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CHARACTERIZING THE “REALISTIC-NESS” OF WORD PROBLEMS IN SECONDARY MATHEMATICS TEXTBOOKS

A DISSERTATION

Submitted to the Faculty of

Montclair State University in partial fulfillment

of the requirements

for the degree of Doctor of Education

by

MARY L. DALTON

Montclair State University

Upper Montclair, NJ

2017

Dissertation Chair: Dr. Eileen Fernandez

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MONTCLAIR STATE UNIVERSITY

THE GRADUATE SCHOOL

DISSERTATION APPROVAL

We hereby approve the Dissertation

CHARACTERIZING THE "REALISTIC-NESS" OF WORD PROBLEMS

IN SECONDARY MATHEMATICS TEXTBOOKS

of

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ABSTRACT

CHARACTERIZING THE “REALISTIC-NESS” OF WORD PROBLEMS IN SECONDARY MATHEMATICS TEXTBOOKS

by Mary L. Dalton

Word problems are an integral part of any secondary mathematics curriculum and one purpose has been to prepare students for the real-world – for everyday events as well as workplace problem-solving. Prior literature suggests that word problems have not met this objective, in part, because the textbook problems do not mirror the kinds of problems commonly found in real life situations.

In this dissertation, I investigate a sample of word problems from two contemporary non-traditional textbooks to uncover the aspects that may influence if and how the problems might be used in the classroom. I utilize a qualitative content analysis with a directed approach, using the literature to guide my initial codes and categories, and allowing other categories and subcategories to emerge during the analysis. I also conduct a numerical analysis of the data to reveal aspects which may be a common thread between the two books. These analyses allow me to answer the research question:

> Given that the two books chosen for this study have different approaches, what aspects of "realistic-ness" exist in the textbooks’ word problems that encourage students to use their real-world knowledge of the context of the problems?

This study suggests that changes to the manner in which problems are presented can be beneficial to re-negotiating the didactical contract. Textbook word problems should be posed in a variety of ways, breaking from the tradition of the three-component structure. Additionally, secondary mathematics textbooks should use scaffolding throughout the curricula to afford students the opportunities to grapple with problems as they would in the real world. This study recommends a digital database to organize and update problems with a real-world context.

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To my daughter Ellen, I say: "You're never too old to learn". Please don't let opportunities pass you by.

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Dedication

To my Dad,

the first mathematics educator in my life:

Lawrence A. Dalton

(1931-1978)

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Characterizing the “Realistic-ness” of Word Problems in Secondary Mathematics Textbooks

Chapter 1: The Research Problem

Word problems are traditionally considered to be a bridge between school mathematics and the mathematics that is encountered in the workplace and in everyday life. Some intentions for using word problems include preparing students for future careers or for the use of mathematics as it arises in everyday occurrences. But do word problems serve these, and other intended, purposes in the mathematics curriculum? Although word problems have been researched and discussed for many years, it seems that these issues remain unresolved.

In this chapter, I provide the reader with a brief history on word problems and their connections to real-life situations, including their development in the mathematics curriculum. I include an overview of the didactical contract which provides the theoretical framework for my dissertation. I conclude this chapter with my research question.

A History of Word Problems in the Curriculum

Although the question of whether word problems model real-life events seems to have emerged in the last fifty to sixty years, word problems have been found on clay tablets dating back to the Babylonians from the period of 2000 to 1600 BC. The tablets appear to have been the textbook of the time; at least one of the tablets contained a “teachers’ list” of values to be substituted into the routine problems of the day. Interestingly, some of the values used in the problems have been disputed as impractical and inconsistent for use with everyday events of that era (Gerofsky, 2004a). A difference in opinion exists among scholars as to whether the word problems on the tablets represent practical problems based in real-life situations. Thus, it seems the question, "Are they real-life problems?" (Gerofsky, 2004a, p. 118) had been asked even of these earliest examples.

In the 1960s, it was noted that most students were not able to apply school mathematics to other classroom experiences or to situations of daily life (Freudenthal, 1968; Lester, 1994). Freudenthal (1968) gives possible explanations for this. He believes that word problems are hastily posed, almost as an afterthought, and that the problems are often unconnected to everyday experiences. He notes that mathematics textbooks focus on the systematic arrangement of mathematical topics, and that the components of the system can become too mechanical – students solve the problems without considering a context or using reasoning. The focus on mathematical topics, in contrast to mathematics applications, leads students to learn mathematics as a closed system, and preempts its experience as a human activity of daily life (Freudenthal, 1968).

Kamii (2003) identifies a phenomenon in word problem development as contributing to students' struggles: in earlier grades, arithmetic begins in a concrete context, with problems that are worded in real-life contexts, such as, "If I have got ten marbles and I give three away, how many are left?" (Freudenthal, 1968, p. 6). If necessary, an elementary teacher can return to similar concrete contexts to help students understand, and give meaning to, these arithmetic concepts. Unfortunately, this arrangement of presenting mathematics via applications does not continue beyond the most basic arithmetic operations (Kamii, 2003).

From a teaching and learning perspective, word problems have been touted as a motivational device to convince students that the mathematics they are learning in school has relevance to their everyday lives and careers (Verschaffel, Greer, & DeCorte, 2000). Word problems have also been considered a device for training students in the practical skills that they will need for their lives or professions. During the 1970s, for example, more everyday situations were introduced into textbook exercises, such as budgeting, bills, banking, salaries, and taxes (Boaler, 1994). This represented a move away from learning mathematics through an abstract lens and toward learning through scenarios from “real life.” The strategy was viewed as a solution to the problem of transfer from abstract mathematics to the mathematics that students would face in everyday life. Boaler asserts her belief:

> that contexts can motivate students, engage their interests and encourage confidence, but they will only enhance learning transfer if they are also able to offer a realistic and holistic view of mathematics which makes sense to students both in the classroom and in the ‘real world’ (Boaler, 1994, p. 557).

Research Question

The context of word problems is essential in motivating students, yet over the years, the use of word problems in school mathematics has fallen short of the goal of preparing students for the mathematics they will encounter in careers and everyday life.

Current trends and standards in mathematics education have influenced the content and organization of recent textbooks, motivating changes in how real-world contexts are presented in textbook curricula. In this study, I focus on word problems, their realistic-ness, and how they are presented in two recent nontraditional textbooks. This focus motivates my research question:

> Given that the two books chosen for this study have different approaches, what aspects of "realistic-ness" exist in the textbooks' word problems that encourage students to use their real-world knowledge of the context of the problems?

I begin Chapter 2 with definitions and examples that will provide a foundation to familiarize the reader with terms that are common throughout the literature. Next, I review the literature on word problems in mathematics. This review includes their characterization, how teachers and students interact with them, and some of the research designs that have generated the findings cited. As a result of this review, I demonstrate that textbooks are deemed to be essential tools for teachers in the mathematics classroom, and that teachers tend to rely on textbook cues for lesson planning and curricular decisions. Additionally, the textbook presentation of word problems and the ways in which they are structured may affect their classroom treatment. Other characteristics of word problems and their "realistic-ness" can also affect teachers' treatment of word problems. This information will be synthesized to motivate my research question and its investigation in the current study.

Chapter 2: Literature Review and Theoretical Framework

In this chapter, I begin by providing the reader with the definitions of terms regarding word problems and their realistic-ness which are used throughout my dissertation. Next, I provide a review of the literature on realistic word problems and their role in the mathematics classroom. I present a view of intentions underlying the use and nature of word problems in textbooks. I describe the didactical contract and present it as one possible theoretical framework for studying word problems. The chapter concludes with a summary of direction for further research on textbook word problems as suggested by the literature.

Definitions

In this section, I introduce the reader to some of the terms that are used in the study. These terms are elaborated in the review of the literature.

Definition of a word problem. In this study, I analyze some aspects of the "realistic-ness" of textbook word problems and their relation to events in the real world. For this reason, I formalize a definition of the word problems I analyze. Word problems have been widely accepted as a type of textbook problem that is presented as a hypothetical situation explained in words to help students relate abstract mathematical concepts to real-life situations (Gerofsky, 2004). For the purposes of this study, a *word problem* is defined as a task, written in text, designed to help students apply their mathematical knowledge to a realistic situation. For example, calculating the number of buses needed to transport a group of people (Palm, 2008) involves applying mathematics to a possible realistic situation. However, problems like the upcoming Complementary

Angles task (also written in text), with no apparent connection to a realistic context have been excluded: "An angle measures 3 degrees less than twice the measure of its complement. Find the measure of its complement" (Burger, et al., 2012, p. 29).

Definition of "real world" and "realistic." In my analysis, I look for problem features that encourage students to make sense of real world contexts and realistic situations.

A definition of "real world" from a general-use dictionary reveals how the school learning environment and everyday life events are commonly viewed as disparate. *Real world* refers to "the realm of practical or actual experience, as opposed to the abstract, theoretical, or idealized sphere of the classroom" (dictionary.com). Every real-world situation presented in word problems should be *possible* in real life, whether or not it has actually occurred or will occur. For example, the Bus Problem would be considered a real-world problem because it depicts a practical event which can possibly occur.

I adopt the Freudenthal Institute's definition of *context* for word problems: a situation from daily life or from a specialized workplace in which a mathematical question or problem is embedded (Utrecht University, 2016). The context of the Bus Problem includes the need for transporting students. In contrast, no context from daily life or from a workplace situation is provided within the text wording of the Complementary Angles task.

Realistic describes those problems and contexts resembling or simulating real-life events. A simulation of a real-life event could be making an order for the buses after calculating how many would be needed to transport students on a field trip (Palm, 2007).

The word *realistic* extends beyond real-world connections; it can also refer to problem situations which can be imagined (Freudenthal, 1978; Utrecht University, 2016). An example of a realistic situation used as a word problem could be: “If there are 14 balloons for 4 children at a party, how should they be shared out?” (Verschaffel, DeCorte, & Lasure, 1994). Students can imagine this situation as if it were real, even though in real life, an adult could probably remedy the situation and be sure that it is possible for each child to receive an appropriate number of balloons. *Realistic situations* can be hypothetical, as long as they represent something that can actually occur in real life.

Definition of Didactical Contract. Students follow rules they have learned in the classroom when solving mathematics problems, and particularly, when solving word problems. In general, when students are able to solve a problem, they are called upon to implement or demonstrate their knowledge; if they cannot do so, the need for some form of teacher intervention becomes evident. These types of teaching situations are often justified by the *didactical contract*. Loosely speaking, the didactical contract governs the specific habits of the teacher that are expected by the student, and the behavior of the student that is expected by the teacher (Brousseau, 1997). These behaviors are reinforced by the presentation of a problem and its solution and through the activity and the discussion that results in a typical classroom setting (Verschaffel, DeCorte, & Lasure, 1994).

Under the didactical contract, the teacher is responsible for ensuring that the student has adequate resources for acquiring the knowledge necessary for solving specific problems. Students are responsible for solving problems even though the solutions may not have been taught to them. Brousseau (1997) asserts that this contract is “doomed to failure” (p. 32) and is meant to be renegotiated. For example, suppose students are unable to complete problems on their own but, according to the contract, the teacher cannot give them more assistance without giving away the solution. The teacher and students must then negotiate a new “contract” which can support the current circumstances and what needs to be accomplished.

Word Problems under the Didactical Contract. Because many teachers and classrooms have operated under the didactical contract as described above, students have learned to solve word problems using what is termed “the rules of the game” (Verschaffel, Greer, & DeCorte, 2000, p. 59). The students have been a part of the mathematics education culture in which they have learned to simply complete algorithms or calculations and not incorporate reasoning of the world around them in solving problems. Students believed that a single numerical answer is the only allowable solution to any mathematics problem, including word problems, regardless of their context.

Textbook writers have often relegated word problems to a secondary status, by placing them at the end of a section of exercises, making them appear as if the inclusion of word problems in a problem set was an afterthought. The context of the problem was sometimes contrived, with ridiculous assumptions or values within the problem. Students and teachers alike would view these problems as nonsensical and lacking intrinsic value. The manner in which the textbooks presented word problems encouraged students to use calculations and rote memorization without any thought to how those approaches might work in the real-world situation.

Literature Review

Importance of Word Problems

There are at least two goals for mathematics instruction: one is to prepare students in approaches for solving mathematical problems; another is to assist students in learning the concepts and skills that are useful in solving everyday life problems (Masingila, 2002). Many consider the link between these two goals to be modeling through word problems (Brown, Collins, & Duguid, 1989; Gravemeijer, 1997; Lave, 1988; Moschkovich, 2002; Schoenfeld A. , 1991; Verschaffel, Greer, & DeCorte, 2000).

Word problems as motivation. Some reasons for using word problems with a basis in real-life scenarios include: clarifying abstract mathematics topics, engaging students in mathematical discourse and activity, and associating classroom mathematics with everyday life mathematics. In these ways, word problems can enable students to discover and create their own mathematical connections to their personal worlds.

The use of word problems based in real-life situations can help to make mathematics concepts clearer to the students (Brown, Collins, & Duguid, 1989). And, some students are more comfortable with word problems that are set in a context than with more abstract problems.

Word problems can motivate and engage students, the intention being to provide students with a purpose and possibly a familiar context for learning and doing mathematics (Moschkovich, 2002). Students can become more confident when they are familiar with the mathematics they will need and when they see connections between school mathematics and their everyday usage of mathematics outside the classroom (Verschaffel, Greer, & DeCorte, 2000).

Word problems as preparation for everyday events. Word problems have been used to evaluate students’ abilities to do certain work (Boaler, 1994), to train students to think creatively, and to develop problem-solving skills for using mathematics effectively within real-life situations (Verschaffel, DeCorte, & Lasure, 1994; Verschaffel & DeCorte, 1997a). Another intention of real world problems is to provide students with experience in solving open-ended problems and problems with multiple solutions (Moschkovich, 2002), similar to those which they may encounter in careers.

Working with word problems can also encourage important features of problem solving (Reusser & Stebler, 1997). Problem solving, as it appears in the workplace and other everyday scenarios, does not prescribe a particular method or concept to be used to achieve a solution. Decision-making occurs and assumptions are made, and these decisions and assumptions need to come from the context or the real-world knowledge of the problem and its accompanying data (Green & Emerson, 2010). According to NTCM, high school students should have significant opportunities “to develop a broad repertoire of problem solving (or *heuristic*) strategies” (NCTM, 2000, p. 335, italics in original). Additionally, the Common Core Standards for Mathematical Practice call for students to “make sense of problems and persevere in solving them” and to “apply the mathematics they know to solve problems arising in everyday life, society, and the workplace” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Learning with real-life contexts through word problems can engage students in interpreting, analyzing, and mathematizing familiar events and examples from the community and everyday events (Boaler, 1994). Problem-oriented experiences where students learn using real-life situations have been effective in helping students associate new knowledge with a situation in which they might use it (Boaler, 1994; Bruer, 1993; Premadasa & Bhatia, 2013). This can help students to view mathematics as a means to interpret daily events in the world around them rather than as a separate body of knowledge (Boaler, 1994).

The Current Nature of Word Problems

Despite the noble goals discussed in the previous section, some mathematics educators question the usefulness of word problems in real-life or the classroom. Gerofsky (2004b) writes:

> The claim that word problems are for practicing real-life problem solving skills is a weak one, considering that their stories are hypothetical, their referential value is nonexistent, and unlike real-life situational problems, no extraneous information may be introduced. Nonetheless, they have a long and continuous tradition in mathematics education and that tradition does seem to matter (p. 41).

Schoenfeld defines a problem as “a task (a) in which the student is interested and engaged and for which he wishes to obtain a resolution, and (b) for which the student does not have a readily accessible mathematical means by which to achieve that resolution” (Schoenfeld, 1989, pp. 88-89). He claims that students need to be engaged in problem-solving and see the task as a problem they want to solve. Whether a task is a problem also depends on student knowledge of both the mathematical content and the problem situation. Bottge and Hasselbring (1993) agree with Schoenfeld that word problems in today's textbooks do not meet this definition, in part, because textbook problems often do not reflect the kinds of situations that students consider to be important or relevant to their own lives. Additionally, most textbook problems can be completed in one step by directly applying a prescribed and preconceived mathematical procedure, which differs greatly from the mathematics that is needed, and how it presents itself, in the workplace and in everyday scenarios.

Textbook Presentation of Word Problems. The activity of solving school mathematics word problems differs vastly from problem solving in real-life situations. Students often believe that the only way to solve word problems is to use the mathematics that they have learned in the classroom. This belief is first encouraged in elementary school as children learn the basic operations, and then the word problems presented to them require the use of those operations (Balacheff, 1986; Verschaffel, Greer, & DeCorte, 2000). This practice continues through middle school and high school as word problems are placed within the textbook chapter or section containing the mathematics topics to be used in their solution (Bruer, 1993).

Some mathematics textbooks simplify word problems to the point of labeling or coding them with an operation so that the student knows beforehand which approach to use (Gerofsky, 2004b). In this way, word problems are presented so that the method for solution is pre-determined (Moschkovich, 2002) and often, the method involves only one step (Bottge & Hasselbring, 1993). This arrangement may show students that mathematics can be useful, but it does not encourage them to think about approaches they would use in solving the problem.

Textbooks are often arranged with word problems placed as the last item introduced in a lesson or unit. This way of organizing a curriculum creates a separation between acquiring knowledge and applying it, between knowing the mathematics and doing the mathematics (Freudenthal, 1968; Verschaffel, Greer, & DeCorte, 2000). Additionally, this organization can relegate the word problem to a secondary status and may even facilitate a teacher’s decision to skip it entirely (Balacheff, 1986; Freudenthal, 1968).

Unsolvable word problems are scarce in textbooks. It is rare for textbook authors to intentionally include problems that have no solution or only optimum solutions (the best result under specific conditions). Yet these kinds of problems are often encountered in the workplace and other everyday situations (Palm, 2008; Verschaffel, Greer, & DeCorte, 2000).

Gazit and Patkin (2012) and Premadasa and Bhatia (2013) agree with Bruer’s (1993) classification of word problems as “…‘the black hole’ of middle school math: a lot of energy goes in, but no light comes out” (Bruer, 1993, p. 99). At any grade level, most problems that are presented to students are not related to any experience the students have had. Textbook problems are often oversimplified or the situations so artificial (Bruer, 1993) that the problems are neither authentic or relevant to the students in their lived experiences.

Three-component structure. To illustrate some of the ideas mentioned above, I consider the research that examines the format of word problem statements. For example, some word problems are constructed according to three components (Gerofsky, 2004b):

A "set-up" component, establishing the characters and location of the putative story. (this component is often not essential to the solution of the problem.)

An "information" component, which gives the information needed to solve the problem (and sometimes extraneous information as a decoy for the unwary).

A question (p. 27, parenthetical passages in the original).

Because of this structure, word problems often include a hypothetical description of a situation that can be restricted and poorly worded (Verschaffel & DeCorte, 1997b). Students are confronted with the word problem and are expected to return a single numerical answer to a question about the hypothetical situation (Verschaffel, Greer, & DeCorte, 2000). There is no reference to real-life people, places, or situations except in "the most arbitrary way" (Gerofsky, 2004b, p. 32). This stereotypical nature of word problems serves to widen the gap between school mathematics and real-world mathematics (Gazit & Patkin, 2012; Gravemeijer K., 1997; Verschaffel, Greer, & DeCorte, 2000). In his commentary on relevance and reality in solving word problems, Gravemeijer (1997) points out that:

> there is an insurmountable difference between solving authentic problems in reality and solving word problems in school mathematics … In the classroom, one

is always dealing with a reduction of reality. The question for the students, however, is what level of reduction is expected (p. 392).

Gravemeijer (1997) suggests that there should be more variety in word problems, in the way they are presented, and in the information they present. For example, some information may be omitted, or some given information may be unnecessary, (Greer, 1997), leaving decisions to the student about which information should be considered and included in the solution. Some problems could be presented more like a workplace problem: there may not be one simple solution, or there may only exist an optimum solution (Green & Emerson, 2010). The context of the problem could be a familiar everyday event or the context could be explained more thoroughly, going beyond the three sentences in the construct described by Gerofsky (2004b). This could encourage students to more deeply consider the problem and its situation in making decisions in how to solve it.

Word problem examples

In this section, I discuss two word problems from the literature to illustrate some of the ideas discussed above. The first problem is The Bus Problem which appears in several variations throughout the literature. The second is the Distance Running Problem, which involves an assumption which is not possible in the real-life situation.

The Bus Problem. The following word problem can be used to illustrate many of the points above. It is sometimes referred to as the Bus Problem: “360 children shall go by bus on a school trip. Each bus can hold 48 students. How many buses are required?” (Palm, 2008, p. 43). According to Gerofsky’s (2004) analysis, this problem fits the three component structure: the “set-up” component of the Bus Problem is that buses are needed for a school trip; the “information” required to solve the problem is that 360 children need to be transported and that each bus holds 48 students; the “question” asks “How many buses are required?”

When presented with the Bus Problem, fifth-grade students typically divide by 48 and come up with an answer of 7.5 buses (Verschaffel, DeCorte, & Lasure, 1994). Their response may be a result of the placement of this problem during or directly after a lesson on division, or the belief that a single numerical answer is expected in the practice problems. But because they do not consider the realistic elements of the situation, the students do not take into account that another whole bus is needed in order to transport the students. Two different qualitative studies substantiate this observation: 49.3% of 67 fourth and fifth grade students (Reusser & Stebler, 1997) and 49.3% of 75 fifth-graders were noted to use their real-world knowledge to solve the Bus Problem (Verschaffel, DeCorte, & Lasure, 1994). That is, about half of the students use their knowledge of the real world situation to make their responses to the word problem.

Carpenter, Linquist, Westina, and Silver (1983) claim that student understanding of mathematical problems should be given more attention; students should be able to see the relationships between the reality of the situation, the data, their computation, and the reasonableness of their answers (p. 657). Teachers should facilitate discussion about a problem’s context, so that students understand that using realistic considerations is welcomed or even expected. Palm (2008) conducted a qualitative study of 161 fifth-graders which showed an increase in realistic reactions in their written responses to tasks that include increased attention to real world details of the problem. When asked to solve a basic form of the Bus Problem, only 33% of the students used reasoning consistent with the reality of the problem situation. In contrast, when students were asked to fill out a requistion sheet to order the buses for a trip, 63% of the students wrote about the reality of the situation. This illustrates how providing more details in a non-traditional form of a word problem can affect how students regard the realistic aspects of the context.

The Distance Running Problem. Some word problems contain unreasonable or unrealistic conditions. Verschaffel, DeCorte and Lasure (1994) examine the following word problem: "John's best time to run 100 meters is 17 seconds, How long will it take him to run 1 kilometer?" (p. 276). This problem may not be relevant to students who are not familiar with the metric system, and they may have difficulty relating meters and kilometers. On the other hand, those students who participate in track and field events may be intrigued by a problem that reflects their interests. This word problem presumes that John can run at the same rate for the entire kilometer as he does for 100 meters, that is, the presentation of this problem assumed that the ratio of distance to time would be constant. School mathematics problems often include the unreasonable caveat "assuming he maintains the same rate" and this caveat may or may not be a realistic condition or consideration in real life. Other problems may also use ridiculously large or small dimensions or speeds which do not reflect events or objects in the real world (Verschaffel, Greer, & DeCorte, 2000).

Textbook word problems v. real-world problems. In sum, real-life situations are not often used to teach mathematics (Verschaffel, Greer, & DeCorte, 2000), and textbook word problems do not mimic the structure of everyday problems (Gerofsky, 2004b; Verschaffel, Greer, & DeCorte, 2000). The word problems in most textbooks usually do not include extraneous information: all numerical data in the problem is considered necessary to the solution, and students are expected to do something with each and every number given (Verschaffel, Greer, & DeCorte, 2000). Students are not given opportunities to grapple with multiple interpretations; they are led to believe that every textbook word problem has exactly one solution and one solution method (Gerofsky, 2004b; Verschaffel, Greer, & DeCorte, 2000).

Analyses of Word Problems

In this section, I review three analyses of word problems found in the literature. The first is Usiskin's (1997) investigation of peer-reviewed articles about problem solving and what he classifies as contrived problems. The second one is Green and Emerson's (2010) argument for the revision of mathematical modeling tasks in an undergraduate business course. The third is a comparative study of elementary textbooks in use in the United States (Ginsberg, Leinwand, Anstrom, & Pollock, 2005).

Word problems in peer-reviewed articles. Usiskin (1997) noted that it is often left to mathematics teachers to incorporate word problems into the classroom curriculum. He notes that many of the problems that are used in high school curricula are contrived: "I have twenty dimes and quarters whose value is $4.40. How many dimes and how many quarters are there?" (Usiskin, 1997, p. 73). Usiskin calls this problem "contrived" as a counterexample to real-world problems because the quantity of each type of coin could be counted at the same time that the value of the coins is noted to be $4.40, and there is no need in the real world to know how many dimes and quarters comprise the total. Further, he defines real-world problems to be those that are encountered by consumers or workers on the job, and notes that such problems are not often encountered by mathematicians or mathematics teachers as part of their jobs or their education preparing them for these jobs (Usiskin, 1997).

Usiskin (1997) analyzed two years of applications in the NCTM’s Mathematics Teacher practitioner journals to compare the number of articles with application examples. He counted articles from 1966 and 1996 to highlight differences that may have occurred after the inception of the NCTM standards in 1989. The NCTM standards promoted the focus of problem solving in school mathematics, which in turn encouraged an increase in word problems in the school curriculum. Usiskin’s qualitative analysis shows an increase in the number of articles with application examples from 16% to 55% from 1966 to 1996. The percentage of contrived problems, as he classified them, increased slightly from 4% of the problems in 1966 to 8% in 1996 (Usiskin, 1997). Usiskin claims that these results would be likely to be replicated in an analysis of textbooks from the same periods. He felt that the increase in the number of real-world applications in mathematics textbooks would continue into the future.

Word problems in undergraduate textbooks. In the second study, Green and Emerson (2010) noted that many undergraduate textbooks are designed to teach basic procedures and techniques of mathematics, but not to develop students’ thinking skills to analyze data and phenomena quantitatively, and thus, prepare students for professional careers. Using a framework based on a revision of Bloom’s taxonomy of educational objectives, they conducted a qualitative comparison of several contextual problems used in undergraduate business courses, using the aspects in Table 1. Most of the aspects in the table are representative of the features of realistic word problems that are recommended throughout the literature. Green and Emerson (2010) note that problem solving in realistic contexts often reaches the upper levels of Bloom's taxonomy: analyzing, synthesizing and evaluating, but also touches on the lower levels including remembering, understanding, and applying.

Green and Emerson (2010) recommend that modeling problems be as realistic as possible, rather than just based on a real-world context. They describe the contrast between the two: the real-world context problem and its counterpart, the realistic problem. In the real-world context problem, students are given limited but sufficient data to solve a problem. For example, in a spreadsheet problem, the precise spreadsheet rows and columns with the necessary data for solving the problem are given, and nothing more. In the realistic problem, the students are provided with an entire Excel spreadsheet which includes an abundance of data. Part of the realistic problem requires that the students sift through and decide which data is appropriate and essential to the problem. The main difference between the problem types in this example is that the realistic problem involves locating and making decisions about the pertinent data.

Green and Emerson recommend changes in the curriculum for undergraduate business students that would emphasize modeling problems rather than mathematical procedures. Realistic contexts provide ambiguity that occurs with real data and require "the use and interpretation of information in a variety of forms, both quantitative and

Aspect	Math modeling approach for preparing business students	Mathematical approach for preparing business students
Nature of evidence needed to solve the problem	Insufficient, possibly contradictory evidence provided	All "evidence" given in either the problem statement, that section of the book, previous math background, or teacher's examples
Connections to mathematical procedures	Unclear, not proscribed	Almost directly given
Types of assumptions made	Required to bridge the real world and the model world, can't make progress or a final decision without them	Only allowed to assert mathematical postulate or theorems (e.g. a quantity is either bigger than 1, equal to 1, or less than 1)
Complexity	Comes from the interaction between the mathematical world, the context/real world and the need to communicate the information	Comes from layering mathematical techniques on top of other mathematical techniques
Uniqueness	More than one possible solution path, each may have a separate justifiable final decision	One solution possible
Solution features being assessed	Does the solution make sense?	Are the steps justified and supported? Were the mathematical techniques performed correctly?
Robustness of strategy for solving the problem	Problem is sensitive to changes in the data or context, often resulting in a completely new problem strategy	Problem is insensitive due to its templated nature
Transferability	High for the general method of solving a problem, but not necessarily for specific solution techniques	Low, both on method and specified techniques because students didn't have to make the connections
Revision	Allows for a deeper understanding of the context, deeper reflection, and learning to occur	Amounts to "correcting mistakes"

Table 1: Comparison of mathematical and math modeling approaches (Green & Emerson, 2010)

qualitative, and a need for communicating the results to an authentic, appropriate audience” (Green & Emerson, 2010, p. 117). The main objectives of the undergraduate course in modeling that they have devised are for students to develop competence in modeling techniques to work with real world data, for students to be able to analyze and interpret data and results in realistic contexts, and for students to communicate their findings in a realistic context. In these ways, students can be better prepared for the kinds of tasks they will encounter in business careers (Green & Emerson, 2010).

