-of-unit project. Group work was a key component of almost every lesson, as students were asked to rely on each other, as well as outside resources and technology, to help them grapple with new ideas and engage with relevant mathematics. The truly unique aspect of this course was the fact that students were actively involved in its planning. Students contributed to the course in the following ways. 1) Several times per unit, students completed written questionnaires that informed me about their interests and opinions regarding the course. I asked students to respond to a few questions, and I shared general themes with students after I collected and analyzed the data. 2) For the first two units, all students participated in large- or small-group discussions, and afterwards, students were invited to participate in voluntary small-group discussions. These discussions were a critical part of the co-construction process, because they gave students an opportunity to reflect on previous units of study and plan for the next. These discussions were semi-structured, as I created guided questions based on the results of the questionnaires, while also leaving space for students to brainstorm and consider new directions. These discussions informed me about students' attitudes and developing QL, but they were also the primary mechanism for me to co-construct the course with my students. During these discussions, we brainstormed various units of study, but since we were unable to reach consensus every time, I often allowed students to explore different subtopics that they wanted to study. Since these discussions were so critical to the co-construction of the course, I held our first one during the first week of school. 3) I gave students the option to speak with me individually. I thought that this would be important early on in the course, particularly if a student did not feel comfortable voicing his or her opinion in a large-group setting, but I found that some students felt much more comfortable talking to me several months into the course. 4) Finally, students were sometimes given a say in the type of assessment they wanted to produce. These decisions were sometimes made during our discussions, but more often they were made once students learned a bit more about each unit.

It was critically important for students to understand exactly how this course would function from the beginning of the year. Students had little experience with this amount of control over their studies, so I had to demonstrate the logistics of the co-construction process from the very beginning. At the beginning of the year, I set aside two class periods for our first large-group discussion. During the first period, I asked students to reflect on their previous mathematics courses, with the hope that students could practice engaging in a whole-class discussion as I learned a bit more about their previous experiences. Afterwards, I modeled what I thought the co-construction process could look like, and I gave them some prompts to consider before we engaged in our first planning session the next day. Once we completed that first planning session, our units settled into a consistent routine. Each unit was designed in such a way that students were asked to complete a quantitative task that was important to their lives or their futures. For example, in our fourth unit, students were required to calculate the nutritional information of a dessert that they cooked, after adjusting the serving size so that each classmate could eat. In order to help students successfully complete these tasks, I tried to create lessons that would give students experience with related content and skills. While students were engaging in these lessons, I asked them to reflect on the course, their attitudes towards mathematics, and their QL, in short questionnaires. Towards the end of each unit, I held planning sessions where students would help to co-construct the next unit. In the early part of the course, I required all students to participate in large- or small-group discussions, but in later units, I made this participation voluntary (for more information, please see Chapter 4). Once I made the small-group discussions voluntary, I implemented a written component to the co-construction, so that I would still hear from every student.

Data Collection

The data collection in this study had two main components: data that I gathered from students, and my own reflections and observations. In terms of the data that I gathered from students, I collected surveys and questionnaires, examined major assessments, and analyzed transcriptions from large- and small-group discussions. In addition, I gathered my reflections and observations in field notes and I kept a research journal. In order to complete my dissertation in a timely manner, I collected data from the beginning of the school year until the middle of February, which corresponded with the end of our fourth unit and the beginning of our fifth. Although I only included experiences that took place during this time period in my dissertation, I continued to collect data for the remainder of the year in order to develop a more thorough understanding of my three research questions.

Surveys

At the beginning of the course and the conclusion of my data collection, I administered short surveys to each student (see Appendix A). The primary purpose of these surveys was to gain some insight into how students' attitudes about mathematics (see RQ2) changed after having participated in my course. The surveys utilized an open-ended format, in order to obtain qualitative data. I shared general themes of the surveys with students, with the idea that students would be better able to contribute to the construction of the course with as much information as possible.

Questionnaires

I administered short questionnaires several times throughout each unit, approximately once a week (see Appendix B). The purpose of these questionnaires was threefold: 1) to give students an opportunity to discuss the course, with the goal of contributing to the evolution of the course (see RQ1); 2) to assess students' developing attitudes about mathematics (see RQ2); and 3) to gain insight into students' developing QL (see RQ2). I generally gave students the last five to ten minutes of class to answer a few questions (although sometimes I distributed questionnaires at the beginning of class), and while this was usually sufficient, I allowed them to take the questionnaires home if they wanted more time. Much like the surveys, I shared general themes from the questionnaires with students.

Assessments

At the end of each unit, students were asked to complete a major assessment in the form of a project. Students always had at least some say in the structure of their major assessment, whether in terms of the topic itself, the form of the final product, or the decision to work alone or in a group. These projects were a regular part of students' coursework, but they also informed the study by providing information on the development of students' QL (see RQ2). At the beginning of the course, I attempted to utilize Boersma, Diefenderfer, Dingman, and Madison's (2011) QL Assessment Rubric (see Appendix C), but I found over time that the rubric was not well suited for this particular course (please see Chapter 4 for more information).

Large- and Small-Group Discussions

Towards the end of each unit, I set aside a period for a large- or small-group discussion. I decided that it would be best to have these discussions near the end of each unit because students would have time to evaluate our current unit of study, but there would still be sufficient time for me to plan the next unit. The purpose of these discussions was to give students an opportunity to contribute to the direction of the course (see RQ1). These classroom discussions varied in structure throughout the year, transitioning from one large-group discussion to several small-group discussions to one voluntary small-group discussion. The discussions served a similar purpose to the questionnaires, because they gave students an opportunity to influence the direction of the course. These discussions added a new element, though, because they allowed students to brainstorm and make suggestions together, rather than with my mediation. For the purposes of data collection, I audio taped each of these discussions and transcribed them shortly afterwards.

Field Notes

During class, I noted instances in which a student discussed the course (see RQ1) or revealed some insight into his or her attitudes about mathematics or QL (see RQ2). As both a practitioner and a researcher, though, I had to create my field notes outside of class. I did this regularly throughout each unit, indicating both descriptions of what happened during class, and my initial reactions to those experiences.

Research Journal

I contributed regularly to a research journal. A research journal "encourages a reflective stance on the part of the writer and can provide a rich source of data on the daily life of a classroom" (Anderson, Herr, & Nihlen, 2007, p. 208). While field notes allowed me to document occurrences in the classroom and my initial reactions to them, a research journal gave me an opportunity to think more deeply about how these instances related to my research study. The purpose of this research journal was twofold: 1) since I was both the classroom teacher and the primary data collector, a journal helped me address concerns about the subjective nature of qualitative research; 2) since practitioner action research involves cycles of planning, acting, observing, and reflecting, the research journal allowed me to document and justify any changes that were made to the research process. This data collection technique was important because it helped me to understand how collaboration with students affected the evolution of the course (see RQ1), and it helped me reflect on what I learned about teaching QL and making mathematics more relevant for my students (see RQ3). Practically speaking, my field notes and research journal existed within the same document, with some sections labeled "field notes" and other sections labeled "research journal." Having the field notes exist in the same document as my research journal facilitated my ability to make connections among my observations, reflections on my teaching, findings regarding my students, and the development of the course as a whole.

Procedure

In order to document how the ongoing co-construction affected the evolution and development of the course (RQ1), I relied heavily on my research journal. While I collected data from students through various instruments, it was my research journal that helped me to reflect on decisions I made, the lessons I planned, and the important changes I implemented along the way. In addition to my research journal, transcriptions from large- and small-group discussions were critically important, because they provided a record of some of the major decisions we made about upcoming units of study.

The task of assessing students' attitudes (RQ2) was a difficult one. Haladyna, et al. (1983) defined "attitude toward mathematics" as "a general emotional disposition toward the school subject of mathematics" (p. 20), and while it sounds nearly impossible to discover a student's "general emotional disposition," there were some cues in the literature about how I could proceed. Aiken (1970) suggested several techniques to measure attitudes, including observational methods, interviews, questionnaires, and content analysis of artifacts, and in a later work he outlined scales that measured enjoyment and value (Aiken, 1974). Hidi and Renninger (2006) argued that "in early phases of interest development, affect may be used as an indicator of interest" (p. 120), so I hoped to extend this idea to attitudes as well. In particular, I hoped that by searching through the data for students' opinions, comments, or reactions to mathematics or mathematics class, I could gain some insight into students' feelings about the subject. Then, by analyzing these data over time, I hoped to be able to draw some conclusions about students' overall attitudes.

The task of assessing students' QL (RQ2) was equally difficult, because like students' attitudes, I needed to collect a corpus of data over a period of time. Early in the course I attempted to use the QLAR rubric from Boersma, et al. (2011), but I found the structure to be too generic for our specific assignments. Instead, I searched through the data for evidence of students' understandings and misunderstandings about quantitative phenomena, and I looked for changes over time. In addition, I included some items on the final survey that asked students to a) reflect on the development of their QL and b) provide some specific examples of what they learned.

Finally, I reflected on my own learning about teaching QL and making mathematics more relevant for my students (RQ3) by consulting my research journal, as well as by looking through my findings on the other two research questions. I found the three research questions to be intimately related, and by answering the first two, I was already finding some major themes regarding QL and issues related to student interest and engagement in mathematics class.

Data Analysis

Anderson, Herr, and Nihlen (2007) described action research as “an ongoing series of cycles that involve moments of planning actions, acting, observing the effects, and reflecting on one’s observations” (p. 3). This process served as the backbone of my research project, as I planned and implemented units of study, observed my students, reflected on my observations and those of my students, and then planned the next unit. Anderson, et al. went on to say that “these cycles form a spiral that results in refinements of research questions, resolution of problems, and transformations in the perspectives of researchers and participants” (p. 3). Thus, these action research cycles not only impacted subsequent units in the Discrete Math course, but they also influenced the direction of the research study and my own perspective as a researcher (and potentially my students’ perspectives as well).

In order to create this Discrete Math course and answer my research questions, I engaged in these cycles of planning, acting, observing, and reflecting. In terms of the course itself, I utilized student data from questionnaires, large- and small-group discussions, and field notes to help me plan subsequent units. For research question 1, my research journal helped me to document the co-construction process, and it helped me to reflect on the various data that I collected from students. For research question 2, field notes, assessments, and questionnaires gave me insight into how the course affected students' QL, and field notes, surveys, and questionnaires helped me determine how the course affected students' attitudes about mathematics. For research question 3, my research journal, as well as my critical friends (who I describe below), helped me to reflect more purposefully on what I learned about teaching for QL and making mathematics more relevant for students.

Since "all qualitative data analysis is inductive and comparative in the service of developing common themes or patterns or categories that cut across the data" (Merriam, 2009, p. 269), I utilized the constant comparative method employed by Glaser and Strauss (1967). In particular, I coded the data, combined events and their properties, defined theory, and put theory into writing. The key process when coding data was to compare new codes with previous ones. Through these constant comparisons, properties of events began to emerge. I documented many of these properties in my research journal, at which point I could compare new codes with the properties. In the process, these properties were either reinforced or refined. The theory began to come into focus once fewer changes to properties became necessary. Finally, with coded data, research notes and properties of events, I put theory into writing. This method was appropriate for this research project because it allowed me to make sense of the development of this

Discrete Math course, as well as its relationship to students' QL and attitudes about mathematics.

In addition, I employed the services of critical friends. A critical friend is a person who can help to "problematize the taken-for-granted aspects" of a given setting (Herr & Anderson, 2005, p. 30). Since a good amount of my research required me to observe my students and reflect on my practice, I needed someone who was not in my classroom to help me make sense of my students' and my own experiences. Herr and Anderson stated that "critical friends often push researchers to another level of understanding because they ask researchers to make explicit what they may understand on a more tacit level" (p. 78). In addition, they can serve as "devil's advocate for alternative explanations of research data" (p. 57). For my research project, I collaborated with several colleagues at Montclair State and one of my colleagues in the high school, and I found that they were able to offer different perspectives that helped me think more deeply about my research data.

Validity and Reliability

Many authors have argued that validity and reliability should be understood differently in an action research study. In particular, Anderson and Herr (1999) discussed five criteria that should be considered when evaluating the quality of an action research project: outcome validity, process validity, democratic validity, catalytic validity, and dialogic validity.

Outcome validity considers the extent to which research cycles lead towards a resolution of the original problem. In other words, do the research cycles in this study provide insight into how a collaboratively constructed QL course shapes the experiences of high school students? In order to improve outcome validity, I relied heavily on my research journal to help me reflect on the research questions and my ongoing research process. I also worked closely with my dissertation committee and critical friends, to ensure that my research project remained focused on my original questions.

Process validity considers whether the methodological choices made in the research cycles were appropriate based on the data presented. In my research study, I used triangulation to inform my methodological choices. My research journal was instrumental here, because it allowed me to process these data and justify how I used them to inform my next research cycle. By providing evidence from multiple sources of data, I was able to make a stronger case for why I made each decision.

Democratic validity considers the extent to which all stakeholders collaborate in, and benefit from the research project. This criterion is extremely important for this particular project, since the course was co-constructed by students. In order to improve democratic validity, I provided multiple means for students to contribute to the direction of the course, including surveys, questionnaires, and large- and small-group discussions. I also engaged in member checks with students throughout the year, particularly during our large- and small-group discussions. I shared the meanings that I made, with the hope of determining whether I had fully considered their thoughts and opinions. In addition, since

I was also a stakeholder in this project, my research journal and critical friends helped me to reflect on my own growth as a practitioner and a researcher

Catalytic validity describes the extent to which a study informs participants about their reality and equips them to influence it. In some ways, catalytic validity is similar to democratic validity, but it goes further because it focuses on how a study empowers participants. This study was designed to do just that, to help students not only develop their QL, but also to understand, and have a say in, how they best learn mathematics.

Dialogic validity considers whether the findings make sense in terms of the data collection and setting (Anderson, Herr, & Nihlen, 2007). This criterion was addressed in some of the ways already mentioned, including the process of triangulation, member checks, and the support of critical friends, but it was further satisfied by the peer review process. For this particular project, the dissertation committee reviewed the methodological choices and the context of the study to determine if the findings made sense.

In addition, my role as both teacher and researcher was bound to impact my research study, in both positive and negative ways. As a classroom teacher, I had insider information that an outsider might not have access to. I was familiar with the culture of the school, students recognized me and hopefully trusted me, and I even had some students in class before. On the other hand, this insider status could have obscured my ability to see the data objectively, which is another reason why critical friends were such an important part of my research. Additionally, my role as a teacher may have prevented students from being fully honest with me. I attempted to create a classroom environment that was open and honest, but I was cognizant of this limitation when performing my data analysis.

Researcher Positionality

In addition to my multiple roles, my positionality also impacted the research design and process. I was constantly reflecting on my positionality throughout the research process, but there were several elements that were certain to impact my study from the start. Firstly, I believe that constructivism best explains how students learn, and I have always tried to structure my classroom in a student-centered, discovery-oriented way. This epistemology undoubtedly influenced the creation of the course and the way I structured my classroom, as well as the way that I analyzed my data and engaged in research cycles. Secondly, I believe very strongly in equity, justice, and the principles behind democratic mathematics education, both in terms of the inner-workings of my classroom and the contexts that we study. This most certainly impacted the way I ran my class, both in terms of the structures I put in place to give students a voice and the daily interactions between me and my students. Finally, my previous teaching experiences definitely affected the research design and process. Throughout my career I have struggled to teach lower achieving upperclassmen, and while these experiences led me to design this research project in the first place, I made a commitment to be mindful of them so that I would not project my preconceptions onto the data.

I conceived of co-construction several months before the start of this course, but really, I had been thinking about it for several years. I had taught for seven years prior to the start of this course, and for many of those years I taught a terminal mathematics course for high school students. Every year, I felt like I was less than effective with students in these courses, because among other things, I struggled to help them understand the material, to get them excited about the mathematics, and to justify why any of the material mattered in the first place. While there were certainly some exceptions, many of these students underachieved, and a number demonstrated poor attitudes towards mathematics and mathematics class. I desperately wanted to improve students' performance and attitudes, but I did not know how to do this if they remained so disinterested in my courses. Eventually, I came across some of the QL literature, and it struck a major chord with me. I was unclear about how to engage students with Algebra II or College Algebra topics, but QL content held much more potential to actually appeal to students. If I could just choose QL topics that were interesting or relevant to my students, then perhaps students would demonstrate higher levels of engagement and take greater ownership over their participation in my class. My concern, though, was that the topics that I thought were relevant and interesting would actually be boring and irrelevant to students (Appleton & Lawrenz, 2011), rendering my QL course just as ineffective as the more traditional courses I had taught in the past. To circumvent this difficulty, I decided that students would need to participate in the planning of the course. Co-construction, then, became the key to this whole endeavor, because it would give students ownership over their experiences, increase the probability that we would study useful and interesting content, and hopefully result in higher levels of achievement and improved attitudes.

I was certainly excited about this initiative, but underneath this enthusiasm lay scars from seven years of failed techniques and less than successful outcomes. Those seven years had instilled in me the belief that students in upper-level electives weren't as well behaved, didn't love mathematics, and weren't as motivated to do work as their higher achieving peers. In the past I had tried every technique I could think of to engage these students, but I was unable to find any silver bullet. I began to dread teaching many of these classes, particularly when there was a student or group of students that was especially disrespectful or indifferent. I wondered why I took these classes so personally, until I realized that it wasn't the students themselves, but it was the truths that these students forced me to face every time I tried to teach them. I am not a great classroom manager, and this defect can be exploited by a group of students that is less amenable to traditional learning. I am not particularly creative when it comes to pedagogy, as I tend to rely on my passion and knowledge for mathematics to carry me through lessons and engage students. These types of students are less impressed with my passion, though, and they don't seem as interested in beautiful mathematical discoveries or impressive mathematical proofs. I consider myself to be an effective questioner that can help students make connections and think deeply about mathematics, but I am naive about how to motivate students to care in the first place. So in reality, I didn't dread these classes because of the students; I dreaded these classes because they made me feel ineffective as a teacher. All of this baggage did not go away when I decided to implement the co-construction process. On the contrary, these past experiences stayed with me for the duration of the course, and they not only influenced the way I taught the course, but also the way I participated in, reflected on, and modified the co-construction process.

Other Ethical Considerations

There are some obvious concerns about studying one's own students, particularly when one considers the unequal distribution of power. As a teacher, I am not only responsible for assigning grades, but also for managing a classroom in which each student is supported, made to feel safe, and encouraged to do his or her best. To make sure each student felt protected, and not coerced, I designed safeguards to ensure that I would not know who agreed to participate until after I submitted their grades. In order to do so, I took the following ethical considerations into account: voluntary participation, informed consent, anonymity, and avoiding preferential treatment.

In order to ensure voluntary participation, one of my colleagues was in charge of recruitment. At the beginning of the year, I stepped out of the room, and he came in to describe my research project. He handed out consent and assent forms, and he came back later in the week to collect the forms. I did not know who decided to participate until after I submitted students' grades. Therefore, all students had the same opportunities during the course, but I only considered data from the participants in my written work.

In order to ensure informed consent, all students under eighteen years of age who volunteered handed in an assent form along with a parental consent form, and all students over eighteen who volunteered signed a consent form. These forms were distributed and collected by my colleague while I was out of the classroom, and I had no knowledge of who agreed to participate until after I submitted students' grades. Once I submitted students' grades and learned who agreed to participate, I spoke informally with various students to see if they would reconsider participating in my study. I recognized that some students might have simply forgotten to have their parents sign the form, so I sought their participation once I no longer had any control over their grades. Despite not having power over their grades, I remained an authority figure in the classroom and the school, so while I asked that they consider participating, I was careful to remind students that they were under absolutely no obligation.

Additionally, all students will remain anonymous in any publications or presentations about this project. Since I am performing research with students in my own classroom, this will be slightly more difficult, but I will obscure any identifiable information to ensure that students cannot be linked with any of their data. In addition, I will keep all surveys, questionnaires, field notes, and assessments in a locked filing cabinet in my classroom, and I will keep transcriptions and my research journal on a password-protected computer.

Finally, I did not give any of my students preferential treatment because of their participation or nonparticipation in this study. While I had no knowledge about who agreed to participate, it is conceivable that I could have treated students differently based on their cooperation or resistance to the co-construction process. Despite this possibility, I committed to treating all students fairly, whether they actively participated in the co-construction process or not. As a further protective measure, I directed students to my colleague for any questions regarding participation, consent or assent forms, or a decision to withdraw.

A Narrative of the Action Research Process as it Relates to the Three Research Questions

In the following two chapters, I will describe my findings in relation to each of the three research questions.

1. How does the ongoing co-construction of a QL course between my students and me affect the evolution and development of the course?
2. How does participating in this course affect students' QL and attitudes about mathematics?
3. Through these experiences, what do I as a teacher learn about teaching for QL and making mathematics more relevant to my students?

I had hoped to present my findings in one chapter, but I concluded that this would be impossible to do, for two main reasons. First, my research questions were structured in such a way that a thorough analysis required an understanding of how the class evolved over time. My first research question in particular asked how the ongoing co-construction of a QL course affected the evolution of the course, and I could find no way of answering this question without taking the reader through some of the key moments in the co-construction process. Secondly, because I engaged in an action research study, I had to document each cycle of planning, acting, observing, and reflecting that took place during the course. The challenge was that these cycles did not take place once a unit, or even once a week, but they happened regularly and consistently throughout the course. As both teacher and researcher, I was constantly engaging in these research cycles, and since I taught two sections of this course, I would often make adjustments between my morning and afternoon classes. While it may not be customary, I have decided that in addition to a traditional findings chapter, I will also provide a narrative account of the co-construction process, particularly as it relates to students' shifting attitudes and QL and my own developing understandings. Structurally, then, I will present my findings on all three research questions together, but I will do so in two distinct chapters. In Chapter 4, I will present my preliminary findings as a narrative, so that I can demonstrate how the co-construction process, students' attitudes and QL, and my own understandings about teaching for QL evolved over time. In Chapter 5, I will describe the major themes that arose over the course of the year, as I take a more global look at my data in an attempt to address my three research questions.

In order to answer my research questions, I consulted questionnaires, surveys, field notes, and projects. These data helped me not only to understand the co-construction process and its influence on the development of the course, but they also gave me insight into individuals' changing attitudes and developing QL. In addition, the small-group discussions played a critical role in the co-construction of the course, and they provided valuable data on students' attitudes and understandings as the year progressed. As such, transcriptions from the small-group discussions feature heavily in these two chapters. Finally, my research journal allowed me to document not only the decisions I made regarding the co-construction process, but also my reflections on students' developing QL and attitudes about mathematics. In addition, my research journal gave me space to process what I had learned about teaching QL and making mathematics more relevant for my students.

When I first attempted to write Chapter 4, I planned on presenting my data in three separate categories, corresponding with each of the three research questions. I found out fairly quickly, though, that many of the data provided insight into more than one, and often all three research questions. For example, students' attitudes and developing QL made a huge impact on the co-construction process, just as the co-construction process influenced students' attitudes and QL. Data that addressed my first two research questions also helped me to learn more about teaching QL, and as a result, I made modifications to the co-construction process and developed new lessons that impacted students' attitudes and QL. The relationship among my research questions is one of the primary reasons that I chose action research for this study, because it was able to capture the dynamic interactions that took place among these interrelated phenomena.

Interconnectedness of my Three Research Questions

I first recognized the interrelated nature of my three research questions after collecting the initial class survey (9/06/2013). I asked students to reflect on their experiences in previous math courses, and their responses made a big impact on me. One student stated: "I do not enjoy math. I have never been good at math. I am never able to learn the material before moving on to the next section." A second student exclaimed: "Right now I feel like I'm horrible and it's humiliating. I just struggle a lot to put it simply." These are just two of many comments, but they represent some of students' biggest fears about math class: it is too hard, it goes too quickly, and it makes students feel bad about themselves. I administered these surveys before we began to construct the course, and while the students did not know it, they were already influencing the course's direction. While I could not say with certainty that I knew their attitudes about mathematics and mathematics class, their responses on the survey were already giving me some indication about how I should structure the course, and more generally, about how best to make mathematics relevant for students. After reading students' responses, I committed to creating a classroom environment where the mathematics would be accessible, where students would have opportunities to engage with material over longer periods of time, and where students would feel comfortable exploring the content because they wanted to, rather than because of pressure or shame. In addition, I recommitted to teaching material that would be relevant to students' lives. One student remarked that "most every math class is the same in that it tends to be tedious and does not focus on how the math skills can be used in daily life." While incorporating students' interests is an integral part of teaching for QL, students' comments confirmed that this approach would be beneficial. In sum, though I administered this survey to learn more about students' attitudes about mathematics and QL, I realized that their comments were actually beginning to shape how I would construct portions of the course. Thus, the process of co-construction had already begun.

Just as students' attitudes about mathematics and QL influenced the co-construction of the course, I found that our attempts at co-construction may also have impacted students' attitudes about mathematics and developing QL. The process of co-construction was extremely personal, as students were given an opportunity to share their interests and goals for the future in a small-group setting with some of their classmates and me. As such, peer pressure played a major role, sometimes by encouraging students and other times by discouraging them. I cannot say for certain that these encounters impacted students' attitudes about mathematics, but they certainly impacted students' behaviors in class. In some cases, students' enthusiasm for a particular topic encouraged others to contribute to the discussion. For example, after one student mentioned the importance of budgeting and saving in her own life, a second student shared the following: "Yeah, no, I'm, with you, I'm trying to like save up for college over the next year or two, and right now I'm trying to save like $100 a week…I thought maybe like balancing and like budgeting might be something worth doing" (Transcription 10/02/2013). This student was clearly building off of his classmate, and it is possible that he may have been more willing to study budgeting because he knew others were interested. On the other hand, students could sometimes influence each other in a more harmful way:

> Student 1: I'm not really so interested in money. I'd like something to learn about, but I don't really know.
>
> Student 2: We should have a big game of monopoly
>
> Student 1: Unless we do that, yeah (Transcription 10/02/2013).

In this situation, I was trying to engage students about their interests regarding money and finances, but the conversation quickly derailed. Student 1 expressed hesitation about what he wanted to study, but Student 2 interrupted with a joke about the game of Monopoly. Student 1 immediately gave his assent, and from that point forward, I was unable to reengage these students. These students were friends, and it became clear to me that certain approaches to co-construction, like small-group discussions, might actually dissuade students from actively participating in mathematics class. These types of interactions could potentially affect students' attitudes about mathematics, especially early in the year when students might be more willing to give mathematics class a chance.

While I cannot say for certain whether our attempts at co-construction impacted students' attitudes about mathematics and mathematics class, I am more confident that they had an impact, either directly or indirectly, on students' QL. In the following examples, the co-construction process gave me an opportunity to address some student misunderstandings:

> Me: Well how do you plan on saving money? Have you thought about it at all?
>
> Student 1: Yeah, I've thought about it. I don't know. I keep my money in a single savings account and I keep a lot of money in cash. I don't like the bank. I don't like the idea of putting my money somewhere that's not…
>
> Student 2: Stick it in a mattress?
>
> Student 1: Yeah
>
> Me: We should probably talk about it then
>
> Student 2: Granted the banks are not exactly reliable right now.
>
> Me: Well, up to a certain amount, they're FDIC insured
>
> Student 2: Yeah, but the interest rates suck
>
> Me: Right. But it's better than the interest rates in a mattress (Transcription 10/02/2013).

I had planned this conversation as an opportunity to see what financial topics would be interesting or useful for students to study, but I stumbled across opportunities where students demonstrated misunderstandings. In the previous conversation, students expressed mistrust in banks, as well as a lack of knowledge about alternate methods of investing money with the potential for a higher rate of return. I did not anticipate that these issues would come up, but I took advantage of these opportunities to help students get a better understanding of the basic principles of savings accounts. In another conversation, students demonstrated a lack of knowledge about life insurance:

> Student 1: Who defines how much, like, your life insurance is? Like, you're worth $2, who says that?
>
> Student 2: People get millions
>
> Me: You can get how much you want. But, you know, I might have cheaper life insurance, I do have cheaper life insurance than my dad
>
> Student 1: Why is that?
>
> Student 3: Because your dad's worth more
>
> Me: Because my dad's sixty eight…
>
> Student 3: No, I thought we were talking about how much you get when you die, like how much your family gets when you die
>
> Me: You decide that
>
> Student 3: Really?
>
> Me: Yeah, so I think I have a one million dollar life insurance policy on myself. So if I die, my wife gets a million dollars
>
> Student 3: But really not a million dollars
>
> Me: No, it is a million dollars (Transcription 10/02/2013).

In both of these examples, our small-group discussions gave me an opportunity to probe student misunderstandings about personal finance and mathematics. As the researcher, I had the responsibility to give students an opportunity for co-construction, but as the teacher, I was equally responsible to help students learn a bit more about these issues. While I am not certain whether they struggled with the concepts themselves or the mathematics behind those concepts, I do believe that in order to be QL, students need to understand both. I took advantage of some teachable moments in these discussions, but I also incorporated some of these concepts into subsequent lessons. Thus, while I may not have affected students' QL in that moment, the small-group discussions did encourage me to craft lessons that would address some of their misconceptions at a later time. In sum, then, students' attitudes and QL played a role in the co-construction of the course, our co-construction may have impacted students' attitudes and QL, and both of these relationships certainly impacted my own learning about teaching for QL. Due to the closely related nature of these phenomena, then, I decided that it would be appropriate to present my findings on these three research questions together.

Additional Insights from the Initial Survey

In addition to the discovery that students' attitudes and QL would be inextricably linked to the co-construction process, I also learned a good deal about the types of attitudes that students were bringing to my class. These data were critically important to my study, because I needed to learn where students were coming from if I had any hope of designing a course that would meet their needs and give them the best opportunity to succeed. Having worked with upperclassmen in a mathematics elective before, I was not particularly surprised by what I found. Though one student really enjoyed math ("Math is the best subject ever!"), the vast majority admitted that math was not their favorite

(Survey 9/06/2013). Many students said that they disliked math because they did not feel like they understood it. One student stated “If I don’t understand what I’m learning then I don’t enjoy it,” while another remarked “I get frustrated easily especially when things don’t make sense.” Many students were aware that a lack of understanding is what drove them to dislike math. One student commented “When I understand something I do [feel like I am good at math], but if I’m struggling with it I can drive myself crazy.” Another stated the following: “I work very slow and classes tend to move very fast and I fall behind easily. The type of person that I am gets stuck in a mindset that I cannot ‘do it’ and I get frustrated and will then fall behind.” These personal reactions were very common, as students were willing to explain just how difficult mathematics had been for them in previous years.

While a lack of understanding was the most common reason for students’ dislike of mathematics class, boredom and bad teachers were also common responses. One student stated that he did not enjoy mathematics because “I’m not good at it and it bores me,” while another said “I tend to get easily bored with the material, and I don’t do well on tests” (Survey 9/06/2013). It is interesting to note that while each student did reference boredom as a reason for their dislike of mathematics class, they also acknowledged that they struggled with the material. Other students blamed poor teachers for their dislike of mathematics: “No [I am not good at math] because I have never had a good math teacher who helps me learn to the best of my ability;” “No, usually I get turned off by it [math class]. I don’t like the way teachers teach it, but I have never found a way I like to be taught;” and “It’s probably the worst class I’ve had and I’m really bad at math but I blame my 7th grade math teacher for that I used to like math once." Notice, again, that even though students are casting blame on a teacher, they are still acknowledging poor performance in mathematics. Not every student who disliked mathematics stated that their lack of understanding was a contributing factor, but it did come up more often than not. Interestingly, relevance did not come up as often, but I found out students' take on that subject by asking them more specific questions later in the survey.