Word problems in elementary school textbooks. In the third study, a group of researchers compared key features of elementary mathematics programs in Singapore and the United States, including textbooks (Ginsberg et al., 2005). The study compared elementary mathematics programs at the textbook level, the lesson level, and the problem level.

At the problem level, the research team investigated the presentation of mathematically challenging exercises in each textbook. They examined one or more of the most difficult problems from the textbooks to appraise the depth of mathematical understanding required by the textbook curriculum. Their small sample set included a total of ten problems from three textbooks: two popular textbooks from U.S. publishers, and a Singapore textbook used in U.S. schools in a pilot project. The mathematical topics chosen were common to the three books.

The researchers categorized each problem in three ways: the number of steps required to solve the problem; whether the expected solution strategy required solving for an intermediate value; and whether the strategy was routine or non-routine in its approach to solving the problem. After a problem was categorized in these three ways, a short discussion of the problem followed. The researchers concluded some elementary textbooks typically used in the U.S. offer opportunities for developing students' mathematical understanding through problem-based learning in real-world contexts. However, the same textbooks fall short in exposing students to multiple-step non-routine problems which are found in the Singapore textbooks (Ginsberg et al., 2005).

Theoretical Framework: The Didactical Contract

Students often solve word problems and seem to understand them without using facts from the real-world situation of the problem (Reusser & Stebler, 1997). The separation between learning and using mathematics has been attributed to the structure and practices of the mathematics classroom (Brown, Collins, & Duguid, 1989; Green & Emerson, 2010). This separation is maintained by treating knowledge as independent of situations in which it can be learned and used. In the classroom, students are encouraged to complete activities without connections to meaning in real-world circumstances. To provide a theoretical framework into how word problems are used and viewed by both teachers and students, I consider the *didactical contract* which currently influences interactions in many mathematics classrooms.

The view that word problem solving has artificial rules and no relationship to reality is, in part, the result of the typical word problems used in mathematics classrooms and the way in which they are presented and treated by teachers (Verschaffel, Greer, & DeCorte, 2000). The *didactical contract*'s rules and expectations govern communication between, and cognitive processes of, the teacher and students in a classroom (Brousseau, 1997). The contract governs the specific habits of the teacher that are expected by the student and the behaviors of the student that are expected by the teacher (Brousseau, 1997). This process is reinforced by the presentation of a problem and its solution and through the activity and the discussion that results in a typical classroom setting (Verschaffel, DeCorte, & Lasure, 1994). These interactions may influence the kinds of tasks the teacher can give to students and what questions and responses can be asked and received.

Teacher-Student Interaction under the Didactical Contract. Under the didactical contract, the teacher and students form a relationship with reciprocal obligations and responsibilities. The teacher is expected to create classroom lessons to impart knowledge to the student, and the student is expected to be able to attain the knowledge and use it. When the teacher recognizes that the student has learned a concept, the teacher will provide the student with new learning opportunities to expand their knowledge base. This didactical relationship is expected to stay in place at all costs under the contract (Brousseau, 1997).

Under this relationship, students are content with a solution that seems mathematical, sometimes at the cost of a solution that actually works or makes sense in the real-world context of the problem (Balacheff, 1986). The didactical contract produces students who wish to be practical and efficient; they want the quickest answers to the questions and problems. Student answers comply with what the students think is expected of them by the teacher, not by the problem situation (Sarrazy & Novotna, 2013; Verschaffel, Greer, & DeCorte, 2000). The goal of most students under the contract seems to be to produce answers, but not necessarily knowledge or solutions based in reality (Balacheff, 1986).

The teacher’s personal attitudes toward the goals of word problems and the problems’ realistic-ness will likely influence their treatment of problem-solving activities in the classroom. Their attitudes are reflected in the types of problems that they select or generate themselves for presentation to the students and the comments and instructions they use to introduce the problems. The students will also pick up on a teacher’s personal thoughts through classroom discussion about the nature of their solving strategies, and the assumptions the teacher allows or encourages in modeling situations. Any feedback that the teacher provides to the students on their thinking processes and solutions will also affect the beliefs, expectations, strategies, and attitudes of the students, and therefore affect student learning (Verschaffel, Greer, & DeCorte, 2000).

Although the use of word problems in the classroom is meant to give students a reasonable facsimile of real-world mathematics, the activities of many students show that they are operating under this didactical contract and that they think of word problems as requiring only the techniques and algorithms they learn in the classroom (Balacheff, 1986; Gravemeijer, 1997; Sarrazy & Novotna, 2013). The teacher’s method of solving a problem is often seen as the official method to which students must conform. Students tend to ignore their real-world knowledge and do not attempt to make sense of their answers with regard to reality (Boaler, 1994; Cooper & Harries, 2002; Palm, 2008; Verschaffel, Greer, & DeCorte, 2000). They become enculturated into the typical practices of a mathematics classroom where attention is given to the correct manipulation of numbers and symbols regardless of the problem situation to which the numbers and calculations are applied (Boaler, 1994; Cooper & Harries, 2002; Schliemann, 2002; Verschaffel, Greer, & DeCorte, 2000). The students adapt to the socio-cultural behaviors of school and resort to rote calculations, and they omit any consideration of the real-world situation. The students display the behaviors that are acceptable in school, resulting in praise and successful grades with as little conflict as possible.

Students have been taught not to ask questions, but rather, to accept the information in the problem purely on trust. The word problem will provide all the data needed to solve it; no outside information will be necessary even if it makes sense within the situation of the problem. Students are to assume that school calculations are different than those in real-life, so any intuition or knowledge of the real world should not be utilized in problem solving in the mathematics classroom (Gerofsky, 2004b; Verschaffel, Greer, & DeCorte, 2000). These "rules of the game" (Verschaffel, Greer, & DeCorte, 2000, p. 59) or assumptions used in problem solving (Gerofsky, 2004b) can be interpreted as part of the didactical contract of the mathematics classroom.

Student View of Word Problems. Even when their school mathematics is presented with a realistic context, students often do not make the connection between the mathematics learned in school and the mathematics they encounter in everyday occurrences. The students often believe that the unrealistic solution is the one that is expected of them (Boaler, 1994; Palm, 2008). A student's failure to provide a realistic solution may also result from: an absence of knowledge on a particular topic (Schliemann, 2002), in particular, age- and grade-specific real-world knowledge

(Verschaffel, DeCorte, & Lasure, 1994). Students bring different perspectives and experience to the discussion, and some may have no experience whatsoever in situations that may be presented in a word problem.

The "suspension of sense-making" (Schoenfeld, 1991, p. 316) is part of the didactical contract which exists in the mathematics classroom. Children build a schema as they gain experience with word problems and this affects how they think and respond to classroom instruction. The students learn to suspend any reality of the problem situation and adhere to the conventions of the mathematics classroom, which have been reinforced over time (Richards, 1991; Verschaffel, Greer, & DeCorte, 2000). The paper-and-pencil method that is used for students to respond to a problem may persuade them to answer in a formal way, conforming to a behavior acceptable to traditional mathematics in the classroom (Verschaffel, DeCorte, & Lasure, 1994).

Gravemeijer (1997) notes that mathematics in the classroom deals with artificial conditions, and it is left to the students to decide what elements of reality should and should not be included in their solutions. Contexts are often oversimplified with assumptions that may propagate the non-reality of the mathematics classroom. This leads students to develop a tendency to ignore any real-world knowledge they may possess; they do not consider the reality of the problem situation in their initial conceptualization of the problem, the mathematical model they may construct, or the interpretation of their solution to the problem (Verschaffel, DeCorte, & Lasure, 1994).

Because they have been indoctrinated in this school routine of mathematics word problems, secondary school students operate under the belief that problem solving is an activity with artificial rules and is separated from the reality of life outside the classroom (Verschaffel, Greer, & DeCorte, 2000). In her study of 25 adult learners, Lave (1988) discovered that the adults use a different method for calculations with a paper-and-pencil test than they do when confronted with arithmetic in their actual experiences in grocery shopping. Transfer is not made between classroom mathematics and how it is presented and mathematics outside of the classroom, because of what is considered to be acceptable in each case. Most shoppers will make approximate calculations with mental math, while a paper-and-pencil test usually requires more accurate solutions; students assume that the latter of these necessitates a particular algorithm.

Reusser and Stebler (1997) conducted an experiment with fourth and fifth graders and found that only 1.5% of the students consistently used realistic reasoning to solve word problems; the students just look for the mathematics to be done. They tend to ignore the nature of the problem situation and the nuances which can affect the answer to a problem. An example of this tendency is students' treatment of the Bus Problem, where they simply divided 360 by 48 and did not consider the realistic need for eight whole buses rather than 7.5 buses (Verschaffel, DeCorte, & Lasure, 1994). Another example involves the following problem: "Steve has bought 4 planks of 2.5 m each. How many planks of 1 m can he get out of these planks?" Many students return an answer of ten planks, rather than eight. However, Cooper and Harries (2002) make the argument that the answer depends on the context: if the question is about building a fence, the acceptable answer is eight, while a response of ten can be acceptable if the question is about installing wood floors.

Textbook Problems under the Didactical Contract. Students tend to follow the rules of the game of word problems (Reusser & Stebler, 1997). They learn to be dependent on classroom and textbook clues in the school situation (Brown, Collins, & Duguid, 1989). In the problem section, they have learned the textbook arrangement of easier questions presented first with the level of difficulty increasing within a problem set. Each solved example is usually followed by exercises to be solved using a comparable algorithm or operation. Most often, textbook word problems are aligned with a particular mathematics topic to use in its solution and the students are led to use pre-determined steps to arrive at an acceptable solution. Word problems are often “simplistic, contrived situations to represent context” (Davis, 2013, p. 20). Any real-world situation may be represented only superficially with emphasis placed on the mathematical content.

The problem-solving strategies and beliefs of students stem from their enculturation in a negotiated didactical contract of the mathematics classroom. Two central contributors to this enculturation seem to be the nature of the word problems given to the students and the treatment of the problems by the classroom teacher (Verschaffel, 2002). Only a handful of word problems that are presented in classrooms and textbooks prompt students to use their personal experiences and everyday knowledge. The majority of textbook problems adhere to the didactical contract in that they are all solvable, every number mentioned in the problem must be used in its solution, and every piece of information needed to solve the problem is included in the problem statement (Reusser & Stebler, 1997; Verschaffel, 2002). This presentation of word problems leads students to develop problem-solving strategies which exclude sense-making of the problem's context or its solutions.

Textbooks are a Valuable Resource. Research shows that many teachers tend to use the textbook as their main curriculum resource (Chval, Chavez, Reys, & Tarr, 2009; Hiebert, et al., 2003; McClain, Zhao, Visnovska, & Bowen, 2009; Nicol & Crespo, 2006). They rely on the textbook for lesson cues and problems in the exercise sets for student work. The Freudenthal Institute for Realistic Mathematics Education (Utrecht University, 2016) maintains that textbooks should be regarded as valuable resources for teachers, especially when textbooks support teachers by providing realistic word problems that can be used within the mathematics classroom. In addition, the NCTM (2016) recently presented their position that "a coherent, well-articulated curriculum is an essential tool for guiding teacher collaboration, goal-setting, analysis of student thinking, and implementation" and that "it is imperative that teachers be provided with curricular materials that clearly lay out well-reasoned organizations of student learning progressions with regard to mathematical content and reasoning" (NCTM, 2016).

Teachers should have a selection of word problems to choose from whose solution goes beyond a simple yes or no answer or a single numerical answer. It is important that textbooks contain the kinds of word problems that can facilitate discussions and dialogue about the reality of the real-life context as well as the mathematical content. Problems that are more consistent with those that are encountered in everyday events tend to engage students in connecting school mathematics to events and situations outside of the classroom.

Verschaffel (2002) suggests improving the quality of textbook word problems using the following goals, which are also suggested throughout the literature:

- Break up the expectation that any word problem can be solved by adding, subtracting, multiplying or dividing, or by a simple combination thereof.
- Eliminate the flaws in textbooks that allow superficial solution strategies to be undeservedly successful.
- Vary problems so that it cannot be assumed that all the data included in the problem, and only those data, are required for solution.
- Weed out word problems in which the numbers do not correspond to real life.
- Accept forms of answer other than exact numerical answers (Verschaffel, 2002, p. 72).

Calling for a move that would precipitate re-negotiating the didactical contract, Verschaffel (2002) proposes that the goals should be negotiated to allow students to use a variety of resources and methods in their interpretation of a word problem and to involve a component of discussion to compare alternatives. Students should be encouraged to use their knowledge about the everyday context of a problem, and possibly be required to communicate their solution and its appropriateness in that context.

To break from the didactical contract as it stands today in reference to word problems, a different socio-mathematical culture would need to be negotiated in the classroom. Textbook resources can help teachers take a more proactive role by providing support and resources to follow when problem solving. Appropriate textbooks can provide a model to establish new guidelines for what counts as a mathematical problem, what counts as a mathematical solution, and what constitutes an acceptable mathematical explanation. By providing pertinent real world problems, textbooks can provide a foundation for solving realistic problems and encourage students to make their own conclusions (Gravemeijer, 2004; Verschaffel, Greer, & DeCorte, 2000).

Possibilities for Realistic Word Problems. In the sections above, we saw how the didactical contract can influence student-teacher interaction in relation to word problems. In the current study, I focus on word problems in secondary mathematics textbooks and aspects of their realistic-ness. I investigate the manner in which they are presented and consider whether the presentation supports re-negotiating the didactical contract in favor of student-centered inquiry and discovery in problem-solving activities.

The NCTM Principles to Actions (2014) call for students to be able to solve problems that resemble those that they would face in real-life settings. Students should know when and in what situations to use mathematical rules and algorithms when faced with them in real life. The mathematics of everyday situations encourages mental math and flexible strategies that are developed as needed to achieve the intended goal (Bottge & Hasselbring, 1993). Students should be encouraged to consider realistic or real-life phenomena when solving school mathematics questions and making connections between what they learn in school and daily situations (Boaler, 1994; Cooper & Harries, 2002). Classroom activities can encourage students to create narratives about the process they utilize to solve problems and to reflect on the efficiency or generalization of approaches to problems:

> by expanding what is considered mathematical to include everyday activities and validating the mathematical aspects of what students already know how to do, classroom teachers can connect students’ practices to the practices of mathematicians . . . teacher can connect mathematicians’ practices to students’ classroom activities by encouraging them to find or pose problems about mathematical objects, make generalizations across situations and construct mathematical arguments (Moschkovich, 2002, p. 9).

An example of a problem which encourages students to provide their own reasoning is the JFK problem:

> You need to arrive at JFK international airport at 7 PM to pick up a friend. At 4 PM, you left for the airport that is 180 miles away. You drove the first 60 miles in an hour. Your friend called you and asked if you can be on time. How would you respond? (Inoue, 2005)

College students in non-STEM majors were asked to respond to the JFK problem in a qualitative study. These students had been chosen purposefully for their ample experience in everyday activities that they could associate with the problem situation, and because they are likely to be better at expressing the reasoning than younger students. None of the students had specialized mathematics training at the college level. It was revealed through clinical interviews that 68% of the students used their knowledge of the real-life activity of driving to the airport to make their responses. The reasoning in their responses included knowledge of the airport and surrounding roads, possible traffic jams depending on the day of the week, variable speeds along the trip, and the amount of time that the traveler would need to pick up luggage and meet the driver at the curb. "Seemingly realistic descriptions of real-life situations in word problems invite students to use their everyday knowledge and imagination" (Inoue, 2005).

Uncovering connections between the students' lived experiences and school mathematics can help bridge the gap between what is truly meaningful and engaging to the student and the mathematical tools that the student can use in sense-making of the problem. The connections will also empower the student with a deeper understanding that can be utilized when they encounter problem situations (Arcavi, 2002; Civil, 2002).

Students often view mathematics as dry and abstract. Boaler (1994) suggests that this perception can be altered by introducing students to problems that will require them to interpret events in their everyday lives and make decisions using mathematics. Problems can seem more realistic to students if they can relate them to familiar life situations (Bonotto, 2013; Carraher & Schliemann, 2002; Cooper & Harries, 2002; Gravemeijer, 1997; NCTM 1989; 2000). Students will be more motivated to learn mathematics with these connections to their everyday world. Mathematics may then be viewed as a way to interpret reality, and not just as a subject that is required in school (Boaler, 1994).

Brousseau and Gibel (2005) present the argument that young children are natural risk-takers; they will take on a task without knowing the outcome, unlike some professionals who may shy away from risks and consequences. For example, a problem may be posed about a sign at an office building elevator: "THIS LIFT CAN CARRY UP TO 14 PEOPLE". The statement of the word problem is: "In the morning rush, 269 people want to go up in this lift. How many times must it go up?” (Cooper & Harries, 2002, p. 7).

This is an interesting problem from the perspective of realistic-ness. Although the context is probably familiar to many students, the reality of the situation may not be obvious. Students who operate under the didactical contract may be compelled to just divide 269 by 14 and return a single numerical solution, regardless of the absurdity of a decimal answer. Some students may consider that element and round up to the nearest whole number. However, as a problem presented with some degree of realistic-ness, a dialogue can be opened so that students may consider explanations of the reality of the situation. This can include, but not be limited to: people may not wait for the elevator to be full to capacity before going up; some of the people may choose to use the stairs instead; more than fourteen people may choose to go up at one time; or, not all of the people will be waiting at the same time to use the elevator. Students should be encouraged to consider the possibility that there may be a range of correct answers with acceptable explanations for a problem of this type (Cooper & Harries, 2002).

Using everyday mathematics from outside of school can show students the ways of using self-invented methods to solve real world problems or approaches that are commonly used because they fit a particular situation (Carraher & Schliemann, 2002). In a qualitative study on using artifacts for problem-posing with fifth-graders, Bonotto (2013) used supermarket circulars to motivate student learning. All of the students in her study were familiar with grocery shopping and supermarket advertising. Bonotto collected data through classroom observations, the written work of students, and audiotapes of classroom discussions. 89% of the students were able to pose questions and insert their knowledge of the shopping experience. They were aware of the need to pay for parking, and that, in their community, shoppers also pay for the shopping bag. The students also demonstrated their awareness of currency denominations. Students created complex original problems that used their experience and knowledge from outside the classroom. Bonotto (2013) believes that flexible thinking and problem-solving skills can be encouraged through less structured and more open-ended tasks. Using artifacts from everyday situations can help to prepare students for events they will face outside of the mathematics classroom. Mathematical creativity is fostered through the use of the knowledge and experiences of the child in the real world.

Schoenfeld (1989) asserts that “a problem is not a problem until you want to solve it” (p. 93). Carraher and Schliemann (2002) also argue that mathematics of the real-world must be meaningful to the students in order to engage them. Brousseau and Gibel (2005) support the use of connections that make sense to the students to enhance their own systematic organizations of knowledge as new knowledge is introduced. “Word problems, instead of being disguised exercises of formal mathematical operations, should become exercises of realistic mathematical modeling” (Reusser & Stebler, 1997, p. 324).

Mathematics problems that use realistic situations should be used only if the students are required to consider the real-world context. The realistic situation allows the students to use their everyday reasoning in conjunction with their mathematical operations to complete the mathematical task (Cooper & Harries, 2002). Students should not be trained to ignore reality, but instead they should be enabled to use the school mathematics-real world connection to tackle problems in real world situations beyond the school environment (Boaler, 1994).

The Literature’s Recommendations for Research

In the words of Verschaffel, Greer, and DeCorte (2000), this review of the literature shows that “something needs to be done about word problems” (p. 158). It is evident in the research that real-life situations are not often used to teach mathematics, and that textbook word problems do not usually mimic the structure of everyday problems (Gerofsky, 2004b; Verschaffel, Greer, & DeCorte, 2000). Verschaffel and DeCorte (1997) argue that realistic mathematical modeling should “permeate the entire curriculum from the outset” (p. 599). They advocate integrating realistic problem-solving throughout the curriculum rather than devoting a separate unit for that purpose.

Verschaffel, Greer, and DeCorte (2000) call for more “attention to . . . carelessly composed questions that are unrealistic in that the numbers or conditions described would not occur in real life” (p. 164). They found that problems often contained ridiculous dimensions that led students to believe that word problems are puzzles of a fictitious nature, rather than a mathematical problem based in reality. They recommend that, when formulating problems, more attention be given to realistic situations.

Although the wording of a problem may not seem realistic, the students should be trained to consider the realistic aspects in their approach to solving the problem (Cooper & Harries, 2002). Palm (2008) argues that students are more likely to consider realistic solutions if the problems give realistic details about a problem situation. Further, Bonotto (2013) believes that “less structured, more open-ended tasks could foster flexible thinking, enhance students’ problem-solving skills, and prepare students to cope with natural situations they will have to face out of school” (p. 53). For example, students should have opportunities to work on problems with a range of possible correct answers, similar to the Lift or Elevator Problem which models an everyday event. Optimizing or estimating under a set of conditions, rather than searching for one single correct solution, can do likewise. Students should also be shown that there may be more than one interpretation for a given situation, and that there may be different solution methods (Green & Emerson, 2010; Verschaffel, DeCorte, & Lasure, 1994).

These suggestions for word problems could support the recommendation for changes to the didactical contract as proposed by Verschaffel, Greer, and DeCorte (2000): to use more realistic and challenging examples and train students to apply their knowledge and experience of everyday events in problem solving. These changes could encourage collaborative work that fosters discussion of the modeling situation, creating a classroom environment which will allow students to develop appropriate beliefs about the mathematical modeling process and encourage students to use judgment and higher-level thinking skills to solve them. These changes create hope for re-negotiating the didactical contract, breaking from the tradition of separating classroom mathematics from real-world events.

The literature’s recommendations for research support the need for an investigation into textbook word problems and those aspects which can shape the manner in which both teachers and students consider the problem, its solution, and the real-world context. My study will address the research question:

> Given that the two books chosen for this study have different approaches, what aspects of "realistic-ness" exist in the textbooks’ word problems that encourage students to use their real-world knowledge of the context of the problems?

In Chapter 3, I lay out my plan for the analysis of word problems in two high school geometry books. I connect the literature review from this chapter to my rationale for this study, using the didactical contract as my theoretical framework. The categories I use in the analysis also correlate with the recommendations for word problems as suggested by the literature.

Chapter 3: Methodology

In this chapter, I use the prior chapter’s literature review to provide a rationale for this study, and a motivation for my research question. A description of the study design follows, in which I explain the data sources and collection of the data, and outline my plan for its analysis. I also discuss the validity and reliability of the chosen methodology, and reflect on the project’s limitations. I conclude this chapter by presenting my position as the researcher in this study.

Rationale for the study of textbook word problems

The literature review and brief history in the previous chapters show that the presentation of word problems has been an issue for many years, perhaps as far back as the ancient Babylonians (Gerofsky, 2004a). Many agree with Bruer’s (1993) assessment that word problems are the “black hole” of school mathematics (Gazit & Patkin, 2012; Premadasa & Bhatia, 2013), and that “something needs to be done about word problems” (Verschaffel, Greer, & DeCorte, 2000, p. 128).

Problems in the workplace or in everyday situations do not necessarily follow the three-component structure of most school word problems (Gerofsky, 2004b). Additionally, real-world problems outside of school are not labeled with a method for solution, and they may allow for alternative or creative methods for their solution (Balacheff, 1986; Bruer, 1993; Hiebert, et al., 2003; Verschaffel, Greer, & DeCorte, 2000). Hence, many word problems in secondary mathematics textbooks have no connection to problems that people need to solve in the real world.

The current didactical contract that is in use in many mathematics classrooms seems to encourage a mechanical handling of word problems: the students act in a way that they believe is expected of them, ignoring any knowledge of the real-world context of the word problem (Balacheff, 1986; Gravemeijer K. , 1997; Sarrazy & Novotna, 2013). They respond with a mathematical answer, whether or not it makes sense in the reality of the situation (Boaler, 1994; Cooper & Harries, 2002; Gravemeijer K., 1997; Palm, 2008). The didactical contract restricts students from making connections between their school mathematics and their out-of-school experiences.

The prior research suggests a need to allow or encourage students to use realistic considerations when faced with solving word problems (Bonotto, 2013; Cooper & Harries, 2002; Green & Emerson, 2010; Palm, 2008; Verschaffel, DeCorte, & Lasure, 1994). A curriculum that encourages students to think in realistic terms and to use their own everyday out-of-classroom experiences to guide their problem-solving will also encourage a re-negotiation of the didactical contract that is present in many mathematics classrooms (Brousseau, 1997; Brown, Collins, & Duguid, 1989; Hiebert, et al., 2003). A new approach can afford the students the opportunities to make use of their real-world knowledge and apply it to problem situations they may face in the classroom and make the connections to problems they face elsewhere.

The review of the literature suggests that textbooks are essential tools for teachers in the mathematics classroom, and that teachers tend to rely on the textbook cues for lesson planning and curricular decisions. The position of word problems in the textbook and in each section of the textbook may affect their treatment in the classroom. In addition, other aspects with regard to the realistic qualities and context of the word problems can also affect teachers’ treatment of word problems.

Current trends and standards in mathematics education advocate a change from teacher lecture and demonstration to student-centered lessons which encourage discovery, cooperative groups, and student discourse. In accordance with these trends, textbook authors and publishers are breaking from the teacher-centered tradition by offering textbooks that support a constructivist view and that emphasize reasoning and problem-solving experiences which engage the student in constructing their mathematical knowledge. Although textbook authors may have similar goals, their approaches to developing curricula can differ.

I have chosen two secondary school textbooks to investigate with regard to their approach to realistic word problems. Both are described as non-traditional by the publishers: however, one uses a discovery approach and the other, a standards-driven approach. In this study, I look for differences as I investigate the aspects of word problems in the two books and how they relate to the conventions of textbook word problems as described in the literature. In addition, I examine the books in tandem with the same conventions in mind. With this scope, I present my research question:

> Given that the two books chosen for this study have different approaches, what aspects of "realistic-ness" exist in the textbooks’ word problems that encourage students to use their real-world knowledge of the context of the problems?

Method

Data Sources

In this study, I conduct an analysis of word problems in mathematics textbooks with regard to their realistic-ness and their connections to situations in problem solving that occur in real life. The data source is chosen purposefully to consist of two textbooks. Both textbooks are intended for use in American high schools for teaching geometry. Keeping the country, classroom use, and grade level constant are intended to facilitate comparison and contrast of the two books.

Discovering Geometry (Serra, 2008) is considered a nontraditional book. A foreword from the author describes the text as presenting "the concrete before the abstract". In fact, the textbook's sequence positions the chapter on "Geometry as a Mathematical System" as its *last* chapter. Many traditional geometry textbooks *begin* with this material, which includes postulates and the basic premises of geometry. The author also claims that students will gain greater understanding by being "actively engaged in the process of discovering concepts for themselves" (Serra, 2008, p. xv). Further, he explains that the students will learn by working in cooperative groups and investigating geometry in ways that will lead to "the discovery of geometry properties" (Serra, 2008, p. xiv). The description of the book's design appears to support teachers and students in breaking the existing didactical contract of most secondary geometry classrooms.

Some textbooks are driven by current standards in mathematics education (Ginsberg et al., 2005). Currently, the Common Core State Standards in mathematics are influential in shaping the content and organization of recently published textbooks. I have chosen a standards-driven textbook as a companion book for my analysis. In particular, I look at the Common Core edition of *Holt McDougal Geometry* (Burger, et al., 2012). This book is described on the publishing company's website as one that "empowers students to develop the core skills they need ... In keeping with the Common Core State Standards, the new, streamlined Student Editions focus on deeper understanding of math strategies and concepts" (Holt McDougal Algebra 1, Geometry and Algebra 2, 2016).

The reader should note that the proclaimed approach of the selected books differ: the first states that the students will be engaged in a discovery approach (Serra, 2008), while the publisher of the second states that the Common Core's set of Mathematical Practices is its priority in preparing students for assessments (Holt McDougal Algebra 1, Geometry and Algebra 2, 2016). Nontraditional textbooks, as defined by Ginsberg et al. (2005), reflect a constructivist philosophy of learning: the textbook curricula emphasize reasoning and problem solving in experiences in which students can construct their own knowledge. According to the publishers' statements, the word problems in these textbooks should be written in a way that students can to apply their mathematical knowledge to real-life situations.

I have chosen geometry books specifically for this study because I believe that there are many real-life situations which correlate to the mathematics that students learn in geometry class. "Know(ing) how to apply geometric concepts in real-world situations" (Educational Testing Service (ETS), 2017) is considered to be an important skill for both teachers and students. It is also my belief that many of the mathematical concepts of geometry can be applied to real-life objects and events that students encounter in their everyday routines.

Sample

The sample for this study is a set of word problems from the selected textbooks. In this section, I describe my rationale for including or eliminating problems from the study’s sample. In past studies of textbook word problems, the analysis was limited in scope to a handful of mathematical topics which were present in all the textbooks used or to a small number of common standards (Ginsberg et al., 2005). My plan differs from this: I plan to analyze those word problems that fit my definition of a word problem for this study: a task, written in text, designed to help students apply their mathematical knowledge to a real-life situation. The word problems that I analyze are those in the Exercises portion of each numbered section of the textbook chapters. These are the problems that are usually intended for the student audience.

The research on realistic problems calls for more problems with a real-world context (Verschaffel, Greer, & DeCorte, 2000). In order for a problem to be realistic, it should be presented or set up using a real-world context (Hiebert, et al., 2003). This further refines the sample of problems that I have chosen for analysis. In particular, I set out to analyze the “best” possible realistic problems presented in the selected texts. With this in mind, I began eliminating problems that did not fit this characterization. For example, a word problem that asks questions that are not necessary in real-life situations – including those that are contrived (Davis, 2013; Usiskin, 1997) – do not prepare students for the mathematics they will encounter in careers and in real life. Thus, the Bus Problem would be included in my sample because it fits the description of a real-life problem: those planning a field trip for students would need to be able to order a reasonable number of buses to transport the students. In contrast, the following contrived problem would not be included in the sample because its question is not necessary to a real-life situation: “Five-foot-tall Melody casts an 84-inch shadow. How tall is her friend if, at the same time of day, his shadow is 1 foot shorter than hers?” (Serra, 2008, p. 599). There is no real-life context mentioned here that would necessitate an answer to this question; in addition, her friend could probably readily answer that question if he was asked directly.

By my preliminary counts, there are 552 word problems in the standards-based book and 306 in the Serra textbook with some connection or mention of real-life events. A mere mention of a real-life connection to the mathematics in a problem is not sufficient for its inclusion in this study. For example, problems in which a real-life context is used as camouflage for straightforward algebra or geometry calculations have been excluded. Problems which include a pre-determined equation are also excluded, as well as those with algebraic or geometric structures superimposed over a real-life context (see Figures 1 and 2).

In several instances in the textbooks, one given real-life context is used to create two or more different, numbered problems (see figure 3). In the example illustrated, this small group is analyzed as a single problem, and is not be counted as four separate

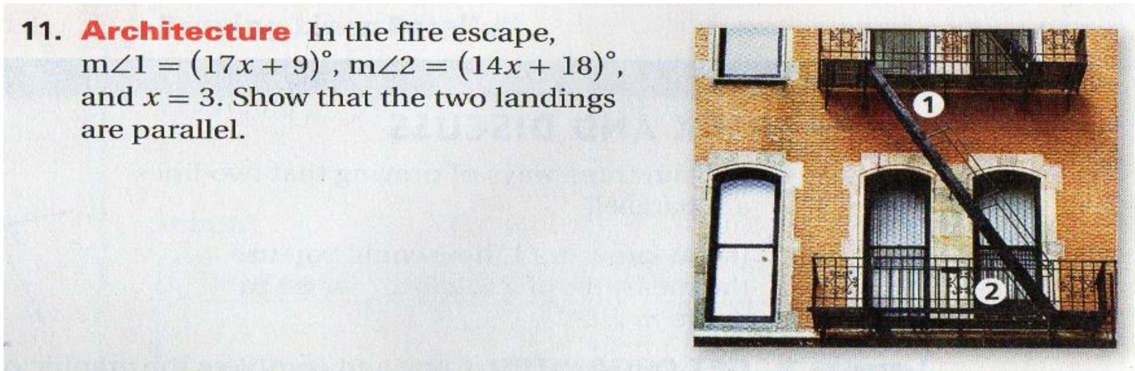

Figure 1. Picture with a superimposed diagram (Burger, et al., 2012, p. 166).
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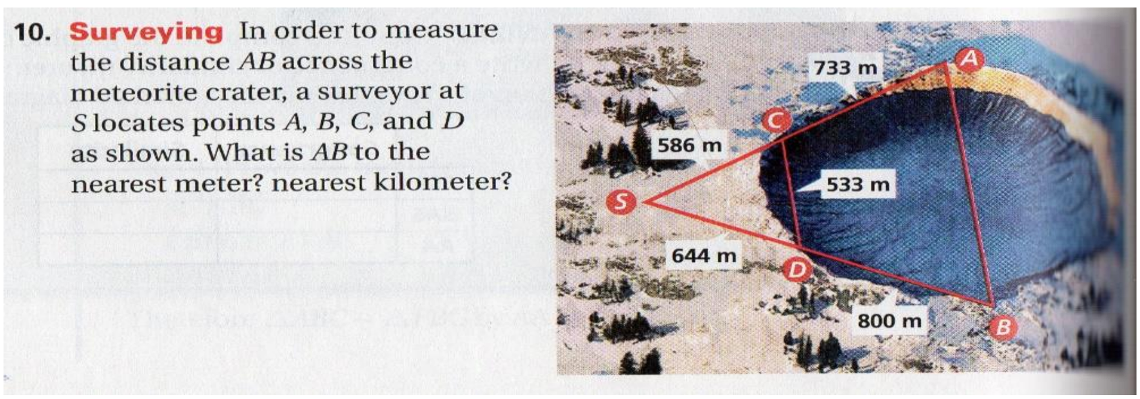

Figure 2. Picture with superimposed geometry diagram (Burger, et al., 2012, p. 486).
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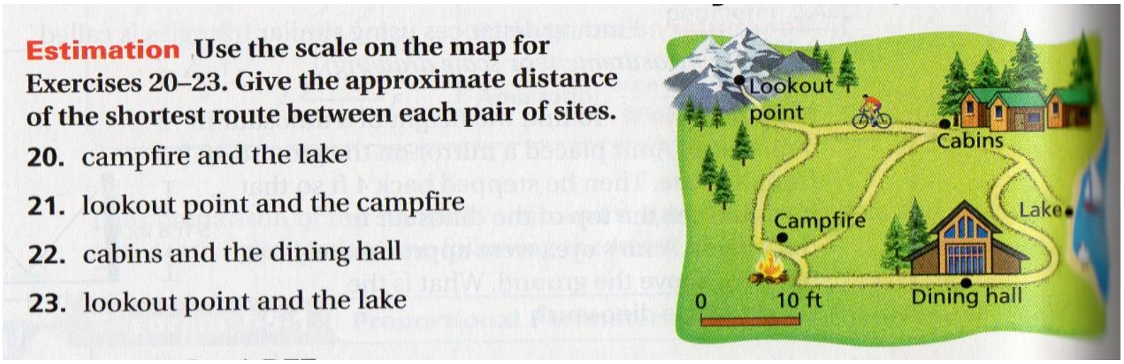
problems. One of the two books also contained an entire chapter on probability, which is not traditionally part of the geometry curriculum; these problems have also been eliminated from the sample.

The discovery book has several sections which exclusively contain word problems in the Exercises. These sections are interspersed between other lessons within the chapter. These sections of Exercises are considered as a part of the sample set since these sections are embedded in the chapters as section lessons (see Table 2).



Estimation Use the scale on the map for Exercises 20–23. Give the approximate distance of the shortest route between each pair of sites.

20. campfire and the lake

21. lookout point and the campfire

22. cabins and the dining hall

23. lookout point and the lake

Figure 3. Example of 4 questions regarding one problem situation (Burger, et al., 2012, p. 506). From *GEOMETRY,* Common Core, Teacher Edition. Copyright © 2012 by Houghton Mifflin Harcourt Publishing Company. All rights reserved. Reprinted by permission of Houghton Mifflin Harcourt Publishing Company.

Table 2: *Chapter sections devoted to word problems - discovery book*

Section	Section Title
1.9	A Picture is Worth a Thousand Words
6.6	Around the World
8.3	Area Problems
9.4	Story Problems
10.4	Volume Problems
12.5	Problem Solving with Trigonometry

The standards-driven book also has several “real-world connections” sections, exclusively composed of word problems. Each section may provide one or two contexts with three or four related problems. These problems were noted to be similar to other problems in the textbook: conforming to the traditional structure of word problems, some being true-or-false statements, most requiring only a single numerical solution. However, these “Real-World Connections” sections are located at the very end of a chapter. In chapter 2, for example, the Real-World Connections appear as the last feature – after the Study Guide and Review, the Chapter Test, College Entrance Exam Practice, Test Tackler, Standardized Test Prep – a full fifteen pages after the end of the last lesson section in the chapter. The Real-World Connections pages appear at the end of chapters 2, 4, 6, 8, 9, and 11. I have not included these problems from the standards-driven book in the count of problems for this analysis, since these problems are not included in the Exercises of a lesson section. I have also omitted the problems in the chapter reviews in both texts. All of these problems were deliberately omitted in my study because they are often intentionally left out by teachers.

The location of these problems can influence teachers’ treatment of them; prior research advocates integrating realistic word problems through the entire curriculum and opposes dedicating a separate section for these problems (Verschaffel & DeCorte, 1997b).

When word problems are spotlighted as the only questions in a section, this creates issues. First, in the interest of time, a section which contains only word problems might be omitted by the classroom teacher. Second, a section exclusively containing word problems upholds a separation between mathematical concepts and their application to real-world situations. Third, the distance of fifteen to twenty pages after the last lesson of a chapter further separates these problems from the rest of the textbook curriculum. This way of organizing a curriculum maintains the separation of school mathematics and mathematics in the real world (Brown, Collins, & Duguid, 1989; Freudenthal, 1968; Green & Emerson, 2010; Verschaffel, Greer, & DeCorte, 2000).

After these eliminations, the sample tallies stood at 208 and 236 for the standards-driven and discovery books, respectively.

Design of study

There are two components to the analysis of the two textbooks. In the first component, I coded each of the word problems in the sample using categories and subcategories. I counted the occurrences of each code and used a numerical analysis to compare the two books and quantify the coding of the sample from both books as a whole. In the second component, I analyzed the data using a qualitative content analysis through a directed approach.

The qualitative content analysis focuses its attention on "the content or contextual meaning of the text" (Hsieh & Shannon, 2005, p. 1278). The directed approach to content analysis uses ideas from prior research to help focus the research question and the analysis. The prior research can also help to determine some initial coding categories for the study (Hsieh & Shannon, 2005). Specifically, the categories in the Ginsberg et al., (2005) study and aspects from the Green and Emerson study (2010) have influenced my initial categories for analysis. Those categories guide the analysis, as well as other categories that emerged during the process of the study. I have chosen particular content to investigate (word problems with a real-life context) and this influences my interpretation of the realistic-ness during my analyses of the textbook word problems and how they relate to the didactical contract.

One particular strength in using the directed approach is that it extends and supports existing theory and prior research. The prior research on the presentation of word problems also guides my discussion of the findings among the data. A directed content analysis often results in findings with supporting and non-supporting evidence as it pertains to the prior research. This could be considered a shortcoming of the directed approach because there may be some bias toward the prior research, and a researcher may be more inclined to find supportive evidence rather than non-supportive evidence. On the other hand, this approach can provide the reader with information on how the coded concepts have developed since they were initially used. To achieve neutral or unbiased results, my researcher's journal accompanies the study as recommended by Hsieh and Shannon (2005).

For the numerical analysis portion, I coded each word problem according to the categories and subcategories in the Qualitative Codebook (see Appendix A). I organized the data from each textbook and calculated totals in spreadsheets (see Appendix B). I report on these and analyze my findings in the Results section in Chapter 4.

Researcher's Journal

The researcher's journal is an essential part of the study where I keep a detailed account of my methods, procedures and decision-making throughout the study. I assess each word problem in connection to initial categories, and I take notice of any themes that may emerge, prompting other categories to arise which could be analyzed. I use rich, thick description as I elaborate on my findings for the qualitative portion of the analysis.

The analysis of the word problems details the data that are collected, how categories and themes are chosen and how they emerge (Merriam, 2009). The researcher’s journal includes memos I wrote as I collected the data; this is a record of my reflections and interpretations of the data and my analysis.

The categories for the analysis of the word problems appear in the qualitative codebook (Appendix A). The codebook provides a systematic approach in qualitative research (Creswell, 2009). For this study, the codebook contains a table that contains columns of predetermined categories and their definitions. The codebook serves to document new categories as they emerged during data collection and analysis.

Data Analysis

The data analysis began with the selection of the word problems from the textbooks that follow the criteria I set for this study: those tasks, written in text, designed to help students apply their mathematical knowledge to real-life situations.

The second step was to code the problems using predetermined categories suggested by the literature. The problems in the sample and their connection to the categories are described and discussed in great detail. Patterns and themes within the data set also inspired new categories and subcategories to emerge during the analysis (Creswell, 2009).

The qualitative content analysis I conduct in this study follows in the tradition of Schoenfeld’s (1983) analysis. Schoenfeld analyzed mathematical problems that he used in a college-level class on problem-solving. These problems were not necessarily word problems, but he characterized them as “problems to make a point” (p. 42) or “training problems" (p. 47). His analyses included classroom discussions, decisions and suggestions for possible approaches and strategies for solving the problems. In Schoenfeld's tradition, I also provide a discussion for each problem which includes my decisions for approaches and strategies. My analyses contrast with Schoenfeld's in that I focus on the aspects of *realistic-ness* of word problems and not on features of problem solving.

I have included detailed narrative passages to relate my findings in the results section of this study. The findings from my directed content analysis provide evidence through descriptions and exemplars to show the ways in which the textbook presentation and problem structure may encourage teachers and students to break the existing didactical contract as well as the ways that may encourage them to continue operating under the existing didactical contract.

The literature influenced my initial categories, as well as the new categories and subcategories. The literature review in Chapter 2 illustrates aspects of word problems that either support or run counter to the existing didactical contract. I analyze the word problems with respect to aspects that may influence a teacher's decisions to use and present problems realistically in a classroom lesson.

Categories. The problems in the sample are set up or presented with a real-life context and are intended to use that context throughout the solving process. Following the lead of Green and Emerson (2010) and Ginsberg et al. (2005), I have chosen categories that appear in the literature to discuss and elaborate on the textbook word problems with a real-world context. During the first rounds of coding, I began with the categories: Location, Variation, Alternative Approaches, and Realistic Considerations.

Codes and categories also emerged during the early stages of the qualitative analysis of individual problems. After the first rounds of coding, I included the category of Support in the Teacher's Edition for instructors. Much later in my analysis, I realized the importance of the support that may be offered for real-world contexts which may be unfamiliar to teachers, and included a subcategory to quantify that support.

Some codes also developed through my actions of writing and reading my researcher's journal. My reflections as both a practitioner and a researcher informed my decisions on coding the textbook problems. Four subcategories for Realistic Considerations emerged regarding the reality of a context and how it may be enhanced or hindered by aspects of the problem.

As I considered aspects of word problems and their realistic-ness, I considered the ways in which I could code and quantify the aspects in relation to this study. One aspect which I have omitted in this study is the placement of the realistic word problems within a problem set. It has been noted that the placement of word problems can affect how students and teacher view them (Freudenthal, 1968; Verschaffel, Greer, & DeCorte, 2000). This aspect of placement within an Exercise set would need to be quantified in a manner that is unlike other aspects in this study, and, therefore, is a possibility for future study.

Label. Some mathematics textbooks label each word problem with a descriptor. Sometimes, that descriptor defines the problem by its context i.e., the Bus Problem or the

Lift Problem. Others, however, label the problem with the operation, algorithm or theorem to apply so that the method is prescribed or pre-determined (Gerofsky, 2004b; Moschkovich, 2002). In this study, any label assigned to a problem is discussed with regard to whether it is labeled by its real-life context or by a preferred mathematical content. For example, the Bus Problem is named for its real-life context rather than instructing students that it is a “Division” or “Remainder” problem. Other real-life context labels for the Bus Problem could be “Student Field Trip” or “Ordering Buses”.

Variation. Many textbook word problems follow the concise three-sentence or three-component structure of: (1) “the set-up”, (2) “information” about the problem, and (3) “the question” (Gerofsky, 2004b). Very few problems encountered in the real world would be presented in such a concise structure. Many problems that need to be solved in everyday life or in the workplace arise from a much larger picture. The data may be given as a part of a conglomeration of data such as a spreadsheet (Green & Emerson, 2010) or the data may need to be found elsewhere. Verschaffel (2002) suggests that problems be varied in a way that students will not assume that all information in the problem statement is necessary, and that this is the only information that is necessary. In this category of variation, I considered whether a given problem conformed to the three-component structure. This process involved consideration of the nuances in the wording of word problems that mimic the structure of problems as they would be represented in real-life situations.

Alternative approaches. Often, textbook word problems are placed in the chapter or section of the book which covers the particular mathematical content needed or intended to be used to solve the problem (Balacheff, 1986; Bruer, 1993; Verschaffel, Greer, & DeCorte, 2000). This arrangement directs the students to use that content and removes any decision-making responsibility from the student (Hiebert, et al., 2003), continuing the tradition of the didactical contract. Therefore, in my study, I coded for the placement of a problem in relation to where its content is first presented. My analyses elaborate on the opportunities for students to use a variety of *alternative approaches*, rather than an approach dictated by placement, to solve a problem.

Realistic considerations. In some word problems, phrases like "assuming that he runs at the same rate" or "ignoring the wind speed" appear. Others use absurdly large or small quantities which are unrealistic in real-life experiences (Verschaffel, Greer, & DeCorte, 2000). These assumptions remove the chance for students to consider the practicality of the context of the problem. In examining the problems, I scrutinize them for ways in which they may compel students to think further or more deeply into this issue. The problems and their solutions are analyzed through thick, rich description with regard to their wording and how this wording encourages students to use *realistic considerations*. This broke down into the following four subcategories:

Problems written in a real-world context (PRW). Some problems in the sample are constructed in a way that they could have easily come straight from an outside-of-school context and been placed in the textbook. These problems have no stated connection to the mathematics. If the problem is not located in a section corresponding to the mathematics content to be used, students would need to make the connections on their own. As a separate entity, the problem is not recognized as a textbook problem; it contains no school mathematics terminology. The Rectangular Room Problem is an example of a problem written in a real-world context: "To make sure that a room is rectangular, builders check the two diagonals of the room. Explain what they check about the diagonals and why this works" (Serra, 2008, p. 295). It is the kind of scenario that could arise in an everyday conversation.

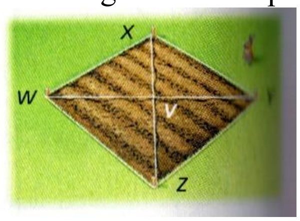
Question about Mathematics (QM). Even though problems may be presented with a real-life context, some of the problems ask questions about the mathematical context rather than requesting information about the real-world context. This kind of wording trivializes the connection to real-life objects and situations and emphasizes the mathematical content instead. The following problem enquires about the area of a pizza without relating it to any real-life need to know the area: "A pizza parlor offers pizza with diameters of 8 in., 10 in., and 12 in. Find the area of each size pizza. Round to the nearest tenth" (Burger, et al., 2012, p. 691). Other than the fact that the pizzas are round and that there are three different sizes available, pizza is not the real subject of this problem. The support for the teacher offers advice about using estimation to determine if the solutions make sense, but does not offer advice about discussing the difference in the areas and which might be the better buy. The question is simply about the areas of the three circles' sizes. I will code problems with questions that focus on the mathematics as QM.

Problems that describe their usefulness (USE). Under this subcategory, I code for problems whose statements convey a purpose for solving. This problem from the sample serves as a counterexample: "Earth takes 365.25 days to travel one full revolution around the Sun. By approximately how many degrees does the Earth travel each day in its orbit around the Sun?" (Serra, 2008, p. 73) This may settle the curiosity of some, but questions are still unanswered within the text of the problem: In what real-world context would the answer to the problem be needed? And why is the degree measure important?

On the other hand, the TV Diagonal Problem is coded as USE. Note its stated purpose in the opening sentence: "Television and computer screens are usually advertised based on the lengths of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch" (Burger, et al., 2012, p. 48). This problem directly states the usefulness of the information (USE): we need to know the diagonal since screen sizes are determined by their diagonals. Problem-solving can be more relevant if the reason for the problem is evident (Gerofsky, 2004b; Moschkovich, 2002).

Technical mathematics content (TMC). If the text of a problem uses technical mathematics content to describe the problem situation, or if it includes a diagram which points to the mathematical content to be used, the problem is coded as TMC. In these TMC problems, school mathematics terms are used rather than everyday terms which are familiar to the lay person. An example of a TMC problem is this:

Gardening. A city garden club is planting a square garden. They drive pegs into the ground at each corner and tie strings between each pair. The pegs are spaces so that $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. How can the garden club use the diagonal strings to verify that the garden is a square?



(Burger, et al., 2012, p. 434)

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The technical mathematics content includes the mathematical symbols for the segments which are not commonly used in a real-life situation. The garden club may measure the diagonals and the angle at the intersection to ensure that the garden is a square, but they probably would not mark the garden with points W, X, Y and Z and refer to the sides as segments with the mathematical notation.

Most realistic word problems. In addition to these four subcategories of Realistic Considerations, I also use the intersection of two subcategories, PRW and USE, to classify a subset of problems in the sample as *most realistic*. I see problems that are written in a real-world context *and* describe their usefulness as those that seem most likely to be stated in everyday routines or in careers in the same manner as they appear in the textbooks; these "most realistic" problems must satisfy both criteria. The problems do not include specialized mathematics terminology, and the purpose of solving the problem is clear within the statement of the problem. An example of a "most realistic" problem is this: "Which is the better buy – a pizza with a 16-inch diameter for $12.50 or a pizza with a 20-inch diameter for $20.00?" (Serra, 2008, p. 646). The purpose is clear;

the solver is looking for the better buy. The question is one that consumers may ask themselves before making a purchase. These problems are coded as PRW and USE, and carry the additional code of PRW/USE (see Appendix B).

In the qualitative analyses portion of the results chapter (Chapter 4), each of the preceding categories is used to analyze a subset of word problems with thick, rich description. The subcategories are discussed if they apply to a particular problem. All of the categories and subcategories are outlined and presented with examples in the qualitative codebook in Appendix A.

Support. In addition to the solutions for the Exercises in the Teacher’s Edition, the side notes sometimes provide information to guide the teacher on how to approach problem solutions. These side notes may offer assistance on the mathematical content or on pedagogy, but some of these notes offer additional information on the real-world context of the word problems. All support for the teacher is noted in the analyses. However, a special focus is given to, and identified for, the support offered for the real-life contexts.

Validity and Reliability

Internal validity or credibility. Qualitative validity refers to the credibility of the findings of the study. Qualitative research investigates “people’s construction of reality – how they understand the world” (Creswell, 2009, p. 214). This means that there may be multiple interpretations of the same occurrence – how a person makes meaning of it and how they understand processes within the occurrence. Internal validity or credibility refers to how the findings match reality.

Since the researcher is the primary instrument of data collection and analysis in qualitative research, the interpretations of the data can be accessed through my researcher's journal (see Appendix C). I have also reflected on my position as the researcher. In my journal, I explain how my biases and disposition may affect my interaction with the data. Reflectivity is a core component of qualitative research (Creswell, 2009). Self-reflection creates an honest narrative and clarifies any biases I may bring to this research (Creswell, 2009). I present all of my interpretations of the word problems, even if they run counter to themes in categories. Presenting discrepancies and negative information provides for reporting bias which is a possible shortcoming of the directed content analysis method (Hsieh & Shannon, 2005). Different perspectives exist in all situations, and a discussion of negative or discrepant information can add to the credibility of the study. Furthermore, this type of information also spurs new categories or codes to emerge.

In a content analysis, the recommended strategies to ensure validity are: using rich, thick description to communicate the findings; clarifying any biases of the researcher; spending prolonged time with the data; and presenting any negative information or discrepancies in the data (Creswell, 2009; Merriam, 2009). My researcher's journal assisted in addressing all four of these strategies.

I evaluated the realistic word problems in both textbooks using all the identified initial categories as well as other categories that have emerged during data collection and analysis. This provides for persistent and prolonged analysis of the word problems to cross-check and to discover prominent themes or patterns that may recur throughout the textbooks (Creswell, 2009).

I have written memos about the categories and definitions as I analyzed the data to provide confirmability. The accounts in my researcher’s journal show that the results are consistent with the data collected.

Reliability or consistency. In qualitative research, reliability refers to the consistency of the researcher’s approach with respect to other researchers and other research projects (Creswell, 2009). An audit trail describes in detail how the data were collected, how the categories were created, and how decisions were made throughout the process of collection and analysis (Merriam, 2009, p. 223). This audit trail is kept in my researcher’s journal, where I wrote my reflections and questions as I collected and analyzed the data. This running record includes documentation of decisions I needed to make about issues and ideas that arose during the analysis of each word problem.

In a content analysis, the main concern for reliability or consistency lies with coding: the definitions of the codes must remain consistent throughout the study. To ensure the reliability and replicability, I defined my categories with care and rigor and use rich, thick description in defining them in my analysis (Guba & Lincoln, 1982).

External validity or transferability. Readers of this study will be able to decide whether the findings can apply to their situations. My obligation as researcher is to provide a sufficient detailed description to make transferability possible. This is achieved through rich, thick description: “a highly descriptive, detailed presentation of the setting, and in particular, the findings of a study” (Merriam, 2009, p. 227). It includes a description of the data sources and sample, and a detailed description of the findings with sufficient evidence provided with examples from the data sources. This description can make the results richer and more realistic to the reader and adds to the validity of the study (Creswell, 2009). The journal can enhance transferability to other textbook analyses (Creswell, 2009; Guba & Lincoln, 1982; Hsieh & Shannon, 2005).

Limitations

My intention is that the method I use will be transferable, but because this study is limited to two textbooks, results are not generalizable. Additionally, the scope of the study is limited to the problems that are available to the students as exercises in the textbook. Since textbooks are developed for use in the classroom rather than for research purposes, the content of the textbook may be driven by popular demand or other needs of the publisher (Merriam, 2009). Another limitation is that my intent in this directed approach to qualitative content analysis is to concentrate on the realistic nature of word problems. Less attention is given to other meanings or intentions within the textbook.

Researcher's Position

I am a mathematics teacher with over thirty years of experience teaching in New Jersey high schools. I have taught in several schools and settings and have experience as a committee member for curriculum writing and textbook selection. Over the years, I have made personal observations with regard to word problems and the differing treatment of word problems by various teachers. Some of the reasons that teachers have cited for their diminished use of word problems can be substantiated by the literature: the problems do not reflect situations that the teachers and students find important (Bottge &

Hasselbring, 1993; Schoenfeld, 1989); the method for solving is prescribed by appearing in the section with a particular mathematical topic (Balacheff, 1986; Bruer, 1993; Verschaffel, Greer, & DeCorte, 2000); and the word problems are seen as unimportant by their placement in the textbook (Balacheff, 1986; Freudenthal, 1968; Moschkovich, 2002; Verschaffel, Greer, & DeCorte, 2000).

On the other hand, my students have often questioned me about the word problems in their textbooks: “Why are there so many word problems in this section, but none in previous sections?”; “Why is the problem labeled with this mathematical operation when I can use a different operation?”; “Why would I want to know the answer to this problem?” Their comments on word problems often lead to discussions on the realistic aspects or the real-world contexts of the problems: “I can’t fit a table on a thirty-square-foot deck let alone fit chairs to go around it!” or “We can’t do this problem if the hypotenuse is shorter than the leg of this right triangle.”

Additionally, I recently encountered an enthusiastic student who shared with the class how he used the mathematics he learned in school to complete a small construction project. He brought in pictures that showed the work in progress, how measurements were taken, and the finished product. He correctly described how the Pythagorean Theorem could be used to ensure that the corners were constructed as right angles. Because he is not adept at written assessments, his sharing of the story was one of the first indications that he understood the geometry he was learning in my classroom.

It is my belief that students need to learn the mathematics that they will use to be successful in their everyday lives, including consumer and workplace mathematics.

Students need to be engaged in learning, and they seem to thrive on familiar contexts when solving word problems. Some problems may require some background information in order for students to be more comfortable with the context. I believe that this study can contribute to the effort to re-negotiate the existing didactical contract in the classroom and encourage students to consider real circumstances when solving any problem.

Chapter 4: Results

In this chapter, I present the results of the numerical analysis of the word problems through my categorization and coding. I coded 208 problems in the standards-driven book and 236 in the discovery book, for a total of 444 problems (see Table 3). Throughout this chapter, the code for each category or subcategory is provided in parentheses. Following the counts of occurrences of the codes in the numerical analysis, I present my interpretative analyses of a subset of the word problems.

Numerical Analysis

It is important to note that the sample drawn from the two geometry textbooks represents a small number of the total Exercises in each book. Each book contains over 850 pages, and several thousand Exercises. These statistics are significant when considering the word problems in the sample and their presentation with regard to their *realistic-ness*. The results are representative of the problems chosen for the sample, and not of the textbooks in their entirety.

In choosing problems for the sample, the first round of selection gathered all of the problems that had some connection to a real-life context. This group of problems was then evaluated and problems were eliminated from the count if the real-life context was not essential to a problem’s solution. The eliminated problems are those that simply mention a real-life connection to the mathematics in the problem or problems with a pre-determined equation, as well as those with an algebraic or geometric structure superimposed over a real-life context. The resulting sample consists of 444 problems from the two textbooks.

Table 3.

Real-world Word Problems v. Exercises by Book

	Discovery	Standards-driven	Full Sample
Real-world problems in sample	236	208	444
Exercises	2176	3795	5971
Pages in student edition	858	926	1784

Label. The category of *label* refers to how the problem is labeled: by its real-life context or by a preferred mathematical content. A label appears explicitly alongside the word problem and the books' language is used to record the label. A past concern with word problems was that problems were often labeled with a mathematical content – an algorithm or theorem to be applied in solving the word problem (Balacheff, 1986; Bruer, 1993; Hiebert, et al., 2003; Verschaffel, Greer, & DeCorte, 2000). For instance, the label Pythagorean Theorem, could direct the solver to use that specific theorem in solving the problem, even if using the ratios of special right triangles or trigonometry could be an option.