I asked students to consider how mathematics was relevant to their lives right now or their future ambitions. For their lives as high school students, students acknowledged that basic mathematics was important: "I love simple math that can be applied to my daily activities;" "I need basic math and certain things but not algebraic equations;" "I've tried to find one, but to me anything over basic math is pointless to me" (Survey 9/06/2013). Built in to these comments is the belief that advanced mathematics had no value for students' lives. One student remarked that he doesn't "see how quadratic equations can help me in everyday life," while another quipped "I am not examining street corners wondering what the degree of the angle is." One student even mocked what she considered to be the absurdity of mathematics problems: "I rarely will ever be ordering pizza for $75 or trying to make a garden with the largest possible area, with a perimeter of 15 feet." Clearly, students did not have the best experience in mathematics classes, and further, they did not believe that much of their high school curriculum was relevant to their lives.

When asked about whether mathematics was relevant to their futures, though, students were split. Some students remarked that mathematics is useful for everyone. One student stated that “math will help me when I am older and making my own money,” while another said that “math is important for everyone’s future” (Survey 9/06/2013). Other students noted that mathematics could be relevant for certain careers: “in the future I will need math to make it through business in college,” or “I want to be a doctor. I love medicine but I’m nervous because I feel like I need to be a whiz at math if I want to become a doctor.” Still others believed that mathematics may or may not be relevant: “Maybe [math will be relevant to my future goals]. I have no idea what I want to do with my life.” A third group of students was convinced that advanced mathematics would have no role in their lives: “No I do not use math nor will I ever that I hope of;” “I don’t think I will be using polynomial functions when I get older;” and “Besides taxes and other run of the mill math no it does not [matter to my future].” From these responses, it would appear that many of my students saw high school mathematics as a group of abstract concepts with little application to the real world. Further, it seems that students did not believe that mathematics is important, nor did they believe that mathematical reasoning skills are essential for an educated adult.

This misunderstanding of what mathematics is and why it is useful was especially evident in two students’ comments. When asked whether he liked mathematics, one student responded: “Yes I do [enjoy math] because I don’t have to read or write essays” (Survey 9/06/2013). A second student remarked that being good at mathematics “depends on how well you retain the information and spit it down to the paper.” These opinions may be relevant for some traditional mathematics courses, but they could not be further from the truth for a QL course. Clearly, I had some work to do. Not only did I have to engage students and help them to learn the mathematics, but I also had to convince them that the subject was even worth their time. This is exactly the challenge I signed up for, though, as I knew coming in that I would be working with a disenfranchised group of students. Not only would I be working with students who struggled with mathematics, but I would also be working against the stereotypes that mathematics is irrelevant, mathematics problems are trite, and higher-level mathematics is not applicable to students' everyday lives.

Introduction and Our First Large-Group Discussion

After I administered the initial survey and gave students an overview of the course, I set out to begin the more formal aspects of the co-construction process. Though the co-construction process really began when I first spoke with students about the course, I wanted to formalize some aspects of this process in such a way that every student would have an opportunity to participate. In order to do this, I decided that we would have planning sessions a week or two before the start of each unit. In order to accommodate this timeline for our first unit, I decided to have a planning session during the first days of school. First, I introduced students to the course and conducted a sample planning session, and then in order to give me time to plan, we spent some time looking at the quantitative reasoning skills that are necessary for understanding various newspaper and online articles (QR in the Media). When I was planning this course during the summer, I remarked in my research journal that "I like this setup…because it eases the students into the course, it gives opportunities for modeling (QR in the Media, large-group discussion), and it also gives students an opportunity to decide if they want to participate" in the class (7/31/2013).

Before we began our dedicated planning sessions, I wanted to have a large-group discussion about students' experiences in previous mathematics courses. Though I would be able to read what students wrote in their surveys, I wondered whether new themes would emerge when students could talk with one another. I asked students to share their experiences in small groups, and then we engaged in a large-group discussion as I took notes on the board. Students shared the following: "don't like math (bad at it, boring); like group work; more real-life situations, anything beyond basic math (basic operations) is useless; and if I understand it, or if it's fun, then I am interested" (Field Notes 9/06/13). Having taught mathematics for several years, I was not surprised by many of these responses. In particular, I anticipated that students who enrolled in Discrete Math (rather than AP Calculus or AP Statistics) would be students who didn't love abstract mathematics or plan on using it in their career.

While students' general dissatisfaction with previous courses was not surprising to me, I did not anticipate that students would be so passive when presented with the opportunity for co-construction. I anticipated that students would relish the opportunity to impact their course of study, particularly when so many of them disliked the material and structure in previous courses. In reality, many students appeared disinterested in co-construction, and some appeared disinterested in the course entirely. I puzzled over these challenges in my research journal:

> I have now seen both Discrete courses, and my first reaction is that they are less interested and excited than what I hoped. I am not sure that they understand what

> is going to take place, and I don't know if they recognize that there are complexities and nuances surrounding QL topics, even if the math isn't too advanced. As I prepare for my first large-group discussion, I wonder how much the students will be able to contribute. They won't necessarily even know where to begin when discussing content, so I might have to guide them a bit early on. Hopefully, the first unit and the first QR in the media assignment will provide some sort of foundation, and then they can really take over (9/10/13).

At this point, I was acknowledging three separate, but related fears. First, I worried that students were not excited about the opportunity to co-construct the course. I knew that many students had poor experiences in the past, but I struggled to understand why they wouldn't want to create a much better learning experience for themselves, now that they had the chance. Second, I was starting to get the impression that some students regarded our mathematics class as a bit of a joke. I had remembered that during the previous year, some of my students remarked that Discrete Math would be an extremely easy course, so they concluded that it would not have to be taken seriously. When I finally met my students, some were excited to learn about real-life applications of mathematics, but "other students thought the idea of real-life applications were a joke, and said we should go on a field trip to a Chinese food restaurant and count the number of grains of rice" (Field Notes 9/10/13). This attitude was already present on day one, and I worried that students who embraced these attitudes would be very difficult to involve in this process. Thirdly, I worried that since students had very little instruction that focused on QL, and little to no experience with the process of co-construction, that they would have a difficult time contributing even if they wanted to. Needless, to say, I prepared for that first planning session with trepidation. I remarked in my research journal: "At this point, I feel horribly underprepared" (9/10/13).

Planning our Units of Study

In order to plan out a scope and sequence, I asked students to discuss topics that interested them in small groups, and we discussed many of these ideas with the whole class. My afternoon class met first, and I made the following remarks in my field notes:

> Our first large-group discussion went much better than I anticipated…In particular, Group A engaged with me about their own interests, and how they might relate to math. Group B, and especially one student, seemed excited to really look at things that they cared about. We discussed music, song length, and pop culture, among other things. While we were discussing, another student came over and started engaging with some of their ideas. Group C also seemed interested, and they called me over multiple times to discuss their ideas (Field Notes 9/10/13).

My comments about the morning class were markedly different:

> Our first large-group discussion went very differently than the other class. Though this class is much smaller, many of the students seem to be less interested in the course…I asked them to brainstorm and then share out, and students started sharing some standard word-problem like examples, such as cooking. One student even asked to study volume and area, but he did not provide the context. I then asked students just to make a list of things they were interested in, and while some took it as a joke, others began to really consider the math that could be related to their hobbies and interests. To close class, I asked students to pick 5 things from our list that they wanted to study, explain why they wanted to study it, and [describe the] specific areas they wanted to look at (Field Notes 9/11/13).

Students in my afternoon class might not have been as focused as they could have been, but they were able to spend a full class period discussing what they wanted to study for the year. My morning class, on the other hand, could not have been more different. Some students demonstrated a lack of interest, while others admitted a lack of understanding and prerequisite knowledge to construct a course. To make matters worse, I was ineffective at moving the class discussion forward and giving every student a legitimate chance to contribute. The strategy I utilized, a large-group discussion, not only favored the more vocal students, but it also introduced an element of peer pressure that may have prevented students from participating more actively. Even in my afternoon class, where the discussion was much more successful, I still noticed that many students were not engaged, or at the very least, were not willing to participate in the discussion vocally. The challenges that I faced with large-group discussions were not limited to the formal aspect of the co-construction process; on the contrary, I experienced similar struggles during normal classroom activities as well. These experiences reminded me of some of the difficulties I faced with large-group discussions in previous years, and I realized that I might have to consider an alternate strategy if I hoped to get greater buy-in to the co-construction process. I remarked to my committee that "I'm not sure that my original large-group discussion is the best way to go…It was nice for an introduction, to brainstorm big ideas, but in terms of actual planning, both for the class and for individuals, writing and small-group discussions may be more appropriate" (9/11/13). As such, I constructed an alternative plan later that day:

> I want students to work together on the co-construction process, and I want to work with them, but I don't think that this is the correct format. Alternatively, I think there needs to be more writing and more small-group discussions. In addition, I think I might need to make more decisions after consulting with each class…. (Research Journal 9/11/13).

While these first large-group discussions were not as effective as I had hoped, we were still able to create a list of potential areas for study. Students suggested traditional topics like probability, game theory, map coloring, statistics, and set theory, common applications involving things like the stock market, population growth, economics, and time management, and more original topics like weightlifting, college admissions policies, Morse code, trends in crime rates, and social networking. After writing down students' suggestions, I attempted to combine related ideas into overarching topics (see Appendix D). I then distributed this list to the students, and asked them to make a case for the five units that they wanted to study.

As opposed to the large-group discussion, this step in the co-construction process went smoothly in both classes, because I asked students to state their preferences in writing. Students expressed a wide variety of interests, but the top five overarching topics were statistics (18 votes), money (17 votes), codes (16 votes), decision-making (12 votes), and probability (10 votes). When providing their rationale, students mentioned both interest and value for the future as major motivations. For example, some students who made their choices based on interest said the following: "Statistics – I love watching sports and examining player stats;" "Probability – It can help you win card games and it's really complex and cool;" "Forensics – Because I enjoy watching all the criminal shows like CSI and criminal minds…how they are able to obtain so much info by the smallest evidence is interesting to me;" and "Genetics – Genetics and genes interest me very much. I love learning about the human body" (Questionnaire 9/13/13). Some students who made their choices based on value for the future said the following: "Money – I personally think that I know how to manage my money but I want to learn more about the stock market and taxes for my future;" "Money – I know that this topic dramatically affects everyday life and is necessary for humans to succeed;" "Decision-making – It is very important to make good decisions that can benefit me [such as] college admission decisions;" and "Forensics – It can come in handy for when I study to be a game warden." Interestingly, only one student stated that he wanted to study a unit because he was "good at it." This is interesting, because the initial class survey revealed that many students disliked math because they felt they didn't understand it. There appears to be a disconnect, then, between what students want from a course at the beginning and what they feel about a course after the fact.

Planning our First Unit – Statistics

After reading students' responses, I decided to focus on statistics for our first unit. In my research journal, I justified this decision in the following way:

> (1) It received the most votes; (2) it is broad enough so that I can differentiate; (3) I find it to be sophisticated enough as to set the right tone for the year. To be fair, money (an alternate choice) can also involve a good degree of sophistication, but I don't want to give off the impression that this is an easy class. We will definitely cover money, but I thought I could hold that for the time being (9/16/13).

In order to continue the co-construction process, I presented a twenty minute overview of possible areas of study in statistics, and I asked students to consider what they wanted to study and the types of projects they wanted to complete. My goal at the time was to incorporate as many of the students' ideas as I could, so that I could pique their interests while also exposing them to a wide variety of topics. At the same time, I wanted to leave some space for students to grow, because I hoped that their interests might shift a bit as they learned more about statistics.

After presenting the overview of statistics, I noted that about half of the class seemed excited to begin their project proposal, but confused about how to begin:

> They didn't seem comfortable with all that control. Some were able to choose a topic or website that they were interested in (fantasy football, number of births per

> day), but they didn't know what sort of product to create around those ideas. I had some interesting discussions with different students about what they could produce, or what sorts of things involved statistics, but they needed much more guidance than I thought they would. I don't feel like they are equipped to handle these sorts of choices. At this point in the year, they are able to let me know their interests, but it would appear that I am going to have to structure units and projects a bit more myself. I hope that I will be able to better equip them to take ownership of their learning along the way. At the same time, they may never have had experience crafting their own learning goals and projects, so I need to help show them how they can do it (Field Notes 9/16/13).

I also noted "many students that seem completely disinterested."At the time, I questioned whether more structure would not only help the struggling students, but also the students who appeared disinterested. This challenge of how to work with such a diverse group was already on my mind in early September, and it would continue to present a challenge throughout the year.

Despite these challenges, students were still able to submit project proposals. When I asked them to justify why these projects would be a valuable use of time, students often cited their interests. For example, students asked to study the following: "fantasy football statistics, to see the relationship between the best players of all time;" "adoption statistics, because it's a topic that to many is important and means a lot;" "politics, [because] on the news, they always seem to give percentage of Obama's approval, and I would like to know how those figures are calculated;" and "fantasy football, [because] it will help create the best fantasy team each week" (Questionnaire 9/17/13). Interestingly, when students were asked to think about the projects they would like to work on, they relied almost exclusively on their interests. This can be juxtaposed with their earlier contributions, when students mentioned interest as well as relevance for the future as a rationale for choosing units of study. Another intriguing fact is that students' level of understanding was not mentioned as a justification, despite the fact that it featured heavily in students' dislike of previous mathematics courses.

After reviewing students' project proposals, I found that students conveyed very different interests. In particular, some students wanted to study surveys and sampling, while others were interested in various phenomena that they wanted to study over time. Additionally, there was a large group of students that wanted to study fantasy football. I documented my decision on how to proceed in my research journal:

> My first reaction was to follow a textbook that looked at statistics, and try to cover all of these topics. When I did that, though, I found a lot on surveys and sampling, a little on regressions, and nothing on fantasy [football]. Additionally, it all felt very traditional – very much like a math textbook. One of my goals in this course is to have students experience math as they would experience it in the real world, and then help them understand what they see. In order to do that, we need to look at phenomena first, and then search for the math. I have to fight the urge to turn to the textbook or the traditional curriculum. With that in mind, I made the decision, after spending a good amount of time planning along the traditional lines, to create three separate tracks for this unit. I am going to create three strands of lessons (fantasy football, sampling, and trends), lasting approximately eight class periods. Then, I will ask them to apply their learning to something slightly different, I will ask them to analyze newspaper articles on related topics, and I will ask them to submit a final project that puts all of their work together. I have reservations, of course, but I think that by allowing students to choose an area of interest, I might be able to convince them of the potential for this course (9/19/13).

Challenges in the Early Stages of the Co-Construction Process

One of the major challenges I faced early in the co-construction process was synthesizing students' suggestions into a coherent list of topics to be studied. Looking back on it now, I did not realize just how influential this list would turn out to be. At the time, I made an honest attempt to combine relevant topics, but now I realize that I unwittingly influenced the direction of the course through my particular choices. At the same time, we were co-constructing the course, and as the teacher, I had to make certain decisions to move the course forward. Even so, I decided to give students an opportunity to revisit this list once they had an opportunity to get more comfortable with the course.

A second challenge I faced involved the actual co-construction process. After attempting to co-construct the course with large-group discussions, I recognized that this approach would not be an effective planning mechanism moving forward. In my research journal, I outlined some of my questions and concerns:

> I began this process thinking we would have a large-group discussion and we'd be able to make decisions not only about content, but about structure, process, etc. The more I think about it, though, I feel that large-group discussions are generally a handful of students talking, mediated by the facilitator (often me)…At some point, someone has to make decisions. Up until this point, that person has been me. Should it be? Should I ask students to come after school and help? Should I split up the class into groups, and have them do some of the work, and then bring it to the larger class? ...I am not sure, but at this point, the single large-group discussion has proven ineffective…I will have to reconsider when we begin to plan for Unit 2, factoring in any literature…and my own (and my students') experiences in Unit 1 (9/16/13).

In anticipation of planning unit 2, I determined that small-group discussions, combined with individual brainstorming and writing, would be a more effective way to co-construct the course.

Finally, I developed some general concerns about the overall structure of the course. By giving students so much of a say, and by differentiating units according to students' interests, I wondered whether I was sacrificing some of the structure that would be necessary to hold students accountable. I reflected on this challenge in my research journal:

> I am nervous about too much freedom. Will students do the work? Will they think it's a joke? How do I teach three lessons at once? Can students handle the freedom that will come with self-guided lessons? …The traditionalist in me is concerned that students will be learning different things, and some students will never really get to study surveys, or trends, or fantasy football. At the same time, my experience has taught me that some students learn very little in a course like this. So wouldn't learning something be a positive? In addition, this is an elective, and students will be choosing their foci next year in college anyway. Why should I treat every student like they're the same? (9/19/13).

Unit 1: Statistics – Experimenting with Choice

Despite these concerns, I began our first unit with high hopes. I had taught units on statistics before, and students tended to like the material. Additionally, I had decided to give students three options, which I thought would be a huge motivating factor. Students would see that their voices were actually heard, and that they had a legitimate say in deciding their course of study. When I presented these options to students, most chose to study fantasy football or trends, while just a handful chose to study sampling. In addition, most students chose to be in groups with their friends, which impacted the classroom dynamic almost immediately. I found that this shift enabled students to take ownership of our classroom space, which had both positive and negative consequences. On the one hand, there was a noticeable increase in students' comfort level, but on the other hand, I had to work a bit harder to get students' attention.

In terms of the structure of the unit, I initially looked to textual and online sources for lesson ideas. For the sampling group, I planned to have students study sampling distributions, margins of error, and estimations, in addition to creating, conducting, and analyzing their own survey. For the trends group, I wanted students to learn about scatter plots, correlation and causation, and regression models, and I planned on having them find and analyze a data set and create models to make predictions. For the fantasy football group, I had to think more creatively about how I could develop students' understandings of how to use and analyze data. I planned on having them evaluate the structure of fantasy football points, compare and contrast football excellence with fantasy excellence, make predictions based on past performance, and compare their predictions with those of the experts. Finally, for all three groups, I wanted to give them an opportunity to apply their skills to a new situation. Therefore, I gave them newspaper articles that addressed each of their topics, and I provided some guided questions to assess how well they understood the nuances of sampling, trend analysis, and sports statistics.

Are Students Learning?

Once we got into a bit of a routine, I asked students whether they felt they had learned anything in the first two weeks. Some responded that they learned a specific mathematical skill, like "how to make an accurate scatter plot" or "how to properly collect data" (Questionnaire 9/23/2013). Others gave responses that focused on content, such as "I learned that forest growth in New Jersey has not changed much since the 1800s as opposed to states such as New York," "I learned how ratings in TV shows vary with seasons and why," or "I learned how stats worked and how you can transfer players stats into a point system." One student indicated that she found the type of learning in our Discrete Math course to be substantially different from her previous mathematics courses: “I learned that there is [sic] a lot of things we could have learned in our other math classes over the years that could have been helpful. I’m happy we’re learning all of the useful stuff now in the class.” Despite the challenges of those first few weeks, most students indicated that they had learned something. Unfortunately, this was not true for everyone. One student responded: “nothing yet,” and she accompanied this with an emoticon of a sad face. This student had struggled to get her project off the ground, and while she did eventually complete it, I was not able to give her the attention she needed early on in the course. This was a common struggle in our first unit of study, as my decision to allow freedom and individual choice may have diminished the level of efficiency that I would have liked.

In addition to asking students whether they learned anything, I asked them to consider whether these new understandings would be helpful in their lives. Once again, students gave several different types of answers. One student stated “I feel like I have learned about crime rates, and this can help me for my future in studying forensics,” while another student with an interest in the outdoors asserted “I gained a vast knowledge about New Jersey’s forest land and population and how it has changed and what the future looks like” (Questionnaire 9/30/2013). These responses indicated that some students appreciated the ability to study content that was relevant to their lives, but not all students felt the same way. One student from the fantasy football group remarked that he did not learn anything useful for his life, because “fantasy football has no meaning in real life unless you are in a league.” Another student exclaimed that she did not learn anything useful because “my project was on statistics and it was pretty basic.” A third remarked that she did learn something useful, but it was important only "for future math classes." These three comments reveal quite a bit about students' thinking. The first student was very interested in fantasy football, but he admitted that it was not particularly valuable for his life. The second student stated that she did not learn anything useful because the material was, in her opinion, too elementary. The third student did learn something valuable, but she saw its value solely in terms of its application to future mathematics classes. These thoughts are incredibly revealing, because I constructed the first unit with students' interests in mind, but it became obvious to me that other factors, such as relevance and apparent rigor, played a role in students' classroom experiences.

On a slightly different note, several students expressed irritation at the difficulty of working with real data. One student commented "I don't like how some of the info we need is hard to find," while a second student found that "sometimes, information on the subject becomes contradicting" (Questionnaire 9/23/2013). Both of these students were acknowledging the difficulties inherent in performing research and using the results to analyze some real-world phenomenon. The challenge of doing research on your own can be frustrating for students, particularly if they find seemingly contradictory information that relies on different assumptions or addresses the same idea from different perspectives. At the same time, it is critically important for them to develop the habits of mind that will allow them to navigate these resources, extract the important and reliable information, and make educated judgments based on their findings. I gave students the challenge of working with real data, and while this task may have made the first unit more difficult for many students, it will hopefully serve to make their educational experience significantly more rewarding.

Striving for Authenticity

Once I began to implement these lessons, I ran into some difficulties. For one, I had a very difficult time planning three separate lessons each day. What was more, I found that despite my individualized lesson plans, many students were still not on task. I reflected on this in my research journal:

> Some students and groups are focused, driven, and inquisitive. I really enjoy the ability to walk around the room and speak with these students, since I can question, they can consider, and we can build towards a plan…At the same time, some students are doing absolutely nothing. Is this my fault? Are my assignments not clear enough? Relevant enough? Did I sacrifice student interest to make the lessons more related to what I want to cover? For example, I have asked the fantasy students to consider the value of the points system as compared to what they would otherwise think of football excellence. I did this because I wanted all of my students to think about data collection, data analysis, and projections (or inference). Should I just have let them do a three week fantasy league? I don't know. Doing a fantasy league would be authentic, and my assignments are not authentic. But I think my questions are important. At the same time, the students aren't taking it that seriously. Is that my fault? Is it just their lack of initiative? I don't know. I am going to do some planning for the remainder of the unit, but these questions are weighing on me. Should I give students authentic experiences? Or should I try to have them meet the same goals, though through different means? (9/21/2013).

I was clearly struggling to balance individual student interests with common class goals. I realized, though, that in order to try to cover similar material, I was sacrificing the very nature of each of the individual areas – particularly for the fantasy football group. In response, I made the decision "to make the tasks more authentic…I hope they see this new format as an opportunity to do work in a more organic way, particularly as they build towards their projects" (Research Journal 9/21/2013). To provide these experiences, I restructured lessons so that each group could focus more clearly on their particular projects. For the trends and sampling group, that meant eliminating some of the extra material so that they could go more in depth into their projects. For the fantasy football group, I decided to let them do a three-week fantasy football simulation. I figured that by letting them conduct a fantasy draft, follow their players and compete against one another, that they would be more willing to engage with the material. I made the decision that engagement had to come first, and only through engagement in the course would students ever even begin to consider the relevant mathematics.

Initially, the results of my adjustments were positive. On the day of their drafts, the fantasy football groups were engaged and excited. They took the entire class period to finish their drafts, and they submitted some written analysis of their picks and those of some of their classmates. The trends and sampling groups also did well, and many students were thankful that I eliminated some of the extra work and allowed them to focus on their projects. Unfortunately, this did not last, especially with the fantasy football groups. I believe that this approach failed for two reasons. First, by adjusting authentic tasks to meet the goals of our math class, I may have taken away some of the legitimacy of the assignments. For example, in order to give the fantasy football students some exposure to statistics, I asked them to compile the data for themselves and determine the winner for each week's matchup. When presented with this task, students did not understand why they couldn't just take this information from the website. After giving it some thought, I realized that they had a point. The task was no longer authentic and interesting to students, and worse, it had even become tedious and tiresome. Secondly, I had given students a good amount of freedom to explore projects of their choosing, but it became clear to me that many of the students were not living up to their responsibilities. I made the following observations in my research journal:

> I painted myself into this corner, because I gave them so much freedom... Is freedom a good thing? I mean, I think choice is a good thing, and I gave them choice…but the responsibility is not there. Should I tighten up next unit? One lesson for everyone, where each person does something related to their own lives? I think a unit on money might be the best option for this, because each person will have their own budget, goals for the future, etc…Based on how this group is reacting, it was a mistake to give them so much freedom right from the beginning. When I was asking students to co-construct the class, I found the same thing. Students did not know how to handle too much freedom. I have repeated the mistake with this first unit. Too much freedom, too little structure. I need to build towards that. I should try something like this again down the road, but for now, I am thinking that we should look at money next unit (09/23/2013).

Students' Reflections

I asked students to reflect on the course in a short questionnaire, and students gave mixed reactions. On the positive end, students appreciated the opportunity to study their interests. One student stated the following: "I liked picking my topic because it was really what kept me going" (Questionnaire 9/23/2013). Other students enjoyed the independence afforded by our statistics unit. One student stated that she liked "how we can study and work on projects that we are interested in and we can work at our own pace but also have a deadline," while another student liked how "the work is very independent compared to my other classes and how I have control over what I do." This independence was a key feature in our first unit, but it was also one of my major goals for the entire course. Unlike some traditional mathematics courses, teaching for QL requires students to work through mathematics in real-world contexts. Therefore, I determined that this course would have to resemble a project-based learning environment, where students would be given everyday situations and asked to work through the corresponding mathematics. A third group of students appreciated the ability to have a real say in their day-to-day activities in the course. One student remarked: “I enjoy the fact that we have control over what [we] get to research.” Lastly, many students appreciated the opportunity to work in groups.

At the same time, many students struggled with the freedom that this unit afforded. Some students decried the lack of structure: “The project we started was very frustrating at first and was hard to plan out,” or “It was a little chaotic in the beginning, but when we got on track it became less chaotic” (Questionnaire 9/23/2013). One student was even more particular: “There is too little teacher interaction and [I] am lost sometimes.” This student pinpointed one of the major challenges that I found in our first unit. In order to provide students with three individualized learning experiences, I could not devote all my time to any one group. While some students thrived with the freedom, many students found it to be a major challenge. A second concern involved the amount of paperwork I required. While I gave students quite a bit of freedom, I required them to submit some written work along the way. One student commented that there was some “unneeded paperwork,” while another stated that “instead of doing the worksheets, I would like to just start the project.” Clearly, I struggled to balance freedom with accountability and structure, and I struggled to find the right mix between whole-group and small-group or individualized instruction. These challenges surfaced in the first unit, but they pervaded the entire course, as I sought to balance individual students' interests with the interests of their classmates, and my own goals for the course.

Planning for Unit 2 – Authenticity and Structure

A week later, I finalized the decision to create a unit on money. I justified this decision in my research journal: "It was the second most popular topic when we voted early in the year, but I also thought that this unit would allow me to experiment with one main lesson for the class, as opposed to three different lessons" (10/04/2013). When presenting the idea for unit 2, I did not give students an overview, because I figured that students knew enough about money and finances without my suggestions. In order to co-construct this unit, I decided to speak with small groups rather than the whole class. I discussed this decision in my research journal: "(1) I thought it would be easier to audiotape; (2) I wanted more people to have a voice, and I figured more people would voice their opinions in a small group; and (3) I thought people would be more comfortable sharing when they're with people (their groups) that they chose to be with" (10/04/2013).

Interest and Relevance

In our small-group discussions, interest and relevance drove our conversations. I asked students to discuss what they wanted to study, and often, students would describe something that was relevant to their everyday lives:

Student 1: Budgeting

> Student 2: Yeah, that's what I was thinking, how to balance your money
>
> Me: Be more specific
>
> Student 1: I just got a job
>
> Me: OK
>
> Student 1: I have a boyfriend. And I have to pay for college
>
> Me: So what…
>
> Student 1: (interrupting) How do I balance my money between everything?
>
> Student 2: Yeah, no, I'm, with you, I'm trying to like save up for college over the next year or two, and right now I'm trying to save like $100 a week…I thought maybe like balancing and like budgeting might be something worth doing (Transcription 10/02/13).

Here is an example of one student's personal interest driving the conversation. Student 1 initiated the conversation with the clear purpose of advocating for a topic to study, and other students weighed in with their own opinions. This pattern recurred throughout these discussions, but there were also occasions where students would reveal their interests in a less conscious way.

> Student 1: How to avoid taxes
>
> Student 2: Wait, are we paying taxes? Since the government's down, do we still, are we paying taxes right now?
>
> Student 3: So wait, what exactly is…
>
> Student 2: So who are we paying the taxes to?
>
> Student 4 (not in group): We are paying taxes to the government…(inaudible) they are taking our taxes but they don't have a plan
>
> Student 1: Oh, that's great. So they're just stockpiling everyone's money

In this example, one student lightheartedly suggested that we should study how to avoid paying taxes. This suggestion piqued Student 2's interest, and he immediately began to wonder whether we have to pay taxes in a government shutdown. This struck me, because this particular group did not demonstrate much of an interest in planning our next unit. Students were making comical suggestions and feigning interest, but the above example shows that interests or relevant topics can still serve to motivate students. Conversations like these inspired me that QL is the right course of action, because even if students appear disinterested, they can still be motivated if given an opportunity to study something they care about.

Students' QL

In addition to describing their interests, these small-group discussions gave me some insight into students' understandings about mathematics. In one discussion, a student described his plans for the future:

> Student 1: (inaudible) After college, I plan on living out of state, so I would like to look at cost of living in different states, just depending on, you know, how much a house costs, how much food costs, how much gas costs in other states, because I mean, it'd be drastically different if you live in New Jersey than if you live in Wyoming.
>
> Me: (interrupting) Great. How much do you know about cost of living? Now.
>
> Student 1: A little bit. Not so much.
>
> Student 2: That New Jersey is a lot
>
> Student 1: Yeah. It's really hard
>
> Student 3: I want to move up North, because it's a lot cheaper

> Student 4: Houses are going to be a lot cheaper anywhere else
>
> Student 2: Because New Jersey is so densely populated…I'd want to get out of New Jersey
>
> Student 4: 5 acres, 5 bedrooms, 2 bathrooms for like the house, the cost of the tiny house up here in Connecticut…
>
> Me: Something else we could look at…cost of living might be higher here, but so are salaries
>
> Student 1: Yeah
>
> Me: So we'd have to balance that
>
> Student 1: Yeah
>
> Me: Like if I had this exact job in…
>
> Student 4: Wyoming
>
> Me: Wyoming…I might make half as much, so that's something else to consider. But the thing I wouldn't have thought to look at that
>
> Student 1: That was my initial thought, if we were to do money that what I would want to do (Transcription 10/02/13).