In this study, I found that each book took a different tack to labeling the problems. Overall, the discovery book was less likely to label the problems, but sometimes the label was simply "Application" (A). Of the 236 problems in the sample from the discovery book, only 74 were labeled (see Table 4). All but two of the 74 were labeled simply as "application"; the remaining two were labeled as "construction", as in geometric construction, meaning that the students should use a straightedge and compass to solve the problem.

On the other hand, the standards-driven book labeled a majority of the word problems in the sample, 145 out of 208, with a real-world context or their connection to another academic subject (RWC). Most of these labels gave no clue on to how to solve the problem. The labels included contexts like Aviation, Carpentry, Graphic Design, Chemistry, and Space Exploration. Eleven more were labeled with mathematics content. Those problems named with mathematics content were also classified in general terms: nine were called “estimation” problems, one was “measurement”, and the last was “probability”. None of these labels prescribes a theorem, algorithm, or content to be applied other than the probability problem, which refers to a previous section on geometric probability to compare areas of a target. Of the remaining labeled problems, eighteen were labeled somewhat generically – three were called “critical thinking” (CT), five were “write about it” (WAI), five were called “multi-step” (MS) problems, five were “short-response” (SR).

Overall, very few problems in these two textbooks are labeled in a way that prescribes mathematical content to be applied in the solution of the problems. Out of the 444 problems in this analysis, only thirteen were found to be labeled directly with a mathematical content (M) to be applied. Additionally, when a problem is labeled as an application, that problem is meant to “help students practice newly acquired skills in a real-world context” (Serra, 2008, p. xxi). The students are expected to apply the mathematical content from the lesson. This leads to the next category: Alternative Approaches and Location.

Table 4.

Labels of Real-world Word Problems by Code

	Discovery		Standards-driven		Total	
	N = 236		N = 208		N = 444	
Code	n	%	n	%	n	%
A	72	30.5%			72	16.2%
CT			3	1.4%	3	0.7%
M	2	<0.1%	11	5.3%	13	2.9%
MS			5	2.4%	5	1.1%
N	162	68.6%	34	16.3%	196	44.1%
RWC			145	69.7%	145	32.7%
SR			5	2.4%	5	1.1%
WAI			5	2.4%	5	1.1%

Alternative Approaches and Location. This category is used to determine whether a problem offers opportunities to use a variety of *alternative approaches* in its solution, rather than a dictated or expected approach. It had been noted that textbook word problems are often placed in the chapter or section which covers the particular mathematical content needed or intended to be used to solve the problem (Balacheff, 1986; Bruer, 1993; Verschaffel, Greer, & DeCorte, 2000) and that this arrangement may direct students to use that content, removing their accountability and flexibility in decision-making (Hiebert, et al., 2003).

Of the 444 problems in the sample, 375 were located (LOC) in the section that first covered the mathematical content intended to be used in their solution: 207 in the standards-driven book and 168 in the discovery book (see Table 5). This means that only one problem in the sample from the standards-driven book is placed elsewhere, possibly giving the students some decision-making power (DMS), as opposed to 68 problems appearing elsewhere in the discovery book. It is important to note that 51 of those 68 problems are review problems, referring to previously learned mathematics topics from earlier chapters and sections.

Table 5.

Location of Real-world Word Problems by Code

	Discovery		Standards-driven		Total	
	N= 236		N = 208		N = 444	
Code	n	%	n	%	n	%
LOC	168	71.2%	207	99.5%	375	84.5%
DMS	68	28.8%	1	0.5%	69	15.5%

Variation. Many problems seem to follow a template for word problems – this has been described as a three-component structure (Gerofsky, 2004b). The template includes a “set-up” – the beginning of the story problem, possibly giving the context. Next, the “information” is presented – the numbers or data needed to solve the problem. The third component is the “question” to be answered (3CS).

One problem from the sample that conforms to this template is the TV Screen Diagonal Problem: “Television and computer screens are usually advertised based on the lengths of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch” (Burger, et al., 2012, p. 48). The “set-up” in this task is that television and computer screens are advertised based on their diagonals; the “information” is that the height is 11 in. and the width is 14 in.; the “question” asks for the length of the diagonal.

An example of a problem from the sample that varies (VAR) from this structure is: “To make sure that a room is rectangular, builders check the two diagonals of the room. Explain what they check about the diagonals and why this works” (Serra, 2008, p. 295). This problem offers no numerical data and there is no calculation to be performed, but students are prompted to make a connection between the properties of special parallelograms and a job-related context.

In my analyses, I found that the numbers of problems in the sample that conform to the three-component structure are 89 in the discovery book and 117 in the standards-driven book (see Table 6). That means that 147 and 91 problems, respectively, did not conform to the template. The totals for the two books are: 206 which conform to the template and 238 which do not.

Table 6.

Variation of Real-world Word Problems by Code

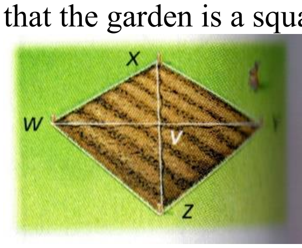
	Discovery		Standards-driven		Total	
	N = 236		N = 208		N = 444	
Code	n	%	n	%	n	%
3CS	89	37.7%	117	56.3%	206	46.4%
VAR	147	62.3%	91	43.8%	238	53.6%

Realistic Considerations. Wording problems so that they are true to real-world events is likely to encourage students to use realistic considerations rather than ignore them (Bonotto, 2013; Cooper & Harries, 2002; Green & Emerson, 2010; Palm, 2008; Verschaffel, DeCorte, & Lasure, 1994). Some problems in the sample are presented with various aspects which may promote making connections between school mathematics and the real-world context, while other aspects may hinder those same connections.

Text of the problem uses mathematics content. Some problems were introduced with real-life context, but then described in terms of mathematical concepts and terms. These concepts and terms are not usually used in everyday conversations outside of school mathematics. Some textbook problems use school mathematics content to describe the problem situation, or include diagrams which will point to the mathematical content to be used (TMC). School mathematics terms are used rather than everyday terms which are familiar to the lay person.

The Squaring the Garden problem appears with a diagram and symbols for students to reference:

Gardening. A city garden club is planting a square garden. They drive pegs into the ground at each corner and tie strings between each pair. The pegs are spaces so that $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. How can the garden club use the diagonal strings to verify that the garden is a square?



(Burger, et al., 2012, p. 434)

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I found that 31 problems in the discovery book and 30 in the standards-driven book are presented with specialized mathematical descriptions of the problem situation, for a total of 61 problems in the whole sample (see Table 7).

Problems written in a real-world context (PRW). These are problems in the sample that are constructed in a way that could have come straight from a real-world situation and been placed in the textbook; the language of the problem includes no school mathematics. These real-world situations include job-training scenarios, career-related problems, and everyday situations for consumers and families. This aforementioned problem serves as an example: “To make sure that a room is rectangular, builders check the two diagonals of the room. Explain what they check about the diagonals and why this works” (Serra, 2008, p. 295). I found that 360 of the problems in the sample fit this description: 207 in the discovery book and 153 in the standards-driven book (see Table 7).

Question about Mathematics (QM). Problems which ask questions about the school mathematics content rather than the real-life situation were coded as QM. These problems seem to leave the real-life situation behind and focus on the mathematics. The following problem asks about the area of a pizza and does not associate it with a real-life need to know the area: “A pizza parlor offers pizza with diameters of 8 in., 10 in., and 12 in. Find the area of each size pizza. Round to the nearest tenth” (Burger, et al., 2012, p. 691). In the sample, I found that 92 of the problems asked about the mathematics rather than the real-world context – 39 in the discovery book and 53 in the standards-driven book (see Table 7).

Problems that describe their usefulness (USE). These are problems that describe their usefulness explicitly in the text of the problem. They are more likely to engage students because the problem provides a reason to know the solution; there is no need for the students to ask why they need to solve it or if there exists a purpose for it in the real world. For example, this problem explains that a television's screen size is based on the length of its diagonal: "Television and computer screens are usually advertised based on the lengths of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch" (Burger, et al., 2012, p. 48). Of the 444 problems in the sample, 155 of them explain why the problem is useful: 98 in the discovery book and 57 in the standards-driven book (see Table 7).

Most realistic word problems. I used the last two subcategories, problems written in a real-world context (PRW) and problems that describe their usefulness (USE), to classify the "most realistic" problems in the sample. These are the problems that seem most likely to be stated in everyday routines or careers in the same manner as they appear in the textbooks. These problems do not include specialized mathematics terminology, and they inform the solver of the usefulness of the problem's solution. The reason for solving is declared within the statement of the problem. The problems I classify as "most realistic" are those in the intersection of the two codes: the problems that meet the criteria for both PRW and USE. The following entry serves as an example of these "most realistic" problems: "According to the Occupational Safety and Health Administration (OSHA), a ladder that is placed against a wall should make a 75.5° angle with the ground for optimal safety. To the nearest tenth of a foot, what is the maximum height that a 10-ft ladder can safely reach?” (Burger, et al., 2012, p. 546).

Table 7.

Realistic Considerations of Real-world Word Problems by Code

	Discovery		Standards-driven		Total	
	N = 236		N = 208		N = 444	
Code	n	%	n	%	n	%
TMC	31	13.1%	30	14.4%	61	13.7%
QM	39	16.5%	53	25.5%	92	20.7%
PRW	207	87.7%	153	73.6%	360	81.1%
USE	98	41.5%	57	27.4%	155	34.9%
PRW/USE	86	36.4%	38	18.3%	124	27.9%

In the entire sample, there are 124 such problems, 86 in the discovery book and 38 in the standards-driven book.

Support. The category of *support* is used to signify whether the teacher’s edition (TE) furnishes explanations to the teacher for the word problem. This support may come in the form of a side note, or a detailed possible solution. Often, a teacher’s edition (TE) may only provide a single numerical solution or the statement “Answers may vary” (NSUP). For this category, “support” is characterized as any information about the real-world context, hints for how to solve the problem, or an articulated possible solution (SUP). From the total sample of 444 problems, 198 of them meet the criteria of support for teachers, 144 from the discovery book and 54 from the standards-driven book (see Table 8).

Of the problems which offer support for teachers, only 61 provide details or information about the real-life context (R) beyond the statement of the word problem. A problem which meets this criterion is:

> A cord of firewood is 128 cubic feet. Margaretta has three storage boxes for firewood that each measure 2 feet by 3 feet by 4 feet. Does she have enough space to order a full cord of firewood? A half cord? A quarter cord? Explain. (Serra, 2008, p. 534).

The answer provided in the TE goes beyond the simple response: “Margaretta has room for 0.5625 cord. She should order a half cord” (Serra, 2008, p. 534). In addition, the TE provides a side note for this problem: “A box is a right rectangular prism. A cord of firewood is measured by rectangular dimensions, typically 8 ft by 4 ft by 4 ft. Gaps are not considered” (Serra, 2008, p. 534).

In the standards-driven book, ten of the problems include support for the real-world context (R), while in the discovery book, fifty-one of the problems do.

Table 8

Support for Teachers for Real-world Word Problems by Code

	Discovery		Standards-driven		Total	
	N = 236		N = 208		N = 444	
Code	n	%	n	%	n	%
SUP	144	61.0%	54	26.0%	198	44.6%
R (subcategory of SUP)	51	21.6%	10	4.8%	61	13.7%
NSUP	92	39.0%	154	74.0%	246	55.4%

Summary. Of the 444 problems in the entire sample, 84.5% of the problems are located in the same section with the mathematics content to be used in its solution (LOC). However, more than half, or 53.6%, vary from the three-component structure (VAR) in their presentation. Only 20.7% ask questions about the mathematics (QM) rather than the real-world context, and only 13.7% use technical school mathematics in the text of the problem (TMC).

Surprisingly, 81.1% of the problems are written using their real-life contexts as their basis rather than the school mathematics of the textbook section (PRW), although only 34.9% express the purpose of finding a solution (USE). Further, it is interesting to note that 27.9% of the 444 problems in the sample fit the profile of "most realistic", classified by both codes: PRW and USE.

With regard to supports available in the Teacher's Edition, this study shows that less than half of the problems (44.6%) offer any advice or assistance for the teacher, and even less (13.7%) support for the real-world context. There is a disparity in the support in the two books: 61.0% of the problems in the discovery book provided support versus 26.4% in the standards-driven book.

Using the percentages for the categories as a guide, the discovery book appears to provide more realistic word problems, especially in the categories of Location, Variation, Realistic Considerations, and Support.

Qualitative content analysis of real-world context problems

In addition to categorizing and coding each of the problems in the sample, I analyzed a selection of problems by solving them and delving into the problem situations as well as the intended solutions. I found several problems contained similar features or aspects that invited detailed analyses of the problems using the categories and subcategories from the coding and numerical analysis, as well as further consideration of the *realistic-ness* of the problems. I present the problems and these analyses in the upcoming sections.

In this qualitative content analysis, I detail the process I use to analyze each of the textbook word problems presented. I include my reasoning, and my ideas and reactions as a practitioner using the textbooks, following the tradition of Schoenfeld’s (1983) problem analyses and discussions. I have also taken into account the approaches that students and other teachers might use for the problems.

In the process of solving the problems and delving into their real-life connections, I found instances where the problems are not written using realistic quantities, where the dimensions given do not promote making connections between school mathematics and the real-life situation. These problems are presented in ways that are not entirely realistic.

I also found problems that need more information to solve them in order to satisfy requirements of the real-world situation. The solutions as offered in the textbooks may satisfy the needs of school mathematics, but a little investigation can uncover realistic aspects that need to be considered if the problem were presented in the real-life context of the workplace or an everyday situation.

Additionally, in this section I present two problems involving quilt-making, with one having more detailed steps than the other for creating a quilt. These two contrasting problems help to illuminate the differences that exist in problems regarding similar contexts, and the manner in which the differences can affect not only the outcome of the solution, but also the extent to which realistic aspects of the problem are considered in the solution.

I also analyze a problem which appears to have a pedagogical purpose: that students should be encouraged to draw diagrams to assist in the visualization of the problem situation. Nevertheless, the purpose is undermined by the configuration of the diagram itself. A slightly different problem set-up would have met the intended pedagogical purpose.

The final problem in this section differs from many word problems in secondary school textbooks: it requests an optimal solution for seating the maximum number of people at a table. The problem breaks from tradition by challenging students to state their opinion and use their own reasoning for the realistic situation.

Problems for which the presentation is not entirely realistic. This first pair of problems I describe are two in which I found that the presentation of the problem was not entirely realistic. In each one, the problem statement may seem straightforward, but the statement comes up short in making the connection to the real-life context. I present each problem, followed by my solution and analysis. A discussion of their shortcomings follows.

The Pet Iguana Pen Problem.

18. *Application*. Ernesto plans to build a pen for his pet iguana. What is the area of the largest rectangular pen that he can make with 100 meters of fencing? (Serra, 2008, p. 426)

My solution for the Pet Iguana Pen Problem:

As a result of my experiences with rectangles and area, I know that the rectangle with the largest possible area will be a square. Others might try different combinations for the length and width of the rectangle, or use an algebraic equation to solve for the maximum lengths possible, but the largest area would be accomplished by using a square.

100 meters of fencing would allow for a square with 25 meters on each side.

The largest area would be:

$A = 25^2$

A = 625 square meters

The textbook solution to this problem is: "For a constant perimeter, area is maximized by a square. 100 m ÷ 4 = 25 m per side.; A = 625 m^2" (Serra, 2008, p. 426). In addition, the side notes for the teacher entitled "Helping with the Exercises" include this advice: "Encourage students to try various bases and heights and to look for a pattern in those dimensions that give the largest area and those that give the smallest, or to graph base versus area. If students graph base versus area, they might notice that the function appears quadratic" (Serra, 2008, p. 427). Both suggestions would produce valid solutions.

Analysis. The problem fits the three-component structure (3CS) of many word problems: the "set-up": Ernesto is planning to build an enclosure for his iguana; the "information": he has 100 meters of fencing; and the "question": what is the largest area he can enclose with the fencing.

This problem is labeled as an application (A). Although the wording of the question within the problem asks, "what is the area", the label itself gives no direction as to what mathematical content should be applied. Perhaps the label of application refers to applying this geometry concept of area to the algebra of quadratic functions and extrema.

It is located in a section titled "Areas of Rectangles and Parallelograms" (LOC). Students would need to remember that a square fits the criteria of a rectangle; this information was presented much earlier in the textbook. This section does not mention squares at all, or their properties in relationship to those of rectangles and parallelograms.

The side notes feature, "Helping with the Exercises" gives support (SUP) for the teacher on this particular problem. Those who have experience in solving maximum area problems usually are aware that, if the perimeter is to remain constant, the area is maximized by enclosing a square. High school geometry students may not have that kind of experience. The textbook's support for teachers in the "Helping with the Exercises" side notes offer an alternative, suggesting that students look for patterns as they guess-and-check or to link this problem to quadratic functions. This also creates an avenue for teachers to accept other approaches that the students may explore.

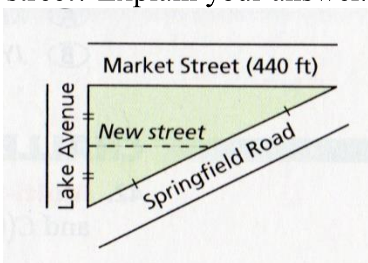
The problem is written as if the real-world context should be considered (PRW). Its set-up does not include school mathematics vocabulary; the question about the area could be classified as being about the mathematics rather than the real-world context. However, it can be argued that knowing the square footage of a habitat, whether for humans or for animals, is part of the culture of the real world. In addition, this problem directly states the usefulness of completing this task: to discover the largest pen that can be created from the given length of fence (USE).

Despite the fact that the problem is written in a way that encourages students to consider the real-world context, this problem is unrealistic in the size of the pen that results from the data provided. Twenty-five meters is approximately equal to eighty-two feet, which would make the iguana pen larger than many homes’ backyards. The recommended size of a cage for an iguana is “at least 12 feet long by 6 feet wide by 6 feet high” (Crutchfield, 2017). The size that results from the calculations in the problem statement would be more reasonable for an iguana habitat at a zoo, rather than a pet enclosure at home.

Additionally, this problem does not mention openings or other features of the iguana pen. Any enclosure for a pet would need a gate or doorway, yet this realistic element has not been mentioned here, in the statement of the problem or in its solution. These are elements of the real-world situation which could be considered in the context of this problem.

The Parallel Parking Space Problem.

29. *Estimation*. The diagram shows the sketch for a new street. Parallel parking spaces will be painted on both sides of the street. Each parallel parking space is 23 feet long. About how many parking spaces can the city accommodate on both sides of the new street? Explain your answer.



(Burger, et al., 2012, p. 337)

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My solution for the Parallel Parking Space Problem:

The length of Market Street is given in the diagram as 440 feet, although it may be difficult to determine where the 440 feet was measured. Since this problem is in the section on the midsegment of a triangle, the 440 feet probably marks the length of the side of the green triangular region between the existing streets. Therefore, the new street will be approximately 220 ft. long. The new street is the midsegment of the triangle formed by Lake Avenue, Market Street, and Springfield Road. The midsegment is parallel to the side of the triangle it is not bisecting, or Market Street. It is half the length of Market Street.

To find the number of parking spaces for one side of the new street, I divide the length of the new street by the length of each parking space.

220 = length of the new street

23 = length of each parallel parking space

$220 \div 23 \approx 9.56$

The spaces would need to be full spaces, so there would be 9 parallel parking spaces on each side of the street, for a total of 18 new parking spaces. The label of estimation for this problem was unclear, other than the question which asks, “about how many parking spaces can the city accommodate on both sides of the new street?” The teacher’s edition (TE) notes:

> Possible answer: about 18 parking spaces; the new street is along the midsegment of the triangular plot of land. The length of the street is half of 440 ft, or 220 ft. Estimate the quotient 220 ÷ 23 by rounding 220 to 225 and 23 to 25. Since 225 ÷ 25 = 9, the city can put about 9 parking spaces on 1 side of the street. So, the total number of parking spaces is about 2(9), or 18. (Burger, et al., 2012, p. 337)

I obtained the same numerical answer, forgoing estimation and using the straightforward calculations; I then rounded the calculated result to full parking spaces that could be accommodated on both sides of the new street.

It is important to note that the three streets that comprise the triangle are two-dimensional in the diagram, showing both width and length, but that the proposed new street is shown as a single dotted line without width.

Analysis. Since the problem is labeled as Estimation (M), the exact dimensions of the proposed street may carry less importance to the solver. This label does not prescribe a mathematical formula or theorem to be applied in this situation; however, its location (LOC) in the section whose lesson is “The Triangle Midsegment Theorem” does just that. Features of the problem that may encourage student choice are the estimation label and the one-dimensional dotted line for the proposed placement of the new street. Although the line is meant to be the midsegment and bisect both Lake Avenue and Springfield Road, it may be unclear whether the midsegment is the center line of the proposed road or if it is meant to be one of the sides. This may be where the estimation comes in.

The problem does not conform to the 3-component structure (VAR). It varies from that structure in that the map diagram shows some of the essential information necessary to answer the question.

This topic provides a connection to high school students in driver’s education, and also a thought-provoking beginner’s lesson in city planning. The problem directly states its usefulness (USE) in its description – the city needs to know how many parking spaces can be accommodated. Twenty-three feet is a reasonable length for a parallel parking space and the width should be about nine feet (New York City Board of Standards and Appeals, 2016). The width of the roadway including the parallel parking spaces on either side of the street could affect the lengths of the street’s sides that are available for parking spaces. The problem’s information is inadequate to consider these options in the calculations.

I found the presentation of the problem to be lacking in other aspects of *realistic-ness*: the map could be reconstructed with a two-dimensional depiction of the proposed roadway, so that the realistic aspects of the situation would be more likely to be taken into consideration. Then, the midpoints of the existing streets would have to be placed by the problem-solver, at their discretion. The location of the midpoints could be a factor of the problem that the solver would need to justify. Another factor to consider would involve the proximity to the intersections and possible fire hydrants and other obstacles like driveway entrances. These factors would need to be addressed had this been a proposal by the city planning board.

Summary. Both the Pet Iguana Pen Problem and the Parallel Parking Space Problem are missing essential elements of *realistic-ness* in their presentation which are essential to the situations in real life. The Parallel Parking Space Problem included a diagram with a one-dimensional street, although the other streets on the diagram were two-dimensional. The solution as given in the TE is a school mathematics solution: it addresses the Triangle Midsegment Theorem. As a real-life problem, the situation is described as a problem that could truly be considered by city planners, but the presentation of this problem falls short in *realistic-ness* of the map diagram. The authors chose an emphasis on the school mathematics topic in favor of the realistic features of the map or schematic of road to be constructed.

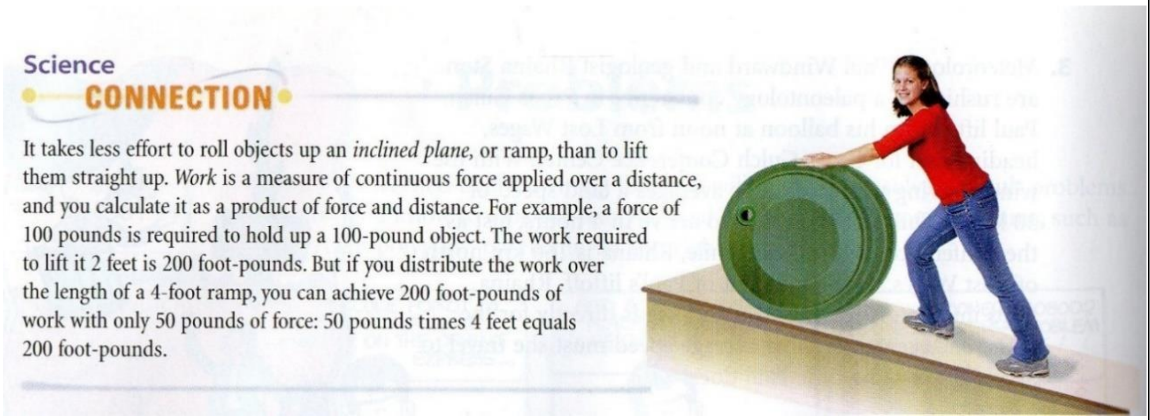
The iguana pen dimensions turn out to be much larger than anyone would expect for a pet iguana to be kept at home. Because the United States still measures in customary units rather than metric units, high school students may not be aware that a 25-by-25-meter square can actually be larger than their own home. In addition, the Pet Iguana Pen Problem does not require solvers to consider how large the square with the maximum area is in actuality, or that there are other elements missing from the pen: a gate or the minimum required height of the pen.

Given the notes in the teacher's edition, it seems that the emphasis of the Pet Iguana Pen Problem is on discovering that the maximum area would be accomplished with a square. From the perspective of *realistic-ness*, high school students should be aware of reasonable sizes for length, area and volume, and they should be given opportunities to investigate size through opportunities to convert from metric to U.S. units. Without textbook support to direct them, teachers may perceive an investigation of the size to be unimportant. Teachers may not know how large iguanas can become, or how large the habitat should be to contain one.

Further Investigation Required to Ensure Realistic-ness. The next two problems contain connections to construction and planning. Both furnish more details in the set-up than the problems in the previous section, creating the impression that all information is sufficient and accurate. Each problem appears to provide enough realistic information, but both result in solutions which are not practical, that is, it is not possible to apply the solutions to the problem's real-life situations.

The ADA Wheelchair Problem.

8. *Application.* According to the Americans with Disabilities Act, the slope of a wheelchair ramp must be no greater than $\frac{1}{12}$. What is the length of a ramp needed to gain a height of 4 feet? Read the Science Connection … and then figure out how much constant force is required to go up the ramp if a person and a wheelchair together weigh 200 pounds (Serra, 2008, p. 499).



Science
CONNECTION

It takes less effort to roll objects up an *inclined plane,* or ramp, than to lift them straight up. *Work* is a measure of continuous force applied over a distance, and you calculate it as a product of force and distance. For example, a force of 100 pounds is required to hold up a 100-pound object. The work required to lift it 2 feet is 200 foot-pounds. But if you distribute the work over the length of a 4-foot ramp, you can achieve 200 foot-pounds of work with only 50 pounds of force: 50 pounds times 4 feet equals 200 foot-pounds.

From *Discovering Geometry: An Investigative Approach* by Michael Serra. Reprinted by permission of Kendall Hunt Publishing Company.

My solution to the ADA Wheelchair Problem:

Note: I offer one solution here. Since the slope of the ramp can be less than $\frac{1}{12}$, there are many possibilities for the length of the ramp. My solution uses the slope of $\frac{1}{12}$ to minimize the length of the ramp. Minimizing the length may minimize cost, which is often a concern in construction projects.

Using the slope of $\frac{1}{12}$, I calculated the run of the ramp to be 48 ft.

$$\frac{1}{12} = \frac{4}{x}$$

$x = 48$

This solution will be using the maximum slope, although other slopes less than $\frac{1}{12}$ are also acceptable (see Figure 4.)

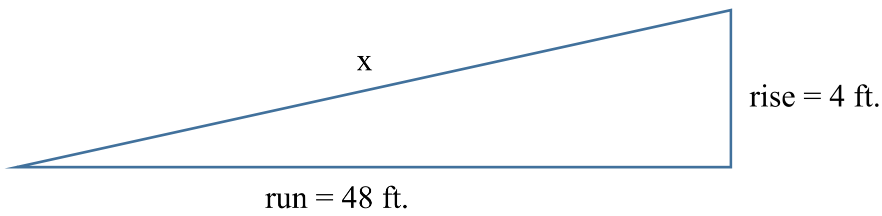


Figure 4. Ramp with maximum slope allowed.

Using the Pythagorean Theorem:

$a^2 + b^2 = c^2$

$48^2 + 4^2 = x^2$

$2304 + 16 = x^2$

$2320 = x^2$

$x = 48.2$ ft.

For the second question, regarding the constant force need to move the wheelchair up the ramp:

Work = force x distance

4 ft. x 200 lbs. = f x 48.2 ft

800 ft.lbs. = f x 48.2 ft

f = 16.6 lbs.

This force represents the constant force to move the wheelchair up the 48.2 ft ramp. A force of 16.6 pounds is required to hold up a 16.6-pound object.

These answers match the teacher's edition solutions: "48.2 ft; 16.6 lb".

The slope of a wheelchair ramp as proposed in the problem matches ADA guidelines (2010 ADA Standards for Accessible Design, 2010). However, ADA guidelines also restrict the rise of any single ramp run to a maximum of 30 inches (see Figure 5).

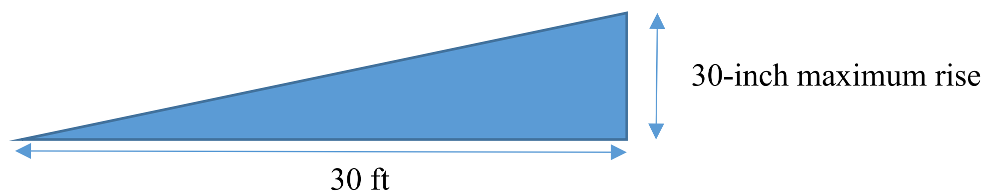


Figure 5. Ramp with maximum rise in a single run.