Student 1 initiated this conversation by describing something that was relevant to his life, but it led into an opportunity for students to share their understandings (and misconceptions). As the teacher, I wanted to know whether students had considered the salary aspect of cost of living, and if not, I wanted to submit that for their consideration. In a way, then, this discussion was not only an opportunity to construct our next unit, but it also served as a teaching moment. This discussion gave students an opportunity to expand on each other's interests, and it gave me an opportunity to deepen students' understandings.

Considerations that Went into the Content for our Second Unit

In terms of the specific content for our second unit, students presented various ideas on what they wanted to study. One student wanted to study wealthy individuals and their net worth because "it could help me with business later in life" (Questionnaire 10/04/2013). Another student wanted to look at the US debt ceiling, because "as we become active citizens it's important to know about our country's debts and the ways to handle it." A third student wanted to look at college loans, because "I will be attending college next year and would like to know what I am looking at." Finally, a good number of students wanted to practice balancing income and expenditures, because "it can help later off in life when you need a budget." In all of these examples, students expressed a desire to study material that related to their interests or their futures, but my challenge was to incorporate these ideas into a cohesive unit of study. I did not want to repeat my mistakes from the first unit, though, so I decided to consolidate the big ideas and create lessons that exposed students to all of these topics. In particular, students expressed a desire to look at (1) governmental finances, (2) finances of businesses, wealthy individuals and television shows, and (3) their own financial lives. I field tested some of these ideas with my morning class, and I found the following:

> Some showed a preference for one of the…topics, and others seemed more amenable to discuss them all. I think I will ask students to learn about all [of them], maybe do something small for each topic, and then choose a final project that goes into greater depth in a specific area of interest" (Research Journal 10/04/2013).

My experiences in the first unit made me think that students needed more structure, but I still believed that students would only engage with material that interested them. I decided, then, that this structure would be an appropriate compromise, as more structure, coupled with some freedom and variety, would likely appeal to a majority of students.

Unit 2: Money – An Experiment with Multiple Classroom Structures

I described in my research journal how I planned to organize Unit 2:

> Part 1: 7 days on the government shutdown and debt ceiling, with students writing a response (from a particular perspective) at the end
>
> Part 2: A 4 day hodgepodge of different things students are interested in, such as cost and revenue for TV shows, counterfeiting, cost of living, net worth, salaries, etc.
>
> Part 3: 10 days on budgeting, both the college student version and the adult version – students will work almost exclusively with Excel during this section (10/09/2013).

These topics represented many of my students' interests, but they also represented topics that I considered to be important for a QL adult. In October 2013, the United States was involved in a government shutdown, and Congress was debating whether or not to raise the debt ceiling. Since these topics were so prevalent in the news, I thought it would be appropriate to study these ideas with my students. Likewise, many students suggested that we study budgeting, and I agreed that the ability to calculate things like taxes, retirement savings, mortgage payments and growth rates were important skills for an educated adult. Furthermore, budgeting would be an excellent way to introduce students to Microsoft Excel, which is an important tool for many adults. As opposed to Unit 1, where I allowed the class to study three different topics, I took what I considered to be the most important issues, governmental and personal finances, and made them a focus for everyone. Then, to allow students some freedom to explore their own interests, I allowed a four-day period where students could study a related issue of their choosing.

In addition to providing various topics for study, this setup also allowed me to diversify classroom structures. I was as yet unsure how to structure my classroom in a way that would best meet the needs of these particular groups of students, so I hoped to learn a bit more by exposing them to various approaches. The first section of this unit would require a more traditional classroom setting, where we would have large-group discussions, look at documents and articles together, and work toward common goals and understandings. The second section of this unit would provide students with a good deal of autonomy, as students would be free to explore a project that met a few general criteria. The third section of this unit would be a marriage of both, where students would learn the same concepts (how taxes are calculated, how monthly mortgage payments are calculated, how much you need to save for your children's college tuition), but they would have to apply these concepts to their own unique situations. I was optimistic that this unit would help me to better understand how to develop my students' QL, particularly in regards to balancing student interests vs. common goals, structure vs. freedom, and whole-class vs. individual/small-group instruction.

Part 1: Debt Ceiling and Government Shutdown – An Attempt at Whole-Class Instruction

In order to help students understand the government shutdown and debt ceiling crisis, I structured a series of lessons where students could discuss their prior knowledge with classmates, read and respond to articles from various sources, and do some reading on their own as the issues unfolded. In addition, due to the technical nature of some of these issues, this series of lessons provided an opportunity for students to learn in a more traditional format, with large-group discussions and direct teacher instruction. While this format made me a bit apprehensive at first, I found that our preliminary large-group discussions went very well. I reflected on this development in my research journal:

> "I am starting to see, especially as I begin unit 2, that different lesson styles meet the needs of different kids. For example, I engaged two students in particular with the content from Unit 2, even though they showed little interest at all in Unit 1" (10/09/2013).

I wondered whether this engagement was the result of a new type of class structure, or if the material was responsible for motivating these students. I distributed a questionnaire to my students to learn more about their experiences with our lessons on the government shutdown and debt ceiling.

In a short questionnaire, I asked students to reflect on their experiences studying the government shutdown and debt ceiling crisis, and in addition, I asked them to think about which unit they preferred and which unit enabled them to learn more. One student claimed that she liked both: "I like what we're learning now because we should know what's going on in our country. I liked last week's project because it focused on something I was interested in" (Questionnaire 10/11/2013). This student's response is revealing, because she articulated her belief that it is important and useful to study different topics for different reasons. Her response is also illustrative of the class as a whole, because she pointed to two very different reasons for her interest in each unit. She liked learning about governmental finances because she thought it was important, but she liked studying statistics because it was interesting to her. These two issues – interest and relevance – came up frequently in students' responses.

Many students claimed that they liked one unit or the other because of their interests: "Statistics project because we could work on things we like and not things that don't interest us;" or "I prefer what we are learning now about the government shutdown, it is more interesting" (Questionnaire 10/11/2013). Several students also cited interest when asked whether they learned more in the previous unit or the current one. Some students claimed that they learned more in the previous unit because it allowed them to study their interests, and one student somewhat sarcastically claimed that he was not learning anything in this second unit because he was not interested in the material. On the other hand, some students said that relevance was a major factor in determining how much they learned: "I learned more this week because the government shutdown and debt ceiling is more important and relevant than the statistics project I did on American Idol;" and "[I learned more in] this unit because you get a feel of what is actually happening in the world today." Additionally, many students stated that this second unit could help them in their lives: "Yes [it will help me] because I feel I have a better understanding of the U.S. government which is necessary for being a U.S. citizen;" "Yes [it will help me] because…we're learning things that are happening in real life and that are important to know;" and "Yes [it will help me because] I am more involved in the political part of the world." Both interest and relevance seem to be important factors in determining students' level of engagement with the material, and as a result, they should be considered when trying to develop students' QL.

In addition to interest and relevance, students referenced understanding and classroom structure as major factors in determining their like or dislike of these two units. One student said that she preferred the previous unit because “I enjoy learning about the government but it is very hard to understand” (Questionnaire 10/11/2013). In this case, a lack of understanding prevented this student from wanting to study government finances. Another student stated that she liked “learning about the government even though it’s the most confusing thing to me.” As opposed to the previous student, this student was interested enough in this topic that she wanted to persevere despite the challenges. It was interesting to hear students talk about their level of understanding, because despite the fact that many referenced it in the initial class survey, few students had talked about it during the first unit.

Along with understanding, students referenced lesson structure as a factor in their learning: “[I learned more] last week because our group projects got us very involved;” “I preferred statistics because it was a little more hands-on;” and “I knew a good amount about both topics but I feel that I learned more the first unit because I taught it to myself. At first I thought I would like to be taught at but I do like teaching myself better” (Questionnaire 10/11/2013). These students appreciated some of the freedoms afforded by the structure of the first unit, where students could work in groups, go into depth in a project of their choosing, and work at their own pace. Not everyone felt the same way, though: “I like the structure of classes this week because I feel like it’s more organized compared to the project.” This student preferred the more traditional, structured classroom environment, and she felt that it allowed her to better learn the material. At the same time, she continued her thought with the following: “I think what we’re learning is more interesting than what I did for my project.” So once again, interest played a role in students’ preferences, and along with relevance, understanding, and classroom structures, it served as an important factor in motivating students to engage with the material.

Based on these responses, it would appear that if I was able to cover material that was interesting and relevant, and help students understand the material using engaging and appropriate classroom structures, then I might be able to reach all of my students effectively. In reality, though, these four categories are not always mutually supportive. Several students preferred learning about statistics in our first unit, but admitted that they learned a good deal more in our second unit. One student stated that she learned more “this week because I didn’t know much about the current government issues,” but “I prefer last week’s classes. I don’t enjoy studying this…” (Questionnaire 10/11/2013). Another student made the same case, that he learned more in our second unit because he had very little prior knowledge about the government. A third student stated that he preferred our first unit “because it was about sports,” but he learned more “this week because I already knew everything last week.” This is an important finding, because it helped me to realize that these four categories – interest, relevance, understanding, and class structure – may not be independent. Students might want to study what interests them, but many may already know so much about it that it wouldn’t be worth their time. Similarly, students might prefer one type of lesson structure, but certain topics might not lend themselves as easily to that structure. Also, I might choose to cover material that is relevant to students’ futures, but many may complain that the information is not relevant to them now. One student acknowledged this fact in her response, when she stated that our second unit "made me an educated voter if I was voting this year." This student was expressing frustration at the unit's lack of relevance to her life right now, but what was I supposed to do? How can I best engage my students when each one has unique interests and understandings, and each one finds different topics relevant and different classroom structures appealing? I puzzled over this quandary in my research journal:

> How then, can you engage more students? Will you just inevitably lose some when the topic is less interesting? Isn't this an argument for student choice, though? How does this information apply to a course with a fixed curriculum? Should you try to differentiate lesson to lesson, or unit to unit, or within a single lesson? One of the many problems, though, is that some content lends itself more easily to lesson structure. I don't really have any answers, just a bunch of questions. I am seeing, though, that different content is bringing different students in (and out), and different class structures are doing the same thing. I have a feeling that I won't find the best solution, but perhaps by mixing things up both from unit to unit, lesson to lesson, and within lessons, then I can capture more students' attentions (10/09/2013).

As such, I determined to try different approaches with my students, with the hope of finding better ways to engage them with the mathematics and develop their QL.

Part 2: Student Selected Projects – An Opportunity to Explore

In the second portion of this unit, I gave students the opportunity to complete a project of their choosing. The only criteria I imposed were the following: (1) the project had to be related to finances or money; (2) the project should not focus on governmental finances (the first part of our unit), or personal budgeting (the third part of our unit); and (3) the project should require students to consider each of the categories from the QLAR rubric: interpretation, representation, calculation, analysis/synthesis, assumptions, and communication. In order to support their creation of this project, I gave students examples of possible topics to study (i.e. cost and revenue for TV shows, counterfeiting, cost of living, net worth, salaries – all suggestions that were made during the co-construction), and I asked them to fill out a project proposal form, where they were asked to describe how they would consider each of the six categories from the QLAR rubric. I then sat and discussed the proposal with each student or group of students, and I gave them three additional days to complete their projects.

At first, I found this second section of our unit to be refreshing. Students worked on their proposals, and I had an opportunity to walk around the room and discuss these issues with various groups of students. Some of the topics included the influence of salaries caps on sports, trends in salaries for various careers, the cost of living in different states, the financial implications of legalizing marijuana, the components of celebrities' net worth (Field Notes 10/21/2013), the implications of funding schools through local taxes, and Enron's collapse (Field Notes 10/22/2013). I found each of these conversations to be extremely rewarding, because I was able to speak with students about their unique interests, ask them questions about the quantitative aspect of those interests, and suggest various directions for them to take their research. At the same time, though, by engaging with individuals and groups of students, I was unable to monitor the class as a whole. As a result, there is a possibility that class time may not have been used as effectively as it could, at least for some, despite the positive interactions between various students and me.

During the four days, I asked students to weigh in on the freedom they were given during this project. Once again, several students preferred this approach because it allowed them to study their interests: “I like this way because we are able to do topics we are interested in;” “I prefer learning and picking my own topic because I like to pick topics that I’m interested in;” and “I do like that we were given the opportunity to choose our own topic and complete a project of interest to us” (Questionnaire 10/25/2013). One student said that he preferred the autonomy of this project, but for a very different reason: “I learn better this way because it teaches us to do things on our own and is getting us ready for the real world.” Not everyone preferred this structure, though, and I would count myself in this group. I found it extremely difficult to hold students accountable when they were doing such different things, and I was unable to manage the class effectively when I was so busy conversing with individuals and small groups about their specific projects. We had just finished a series of lessons that utilized a traditional format, and I wonder if I overreacted and moved too far in the direction of freedom and autonomy. I contemplated the consequences of this decision in my field notes:

> The class was unmotivated today. I am giving them an opportunity to self-pace, to do research on something they want to study, and still there is little motivation…Do I have to provide more structure? It would appear so. But I want to provide structure in a way that also allows students to do work themselves. But then do I have to collect the work at the end of the class to ensure that they are working? This is the premise that I have been operating under, that I need to provide some structure, give students time to work, and then collect their work to hold them accountable. Unfortunately, it hasn’t worked in previous classes. I will try something along those lines in the 3rd portion of this unit, so we will see (10/23/2013).

Part 3: Budgeting – Finding a Balance between Common Learning Goals and Individual Interests

In the first section of the unit, I utilized a more traditional learning environment, and I noticed that some students were not engaged. In the second section of the unit, I gave students an incredible amount of freedom, and I felt that there was too little structure to ensure that sufficient learning took place. Consequently, I tried to structure the third portion of the unit in a way that held students accountable through common learning goals, but also provided opportunities for individual exploration. I tried to find a classroom structure that would balance individualized learning with common class goals, while also appealing to students' interests. I believed that budgeting would be an appropriate topic for this technique, because it would allow students to apply new understandings to their unique life situations.

In addition to classroom structures and students' interests, I also considered relevance when designing this part of the unit. I recognized that students were coming in with very different work and financial experiences, but I knew that most would be attending college the following year. Therefore, I designed the first half of this mini-unit around some of the basics of budgeting, with a particular focus on issues that affect college students. At the same time, I didn't want to ignore some of the other important issues that affect people's financial lives, such as saving for retirement, paying taxes, and acquiring housing. Thus, I designed the second half of this mini-unit around the financial issues that students would face if they were to obtain a full-time job, buy a house, or start a family. I believe that these topics are critically important for any QL adult, and even though my students were only juniors and seniors, I wanted to prepare them for life after high school. Additionally, I decided to utilize Microsoft Excel to help students' engage with this mini-unit, because I have found that a good understanding of Excel can be an important asset when working with quantitative information.

First Part of our Mini-Unit – Basic Budgets and an Introduction to Excel

For the first part of this mini-unit, I wanted to cover some of the major financial decisions that students would face in their first years of college. In particular, I decided to cover basic budgets, credit cards, loans, and savings. To begin with, I asked students to create a budget that described their income and spending over the previous four months. If students did not have a job, I asked them to simulate what their first four months of college might look like. Students came to this lesson with very different levels of knowledge about Microsoft Excel, so students with more experience began to assist their classmates. Additionally, the students who had jobs and were already paying some of their bills seemed to be more invested in this activity than students who were speculating on their income and savings for the following year. Many of these students immediately set out to organize their income and expenditures, while some of the others appeared to have less focus about how to approach the project. So while there was a wide range of ability levels and experience, some students felt that this project was extremely valuable. One student commented that he was impressed that he was able to save so much money from month to month (Field Notes 10/30/2013). Even though he had already been saving this money, the budget assignment helped him to take a longer view of his spending and savings habits. This was an important development for this student, because it allowed him to consider his finances more purposefully, whereas in the past he may have just spent and saved without an end goal in mind. This assignment may not have been as important for other students, but at the very least, it allowed them to refine or improve their skills with Microsoft Excel.

In addition to the basic budget project, I gave students a two-day credit card assignment (adapted from Rubenstein, et al., 1992). In this assignment, students were asked to consider what would happen if they bought a $1000 laptop and only made the minimum monthly payment every month. Students were presented with the fine print from a credit card company, and they were given guided instructions to create a spreadsheet that documented their initial balance, interest, and payment for each month. Students were then asked to calculate how much they would pay, in total, for the laptop, as well as how long it would take to pay off their bill in full. As an extension, students were asked to consider how things would change if they missed a payment and went into default. I worried that students wouldn't really learn the big ideas if I modeled some examples for them, so I decided to present this assignment in a written format. While I knew that this would be challenging for some students, I decided that this would give students the best chance to construct the knowledge for themselves. In practice, this worked out better for some students than for others. I made the following observations in my field notes:

> [One student] was really impressive. He came up with a new way of organizing the information, and he did it by himself, and helped his classmates around him. He did make some mistakes (i.e. not dividing the APR by 12), but he had a strong

> knack not only for Excel, but for the concepts behind the project. After I questioned him, he explained himself clearly and effectively (10/31/2013).

For most other students, though, I found myself walking around the room and providing a good deal of hands-on support. I found this method of support to be very successful with students, but unfortunately, it was time consuming. At first, many students wanted to be told how to complete the assignment, but I made the commitment to sit with individual students or small groups and ask them questions until they figured out how to construct the spreadsheet themselves. To be honest, it was frustrating to go through the same routine several times in a row with different students, but I do think this approach was necessary. I reflected on this choice in my field notes:

> Should I teach the entire class? But it is so hard to hold their attention. Should I teach small groups? But then I cannot monitor the other groups of students. This is the big problem in a course like this. It is so hard to teach an entire group of students anything. Either they will talk or they will zone out. I believe you learn by doing, but how do I get [the students that I am not working with] to do something? (11/01/2013).

I eventually made the decision that individual and small-group instruction was worth the investment of time because it would force students to think for themselves. In the past, I found that many students sat on the sidelines during large-group discussions and waited for others to tell them how to do the work. In small groups, on the other hand, I can ask students questions and guide them to discover the best approach for themselves. I believe that in order to learn mathematics, you have to do mathematics, and therefore, I decided to sacrifice some time, as well as efficiency, to help students really engage with this assignment. Unfortunately, in order to accomplish this goal, I ended up spending almost twice as long on the credit card assignment as I had planned, and as a consequence, I could only give short treatment to loan amortization and savings. So even as I was finding individual and small-group instruction to be a powerful way to develop students' QL, I was as yet unable to do this in a way that efficiently used our class time, and was effective for all students. Therefore, I decided that during the co-construction of our third unit, I would ask students to reflect not only on the content that they wanted to study, but also on the class structures that best supported their developing understanding.

A Break for Co-Construction – A Shift toward More Genuine, and Voluntary, Collaboration

In the middle of this mini-unit on budgeting, we paused for a day to co-construct our third unit. Though this did interrupt our work with Excel, I found the break to be useful for my own planning, because I was struggling to find the type of lesson structure that would best support students' developing QL. I hoped that during this co-construction session, I could ask students to provide input not only into the content for our third unit, but also the lesson types that would help students be more successful. Unfortunately, I really struggled with the small-group discussions in my morning class. Paradoxically, many students were unable or unwilling to engage in a conversation that was designed to learn more about how to engage students. I reflected on this in my research journal:

> Students can even deflate a small-group discussion…Maybe if I got a small group of students (who were actually interested) together, we could actually make some decisions. That would also serve to allow students who would drag down the process to opt out. Plus, we would be able to co-construct at a more substantive level, rather than just by sharing various ideas and then leaving it to me to do the rest (11/11/2013).

This journal was prompted by my morning class, where I found that many students were not particularly interested in participating in the small-group discussions. While some students said nothing, others filled the space by making comical suggestions. As a result, our conversations got sidetracked, and it was very difficult to bring students back to the topic at hand. Furthermore, these attempts at humor set a tone that these discussions were not to be taken seriously, and I wonder if this dissuaded students who might have wanted to participate. After seeing two of the three discussions not go well in my morning class, I wondered how much value these discussions added to the process. Furthermore, I worried whether my afternoon class would be any different. I wrote the following in my research journal:

> I was dreading having these [next round of] conversations. I know many of these students are not interested, and they know it, too…For the next round, I would definitely like to invite students into the conversation who want to be a part of it, and allow the others to opt out" (11/11/2013).

I decided to try this immediately with my afternoon class. By making participation voluntary, I hoped to have more productive, focused conversations. Additionally, the time that would be gained by eliminating additional small-group discussions would allow the volunteer group to take a much more substantive role in the co-construction process. I justified this decision because any student was welcome to volunteer, and even if they chose not to, they would still be asked to submit their thoughts in writing. Thus, our small-group discussions would be reserved for students who committed to engage in a productive, lengthier conversation, and all other students would influence the co-construction process through reflective writing.

Planning the Content

For our third unit, I decided that we would study identification codes and cryptography. I made this choice for two reasons: (1) during the first week of school, students selected statistics, money, and code breaking as their top three choices for units of study; (2) secondly, since we had studied somewhat traditional topics in statistics and finances, I wanted to give students an opportunity to pursue something that was more unique. While I did select their top three choices for our first three units of study, I told students that we would revisit those choices after the unit, since students' opinions and interests were likely to change as a result of their experiences in the course. I asked students to submit a project proposal related to identification codes or code breaking, and I held three small-group discussions in my morning class and one small-group discussion in my afternoon class for more in-depth co-construction. Despite some of the challenges in the morning class, one of the three groups was still able to make some real progress in the co-construction.

When asked to discuss what type of lessons or topics they wanted to study, some students indicated that codes were of little interest to them:

> Student 1: I don't know. I don't know. Codes, like bar codes, I think that's, personally, I think it's a little silly. I don't know.
>
> Me: Like boring?
>
> Student 1: Yeah. I don't know. I feel like that's…
>
> Student 2: You scan, and then you need to know the UPC number, which is like the last four digits of, like, the bar code itself…
>
> Student 1: So you type in the numbers. It's just like, just like system recognition, I don't know, I feel like it's not too…

> Student 3: Complicated
>
> Student 1: It's not too complicated and it's also not that exciting (Transcription 11/11/2013).

Later, Student 1 reiterated his position:

> Student 1: Yeah, I understand, I understand the importance of computer programming. I understand the importance of bar codes. I understand the importance of codes encrypted on dollar bills, and codes encrypted on my drivers' license, but, but…
>
> Student 5: Yeah, it's not…
>
> Student 1: It in no way interests me

I was under the impression that students would be interested in studying codes and code breaking, so this discussion concerned me. One of the major reasons I decided to go ahead with a unit on codes was because I thought it would be interesting to students. I did not believe that the content itself would be useful for most students, but I thought it would still be beneficial because the lessons required students to use mathematical thinking (i.e. students would have to verify check digits, consider the most efficient ways to send information, use logical reasoning to decrypt ciphers, etc.). If they weren't interested, though, I wondered whether I should have intervened and suggested a different unit, because there are many relevant topics that can also develop students' mathematical thinking. Thankfully, other students in this group displayed a higher level of enthusiasm:

> Student 3: There was this one thing that I watched, it was, like, America's like top, like, top unsolved crimes. And one of them was like this murder, this guy had all these messages in his pocket about all these, they were just like in code, and like until today, like nobody has ever been able to crack those…
>
> Student 4: I read that

> Student 3: Figure out what they are and there was like one with like the Zodiac killer, and all that, so it's just like, things that have never been, like, you know,
>
> Me: That's interesting, so looking at sort of famous events
>
> Student 3: Yeah, crime and stuff like that…
>
> Me: And you want to do something with forensics, or crime, right?
>
> Student 3: Yeah. Yeah

This particular student, Student 3, was interested in pursuing a career in forensics or criminal justice. Therefore, a unit on codes was right up her alley, as it married her interests with her desired career goals. The challenge, then, was to try and find some aspect of this unit that would be valuable for other students, particularly for Student 1. Thankfully, the small-group discussion eventually turned to a topic that was engaging to a larger audience:

> Student 1: I, I personally, out of all those things, I think that Morse code is pretty sweet
>
> Student 5: Yeah, I want to actually translate one. I want to know how that works
>
> Me: How to translate Morse code?
>
> Student 5: Yeah
>
> Student 1: Yeah. I would like to do Morse code
>
> Me: But isn't Morse code just another example of…
>
> Student 1: But I think there's a practical application…like if you don't know Morse code and like, you need to, like, convey a message to [another student], but, say that there's a…we can't talk to each other
>
> Student 3: That's why we have phones, and text messaging. Not to shoot down your idea

Student 1: I'm just saying, I'm going to live in the forest and I just think Morse code

Me: Right, so it's specific to your interests

Student 1: It's extraordinarily specific to my life

In this exchange, it became clear that Student 1 did find some value in studying codes. In particular, he wanted to study Morse code, because he thought it would be valuable for his future. On other occasions, this student talked about his interest in forestry, agriculture and the environment, and in this conversation he speculated that some knowledge of Morse code would be valuable for his future. Interestingly, both Student 1 and Student 3 became interested in codes because of their relevance to future career goals. Though the conversation began with students talking about their interests, it quickly shifted to a discussion about relevance. This conflict between interest and relevance undercut our whole conversation, and perhaps not surprisingly, it also came up in my afternoon class.

In the afternoon class, I asked for volunteers to have a small-group discussion, and eight students decided to participate. As opposed to the morning class where students demonstrated varying degrees of interest, students in this session seemed genuinely interested in co-constructing the course. Much like the morning class, though, this afternoon group struggled to find a balance between interest and relevance.

Me: There doesn't seem to be high energy about this topic

Student 6: Well, it's a little, it's a tough topic

Student 7: Cause you really don't use it unless you're going into that field

Student 6: And I don't know anything about it

Student 7: So it's like, to be interested in it, I mean, I can…

Student 8: I mean, I'm interested…I don't really have plans on going into…firewalls

Student 9: They're interesting, those are interesting. Who cares about 0s and 1s (to Student 10)?

Student 10: (smacks table) 0s and 1s are what run your life

Student 7: Really, they do

Student 6: Yeah, but it's not going to change anything

Me: Right, but the impression I'm getting is that this is an interest for a lot of people and it's not necessarily a career path…for many, not for all. I mean, some people might want to…

Student 7: No, it's cool, like, to know… (Transcription 11/11/2013).

This excerpt was extremely rich, because it demonstrated how students can wrestle with different reasons for studying any given topic. First, Student 7 commented that knowledge of codes is not particularly useful unless you study a related field. Much like some students in the morning group, she did not see much value in this unit if it didn't relate to her life or her future. Student 8 saw it differently, because he acknowledged that certain aspects of this unit might be interesting, and therefore worth studying, even if they weren't relevant to his future plans. Student 9 seemed to take a different tact, as she was mainly concerned about whether the topic caught her attention. As opposed to Student 7 or Student 8, she didn't seem to consider relevance when determining whether a topic was worth studying. So even within this one group, there were divergent ideas about whether interest or relevance should determine if a topic is worth studying.

An additional element of this conversation that is worth mentioning centers on Student 10's comment. Student 10 suggested that knowledge of some of this material, and particularly 0s and 1s (a reference to binary code), is important because the knowledge itself has important applications. Since binary code is used by computers to transmit information, Student 10 saw it as something that was worth knowing. Student 6, on the other hand, commented that even if you had this knowledge, "it's not going to change anything" (Transcription 11/11/2013). This was the crux of the disagreement in this group: if we can agree that knowledge is important, does that mean that everyone should know it? Alternatively, since students would only be learning the basics of binary code, would it be worth the time? This conversation really struck me, because it made me consider what should be understood by all educated adults, and what should be left to specialists. Really, what the students were discussing was what it means to be QL. Are there certain skills and understandings that all adults need to have, or do they vary based on each person's unique circumstances? Looking back on the first two units, I would argue that educated adults should understand sampling, correlation, budgeting and finances, but I doubted whether I would include codes in the same category. At that point, I wondered whether to even proceed with a unit on codes, since I didn't consider it to be essential for a quantitatively literate adult. Nevertheless, I decided to move forward with the unit, and I hoped to see whether students would engage with the material or be turned off by its perceived lack of relevance or specialized nature.

After the small-group discussions, I opened the co-construction to the entire class by asking each student to write about what they would like to study. Students who were excited to study codes submitted responses like the following: “I would like to study Morse Code because it has always intrigued me;” “I would like to study code breaking because it seems pretty interesting. Also, I don’t really know much about it and would like to learn;” and “I would like to talk about code breaking. I’m very interested in learning how people can decrypt codes and encrypt them” (Unit 3 Project Proposal 11/12/2013). There were also some students who were not particularly fond of our third unit. For example, one student wrote “I don’t feel like codes are a subject of interest.” Whether students wanted to study codes or not, the vast majority justified their responses by discussing their interests. While one student did say he wanted to study Morse code because he thought it would “come in handy,” no other student justified his or her response by addressing the content’s relevance to their lives. In some ways, this was similar to the small-group discussions, where students initially made suggestions based on their interests. The major difference, though, was that in the small-group discussions, students eventually began to consider the relevance and importance of learning the given content. The power of the small-group discussions as a tool for co-construction was the fact that students were given an opportunity to think through these units, get feedback from one another and from me, and consider the course in a more meaningful way, whereas the written responses were only able to capture a quick snapshot of students’ thinking.

At first glance, then, it would seem that small-group discussions would be a more appropriate method for co-construction. In reality, though, I found that only two of the small groups saw students discussing mathematics in a thoughtful and insightful way. In the other two discussions, students approached the subject in a cursory manner, much like they did when writing their proposals for Unit 3. So while small-group discussions do have the potential to allow for meaningful co-construction, it would seem that students' willingness to participate and engage in this process is also a contributing factor.

Planning the Classroom Structures

In addition to planning the content for Unit 3, I used this round of co-construction to learn a bit more about the types of classroom structures that would best support students' engagement with mathematics. Two experiences in particular compelled me to solicit this feedback, and both occurred on the last day of the first half of our mini-unit on budgeting. This day also happened to be our planning day, when students were asked to submit their budget assignments and engage in the co-construction process. I was under the impression that this was one of my better assignments, because it drew on students' individual life experiences, it utilized a powerful tool like Microsoft Excel, and it required only a basic application of mathematics to calculate total income, total spending, and savings. Unfortunately, only seven of the eighteen students in my morning class turned in the assignment on time (Field Notes 11/11/2013). To be honest, I was a bit hurt by this, because I had worked hard to construct a unit that would be useful and appropriate for this particular group of students. In order to understand why this happened, I walked around the room and spoke with some students. One student in particular helped me to realize that I needed to reconsider the way I was teaching.

This particular student, who by many accounts was successful in her other classes, told me that she preferred to have written instructions to complete her work (Field Notes 11/11/2013). My first reaction was very defensive, because I thought about all of the times that I had worked hard to give the class useful lectures, the many discussions that I tried to have with the class, and the daily reminders that I gave students about when things were due. At the same time, I knew that she had a point, because it was becoming obvious to me that whenever I would speak in front of the class, only a portion of students would actually listen to me. This was especially true when we held class in the computer lab, as some students might have had a difficult time controlling themselves when faced with so many distractions. I wrote about these experiences in my field notes:

> I am having a really difficult time teaching in front of the entire class. But when I give instructions on a worksheet, students complain that I am not teaching. I think that I am going to try and give out [really detailed, well thought-out] instructions for each class period. I believe that students learn by doing, and right now they are not doing. Further, I cannot stand up in front of a room and try to explain something when so few of my students are listening. How can I even get at QL if students are not engaged? I thought QL would be a means to spur engagement, but for many students, relevant concepts are still not enough to motivate them to do work (11/11/2013).