The rise of 48 inches, exceeds the maximum and therefore, cannot be completed in a single continuous incline. One way to accommodate these realistic considerations (as necessitated by the 2010 regulations), as well as the 48-inch rise in the problem, is to build two ramp runs and a landing. The minimum length for a landing is sixty inches, according to ADA standards. This second solution (see Figure 6) shows a straight run, only one possible configuration for the two ramps and landing.

In this configuration, using the slope ratio of $\frac{1}{12}$, each inclined portion requires a length of 24 feet and the landing requires 5 feet, for a total run length of 53 feet.

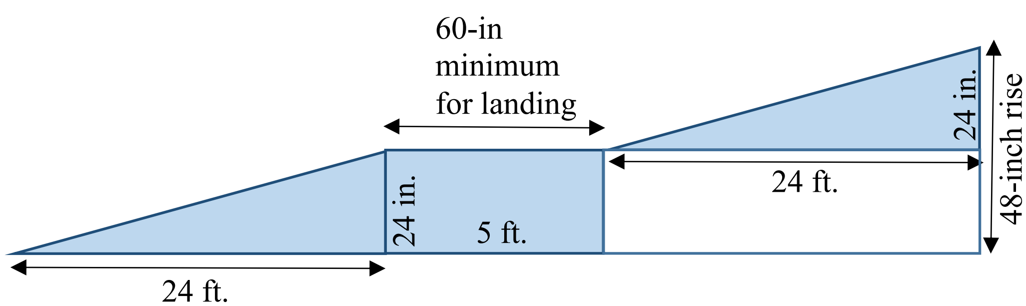


Figure 6. Ramp with required 60-inch landing for a rise above the 30-inch maximum.

Using the Pythagorean Theorem:

$a^2 + b^2 = c^2$

$24^2 + 4^2 = x^2$

$576 + 16 = x^2$

$592 = x^2$

$x = 24.3$ ft.

Each inclined portion has a ramp length of 24.3 feet.

To calculate the force needed to move a wheelchair up the ramp:

Work = force x distance

2 ft. x 200 lbs. = f x 24.3 ft

400 ft.lbs. = f x 24.3 ft

$f = 16.5$ lbs.

The force needed to move the wheelchair up the ramp decreases slightly for each inclined portion.

The solution shown above is just one configuration for a wheelchair ramp in this situation. Depending on space available, the ramp may need to be built with a turn or two included.

It is unclear why the students are asked to find the constant force needed to move the wheelchair up the ramp. There is no context given to express a need for this data.

Analysis. The problem is labeled as an Application (A). It is located in a section of "Story Problems" which begins by reminding students to draw diagrams and "to look for any special relationships in your diagrams, such as congruent polygons, parallel lines, and right triangles” (Serra, 2008, p. 498). No clues are given that would direct students to use particular mathematical content to solve the problem, other than this section’s inclusion in the chapter on the Pythagorean Theorem (LOC).

The problem is presented using the three-component structure: the “set-up” is the slope requirement of the ADA; the “information” is that a ramp is needed to gain a height of 4 feet; the “question”: what is the length of the ramp? (3CS). It includes a reference to a Science Connection to apply to the problem situation, although this connection seems artifical. The students are asked a follow-up question beyond the mathematical content, which seems to have no bearing on the problem or its real-life context. There is no reference to force guidelines in the ADA reference (2010 ADA Standards for Accessible Design, 2010).

The problem may be a bit confusing in the way that it is posed. The question asks “What is the length of ramp needed?” The length of ramp refers to the length of the sloped surface of the ramp rather than the run. This may cause students to incorrectly use the ratio of rise-to-run to calculate the run of the ramp, instead of using the Pythagorean Theorem to calculate the length of the inclined surface of the ramp. Support for teachers to handle this possible misconception is not included in the side notes in the teacher’s edition (NSUP). In fact, the textbook includes no support for the teacher at all for this problem.

The wording of the problem is only *partially* accurate in its reference to ADA guidelines. ADA regulations require a slope that does not exceed $\frac{1}{12}$. However, a look into additional regulations uncovered a stipulation that the rise of any wheelchair ramp cannot exceed 30 inches. This will restrict any single ramp run to 30 feet. If the problem had included this information, then other acceptable solutions for the ramp configuration could be considered. This could, in turn, engage students in decision-making and considering alternate configurations for the ramp that are less conventional from a mathematical and realistic perspective.

Ignoring the Science Connection about constant force, this problem provides its usefulness in a real-world context – a wheelchair ramp must be constructed according to the slope requirements of the ADA. All details are not given in the text of the problem; if students are encouraged to investigate the ADA regulations and guidelines, this problem can become more realistic; the task becomes more like an on-the-job project that must be designed and proposed.

The Circular Table Problem.

> 7. Application. Zach wants a circular table so that 12 chairs, each 16 inches wide, can be placed around it with at least 8 inches between chairs. What should be the diameter of the table? Will the table fit in a 12-by-14-foot dining room? Explain. (Serra, 2008, p. 343)

My solutions for the Circular Table Problem:

Option 1:

What is the largest table that would accommodate 12 chairs, allowing space for each chair as specified?

12 chairs at (16 + 8) inches each

12 (16 + 8) =

12 (24) = 288 inches

The circumference of the table would need to be at least 288 inches.

$C = \pi d$

$288 = \pi d$

$\frac{288}{\pi} = d$

$91.67 = d$

91.67 inches or 7.6 feet in diameter

A table would need to have a diameter of 7.6 feet to accommodate 12 chairs with 8 inches of space between them.

Option 2:

I considered this problem in a different way by inscribing the round table in a dodecagon (see Figure 7). This gives 12 sides with space for the 16-inch-wide chairs and eight inches between chairs. Each side of the dodecagon is 24 inches long.

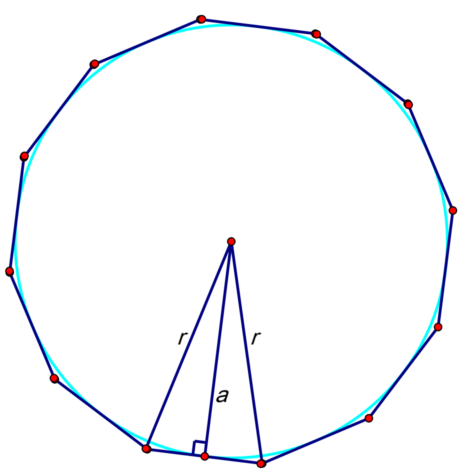


Figure 7. Round table inscribed in a dodecagon.

The apothem of the dodecagon is labeled *a* and the radius of the dodecagon (and its circumscribed circle) is *r*.

The triangle in Figure 8 is one of twelve isosceles triangles in the dodecagon.

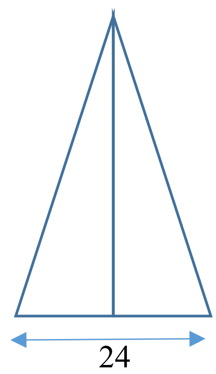


Figure 8. One of twelve congruent isosceles triangles of the dodecagon.

Each central angle of the dodecagon is 30° since 360 ÷ 12 = 30.

In one of the right triangles, that angle is bisected, so the resulting angle is 15° (see Figure 9).

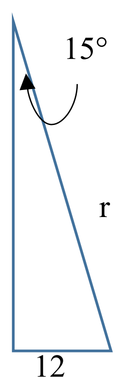


Figure 9. One-half of the isosceles triangle from Figure 8.

$$\tan 15 = \frac{12}{a}$$

$$a \tan 15 = 12$$

$$a = \frac{12}{\tan 15}$$

a = 44.78

The apothem of the dodecagon is the radius of the inscribed circle.

The diameter of the inscribed circle is: 44.78 x 2 ≈ 89.57

This means that the diameter of the table would be approximately 89.57 inches or 7.5 feet in diameter, rounded to the nearest tenth.

This option gives a slightly smaller diameter for the round table.

No guidelines were given in the statement of this problem with respect to allowances for space around the table or floor space for chairs and walking around the table. I investigated consumer information on the internet: there should be a minimum of 36 inches to allow for moving the chairs in and out and for walking around the table (Mayhugh, 2013). This means that, in addition to the 7.5-foot diameter of the table, 3 feet more is needed around the table on all sides (see Figure 10).

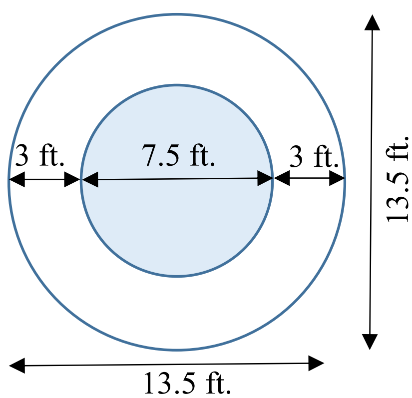


Figure 10. Table with 3 feet of space around it.

Both the length and the width of the room should be at least 13.5 feet to meet the minimum space requirements. The dimensions of the room are 12 feet by 14 feet, so a table of that size is larger than recommended for the room's size (see Figure 11). Considering that most home dining rooms also contain other furniture, this table is too large for the room. Other obstacles and doorways and walking space should also be taken into consideration.

Another point to consider is whether a 7.5-foot (or 90-inch) diameter table is available or feasible. Most ready-made round tables have a smaller diameter, between 48 to 72 inches. To expand them for more seating, rectangular leaves are inserted to make the table longer. As for round tables, I have not found any tables available that are larger than 72 inches, or six feet, in diameter. This raises questions about the answer to this

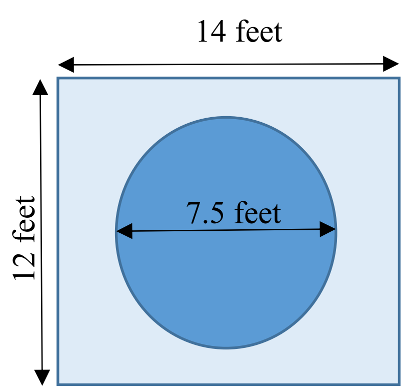


Figure 11. Table placed in 12-by-14 room.

problem. Is a 90-inch diameter table too large to be stable on a pedestal base? Other than its massive size, is there any other reason that they are not readily available? A custom table could be ordered; nevertheless, a 90-inch-diameter table is too large for the dining room with the dimensions given in the problem.

Analysis. This problem is positioned in a section titled "Around the World", which opens by declaring that the set of application problems is related to pi and involves the circumference of circles. The section immediately preceding this one was titled “The Circumference/Diameter Ratio”, in which students investigated the ratio by measuring circular objects and recording their findings to approximate a value for pi. Although the problem is labeled simply “Application” (A), its inclusion in this section does direct the students to the mathematical content to use in its solution (LOC).

The structure of this problem matches the three-component structure (3CS) (Gerofsky, 2004b). The “set-up” explains that “Zach wants a circular table”; the “information” given is that he wants to fit 12 chairs, with dimensions for the chair size and space between them; and “the question” asks how large the table should be. This problem gives all the information necessary to solve it within the guidelines given in this textbook section.

Given the strict confines of the “information” provided within the wording of the problem, and the placement of the problem in this section of the textbook (LOC), students are not invited to use other mathematical content. This problem is situated in a way that directs the students to use particular content – the only example in this section illustrates the use of the equation for circumference – and compromises responsibility for decision-making from the student.

With regard to realistic considerations, this problem is presented in a way that sounds like a conversation (PRW), even though it meets the three-component structure criteria. What should the diameter be? Will it fit in the dining room? The information given in the problem and the resulting questions, are not typical of traditional school mathematics word problems.

The answer given in the teacher's edition is: "d ≈ 7.6 ft. The table will fit, but the chairs may be a little tight in a 12-by-14 room. 12 chairs = 192 in., 12 spaces = 96 in., C=288 in, d = 91.7 in. ≈ 7.6 ft." (Serra, 2008, p. 343). Although the table is too large for the room, the problem also does not mention doorways, other obstacles or furniture that may be constraints on a table shape or size for a room. A scale drawing or floor plan of the room could have provided a more realistic view of this context.

Summary. Each of the two preceding problems seemed to provide accurate information for the solver, especially in the wheelchair problem where the ADA regulations are referenced. A closer inspection, however, reveals that the information was not quite what it seemed. This analysis suggests the value of having high school students research topics and find accurate and reliable information to be used in problem-solving. Students could include these extra elements in their solutions. Additionally, using all of the regulations and guidelines would be essential to problem-solving at home or in the workplace.

The difference that realistic considerations can make. I analyzed the next two problems in the order that they appear here. This pair of problems illustrates how differences in the realistic-ness in the problem presentation can affect rigor and expectations for problem-solving for high school students. Both problems calculate fabric area for creating quilts, but the questions are posed in ways that produce different results in the solutions of the problems.

The Quilt Triangles Problem.

13. **Crafts.** The quilt pattern includes 32 small triangles. Each has a base of 3 in. and a height of 1.5 in. Find the amount of fabric used to make the 32 triangles.

(Burger, et al., 2012, p. 38)

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My solution:

Using the information as given in the wording of the problem: each triangle has a base of 3 in. and a height of 1.5 in. However, each triangle appears to be an isosceles right triangle, resting on one of the congruent legs. Some solvers may use the legs as the base and height, but given that the base and height are different, I deduced that the hypotenuse is considered to be the base (see Figure 12). Therefore, the height is the length of the altitude to the hypotenuse.

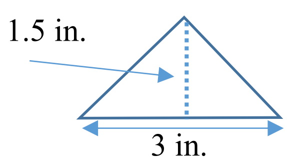


Figure 12. Right triangle rotated to rest on the hypotenuse as base.

Area of a triangle: $A = ½ bh$

$= ½ \cdot 3 \cdot 1.5$

= 2.25 square inches for each small triangle.

There are 32 triangles, so 32 x 2.25 = 72 square inches.

The problem asks for the amount of fabric used to make the 32 triangles. This solution does not appear to include seam allowances for sewing the pieces together. To calculate that amount, each piece must include at least ¼ inch of extra fabric on each edge (An accurate seam allowance, 2017). The area of 72 square inches only includes the area of the triangles on the surface of the quilt square. Including the seam allowances would add on the shaded areas on the edges of each triangle, as shown in Figure 13. The width of these thin rectangles is equal to the seam allowance, and the lengths are equal to the sides of the triangle.

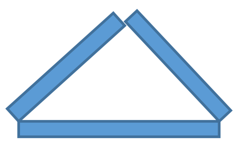


Figure 13. Quilt triangle with seam allowances shown.

For the base (or hypotenuse) the area of this rectangle is:

A = bh

= 3 (¼)

= ¾ square inch

For the other two sides of the triangle:

The lengths of the legs of the isosceles right triangle are: $\frac{3}{\sqrt{2}} \approx 2.12$

The area of the seam allowance on each of these sides is: 2.12 x .25 = .53 square inches.

The total for the triangle plus the seam allowances is:

2.25 + .75 + 2(.53) = 4.06 square inches

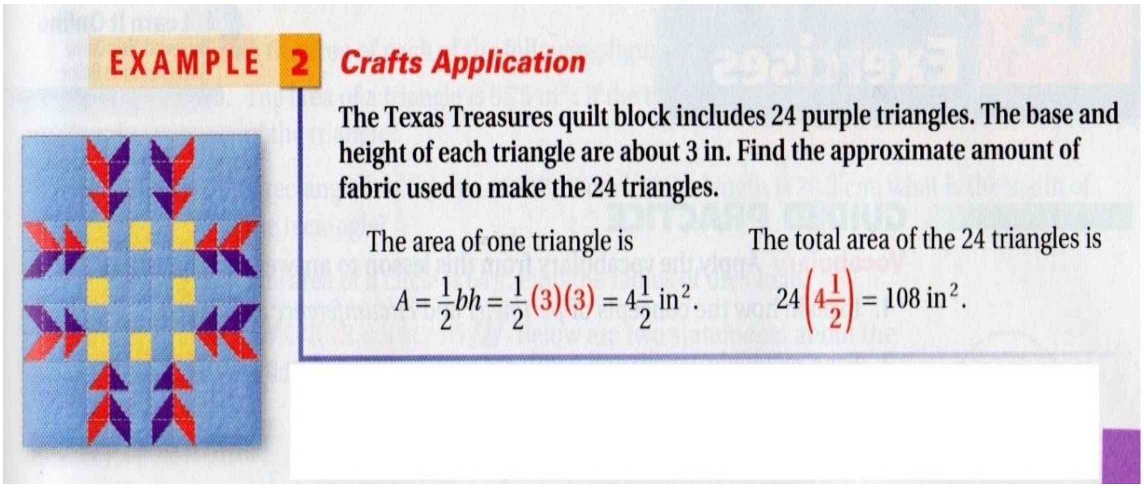
For 32 small triangles, the total area would be 4.06 x 32 = 129.92 or approximately 130 square inches.

At the very least, 130 square inches would be needed to create this quilt pattern. The TE solution states “72 square inches” (Burger, et al., 2012, p. 38). The solution would be correct if the problem used tiles or tangram pieces. However, the question asked for the amount of fabric used to make the triangles. If the task is to find the amount of fabric *shown* in the picture, 72 square inches would be a correct response.

Analysis. The problem is labeled by its real-word context of “Crafts” (RWC). It is located in the section titled “Using Formulas in Geometry” (LOC). The formulas in the lesson include perimeter and area of squares, rectangles, and triangles, as well as the circumference and area of circles. Within the lesson, a similar Crafts Application given as an example (see Figure 14) shows the calculation of the area of one triangle using the base and height measures and then multiplying by 24 for the total area of 24 triangles.

The solution in the lesson example does not refer to the real-world context, nor does it model the connection between the context and mathematics throughout the problem. The solution to the example merely states “The total area of the 24 triangles is 24(4 ½) = 108 in^2” (Burger, et al., 2012, p. 37). If a student were to refer to this lesson example, he would be inclined to use that model to determine the solution to the assigned problem. This example problem asks for the amount of fabric used to make the triangles;

yet the solution given is the total area of the triangles. The problem asks about the real-life context, but the textbook's acceptable solution relates only to the school mathematics.


EXAMPLE 2 ***Crafts Application***

The Texas Treasures quilt block includes 24 purple triangles. The base and height of each triangle are about 3 in. Find the approximate amount of fabric used to make the 24 triangles.

The area of one triangle is

$A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4\frac{1}{2}\text{ in}^2.$

The total area of the 24 triangles is

$24\left(4\frac{1}{2}\right) = 108\text{ in}^2.$

Figure 14. Crafts application example (Burger, et al., 2012, p. 37).
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A picture of a quilt square accompanies the Quilt Triangles Problem, but the picture does not enhance the problem in any way. Students can easily answer this question without using the diagram since all of the information is given within the problem's text. Students may consider the two legs of the triangle as the base and height, and assume that the two quantities are equal. The picture could serve as a learning experience for students, to show that the triangle's base could be any side of the triangle, and that in this case, the hypotenuse of the right triangle is used as the base. In the real-life experience, the picture is useful for quilters so that they can visualize the pattern and how the pieces fit together.

All of the information is included in the problem's text: the "set-up": the quilt pattern includes 32 small triangles; the "information": the base and height of each triangle; the “question”: Find the amount of fabric needed. The problem’s text conforms to the three-component structure (3CS).

The text of the word problem specifies finding the areas of the 32 small triangles, and the problem is located in a section on area (LOC). Additionally, the information given in the problem does not transfer to the picture easily, so some solvers could conform to using the information as stated. Once they realize that all of the necessary information is in the problem statement, they may also realize that there is no need to refer to the picture, so they may ignore it. The lesson example also does not model making those connections, possibly setting a precedent for teacher expectations for student work.

The base and height of each triangle is given in the statement of the problem (TMC). The task requests the amount of fabric for all of the triangles, but there is no need to know the area of all of the small triangles in the real-life situation. Also, without extra fabric, no room has been left for sewing the pieces together. The reality of the situation was not used in writing this problem, so the writing does not invite realistic considerations to be used by the student.

The solution modeled in the example does not truly consider how much fabric is used to make the 32 triangles. The amount of fabric used must be more than what is seen in the picture. The lesson includes the formulas for the area of a rectangle, and adding the seam allowances is within the capabilities of high school students. This is especially relevant given that the Common Core State Standards recognize “solv(ing) real-world and mathematical problems involving area, volume and surface area of two- and threedimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) as a *seventh-grade* standard, and the current text is a high school text.

The support offered for this problem does not go beyond the lesson example for the Crafts Application. Neither the TE solution nor the example emphasize the connection to the real-world context, especially since the solutions do not communicate that connection: “The total area of the 24 triangles is 24(4 ½) = 108 in^2” (Burger, et al., 2012, p. 37).

A more realistic problem would involve areas of the triangles separated by color, since the triangles create the quilt pattern through the use of different colors. A quilter would need to know how much fabric of each color is necessary to create the pattern. If all of the triangles were yellow, for example, then the pattern would not be as interesting or as eye-catching (see Figure 15). The pattern would not need to consist of triangles of fabric, but squares (see Figure 16). This changes the conditions of the problem.

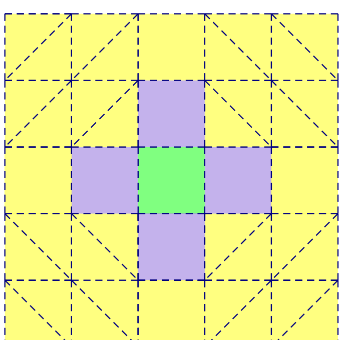


Figure 15. Quilt square with all triangles yellow.

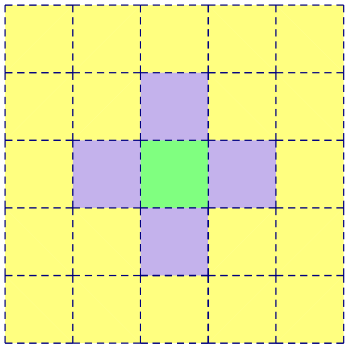


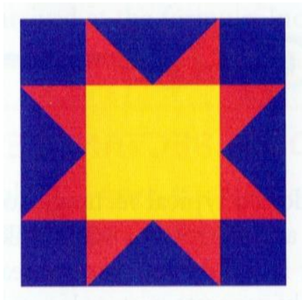
Figure 16. Quilt square with adjacent triangles shown as squares.

This problem requests calculations about the quilt top only – the decorative pattern that is shown in the diagrams. High school students, however, could be asked how much fabric of each color should be purchased to create an entire quilt comprising a dozen of these squares, perhaps to cover a bed. Fabric is available in varying widths (36, 45, 54, 60, 72, and 108 inch widths). The problem could include calculating the amount of fabric needed for the backing of the quilt, usually one fill sheet of fabric the same size as the pieced top sheet. Some quilters choose to create their own patterns; in this case, the quilter would need to complete these calculations for their own personal use. Others use a published pattern which includes this information. Quilt designers who publish their patterns would need to include this information for other crafters.

The Ohio Star Quilt Problem. The following problem appears in the discovery book:

25. *Application*. The Ohio Star is 16-square quilt design. Each block measures 12 inches by 12 inches. One block is shown above. Assume you will need an additional 20% of each fabric to allow for seams and errors.

a. Calculate the sum of the areas of all the red patches, the sum of the areas of all the blue patches, and the area of the yellow patch in a single block.
b. How many Ohio Star blocks will you need to cover an area that measures 72 inches by 84 inches, the top surface area of a king-size mattress?
c. How much fabric of each color will you need? How much fabric will you need for a 15-inch border to extend beyond the edges of the top surface of the mattress?



(Serra, 2008, p. 427)

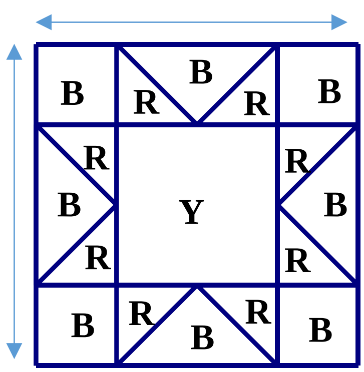


Figure 17. A re-creation of the picture provided with the problem. The colored pieces have been denoted with R for red, B for blue, and Y for yellow pieces.

My solution:

Part a. Since each side of the quilt block is 12 inches in length, and the quilt block is comprised of 16 squares equal in size to each of the corner blue squares, each blue (B) square has side lengths of three inches. Each blue triangle has a base of six inches and an altitude of 3 inches (see Figure 17).

Area of each square: $A = s^2$ where s is a side length.

$$A = 3^2 = 9$$

Each blue square has an area of 9 in^2. The total for the four blue squares is 36 in^2.

Area of each blue triangle: $A = \frac{1}{2}bh$ where b is the base and h is the altitude.

$$A = \frac{1}{2} \cdot 6 \cdot 3$$

$$A = 9$$

Each blue triangle has an area of 9 in^2. The total for all 4 of the blue triangles is 36 in^2. The total for all of the blue pieces, both squares and triangles, is 72 in^2.

There are 8 red triangles, each with a base of 3 inches and an altitude of 3 inches.

Area of each red triangle: $A = \frac{1}{2}bh$

$$A = \frac{1}{2} \cdot 3 \cdot 3$$

$$A = 4.5$$

Each red triangle has an area of 4.5 in^2. The eight red triangles' area total is 36 in^2.

The yellow square at the center of the Ohio Star pattern has a side length of 6 inches. Its area is 36 square inches.

The sum of the areas of the patches is 72 in^2 of the blue fabric, 36 in^2 of the yellow fabric, and 36 in^2 of the red fabric.

Part b. To cover the top of the king-size mattress that measures 72 inches by 84 inches, six 12-by-12 blocks would fit across to cover 72 inches and seven blocks down the length of the mattress. To cover the entire top surface of the mattress, 42 blocks would be needed (see Figure 18).

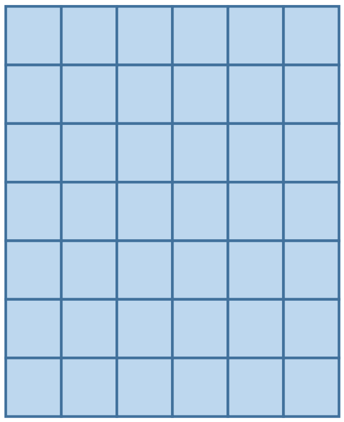


Figure 18. Squares to cover the top of a king-size mattress.

Part c. The quilt blocks covering the mattress surface now form a rectangle 72 by 84 inches long. Adding a border which is 15 inches wide would increase each edge length by 30 inches, so the quilt would now be a rectangle which measures 102 by 114 inches (see Figure 19).

The pieces of the border will be two rectangles that are 84 by 15 inches and two rectangles of 102 by 15 inches.

The areas of the rectangles would be:

102 x 15 = 1530 and 84 x 15 = 1260

There are two of each size: 2(1530) + 2(1260) = 3060 + 2520 = 5580 in.

To make the border, I need to include the extra 20% for errors and seam allowances: 20% of 5580 = 1116

5580 + 1116 = 6696 in^2.

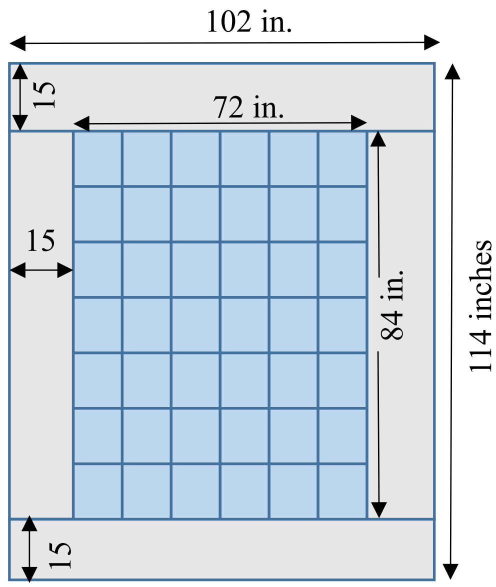


Figure 19. Quilt with 15-inch wide border.

Analysis. This Ohio Star Quilt Problem from the discovery book has some elements of realistic-ness that are lacking in the Quilt Triangles Problem from the standards-driven book. This application directs the solver to incorporate an assumption of an extra 20% of fabric for seam allowances and errors, while the standards-driven book disregards the seam allowances entirely. The problem further guides the solver to calculate how much of each fabric color is needed, and involves adding a border to complete the bedspread for a king-size bed.

The problem is labeled as an application (A); this book refers to applications as problems to “help students practice newly acquired skills in a real-world context” (Serra, 2008, p. xxi). Its location in the section on Areas of Rectangles and Parallelograms may persuade students to find a way to use those formulas (LOC) or they may choose to use the area of a triangle, which is one of the topics in the section following this one. The textbook’s lesson also discusses ways to estimate the areas of different shapes by using a square grid, which may encourage this method of counting squares rather than using formulas. In either case, the students might be using substitute calculations, but the overall approach would be to use area formulas.

This application is presented with several steps, all related to a quilting project. Each step creates a new part of the total quilt project of a comforter or bedspread. Although it is written in the format of a textbook word problem, it could easily be seen as the steps that would be found in a quilter’s pattern book. Although each part of the problem conforms to the three-component structure, the overall problem varies (VAR) from the traditional three-component structure: its structure includes adding more information with each step of the quilt-making process. Its structure can engage students in a more complex task than can many three-component structure problems, yet it is still within the capabilities of high school geometry students.

Realistic considerations are encouraged in the structure of this problem. The structure leads from one step to another, building the quilt from one square to the area of the top surface of the mattress to the border to complete the bed covering. The elements of this task appear to be realistic (PRW). You can get caught up in the planning as if you are actually engaged in making the quilt, following its step-by-step construction. After reading the entire task, the reasoning behind the calculations is evident, providing the problem-solver a purpose (USE).

The TE solution is as follows:

In one Ohio Star block, the sum of the red patches is 36 in^2, the sum of the blue patches is 72 in^2, and the yellow patch is 36 in^2.

42

About 1814 in^2 of red fabric, about 3629 in^2 of blue fabric and about 1814 in^2 of yellow fabric. The border requires 5580 in^2 (if it does not need the extra 20%). (Serra, 2008, p. 427)

This TE solution to this problem falls short in support for *realistic-ness.* First, the fabric needed for the backing of the quilt has been omitted from consideration. Second, the amount of fabric needed to complete the decorative quilt top was not converted to quantities used for purchasing fabric. Third, the batting, or filling for the quilt, was also omitted; however, batting is sold in sizes to fit standard-sized beds. A quilter need only know the size of the quilt being made to purchase the correct length and width of batting to construct a quilt. Fabric is usually sold by the yard or fractions of a yard according to the width of the fabric. In addition, even though 20% extra may be excessive, the border fabric will require a seam allowance.

The TE provides support for the teacher in its side notes (SUP). The lesson not only encourages estimation of areas by counting squares on a grid, but also suggests having graph paper available to make accurate diagrams as part of a solution. The answers for parts a and c are not as concise as for part b; the full sentence in part a and the parenthetical caveat in part c provide more depth; this may foster more support and encouragement from teacher to student.

The Ohio Star Problem uses an actual quilt pattern with a long history in the United States. At different times and places, it was known as *Tippecanoe and Tyler Too* after the presidential campaign of 1839 and 1840, and *Texas Star,* or *Lone Star,* during Texas annexation period of the mid-1800s (Allen, 2009). The Quilt Triangles Problem, on the other hand, does not name its quilt pattern.

Summary. In comparing these two quilt problems, there are significant differences in what students are expected to do. The NCTM position on high expectations states that teachers should recognize that students are “able to solve challenging mathematical tasks” (NCTM, 2016). Given that the Quilt Triangles Problem is written at a level that meets a seventh-grade standard (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), it does not meet the rigor expectations for high school students. The problem does not include seam allowances or an overage amount to allow for them, although it asks for “the amount of fabric used to make the 32 triangles” (Burger, et al., 2012, p. 38). The Ohio Star Problem, on the other hand, comes closer to meeting that expectation by including several steps of the quilt construction process and by including the 20% extra to allow for seams and possible errors. The difference between the objectives of the problems also illuminates the realistic quality of the Ohio Star Problem. Finding the total area of all thirty-two triangles, regardless of their color, is a much less rigorous task than calculating the amount of fabric by color to complete the entire quilt, including borders. Although both problems focus on the decorative top of the quilt, the Ohio Star Problem finishes only a few steps away from purchasing the fabric in a real-world situation, while the Quilt Triangles Problem does not provide a practical solution for any real-life scenario.

A Problem with a Pedagogical Purpose. This next problem is from a section called "A Picture is Worth a Thousand Words". The lesson attempts to impel students to draw diagrams to "apply visual thinking to problem solving" (Serra, 2008, p. 81). In this instance, however, diagram drawing does not appear to affect the outcome of the task.

The Perimeter Fence Problem.

4. Mary Ann is building a fence around the outer edge of a rectangular garden plot that measures 25 ft by 45 ft. She will set the posts 5 feet apart. How many posts will she need? (Serra, 2008, p. 84)

My solution: I have included a diagram to depict the rectangular garden plot (see Figure 20).

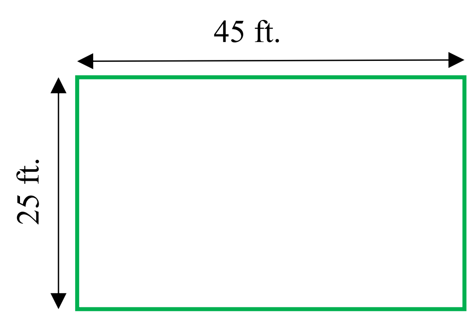


Figure 20. Rectangular garden plot.

Perimeter of the garden: $P = 2(l + w)$

$$P = 2(45 + 25)$$

$$P = 2(70) = 140 \text{ ft.}$$

To find the number of posts: divide the number of feet in the perimeter by 5 feet for the spacing: $140 \div 5 = 28$ posts (see Figure 21).

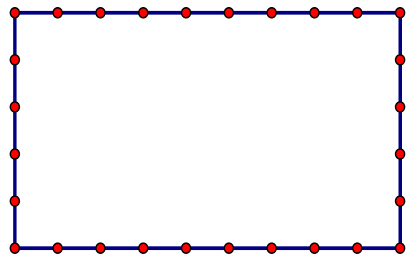


Figure 21. Rectangular garden plot with fence posts.

The directions for the set of problems state: “Read each problem, determine what you are trying to find, draw a diagram, and solve the problem” (Serra, 2008, p. 84). This problem would have a different solution if the fence were to be assembled as a single linear run.

For example, a 10 ft by 15 ft rectangle would require 10 posts.

Perimeter: P = 2(l + w)

P = 2(10 + 15)

P = 2(25)

P = 50 ft.

To find the number of posts needed: 50 ÷ 5 = 10 posts (see Figure 22).

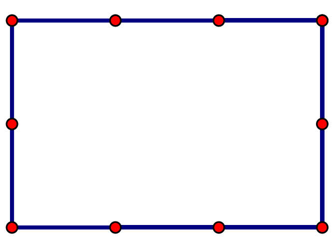


Figure 22. Rectangular garden plot with 50 feet of fencing.

However, if the problem asked about 50 linear feet in a single run, eleven posts would be required to anchor both ends of the fence (see Figure 23).

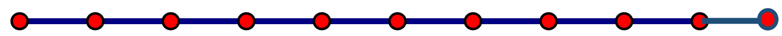


Figure 23. Linear fence with 50 feet of fencing.

Students could easily stumble upon the correct answer for the textbook problem as it is stated with the rectangular region enclosed, even without drawing a diagram. The first post needed will also anchor the last section. However, it is important for them to be able to visualize the second situation for applying this to a real-life situation. To relate this to a real-life context, there is nothing a “weekend warrior” hates more than another trip to the home improvement center because of a miscalculation.

***Analysis*.** With regard to variation, the problem is presented in the familiar three-component structure(3CS): The “set-up” is that Mary Ann is building a fence around the outer edge of a rectangular garden. The “information” gives the dimensions, and that is followed by the expected question: “How many posts will she need?”

The problem encourages realistic considerations by presenting this as a real-life situation (PRW) that can be imagined by the students (Freudenthal, 1978; Utrecht University, 2016). Its usefulness is also explicitly stated in the problem (USE). Most high school students would have experienced a construction site at least by observation. Furthermore, the size of the garden plot proposed is a manageable size as a project for students of this age group. Regardless, there is no mention of a gate or opening for this perimeter fence. At 25-by-45 feet, this is a sizeable garden plot, and likely not a small flower bed with a decorative fence border that one can step over.

The problem is located in a section of word problems titled, “A Picture is Worth a Thousand Words” (LOC). Diagrams are not provided, but are required as part of the solution. In this problem, however, the expected solution can be obtained by simply finding the perimeter and dividing by 5 for the five-foot distance between the posts. This problem does not accomplish the section's pedagogical purpose: to encourage students to draw diagrams. The problem may accomplish this pedagogical goal of drawing diagrams if the question asked about a linear run of fence rather than enclosing a rectangular region.

The problem was not labeled (N) at the onset. In fact, another wording of the problem replacing "outer edge" with "perimeter" would have directed solvers to the formula or concept to apply in this situation. For example, the problem states "Mary Ann is building a fence around the outer edge", but it could have read "Mary Ann is building a perimeter fence" or "Mary Ann is building a fence along the perimeter of the rectangular garden".

Support was provided in the TE for the group of exercises in this section, but only in general terms (NSUP). The help offered suggests that students work in pairs, with one student being the reader and the other drawing the diagram. It is suggested that this will help "students move away from thinking of the exercises only as word problems. The figures help them build the needed connections" (Serra, 2008, p. 84). These suggestions are more pedagogical in nature, and do not address the diagram or other possible configurations for the fence.

Summary. This problem was written with a purpose in mind, as evidenced by its location in a section about visualization, "A Picture is Worth a Thousand Words" and the textbook's pedagogical suggestion. This particular problem does not measure up to that purpose, being that students can easily stumble upon the correct answer to the problem without drawing a diagram. It might have been more appropriate to use the linear run of fence for the purpose of encouraging students to use diagrams. The answer may be surprising to some students, and that could inspire them to use visualization in their problem-solving schema.

A problem with an optimal solution. The following problem is one which is presented in a more realistic manner than many textbook problems. Forgoing the label and the imperative "Explain your reasoning", this task reads as a question that may be asked of a reception planner or a banquet hall manager. The question does not ask for an absolute answer, but instead, for an opinion of what may be optimal for table seatings.

The Table Choice Problem.

41. *Critical Thinking*. Which do you think would seat more people, a 4 ft by 6 ft rectangular table, or a circular table with a diameter of 6 ft? How many people would you sit at each table? Explain your reasoning. (Burger, et al., 2012, p. 693)

My solution:

I begin by providing diagrams for the two tables (see Figure 24).

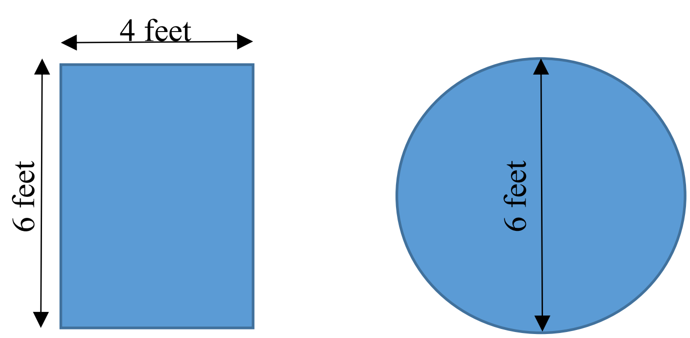


Figure 24. The two tables with dimensions as given in the problem.

The perimeter of the rectangular table:

$P = 2\ (l + w)$

$P = 2\ (6 + 4)$

$P = 20$ feet

$P = 20\ (12) = 240$ inches

Circumference of the round table:

$C = \pi d$

$C = 6\pi$

$C \approx 18.85$ feet

$C \approx 226.2$ inches

Comparing the perimeter of the rectangular table to the circumference of the round table, we can see that there is more linear "space" around the rectangular table. Since no guidelines are given in the problem for the amount of space to be allowed for each seat, I determined an arbitrary amount of space of 22 inches while sitting at a table. I measured the amount that I felt was reasonable for a comfortable seating space at a dining table. This is 4 inches wider than a standard placemat.

If you wish to give each person about 22 inches of space, at the rectangular table, the number of seats could be calculated:

Number of spaces = $P \div 22$ inches

$= 240 \div 22$

$= 10.9$ or about 11 people.

However, the perimeter is not “continuous”, meaning that, in using this arrangement, one person would need to sit at a corner, which can be a very uncomfortable position.

Therefore, it is more reasonable to give each person 24 inches of linear space. Given the dimensions of four feet by six feet for the rectangular table, this would mean that two people could be seated at each four-foot side, and 3 people on each six-foot side for a total of ten people (see Figure 25).

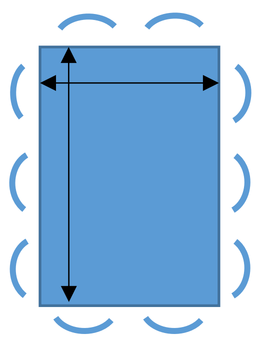


Figure 25. Rectangular table with chairs shown.

In comparison, the circumference of the round table is about 226.2 inches. Using the 22 inches I arbitrarily chose for spacing, the number of spaces at the round table would be:

Number of spaces = C ÷ 22

≈ 226.2 ÷ 22

≈ 10.28 or about 10 people around the table.

My calculations show that an equal number of people can be seated at each table. Each table would comfortably hold 10 people (see Figure 26).

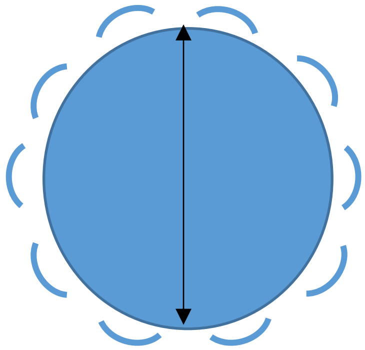


Figure 26. Round table with chairs.

Analysis. The problem is labeled as “critical thinking” (CT), using neither a mathematical operation, algorithm or theorem, nor a real-life context to introduce the situation to the reader. The label of “critical thinking” seems to offer students the opportunity to consider the problem and its real-world context, and choose an approach of their own for solving it. The problem leaves the question of how much elbow room to allow to the solver.

With respect to variation, this problem does not conform to Gerofsky’s (2004) three-component structure of (1) the set-up, (2) “information” about the problem, and (3) “the question”. The question is posed almost as a conversation question in a real-life situation: Which table would work best for certain needs at home? Which allows the most room for guests at the table?

The problem invites students to consider the reality of the situation and use decision-making by asking the question, “Which do you think …?” The problem appears as an exercise in the section on “developing formulas for circles and regular polygons”

(LOC), but no direction is given for its solution. The teacher’s edition characterizes this as a possible answer:

> “The circular table would fit at least as many people as the rectangular table. At the rectangular table, 2 people would fit at each of the 4 ft sides and 3 people would fit at each of the 6 ft sides, for a total of 10 people. Each person would have 2 ft of space. If 11 people sat at the circular table, each person would have about 1 ft 9 in. of space” (Burger, et al., 2012, p. 693).

The seating arrangement at the rectangular table matches the one I presented in my solution, but without any explanation other than giving each person 2 feet of space. It is curious that, at the round table, the amount of space is reduced by 3 inches to accommodate an extra person. Although not stated in the problem or the solution, this solution answers the question, “which table could comfortably accommodate one more person if necessary?” The given solution for eleven people gives only one inch less per person than my arbitrary amount. However, the solution does begin with the fact that the round table “would fit at least as many” as the rectangular one. It can also be argued that the round table, with its “continuous” space allows for more flexible seating arrangements than the rectangular table.

Other than the possible solution offered in the teacher’s edition, no guidelines are offered for solving this problem. This, and the wording of the question, “Which do you think…?” may prompt students to consider space at the table in their own home or in the school cafeteria. The given dimensions are reasonable for tables in the students’ lived experiences. In the textbook’s solution, the space allowed for each person is not the same for the two tables, yet each is a reasonable amount. Students may use their real-world experiences and consider seating arrangements in their home or use less space per person for children. Additionally, some may consider benches for seating, rather than chairs, as often used in banquette, school cafeteria, and picnic tables. More children than adults can be seated using this type of seating arrangement. The possible solution illustrates the encouragement of realistic considerations in pondering this problem.

Chapter 5: Discussion

In this chapter, I discuss the results of the study as presented in the previous chapter. I begin by reviewing some of the concerns addressed in the current literature which both books appear to address. I then move on to aspects that still need improvement. In this first part of the review, I consider the entire sample using my numerical analysis, including the problems from both books. I then move on to consider how the books’ different approaches may relate to my findings. This discussion will focus on answers to the research question:

> Given that the two books chosen for this study have different approaches, what aspects of *realistic-ness* exist in the textbooks’ word problems that encourage students to use their real-world knowledge of the context of the problems?

Discussion of numerical analysis of word problems

In this section, I discuss the coding and characterization of the word problems through the results of the numerical analysis.

Label. The existing literature indicates that the labeling of word problems with an operation or mathematical content so that students know beforehand which algorithm to use to solve the problem was a concern (Gerofsky, 2004b; Moschkovich, 2002). This study indicates that labeling is not an issue in the books used as data sources in this study – only thirteen of the problems in the entire sample of 444 were found to be labeled with a mathematical content.

It can be argued that the mathematics labels used in the sample for those thirteen problems, however, are non-specific. The label of “Estimation” is used for nine different problems in the standards-driven textbook. The “Measurement” problem uses shadows and similar triangles for indirect measurement. This leaves just three problems with specific labels – two labeled “Construction” and one labeled “Probability”. The labeling for the problems in this sample was generic – there was no pre-determined plan spelled out at the onset of each problem. Using labels such as “Estimation” or “Measurement” are not as specific as the labels used in previous textbooks.

Problems written in a real-world context. The literature suggests that word problems have traditionally been hypothetical in nature (Verschaffel & DeCorte, 1997b), and that often there is no reference to real-life people, places, or situations (Gerofsky, 2004b, p. 32). The literature also recommends that problems be presented more like problems that occur in the workplace or everyday life situations (Bonotto, 2013; Green & Emerson, 2010; Inoue, 2005; Palm, 2008).

Those problems that are written as if they came directly from a real-world context (PRW), whether job-related or those from everyday events, offer opportunities for students to immerse themselves in the real-world context – they may be able to imagine themselves solving these problems in an actual situation. When no school mathematics is stated inherently in the problem, students are compelled to make the connections to the real-world situation on their own.

Of the 444 problems in the sample, 360 fit this description. This large number is most likely due to the sample selection process – the problems were chosen for the real-world connections, and during subsequent stages, some were eliminated for their unbalanced focus on the school mathematics rather than on the real-world context.

Nevertheless, the word problems in the sample are a subset of the 858 word problems that mentioned a real-life context in the chosen textbooks. The number of problems that are categorized as PRW amounts to almost 42% of the 858 word problems.

Still, some of the tasks are posed using technical school mathematics (TMC) or the question to be answered is about the mathematics (QM) rather than the real-world objects or situation, or both. Only 14% of the problems in the sample are coded as TMC, 21% as QM, and 6% as both TMC and QM. Most of the problems in the sample focused more on the real-world context, making the tasks more realistic, rather than making superficial connections between the school mathematics and the real-world context.

This supports the role of *realistic-ness* and encouraging students to use realistic considerations and making connections between school mathematics and everyday contexts. Most of the problems in the sample focused more on the real-world context, making the tasks more realistic rather than making artificial or superficial connections between school mathematics and the real-world context.

Usefulness. Too often, students wonder why they need to learn the mathematics that is taught in school. The connection to a real-world situation may not be evident, even in tasks that relate to real-world contexts. Students are left to wonder why they need to solve these problems. Of the 444 problems in the sample, 35% of them explain their purpose within the text of the problem. Knowing the usefulness of a problem and its connection can motivate students (Gerofsky, 2004b; Moschkovich, 2002). It is another strategy to engage them in a particular task.

It seems that progress has been made in the realistic presentation of word problems in textbooks, particularly in regard to labeling. Many word problems are presented using realistic contexts without explicit connections to school mathematics, and relatively few depend on the technical mathematics language used in school to present the realistic situations. On the other hand, more headway is needed in the following areas: location, support for teachers, and variation.

Location. In real life, problems are not given with prescribed methods for solving them – that is why they are problems in the first place. If a prescribed method could be given, then the task would not be a problem – it is more easily resolved (Schoenfeld, 1989). In real-life situations, alternative approaches are encouraged and expected. Problem solvers, in careers and in on-the-job situations, are those who can see alternative approaches, as well as alternative solutions.

Almost 85% of the word problems in the sample are situated within the textbook section conveying the mathematics topic to be used in its solution. Examples in the section’s lesson are followed by exercises to be solved using that topic. This is a concern because students can become dependent on these cues. Also, students may be led to believe that the only way to solve a problem is to use the mathematics that they have most recently learned (Balacheff, 1986; Brown, Collins, & Duguid, 1989; Bruer, 1993; Verschaffel, Greer, & DeCorte, 2000). Students need to be given more opportunities to grapple with the contexts and make decisions when applying solutions (Schoenfeld, 1989).

Another 11.5% were found to be review problems, with a note to use previously covered topics. This means that only about 3% of the problems in the sample encourage students to consider alternative solution strategies. The location of a problem in a particular textbook section removes the decision-making power of the students. It also reduces a problem to an exercise where the work is routine and the elements of the real-life context are not necessary. When the location of the problem prescribes a mathematical context to be applied, the students will not make the connections between school mathematics and the context.

Variation. Two hundred thirty-eight of the 444 problems in the whole sample are written in three sentences or less and fit the profile of the three-component structure (Gerofsky, 2004b) of traditional mathematics word problems. This may be due to space constraints, especially given the number of exercises throughout each textbook, and the number of pages per book. Most problems in real-world situations cannot be presented in the same concise manner as in mathematics textbook problems. Some real-world problems, such as design or architecture projects, are often not envisioned as mathematical problems at all. And issues that arise during a design project can change many factors in the project.

For instance, a family may be designing a new home. The information for this project may not be presented all at once; often, the customer adds new details as the project continues. It may begin as a 3-bedroom, 2-bath home. Then, add in an eat-in kitchen as well as a dining room, a family room and a living room, access to the garage through the laundry/mudroom, and so on and so on. There may be issues with accessibility for certain family members. There may be a need for other design changes. An architectural design problem cannot be simply stated in three sentences. This is the nature of real-world problems. As another example, creating a work schedule that meets the needs of all of the employees at a fast-food restaurant or a small retail store is no small task. The “information” part of this problem comes from many sources – every employee who needs to be scheduled. And, the solution to this problem is not a single numerical answer. In fact, the solution may only be a “best solution”; it is possible that every worker will not be content with the solution which may be optimal for the scheduler. Do we prepare students for these types of real-world problems? This study’s analysis suggests the answer is, “No.” Students are not regularly given opportunities to solve these problems. The concise format of most textbook problems will not prepare students for the kinds of problems they will face in their future situations outside of school.

Nonetheless, the question of why so many problems are still written so concisely remains. Perhaps the reason is constraints by the publisher due to space issues and the page volume of the textbook. The content of the textbook may also be driven by popular demand (Merriam, 2009). Given the technological advances of recent years, it is conceivable that online resources in the form of a database of word problems and realistic tasks and projects can be made available. This database can supplement the current static resources of presentation materials, worksheets and assessments available from textbook publishers. I envision a live database of problems that can be continually updated with the most current information while eliminating obsolete tasks and data.

Support. It seems that some textbook authors and publishers assume that all teachers are able to see every possible approach to a problem. There is no limit to what students may claim works to achieve a task's objective. The task itself may demand that the students explain their work, but the teacher support, the TE, only offers "Answers may vary" or "check student's diagram".

Since textbooks provide teacher's editions and teacher resources in many forms, then the TE should provide assistance to teachers for word problems. Word problems are often omitted, and lack of teacher support for each problem could be the justification that teachers use in their decision to omit them.

The two textbooks provided support for teachers for less than half of the problems in the sample, 198 out of 444. Of the 198, only sixty-one provide some type of support for the real-world context. This support is essential as teachers may not possess the experience or knowledge of a real-life context, whether it is a career-based or an everyday task. Additional information can familiarize the teacher with the connection between the school mathematics and the real-world context. It can help the teacher to moderate a class discussion and validate facts about the context.

Working through a word problem can cause anxiety, even for mathematics teachers, especially if they are unsure that they are on the right track. Teachers who are not risk-takers may not begin or assign a problem without these supports. Additionally, in the interest of time management, high school teachers usually need to prepare for 5 classes per day, and possibly two or three different courses.

The discovery book provides more support for teachers on individual word problems (with 144) than does the standards-driven book (with 54), at a ratio of 8 to 3. This disparity may be attributed to the different intentions of the textbook curricula. The discovery book is purposefully written as an alternative to other textbooks and their "teacher-centered" and "lecture-driven" curricula (Serra, 2008, p. xv). Perhaps offering teachers more support opens up space for them to consider a more student-centered curriculum focusing on discovery. The standards-based book, on the other hand, is intended to "empower students to develop the core skills they need" and "focus on deeper understanding of math strategies and concepts" (Holt McDougal Algebra 1, Geometry and Algebra 2, 2016). The focus of this book is similar to more traditional textbooks, and the authors may not see a need for teacher support related to such skills.

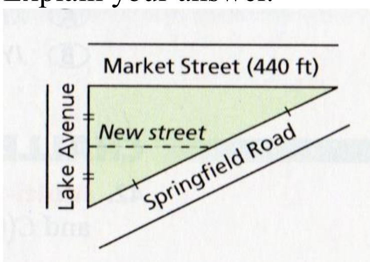
Discussion of qualitative content analysis of word problems

In addition to categorizing and coding the features of word problems as stated, I also analyzed a number of problems using in-depth solution analysis and investigating aspects of the real-life contexts. In this section, I discuss these findings.

Presentation is not fully realistic. Some problems seemed to be realistic in their presentation, but in attempting to solve them, they were found to be less than fully realistic. I refer to the Pet Iguana Pen and the Parallel Parking Space Problems, repeated here for the reader's convenience:

Pet Iguana Problem Ernesto plans to build a pen for his pet iguana. What is the area of the largest rectangular pen that he can make with 100 meters of fencing? (Serra, 2008, p. 426)

Parallel Parking Space Problem
The diagram shows the sketch for a new street. Parallel parking spaces will be painted on both sides of the street. Each parallel parking space is 23 feet long. About how many parking spaces can the city accommodate on both sides of the new street? Explain your answer.



(Burger, et al., 2012, p. 337)
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Both problems are missing elements of *realistic-ness* in their presentation which are essential to the situations in real life. The iguana pen dimensions turn out to be much larger than anyone would expect for a pet iguana to be kept at home. If students accept these dimensions on trust, it can incite the suspension of sense-making (Schoenfeld, 1991) when, instead, a critical analysis of the information should be inspired. Students may not be encouraged to consider the practicality of their own solutions when the problem they are given to solve uses such absurd quantities (Verschaffel, Greer, & DeCorte, 2000). It is also possible that since the United States still measures in customary units rather than metric units, some high school students may not realize that a 25-by-25-meter square can be larger than their own home. It is the position of the NCTM that "students need to develop an understanding of metric system units and relationships, as well as fluency in applying the metric system to real-world situations" (NCTM, 2016). The solution of the iguana problem compromises this position of the NCTM.

The Parallel Parking Space Problem included a diagram with a one-dimensional street, although the other streets on the diagram were two-dimensional. The situation is described as a real problem that could be considered by city planners, but the presentation of this problem falls short in *realistic-ness* of the map diagram. The location of the problem in the section on the midsegment of triangles encourages the student to use that particular theorem, that is, the students know which approach they are expected to use (Gerofsky, 2004b; Moschkovich, 2002). The presentation of the problem, with both one- and two-dimensional streets, should give students the opportunity to consider possible configurations for the new street to be constructed as well as alternate solutions to the problem.

Both problems could have been presented with more realistic details. Alternatively, students could be encouraged to spend more time on each problem and delve into the details. Although they may not voice their concerns, I know that some high school students realize the absurdity of some problems that they are asked to solve. Others solve the problems without thinking about the details, because this is what has been expected of them. In the tradition of the didactical contract, they will carelessly come up with a solution and disregard the details and the reality of the situation (Gravemeijer, 1997; Richards, 1991; Verschaffel, Greer, & DeCorte, 2000). Students need to be encouraged to question, "Does this make sense?"

Problems which do not consider all *realistic-ness* in their solutions. Some of the problems analyzed in this study appeared to come from real-world contexts, yet the expected solutions may not have followed through to meet regulations or specifications that are realistic. I refer to the two problems from the sample and analyses to illustrate this point: the ADA Wheelchair Problem and the Circular Table Problem.

ADA Wheelchair Problem According to the Americans with Disabilities Act, the slope of a wheelchair ramp must be no greater than $\frac{1}{12}$. What is the length of a ramp needed to gain a height of 4 feet? (Serra, 2008, p. 499)

Circular Table Problem Zach wants a circular table so that 12 chairs, each 16 inches wide, can be placed around it with at least 8 inches between chairs. What should be the diameter of the table? Will the table fit in a 12-by-14-foot dining room? Explain (Serra, 2008, p. 343).

Each problem appears to include realistic information in its wording. My solution and investigation into the context uncovered that the wheelchair ramp will not meet ADA regulations if it is constructed in a single run. The regulations require a landing. The circular table problem does not take into consideration that a 92-inch table is not readily available, and may not be realistically feasible. In fact, a 72-inch diameter is considered the maximum recommended for practical conversation across the table (Mayhugh, 2013).

These problems lack *realistic-ness* in their impact on the final results. Each problem appears to give ample, reliable information. The textbook information in both cases coincides with the solutions as given in the Teacher's Edition. It was only through some legwork into readily available information on reliable websites that the lack of follow-through becomes evident. However, problems such as these two are representative of the kinds of problems that can challenge high school students (Gravemeijer, 1997; Green & Emerson, 2010). They are not ordinary problems, nor are they like traditional word problems. If students are given an opportunity and the time to scrutinize the realistic-ness of a problem and its situation, they can do the legwork by checking current regulations and recommendations online and producing realistic results (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). It may not be something they are accustomed to doing in mathematics classes, but today's students are gaining experience in researching websites for all subject areas (Purcell, et al., 2012).