This was a difficult pill for me to swallow. It was very humbling for me to admit that I could not engage the class by lecturing, holding large-group discussions, or trying to instruct them in a large-group setting for an extended period of time. Furthermore, it was confusing to me, because I rarely struggled with whole-class instruction in my other classes. I expressed this confusion in my field notes when I remembered that "I brought up population growth and demographic changes in my precalculus classes [that same week], and I couldn't get them to stop talking about it" (11/11/2013). Why was it that I could utilize certain strategies with my precalculus students and they would be wildly successful, but yet I would fall flat on my face when I used those same techniques with these students? Looking back on my years of teaching, this was the exact same problem that I found in many of my senior-level elective classes. Had tracking, which had relegated the majority of these students to the lowest-level math classes, taken such a toll on these students that they could not operate in a traditional classroom setting? Had students become so accustomed to procedural, "do as the teacher does" style work that they were not comfortable participating in a course that required critical thinking and problem solving? Was peer pressure from students who were not interested in the course so powerful that it prevented otherwise capable students from succeeding? I had theories, but no answers, and despite my desire to blame this problem on systemic injustices, I realized that for this particular group of students, it was my responsibility to make things better. So even in this challenging situation, and even if I could only make a small amount of progress, I decided that I had to try. For that reason, I chose to ask students how I could better serve them. By engaging students in this discussion, I hoped for two outcomes: (1) that students would give me practical suggestions on how I could better support their developing QL; and (2) that students would recognize that I cared about them, and as a consequence, that they might reconsider their effort and their willingness to engage in the course.

In order to get students' feedback on how to best structure our classes, I first spoke with the afternoon small group after we had finished discussing the content for

Unit 3. I asked them how I could better meet their needs, and one student spoke for many when he said that "I think, I think it'd be easier if we had like, the outline, or at least something we could follow, to help us…" (Transcription 11/11/2013). For the first half of our mini-unit on budgeting, I gave some instructions in writing (i.e. the credit card assignment), and for others, I modeled the assignment for the class, but it became clear to me that these approaches were ineffective when so many students handed in their projects late. This was hard for me to digest, though, because I knew that I had explained the project in detail. Further, I had introduced the budget assignment slowly, I had provided time for students to apply their knowledge, and I had walked around the room to assist students with their assignments. I hoped that students would be able to thrive in an environment like this, and in all honesty, I thought I might have even enabled them a bit, by giving them so much direction. Therein lay my quandary: how could I teach students the material, while also equipping them with the skills they would need to be successful in life after high school? Both goals were important to me, and I wondered how I could possibly meet one without sacrificing the other.

Students went on to offer more suggestions:

Student 1: I can't, I, it's really hard to, like, come up with something on my own

Me: Like, more, more support, from me?

Student 1: Yeah

Student 2: Yeah. Like I can't work without directions

Student 3: Like when…we picked our own topic

Student 4: But that was, that wasn't that bad. Cause you gave us, like an umb…, like a whole topic and then you're like, pick one

> Me: But you didn't like that, Student 3? You wanted more structure? ...
>
> Student 3: Like when I was handing it in, I didn't know if it was everything (Transcription 11/11/2013).

In this segment of the conversation, students were alluding to the second part of the unit, when I gave students an opportunity to research a topic of interest and complete a related project. Student 1 stated that he felt uncomfortable with this process, and Student 3 said that she was unsure whether she was meeting the guidelines of the project. Student 2 might have exaggerated a bit, but her point was well taken that this particular project gave students too little structure. I had originally made the decision to give this project for two reasons: (1) I wanted to give students an opportunity to explore their interests; and (2) I didn't want anyone to feel like I was ignoring their suggestions. Therefore, I decided that the way to include everyone's suggestions was to create a project where students could choose their own focus. Looking back on it, though, I realize that students may have been uncomfortable with all that freedom. Students were almost certainly used to strict requirements, and in this mini-unit, I gave them anything but. Perhaps it was unfair to ask students to design their own research project, where they would consider an area of interest from a quantitative perspective and then ask (and answer) important questions about the topic. This is a key facet of being quantitatively literate, and I hoped that by giving them this lofty goal, while also providing support, that students would be able to rise to meet it. It turns out that I asked too much, too soon. Students in this particular small group let me know that they wanted more structure and more detailed, written instructions, and it was up to me to balance this request with my goal of developing their QL skills. Perhaps by giving them a bit more support and time, students would be able to develop the skills that could help them analyze and understand the quantitative phenomena in the world around them.

In addition to the small-group discussion, I administered a short questionnaire that asked students to describe what sort of classroom environment they would need to be successful. After collecting the questionnaires, I wrote the following in my field notes:

> I glanced at the responses, and some took it seriously, and some did not. Actually, when I was asking students to do this at the beginning of class, some students were staring at their computers. I then asked them to get to work, and a few looked at me, clearly not having heard the instructions. This is a microcosm of our bigger problem, that some students have a really hard time paying attention at all (11/13/2013).

Despite some students' lack of attention, I walked around the room and encouraged them to give me feedback. While it was frustrating that I had to explain the instructions again, it was important that I hear those students' opinions, because I clearly wasn't engaging them appropriately. Students offered myriad suggestions, covering issues such as technology, interest, understanding, and group work.

Some students mentioned the importance of technology, and one said that "working in this computer lab is an effective way to work…[and]…using the programs on the computer such as Excel is helpful" (Questionnaire 11/13/2013). Other students stated that class needs to be interesting, and one student remarked that in order to be engaged, class would need to capture him "within the first 5 minutes." Three students mentioned the importance of understanding what is being taught, and while the first did not offer suggestions on how to accomplish this, a second stated that she liked "being able to take notes" and a third said that he preferred "topics we know stuff about." Still other students described their preference for group work, for reasons such as "we help each other when we don't understand," "I can feed off my partners and get work done," or "I learn better with my friends." Two students responded that the class was already engaging, although I'm not sure if they said this to satisfy me, or because this is how they honestly felt. All of these suggestions were helpful, but they touched on things that I was already considering. The QL literature had emphasized the importance of technology and collaboration, and I was already working to create lessons that best supported students' understanding of the material. What struck me, though, were two other categories of responses: one that involved students' concerns about their peers, and another that called for clearer, written instructions.

Several students said that they would be more engaged if their classmates acted differently. One student stated "I think we should get rid of people like…," and another suggested that we "keep the class the same; just because a certain number of students do not do their work does not mean we should change the way we have done things already" (Questionnaire 11/13/2013). Some students went a step further, as they offered suggestions on how to best engage those students who had trouble staying focused. One student asked for "more class discussions since a majority of the class is not tuned in on what to do." A second student said something similar: "I like how everyone can do their own things but I feel like we should work on everything as a class to keep everyone focused and that way we will understand everything better." I thought these comments were interesting, because from my perspective, I found that students zoned out during lectures and whole-class discussions. In addition, I felt that students learned less when I utilized these techniques, because if they weren't participating in the class discussions, then it was unclear to me whether they were doing anything at all. I imagine that these two students felt that the class was quieter during whole-group instruction, and perhaps they were better able to learn in this type of environment. These suggestions really challenged me, then, because students were requesting a type of instruction that I only found to be appropriate in certain situations. Another student gave an even more detailed recommendation:

> …I don't know if this would be possible or not, but a lot of the stuff I am interested in is from the TV show Numbr3s (game theory, cryptography, information theory, graph theory, etc.). Maybe if the whole class was able to watch an episode or two of Numb3rs, they would become more interested in those aspects of Discrete Math, and could figure out what they find interesting after seeing some aspect of how they are used…

I really felt for this student, because she was highly successful in her previous mathematics courses (including one with me the previous year), and she enrolled in Discrete Math because she wanted to engage with complex, nontraditional mathematics. Unfortunately, because most of her classmates either didn't like mathematics or didn't feel like they were good at mathematics, I had to structure the course in a way that met the majority of the students' needs, rather than hers. This student's presence in my class really made me want to focus more on student choice, because this student deserved an opportunity to study her interests at a level that was appropriate for her. As much as it was easier for me to streamline lessons, I had to remember that I was working with an incredibly wide range of students with varying interests, levels of understanding, and goals for the course. For that reason, I could never rely on whole-class techniques for an extended period of time, because students in my class were not only so different, but they also had different goals and objectives for the course.

A final group of students asked for clearer, written instructions for all of our projects and assignments. One student stated that she needed “to have a list of directions directly in front of me so that I know exactly what to do and am focused 100%,” while another said that for projects, she needed “clear written out instructions” (Questionnaire 11/13/2013). A third student exclaimed: “Written instructions! Please!!” These responses were humbling, because they pointed to my own failure to effectively support my students. While I often gave students typed-out instructions, it is true that I would sometimes give instructions orally or write them on the board. Furthermore, according to some students, the typed-up instructions I distributed were not always detailed enough. This lack of detail was problematic, according to some, because it prevented me from answering questions from a greater number of students. One student said that “instructions on paper would help because everyone wouldn’t be asking you for help,” and another said that “the class needs to be a healthy balance of teacher and student involvement. I believe that the teacher needs to be more involved with the students to keep us on track.” Clearly, then, students wanted greater detail in written instructions, both to help them complete assignments, but also to free me up to provide more one-on-one and small-group support. My challenge, then, was to provide the support students wanted while still creating lessons that required students to think for themselves. I wonder if some students said they wanted detailed, written instructions, when really they wanted me to show them how to do all the problems. I think that some students are uncomfortable thinking through problems by themselves, particularly when they came from classes where they were rarely asked to do so. Being able to think critically about unfamiliar situations is an important part of being QL, so it would have been counterproductive for me to explain exactly how to approach each problem. In the past, I tried to find a balance between supporting students and requiring them to think for themselves by walking around the room and asking students questions until they were able to construct knowledge for themselves. Based on students' responses, then, my task was to ask these questions and provide more support in a written format, while still requiring students to think and reason through each situation on their own.

Second Part of our Mini-Unit – Putting Students' Ideas into Practice through a Budget Simulation

For the second half of our mini-unit on budgeting, I asked students to use Excel to run a budget simulation for their 20s. I began by asking students to research the starting salary for their dream job, calculate yearly 3% raises, and factor the salary of a potential spouse as well. I provided detailed, written instructions, as per the request of many students, and I circulated the room to provide support. I documented students' response in my field notes:

> Students were on task, perhaps more than usual…I think they liked the opportunity to think about their actual job, and how much money they would make. This was more of a personal, creative lesson, but we'll see how the next lesson goes, when we get more technical (11/13/2013).

I was unsure how the next phase of this mini-unit would go, because I was planning to cover some challenging concepts like calculating taxes, buying a house and a car, and saving for retirement and their children's college tuition. What I found, though, was that throughout this mini-unit, students were working, they were engaged, and they were excited to consider the financial possibilities of life after college.

On the third day of the unit, I noticed the following:

> Some students who are not usually interested were discussing what house they would live in, car they would drive, etc. It might not be exactly what I had planned on them doing, for as long as they were doing it, but they were considering the financial implications of these decisions (Field Notes 11/15/2013).

This was a welcome development, especially for my morning class. Students were excited to speculate about their futures, and I noticed a climate shift in the room, where it became acceptable to talk about one's own project and ask others about theirs. Just a few days later, I made the following pronouncement:

> Today might have been the best day so far this year. A good number of students have really bought into this project, and they are excited to do research on home prices, salaries, and prices of cars. Since so many of the students have bought in, I think it has pressured the rest of the class to get on board as well. We're not at 100%, but we are close – maybe 2 or 3 students in each class have yet to engage (Field Notes 11/19/2013).

I speculated later that day as to why this mini-unit had been so successful:

> One reason may be because students are interested in their futures. They like to think about what they could be. Even though it is not relevant to them at this moment, it is relevant to their futures. Secondly, I typed up instructions, and basically created these lessons through a self-paced, self-guided unit. Students have everything written in front of them, so they can move at an appropriate pace, without having to wait for me, listen to me, or wait for the class to be quiet. This is extremely encouraging…"

I went on to note that one student finally engaged during this class period, even though he had likely not done "more than 30 minutes of work all year" (Field Notes 11/19/2013). I observed that he worked hard for the full period. Unfortunately, though, another student bumped into his computer near the end of class, and accidentally unplugged it. He was understandably mad, and worse, I worried that I had just lost my one opportunity to engage him with the mathematics. Thankfully, though, I worked with him again just a couple of days later, and I found that he got right back to work. In addition, I found that he actually had a strong grasp of percentages, and an impressive ability to do mental mathematics:

> I worked with [this student] today, who did next to no work all year… Today, we talked about taxes, and I was impressed by his ability to do mental math. I didn't force the Excel formulas on him, because he seemed to be able to process it more effectively in his mind. For example, he [projected himself] in the 25% tax bracket, and he said: 'Well, if I make $3000 more next year, then I will just add $3000 to this number (the previous amount that was to be taxed at 25%), and find 25% of that.' I'm impressed. I'm really impressed (Field Notes 11/22/2013).

This was one of my most rewarding experiences of the year. It confirmed for me that if I could create the right type of lesson, then even the most challenging and seemingly apathetic students might be able to engage in complex, extended mathematical thinking. I really liked the structure of this unit, how students were able to engage with material over a longer period of time, how lessons were self-paced, and how students were responsible for a final product at the end. I believe that this second half of the mini-unit on budgeting was my most effective series of lessons to that point, and I hoped to replicate that experience in our subsequent unit on codes.

Students' Reactions to Unit 2

On a short questionnaire, I asked students to discuss what they learned in Unit 2. One student responded that "balancing a budget is very difficult," but he said it was important to know "so you are prepared for the real world" (Questionnaire 11/22/2013).

Another student exclaimed "Being an adult sucks…It is crucial to manage your money." In response to whether this knowledge was important, she responded "Yes. Very. Because everything we looked at are all things we need to think about later in life." When I asked students whether they felt more quantitatively literate as a result of this assignment, students made the following comments: "The Excel spreadsheet enabled me to feel more comfortable with numbers. It makes it a lot easier to calculate numbers;" "I have a better understanding of money and how much everything costs – new perspective;" and "I feel like I have a much better understanding on how to budget money without going into debt."

During this mini-unit, students complained that some of the tasks were extremely difficult, but yet the vast majority of students persevered in completing them. I found that most students developed real ownership over their projects, and I have a feeling that they wanted to be successful not only for the grade, but also because the project meant something to them. After students completed their projects, I asked them to write a few sentences on what they learned from this particular project. Here are some of their responses:

> Student 1 – This assignment has shown me the complexity of balancing a personal budget. I now can see the major difference in saving for my retirement at 22 rather than 35. I have also been able to see what is most important to spend my money on and what I would be able to hold off on. This assignment has made me better at balancing a personal budget as well as using Excel in general.
>
> Student 2 – This assignment showed me that it is best to start saving for things very early, because it will pay off in the long run. For example, starting to save for retirement at age 22 instead of age 35 makes the amount to save per year significantly less – it is still a lot of money, but it is more manageable.

> Student 3 – From this assignment, I learned that dealing with savings is a lot more complicated than I thought. I did not realize how many extra expenses there is [sic], such as car insurance, gas, cable, taxes, and rent. It is a lot to take into consideration and this project definitely helped me better prepare for the future and have a better outlook on what I need to prepare for in the future.
>
> Student 4 – This assignment is pretty much a wakeup call for how expensive life is. I need to save money (mom's looking into a retirement account for me now) and I know that I have a lot of responsibilities that I'm currently taking for granted (Budget Simulation Comments 11/21/2013).

Challenges with this Approach

Despite what I consider to be some major successes in this mini-unit, there were also some areas where I fell short. One area in particular has to do with how I asked students to calculate their taxes. I distributed detailed, written instructions for how to complete this assignment, and I tried my best to give students the support they would need to be successful without doing the thinking for them, but based on students' reactions, I must have done an inadequate job. In the written instructions, I wrote the following:

> Visit http://www.bankrate.com/finance/taxes/tax-brackets.aspx to find out the tax brackets for 2013…Create a column for taxes, and calculate your total taxes for each year in your 20s. This is slightly challenging to do, so let me give you an example. Let's say that you and your spouse make 175,000 together. You will be charged 10% on your first $17,850, 15% up until $72,500, 25% up to $146,400, and then 28% beyond. The mathematics here will be as follows: .10(17850) + .15(the difference between 72500 and 17850) + .25 (the difference between 146400 and 72500) + .28(the difference between 175,000 and 146400). If you need help, please ask me or consult http://www.moneycrashers.com/calculate-federal-income-tax-brackets-rate-tables/ (Instructions for Budget Assignment Part 2 11/12/2013).

At the time, I thought that I was giving them too much information. I debated whether or not to include an example, but I decided to go ahead and give students the extra support.

After all, the numbers I used were just an example, and students would have to apply this technique to their individual tax situation. Based on students' reactions, though, I think I provided them with far too little support:

> I spent a lot of time walking around among the groups, explaining not only how to do [their taxes], but why this graduated system was needed. I feel like it would have been much more efficient to do this with the whole class, but I just feel like many of the students wouldn't have listened. Part of the efficiency problem is that I won't tell students how to do [these calculations]; [rather,] I will ask them questions to help them arrive at the solution. This proved to be a bigger problem in my afternoon class, where I really only had time to work with three, maybe four groups (Field Notes 11/14/2013).

I really wanted to support these individuals and small groups, but I couldn't help but wonder if I was being horribly inefficient. The problem, though, is that I wanted to elicit understanding from students in order to help them construct the knowledge for themselves, and this would have been nearly impossible in a whole-class discussion if only one or two students participated. I decided, then, that individual and small-group instruction was more appropriate, but in order to make it work, I would have to create an even more detailed set of instructions. In order to be a bit more efficient, I decided to seek a new middle ground between whole-group and individual instruction, by providing ample support, in writing, to allow students to learn in a self-guided way. I had been successful with this technique for other aspects of the budget simulation project, but I needed to find a more effective way when dealing with more complicated topics. After all, I was used to teaching classes orally, so I had very little experience creating effective lessons in a written format. In anticipation of our upcoming unit on codes, I committed to improving this technique:

> Next time, I will scaffold even more, and I will provide a clearer example. I don't want to do the thinking for them, but I also don't want to provide insufficient support. I have had to walk around and explain the same idea to far too many students. I could have done a better job with the worksheet. Looking forward to codes, I want to try something similar. Instructions and support on a worksheet, self-guided and self-paced work, and checkpoints along the way to ensure that students are working (Field Notes 11/19/2013).

Unit 3 – An Attempt to Replicate Unit 2's Success

After consulting students' project proposals and reviewing transcriptions from our small-group discussions, I found that students wanted to study the following: secret and unsolved codes, the history of the Allied code breaking efforts in World War II, cryptography, Morse code, NSA spying, the group Anonymous, and hacking. Students appeared really excited to learn about these topics, but with the exception of some general knowledge about World War II, I knew very little about any of them. While we were co-constructing, one student sent me a link to a website that described some famous unbroken codes, so I decided to present these scenarios to students in order to pique their interest and demonstrate the relevance and continuing importance of these ideas. In the meantime, I had to scramble not only to prepare lessons for this upcoming unit, but also to learn the mathematics that I was about to teach! Thankfully, when I was doing some research on the Allied efforts during World War II, I came across the resource-laden website for Bletchley Park (Centre for Innovation in Mathematics Teaching & Bletchley Park Trust, 2014).

Bletchley Park is the name of the organization that broke codes for Great Britain during World War II, and I found that they had a well-structured resource page with mathematics lessons for students. These lessons introduced students to topics in cryptography, and they also explored related concepts like Braille, Morse code, and bar codes. The cryptography lessons were particularly robust, because they provided detailed background information, example problems with worked-out solutions, and practice problems. In addition, they were constructed in such a way that each lesson built on the knowledge students would gain in previous lessons. I wrote the following in my research journal:

> This discovery solved two major problems for me: (1) these lessons will help to support my own naiveté about cryptography; and (2) these lessons will provide a framework through which I can give students handouts and allow them to work in a self-guided, self-paced manner" (11/23/2013).

While the former point was important, the latter ended up convincing me that we should take advantage of the Bletchley Park resources. Practically speaking, it would have been extremely difficult to create lessons that were sophisticated enough to help students understand cryptography, but Bletchley Park had already done that for me. Furthermore, the budget simulation activity had shown me that students could work effectively and engage in important mathematics when they were given carefully crafted, typed-out instructions. The Bletchley Park resources provided me with a similar type of lesson structure, where students could have all of the information in front of them and work at their own pace, while I would be free to move from group to group to provide additional support.

I decided to make some of the fundamental cryptography lessons mandatory, such as substitution ciphers and transposition, while allowing students to choose from others, such as binary codes, genetic fingerprinting, and ISBN numbers. As a culminating project, I decided that students could attempt to break some codes from the FBI website, transmit and receive messages using Morse code, or create their own ciphers and decode hidden messages from their classmates. I was excited to begin this unit, but in all honesty, I was nervous to engage with material that I knew so little about. I wrestled with these issues right from the start, but I committed to move forward nonetheless. I wrote in my research journal that codes “wouldn’t have been my first choice for a unit, but there is mathematics and logical reasoning involved, so if students are interested in it, then I think it is worth the time” (11/23/2013). I decided to take a chance, hoping that my own ignorance and relative lack of interest would be offset by Bletchley Park’s well-crafted lessons and students’ apparent interest in learning about cryptography.

A (Mostly) Failed Attempt at Piquing Students’ Interests

When I asked students to describe their prior knowledge of codes or cryptography on a pre-unit questionnaire, the vast majority of students said that they knew very little. Of the students who did have some knowledge, one student stated that he knew “SOS and the very basics of Morse code” and had “broken code before but nothing hard” (Questionnaire 11/27/2013). A second student stated that he could “use a program hash-cat that can break the hash combo and give [him] the characters that were used to make a password.” This student was especially excited when he learned that we would study codes, but I worried that the activities we would engage in would not be authentic enough to compare with his real-life experience.

In addition to inquiring about their prior knowledge, I asked students to reflect on their interest level regarding our upcoming unit. Students' responses varied, but they primarily reflected the content's appeal, importance, relevance, and difficulty. One student touched on her own curiosity about cryptography, particularly in regards to some unsolved codes:

> I think code breaking is interesting because it can either be easy or very difficult and the way people try to break them is interesting. Reading the story of the Somerton man and seeing how people still haven't solved the mystery…is amazing" (Questionnaire 11/27/2013).

Others noted the importance of codes, and specifically binary code, as it related to computers. One student said that he would like to study coding because "computers are the future," while another, recognizing that we wouldn't actually be using computers, stated that "if we're not learning to code on a computer it's not that helpful." Another group of students ascribed their interest in codes to the subject's relevance. One student attributed his interest in Morse code to his future career working in the outdoors, while another credited his interest in code breaking to his desired career as a law enforcement officer. Finally, some students linked their interest in this unit with their level of understanding, as one student said that he was not interested "because I don't know about it," while another said that he was interested in code breaking "only because I can do it."

Students expressed multiple rationales for their interest in codes, and they also gravitated towards certain sub-topics under the umbrella of codes. Some students were more interested in binary codes and computers, while others wanted to study Morse code. Many students wanted to learn about famous unsolved codes, while still others wanted to learn basic cryptography. If this was not confusing enough, I was also unclear about the specific learning objectives for this unit. I think that I wanted students to learn how to do basic cryptography, but I also entertained students' requests to study Morse code or read about the history of unsolved codes. I think that I was blinded by the novelty of a unit on cryptography, and I allowed myself to be pulled away from some of the principles that guided the entire course. I had created this course to help students develop their QL skills, and I had initiated the co-construction process to learn more about the content and skills that would be most beneficial for students. This had worked really well for our first two units on statistics and financial literacy, because students' interests had lined up with topics that were relevant to their lives. Our third unit was different, though, and I found out all too quickly that the novelty of studying cryptography would not last long.

While many students were engaged during the first lesson or two, I soon realized that students were not enjoying themselves, nor were they learning very much mathematics. A week into the unit, I noted that in my morning class "some students did little to nothing, and several students started packing up with ten to fifteen minutes left" (Field Notes 12/05/2013). While my afternoon class went a bit better that day, it was not long before they started to lose interest as well:

> Today's class was not as productive as others. Today, students were working on public key cryptography, which requires a basic understanding of modular arithmetic. The underlying principles are fairly technical, but the [method of encryption and decryption] is fairly procedural. Looking back on it, I don't know if this was the best lesson to give students. As opposed to the substitution ciphers, which relied on translations and letter frequencies, I could tell that students were just following a procedure without understanding the underlying relationships. Students still worked, but the energy was much lower. The work felt tedious today, as opposed to last class, when students were engaged and excited (Field Notes 12/09/2013).

Students in the morning class also complained about this particular assignment, and I wrote in my field notes that "I don't blame them. It is modular arithmetic at a pretty high level, so I don't know how understandable, or valuable, it is" (12/11/2013). I began to wonder at that point whether I had made a mistake by allowing students to study cryptography.

In order to get a better sense of how students were experiencing this unit, I walked around the room to do an informal check-in. I spoke with one student who hadn't turned in any assignments, and I reflected on his response in my field notes:

> He responded that he didn't think this was useful or interesting. He commented that it is not hard to shift letters by 3 or 4, nor is it challenging to follow an algorithm where you raise numbers to the third power or the seventh power. Further, he said that this had no value for his life. I told him that this didn't give him an excuse to not do the work, but after thinking about it, I realized that he may be right. The whole purpose of this course is to develop students' QL skills, and this unit is not doing it for this student, or more than likely, many others as well. I chose this unit because students demonstrated interest, but maybe I made a mistake. Students may be interested in something, but what if it isn't valuable for their futures? I guess this could be great if students are going to work for the FBI, but the vast majority will not…In this unit, I submitted to interest at the expense of relevance for the majority of the class (12/11/2013).

This student made me think more profoundly about the co-construction process. Co-construction was a tool I employed to help develop students' QL, but here was an example where students' interests moved us away from relevant mathematics. I decided to let this student go off on his own to explore topics that were more valuable for his life, but I wondered whether other students were feeling the same way. I decided to check-in with some students in my afternoon class, and two students in particular compelled me to make a change.

I paraphrased one student's comments in my field notes: "Do we really have to do this? What's the point? I liked it better when we did budgets, because I will actually need to know that stuff" (12/11/2013). When I asked a second student how he was experiencing our third unit, he stated that he was "getting an Algebra II vibe." I wrote this down as soon as I heard it, because it truly exemplified how far we had moved from teaching for QL. I had spoken with this student about Algebra II before, and he told me that he found it to be irrelevant and unnecessary for his life. I didn't disagree with him, because while Algebra II can be a worthwhile class for many students, it often misses the mark for students who do not have the prerequisite knowledge or the desire to pursue mathematics at a higher level. When he claimed that our unit on cryptography was similar to Algebra II, I concluded that it might be time for a change. I had been feeling that this unit was taking us off track for some time, and I wrote in my field notes that "it is hard to convince students to do something if you're not convinced yourself." In response, I decided to give students a questionnaire to get a better gauge on their feelings about our unit on cryptography.

Recognizing that Students are not Learning

I first asked students to consider whether the lesson structure was helping them to learn. I was particularly interested to hear their responses because I had success with a similar structure during our budget simulation in Unit 2. On the questionnaire, many students stated that they found the lesson structure helpful because it gave them control over their learning and allowed them to move at their own pace. One student mentioned that she liked this format better "because I have exactly what I need to do right in front of me," while another thought it was better "because it gives me a chance to prepare what is going to come" (Questionnaire 12/12/2013). These responses were in line with students' responses from the previous unit, when I first learned that many students preferred typed-up assignments with detailed instructions. Though the responses were generally positive, some students remarked that the activities did not contain detailed enough explanations. One student stated that he liked "how they are typed up but you need to make more detailed [sic] on what you want us to do," while another contended that "codes aren't taught well following the exact wording of the packet." One student leveled an even stronger criticism: "I understand that in college we might need to teach ourselves but I have no idea what's going on." These responses really hit me, because they spotted a weakness in this unit that I had worried about from the beginning: namely, that I was effectively relying on an outside resource to teach my students. To be fair, I think that the Bletchley Park resources are extremely well done, but without additional scaffolding and more individualized support, the material may have been too much for my students. I believe that my students' criticism had more to do with my failure to adapt these lessons than the format itself, because if I had created the activities with greater care, then maybe they would have developed a deeper understanding. I had already concluded that large-group instruction was ineffective with these two classes, and now I was finding that detailed, typed-out instructions could be an effective alternative, but only if I tailored them to the specific needs of my students.

A pair of students did not like the fact that I gave them several lessons in one large packet. At the time, I felt like I was just saving time and paper, but the students commented that they felt "overwhelmed" and thought it was "intimidating to see so much work that needs to be completed" (Questionnaire 12/12/2013). This might seem like a trivial recommendation, but these are the types of things that can either encourage students to participate or convince them to shut down. Once again, the lesson structure was not the problem; rather, my inability to tailor the assignments to my students' unique needs had brought about these negative reactions. I realized at this point that I needed to take greater care when creating my lessons, because poor formatting or unclear directions could create a barrier to prevent students from engaging with the content. Finally, one student made a comment that really struck a chord: "I don't like the typed format of all paper. I prefer projects more than this format." At first glance it appeared that this student was just stating his preference for projects over worksheets, but I think he was really hinting at a much bigger issue. It may be that he was questioning whether daily worksheets with example problems and activities were as valuable as a project-based learning approach, particularly in a QL classroom. I wondered whether other students felt similarly, and thankfully I was able to gain some insight from other items on the questionnaire.

I asked students to reflect on what they learned, and to discuss whether these new understandings were preparing them to be quantitatively literate. Some students tied their learning to interest or ease of understanding, as I had observed in previous units, but others made a judgment on the value on these assignments and the unit as a whole. Some students defended these activities in this unit, because they challenge “a different part of [the] brain” or “in life you need to break down problems and be able to solve them,” but the vast majority did not (Questionnaire 12/12/2013). One student stated that he “may have learned very little; however, [he does] not see the real world application [he] will be able to use in the future.” Other students shared a similar sentiment, as they stated that their desired careers or fields of study were unlikely to require code breaking. One student went a step further when he said that “I feel like I haven’t learned much because it is all strategies and following directions and it gets frustrating.” He didn’t say it in so many words, but this student was really challenging whether these lessons could lead to genuine learning, since according to him, they merely required “following directions.” Two other students were a bit blunter, as one said that this unit was not preparing him to be quantitatively literate because “I am just answering questions on the same stuff repeatedly,” while another said that “this unit is all busy work with little to no value.” These students did not hold back, and I grateful that they didn’t, because I needed to know that these lessons were not helping students develop their QL. Clearly, our unit on codes had missed the mark, and it was obvious to me that I had to make a change.

Changing Course

I reflected on this moment in my research journal:

> I wonder if I wasn’t as creative as I could’ve been when creating these lessons. To be honest, I wasn’t interested in the material, and I didn’t know anything about codes or code breaking. Interestingly, this mirrors the experiences of [many] students, doesn’t it? I wasn’t interested, and I didn’t understand the material. As a result, I wasn’t successful. While it is hard to admit that I messed up, and I am

> tempted to just push through the rest of this unit, I decided to create an alternate assignment (12/12/2013).

I wanted to give students the opportunity to complete an alternate assignment that had real value for them, but was also short and understandable enough that they could complete it while the rest of the class finished their work with codes. Since many of my students were fully immersed in the college application process, I modified an assignment from Chelst and Edwards (2005) that investigated decision-making using multiple criteria. The assignment asked students to come up with criteria for their ideal college, create measures that could be quantified, convert all the measures to common units, and calculate a number that could be used to compare schools. I chose this assignment with the advice of a colleague who used to teach the upper-level mathematics electives, and he said that it was a nice activity to show students how mathematics can be used to make important decisions. In an attempt to avoid my earlier mistakes, I created an assignment that carefully built one idea onto the next, that provided examples and additional background information, and could all fit on just a few sheets of paper. I wished that I didn't have to split the class into two groups with two very different assignments, but I felt that I had no choice. A handful of students were really thriving with the cryptography assignments, but I couldn't ignore the fact that the rest of the class was not.

I presented this assignment to my morning class, and of the sixteen students who were working on codes (two were already working on an alternate assignment), only five decided to switch. I was startled, to say the least, and I reflected on this incident in my field notes:

> I was surprised, because several students had remarked that the codes assignment was not useful to them, or it was too hard. One student who complained that it was too hard ended up sticking with it, and [I had to convince] another student who stated that he really hated this assignment…to switch… I didn't understand why more students didn't switch. Here are some of my thoughts: (1) they wanted to take the easiest route, and they thought codes would be easiest; (2) their friends weren't switching; (3) they didn't actually care about the relevance of the assignment, so they…just stayed put (12/12/2013).

To be fair, some students were doing really well with the codes assignments, and one student in particular was thriving more in this unit than at any other point in the class. As for the rest of the class, I began to wonder whether their complaints about the unit were merely a smokescreen because they didn't want to do anything, regardless of the topic. Interestingly, my three top performers, based on their grades at the time, all switched. This made me wonder whether there was anything I could do for the students who were underperforming. I commented on this later in my field notes:

> Even if not many switched, I still think it was worthwhile to offer an alternative, because choice allows students to feel some ownership over their work, and it also eliminates an excuse. At the same time, the one student who I convinced to switch made it clear to me that he didn't like either project, and throughout the year, he has expressed that he is not interested in anything I have proposed. Some students will just be difficult. Some students don't like school. This is an important lesson for me. I can't beat myself up over not reaching every student.

This was a dilemma that I struggled with throughout the year, that I can only control so much of students' educational experiences. No matter how hard I try, I cannot choose the mix of students that are in my room, nor can I be motivated for them. While there is definitely more that I can do, I have to be realistic and reach the students I can reach, and not give up when I don't reach everyone.

When I next saw my afternoon class, one student who was particularly frustrated with the codes assignment "walked in and made a resigned comment about how she wasn't looking forward to this" (Field Notes 12/13/2013). When I told her that I created another option, she "immediately brightened up." I began to address the class by telling them that the questionnaires had revealed that many students didn't like the unit, and one student interrupted and said "because it's useless." The rest of the class went silent and looked at him, and much to their surprise, I did not disagree. I reminded students that even though they might not be interested in everything we do in class, it was essential that our units be relevant either to students' lives or the lives of most adults. I explained the assignment, and twelve students decided to switch, while the other twelve stayed with codes. Much like the morning class, I found it interesting that certain students, and particularly one group of boys, decided to stay with codes, "even though some of them had complained vociferously during the last class." One of these students is the boy who remarked that "this unit is all busy work with little to no value" (Field Notes 12/12/2013), but yet he decided to stay with it. Another pair of students who originally switched to the college project decided to switch back, and when I asked why, one student responded that he did so "because [the other student] said it would be easier" (Field Notes 12/16/2013). I was shocked, to say the least, because I worried that there were students who simply wanted to get out of doing work, and some of them were exploiting the very real concerns of their classmates. I believe that this may have led to a more hostile learning environment, and it is unfortunate that this had to take place. At any rate, by giving students the option of which assignment to complete, I had forced them to make a choice, and hopefully they would take some ownership over their work.

Moving Forward with Multiple Assignments

Giving students a choice in what they wanted to study impacted the classroom environment, particularly in my afternoon class. Several students really began to do good work on the code activities, and many of the students who chose to switch to the college choice assignment were thriving. For the project phase of the unit, the codes group had to decipher three FBI practice codes and make an attempt at an unsolved code, the college group had to complete their rating system and defend their choice of an ideal college, and a pair of students that had broken off earlier in the unit decided to create a presentation on the mathematics of the interstate and US highway systems. I found that some students in the codes group had trouble with the end-of-unit assignment, but for very different reasons. One particular student was unable to crack the FBI codes, even though they required basic skills that students had learned in their first lesson on substitution ciphers. She was frustrated that I did not provide more guidance, which I had done purposely to give students a chance to transfer their skills to a novel situation. Another group of students worked through the first three codes in a day, but then completely stalled with the unsolved code. I thought that I was motivating them by asking them to attempt an unsolved code, but in retrospect, I think I actually dissuaded them (Field Notes 1/08/2014). In retrospect, I think that I failed to create an effective assignment because I was unable to find a task that truly represented a real-world situation. Cryptographers would have access to technology and techniques that far surpassed what we did in class, so the best I could do was to simulate a basic code breaking experience.

Students in the college group faced a different set of challenges, and these were more mathematical in nature. In order to assign a numerical value to criteria like distance from home, environment, and reputation, I suggested that students make their ideal choice in each category a 1.0 and their least desirable option a 0.0, with all others spaced proportionately in between. One student scoffed at this suggestion, telling me that I was overcomplicating everything. I suggested a 0 to 1 scale because I thought it would be easier to convert numerical data, like SAT scores, to that same scale, thus allowing every criterion to have common units. I didn't think it would be a problem at the time, but two other students expressed similar concerns and asked if they could convert all the scores to a 1 to 10 scale. I began to wonder whether students had a fear of decimals, and I wrote down some impressions in my field notes:

> It became clear to me that decimals were really causing a problem for these students. I was more familiar with fear of fractions, but decimals as well? Clearly, I need to do some work with these students, where they [can] experience a situation where decimals are useful and necessary. I would think that our unit on money would have done that, but I guess these fears are fairly deep-seated" (1/08/2014).

I recognize now that I was sometimes more focused on finding engaging and relevant contexts than in developing basic skills that would help students later in life. The fact that I didn't learn about students' fear of decimals until January is telling, and it shows that there is a down side to giving students too much control in the co-construction process. With my knowledge of mathematics and additional years of experience, I should have set more boundaries and established more requirements in the co-construction process. In an attempt to give students a voice, I may have erred on the side of too much student control, thereby losing the healthy balance that would engage students through relevant contexts while still equipping them with important QL skills, like the ability to use and understand decimals.

A third group of students had confronted me early in December with a request to work on something that was of greater interest and value to them. These two students were generally hard-working and responsible, so I honored their request. I asked them to write a project proposal that investigated a quantitative issue of interest to them, and to prepare a PowerPoint presentation that they would share with the whole class. They chose to do research on the interstate highway system, and to present its structure and relationship to US routes. At first, I thought that the related mathematics might be somewhat trivial, but I was willing to give them a chance to explore their interests. On the day of their presentation, the students described how the interstate highway system was a grid structure that utilized odd multiples of five (running north and south, starting in the West) and multiples of ten (running east and west, starting in the South) that overlaid a US route system that was planned with an opposite orientation. I described the class's reaction in my field notes:

> The class was very well-behaved, and at least some students were paying close attention. I had a lot of questions, as did another student, as we tried to understand the implications of these grids on our own state. The assistant principal came in about halfway through, and as a former history teacher, he volunteered some additional information to help us better understand why this grid was created…I thought this was an excellent example of interdisciplinary work that resulted from a particular student's interests (1/15/2014).

I was impressed by these students' work, because the majority of my morning class, which was often tough to engage, seemed really interested in the presentation. The topic might not have been my first choice, but the students learned about numbering systems, they explored patterns and multiples, and they considered basic principles of graph theory. In addition, we were fortunate enough to learn more about the history of the interstate highway system, which may have appealed to a different subset of students. All told, this experience demonstrated that some students are capable of finding mathematics in things that they encounter every day. The challenge moving forward, then, was to see if these types of assignments could support a broader range of QL skills, rather than the very specific knowledge that was acquired during this particular project.

I asked students to reflect on their experiences in a post-unit questionnaire, and two comments in particular demonstrated that despite some of the challenges, there was also some real growth in Unit 3. One student commented that she learned “what really matters to me when it comes to picking a college” (Questionnaire 1/13/2014). This student may not have articulated it this way, but her response indicated that mathematics had helped her to get a clearer understanding regarding a major life decision. She had used mathematics as a tool to help her make an important decision, and hopefully, she would continue to use mathematics as a tool in life after high school. A second student stated that “learning codes made me have an aspect [sic] on how math can just exceed the x and y concept.” This student was acknowledging the fact that mathematics was much broader than the typical high school curriculum. This was an important takeaway, because some students might believe that mathematics is nothing more than their high school’s rendition of Algebra I, Geometry, and Algebra II. If students leave high school with that belief, then those who have poor experiences might carry negative feelings about mathematics into life after graduation.

I also asked students to reflect on their interest level in our Unit 3 topics as well as the utility of those topics, and I got a variety of responses. A handful of students stated that they liked learning cryptography, while others asserted that the college project was easier to do, and much more helpful. One student exclaimed that she "loved the codes," but when responding to whether she found the topic to be useful, she said "not really, but it's really fun" (Questionnaire 1/13/2014). This theme permeated many of the students' responses, as they responded that codes would be useful "only for school," or only "if you're…going into a career with this." Students who worked on the college project expressed very different sentiments, as they claimed that these techniques were useful "because I will be choosing a college very shortly," "because it is an important part of my life at the moment," and "because I may go to one of the schools." While I cannot say that all students developed a sufficient understanding of the mathematics that was involved in the project, these responses do suggest that relevant topics might serve as better motivators than topics that are merely interesting. This balance between relevance and interest factored heavily in our discussions and in my own reflections as we set out to construct our next unit.

Preparing for Unit 4

I had wondered in previous small-group discussions whether the categories I created early in the year were having a detrimental effect on students' participation in the co-construction process. For example, in one small-group discussion, a student repeatedly brought up Morse code, even though other students in the group wanted to take the conversation in a different direction. At first, I thought that this student was just interested in studying Morse code, but then I starting taking note of similar experiences at various other times during the co-construction process. Even if discussions were moving in one direction, students often felt compelled to adhere to the categories and subtopics I created. I wrote about this several times in my field notes and research journal that week: “These umbrella topics I chose have had a huge effect on our discussions (Research Journal 11/11/2013); categories had a huge influence on students (Field Notes 11/12/13); and *“*Clearly, these words, these categories are dominating and directing our discussions” (Notes on the Transcription 11/11/13).

This amount of influence was problematic, because there was no science behind the method I used to create these categories. For example, I lumped “Sports, Psychology, Politics/War/Religion, Beard growth, Pop culture (Music, Movies, TV shows), GPA, Test Scores, Crime rates and cold cases” all into one category that I called statistics (Categories 9/11/2013). It is no wonder that students chose this as their first unit of study, since it had topics that could appeal to many different groups of students. While I stand by my opinion that all of these topics could be studied in a statistics unit, in reality it would only be feasible to study a few. A similar thing happened in our second unit, where I combined “Taxes, Stock Market, Investing/Interest, Inflation, Loans, Work, Net worth, Currency, Economics, and Finance” all under the umbrella of money (9/11/2013). Now we covered more of these topics in Unit 2, but we still didn’t address them all, nor did we cover them to the extent that students might have wanted. When I noticed the same thing happening during the co-construction of our third unit, where students who wanted to study Morse code were being asked to co-construct a unit with students who wanted to study cryptography, I realized that something needed to change. As I was reflecting on this, I wrote that "Next time, I want to return to the original list and start from scratch – topics – a single unit – project – lesson structure" (Notes on the Transcription 11/11/13).

I asked both classes for volunteers to help me co-construct Unit 4, and five students from each class agreed. I began our discussions by explaining how I wanted to do things a bit differently this time:

> Me: OK, so what you're looking at is the original list of topics…
>
> Student 2: Wow
>
> Me: And next to each number I tallied the total number of students that picked that topic. So if you notice, at statistics, had 18, money had 17, and then Morse code and code breaking had 16. So those are the top three choices. But, what I'm starting to realize is that I lumped these categories in myself, so I might have unduly influenced, um, people. Like, for example, statistics, I put sports and politics and beard growth and crime rates, I put them all together. You didn't do that. So I don't know if…
>
> Student 3: You're saying that you don't know if these are the categories that we would necessarily pick, you just picked the actual…
>
> Me: Right
>
> Student 3: Specific, yeah
>
> Me: Right, you might not have wanted to study statistics, but maybe you really wanted to look at crime rates, so you picked statistics.
>
> Student 3: But now you're lumping it in
>
> Me: Yeah, so I don't know if I influenced you too much, so I wanted to give you guys the original list, and I don't want to decide what unit to study next. I want you guys to take a look at it and talk about it, and I also want you to feel comfortable, if you want, pulling things out, or reorganizing them in any way that you want. But the goal for this short conversation is maybe that the six of us can agree on a unit, and then we can talk about some specific things to focus on in that

> unit. And you can use this list as a guide, or you cannot use this list at all, that's up to you.
>
> Student 1: So what are we, like a council, here?
>
> Me: Yeah
>
> Student 1: Wow (Transcription 1/02/2014).

I think that students were honored with the amount of control I gave them, particularly because their decisions would be so consequential for their classmates. While I sensed that some students felt an additional burden because of this, one student remarked that she didn't "think [her other classmates] really care" (Transcription 1/02/2014). This theme came up several times in the morning discussion, as students tried to construct a unit that was useful for them while also recognizing the reality of the classroom dynamic around them. This challenge of how to work with the rest of the class came up again when I asked the group whether one of them wanted to introduce our plan of what to study. The following dialogue ensued:

> Student 5: No, you (talking to me) gotta introduce it
>
> Student 3: Yeah, you (talking to me) have to do it. Nobody already…
>
> Student 2: I think the one they would look to the most is Student 5
>
> Student 4: I'll just start screaming at people
>
> Student 3: Yeah, let's all just run
>
> Student 4: Yeah
>
> Student 3: Yeah, so you (talking to Student 5) should probably do it
>
> Student 5: I'm not doing that. I'll get made fun of…they wouldn't listen to me at all.

I knew that peer pressure was a factor in this class, but I had never seen students talk about it so openly. Student 5 seemed to be a popular person, but he was clearly taking a risk by participating in this small-group discussion. He was unwilling to try and encourage the rest of the class, and like other members of this small group, he had decided to tread lightly around these more difficult students. This theme came up once more later on in the discussion, when we broached the subject of class structure and whether my approach was appropriate for this group of students. Student 5 commented that "the problem is just, like, the people that don't do the work, like, in general, no matter the topic. Like there are some people that just like, absolutely do no work. Like, nothing" (Transcription 1/02/2014). I struggled with my morning class all year, but it was striking to hear students reflect on some of the same issues. It was helpful to be reminded that in the midst of trying to improve students' QL, create engaging and relevant activities, and balance student choice and clearly defined goals and objectives, that I was working with high school students who didn't always want to learn, who struggled to feel accepted, and who were sometimes preoccupied with fitting in. Nevertheless, every student deserves a chance to learn, and QL is important enough that I had to persevere in creating lessons that could somehow engage even the most resistant students. So the students and I immersed ourselves into the co-construction process, and we navigated through constraints, differing opinions, and various compromises to prepare for Unit 4.

Operating within Constraints

One of the major constraints that students grappled with was the fact that we were limited to topics that contained meaningful and appropriate mathematics. In my morning class, students revisited the list of topics that we created in the beginning of the year, and students struggled to see the connection between mathematics and some of these topics: “What would we do with nature and outdoors? What does that even, like, how would we study that?” (Transcription 1/02/2014). One student attempted to make a case for some of these topics, but he did not offer any specifics: “Surfing has a lot of math involved. Farming has a lot of math involved. Forestry has a lot of math involved.” As I listened to students discuss these issues, I could not help but worry that they would choose a topic that either had little in the way of relevant mathematics, or was so foreign to me that I would have to learn it in just a few short days. The QL literature suggested that students should be given an opportunity to explore their interests and seek answers to questions with unknown answers, but I worried that this would not be appropriate for many of my students. Some students could surely engage in that type of independent, self-structured work, but others would likely need much more support.

A similar discussion came up in my afternoon class:

> Student 6: My question would be, though, if we were to do something like cooking or nature/outdoors, what, how would we set something like that up, like, what would we, how would we, eh, how would we relate that to math?
>
> Student 7: Ingredients…
>
> Student 6: Like cooking, I know you’re measuring everything, um, nature/outdoors, I mean…
>
> Student 8: I don’t know

Me: Yeah, for cooking there are a lot of conversions.

Student 6: Weight (Transcription 1/02/2014)

Once again, students chose topics of interest to them, and they discussed whether each topic had sufficient mathematics to warrant its inclusion in the course. These discussions had the potential to be a powerful learning experience for students, because QL requires an understanding of how mathematics plays a role in everyday life. By participating in these discussions, then, students were presented with an opportunity to think deeply about the role of mathematics in their lives, and thus further develop their QL.

In addition to wrestling with the mathematics involved in each topic, students also had to navigate the specific boundaries that were placed on them by me and other authority figures in the school. The following dialogue illustrates one such exchange:

> Student 1: Yeah, but I have a question. What…how. OK. I guess I don't fully understand how the class works. Does there need to be, like on your end, does there need to be a unit that the whole entire…like does there need, you need, you have to say we are studying statistics as unit 2, or whatever, and then have everyone pick something under that? Or can everyone pick their own thing? Or would that be too…
>
> Me: Well, the challenge is how capable am I of keeping up with that. So as long as I can manage it. If you remember our first unit in statistics, we basically had three different classes. We had a fantasy football…
>
> Student 2: That was so long ago
>
> Student 3: But how did it work, how did it go?
>
> Me: I think you guys can answer that. It was hard for me.
>
> Student 3: OK. So would you be more comfortable if we had more of a common theme going on throughout the classroom? (Transcription 1/02/2014).

In this segment, students seemed to be empowered by the amount of control I was giving them, but they also recognized that they had to operate within certain boundaries and constraints. Student 1 wanted to know how much freedom they had, and his tone indicated that he was really questioning whether I was working under my own set of external constraints. Student 3, on the other hand, clearly saw me as the authority figure, and she was concerned primarily with satisfying me. This was an interesting exchange, because it demonstrated that different students can operate with different understandings about the factors that influence what takes place in a classroom. While this was only a small segment of the discussion, Student 3 operated under the assumption that I had final say in what took place, while Student 1 questioned whether there might be other forces at play. These are the types of discussions that don't always come up in a classroom setting, but when students are given the power to make decisions, then they have to wrestle with some of the outside forces that teachers negotiate every day.

The third major constraint that students wrestled with was much more logistical in nature. As they worked through each other's ideas and talked about the mathematics involved, students also had to consider whether certain activities would be feasible in a classroom setting. In one such example, students discussed the possibility of doing a unit on cooking, and one student in particular struggled to imagine how we could achieve this in a classroom setting. First, she stated that it was "insane," and when I asked her to explain, she pointed out the difficulty of finding a location to do the cooking (Transcription 1/02/2014). When I validated her concern, she went further by saying that if we couldn't actually cook, then the unit "wouldn't be particularly hands on." With this comment, this student addressed the challenge of trying to recreate an authentic experience in a classroom. While we could never have a truly authentic experience while inside a mathematics classroom, I wondered how much these experiences could be modified before students would dismiss them as phony. In her opinion, any cooking simulation that didn't actually take place in a kitchen would have automatically rendered the unit inauthentic, and therefore, not worth doing. Students wrestled with many of these challenges throughout the co-construction process, but they also struggled to work through the differing opinions of their classmates.

Working through Differing Opinions

During this round of co-construction, it became apparent that students were not only working against various external constraints, but they were also vying with each other to have their voices heard. I found that as students shared their ideas, certain individuals were able to influence the direction of the discussion. In my morning class, Student 1 represented this dominant voice:

> Student 1: I think a good idea is…to look at…I want to move in the direction of looking at numbers that, more of understanding numbers. Like we're looking at the interstate system. And that these are just numbers that are, literally everywhere. And you look at, you look, driving to school you see them, you see county roads, you see interstate roads, you see, um…
>
> Student 3: Turnpike
>
> Student 1: Turnpikes
>
> Student 2: Parkway
>
> Student 1: You see parkways. You see exit numbers. And what do these mean? Like these are just numbers that (inaudible) daily. And I think, I mean that's only

> the most that I'm saying. There are numbers that you see every single day that you just don't think about, and I think that something useful would be to…
>
> Student 5: Understand why…
>
> Student 1: Understand why there are, why math exists in…
>
> Student 5: Wow
>
> Student 4: Especially with, like, driving
>
> Student 5: He's getting deep now (Transcription 1/02/2014)

Before this segment, Student 1 had given his classmates an opportunity to express their ideas. During this interchange, though, he made the case for why we should study numbers that we encounter in our everyday lives. Student 1 was well-respected by his peers, so they tended to pay attention when he made a point. In addition, he was an effective speaker, and he knew how to make his peers feels included in his ideas. Later in the discussion, Student 1 once again demonstrated a masterful ability to include others in his plans:

> Student 1: I think, I know that, because, that would, I feel like that would be your umbrella, and I'm not, I'm not saying that I have, I haven't thought about it, I could think of some more, and all of us could think of some more ideas with numbers come into play every day, because…
>
> Student 3: How about we do something though, like, everyday numbers. That would be the category. Then people can kind of pick what everyday numbers they want to look into
>
> Student 2: We've come up with, what, 5 ideas from 5 people? Like if we talked to everyone else, like what numbers do you see every day that you just don't understand? Like why is there no Room 1 in [our] high school? (Transcription 1/02/2014)

In this exchange, Student 1 invited his peers to think of some other numbers that we see in everyday life, and Student 3 immediately formalized Student 1's idea. Student

2 then affirmed Student 1's suggestion that everyone should get involved, and she extended his suggestion to the entire class. It is impressive how Student 1 helped his peers feel ownership over this idea, even though it was his to begin with. Interestingly, the afternoon class also saw one student rise up to take control of the discussion, but he did so in a very different way.

> Student 6: I think cooking would be fun. I think people would actually get into it, because it's something they can do
>
> Student 8: Yeah, but it depends, like, how…
>
> Student 9: We can't exactly bring a grill into the classroom
>
> Student 8: Like, when people think of cooking, people are going to want to make stuff. When they find out they're not going to be able to…
>
> Student 6: I was talking…
>
> Student 8: What?
>
> Student 6: The environmental science club, we've already been talking, because we're going to start using some of our crops to go into…
>
> Student 9: Yeah
>
> Student 6: And she said she would teach the environmental club on, like, how to cook, and measuring everything out.
>
> (short back-and-forth regarding this woman's name)
>
> Student 6: I feel bad that I don't remember her name. But, I mean, she's nice and she's, she's willing to teach people how to do this and so if we, like, I don't know, I know we'd have to get permission from the school and everything and her permission, but I think we could set up a pretty good thing. And then at the end of the, like as, like, our final thing for it, maybe cook something, like do groups, that kind of deal would be... and each make, like the same… (Transcription 1/02/2014).

Much like Student 1 in the morning class, Student 6 had a clear idea about what he wanted to study. Unlike Student 1, though, Student 6 took a more individualized approach, as he became the primary advocate for his idea, as well as the primary opposition for others' ideas. For example, when one student suggested that we study forensics, Student 6 stated "I don't know how many people who would get into it" (Transcription 1/02/2014). Later, a different student suggested that we study fitness, and Student 6 commented "That has a lot to do with the individual, though." Student 6 had a clear vision of where he thought the class should go, and he did what he could to convince his peers. Towards the end of the discussion, he took a slightly different approach, because rather than trying to convince others that his idea was best, he offered a compromise: "I mean, cooking, if we were to do it, could we add a, um, like a health/fitness thing into it?" This suggestion involved a different tactic than he used previously, but it certainly was effective. By combining fitness and nutrition into his cooking unit, Student 6 was able to make the unit sound much more attractive to his classmates. When I polled the class, fifteen out of seventeen students chose cooking/fitness, so Student 6's approach turned out to be successful.

The emergence of individual students as key contributors in the co-construction process was a development that I had not anticipated when I designed this study. I had thought that by inviting students to co-construct the course with me, that I would be giving them all a real opportunity to influence the development of our course. I knew at the time that I would have to find the right balance between my own participation and that of my students, but I did not anticipate the degree to which individual students could take control of the co-construction process from their peers. I had already faced the idea of unequal participation when some students showed little to no interest in the co-construction process early in the year. I justified this inequality, though, because students were free to choose whatever level of participation they wanted. Further, I always asked students to share their thoughts with me in writing, and this made me feel like I was still receiving input from all students. The experience with Student 1 and Student 6 made me reconsider all of this, though, because even though there was no ill will on their parts, they had effectively planned the subsequent unit for their respective classes. This was concerning to me, because their dominance in those discussions had prevented others from participating to a larger extent. This also led me to consider the class as a whole, especially the 75% that had not volunteered for these small-group discussions. Did they really have no opinion, or was peer pressure influencing their level of participation, just as Student 6 and Student 1's rhetoric was influencing the 25% who did participate? I realized at this point that while I had learned a great deal about co-construction and its impact on the development of this course, I was only scratching the surface when it came to students' influences on each other. Though I was only beginning to understand the psychological and sociological aspects of co-construction, they clearly played an important role in how our course evolved over time.

Reaching a Compromise

I made a decision early on that I would not prepare two different units for each of my Discrete Math classes, so I was faced with the difficult decision of how to proceed.

My morning small group decided that they wanted to study everyday numbers, while my afternoon small group decided to study cooking and nutrition. In order to help me with this decision, I spoke with both classes after each small-group discussion. In the morning, I summarized the content of our small-group discussion, and I asked students to write some examples of everyday numbers that they wanted to understand in more detail. One student immediately called out something to the effect of “I am not even a little bit interested in that” (Field Notes 1/02/2014). While this student regularly reminded his classmates that he was not interested in any of our units of study, I could tell from the looks on their faces that others felt the same way. I asked the ten students who did not co-construct with me to write down their ideas, and only five of them responded. Their answers were lukewarm, as they responded with suggestions like “maybe why items have specific model numbers or why the school room numbers are the way that they are” or “why there is no room #1 at the high school” (Topics for Unit 4 1/02/2014). Students’ unenthusiastic reactions just added to my own mixed feelings about this unit, because I was already concerned that this unit would not cover the type of mathematics that I felt could develop students’ QL.

When I spoke with the afternoon class, they seemed much more excited about studying cooking and nutrition than my morning class felt about everyday numbers. Students in the afternoon class wrote responses such as “I think cooking would be cool and [a] very useful skill to learn,” “I think it would be interesting to learn about anything to do with fitness because I am very into sports,” and “I’m not good at cooking so it would probably help to learn about the measurements” (Topics for Unit 4 1/02/2014).

After giving it careful consideration, I decided to study cooking/nutrition/fitness first. At the same time, I wanted to respect the input of the morning small group, so I decided that we would study a version of everyday numbers as our fifth unit. I checked in with my morning class, and in particular, with each participant in our small group, to explain my rationale for choosing the cooking unit first. Each participant seemed to be on board, particularly when I told them that we would have access to the school's kitchen. I reminded them that their contributions were important, and I asked them to continue thinking through their ideas about everyday numbers for when we explored those ideas further in Unit 5 (When we eventually did return to their ideas, students and I modified the theme of everyday numbers to focus primarily on the mathematics of road trips. While I will not include a detailed narrative of the road trip assignment in this Chapter, I do reflect on it as I make meaning across my entire data set in Chapter 5).

Unit 4 – Exploring the Relationship between Mathematics and QL

One of the things I struggled with throughout the course was trying to understand the relationship between QL and mathematics. The literature provided some insight into the theoretical nature of this relationship, but I was still unclear about the practical role of mathematics in a QL classroom. What made this struggle even more prominent was the fact that I continued teaching three precalculus classes for the duration of this study. In my precalculus classes, I approached mathematics as an end in itself. While I certainly highlighted various applications of the content, I primarily saw my job as helping students grapple with complex mathematical concepts. In my Discrete Math classes, on the other hand, I had started to see mathematics as a tool, rather than an end in itself. Now, I certainly wanted students to understand mathematical concepts, but as the course wore on, it became increasingly clear that the mathematical content was not my primary focus. In precalculus, I wanted students to understand mathematics for its own sake, but in Discrete Math, I wanted students to understand certain mathematics because of its utility for a specific context or application.

One way to understand this distinction would be to look at a concept that was covered in both precalculus and Discrete Math; namely, exponential functions. In precalculus, we studied the behavior and characteristics of exponential functions by analyzing their domain and range, end behavior, asymptotes, and intercepts. We examined the relationship among algebraic, graphical, and numerical representations of exponential functions, and we considered the relationship between exponential and logarithmic functions. In Discrete Math, on the other hand, we learned how to budget our money, and since money and prices grow exponentially, we explored how this relationship would impact savings and retirement. The exponential relationship was important to understand, but only inasmuch as it impacted our understanding of budgeting. In essence, exponential functions were used as a tool in my Discrete Math classes, whereas they were a focus in and of themselves in precalculus.

Our fourth unit was another opportunity to see how our QL class could use mathematics as a tool. In this unit, we studied nutrition, fitness, and cooking, and we explored the mathematics that accompanied these topics. The mathematics might not have been overly complex (it included topics such as basic conversions, the use of appropriate algorithms, scaling, and measurement), but it did provide important insights into the topics we were studying. Teaching for QL does not require the development of increasingly sophisticated mathematical ideas; rather, it requires teachers to help students understand the quantitative aspects of everyday life.

I split this unit into three sections: during the first week, students analyzed their consumption habits for a week, both in terms of total number of calories and specific types of food; during the second week, students documented all of their activities and calculated the number of calories they burned using multiple algorithms; and during the final week, students modified a dessert recipe to account for the number of students in their class, calculated nutritional information based on their scaled-down serving size, and prepared and cooked the dessert. I liked this unit because even as each assignment was the same for each student, "each one [was] different because it [focused] on each student's life" (Research Journal 1/09/2014). I thought that the individualized aspect of this assignment would motivate students in much the same way as the budget simulation in Unit 2. I wrote about my hopes for this unit in my research journal:

> I like this unit, because it plays on students' interests, it requires students to use mathematics on real data, and these data are unique to each student. The content relates to students' everyday lives, and the knowledge they will gain will not only be valuable for their QL, but it will also help them to live healthier lives. Furthermore, cooking will allow us to spend some time with fractions, which I imagine are very difficult for some students. Based on my recent discussions with students about decimals, I fear for how students think about fractions. Hopefully, by showing students the importance of fractions with cooking, and then giving them some hands-on experience with fractions, then they will get a bit more comfortable, and perhaps find a bit more value in the utility of math in everyday situations (1/09/2014).