Problems which do not challenge the capabilities of high school students. Some problems in the sample were simplistic in their presentation, despite the connection to a real-world context. One such problem is the Quilt Triangles Problem:

Quilt Triangles Problem
The quilt pattern includes 32 small triangles. Each has a base of 3 in. and a height of 1.5 in. Find the amount of fabric used to make the 32 triangles.

(Burger, et al., 2012, p. 38).
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This problem simply asks for the area of all of the small triangles, all of which are the same size. Students can find the area of one of these, and then multiply by 32 to get the requested answer. My analysis pointed out the shortcomings of this solution (including all colors in one lump sum), but it should be further emphasized that this problem as stated underestimates the capabilities of high school geometry students. Finding the area of a triangle and relating it to real-life problems is a seventh-grade Mathematical Content Standard (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). High school geometry students are capable of answering more in-depth application questions, such as “how much fabric of each color is needed for the entire quilt top?”

In the comparison of the Quilt Triangles Problem to the Ohio Star Problem, I found that the more detailed problem was presented in a way that could engage students in making connections between the mathematics and the real-life context. The Ohio Star Problem presented the steps to creating the entire quilt top, while the Quilt Triangles Problem was presented very simplistically, asking for the total area of 32 congruent triangles. The Ohio Star Problem, on the other hand, presented details of the real-world context.

Many high school students have become accustomed to one- or two-step problems (Bottge & Hasselbring, 1993; Ginsberg, Leinwand, Anstrom, & Pollock, 2005) and they may balk at the thought of a multi-step or multifaceted problem. This is a result of the didactical contract (Brousseau, 1997; Davis, 2013; Verschaffel L. , 2002) and in order to break it, the kinds of problems that are presented to students must change. To negotiate a new contract, students can be presented with problems that challenge them rather than questions they can easily answer. Students need the opportunities to “make sense of problems and persevere in solving them” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Problems with a Purpose. Some word problems are placed in a textbook curriculum to serve a particular purpose (Moschkovich, 2002). That appeared to be the case with The Perimeter Fence Problem: it was placed in a section that encouraged students to draw diagrams to apply visual thinking to problem solving. However, this particular problem does not need a diagram to reach the solution. Students are able to arrive at the number of fenceposts needed simply by dividing the perimeter by the distance between posts.

If the purpose of the problems in a section is to encourage students to draw diagrams, then the diagrams should have a meaningful role in the solutions of the problem. Students should have opportunities to solve problems in which the solutions may not be evident and may require considering all aspects of the real-life situation.

Answers to the Research Question. The research question for this study asks: Given that the two books chosen for this study have different approaches, what aspects of *realistic-ness* exist in the textbooks' word problems that encourage students to use their real-world knowledge of the context of the problems?

This study found that the *realistic-ness* of word problems in this sample is enhanced by several aspects that were analyzed. In both textbooks, the problems in this sample were found to be labeled more generically, if labeled at all. Only a handful of problems were labeled with mathematics to be applied.

Also, the problems are often written as if they came directly from a real-world context rather than from a mathematics textbook (PRW). Although the problems were chosen using specific guidelines, the number of problems that are written in this manner is significant – 87.7% of the problems from the discovery book and 73.6% of those from the standards-driven book. The larger percentage in the discovery book is probably due to the intentions of the author and of nontraditional textbooks: to emphasize reasoning and problem solving in student-centered experiences (Ginsberg, Leinwand, Anstrom, & Pollock, 2005).

On the other hand, the study also shows that more work needs to be done with regard to other aspects of realistic word problems. For instance, although I found that neither book labeled word problems in a way to prescribe the mathematics to be applied, the location of the problem within a particular section has taken on that role. The standards-driven book prescribed a method through the location of all but one of the problems in the sample, and the discovery book did likewise for 71.2% of its problems in the sample. The reasoning behind this may be that they are providing the connections between the mathematics and the real-world context, and this placement makes those connections clear. Nonetheless, students should be given more problems that require them to consider options for problem-solving strategies. Placing problems in a different section would at least give them opportunities to recall previously-learned methods and topics that may be applicable.

Word problems written in the concise three-component structure (Gerofsky, 2004b) were more likely to be found in the standards-driven book than the discovery book. The findings show that 56.2% of the sample from the standards-driven book and 37.7% from the discovery book fit this pattern. The concise format may be due to space constraints on the page or constraints on the total number of pages in the textbook. After all, both books are quite lengthy: the student edition of the discovery book has 858 pages, and the standards-driven book weighs in with 926. Regardless, today’s technologies offer space in online venues which would not limit the volume or depth of the content or length of a word problem.

Findings also show that the word problems and TE solutions lack connections to real-world data and conditions. Neither book provided much teacher support for real-world contexts. Of the problems that provide support for the teacher, only 51 problems in the discovery book as opposed to 10 in the standards-driven textbook supply details about the real-world context. Neither textbook offers teachers the kinds of supports they require to be successful in implementing the real-world word problems in their classrooms.

Chapter 6: Implications

This final chapter begins with the limitations of this study. I then discuss the implications for textbook authors and publishers, followed by implications for teacher educators and for classroom practice. I offer recommendations for future research on textbook word problems. I conclude with some final remarks about the importance of providing students with the experiences they need to be successful in their future.

Limitations. This study was limited to a data source of only two textbooks. I restricted this analysis to a specific sample of word problems within each book: those problems in the exercises in each regular section of the two textbooks that were deemed to have a real-life context. This sample included 444 problems that were considered to be the best fit for the intended analysis. My intention is that the method I used can be transferred to study other textbooks or word problems, but my results are not generalizable because of the small number of books investigated.

Before I began the analysis, the preliminary counts of the word problems with a mention of real-life context stood at 858 problems between the two textbooks. Through the categorization and analysis, problems were eliminated from the sample for their weak presentation or lack of connection between the mathematics and the context. In this study, I analyzed and categorized about 51% of that total. The resulting sample represents a “best case” pool of realistic word problems for analysis and discussion. It is important to note that the results may have differed if all 858 problems had been included.

Additionally, the two books that served as the data sources for this study are both geometry books intended for high school students. The results may have differed if the subject matter under investigation had been algebra, statistics, calculus, or general mathematics. Additionally, a change in grade level also may have altered the results.

Another limitation is that the directed approach to qualitative content analysis that I used in this study emphasized the realistic nature of word problems. The analyses unearthed many aspects of word problems, but less attention was given to other meanings or intentions within the textbook (Hsieh & Shannon, 2005). A different qualitative content analysis could have focused on social issues or the language used in word problems. For example, a focus on language might inspire students to consider a problem in different ways or some words may impel them to think or investigate more deeply than others. The language of a word problem may interfere with the gist of the problem, causing the intent of the textbook author to be lost.

Implications for textbook authors and publishers. The current study shows that many of the problems are still written in the concise three-component structure. The story, or context of the problem is meant to be a hook to engage students in solving that problem. Although they are sometimes called story problems, it seems that the story is short, if present at all, compromising student engagement. Textbooks need to include problems that go beyond the basic three-component design, particularly since problems in the real world rarely conform to this structure. Students need to have opportunities to read through a problem situation, to discern what given information may be necessary, and to decide if further investigation into the context is needed.

Prior research shows that one of the intentions of word problems is to train students in practical skills that they will need in college, careers, and everyday life (Boaler, 1994). However, some textbook problems do not serve this intention. Textbook word problems are often strategically placed in the textbook, leading students to the solutions without making sense of the problem and its context. Textbooks should use scaffolding throughout the textbook curriculum and provide problems that offer opportunities for students to choose a solution method and to make connections to previous topics and to real-world contexts. In this way, more multifaceted problems could be introduced, so that students will have experience in solving problems that are more representative of what happens in real life.

Another important intention of word problems is to provide a purpose and a familiar context (Moschkovich, 2002). The NCTM’s Principles to Actions call for encouraging students to become problem solvers, giving them opportunities to make sense of each problem, and discuss the process of their solution (National Council of Teachers of Mathematics, 2014). The Common Core State Standards (CCSS) for Mathematical Practice reiterates this call for students to “make sense of problems and persevere in solving them” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Most of the problems in this study appear to have enough information to familiarize students with the context, but only some of them explicitly state their purpose, making it clear to students why they are solving a given problem. In order to persuade students to persist in making sense of problems, the usefulness of the problem should be clearly stated, especially if it is not a common or familiar setting for the students. Using accurate information of a real-life situation can persuade students to consider whether solutions make sense.

Additionally, given that mathematics teachers come from a multitude of different preparations and backgrounds, it is essential that the textbook offers the support they need to deliver a curriculum. Less than half of the problems in the sample were found to have any support for teachers at all. Besides their teacher education, mathematics teachers have varying degrees of experience with real-world contexts. Textbook authors and publishers should ensure that each real-world problem, especially the multi-step or multifaceted problems, provide teachers with more than a single numerical value or a phrase such as “answers may vary” or “check student work”. Analyses or suggestions that incorporate realistic-ness can be included as part of this support. In addition, the teacher’s edition can direct teachers to have students investigate contexts further for the most accurate and up-to-date data and regulations.

Textbook publishers have many online resources available for presentation, for remediation and review, and for extra practice. Many of the resources are presented in a static form. Given the emerging technologies of this Information Age, publishers could include a “living” database of realistic problems or tasks. This database could be maintained and updated as needed, providing external links to relevant, reliable websites for accurate, current information. Problems can be updated to give the most accurate information at the time of use. A database is not held to the same space restrictions as a print publication, so the space constraints which may have been an argument for the three-component structure would also no longer be an issue.

These implications would help to create a textbook curriculum that is less teacher-directed and more student-centered, giving teachers a solid resource toward re-negotiating the didactical contract. Giving students the opportunity to choose strategies and make connections will encourage students to persevere in problem-solving, and empowers them to become independent mathematical thinkers.

Implications for teacher educators. An integral part of the skill set of a proficient and effective teacher is the ability to “engage students in learning and developing mathematical reasoning and problem-solving skills” (Stanford Center for Assessment, Learning, and Equity (SCALE), 2016). Teachers are “expected to solve problems by integrating knowledge from different areas of mathematics, to use various representations of concepts, to solve problems that have several solution paths, and to develop mathematical models and use them to solve real-world problems” (Educational Testing Service (ETS), 2017). These skills include being able to make connections between new learning and students’ previous experiences in their academic and personal lives. Teachers need to use tasks and problems that can help students make clear connections between their school mathematics and real-world contexts.

This study’s qualitative content analysis can serve as a model for teacher educators to prepare and equip pre-service teachers with these skills. The analysis shows the importance of delving into a problem with a real-world context and investigating both the mathematical content and the real-life context beneath the surface of the problem. Pre-service teachers may have been students under a traditional didactical contract, and may need to be introduced to ways of re-negotiating that contract in their own classrooms. Taking the time to delve into the details of realistic-ness in a problem-solving lesson may be the catalyst they need to forego the old contract and lead off with a new one that focuses on student-centered lessons promoting inquiry and discovery.

Implications for classroom practice. This study shows that textbook word problems have some aspects of *realistic-ness,* but that some need tweaking to encourage students to use the information available to them. The fact that problems do not follow through on using realistic values or aspects throughout keeps the existing didactical contract in place: students expect that word problems can be solved by one or two simple operations, and they hesitate when faced with a more complicated task (Verschaffel, 2002). To break this didactical contract, and negotiate a new one, more complex real-world problems need to be introduced into the high school curriculum. Choosing an appropriate textbook is the first step, since it facilitates teachers' implementation of a more realistic curriculum in the classroom.

One of the objectives of the Common Core State Standards is to prepare students for college and careers (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). More problems used in high school curricula should mimic the kinds of problems that students would be expected to solve in their possible future experiences. Most of the problems in this study's sample were found to be written in a real-world context. Some, however, still focus on the school mathematics rather than relating the problem to conditions of the context. Word problems need to be more complex tasks, possibly requiring hours or even days to solve. These are the kinds of problems that students may face in their futures, whether in a career or in everyday life. Teachers should give their students opportunities to work on such realistic and complex problems cooperatively with their classmates.

The CCSS suggest that “mathematically proficient students…are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Using relevant websites can promote the *realistic-ness* of the problem and its real-world context. As I solved and analyzed problems in the qualitative content analysis portion of this study, I confirmed information stated in the problems by checking reference on trustworthy and easily available websites with accurate information i.e., ADA regulations and standards, the construction of quilts, and practical sizes of tables. In verifying the accuracy of the stated problem, more information came to light which would affect solutions and outcomes in the real-world contexts. Students can be encouraged to use up-to-date information during problem-solving activities; for example, they can become aware of changes in safety regulations and recommendations. They can use information they find on reliable and accurate websites and check the *realistic-ness* of their solutions with current data.

They can be given problems that involve more real-life contexts, and they can investigate and engage in fact-finding on reliable websites. These activities are familiar to students and are part of their prior knowledge and experiences (Purcell, et al., 2012) and can increase the rigor of the problems they solve.

This study’s qualitative content analysis provides teachers with a model for classroom presentation and discourse on word problems and their solutions. This model calls for more elaborate presentations of word problems and their realistic contexts. In order for students to consider the reality of the problem’s context, the teacher must model these practices first. This is an important activity in a mathematics class; students often see the refined finished product of a solution, but do not witness the grappling and messiness of the work that mathematicians do to solve problems (Schoenfeld, 1989). Since students often follow the model of their teacher in solving problems, the teacher should model the activities that are expected of the student: discussing the problem situation, investigating details on reliable websites, making sense of the connections between the mathematics and the real-world context, and checking that the solution that results is a realistic one. This can also stimulate the re-negotiation of the didactical contract into a more student-centered, inquiry- and discovery-infused activity.

Then, rather than giving students a set of exercises or word problems to complete as an assignment, the teacher can assign just one problem. With a focus on one problem, students can investigate the real-world context more deeply, perhaps discovering important features or aspects of the particular context or problem situation. Instead of actually solving the problem for homework, the students could be asked to consider the problem from different perspectives and to investigate the problem and its context on the internet. Students should know that they do not need to accept the information in a word problem purely on trust. They can use reliable websites on the internet to confirm accurate information and to investigate both the real-life context and the school mathematics to enhance their understanding of the connections between the two. They could bring this preliminary work to class, then work collaboratively in small groups to formulate and present a proposal for their solution.

Although many problems are written with connections to daily life or job-related contexts, it would not be necessary to treat every one of these problems as a group project. But, occasionally, if time is spent to allow students to consider the reality of the contexts, they may begin to make better sense of the problems they encounter throughout their school mathematics experiences.

Recommendations for Future Research. In this study, the integration of the word problems within a set of Exercises was not part of the analysis. If the word problems are positioned together at the end of the set of exercises, this may affect their treatment in the classroom and whether these problems will be included as part of a classroom lesson or student assignment. This analysis of the integration of the word problem within the problems set is a possibility for future research.

The word problems in a geometry textbook lend themselves easily to a characterization of their realistic-ness and connection to real-world contexts; many of the problems deal with finite plane and solid figures which easily model objects in real life. Possibilities for future research could include analyzing problems in textbooks covering algebra, calculus, or statistics, for example.

A study of textbook word problems could be conducted with a focus on real-life situations to characterize the realistic-ness of workplace problems versus problems that occur in everyday life events. The categories and codes of this study could serve as a model, though the specified focus may demand that additional categories and codes emerge.

A study of word problems could be conducted using a social justice framework. A new set of codes and categories may be necessary to investigate social justice issues that can inform decision-making in the selection of textbooks for a school district.

Future research could also compare the word problems with a real-world context in middle school textbooks to those in textbooks intended for high school students. This kind of research may explore the aspects of *realistic-ness* that are addressed by textbook curricula at different grade levels. An investigation into a series of textbooks that a student may use over the course of several years may also uncover implications for classroom practice as well as inform decisions for adoption and implementation of a longitudinal curriculum. This type of study may find that the treatment of real-world contexts may evolve through the stages of the curriculum, from elementary grades through high school.

Conclusion

The manner in which textbook problems are used by the teacher in their instruction is likely to affect how students will perceive the problems. By encouraging sense-making and realistic considerations, teachers can help students to leave learning by rote memorization and the expectations of the former didactical contract behind. Teachers should allow and encourage students to use realistic considerations so that students will apply their real-world knowledge and not restrict themselves to the confines of school mathematics and its algorithms. These implications would help to make school mathematics more student-centered, especially at the high school level, when students should be prepared to be more independent and learn through understanding. These changes would stimulate a re-negotiation of the didactical contract and help students to become successful, productive members of society.

As mathematics educators, if we want students to become better problem-solvers, we need to give them problem-solving experiences with context and purpose. They need opportunities for practice in problem solving: to work collaboratively on problems, to consider different options for approaches as well as solutions, and to decide whether an approach or solution makes sense in the real-world context. If we want to prepare students for problem-solving so that they may succeed in their college and career experiences, we need to model the kinds of problems they will face in those experiences. Students need to be given the opportunities to make the connections between the mathematics they learn in school and the real-life experiences of their futures.

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Appendix A: Qualitative Codebook

The qualitative codebook contains the categories and subcategories that were used in the coding of the word problems in the sample. It has evolved through the process of the numerical and qualitative content analyses of this study.

Category	Code	Definition
Label		Signifies how the problem is labeled: by its real-life context or by a preferred mathematical content.
	A	labeled as *Application*
	CT	labeled as *Critical Thinking*
	M	labeled with mathematics to be applied
	MS	labeled as *Multi-Step*
	N	no label
	RWC	labeled with a specific real-world context
	SR	labeled as *Short Response*
	WAI	labeled as *Write About It*
Variation	3CS	The word problem follow the concise three-sentence or three-component structure of: (1) “the set-up”, (2) “information” about the problem, and 3) “the question” (Gerofsky, 2004b). (*See example below.)
	VAR	Presentation deviates from Gerofsky’s 3-component structure. Problem may have a more complex arrangement or display, or simply differs from the structure. The solution may not be a single numerical value.
Alternative Approaches	LOC	The problem is located in the section that covers the particular mathematical content intended to be used to solve the problem.
	DMS	Decision-making is left to the student, since the location does not dictate a particular content to be used.
Support	SUP	The TE provides support with explanations of a possible solution to the word problem; the answer is something other than a single numerical solution or “Answers may vary”. Support may also be available as advice in the side notes of the TE.
	R	(A subcategory of SUP.) The TE provides support for the real-world context of the word problem.

	NSUP	No support is available for the problem.
Realistic Considerations		The wording of a problem is true to real-world events, encouraging students to use *realistic considerations* rather than ignore them. The following subcategories are not mutually exclusive: (**See examples for each below)
	TMC	The text of the problem uses school mathematics content to describe the problem situation or includes a diagram which will point to the mathematical content to be used. School mathematics terms are used rather than everyday terms which are familiar to the lay person.
	QM	The question is about the mathematics, and does not encourage connections to its real-world context.
	PRW	The problem is written as if it is from a real world situation that is placed in the textbook. The problem reads as a question within a career or everyday situation. No textbook mathematics is part of the wording. ("Explain your reasoning" and "Round to the nearest tenth" will be ignored.)
	USE	The usefulness for the task is given within its text. Usefulness addresses the purpose of the task in a real-life situation – who would need to know this information and in what context.

*3CS example:

Television and computer screens are usually advertised based on the lengths of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch (Burger, et al., 2012, p. 48).

The "set-up": Television and computer screens are advertised based on their diagonals;

The "information": the height is 11 in. and the width is 14 in.;

The "question": what is the length of the diagonal?

** Examples for Realistic Considerations subcategories:

TMC

Birdy McFly is designing a large triangular hang glider. She needs to locate the center of gravity for her glider. Which point does she need to locate? Birdy wishes to decorate her glider with the largest possible circle within her large triangular hang glider. Which point of concurrency does she need to locate? (Serra, 2008, p. 188)

(The text contains mathematical content: *point of concurrency*)

QM

A pizza parlor offers pizza with diameters of 8 in., 10 in., and 12 in. Find the area of each size pizza. Round to the nearest tenth (Burger, et al., 2012, p. 691).

(The question is about mathematics: *Find the area.*)

PRW

Which do you think would seat more people, a 4 ft by 6 ft rectangular table, or a circular table with a diameter of 6 ft? How many people would you sit at each table? Explain your reasoning (Burger, et al., 2012, p. 693).

(Problem from the real-world; no classroom mathematics is mentioned, other than "Explain your reasoning.")

USE

To make sure that a room is rectangular, builders check the two diagonals of the room. Explain what they check about the diagonals and why this works (Serra, 2008, p. 295).

(Usefulness: *To make sure the room is rectangular.*)

Appendix B: Spreadsheets

These spreadsheets contain the raw data from each textbook, showing the coding for each of the problems in the sample.

Coding Raw Data
Discovery Book

Chapter	section	problem #	Label	Location	Variation	Realistic Considerations				Support	
0	1	8	N	LOC	VAR	QM				SUP	R
0	1	10/11	N	LOC	VAR	TMC	QM	PRW		NSUP	
0	2	5/6	N	LOC	VAR	TMC	PRW	USE		SUP	
0	2	7	N	DMS R	VAR	QM				SUP	R
0	4	5	N	DMS	VAR	TMC	QM			SUP	R
1	2	44	N	LOC	VAR	TMC	PRW			SUP	R
1	2	47	N	DMS	VAR	PRW	QM			NSUP	
1	3	27	N	DMS R	VAR	TMC	QM			NSUP	
1	5	23	N	DMS	VAR	PRW				NSUP	
1	7	27/28	N	LOC	VAR	QM				NSUP	
1	8	7	N	LOC	VAR	QM	TMC			NSUP	
1	9	1	N	DMS	VAR	PRW	USE			NSUP	
1	9	2	N	LOC	VAR	PRW				SUP	
1	9	3	N	LOC	VAR	PRW				SUP	
1	9	4	N	LOC	3CS	PRW	USE			SUP	
1	9	5	N	LOC	3CS	PRW				SUP	
1	9	6	N	LOC	3CS	PRW				SUP	
1	9	11	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
1	9	17	N	LOC	VAR	TMC	QM			SUP	R
2	2	9	A	LOC	VAR	PRW				NSUP	

Coding Raw Data
Discovery Book

2	2	17	N	DMS R	VAR	PRW	QM			NSUP	
2	3	8	N	LOC	VAR	PRW	USE			SUP	R
2	3	9	N	LOC	VAR	PRW	QM	USE		SUP	R
2	3	10	N	LOC	3CS	PRW				SUP	
2	4	13	N	DMS R	VAR	PRW	QM			SUP	R
2	5	7	N	LOC	3CS	PRW				NSUP	
2	5	8	N	LOC	VAR	QM				SUP	
3	1	13	N	DMS R	3CS	PRW				SUP	
3	2	11	N	LOC	VAR	PRW				SUP	
3	5	13	N	DMS R	3CS	PRW	USE		PRW/USE	SUP	R
3	7	5	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
3	8	1	N	LOC	VAR	TMC	QM			SUP	
3	8	12	A	DMS R	VAR	PRW				SUP	
3	8	15	N	DMS R	VAR	TMC	QM			SUP	
3	8	16	N	DMS R	3CS	PRW				NSUP	
4	3	18	N	LOC	VAR	TMC	QM			NSUP	
4	4	11	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
4	4	20	N	LOC	VAR	PRW	USE		PRW/USE	SUP	R
4	5	28	A	DMS R	VAR	PRW	USE		PRW/USE	SUP	R
4	7	15	N	DMS R	VAR	TMC	QM			SUP	
4	8	13	N	DMS R	VAR	PRW				NSUP	
4	8	17	A	DMS R	VAR	PRW				SUP	

Coding Raw Data
Discovery Book

4	8	18	A	DMS R	VAR	PRW				NSUP	
5	1	15	N	LOC	VAR	TMC	PRW			SUP	
5	1	16	A	LOC	VAR	TMC	QM	PRW		SUP	
5	3	17	A	LOC	VAR	TMC	QM			SUP	
5	4	10	N	LOC	VAR	TMC				SUP	
5	4	11	N	LOC	VAR	QM				SUP	R
5	4	12	N	DMS R	VAR	TMC	QM			SUP	R
5	5	14	N	DMS	VAR	QM				SUP	
5	6	21	N	LOC	VAR	PRW	USE		PRW/USE	NSUP	
5	6	22	N	DMS	VAR	PRW	TMC	QM		SUP	
5	6	23	N	LOC	VAR	TMC	QM			SUP	R
5	7	10	N	LOC	VAR	TMC				SUP	R
5	7	12	N	DMS R	VAR	PRW				SUP	R
5	7	15	N	DMS	VAR	QM				SUP	
5	7	16	N	DMS R	VAR	PRW				SUP	R
6	1	6	N	LOC	VAR	PRW				SUP	R
6	1	7	N	LOC	VAR	PRW				SUP	R
6	1	16	N	LOC	VAR	TMC				SUP	R
6	1	19	N	DMS R	VAR	PRW	QM			SUP	
6	2	18	M	LOC	VAR	PRW	USE		PRW/USE	SUP	
6	2	19	M	LOC	VAR	PRW	TMC			SUP	
6	2	25	N	DMS R	VAR	PRW	USE		PRW/USE	SUP	

Coding Raw Data
Discovery Book

6	2	27	N	DMS R	VAR	PRW	USE		PRW/USE	SUP	
6	3	20	A	LOC	VAR	PRW	QM			SUP	
6	5	11	N	LOC	3CS	PRW				NSUP	
6	5	14	N	DMS	VAR	PRW	QM			NSUP	
6	5	15	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
6	6	1	N	LOC	3CS	PRW				NSUP	
6	6	2	N	LOC	VAR	PRW				NSUP	
6	6	3	N	LOC	VAR	PRW	QM			NSUP	
6	6	4	N	LOC	3CS	PRW	QM			NSUP	
6	6	5	N	LOC	VAR	PRW	QM			NSUP	
6	6	6	N	LOC	VAR	PRW	QM			NSUP	
6	6	7	A	LOC	3CS	PRW				SUP	
6	6	8	N	LOC	3CS	PRW				NSUP	
6	6	16	N	DMS R	3CS	PRW				SUP	
6	7	9	N	LOC	3CS	PRW				SUP	
6	7	10	N	LOC	3CS	PRW				SUP	
6	7	11	A	LOC	3CS	TMC				SUP	
6	7	12	N	LOC	VAR	QM	USE			SUP	R
6	7	13	N	DMS R	3CS	TMC	QM			SUP	
6	7	14	N	DMS R	3CS	TMC	QM			SUP	
7	2	9	N	LOC	3CS	PRW	USE	TMC		NSUP	
7	2	10	N	LOC	3CS	USE	TMC			SUP	

Coding Raw Data
Discovery Book

7	2	11	N	LOC	3CS	TMC	USE			NSUP	
7	2	12	N	LOC	VAR	USE	QM			SUP	R
7	4	18	N	DMS R	VAR	USE	PRW		PRW/USE	SUP	
7	6	11	N	DMS R	VAR	PRW	USE		PRW/USE	NSUP	
7	6	14	N	DMS R	3CS	PRW				SUP	
7	7	12	N	DMS R	VAR	PRW				NSUP	
8	1	17	N	LOC	VAR	PRW	TMC			NSUP	
8	1	18	A	LOC	VAR	PRW				SUP	
8	1	19	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
8	1	20	A	LOC	3CS	PRW	USE		PRW/USE	NSUP	
8	1	25	A	DMS	VAR	PRW	USE		PRW/USE	SUP	
8	2	19	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
8	2	20	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
8	3	1	A	LOC	VAR	PRW	QM	USE		SUP	
8	3	2	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
8	3	3	A	LOC	3CS	PRW	USE		PRW/USE	SUP	
8	3	4	A	LOC	3CS	USE	PRW		PRW/USE	SUP	R
8	3	5	A	LOC	VAR	TMC	USE			NSUP	
8	3	6	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
8	3	7	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
8	4	15	A	LOC	3CS	PRW	USE		PRW/USE	NSUP	
8	5	12	A	LOC	3CS	PRW				SUP	

Coding Raw Data
Discovery Book

8	5	13	A	LOC	3CS	PRW				SUP	
8	5	14	N	LOC	3CS	PRW				SUP	
8	5	15	A	LOC	VAR	PRW				SUP	
8	5	22	N	DMS R	VAR	PRW				SUP	R
8	6	13	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
8	6	14	N	DMS	3CS	PRW	USE		PRW/USE	NSUP	
8	7	12	A	LOC	3CS	PRW	USE		PRW/USE	NSUP	
8	7	16	A	DMS R	3CS	PRW	USE		PRW/USE	NSUP	
9	1	12	N	LOC	3CS	PRW				SUP	
9	1	16	N	LOC	3CS	PRW				SUP	
9	2	8	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
9	2	14	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
9	2	16	N	LOC	3CS	USE	PRW		PRW/USE	SUP	
9	2	17	N	LOC	3CS	PRW				SUP	
9	4	1	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
9	4	2	N	LOC	3CS	PRW	USE		PRW/USE	SUP	
9	4	3	N	LOC	VAR	USE	PRW		PRW/USE	NSUP	
9	4	4	N	LOC	3CS	PRW				SUP	
9	4	5	N	LOC	3CS	PRW	USE		PRW/USE	SUP	
9	4	8	A	DMS	VAR	PRW				NSUP	
9	5	22	N	DMS	VAR	PRW	USE		PRW/USE	NSUP	
9	6	9	N	LOC	VAR	PRW	USE		PRW/USE	SUP	