Before starting this unit, I gave students a questionnaire that asked them to reflect on the importance of nutrition, fitness, and cooking. Several students stated broadly that these topics were important, as one student said that “these will always be important in life” while another said he was interested because the topics were “practical” (Questionnaire 1/13/2014). Other students described their importance in more individualized terms: “Yes [I am interested in cooking] because there will come a time where I will live on my own and I’ll need to cook-bake myself;” “Yes, [I am interested in] nutrition more so I stop eating so much garbage;” and “Yes. It is always important to know how to stay healthy (nutrition) and how to make good foods (cooking) especially with college coming soon.” Two other comments caught my attention because they touched on slightly different themes. One student said that he was interested more in cooking, “as it is a topic the school does not normally cover.” This student seemed to appreciate the opportunity to do something that was atypical, so much so that he later asked whether we could do something similar once a month for the rest of the year. The chance to give students novel classroom experiences is important, particularly in terms of authentic instruction. Another theme was alluded to by a second student, who commented that he was “interested in cooking because there is a payoff.” I thought this comment was telling, because he acknowledged that there was a certain amount of value in learning how to cook that may not be present in other academic activities. This was an important feature in some of my most impactful lessons, and not surprisingly, it is also referenced as one of the biggest benefits of teaching for QL by the literature.

Caloric Intake and Expenditure – Turning My Understanding of QL and Mathematics on its Head

Though I constructed this unit to have three distinct segments, I found that in practice there were actually only two. The first required students to keep track of their caloric intake and expenditure, and the second asked students to modify, cook, and calculate the nutritional information for a dessert. When I introduced the first assignment, I was a bit nervous, because it required students to keep a detailed accounting of the food they ate at home as well as at school. In order to do this, then, students would have to apply themselves outside of the classroom. Unfortunately, just as in so many of our previous activities, I found that too many students refused to engage. This turned out to be especially problematic for this activity, because I had made the critical error of confusing what students could do in class vs. what they should do at home. Many students were unproductive in class because they argued that they could only do it at home, but at the same time, past experience had taught me that these students were unlikely to do anything unless they were in class. On the first day of this assignment, I wrote that "I still have faith in this project," but I soon found myself wishing that it would come to an end (Field Notes 1/10/2014).

Despite what I now consider to be a poor assignment, there were still some important developments. For one, I was able to have some interesting and significant discussions with students. I wrote about one student in my field notes:

> One student in particular had been told by a coach to lose some weight, and he had been wondering why he was slightly overweight, even though he had been exercising. He came and saw me earlier in the day to tell me how excited he was, and he got right to work in class, and also began to discuss nutrition with his

> group mates. He was really a positive influence, and while he can often distract, in this case, I think he motivated others to stay on task (1/14/2014).

I noted several times in my field notes how impressed I was with this student. Clearly we had arrived at an important topic for him, because before this activity, he had been unable to understand why he wasn't losing weight. The mathematics of this assignment then provided him with the tools he needed to not only understand his current eating habits, but also to set some specific goals that he could document, measure, and track over time.

I had the opportunity to work closely with a second student, Student 2, "which was great, because he usually tells me that he has no interest in what we are doing" (Field Notes 1/15/2014). During previous lessons, he argued that he ate several thousand calories but gained no weight, but when I asked him to verify this fact with some data, he replied that he "just knew." I wrote in my field notes that while he was interested in the topic, "he did not want to acknowledge that a mathematical analysis might help him reach a better understanding." I was appreciative, then, when I had an opportunity to speak with him and try to help him understand how mathematics could help him explain this phenomenon. The way that I eventually reached him was through no skill of my own, but rather through the intervention of one of his good friends, Student 3. Student 3, who I also struggled to reach all year, was extremely interested in fitness and nutrition, and he explained to Student 2 how he used to count calories and was now working methodically towards a specific weight gain goal. The short discussion among Student 2, Student 3 and myself left a lasting impression on me, and I wrote about it in my field notes:

> It was really nice to hear Student 3 offer something really useful to a discussion, and he also served as a nice foil to Student 2, who hesitated to admit that a careful accounting [of calories] might be beneficial. This was one of the more interesting

> discussions I've had this year, and it also might turn out to be extremely important. I have had very little success with either Student 2 or Student 3 so far this year, and this conversation saw Student 3 as an expert, and it allowed Student 2 to at least consider the benefits of this class, even if he wasn't convinced. I hope to build on this experience in future classes…

It would be hard to describe the difficulty I had with these two students over the course of the year, so this conversation genuinely energized me. I had been worried that Student 3 felt overwhelmed by mathematics for most of the year, so it was great to hear him speak with confidence and conviction about a quantitative idea. Student 2 had frequently broadcast his lack of interest in anything related to mathematics, so it was important that he acknowledged, with the support of Student 3, that mathematics was an important piece of one of his interests. These experiences may seem inconsequential in the grand scheme of the course, but they could turn out to be very important for students who hadn't had much success, or who didn't find much enjoyment in mathematics.

Aside from these positive encounters with students, the second interesting development that took place during this unit involved a smart phone app called My Fitness Pal. Apparently, if you type in the food and serving size of anything you eat, this free app will search for the complete nutritional information, display it, keep track of what you eat every day, and compare it with what you should be eating. Several students told me about this app, and they asked me the logical follow-up question of "Do I still have to keep a log of everything I eat and calculate the total calories by hand?" In reality, this app is a much more thorough version of what I wanted students to do for this assignment, but it took away some of the mathematics that was involved (students still had to calculate serving sizes for each meal, but they no longer had to do any of the arithmetic to calculate their total calories). I reflected on this piece of technology in my field notes:

> This makes me wonder…whether I should even be giving this assignment at all. If an app can do it, and do it much better, then why should students have to do it? Wouldn't it be better to have them download the app, experiment with it, and then offer critiques? This is an interesting theme…I have utilized technology, but I've used it to support learning, as I did with Excel. Here is an example of technology that might replace the necessity for the particular skills students are developing, namely, the ability to read nutritional labels…and then calculate total and average calories per meal and per day (1/14/2014).

This goes back to one of the major themes of this particular unit. Is mathematics the objective, or is it a tool? On the one hand, I am a researcher that is trying to teach for QL, but on the other hand, I am a teacher who wants students to learn mathematics. I chose this activity partially because it could develop students' QL and strengthen their mathematical skills, but if I was only focusing on QL, then wouldn't the app make much more sense? It is more powerful than pencil and paper, it is more convenient, and it is certainly more likely that a student would actually continue using it once the unit ended. Furthermore, students still need QL to understand the app, to estimate or calculate serving sizes, and to adjust their consumption habits to better meet the recommended daily allowances. Looking back on it, I can certainly understand the argument for downloading and utilizing this app, but at the time, I told students that while they could use it, they still had to submit their daily logs with pencil and paper. Maybe the rest of the class didn't notice it, but for the students who used My Fitness Pal, I had likely limited the relevance and utility of this assignment. In this situation, I had not used technology to support learning, but instead, I had put limits on it. As a consequence, I had taken an authentic context and structured an activity that had rendered the task inauthentic, at least for these students.

The second assignment was in many ways similar to the first, since I asked students to document all of the activities they engaged in for a week, with an eye towards calculating their daily caloric expenditure. Unlike the previous activity, though, this assignment required students to reason mathematically, and not just utilize certain mathematical tools. I wrote the following in my field notes:

> I was happy to see that students were really questioning the formula I gave, as well as the scale factors that they were using for various activities. Students had to perform various conversions to utilize the formula (pounds into kilograms, inches into cm), [but they were also] grappling with the reasons why formulas looked a certain way. I think this is a better exercise, because students are looking at something that is important (fitness), that is quantitative, and that requires them to think in a more mathematical/logical way. That is really my goal here, [to] help students to think more logically and consistently about what they eat and how they exercise (1/28/2014).

Just like for the previous activity, mathematics was being used as a tool, but the reasoning that was necessary to complete this task was more sophisticated, and more important for developing QL. Students were not simply using arithmetic to perform basic calculations, but they were judging the accuracy of multiple formulas and evaluating which one would best explain their own activity. I thought that these activities were beneficial for my students, but to get a more detailed picture, I distributed a questionnaire that asked students to reflect on these two assignments.

Many students indicated that they learned a good deal from these two activities, and some even said that they planned to use this knowledge to make a lifestyle change. For example, one student said that “it taught me to look at what I eat and actually cause

[sic] me to exercise more," while another commented that she thought "these lessons were valuable because I now eat healthier and I learned a lot about calorie intake that I never knew about before" (Questionnaire 2/04/2014). The vast majority of students appreciated the opportunity to learn about nutrition or fitness, but few students made any reference to new mathematical understandings. I was initially a bit surprised by this, and I wrote the following in my field notes:

> "Am I teaching them QL? Am I teaching them mathematics? Or are we solely using mathematics as a tool, and I am just teaching them other content? Should I be teaching them math, or exploring areas to study that need some math?" (2/12/2014).

This debate over mathematics and QL continued to play out in my mind, as I struggled to understand how this complex relationship should play out in my classroom.

Two other students helped me to gain some clarity about how mathematics should look in a QL classroom. Referring to the caloric expenditure activity, one student commented that "this lesson was much more valuable than previous ones" (Questionnaire 2/04/2014). This comment seemed very straightforward, but when taken in the context of this student's particular struggles throughout the year, it revealed some of the challenges, and triumphs, that are possible with a QL course. Unlike a traditional mathematics course, where teachers have to follow the prescribed curriculum, teaching for QL offers opportunities to study almost any context or scenario where mathematics is used. This can be especially important for students who underachieve, because it gives teachers the flexibility to cover content that might engage even the hardest to reach students. If a QL classroom is to respond to individual students and engage them in ways the traditional classroom cannot, then a critical component of teaching for QL must be teachers' ability to cover any topic with an appropriately-leveled quantitative context. Consequently, then, it would seem that mathematics becomes the criterion that limits the field of topics, but the contexts become the true focus of study. Could this be? Could the true role of mathematics in a QL classroom be nothing more than a filter and helpful tool? I'd like to think that I never fully ascribed to this belief, but thankfully a second student's comments reminded me of the much more significant and intricate relationship between mathematics and QL.

This student remarked that "these lessons are very valuable and it's nice to put numbers to things that normally don't have numbers" (Questionnaire 2/04/2014). I believe that this comment, even more than the first, gets at the true heart of the relationship between QL and mathematics. Caloric intake and expenditure are certainly quantitative phenomena, but this student had not thought of them from that perspective. This student's accomplishment was not the fact that he developed a better understanding of nutrition or fitness; rather, his true achievement lay in the fact that he understood how quantitative reasoning pertained to these two topics. At the beginning of the year, this student commented that "besides taxes and other run of the mill math… [mathematics] does not [matter to my future]," so he had clearly begun to expand his understanding of how mathematics relates to everyday life (Survey 9/06/2013). This student helped me to see that the goal of a QL classroom is to help students reason quantitatively about topics like nutrition and fitness, and more generally, to understand how quantitative arguments, approaches, and tools are used in everyday life. This finding was a revelation to me, because I was starting to think that mathematics was nothing more than a tool to understand real-life contexts or a filter to be used in a QL classroom. This student reminded me that the individual contexts were the tools to help students develop their quantitative reasoning and QL skills. We chose to study statistics, finances, codes, and nutrition because they all contained a quantitative element, but knowledge about these topics was not our primary concern. The primary objective was to improve students' ability to reason with quantitative information. I did not realize at the time why I preferred the assignment on caloric expenditure, but I realize now that this assignment did not just use mathematics as a tool. On the contrary, its primary purpose was to improve mathematical reasoning, and knowledge about fitness and practice with mathematical algorithms were simply an added bonus. Students may have learned about each context that we studied, and some may also have improved their fluency with basic mathematics, but neither of these was the primary goal of the course; rather, the principal focus was to improve students' quantitative reasoning and QL.

Our Cooking Activity and one Final Attempt at Authentic Instruction

As this unit progressed, our area was hit by several bouts of severe winter weather. This proved to be problematic, particularly since a good deal of planning was necessary to coordinate our class schedules, the school calendar, and the availability of the cafeteria. In addition, students needed to be prepared with the necessary ingredients and cooking utensils, so a snow storm could have really caused a problem if it struck on the day we were scheduled to cook. One week before our scheduled day in the cafeteria, some forecasters were projecting the possibility of a crippling snowstorm that could drop more than two feet of snow. This scenario concerned me, so I asked my students whether they would be willing to do a cooking simulation in the event of bad weather, rather than actually going into the kitchen. I wrote about their reactions in my field notes:

> To be honest, I expected that they wouldn't care that much, but I got a very different response. One student said [sarcastically]: "How do you do a cooking *simulation*?" Then a handful of others said that they really wanted to cook if they had an opportunity. The room felt…deflated when I asked them to consider a simulation, and I recommitted at that point to making sure that I got them into the kitchen, snow day or not (2/04/2014).

I told students that we would find our way into the kitchen regardless of the weather, and this definitely improved the energy level in the room. After class, I was surprised to see a student come and ask me if he could video tape our cooking lesson for another one of his classes. I had struggled with this student perhaps more than any other over the course of the year, and to be honest, I didn't think he assigned much value to what we did in class. It appeared from this request that he did care, and further, that he thought the lesson was important enough to video tape and show to another class. These were the types of encounters that I had hoped for when I committed to this activity. Word got out that the Discrete Math classes were going to cook in the cafeteria, and students in other classes got excited, and perhaps a bit jealous of my students. It was nice that my students, who normally ascribed little value to mathematics class, could now have a novel and hopefully very positive classroom experience.

The activity itself required students to find a dessert recipe, adjust the serving size to reflect the number of students in their class, modify one ingredient to make the dessert healthier, and calculate the nutritional information, but the real highlight was the opportunity to actually prepare and cook these recipes. Once I set up contingency plans with the cafeteria, I spoke with more enthusiasm and conviction about this activity, and students really invested in the project. Students talked excitedly about potential recipes, and some groups even asked if they could do additional preparation before school or the night before. I wrote in my field notes that one student who was often unmotivated "exclaimed out loud how 'pumped' he was for this assignment" (2/04/2014). I had never seen this group so excited about an assignment, and I hoped that this energy would translate into a successful day in the cafeteria, and more generally, to higher levels of QL.

I was equally excited about this activity, but also a bit nervous, because I realized that the success of this assignment depended on students shopping over the weekend and remembering to bring in their ingredients. If students failed to do this, then the whole class would be a failure, and I had no backup plan. Driving to school on the morning we were scheduled to cook, I wondered how these activities would play out. I had never done anything like this in my teaching career, and I had never built a lesson plan that relied entirely on students' ability to provide the necessary materials. I was further concerned because I would meet my morning class first, and they had generally been the class that was less likely to follow through on their commitments. I wrote about my impressions as I entered the cafeteria:

> When I first arrived in the cafeteria, I noticed that only a handful of students had their ingredients. A pit feeling rose up in my stomach, as I worried that the class had not followed through on its commitment. After walking from group to group, though, I found that four groups had brought most of their materials, and only two did not. Of those four groups, two did a really excellent job, they were completely well prepared and on point, and they completed their recipe effectively and efficiently. The other two groups struggled, but also finished their desserts (Field Notes 2/10/2014).

The class was certainly not as prepared as I would have liked, but they did enough to make our time in the cafeteria productive and worthwhile. I found students to be excited and engaged, and even the students that didn't bring in their ingredients joined their classmates and helped with the cooking. In my afternoon class, every group had their ingredients, and most had all of their cooking supplies as well. One group was ready to put a batch of cookies (their first of three) in the oven in the first several minutes of class, and every group ended up completing their dessert on time. Due to school policy, I had to manage the oven, but I found that students had a hard time staying away. Students began to form a semicircle around the oven, and they frequently asked me to remove their desserts so they could check on the progress. I wrote in my field notes that "this lesson was definitely a high point of the year," because almost every student had taken ownership over the outcome of this activity. My only concern, then, was whether this activity had actually improved students' QL, or if it had just been a fun diversion with little mathematical value.

I wrote several times in my field notes about my concerns regarding this activity: "I am not entirely sure how deeply they were digging into the mathematics, but they were surely intrigued by the opportunity to cook" (2/07/2014); "What remains to be seen, though, is…whether students will improve their mathematical understanding to any measurable degree" (2/10/2014); and "I still worry that students didn't take away the mathematics that I hoped they would" (2/11/2014). There is always a fear when you create a lesson that students will not take advantage of its potential to improve their mathematical understanding. This is the primary challenge when you believe in a constructivist theory of learning, though, because the teacher can only create an environment that reflects students' interests and builds on their prior knowledge, but it is up to students to make the connections and build new understandings for themselves. My impression is that some students did engage deeply with the mathematics, while others did not. Interestingly, many of the students who were not entirely prepared for class were forced to use quantitative reasoning skills to successfully prepare their desserts. One group in particular did not come prepared with appropriate measuring tools, so they had to perform conversions to measure certain ingredients, use estimation techniques to calculate others, and modify the baking instructions to account for their different-sized pan. In reality, all students had to make adjustments, since they were required to cook their desserts in a convection oven that was set to a temperature they could not change. In my opinion, these real-life limitations forced students to think quantitatively about their recipes, their measurements, and their baking time, but to get a better sense of how students understood their experiences, I distributed a post-unit questionnaire.

I asked students to describe what they learned about cooking, and I was pleased to see that many referenced the underlying mathematics. While some students talked about scaling or the effect of certain ingredients on total calories, most referred to the importance of measurement and precision. One student said that she "learned how to accurately measure things," a second stated that she "learned how to use appropriate doses of certain ingredients," and a third remarked that "measurements are crucial in cooking" (Questionnaire 2/11/2014). One student summed up this general sentiment when he stated that "everything needs to be precise." It would seem, then, that students did develop a deeper understanding of mathematics, particularly with regards to precision and measurement. Since students also seemed to be highly engaged in our most recent activity, then, I wondered whether this unit also changed their perceptions of mathematics.

While some students asserted that this unit had no impact, many responded that it helped them to understand how mathematics is used in everyday activities. For example, one student stated that she “never thought of nutrition as a math topic but it’s super helpful,” a second said that “this unit opened up my eyes to the fact that there is a lot more math in cooking than I first thought,” and a third said that this unit made her “appreciate math more because it related to scenarios I personally care about daily” (Questionnaire 2/11/2014). These students learned that mathematics was more than just algebra or geometry, and they began to appreciate its utility in their everyday lives. This question about the nature of mathematics came up in three other students’ comments, where they shed some light on their developing understandings of mathematics and QL.

Student 1 remarked that the unit did not change his perceptions, but he continued by saying “it’s still just math but it could actually be useful” (Questionnaire 2/11/2014). While he was not ready to admit that his perceptions changed, he did acknowledge that there was at least a subset of mathematics that could be valuable for his life. Student 2 expanded on this idea when he said that this unit “simply used a more practical form of math rather than the useless types.” This student articulated his belief that mathematics was a much larger discipline than what was typically taught in schools, and further, that some of the topics were useful while others were not. Student 3 was a bit more specific when he pronounced that “math is not all hard equations.” Taken together, these statements shed some light on a common belief that mathematics is difficult, useless, and irrelevant to most students’ lives. At the same time, there is hope, because each of these students was able to concede that certain types of mathematics could be valuable, both generally and in their own individual lives. Teaching for QL has this potential, to help students find value in mathematics, even if they had been unsuccessful or unexcited by previous courses. This was the reason I conceived of this course in the first place, to give upperclassmen who had previously underachieved a chance to reengage with mathematics class. My sincere hope, then, is that these students’ renewed engagement with mathematics would be accompanied by an increased capacity for learning, as well as a higher level of QL.

Findings

In the previous chapter, I presented a narrative that described the evolution of the class, my students, and my own thoughts and goals regarding co-construction and teaching for QL. The narrative documented some of the key events that took place during the course of the year, particularly with regards to the co-construction of the course, students' attitudes towards mathematics and their developing QL, and some of the things I learned as both teacher and researcher. The narrative also provided context for some of the major decisions I made during this process, both by highlighting the factors that led to those decisions and describing the impact of those decisions on students, on me, and on the course as a whole.

In this chapter, I distill some of the common themes that emerged from the data I collected over the course of six months. I connect important findings from my narrative to one another and to my research questions, and I attempt to make meaning of my data in relation to these major themes and the previous literature.

Theme 1: The Ramifications of Co-construction

Taken as a whole, the data helped me to understand some of the challenges and benefits involved when including students in the co-construction process. I learned about the tensions that might arise in the context of shared decision-making, particularly in regards to power relations and classroom roles. I also found that co-construction can be a type of instruction, because students have an opportunity to think more deeply about mathematics and develop important analytical skills as they examine and evaluate the merits of different content and classroom structures. Structurally, I will present evidence for each of these components first, and then I will look across these elements to try and make meaning of co-construction as a real-world classroom practice.

Shared Decision-making

I initiated the process of co-construction with the literature on democratic mathematics education in mind, since it stressed the importance of having students participate in decisions that affect them and their classroom (Ellis & Malloy, 2007). Though I imagined that this process could be difficult for students, I found that co-construction was also trying for me, and it forced me to rethink the way I planned for instruction and organized my classroom. Since my students and I were learning how to co-construct together, they worked alongside me and offered feedback and suggestions for new directions as I struggled to find my footing. As I mentioned in Chapter 4, students informed me when my assignments were too tedious (sampling in Unit 1; codes in Unit 3) or too difficult (credit card assignment in Unit 2), when my directions were too vague (second mini-unit in Unit 2) or too painstaking (caloric assignments in Unit 4), and when the projects were too advanced (government shutdown paper in Unit 2) or too irrelevant (codes in Unit 3). It was informative and humbling to hear students' honest opinions, but it was a necessary byproduct of giving students a true voice in how our classroom would function. After engaging in the co-construction process for six months with two different classes, I found the traditional classroom roles of teacher and student were challenged as we negotiated a new balance of power. This renegotiation of power took into account students' experiences and beliefs about mathematics and mathematics class, my own objectives, limitations, and beliefs about education, and our ability to work together within the confines of our school's norms.

Students' Willingness to Participate

Initially, students were unsure how to handle the co-construction process, which is understandable, given that they likely had not had similar opportunities in previous courses. I found that students struggled to handle greater decision-making power for three reasons. First, students were unfamiliar with the content. I recognized this early in the year, as I wrote in my research journal that I was unsure "how much the students will be able to contribute. They won't necessarily even know where to begin when discussing content" (9/10/2013). One of the principles of a QL classroom is the fact that the curriculum should be "driven by issues that are important to people in their lives and work," but my students were not yet aware that mathematics could be relevant to their everyday lives (Quantitative Literacy Design Team, 2001, p. 18). As a result, when I asked students to brainstorm possible areas of study, students referenced traditional topics like volume and area, rather than things that were more relevant to their everyday lives (Field Notes 9/11/13). Students may have responded in this way because they thought this is what I wanted to hear, or perhaps because they'd never really been asked this question before. In either case, this lack of knowledge about mathematics content and its relevance to their lives certainly put students at a disadvantage, and it may have prevented some from participating more fully in the co-construction process. Secondly, students were understandably limited in experience and expertise regarding curricular planning. As a result, students began the year expecting me to come up with useful and engaging activities, rather than considering these things on their own. Finally, students were uncomfortable with the nontraditional roles afforded to them, and me, during the co-construction process. For some students this had to do with their innate desire to please the teacher, for others, it may have resulted from laziness or a preference to not think too deeply, while for others it just involved a general level of discomfort with these unfamiliar roles. While it was difficult to understand the motivations on a case by case basis, instances of this discomfort were frequent in my data. For example, when planning Unit 4, one student commented that “I don’t think Mr. Russo…should give the class that power to pick what they want all the time, I think he should say, this is what we’re doing, we’re going to do this topic” (Transcription 1/02/2014). This student was not alone in her desire to defer to me, but thankfully, students became more willing to work alongside me as time went on.

As the year went on, students became more willing to engage in co-construction in one capacity or another. For example, some students were more willing to participate in small-group discussions, others felt more comfortable offering substantive comments on questionnaires or project proposals, and still others began to speak with me about their likes and dislikes of the course. This was in part a result of students starting to believe that their voices actually did matter, and I believe that this change took place for two main reasons. First, I think that by sitting down with students and listening to their ideas, I was able to convince students that I actually cared about their ideas. Secondly, as the year went on, I was able to implement more students' ideas into lesson plans, which reinforced the fact that their suggestions and contributions actually mattered. This change in students' attitudes towards co-construction was particularly evident in our unit on cooking, where one student "exclaimed out loud how 'pumped' he was for [the cooking] assignment," and others expressed surprise that I had followed through on their idea to bring our class into the kitchen (Field Notes 2/04/2014). One student summed up this sentiment when he said that "it was good to have a teacher who was willing to actually listen and take note of what we wanted to learn in the class" (Survey 2/11/2014). Another factor that may have contributed to student participation was my decision to give students more control over course content. Early in the year, I did not deviate from a short list of traditional Discrete Math topics that I had generated, but over time I realized that students would be more willing to participate if I would allow them to study their own interests, rather than modifications of my own.

The Zone of Proximal Development

I found that over the course of six months I had to move towards students to engage them in the co-construction process. At first, I thought that I would just "have to guide them a bit" (Research Journal 9/10/2013). For example, during the co-construction of our second unit, I pushed students to think about some of the financial realities they would face after graduation.

Student 4: Graduate? My parents still help me with money.

Student 5: Yeah, that's true

> Student 4: I don't know what to do with my life after that.
>
> Me: So when we're talking about graduating college, we're talking about renting an apartment or buying a house.
>
> Group: Yeah
>
> Me: We're talking about life insurance (Transcription 10/02/2013).

Students in this group were not interested, nor were they necessarily ready to talk about life insurance, but I proceeded to give them information about the importance of life insurance and the relationship between cost and age. I pressed students to consider a topic that was important for someone my age, and I directed the conversation where I wanted it to go. At first I thought I was operating within students' ZPD, because I was trying to learn about their life experiences so that I could bring students where I wanted them to go. As the year went on, though, I realized that the ZPD might have to be re-conceptualized for a QL classroom, because QL relies not only on the experiences and interests of the student, but also on their unique goals and objectives. Therefore, my role as the "knowledgeable adult" was not to direct where students should go, but rather, to help them get where *they* wanted to go. For example, in one instance of co-construction a student stated the following:

> Right now I have a [poor grade] in this class because I'm not doing all the work because [codes are] something that doesn't interest me…like, do I want to [work on codes], when I think I have been very active at looking at [other] things that interest me, but that's not always in my best interest" (Transcription 1/02/2014).

This student admitted that he was not completing the work because codes weren't interesting to him, and he further suggested that there was a disconnect between active learning and success in my class. I realized that I was misunderstanding how to operate within this student's ZPD, because even as I was learning a good deal about his experiences, he had no interest in going where I wanted him to go. I realized that true co-construction required students to set the objectives, and not just the process by which we get to my objectives (As an aside, this would have been much more difficult if the student didn't want to study budgeting or statistics, two topics that I consider to be important for a quantitatively literate adult. If that were the case, I would have tried to convince him of the importance of those topics, but I believe that if students do not believe in the value of a subject, then they are less likely to learn about it).

In response to this student's concerns, I decided to initiate an individualized co-construction process where I could work alongside him as he set the objectives that would help develop his QL. I did this first by validating his concerns, as I admitted that while this unit might be valuable for others, it was not ideal for him. Secondly, I acknowledged my own goal of promoting QL through contexts that were important to students, and I asked him to help me brainstorm an assignment that could meet my goals for QL as well as his perfectly legitimate need to study content that was relevant and valuable to him. Previously, I had considered the ZPD mainly in terms of "actions the child can actually carry out," but now I was finding that it also could be considered in terms of "what the child wants" (Van Oers, 1996, p. 97). I began to place a greater emphasis on students' ZPDs in each round of co-construction, but I made them a primary focus when we planned for Unit 4.

When planning for Unit 4, I invited students to look at our original list of topics and think creatively about our next unit of study, and in both classes, students chose topics that I not only wouldn't have thought of, but also that I initially didn't think would be very successful. In one class, students decided to pursue a unit on everyday numbers, and I worried that the content wouldn't be rigorous enough for seniors in high school. While I let the conversation go, I later wrote incredulously on the transcription "Is this math?" and "Are students actually interested in this…?" (Transcription 1/02/2014). In the second class, students decided that they wanted to study cooking, and it was difficult for me not to jump in and stop them. I was uneasy with the idea of cooking and all of the paperwork, permission slips, ingredients, and allergies that would go along with it, but I held my tongue, because I recognized that some of the most profound classroom experiences had occurred when students were able to exercise control over the curriculum. This had mostly taken place in more informal settings (individually during the budget simulation, informally during the college choice assignment), but I now recognized an opportunity to transfer this experience to the more formal co-construction process. I decided to relinquish control over the types of content we could cover, and as a result, we co-constructed a unit that I hadn't authorized in advance (as opposed to our previous three units, where students provided input, but I eventually chose the unit to submit to co-construction). Consequently, I believe that students developed aspects of ownership over each of these units that they had not experienced before, because the co-construction took into account their interests, their ideas, and their ZPDs. My decision to give students more control had a significant impact on co-construction for the following three reasons: first, two units were created that never would have been part of the curriculum; second, students demonstrated higher levels of engagement, as almost every student took ownership over the outcome of the activities (as referenced in Chapter 4 and Field Notes 2/10/2014); and third, students were able to articulate the value of certain types of mathematics and mathematical thinking for their everyday lives (as referenced in Chapter 4 and Questionnaire 2/11/2014).

There was an additional challenge during our ventures into the ZPD, and this related to the nontraditional nature of our roles. Typically, the ZPD is conceptualized with a more knowledgeable adult in the role of teacher, and the less knowledgeable child in the role of learner. The challenge for my students and me, though, was that while I was more knowledgeable about mathematics and mathematics content, I too was learning how to operate in a QL classroom and co-construct a course. This created some tension during our small-group discussions, because students often looked to me to direct the co-construction process in much the same way as I directed their learning with mathematics. This was impossible, though, both because I was no more an expert in co-construction than they were, but also because co-construction necessarily required substantive contributions from all members, and not just me. Therefore, the analogy of the ZPD only goes so far, because in some ways I was the more knowledgeable adult, and in others, I was just another participant in the process. There was no easy solution to this difficulty, so we just lived with the tension, as we struggled to navigate our multiple roles and carve a path that would lead to more meaningful experiences in mathematics class.

Balancing Structure and Freedom

As I strived to move closer to students, I found that we had to negotiate the right balance of teacher and student control, as well as structure and freedom. This negotiation was dynamic and ongoing, and different amounts of control and freedom were necessary for different times and in different contexts. For example, I gave students a great deal of freedom by letting them study three unrelated topics during our statistics unit (because I wanted to encourage student buy-in and demonstrate the relevance of mathematics), while I exercised more control in the third unit when I created an alternative assignment with no student input (because I needed to act quickly to salvage what was quickly becoming a lost unit for many students). Students were also involved in this balancing act, as they advocated for freedom and structure at different times during the course:

> Student 1: And also when, when we first started, like the first couple of classes, when you told us we were going to co-construct it, I think that, kind of, like everyone was, like what the hell are we doing? Cause obviously we've never done it before.
>
> Student 2: I feel like we still have more freedom
>
> Student 1: …That, that leaves the class kind of like, um, like lost, cause, you think, I don't know
>
> Me: The freedom structure thing, right?
>
> Student 1: Yeah
>
> Me: So we want freedom, but we want structure
>
> Student 3: I think, too, like for high school, we have too much freedom. I know that sounds like so weird, because that's all we want
>
> Student 1: I think that…

> Student 3: But like in college, it's OK, it's due this day and you don't have it, sorry.
>
> Student 1: I think the freedom is a good thing, like picking what we learn and what we, all that stuff. But the rest of it, like, us creating our own projects and our own assignments…(inaudible)…who actually here knows how to do that? (Transcription 11/11/2013)

This conversation is a great example of how students and I worked together to ensure the right mixture of structure and freedom. For example, Student 3 alluded to the fact that students say that they want freedom ("that's all we want") when they actually want structure ("we have too much freedom"), even though they know that freedom will best prepare them for college ("but like in college…it's due this day and you don't have it, sorry") (Transcription 11/11/2013). Earlier in the year, I summarized a conversation in my research journal during which this student told me that seniors want "to maintain their childhood before they go into the real world," and that "she will have to learn about [things like the government shutdown and debt ceiling] eventually, but…she doesn't want to do it now" (10/24/2013). This comment underscores the unique developmental stage of many of my students, because they were finishing a highly structured period that didn't provide them with the freedoms they thought they deserved, while they were about to enter a freedom-filled world that came with certain consequences that they might not have been ready to accept. So in one sense, students had a foot in each world, while in another, they were really stuck in between the two.

Student 1 provided a different perspective on the balance between freedom and structure, because while she seemed incredulous that students should contribute anything to the construction of projects or assignments, she felt completely comfortable giving input into our units of study. This opinion provides insight into some of the uneasiness shared by many students, but there was also a vocal minority that wanted more input into the construction of their day-to-day learning experiences. For example, another student (Student 4) agreed that students should have a say in what they study, but he believed that students should have a much bigger role in the construction of their academic experiences. I sat down with Student 4 later in the year, and we discussed the pros and cons of giving students the freedom to construct their units of study and corresponding assignments. I summarized this conversation in my field notes:

> He said that all students were capable of choosing an area of interest, asking good questions about it, and then finding those answers. I agreed with him, but I remarked that many students need to grow into that, and not all students were ready for that type of freedom yet…I want to provide students with freedom, but I believe that has to be done within the confines of very clear guidelines and related content. At the same time, there are some students who can thrive in the type of environment that Student 4 is suggesting, and I want to give those students opportunities as well. So just as I allowed Student 4 to move in a different direction in Unit 3, I will offer that opportunity again if he struggles to engage (1/08/2014).

What Student 4 mentioned was my ideal classroom, where students would find a QL topic of interest, ask deep and meaningful questions about it, struggle to find answers, and then present their findings to the rest of the class. Unfortunately, not all students were capable of doing this kind of work, at least not without support from me or their peers. What Student 4 didn't account for in describing this idealized model was that I had to respect each student's ZPD, and not all students were as equipped as he was to operate effectively with so much freedom. Students had different levels of mathematical readiness, some were more aware of their interests and future goals than others, a small number were able to work effectively with their peers and me in the co-construction process, and a scant few were capable of co-constructing lessons and projects with me while the rest of the class worked on something else. Students and I went back and forth on how much structure I should provide in the course and the co-construction process, but it was likely impossible for us to reach a consensus that worked for all students. From a Vygotskian perspective, students would only be able to learn mathematics and participate in the decision-making process if the co-construction process accounted for their unique ZPD. Since each student brought a unique set of experiences, understandings, and interests, then, I realized that I would have to differentiate the co-construction process for each individual in my class. The practice of customizing the co-construction process became an important part of the course, which was interesting because it was very different from any techniques I had found in the literature. Consequently, it had important implications for the themes discussed later.

Working through Tensions alongside my Students

In addition to the challenges that students and I faced, the co-construction process also demanded a lot of me as an educator and a researcher. This learning curve would have been difficult in its own right, but tensions were further magnified by students' close proximity to me during the decision-making process. My data showed that three of the primary tensions that we faced involved my evolving understanding of how to do co-construction, my ability to manage the co-construction from a logistical perspective, and my desire to share power with students despite an inherently unequal power relationship.

The first tension came as a result of my slowly developing understanding of the co-construction process. My primary objective for the class was to improve students' QL, but the means by which I hoped to accomplish this goal shifted as I learned more about co-construction and my particular group of students. For example, I chose our statistics unit partially because it was "broad enough" to allow me to differentiate (Research Journal 9/16/2013), but I chose our unit on money in part because it "would allow me to experiment with one main lesson for the class" (Research Journal 10/04/2013). As I mentioned in Chapter 4, I crafted the third unit on codes under the assumption that I knew students' interests, but I let students craft Units 4 and 5 because I conceded that they were the best judges of what was interesting and relevant. My ideas about how to best carry out the co-construction shifted as I learned what worked and what didn't work, but also as students became more comfortable and confident in their own roles. As they learned more about the impact of their contributions, they adjusted their level of participation, which in turn affected that way I conceived of co-construction. This led to the twists and turns that I mentioned in Chapter 4, as students and I plotted a course in a vessel that we were only just learning how to sail. This created some tension, but also some excitement, as students evolved along with me during each round of co-construction.

The second tension involved my failure to set proper boundaries, specifically in terms of my limited time, energy, and knowledge. My limitations of time and energy were on full display in my struggles over when to co-construct units, because I felt like I was either doing a disservice to students by asking them to plan too soon (before they had more experience in the current unit of study), or I was doing a disservice to myself by failing to leave adequate time for planning the next unit. My limitations of knowledge came into play when I realized that I could not create effective units if I allowed students to choose topics that I didn't fully understand. This was especially true when we planned our unit on codes, because students' interests centred on content that I did not know very well. As a result, I was forced to learn the material just a couple of days before the students, and I was left with the unenviable task of scaffolding content that I didn't fully understand. This proved to be impossible, because as obvious as it may sound, the co-construction process had to work for me, too, and I had failed to define proper boundaries for myself as a key participant in the process. I found that shared decision-making requires a great deal of energy on everyone's part, but in order to sustain it over any extended period of time, it has to work for the teacher as well.

The third tension involved the friction between the co-construction process and the fact that school structures had endowed me with considerable amounts of power. By welcoming students into the decision-making process, I attempted to expose them to some of the constraints that I faced (such as "we can't exactly bring a grill into the classroom" (Transcription 1/02/2014)), but there were additional constraints on students that I was unable to alleviate. In particular, I wanted students to feel like equal participants in the co-construction, but in reality, I held grading power, institutional power, and more knowledge and experience than most, if not all of my students. First and foremost, I assigned students their grades, and I noticed that while students didn't always care very much for the day-to-day classroom activities, they did pay close attention to their final grades (Field Notes 11/15/2013). This impacted the way students acted around me, and it was a challenge for me to balance this role with my desire to communicate with students about their likes and dislikes of the course. In addition to my power over grades, students were also aware of my ability to contact parents or assign detentions, which further delineated my role from theirs. Finally, I had more knowledge about mathematics and applications of mathematics than my students, so even as I welcomed their input and sought out their experiences, many continued to defer to me about appropriate content or lessons types. As much as I wanted to welcome students as equal contributors to the course, my role as teacher was unmistakable, and so we sat with the tension of trying to share power in a context where power could never fully be shared.

Co-construction as Instruction

In addition to its benefits for democratic mathematics education, I found co-construction to be an effective form of instruction in its own right. First, as I mentioned in Chapter 4, some of our small-group discussions provided me with opportunities to develop students' QL and clear up misconceptions about mathematics or topics of interest in students' lives. Secondly, the process of co-construction helped students to be reflective, to consider how they learned best, and to contemplate the value of their education both at the time and for their future. Thirdly, co-construction helped students to develop other important life skills, such as decision-making and the ability to work independently.

Small-Group Discussions as a Means for Developing Understanding

I found the small-group discussions to be an excellent medium through which to instruct my students, because they provided me with an opportunity to learn about students' interests in their ZPD. The co-construction of Unit 2 was particularly fruitful in this regard, since I was able to help students grapple with some of the complexities of the government shutdown, the nuances of taxes for different types of jobs, and the value added of banks, particularly in terms of interest rates and FDIC deposit insurance. As referenced in Chapter 4, I found that in one small-group discussion I was able to help students develop a more nuanced understanding of cost of living. At the beginning of the discussion, one student wondered whether it might be beneficial to move across the country, because the "cost of living would be drastically different if you live in New Jersey than if you live in Wyoming," but after some conversation back and forth, he came to understand how income might also factor into the equation (Transcription 10/02/2013). I found that these teachable moments arose in many of the small-group discussions, but interestingly, they seemed to happen more frequently at the beginning of the year than towards the end. Perhaps my decision to give students more control in the co-construction compromised some of my authority in the typical teacher-student relationship, which made it more difficult to take advantage of each student's ZPD. In effect, co-construction might have been more of an instructional tool when I exercised greater control, but it was better able to engage students when I gave them more freedom. In both situations I was able to operate in each student's ZPD, but in the former I was able to help students build understanding, while in the latter I was able to help them develop interest.

Individualization as an Approach to More Meaningful Learning

Over the course of the year, I found that individualized or small-group co-construction was not only an effective way to improve students' participation in the planning process, but it was also a mechanism through which I could help students think more deeply about their educational goals and their relationship to mathematics. I was initially uncomfortable allowing students to create alternate assignments, particularly after I struggled to manage the three different groups in our first unit on statistics. My attitudes changed when a student informed me that despite his best efforts, he could not engage with an assignment that he considered inconsequential for his life. My first reaction was to lecture him about the importance of hard work and perseverance, but after composing myself, I responded that the goal of this course was to improve students' QL, and I would be willing to consider an alternative if the current assignment was incompatible with this goal. I took a big chance when I made this decision, because I trusted that this student would be able to complete his new assignment (on the US highway system) with very little support, and also that his classmates would accept my decision without begrudging him, or me. Thankfully, neither of my fears was realized, because this student's level of engagement spiked, he delivered a presentation to the class that demonstrated a sound understanding of the topic, and his classmates were impressed with his work and thankful for his contribution (Field Notes 10/29/2013). This experiment was a success in part because of the conscientious nature of this student, but also because this student had an opportunity to reflect on the relationship between mathematics and his interests and craft an assignment that would promote both of these ends. I was so impressed by this experience that I offered the option of individual co-construction to the rest of my students, but as I stated earlier, students were at different places in this process, and only a few acquiesced over the next several months. While individual co-construction may not have been for everyone, I found that students could still obtain similar benefits by reconstructing portions of existing assignments, rather than constructing new ones from scratch.

For example, in unit 2, students had to create a budget simulation that was appropriate to their unique life circumstances, and some students asked me to loosen the rules to make the assignment work better for them. For example, I asked students to research an average starting salary and then give themselves 3% raises annually, but one student said that as a future firefighter, it would be more appropriate to consult the salary guide and apply his raises accordingly. Similarly, I gave students a handful of requirements for the road trip assignment in Unit 5, but some students asked if they could rent an RV, assuming they could find a place to rent it, calculate the costs of the rental, and then find gas stations along their route that sold diesel gasoline. These assignments may not have been uniquely designed by these students, but their modifications did allow students to consider the relationship between mathematics and their lives, interests, and ideas. The product of these modifications was an assignment that was more meaningful and engaging to students, but the process of adjusting these assignments was equally important, because it required students to think about how their own situations related to the mathematics, or alternatively, how mathematics interacted with their understanding of each particular situation.

Other Important Life Skills

In addition to helping students construct new and more meaningful knowledge, individualized and small-group co-construction also helped students to develop other important life skills. Specifically, I found that students were given an opportunity to improve their decision-making skills, as well as their ability to work effectively in a student-centered environment. Firstly, participants in small-group discussions were given the responsibility of planning a course with a given set of constraints, so they were forced to weigh pros and cons, consider alternatives, and make decisions that would bring the maximum benefit to the largest number of people. This was nowhere more obvious than when students considered how they would bring their potential unit of study to the class as a whole. Since students knew that they had to construct a unit that would appeal to their classmates, they were very careful about vetting each topic under consideration. In one small-group discussion, students repeatedly considered whether their classmates would “get into it” (Transcription 1/02/2014), and decisions were often made with this concern foremost in their minds.

Secondly, co-construction provided a means through which students could engage in more student-centered, self-driven activities. One student commented on this aspect of our class:

> Like, my brother’s in college, and he’s living at home. When I talk about this class with him, he’s like that’s what I needed with math in high school, and in college I didn’t get it. Like the ability to, like free exploration and what we’re doing. And I feel like this is a very, like, college-level class, when it comes to, like, form (Transcription 1/02/2014).

This student was trying to articulate the benefits of learning how to succeed in a college-like setting, where students are given more leeway to complete their assignments and do their work on time. She found it beneficial to gain some experience with this type of course in high school, so that she could be better prepared to succeed in college. In sum, then, co-construction was able to not only support students' learning of mathematics, but it also furthered students' decision-making skills, their ability to function in a student-centered environment, and their understanding of how mathematics relates to their own unique lives.

Co-construction for the Real World of Teaching

The previous analysis gave me much needed insight into the co-construction process, but what is left to do is to consider how this process could be employed by mathematics teachers in other contexts. At first glance, this venture may seem a bit overwhelming, particularly considering the amount of time and energy that my students and I spent either overcoming or mitigating each of the hurdles that crossed our paths. At the same time, the hard work we did over the course of six months may be able to offer some guidance moving forward, so that other classes of students can experience the benefits of co-construction with fewer initial growing pains. The primary challenge that we had to overcome involved getting students to buy-in to the co-construction process, and this required students to both feel competent (knowing the material, knowing how to plan lessons, etc.) and confident (feeling comfortable with their new roles, understanding that we were equal participants in co-construction but unequal in other areas, etc.). Two techniques that promoted students' feelings of competence and confidence included full implementation of students' ideas and a re-conceptualization of the ZPD.

The first technique involved taking a student's suggestion and implementing it in a way that adhered as closely as possible to the student's original intent, within the confines of the broader goals of the course. This technique took me several units to learn (just as it took students several units to feel comfortable enough to make these suggestions), because I originally ran co-construction by choosing a unit of study, and then asking students to brainstorm different lessons and activities that related to that unit. I eventually came to realize that students didn't feel like they were doing anything, because I was not only planning individual lessons and assignments (which I had to do as a teacher), but also suggesting the broad themes that we would study. I relinquished some control for our final two units, and I was amazed at the surprise and delight of some students when they saw their actual ideas come to fruition (such as in the cooking activity and road trip assignment). I believe that students felt more confident because I substantiated their ideas, and more competent because they were now an "expert" in this particular field of study. While this might be more difficult for some teachers than for others, I think it is important to take students' input and incorporate it into the class as often as possible, both to affirm the students whose ideas were selected and to let their classmates know that their participation is important and their ideas are valuable.

The second technique involved a re-conceptualization of the ZPD, where I focused not only on the actions of students, but also on their desires, interests, and objectives. Unlike the traditional understanding of the ZPD where the teacher learns about students' interests and understandings and then directs them to some predetermined objective, I allowed students to select that objective and then facilitated their journey from point A to point B. I found that this increased students' confidence because it allowed them to have a direct say in their classroom experience, and it also helped them feel competent because they could study an area of interest or a particular strength, rather than something that they might value or understand very little. The only way that I could implement this technique was to allow for some individualization, and while it might seem like this would be unmanageable, I found two distinct approaches that worked well for my class.

One way that I allowed for individualization was to let students determine exactly how they wanted to engage in co-construction (a sort of meta co-construction), as some preferred to complete a general class assignment, others decided to break off in small groups, and still others had very specific objectives that were only appropriate for them. This was not as difficult as it sounded, because very few students actually requested alternate assignments. The students that did make these requests cared enough about their learning experience to put in the work, so I was able to lean heavily on them to prepare a proposal, sit down and discuss it with me, make revisions, set benchmarks, and then present some final product either to me or to the class as a whole. My challenge was not so much with classroom management as it was with the idea that students were completing vastly different assignments, but I overcame this concern in two ways. First, I constantly reminded myself that our common goal was QL, and just like no single novel has a monopoly on teaching critical reading skills, no piece of content has a monopoly on

QL. The second piece was a bit more nebulous, because I had to determine if students were completing assignments that were equally rigorous, equally time-consuming, and able to be graded fairly. Once again, I learned over the course of the year that this was less of an issue than I had originally thought, because students that opted for individualized assignments tended to welcome additional work and rigor. I found this to be true in my precalculus classes as well, that student choice did not promote inequality, but rather, it allowed students the freedom to challenge themselves if they were so inclined. Practically speaking, though, I never allowed individualized assignments to be less rigorous or less demanding than the general class assignment, but I rarely had a student opt out of our co-constructed assignment once he or she began the process.

A second way that I individualized instruction might be more reasonable for many teachers, as it involved the creation of rich assignments that allowed for modifications according to students' unique skills and interests. The best example I can give is the budget simulation, where students were able to individualize the assignment to reflect their personality and specific life goals. This approach is less radical in many ways, because students work on the same assignment and meet the same learning objectives, but it also takes an incredible amount of creativity and planning, and maybe a bit of good fortune as well. I found my budget simulation to be so successful partially because it allowed students to inject some of their own personalities into the project, but also because it met students at a developmentally appropriate place in their lives. Unfortunately, not every lesson hits the mark, so I decided that if a student ever felt like an assignment was not meeting their needs, and if they could convince me of a suitable alternative, then I would be willing to let them construct a new project.

I believe that my decision to allow for individualization was an important one for four main reasons: (1) it empowered students to take ownership over a class that they had previously struggled with; (2) it motivated these students' classmates, both because it demonstrated that I respected them enough to give them a choice and it took away the excuse that assignments were not valuable to their lives; (3) it did not add an unbearable burden for me, because the students who took advantage of this opportunity tended to be hard-working, responsible, and thoughtful; and (4) it removed the negative influence these students might have had if they were forced to complete the original assignments. It was not easy for me to accept this new method of planning, but after experiencing it for six months, I plan to incorporate it in some capacity in all future classes. I found that co-construction not only produces lessons that facilitate learning, improves students' attitudes towards mathematics, and enhances the classroom environment, but it also serves as a type of instruction in itself, by equipping students with the skills they need to make decisions, evaluate multiple alternatives, convince their peers, and take ownership over their choices.

Theme 2: Student Interest

While small-group discussions were certainly one example, there were additional classroom structures that had an impact on students' level of interest. In order to better understand the relationship between these practices and students' interest, I utilized Hidi and Renninger's (2006) theoretical model of interest development. Specifically, I analyzed any external stimuli that triggered students' situational interest, and I documented any classroom structures that supported students' development of individual interest. Before I could engage in that analysis, though, I considered how the learning culture may have affected students' level of interest.

The Learning Culture

My data showed that there were students who demonstrated a low level of interest throughout the year, despite my attempts to appeal to their ZPD. For example, when I asked students to describe how the co-construction process affected their classroom experiences, one student responded "Eh. I think it was a nice attempt to improve my experience. Maybe if I took it more seriously it would've bettered my experience even more" (Survey 2/11/2014). This quote gives a window into one aspect of our school's learning culture, and while it varied across different students and different groups of students, a culture of indifference was prevalent among these particular classes. This undercurrent of indifference was present throughout the year, and it manifested itself in certain ways inside the classroom and inside the school.

Inside the Classroom

While students certainly provided support and encouragement for one another, there were certain types of interactions that may have limited students' ability to display interest in the co-construction process, and as an extension, in the course itself. As I mentioned in Chapter 4, the exchange about Monopoly was one example where a student's natural interest was stymied when his friend insisted on making light of the discussion. Another example took place during our small-group discussion for Unit 3, when a student stated that "Codes, like bar codes, I think that's, personally, I think it's a little silly. I don't know" (Transcription 11/11/2013). Neither of these comments was necessarily malicious, but they both may have impacted students' willingness to demonstrate interest in a mathematical topic. These comments point to the significance of students' impact on one another, and in particular, to the influence of dismissive comments on students' interest.

A slightly different aspect of the learning culture became apparent in another small-group discussion referenced in Chapter 4, where students worried that they would "get made fun of" when presenting their ideas to the class (Transcription 1/02/2014). This encounter revealed an environment that was much more toxic than I ever would have believed, because students felt compelled to downplay their interest in mathematics, rather than appearing like they were too involved in the course. While student-to-student interactions were not the focus of this study, they did play a role in the evolution of our course, although likely in multifarious and complex ways. To add to this complexity, I found that there were additional forces originating outside my classroom that also may have inhibited students' interest in the course.

Inside the School

I found that forces outside of my classroom may have had a significant impact on students' learning, and in particular, my data showed that tracking, aspects of school culture, and course scheduling all had an effect on students' level of interest in my course. Tracking was an established practice for mathematics classes in my district, and I believe that it impacted students' participation, interest, and ultimately their performance in my course. Starting in middle school, students in my district are tracked into three levels, so that by the time they are seniors in high school, most students that enter Discrete Math have been in the lowest track for seven or eight years. The negative outcomes of tracking have been well documented, but in my high school, the problems were exacerbated by a district-sponsored initiative to increase advanced placement enrollment. I spoke with a colleague who taught multiple senior elective courses, and she noticed that ever since our school started pushing for more students to take advanced placement classes, the AP "types of students" no longer opted for her course (Field Notes 1/14/2014). A former student substantiated this claim when he said that "he wished he could take Discrete, but he was currently enrolled in AP Stats (and took AP Calc the previous year)." He went further, though, by providing an additional, and perhaps more troubling rationale for not enrolling in the course:

> He said that it would not be feasible for him to take Discrete because of APs, but he also recognized that he would not fit in very well with the type of student that would take Discrete. He said that it was a shame that we wouldn't be able to cover what we could cover, and therefore, that Discrete wouldn't be the class that it could be, because of the level of student.

Unfortunately, this student had a point, because tracking and the push for advanced placement courses had displaced many of the higher achieving and highly motivated students, which left courses like Discrete Math with lower achieving students, but also with the stigma of being the lowest level mathematics class. Tracking and advanced placement courses were certainly a part of the problem, then, but this perception about senior electives and students' resulting self-selection out of them exacerbated the impact on courses like Discrete Math.

As a result of these practices, senior electives developed a reputation for being easy, which likely impacted students' level of participation and interest in the courses, as well as teachers' treatment of these courses. My data provided several examples of students' attitudes towards elective courses, such as when a student argued that a sociology teacher "was giving way too much work for an elective" (Field Notes 12/16/2013), or a different student justified taking Discrete Math because "he thought there would be less work" (Research Journal 1/31/2014). This is consistent with many students' actions while in my class, as evidenced by a student's reaction ("Like there are some people that just like, absolutely do no work. Like, nothing" (Transcription 1/02/2014)), as well as my own ("About half of the class puts in little to no effort. Students do not listen, they put in no work, and they seem to not care at all" (Field Notes 9/17/2013)). It would be easy to blame this solely on the individual students, but if our school didn't allow for academic programs that were loaded with study halls, lunch periods, and these "easy" courses, then maybe this toxic culture around elective courses wouldn't exist. The stigma associated with electives reflected more general problems with scheduling, where courses like Discrete Math became a receptacle for every student who dropped a “higher-level” mathematics course or came to school in the middle of the year. It is no wonder, then, that some students were apathetic towards a course that was held in such low esteem by the school community.

Triggering Situational Interest

While these external factors certainly put my students at a disadvantage, my data showed that I was still able to implement techniques that triggered students’ situational interest. This is consistent with the first two phases of Hidi and Renninger’s (2006) four-phase model, as situational interest “is triggered in the moment by environmental stimuli” and not by the students themselves (p. 113). In general, I found that developmentally appropriate lessons and assignments that enabled students to demonstrate expertise promoted higher levels of situational interest.

A Developmentally Appropriate Assignment

The budget simulation was an excellent example of an assignment that triggered students’ situational interest because it intersected with students at a pertinent place in their development. My data showed that unlike some other assignments, a critical mass of students really invested in this project, as students who were “not usually interested were discussing what house they would live in, car they would drive, etc.” (Field Notes 11/15/2013). This is consistent with my understanding of the ZPD, because I was able to construct an assignment that really took into consideration students’ interests. The majority of my students were in the midst of the college application process at the time, so they were thinking in earnest about what they wanted to make of their lives. So even though the assignment asked students to consider their financial lives in their 20s, it may have resonated with some of the questions and concerns that were at the forefront of students' minds. As I mentioned in Chapter 4, this higher level of interest transformed the atmosphere in the room, and it likely had a positive impact on student learning. One student's reflection illustrates this point:

> When doing this assignment I learned a lot. I learned that it is very very very expensive to live each year, and even more expensive if I were to have children. That was definitely one reason why I was turned away from having children. I definitely did not know the reality of how much my parents spend each year on everything. One thing that shocked me was the price of gas a year. In this plan we only had one car but in real life my family has four cars in which my parents pay for gas, car insurance, and any leases and repairs. This project definitely made me realize how expensive it is to live in a nice community. This definitely opened my eyes about college. I got lucky to receive a scholarship and I was very proud knowing that the cost of college a year was going to be significantly less for my family. However, it shocked me to see that college is definitely a lot more than I thought and loans are definitely going to be needed (Budget Simulation Comments 11/21/2013).

It appears that the budget simulation stimulated this student's situational interest, both because it drew on her experiences and things that mattered to her and it extended her knowledge in meaningful and tangible ways. This assignment was unique in its ability to engage the majority of the class for an extended period of time, and I believe that this was due to an opportune intersection between the assignment's objectives and students' concerns at the time. In some ways, then, the characteristics of this assignment were not dissimilar from another successful technique, where I attempted to stimulate students' situational interest by tapping into their areas of expertise.

Assignments that Acknowledged Student Expertise

I found that students' situational interest increased when assignments aligned with their individual fortes. Two examples from Chapter 4 demonstrate this point, as one student who was skillful with Excel helped his friends navigate the difficult credit card project (Field Notes 10/31/2013), while another student who was deeply withdrawn demonstrated an impressive amount of knowledge regarding fitness and nutrition (Field Notes 1/15/2014). In the first example, a student was able to use his expertise to help his classmates complete an assignment on Excel, while in the second example, a student convinced his friend that a careful accounting of caloric intake and expenditure was an important element of a healthy lifestyle. In each of these situations, students were able to study an area of strength, but in addition, they were able to share their expertise with a classmate, with me, or with the class as a whole. This is consistent with my findings regarding the budget simulation, because these assignments drew on knowledge and skills that were uniquely appropriate for certain students.

Developing Deeper Interest

I found that the development of individual interest required a more differentiated approach, and further, that this differentiated approach was already available to us through the individualized co-construction process. The promotion of individual interest was more difficult than that of situational interest, because this type of interest is often self-generated, rather than triggered by external stimuli (Hidi & Renninger, 2006). In order to determine if any students had developed this level of interest, I analyzed my data using Hidi and Renniger's indicators to see if anyone took advantage of "the opportunity to reengage tasks related to his or her emerging individual interest and [opted] to do these if given a choice," generated "his or her own 'curiosity' questions about the content of an emerging individual interest," or exceeded "task demands in their work with an emerging individual interest" (p. 115). These indicators reminded me immediately of certain students in my classes, and particularly those that initiated an individualized co-construction process with me. As I mentioned earlier, the individualized co-construction process took advantage of students' ZPD to advance their understanding, but according to the authors' indicators, it may also have encouraged the development of their individual interest. This finding reminded me of my conversation with Student 4 that I mentioned earlier, where he suggested that students "were capable of choosing an area of interest, asking good questions about it, and then finding those answers" (Field Notes 1/08/2014). Not all students may have been ready for that responsibility, but many of the students who engaged in individualized co-construction were. There appears to be a correlation, then, between students who participated in this process and those that demonstrated individual interest, but it remains unclear whether individualized co-construction promoted students' interest, or if the converse was true. Additional research will be necessary to determine whether there is causation between co-construction and individual interest, or merely correlation.

Relationship between Interest, Attitudes, and Engagement

At the end of the data collection period, I asked students to reflect on their experiences in Discrete Math to that point, and the data revealed that students' level of interest impacted their attitudes towards mathematics as well as their level of engagement. This is consistent with the interest literature, which emphasizes the positive relationship between interest and achievement (Koller, Baumert, & Schnabel, 2001; Schiefele & Csikszentmihalyi, 1995) and interest and attitudes (Ma, 1997). For example, one student noted that "this class has been very different, and enjoyable. We have learned real life skills, which is something most math classes do not provide…because of this I have gained a more open mind towards Math in general" (Survey 2/11/2014). This student established a causal link between the enjoyment he felt during the class and his attitudes towards mathematics. A second student established a similar link between interest and engagement when he stated that "I have been interested in all topics studied so far. As opposed to subjects studied before Discrete Math [where I was] bored with many topics and would not put forth the same effort [as I am] in this class." While I cannot generalize this sentiment to all students in my class, I think it is still instructive to note that some students found that our course had a positive impact on their attitudes and level of engagement. While co-construction was likely a factor in this relationship, I found that my third theme, teaching for QL, also may have impacted students' attitudes and level of engagement.

Theme 3: Teaching for QL

While co-construction and student interest ended up being important goals in their own right, each of these themes was initially a mechanism through which I hoped to accomplish my primary objective for the course: to develop students' QL. I found that over the course of the year I gained a great deal of insight into the relationship between QL and mathematics, and I also learned some strategies for teaching QL in a classroom setting.

Relationship between QL and Mathematics

The QL literature presented differing viewpoints on the relationship between QL and mathematics, and my experiences with Discrete Math only reinforced this conflict. Two tensions that emerged between my students and me involved our challenge to reconsider mathematics from the perspective of QL, and our struggle to define exactly whose mathematics should be paramount.

Learning to See Mathematics in a New Light

Though we initially struggled to re-conceptualize our preconceptions about school mathematics, I found that over the course of the year, some students and I became increasingly capable of seeing the value of mathematics from a QL perspective. This was certainly not the case for all students, though, as my data show that some continued to think of mathematics as being completely separate from our work in Discrete Math. At the beginning of the year, I was not surprised that students espoused some of the more stereotypical conceptions of mathematics class, such as the idea that you “don’t have to read or write essays” or the misconception that your success “depends on how well you retain the information and spit it down to the paper” (Survey 9/06/2013). I was more surprised to find that some students retained these beliefs more than halfway through the year. For example, one student stated that she liked “how math has legitimate answers…how there is no in between,” while another remarked that “math is all numbers, so it doesn’t need explanation in words” (Survey 2/11/2014). I wondered if these students considered our course to be something other than mathematics, because every assignment we did involved some sort of ambiguity or “in between” and required lengthy explanations “in words.” My suspicion increased with a third student’s comments, as she admitted that she “could learn to enjoy math like what we’re learning” but that our course was “not ‘Math,’ it’s life.” The premise that students didn’t believe our course to be mathematics was further supported by two of my highest achieving students, who not only received outstanding grades throughout the year, but also asked meaningful questions, made connections between the assignments and their lives, and mentioned on several occasions that they enjoyed the work we were doing together. When I asked them if they felt like they were good at mathematics, one responded “No, I don’t think I am very good at math,” while the other commented that “No, I suck at math.” It would seem that even as students’ attitudes towards my class began to improve, they still considered mathematics primarily in terms of Algebra I, Geometry, and Algebra II. I concluded that it would likely require a more comprehensive effort to expand students’ understanding of what constituted mathematics, rather than one course at the end of their high school careers.

Students were not the only ones who had trouble re-conceptualizing mathematics, as I found myself struggling to understand the exact relationship between mathematics and QL. As a mathematics teacher for almost a decade, I thought I knew my subject really well, but my exposure to QL made me question the very nature of mathematics. My data show that during several points throughout the year, I questioned whether a certain topic or project involved mathematics, QL, or something entirely different. For example, when I first read through some students' suggestions from one of our small-group discussions, I wrote comments such as "Is this math?" and "What is math, really? Is this math? Is it not?" (1/02/2013). I documented some of my meanderings in Chapter 4, where I first considered whether mathematics was simply a filter or a tool before I finally concluded that mathematical reasoning was the key to understanding "how quantitative arguments, approaches, and tools are used in everyday life." While mathematics and QL certainly share some content, the more meaningful connection involves the reasoning skills that help individuals be successful in both areas. I found that many students saw a relationship in terms of shared content, but I was unable to say for certain whether students saw a connection with reasoning skills as well.

When searching through my data, I found that students were much more comfortable reflecting on content, rather than reasoning, when describing their impressions of the course. For example, as I mentioned at the end of Chapter 4, several students found that QL offered "a more practical form of math" that could actually be useful for their lives, and that math was "not all hard equations" (Questionnaire 2/11/2014). It was much more difficult to ascertain students' understanding of mathematical reasoning, but two examples from my data show that even if they were not aware of it, students were applying these reasoning skills to QL situations, particularly towards the end of my data collection period. The first example took place during the assignment on caloric expenditure, where I provided students with different and sometimes contradictory approaches for calculating the number of calories they burned during various activities. I observed students' interactions and wrote the following in my field notes:

> I was happy to see that students were really questioning the formula I gave, as well as the scale factors that they were using for various activities. Students had to perform various conversions… [but they were also] grappling with the reasons why formulas looked a certain way (1/28/2014).

I believe that students were utilizing mathematical reasoning skills in this instance, and further, that they were using them to consider a topic that was important to their lives. The second example comes from our fifth unit, when many students relied on Google Maps to help them calculate driving times for the road trip assignment. I pushed students to question the veracity of these calculations, and students were able to ask insightful questions related to Google's methods for calculating time, the impact of stops on Google's algorithms, and the influence of time of day, traffic, and individual drivers' characteristics on each of these calculations. So even if they didn't articulate it, students had begun to employ mathematical reasoning to help them solve QL problems. The next step, then, would be to help students become aware of this association, so that they could use mathematics more purposely to solve problems in their everyday lives.

Whose Mathematics?

Despite an increase in most students' appreciation for the value of mathematics, I found that students and I still struggled to determine which aspects of mathematics, or more precisely, whose mathematics was the most worthwhile. This back-and-forth was one of the key themes for the entire study, because it underscored the link between the co-construction process and QL. It was only through the co-construction process that I discovered the tension between students' preferences for mathematical content and my own. The data show that this tension was further amplified by our age differences, because things that students thought were significant were often different from what mattered to me, and they were often nothing like what was covered in textbooks. As I mentioned in Chapter 4, I emphasized content such as interest rates, retirement savings, life insurance, and the government shutdown, and while students were able to state the headlines, I found that they hadn't "internalized the meanings, or applied those meanings to their lives" (Research Journal 10/21/2013). I wanted to prepare students to be knowledgeable about the quantitative elements of society as a whole, but what I failed to understand is that some of these concepts might have to wait, because they were not especially relevant to students' lives at that particular time.

If the previous tension was not enough, there was the added challenge of navigating the mathematics that students thought was important and the mathematics that had a significant impact on the world around them. My data show that these issues surfaced primarily during the co-construction process, because students' primary task during these discussions was to evaluate the mathematical topics that would be included in our course. To provide an example, a segment from a longer dialogue mentioned in Chapter 4 is pertinent, because students debated the value of studying important, but in some students' opinions, irrelevant mathematics:

> Student 9: Who cares about 0s and 1s (to Student 10)?
>
> Student 10: (smacks table) 0s and 1s are what run your life
>
> Student 7: Really, they do
>
> Student 6: Yeah, but it's not going to change anything (Transcription 11/11/2013).

This idea that "it's not going to change anything" is so interesting, because it highlights the conflict between mathematics as a discipline in itself and mathematics as a tool that can help an individual student. As I mentioned earlier, these tensions boil down to the question of whose mathematics is important. The students'? The teacher's? The mathematics community's? The free market's? This question is further complicated by the concept of QL, because on the one hand, students should be studying things that are relevant to their lives, but on the other, they do not necessarily know what types of quantitative situations they will face once they graduate high school. Furthermore, unlike in traditional mathematics courses where students needed to learn the language and practices of the mathematics community, QL courses require no such conformity, because knowledge depends on the individual and things that matter to him or her, rather than anyone else. Therefore, there was no way to ever resolve this tension, but I believe that our repeated attempts were still important, primarily because they helped students understand the importance of mathematics in their own lives, in the lives of their peers, and in society as a whole.

Teaching for QL in a Classroom Setting

In addition to the previous challenges, I also struggled to effectively teach for QL in a classroom setting. In particular, I found that teaching for QL required a form of authentic instruction that was extremely difficult to implement in my classroom, and presumably most other classrooms like it. In addition, teaching for QL was often more successful when I relied on an individualized approach, which added several layers of complexity to an already demanding course.

Authentic Instruction

I found that my Discrete Math course was a perfect case study for Ainley, Pratt, and Hansen's (2006) planning paradox, because I struggled to incorporate authentic tasks while also crafting well-structured mathematics lessons. My data show that many of my less successful lessons fell victim to this planning paradox, either because the tasks were unrewarding due to "tightly focused learning objectives" (i.e. such as the extra paperwork I required in the sampling activity) or because the learning was "less focused [and] difficult to assess" (i.e. the three week fantasy football league) when I focused on engaging activities (p. 24). I wrote in my research journal that I experienced "a legitimate struggle between authenticity and pedagogical structure," but thankfully I was able to implement at least a couple of authentic assignments in my classroom without rendering them completely hollow (1/31/2014). Two assignments that stand out are the road trip assignment and the budget simulation, because both experiences required nothing more than a computer and some time, which I made available to students in both situations. For an authentic assignment to be successful, students needed the proper tools for the task, and for topics like codes (special encryption and decryption machines) and nutrition (monitors that measure physical activity), this was simply impossible. I found that while authentic instruction was difficult, it was not impossible, provided that the assignment lent itself well to the tools available in the classroom. What made the implementation of authentic assignments especially difficult was the availability of new technologies, which made some traditional mathematics appear tedious, and perhaps even obsolete.

Technology had a huge impact on the authenticity of my classroom, as various applications had supplanted the need for certain types of mathematics. Two instances from my data are especially pertinent, and they convinced me that I would have to let students use the proper tools if I wanted to develop their QL. The first example involves a student who questioned why he had to calculate nutritional information when a website could do it for him. The second example took place a couple weeks earlier in our nutrition unit, when students informed me that an app was capable of doing everything I had asked them to do in their assignment. I wrote in my field notes that "if an app can do it, and do it much better, then why should students have to do it? (1/14/2014). I realized that I could no longer censor technology, because teaching for QL had to prepare students to solve real world problems with real world tools, not with pencil and paper in a mathematics classroom. This does not imply that mathematics is not necessary, though, and this point is essential to this study. Mathematical computation might not be as important as it once was, but as I mentioned earlier, mathematical reasoning takes on even more significance. Students now have the responsibility of evaluating these new technologies, so rather than shunning them, I encouraged students to use them, understand them, and then evaluate their validity and precision (as with the road trip assignment and Google Maps, mentioned earlier). Returning to an earlier point, I found that authentic assignments were much more successful if I provided students with the tools they would use in real life, and technology had become an important part of this equation. This is one of the reasons why the budget simulation was such a success, because I provided students with the exact technological tools (Excel) they would use if they were performing this task in their adult lives.

Individualization

In addition to the importance of authenticity, I found that individualization was an equally significant component of a QL classroom. This finding is an extension of my previous two themes, because while individualization became a key component of the co-construction process and served to increase students' interest, it also turned out to be an important stimulus for developing students' QL. The co-construction process was meant to incorporate students' ideas in the planning of the course, which I felt was a necessary prerequisite to develop a course for students' QL. Unfortunately, as I mentioned in Chapter 4, only a fraction of each class participated in the small-group discussions, and of that fraction, only a handful exhibited the eloquence and the persuasiveness to successfully advocate for their interests. The result was a situation where certain students fell between the cracks, and one student in particular demonstrated the harmful impact that could result:

> Personally, I did not find that the co-construction process improved my experience in this course because although the topics we studied were interesting and useful, I felt that I was interested in different things than the rest of the class (Survey 2/11/2014).

This comment broke my heart, but it spoke to the fact that QL means different things to different people, and if I wanted to teach for QL, then I would have to take students' individual interests and experiences into account. This particular student wanted to major in engineering in college, so topics like nutrition and road trips may have been interesting to her, but they would not be as useful for developing her QL as other, more advanced mathematics topics. I wrote in my research journal that if I "wanted to develop students' QL skills, some of those will be general (reading the newspaper, understanding sampling, financial literacy), but some will be more specific, depending on students' interests and career goals" (1/31/2014). This task would require individualized co-construction, because it would be nearly impossible to plan a program based on the unique interests and career goals of every student, particularly when these interests might be at odds with one another or change over the course of the year. As an extension, these assignments would have to develop students' interests, and in particular, their individual interests. Then, students would have an opportunity to construct new knowledge about a topic of real value to them, and as a result, they would develop the skills and knowledge they would need "to engage effectively in quantitative situations arising in life and work" (Alsina, 2002, pp. 2-3).

Discussion

In the previous two chapters, I attempted to answer my research questions by demonstrating how the ongoing co-construction of a QL course affected the evolution and development of the course, how students' participation in this course affected their QL and attitudes about mathematics, and how my participation in and reflection on this process informed my developing understandings of how to teach for QL and make mathematics more relevant to my students. In this chapter, I will show the significance of my findings by dialoguing with and expanding on the literature, I will demonstrate the implications for practitioners in mathematics and other disciplines, and I will pose queries for future research.

Significance for the Literature

The findings of this study contribute to the literature in many ways, and in particular, to the literature on co-construction, interest, QL, and action research. There was a dearth of literature on whole-class co-construction, and my study fills the gap by describing how a high school mathematics class can be co-constructed by students and the teacher. Other studies described the classroom environment when teachers responded to students (Maskiewicz & Winters, 2012) or students created midterm questions (Ahn & Class, 2011), but this study examined how students and the teacher attempted to share equal decision-making power over the entire course. Therefore, this study extends the findings of Ahn and Class to a co-construction situation for an entire course, specifically regarding the unique approaches taken by different co-constructing groups, the ability of co-construction to provide "an entry point for every student," and the struggles faced by the teacher when shifting control to students and treating them "as partners" (p. 274). Given the fact that many of my students required time to get used to co-construction, my study makes the case for sustained exposure to the process over a longer period of time. Similarly, my study suggests the use of co-construction in a QL classroom, thereby making an important connection between the literature on equal participation in decision-making (Allen, 2011; Ellis & Malloy, 2007) and the QL literature.

The findings of this study also contribute to the literature on interest, and in particular, to Hidi and Renninger's (2006) four-phase model of interest development. My findings corroborate the suggestion that students' situational interest is generally triggered by an affective response, and they also show that additional elements are necessary for this interest to be maintained (as evidenced by the fantasy football and codes assignments). Hidi and Renninger found that students generated curiosity questions when they approached the later stages of interest development, and this was especially apparent in my study when students were able to ask thought-provoking questions during the process of individualized co-construction. Additionally, my study was able to evaluate the following proposals for how classroom teachers can promote interest:

> Offering choice in tasks, promoting a sense of autonomy, innovative task organization, support for developing the knowledge that is needed for successful task completion, …building a sense of competence[,] …project-based learning that includes students' work with peers or other social situations, computer environments that are attractive, and word problems or passages that have contexts specifically addressing students' individual interests (p. 122).

My study employed each of these techniques at various times throughout the year, and while certain students demonstrated signs of individual interest, others remained mostly at the level of situational interest. My findings show that these techniques worked for some students, but not for all, and one of the primary reasons for these mixed results was the variable impact that school culture had on certain students. My findings show that the school's learning culture had a significant impact on these students, and in particular, on their ability to develop individual interest. Middleton and Spanias (1999) stated that by implementing appropriate instructional practices consistently "over a long period of time" the "more general goals of schooling can be restructured and reinvented with a fair degree of success so that the school culture becomes conducive for student learning and motivation" (p. 82), but I found that my work in Discrete Math was unable to overcome habits formed over many years and reinforced by experiences in other classes throughout the day. Further research will be necessary to determine if individual interest can be promoted if multiple teachers implement these techniques, or if a school embraces this practice in other ways.

This research study contributes to the literature on QL primarily by filling two important gaps. To my knowledge, this study was the first to examine a full-scale QL course at the high school level, and it was one of just a few (Catalano, 2010; Madison, 2006; Madison & Dingman, 2010) that studied a QL course using qualitative techniques. Therefore, this study represents an important new direction for the QL literature, which had focused largely on undergraduate courses and quantitative methodologies. Furthermore, this study offers a distinctive approach to teaching for QL by employing the process of co-construction, which could invite dialogue between the QL literature and the literature on democratic mathematics education. In addition to these contributions, my findings also dialogue with some other important themes in the QL literature.

I found that several of the literature's curricular suggestions for high school were appropriate, particularly the recommendation to include financial literacy (Lusardi & Wallace, 2013), to organize education around different students' interests (Hoachlander, 1997), and to highlight QL tasks in a senior elective (Madison & Steen, 2008). My findings suggest that quantitative reasoning was the primary conduit between school mathematics and QL, which adds to the dialogue on this important issue (Cobb, 1997; Ellis, Jr., 2001; Hughes-Hallett, 2001; Madison & Steen, 2009; Manaster, 2001; Orrill, 2001; Porter, 1997; Steen, 2001b). Finally, this study can respond to four of the difficulties in teaching QL that were discussed in the literature.

First, one of the primary benefits of teaching for QL is the fact that students will learn the importance of mathematics, but Ma (1997) wrote that "successful efforts in bringing students to a better awareness of the importance of mathematics may not automatically improve other attitudinal aspects" (p. 228). My findings were consistent with this assertion, as students' beliefs in the importance of different topics did not necessarily correspond to their feelings of difficulty or enjoyment with those same topics. Second, I found that Jurdak's (2009) assertion that situated problem solving that is "constrained by school rules, norms, and expectations" is much different from decision-making in real life to be somewhat true, but much less so when I allowed students to use any and all modes of technology to help them complete assignments (such as My Fitness Pal, Excel, etc.). In particular, when students were presented with tools that were similar to those they would use in a real-life scenario (such as when they used Google Maps during the road trip assignment), I found that they were much less constrained by "school rules, norms, and expectations." Similarly, Ainley, Pratt, and Hansen (2006) suggested that "the use of 'authentic' settings…will fail to resolve the planning paradox," or the tradeoff between clear learning objectives and engaging tasks, but I found that lessons with correct and appropriate technology (like the budget simulation) were able to have both "clear mathematical focus" and "socially meaningful contexts" (p. 27). Finally, I found difficulties similar to those in the literature regarding assessment, and while I found Boersma, Diefenderfer, Dingman, and Madison's (2011) QLAR rubric to be generally helpful, I did not find it specific enough to highlight key elements in many of my QL assignments.

In addition to addressing literature from each of these content-related issues, my research study also contributed to the growing body of work on action research. This study can be useful for high school practitioners, and particularly for those who work with similar types of students. Hopefully, this study will convince teachers that they too can engage in similar projects, which can add to the dialogue about how to best meet the needs of this particular population. Secondly, this action research study is distinctive because it asked students to do their own "micro action research" projects, as they too engaged in cycles of planning, acting, observing, and reflecting in a systematic way. In a way, then, this study had aspects of participatory action research, but in a more informal way with students as the co-researchers. Finally, this study suggested the importance of documenting each research cycle in greater detail, with the hope of not only answering the research questions, but also of improving outcome validity, process validity, and dialogic validity (Anderson & Herr, 1999). This study may become a model for a different type of action research, particularly if the researcher wants to document his or her methodological choices in greater detail.

Implications for Practitioners

This study also carries implications for practitioners, particularly for those that work with traditionally lower achieving students, students who are not on a calculus track, or seniors who might be lacking in motivation. I believe that some of the lessons I learned might have value, since like many teachers, I faced challenges as a result of tracking, scheduling difficulties, and other aspects of school culture that were out of my control. I realized that students wouldn't learn in these contexts if I didn't operate within their ZPD, not only in terms of the "actions the child can actually carry out," but also in terms of "what the child wants" (Van Oers, 1996, p. 97). I found that this shift in focus helped me to engage a good number of students, and I did this through whole-class co-construction, as well as through co-construction of individual assignments. In addition, I found that this study had additional implications for teachers in a QL classroom.

Whole-Class Co-construction

When learning objectives focused on general skill development (such as quantitative reasoning skills), I found whole-class co-construction to be a powerful technique to promote participation, interest, and ownership. Co-constructing a course with an entire class of students might seem excessive to some practitioners, but at least in my context, it proved to be largely effective. I found that by using this approach, many students participated in varying degrees over the course of the year, I was able to trigger situational interest for most and promote individual interest in some, and many students felt empowered by the opportunity to study material that was relevant to their lives and their futures. It might not have been the easiest technique, and I certainly made my fair share of mistakes along the way, but when I compare it to previous experiences with similar populations, I found this approach to promote higher levels of engagement for a greater number of students. My findings may be valuable to practitioners who want to attempt this approach, because they can learn about some of the effective practices (i.e. small-group discussions, written and verbal co-construction, careful attention to unit categories, leaving oneself enough time) that took me some time to discover. In addition, practitioners can consider how individualized co-construction can augment the whole-class co-construction process, and while this method might not be appropriate for every teacher or every classroom, it provides some insight into practices that at least for me, elicited greater participation and buy-in to the course.

Co-construction of Individual Assignments

Alternatively, if practitioners are not comfortable with whole-class co-construction, but they still want to increase engagement by giving students more control over the curriculum, then my findings regarding the use of well-crafted assignments that allow for self-pacing might be of some value. I created a handful of lessons that engaged a good number of students (notably the budget simulation and the road trip assignment), and my findings regarding these lessons point to the importance of incorporating the following techniques. First, as I discussed in Chapter 5, they met students at a developmentally appropriate place in their lives. As opposed to some of my failures with catchier ideas (codes, fantasy football), lessons like the budget simulation appealed to students because they related to issues that students cared about deeply, such as their lives after college. Secondly, I found that self-paced assignments with clear instructions were most effective for my group of students, because they allowed me to circulate the room and work with students in small groups, they mitigated tensions that could have resulted if I had to reprimand students to get their attention, and it reduced my anxiety, because I could focus on helping students who wanted to learn rather than constantly correcting students who didn't. Thirdly, these lessons incorporated appropriate technology, which added to the authenticity and relevance of the assignments for many students. Finally, each of these assignments allowed for a degree of differentiation or individualization, which promoted ownership and limited cheating. I found that each of these techniques improved the quality of these assignments, and I have been able to apply them not only in Discrete Math, but in my other classes as well.

I do not believe that this approach worked for every student, but I found it to be much more successful than whole-group approaches that I attempted in the past. These self-paced assignments gave the more interested students a chance to engage, and they freed me up to support and encourage those who were on the fence. While this approach may not have engaged the most difficult students every time, it did prevent them from holding the rest of the class back, since each student could continue working at his or her own pace. I found that it was nearly impossible to engage every student on any given day, and I hope this offers some encouragement to teachers who work with similar groups of students. As I mentioned in Chapter 4, I found a disconnect between students' macro feelings about mathematics class and their attitudes on a day-to-day basis, which helped me realize that even if I could improve students' attitudes towards mathematics, I would never be able to control how students felt on any given day. The recognition that every student would not be at their best every day was incredibly liberating to me, and it helped me to focus more on a learning trajectory over the course of the year, rather than on any daily checklist to measure participation, interest, or engagement.

Teaching for QL

This study also has implications for mathematics teachers who want to teach for QL. Teachers and students can develop overly rigid conceptions of what constitutes mathematics, and I found that teaching for QL enabled us to rethink the way that a mathematics classroom should look, particularly in terms of what we studied and how we studied it. My findings suggest that whether teachers choose contexts (as suggested by some of the literature) or teachers and students co-construct contexts (as shown in this study), QL classrooms should focus primarily on quantitative reasoning. As opposed to units that focus primarily on content, quantitative reasoning skills can be applied to many diverse situations, so they are more likely to appeal to a broader range of students. In order to help students develop these skills, I found that "authentic" assignments that utilized current and appropriate technology were the most effective. By giving students the tools they would use outside of school to complete the same task inside the classroom, it reinforced the relevance and utility of these skills while also increasing student interest and engagement. Additionally, I found that the co-construction process helped me to more effectively teach for QL, because it enabled me to differentiate the course based on students' interests and individual lives. I believe that the co-construction process enabled students to develop ownership over their work and cultivate skills that were uniquely appropriate for them and their lives. While whole-class co-construction was one way to achieve this end, I believe that teachers could also allow students to co-construct their own assignments, assign projects with various types of choices, or give students assignments that invite them to show their unique personalities and interests.

Challenges of Teaching for QL Using Co-Construction

It is also important to understand the challenges that may emerge from this process, particularly for teachers who want to attempt something similar in their own classrooms. As for the co-construction process, teachers should be prepared to invest a significant amount of time, because students (and teachers) will likely need time to adjust to their new roles in the classroom. Similarly, teachers will need to carefully manage interactions between students, so that all students, and not just the most outgoing or persuasive ones, feel encouraged to participate. In terms of teaching for QL, my findings point to the challenges that teachers can face when managing multiple assignments in the classroom. Differentiation is an important part of a QL classroom, but it can also be extremely difficult, particularly for teachers who are more comfortable implementing a single lesson plan.

Additionally, there are unique challenges for teachers who want to combine teaching for QL and co-construction. For one, many students have never studied QL topics in a mathematics course, and they might be unfamiliar with any mathematics topics that are relevant to their lives. As a result, students may not be able to provide much input about what they want to study, particularly in the beginning of the year. In addition, school culture has the potential to impede students' willingness to participate in the co-construction of a QL course, and this needs to be considered before attempting this type of project. If possible, it may be helpful to consider this type of project in collaboration with other teachers or administrators, so that students receive a unified message about how they should participate in their courses. Hopefully, this study has provided some guidance on how to handle these challenges, so that teachers who attempt to teach for QL using co-construction can be prepared when faced with similar challenges in their own classrooms.

Limitations

Despite the significance of this study, there are important limitations that need to be considered. First and foremost, this study was not intended to be generalizable, so practitioners and researchers should consider their own contexts when evaluating the findings. Secondly, as I described in Chapter 4, students and I learned how to co-construct as the year progressed, so it took us some time to work through the procedural difficulties. The next time I do this, I will have a much better sense of how to implement the co-construction process, so I will be able to avoid many of these issues. There were also several logistical limitations to this study. Firstly, new students enrolled in the course as late as January, which impacted my ability to cultivate a classroom environment. Secondly, it was often difficult to secure a room with computers, which affected my planning and our use of technology. Thirdly, I struggled to distinguish students' voices on audiotapes from our first large-group discussion, which impacted my ability to analyze data from the first week of school. In addition, due to Institutional Review Board requirements, students had to write their names on questionnaires and surveys, which may have impacted their willingness to give me open and honest feedback. Another limitation involves my decision to end data collection in the middle of February. While I feel comfortable that I stopped collecting data after reaching a point of saturation, I certainly could have lost some important information as a result of that decision. Finally, my choice of practitioner action research was an incredible strength of this study, but it was also a limitation, because I was restricted by my own time and energy.

Implications for Future Research

Future research may be able to not only address some of these limitations, but also to expand on this study in several important ways. My study utilized co-construction to develop a course from the ground up, but future research should consider how co-construction could play a role in a course that is limited by a set curriculum or content standards. This would be an extremely significant study, because most teachers are now required to align their courses with the Common Core State Standards for Mathematics. It would also be interesting to study the relationship between individualized co-construction and individual interest. My data showed that students who participated in individualized co-construction often demonstrated signs of individual interest, but future research should examine whether this is a causal relationship, a reverse causal relationship, or something else. Similarly, I focused primarily on the impact of individualized co-construction on students' individual interest, but less on the impact of students on one another. Future research could consider how students support or oppose one another in a co-construction setting, particularly in terms of each student's unique ZPD.

Further research would also enhance our understanding of teaching for QL. While this was the first study that I am aware of to take an in-depth look at QL in high school, additional research could consider alternative approaches at the high school level. For example, researchers could consider the efficacy of a high school QL course that does not utilize co-construction. Alternatively, researchers could examine how QL topics could be infused into a course with a predetermined curriculum. A third option would be to examine assessment in a QL course, particularly at the high school level. In addition, it would be interesting to study the impact of a QL course on students' attitudes towards mathematics, as well as their achievement in future mathematics courses.

In addition to co-construction and teaching for QL, a third avenue for future research revolves around school culture. I found that aspects of school culture seemed to have different effects on individual students, and this may have impacted their willingness to participate in the co-construction process. Future research should explore how school culture is mediated by co-construction or other techniques that seek to empower students, particularly in mathematics classrooms. Further research is also needed to better understand this demographic of students, so I encourage practitioners and researchers to consider how alternate courses or nontraditional approaches could better improve these students' achievement and attitudes towards mathematics. Finally, it would be useful to determine the impact of co-construction on students in the long run, particularly in terms of their willingness to enroll in college-level mathematics courses. While this would be difficult to determine, it could go a long way toward assessing the impact of co-construction and teaching for QL on students' long term engagement with mathematics.

Conclusion

I believe that the findings of this study were significant, and they certainly helped me to gain a better understanding of how to make mathematics more meaningful for my students. This study filled a gap in the QL literature by examining a high school class using qualitative methods, and it filled an additional gap in the literature by exploring the impact of co-construction on an entire course. This study has important implications for practitioners both inside and outside of mathematics education, as well as for researchers who want to study co-construction, QL, or democratic mathematics education. In the end, this study describes my attempt to improve the experience of a traditionally underserved group of students through the process of co-construction and teaching for QL, but my hope is that it will inspire others to try something similar, or something completely different, so that we could get a better idea on how best to encourage, inspire, and engage all of our students with important mathematics.

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Appendix A

Students will be asked questions similar to the ones listed below.

1. To what extent do you enjoy math? Please explain, providing examples if possible.
2. To what extent do you enjoy math class? Please explain, providing examples if possible.
3. How is what you learn in math class relevant to your life right now? Please explain, providing examples if possible.
4. How is what you learn in math class relevant to your future goals? Please explain, providing examples if possible.
5. To what extent do you feel like you are good at math? Please explain, providing examples if possible.
6. To what extent has our Discrete Math course changed your opinions about math or math class? Please explain, providing examples if possible.

Appendix B

Students will be asked open-ended questions similar to the ones listed below. I will select the questions based on the focus of the particular lesson or assignment about which I am seeking input.

1. Describe the point of this assignment.
2. Name one mathematical topic and one non-mathematical topic you learned during this assignment?
3. Has this assignment changed how you understand the topic at hand? Explain.
4. What is the relationship between mathematics and the topic at hand?
5. Reflect on the value of this assignment for your life.
6. Where do you see this assignment in the context of our course?
7. What would you have changed about this assignment to make it more useful/relevant to your life?
8. Was there any part of this assignment that was particularly interesting to you? Explain.
9. Based on your experiences with this assignment, what areas would you like to explore in more depth?

Appendix C

Quantitative Literacy Core Competency	Achievement Level 3	2	1	0
Interpretation *Ability to glean and explain mathematical information presented in various forms (e.g. equations, graphs, diagrams, tables, words)*	Correctly identifies all relevant information.	Correctly identifies some, but not all, relevant information.	Some relevant information is identified, but none is correct.	No relevant information identified.
Representation *Ability to convert information from one mathematical form (e.g. equations, graphs, diagrams, tables, words) into another.*	All relevant conversions are present and correct.	Some correct and relevant conversions are present but some conversions are incorrect or not present.	Some information is converted, but it is irrelevant or incorrect.	No conversion is attempted.
Calculation *Ability to perform arithmetical and mathematical calculations.*	Calculations related to the problem are correct and lead to a successful completion of the problem.	Calculations related to the problem are attempted but either contain errors or are not complete enough to solve the problem.	Calculations related to the problem are attempted but contain errors and are not complete enough to solve the problem.	Calculations given are not related to the problem, or no work is present.
Analysis/Synthesis *Ability to make and draw conclusions based on quantitative analysis.*	Uses correct and complete quantitative analysis to make relevant and correct conclusions.	Quantitative analysis is given to support a relevant conclusion but it is either only partially correct or partially complete (e.g. there are logical errors or unsubstantiated claims).	An incorrect quantitative analysis is given to support a conclusion.	Either no reasonable conclusion is made or, if present, is not based on quantitative analysis.
Assumptions *Ability to make and evaluate important assumptions in estimation, modeling, and data analysis.*	All assumptions needed are present and justified when necessary.	At least one correct and relevant assumption is given (perhaps coupled with erroneous assumptions), yet some important assumptions are not present.	Attempts to describe assumptions, but none of the assumptions described are relevant.	No assumptions present.
Communication *Ability to explain thoughts and processes in terms of what evidence is used, how it is organized, presented, and contextualized.*	A correct and complete explanation is clearly presented.	A partially correct relevant explanation is present, but incomplete or poorly presented.	A relevant explanation is present, but is illogical, incorrect, illegible, or incoherent.	No relevant explanation is provided.

(Boersma, et al., 2011)

Appendix D

1. Probability
- Counting cards/board games
2. Statistics
- Sports
- Psychology
- Politics/war/Religion
- Beard growth
- Pop culture (Music, Movies, TV shows)
- GPA, Test Scores
- Crime rates, cold cases
3. Decision Making
- College Admissions
- Game theory
- Time management
4. Money
- Taxes
- Stock Market
- Investing/Interest
- Inflation
- Loans
- Work
- Net worth
- Currency
- Economics
- Finance
5. Science/Physics Applications
- Speed, cars
- G-force
- Music theory
- Instruments
- Genetics
6. Morse code, Identification Codes (Driver's license, bar codes), Code breaking
7. Voting
8. City planning, engineering, grid systems
9. Set theory
10. Forensics
11. Weightlifting/exercising/fitness/healthy eating
12. Graph theory, Map coloring, One T Draw (app for IPhone)
13. Number theory/pi/history of mathematics
14. Cooking
15. World Travel/Cultures/Population growth
16. Nature/Outdoors

- Forestry
- Woodwork
- Hiking
- Fishing
- Farming/Gardening
- Surfing
- Canoeing
- Go-carting
- Jetskiing
- Frisbee
- Golf

17. Socializing/social networking