Coding Raw Data
Discovery Book

9	6	16	A	LOC	3CS	PRW	USE		PRW/USE	NSUP	
9	6	21	A	DMS	VAR	PRW				SUP	R
10	2	14	A	LOC	VAR	PRW	USE		PRW/USE	SUP	R
10	2	15	A	LOC	VAR	PRW	USE		PRW/USE	SUP	R
10	2	16	N	LOC	3CS	PRW				SUP	
10	2	17	N	LOC	VAR	PRW				NSUP	
10	2	18	N	LOC	VAR	PRW	USE		PRW/USE	SUP	R
10	2	23	N	DMS R	VAR	PRW				NSUP	
10	3	13	N	LOC	3CS	PRW				NSUP	
10	3	14	N	LOC	VAR	TMC	PRW			NSUP	
10	3	15	N	LOC	VAR	PRW				NSUP	
10	3	18	A	DMS R	VAR	PRW	USE		PRW/USE	NSUP	
10	3	19	A	DMS R	VAR	PRW	USE		PRW/USE	NSUP	
10	3	20	N	DMS R	VAR	PRW	USE		PRW/USE	NSUP	
10	4	1	N	LOC	VAR	PRW				NSUP	
10	4	6	N	LOC	VAR	PRW				SUP	
10	4	7	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
10	4	8	N	LOC	3CS	PRW				SUP	
10	4	9	N	LOC	VAR	PRW				NSUP	
10	4	10	A	LOC	3CS	PRW				SUP	
10	4	11	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
10	4	12	N	LOC	VAR	PRW	USE		PRW/USE	SUP	

Coding Raw Data
Discovery Book

10	4	13	N	LOC	VAR	PRW				SUP	
10	4	14	N	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	4	15	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	5	1	N	LOC	3CS	PRW				NSUP	
10	5	2	N	LOC	3CS	PRW				NSUP	
10	5	3	N	LOC	3CS	PRW				NSUP	
10	5	4	N	LOC	3CS	PRW				NSUP	
10	5	5	N	LOC	3CS	PRW				NSUP	
10	5	6	N	LOC	3CS	PRW	USE		PRW/USE	SUP	
10	5	7	N	LOC	3CS	PRW				NSUP	
10	5	8	N	LOC	3CS	PRW				SUP	
10	5	9	N	LOC	VAR	PRW	USE		PRW/USE	SUP	R
10	5	11	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	6	8	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
10	6	9	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
10	6	15	N	LOC	3CS	PRW				NSUP	
10	6	16	N	LOC	3CS	PRW				SUP	R
10	6	17	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	6	18	N	DMS R	VAR	PRW				NSUP	
10	6	19	N	DMS R	VAR	PRW	USE		PRW/USE	SUP	R
10	7	8	N	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	7	9	A	LOC	3CS	PRW				SUP	R

Coding Raw Data
Discovery Book

10	7	10	A	LOC	VAR	USE	PRW		PRW/USE	SUP	R
10	7	11	N	LOC	3CS	PRW				NSUP	
10	7	12	N	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	7	13	N	LOC	3CS	PRW				SUP	
10	7	14	N	DMS R	3CS	PRW				NSUP	
11	1	17	A	LOC	VAR	PRW				SUP	R
11	1	22	A	DMS	VAR	PRW	USE		PRW/USE	SUP	R
11	2	17	A	DMS	3CS	PRW	USE		PRW/USE	SUP	R
11	2	18	A	DMS	VAR	PRW	USE		PRW/USE	SUP	R
11	2	22	N	DMS	VAR	PRW				SUP	R
11	3	1	N	LOC	3CS	PRW				NSUP	
11	3	4	N	LOC	3CS	PRW				NSUP	
11	3	5	A	LOC	3CS	PRW				SUP	
11	3	6	N	LOC	VAR	PRW	USE		PRW/USE	SUP	R
11	3	8	N	LOC	VAR	PRW				SUP	
11	3	9	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
11	3	10	N	LOC	VAR	PRW				SUP	
11	5	8	A	LOC	VAR	PRW	USE		PRW/USE	SUP	R
11	5	9	N	LOC	3CS	PRW				NSUP	
11	5	17	N	DMS R	VAR	PRW	USE		PRW/USE	NSUP	
11	6	8	A	LOC	3CS	PRW	USE		PRW/USE	SUP	R
11	6	9	A	LOC	3CS	PRW	USE		PRW/USE	SUP	R

Coding Raw Data
Discovery Book

11	6	10	N	LOC	VAR	PRW				SUP	
11	6	11	N	LOC	VAR	PRW				SUP	
11	6	12	N	LOC	VAR	PRW	QM			SUP	
11	6	13	N	LOC	VAR	PRW	QM			NSUP	
11	7	24	A	DMS R	VAR	PRW	USE		PRW/USE	NSUP	
12	1	25	A	DMS R	VAR	PRW	USE		PRW/USE	SUP	
12	1	26	A	DMS R	VAR	USE	PRW		PRW/USE	SUP	
12	2	10	N	LOC	3CS	PRW	USE		PRW/USE	SUP	R
12	2	11	N	LOC	3CS	PRW				NSUP	
12	2	12	A	LOC	3CS	PRW				NSUP	
12	2	13	A	LOC	3CS	PRW	USE		PRW/USE	NSUP	
12	2	14	A	LOC	3CS	PRW				SUP	
12	2	15	A	LOC	3CS	PRW				NSUP	
12	2	16	A	LOC	VAR	PRW	USE		PRW/USE	SUP	R
12	2	22	N	DMS R	VAR	PRW				SUP	R
12	3	12	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
12	3	13	N	LOC	3CS	PRW	TMC			SUP	
12	3	14	N	DMS R	VAR	PRW				NSUP	
12	3	15	N	DMS R	3CS	PRW				SUP	
12	4	10	A	LOC	VAR	PRW	USE		PRW/USE	SUP	
12	4	11	A	DMS R	3CS	PRW	USE		PRW/USE	NSUP	
12	4	12	A	DMS R	3CS	PRW				NSUP	

Coding Raw Data
Discovery Book

12	4	13	A	DMS R	3CS	PRW	USE		PRW/USE	NSUP	
12	4	14	A	LOC	VAR	PRW	USE		PRW/USE	SUP	R
12	5	1	A	LOC	3CS	PRW	USE		PRW/USE	NSUP	
12	5	2	A	LOC	3CS	PRW				NSUP	
12	5	3	A	LOC	3CS	PRW				NSUP	
12	5	4	A	LOC	3CS	PRW				NSUP	
12	5	5	A	LOC	VAR	PRW	USE		PRW/USE	SUP	R
12	5	7	N	LOC	3CS	PRW	USE		PRW/USE	SUP	
12	5	8	A	LOC	VAR	PRW	USE		PRW/USE	NSUP	
12	5	9	N	LOC	3CS	PRW				NSUP	
12	5	13	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
13	1	18	N	LOC	VAR	PRW				SUP	R
13	1	26	N	DMS R	VAR	PRW	USE		PRW/USE	SUP	
13	1	28	N	DMS R	3CS	PRW				SUP	R
13	2	18	N	DMS R	VAR	PRW	USE		PRW/USE	SUP	R
13	4	16	N	DMS R	VAR	PRW				SUP	R
13	5	1	N	LOC	VAR	PRW				SUP	
13	7	21	N	DMS R	VAR	PRW				SUP	
			A	DMS R 51		PRW	USE	TMC	QM	SUP	R

Coding Raw Data
Discovery Book

			72	LOC	3CS	207	98	31	39	143	51
			N	168	89					NSUP	
			162	DMS	VAR					93	
			M	17	147				PRW/USE		
			2						86		
		Total	236	236	236					236	

Coding Raw Data
Standards-driven Book

Chapter	section	#	Label	Location	Variation	Realistic Considerations				Support	R
1	1	46	CT	LOC	VAR	TMC	USE			SUP	R
1	2	8	RWS	LOC	VAR	PRW				NSUP	
1	2	16	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
1	2	42-43	RWS	LOC	VAR	PRW				NSUP	
1	5	6	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
1	5	13	RWS	LOC	3CS	QM				SUP	R
1	5	26	RWS	LOC	3CS	TMC	QM			NSUP	
1	5	30	N	LOC	VAR	QM				NSUP	
1	5	32	RWS	LOC	VAR	PRW				SUP	R
1	5	44	N	LOC	VAR	QM				NSUP	
1	5	54	M	LOC	3CS	TMC				SUP	
1	6	11	RWS	LOC	VAR	QM	USE			NSUP	
1	6	21	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
1	6	28	RWS	LOC	3CS	PRW				NSUP	
1	7	7	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
2	1	7	RWS	LOC	VAR	PRW				NSUP	
2	1	16	RWS	LOC	3CS	TMC				NSUP	
2	1	28	M	LOC	VAR	PRW				NSUP	
2	1	34	CT	LOC	VAR	PRW				SUP	
2	1	41	RWS	LOC	VAR	PRW				NSUP	

Coding Raw Data
Standards-driven Book

2	1	42	RWS	LOC	VAR	PRW	QM			NSUP	
2	3	8	N	LOC	VAR	PRW				NSUP	
2	3	13	N	LOC	VAR	PRW				NSUP	
2	3	14	RWS	LOC	VAR	PRW				NSUP	
2	3	15-18	N	LOC	VAR	PRW				NSUP	
2	5	34	RWS	LOC	3CS	QM				NSUP	
2	5	35	RWS	LOC	3CS	PRW	QM			NSUP	
3	3	40	RWS	LOC	3CS	PRW				SUP	
3	4	28	WAI	LOC	VAR	PRW				NSUP	
3	5	25	N	LOC	VAR	PRW				NSUP	
3	6	12	RWS	LOC	3CS	PRW				SUP	
3	6	23	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
3	6	45	RWS	LOC	3CS	PRW				NSUP	
4	1	12	N	LOC	VAR	TMC				NSUP	
4	1	25	RWS	LOC	VAR	QM				SUP	
4	1	28	RWS	LOC	VAR	QM				SUP	
4	1	32	RWS	LOC	VAR	PRW	TMC			NSUP	
4	1	38	RWS	LOC	VAR	PRW	TMC			SUP	
4	2	11	RWS	LOC	3CS	TMC	PRW	USE		SUP	
4	2	22	RWS	LOC	3CS	TMC	USE	QM		NSUP	
4	2	33	RWS	LOC	3CS	QM				NSUP	

Coding Raw Data
Standards-driven Book

4	3	15	RWS	LOC	3CS	TMC				NSUP
4	5	25	RWS	LOC	VAR	QM	USE			SUP
4	7	23	WAI	LOC	VAR	QM				NSUP
5	1	18	RWS	LOC	VAR	PRW	USE		PRW/USE	SUP
5	1	37	SR	LOC	VAR	PRW				SUP
5	2	11	RWS	LOC	VAR	PRW	USE	QM		SUP
5	2	20	RWS	LOC	VAR	PRW	USE		PRW/USE	SUP
5	4	29	M	LOC	VAR	USE				SUP
5	5	15	RWS	LOC	VAR	QM				NSUP
5	5	59	N	LOC	VAR	QM	USE			NSUP
5	6	15	RWS	LOC	VAR	QM				SUP
5	6	33	SR	LOC	VAR	PRW				SUP
5	7	5	RWS	LOC	3CS	PRW				NSUP
5	7	18	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP
5	7	28	RWS	LOC	VAR	PRW	USE		PRW/USE	SUP
5	7	36	RWS	LOC	3CS	PRW				NSUP
5	8	8	RWS	LOC	3CS	PRW				NSUP
5	8	12	RWS	LOC	VAR	TMC				NSUP
5	8	16	RWS	LOC	VAR	PRW				NSUP
5	8	22	M	LOC	3CS	PRW				NSUP
6	3	25	RWS	LOC	VAR	QM				SUP
6	3	38	WAI	LOC	VAR	PRW				SUP

Coding Raw Data
Standards-driven Book

6	4	33	RWS	LOC	VAR	TMC	QM			SUP	R
6	5	1	RWS	LOC	VAR	TMC				SUP	
6	5	6	RWS	LOC	VAR	USE				SUP	
6	5	44	RWS	LOC	VAR	QM				SUP	
6	6	3	RWS	LOC	VAR	TMC	USE			NSUP	
6	6	13	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
6	6	26	M	LOC	VAR	USE	PRW		PRW/USE	NSUP	
7	1	6	RWS	LOC	3CS	TMC				NSUP	
7	1	11	RWS	LOC	3CS	PRW				NSUP	
7	1	21	M	LOC	VAR	PRW				NSUP	
7	1	26	N	LOC	VAR	TMC	QM	PRW		NSUP	
7	1	30	RWS	LOC	3CS	QM	PRW			NSUP	
7	2	13	N	LOC	VAR	PRW	USE		PRW/USE	SUP	
7	2	23	RWS	LOC	VAR	PRW	USE		PRW/USE	SUP	
7	3	19	RWS	LOC	3CS	TMC				NSUP	
7	4	5	RWS	LOC	3CS	PRW				NSUP	
7	4	28	RWS	LOC	VAR	PRW				NSUP	
7	5	2	RWS	LOC	VAR	PRW				NSUP	
7	5	7-9	MS	LOC	VAR	PRW				SUP	
7	5	12	M	LOC	3CS	PRW				SUP	
7	5	15-17	MS	LOC	VAR	PRW				SUP	

Coding Raw Data
Standards-driven Book

7	5	20-23	M	LOC	VAR	PRW				NSUP	
7	5	27	RWS	LOC	3CS	PRW				NSUP	
7	5	30	RWS	LOC	VAR	PRW				SUP	
7	5	31	N	LOC	3CS	PRW	QM			NSUP	
7	5	32	M	LOC	VAR	PRW				NSUP	
7	5	33	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
7	5	43	RWS	LOC	3CS	PRW				NSUP	
7	6	3	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
7	6	10	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
8	1	40	RWS	LOC	3CS	TMC	PRW			SUP	R
8	2	43	RWS	LOC	3CS	PRW				NSUP	
8	2	48	RWS	LOC	3CS	USE	PRW		PRW/USE	NSUP	
8	2	55	RWS	LOC	3CS	TMC				NSUP	
8	3	20	RWS	LOC	3CS	PRW				NSUP	
8	3	38	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
8	3	47	N	LOC	VAR	PRW	USE		PRW/USE	NSUP	
8	3	52	RWS	LOC	3CS	PRW				NSUP	
8	3	68	N	LOC	3CS	PRW				NSUP	
8	3	74	N	LOC	3CS	PRW				NSUP	
8	4	8	RWS	LOC	3CS	PRW	USE		PRW/USE	SUP	
8	4	9	RWS	LOC	3CS	PRW				NSUP	

Coding Raw Data
Standards-driven Book

8	4	14	RWS	LOC	3CS	PRW				NSUP	
8	4	15	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
8	4	16	RWS	LOC	3CS	PRW				NSUP	
8	4	25	MS	LOC	3CS	PRW				NSUP	
8	4	26	WAI	LOC	VAR	PRW				SUP	
8	4	27	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
8	4	30	SR	LOC	VAR	PRW				SUP	
8	4	31	N	LOC	3CS	PRW				NSUP	
8	4	32	N	LOC	3CS	PRW				NSUP	
8	4	33	N	LOC	3CS	PRW				NSUP	
8	4	34	N	LOC	3CS	PRW				NSUP	
8	5	16	RWS	LOC	3CS	PRW				NSUP	
8	5	62	MS	LOC	3CS	TMC	PRW	QM		NSUP	
8	5	64	RWS	LOC	3CS	PRW				NSUP	
8	6	17	RWS	LOC	3CS	PRW	QM			NSUP	
8	6	31	RWS	LOC	3CS	QM	PRW			SUP	R
8	6	42	RWS	LOC	3CS	TMC	QM			NSUP	
8	6	67	RWS	LOC	3CS	PRW				NSUP	
8	6	68	N	LOC	3CS	PRW	QM			NSUP	
9	1	8	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
9	1	19	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
9	1	27	RWS	LOC	VAR	PRW				NSUP	

Coding Raw Data
Standards-driven Book

9	2	20	RWS	LOC	VAR	USE	TMC	QM		SUP	
9	2	28	RWS	LOC	VAR	USE	PRW		PRW/USE	SUP	R
9	3	11	RWS	LOC	VAR	USE	TMC	QM		NSUP	
9	3	22	RWS	LOC	3CS	TMC	QM			NSUP	
9	3	35	RWS	LOC	VAR	USE	PRW		PRW/USE	NSUP	
9	3	45	RWS	LOC	3CS	USE	QM			NSUP	
9	4	11	RWS	LOC	VAR	PRW	USE		PRW/USE	SUP	
9	5	19	RWS	LOC	VAR	PRW				NSUP	
9	5	43	RWS	LOC	3CS	QM				NSUP	
9	5	45	RWS	LOC	VAR	QM				NSUP	
9	6	3-5	RWS	LOC	VAR	QM				NSUP	
9	6	15-17	RWS	LOC	VAR	QM				NSUP	
9	6	43	RWS	LOC	VAR	PRW				SUP	R
9	7	8	RWS	LOC	3CS	PRW				NSUP	
9	7	19	RWS	LOC	3CS	PRW				NSUP	
9	7	30	N	LOC	3CS	TMC	PRW			NSUP	
9	7	32	RWS	LOC	3CS	TMC	PRW			NSUP	
9	7	35	RWS	LOC	3CS	PRW				NSUP	
9	7	49	SR	LOC	3CS	PRW				NSUP	
10	1	10	RWS	LOC	VAR	TMC	QM			NSUP	
10	1	29	N	LOC	3CS	PRW				SUP	

Coding Raw Data
Standards-driven Book

10	1	48	RWS	LOC	3CS	PRW				NSUP	
10	1	49	RWS	LOC	3CS	USE	PRW		PRW/USE	NSUP	
10	1	56	N	LOC	3CS	USE	PRW		PRW/USE	NSUP	
10	2	5	RWS	LOC	3CS	QM				SUP	
10	2	13	RWS	LOC	3CS	QM				NSUP	
10	2	32	RWS	LOC	3CS	TMC	PRW			NSUP	
10	2	38	MS	LOC	3CS	PRW				SUP	
10	2	39	N	LOC	3CS	PRW				NSUP	
10	2	40	RWS	LOC	3CS	PRW				NSUP	
10	2	41	CT	LOC	VAR	PRW	USE			SUP	R
10	3	6	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
10	3	13	RWS	LOC	3CS	USE				NSUP	
10	3	21	RWS	LOC	VAR	PRW				NSUP	
10	3	23	N	LOC	VAR	PRW				NSUP	
10	5	7	RWS	LOC	3CS	PRW	USE			SUP	
10	5	14	RWS	LOC	3CS	QM				NSUP	
10	5	23	RWS	LOC	3CS	PRW				NSUP	
10	5	29	N	LOC	3CS	PRW				SUP	
10	6	6-7	RWS	LOC	3CS	USE				NSUP	
10	6	20-22	RWS	LOC	3CS	PRW	USE		PRW/USE	SUP	
10	6	38	RWS	LOC	3CS	PRW	USE		PRW/USE	SUP	

Coding Raw Data
Standards-driven Book

10	6	44	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
10	6	48	SR	LOC	3CS	PRW				NSUP	
10	6	50	N	LOC	3CS	PRW				NSUP	
11	1	11-12	RWS	LOC	VAR	PRW				NSUP	
11	1	22-23	RWS	LOC	VAR	PRW				NSUP	
11	1	37	N	LOC	VAR	PRW	QM			SUP	
11	1	48-50	RWS	LOC	VAR	PRW	USE		PRW/USE	NSUP	
11	2	5	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
11	2	16	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
11	2	25	RWS	LOC	VAR	PRW	USE	QM		NSUP	
11	2	26	N	LOC	3CS	PRW				NSUP	
11	2	31	N	LOC	3CS	PRW	USE		PRW/USE	NSUP	
11	2	32	RWS	LOC	3CS	PRW	USE		PRW/USE	NSUP	
11	2	33	RWS	LOC	3CS	PRW				NSUP	
11	2	34	RWS	LOC	3CS	PRW				NSUP	
11	3	5	RWS	LOC	3CS	PRW				NSUP	
11	3	16	RWS	LOC	3CS	PRW				NSUP	
11	3	41	N	LOC	3CS	PRW				NSUP	
11	4	16	RWS	LOC	3CS	QM				NSUP	
11	4	28	M	LOC	3CS	PRW				NSUP	

Coding Raw Data
Standards-driven Book

11	4	29-32	RWS	LOC	VAR	PRW	QM			NSUP	
11	4	33	RWS	LOC	3CS	PRW				NSUP	
11	4	34	RWS	LOC	3CS	PRW				SUP	R
11	4	35-38	RWS	LOC	VAR	PRW				NSUP	
11	4	41	N	LOC	3CS	PRW				SUP	
11	4	45	RWS	LOC	3CS	QM				NSUP	
12	1	8	RWS	LOC	3CS	PRW				NSUP	
12	1	15	RWS	LOC	3CS	PRW				SUP	
12	1	43	RWS	LOC	3CS	QM				NSUP	
12	2	36	RWS	LOC	VAR	TMC	QM			SUP	
12	2	46	WAI	LOC	VAR	QM				NSUP	
12	3	5	RWS	LOC	3CS	PRW				NSUP	
12	3	15	RWS	LOC	3CS	QM				NSUP	
12	3	29	N	LOC	3CS	QM				NSUP	
12	3	37	M	DMS R	3CS	PRW				NSUP	
12	5	11	RWS	LOC	3CS	PRW				NSUP	
12	6	15	RWS	LOC	VAR	USE				NSUP	
12	6	26	RWS	LOC	3CS	PRW	QM			SUP	
12	7	37	RWS	LOC	VAR	TMC	QM			NSUP	

Coding Raw Data
Standards-driven Book

			RWC 145	LOC 207	3CS 117	PRW 153	QM 53	TMC 30	USE 57	SUP 55	R 10
			CT	DMS R	VAR					N	
			3	1	91					153	
			WAI						PRW/USE		
			5						38		
			MS								
			5								
			SR								
			5								
			M								
			11								
			None								
			34								
		Total	208								

Appendix C: Researcher's Journal

The researcher's journal contains my reflections and interactions with the data. I have written in the journal about the coding – how the categories and subcategories emerged. I have also written about some of the experiences I have had as a practitioner and other information that may be found to be pertinent to this study.

4/23/2014

Students do not always have the same outlook on a word problem. For example: suppose you are discussing a word problem on area of a deck and you ask the class why they might need to know the area of the deck. I teach in a school in a more affluent school district, and a student answered “Bragging rights. The owner can brag about how large his deck is.” (The deck in this instance was only 30 square feet. I believe he had no perception as to the actual size the dimensions represent.) When pressed further, “what about painting or staining the deck?” these same students answer “We have a guy for that. . . I don’t need to know. . .” I continued my questioning further, “What if the worker buys 10 cans instead of three and charges you for all of them?” Their answer was: “He works hard, he deserves the extra cash.” (!!!) I realize that students from other life experiences will have different points of view. Other students who might do the manual labor would take more interest in the problem.

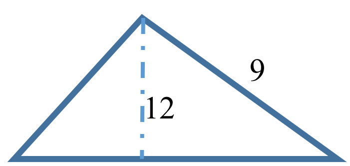
4/29/2014

I read an article in 5/2014 Mathematics Teacher on authenticity (Tran and Dougherty). Interesting word problem: If 360 students need to go on a bus trip, 48 students per bus, how many buses are needed? The claim in the article is that 7.5 is the correct number of buses, but how do you order ½ of a bus? It was never explained in the article that the students needed to know to order 8 buses, or at least 8 drivers if you count a van or short bus as a half of a bus!

Palm (2008) Framework for authenticity (p.40) distinguishing between tasks in terms of: 1. authenticity and 2. For developing tasks at the highest possible authenticity.

I recall problems from Unit 4 in our current high school geometry curriculum: Students find the angle of elevation to the top of a tree, given the height of the tree and the distance from the tree. Or a shadow problem, given the height of the tree, find the height of the building. My contention is that the height of the tree is constantly changing if it is a live specimen. Furthermore, in different seasons it can appear taller or shorter depending on leaf growth. It would seem more reasonable that the problem should ask for the height of the tree, perhaps to track its growth over time. The heights of many buildings are available online (Wikipedia, etc.)

Another problem I recently encountered was a problem regarding a triangular sandbox where the dimensions given were impossible: the height of the triangle was longer than the sides of a scalene triangle, similar to the following diagram:



In writing the literature review, I was reminded of a problem I assigned to my Honors pre-calculus class several years ago. The problem situation involved a quarterback throwing a football in a path with the shape of a parabola. Students got an answer that showed that the Quarterback could throw the football over 3 million feet! Of course, this led us to a discussion about reasonableness of solutions. This shows that the students are not engaging with the real-world scenario as they solve the mathematical problem.

4/30/2014

I am also reminded of a poster that was popular a few years back “When are We Ever Gonna Have to Use This?” I will admit to having this poster in my classroom for quite a few years, but I guess I have become less complacent over the years. I have issues with the word “gonna”, apparently now accepted as a word.

Beyond that, the poster is merely a list of occupations with corresponding mathematical topics which may be used in each occupation. The poster must be outdated – the book it is based on was last updated in 1988 – yet it is still available. Although the poster was meant to motivate student interest in mathematics by connecting it to possible careers, the poster provided no support for teachers in engaging students in discourse.

5/1/2014

Some possible personal outcomes of my research:

Authentic problem-solving examples in workbook form,“how-to” create authentic problem-solving examples to use in class, reasons for authentic problem-solving with justification for different career paths (college or other),teacher preparation programs in higher education.

5/4/2014

Realistic situations for word problems

Pizza: Which is the better price for pizza, large or extra-large pie? Choose your toppings, Decide between Sicilian or regular, Decide between 2 pizzerias

Area of a deck: Cost of painting or staining, Perimeter for fence or railing

Area of a field to re-seed

Compound interest problems

Number of buses (Palm)

Find the height of a tree (not the angle of elevation)

Estimate the height of a tree using shadows (not height of a building using the tree's height)

You want to play a prank on your brother and fill his room with balloons. How many balloons would be needed to successfully pull off this prank?

Post-it's to "wallpaper" someone's office or school locker (as seen on the internet)

Whether or not these problems will be used in real life, these are reasonable problems to solve. No Super-Quarterback throwing a football for three million feet (over 568 miles).

No problems where Superman saves the day by traveling faster than the speed of light.

Real world problems will:

Be relevant

Make students read and make sense of the problem (how often have we heard that "I didn't read the problem. I didn't know it was supposed to be a sandbox."

5/12/2014

Meeting with Dr. Fernandez. different options were discussed as possible research paths –

Option 1: Action research: A study including: lesson prep, knowledge domain, topic selection, lesson implementation, student questions, ability to respond, reflection

Data triangulation – someone else to observe

Option 2: Curricular Analysis

Analyzing textbook

Teacher knowledge

6/27/2014

Question: can I analyze my teacher knowledge and use of reasoning in the classroom?

For example, I usually begin the geometry class with questions like:

“How much dirt is in a hole 6½ feet wide by 3¾ feet long and 5½ feet deep?”

I also do not advocate students memorizing “tricks”:

Memorizing the coordinate notation for reflections over the x- and y-axes, or for 90-, 180- and 270-degree rotations about the origin. I prefer that they understand and reason through the transformations as they complete them.

10/1/2016

In determining the data sample, (the word problems with real-world context), I read through every problem in the Exercises of each section of the two textbooks.

In the “Discovering Geometry” textbook (Serra, 2008) there were 303 problems with a real-life connection. The Holt McDougal textbook (Burger, et al., 2012) has 552 such problems.

However, after considering the problems a bit further, I contend that some of the problems only mention a real-life context, but do not require student solvers to engage with the context. Some problems do not delve into the real-life connection and so, neither will the students. In actual practice, I believe that students would read the problem, solve the mathematical question, and never consider the real-life aspects of the problem. Examples of these types of problems are given in Figures 1 and 2 in chapter 3 of my dissertation.

I have decided that there is a large enough sample that these problems can be excluded from the analysis. After all, they will not encourage students to use *realistic considerations*.

In some of the problems, an algebraic expression or equation is given to represent the measure of a segment or angle. No reasoning or explanation is given for the assignment of the algebraic expression to the object. Again, students are not given the opportunity to work through the "procedural complexity" (Green & Emerson, 2010) and these problems are reduced to one or two steps.

On another note, some of the real-life word problems I chose to include in the sample have multiple questions about a single problem situation. These were numbered as a single problem with parts a, b, and c. However, in at least two or three instances, a problem situation was presented followed by 4 to 12 different numbered questions that were related to the situation. I have chosen to consolidate the problems from the latter situation into one problem to match criteria from the majority of the sample set. An example of this type of problem cluster appears in Figure 3 in Chapter 3.

This consolidation will affect the tally of the problems in each textbook. The current tallies are Serra: 262 and Holt McDougal: 345.

10/4/2016

The problems for discussion, description, and elaboration in the analysis have been chosen. These problems will be analyzed along several categories as posed by my initial coding scheme: *realistic considerations, alternative approaches, procedural complexity, label,* and *variation.*

Other categories may emerge during my analysis of the data (the word problems).

10/19/16

Each problem will be discussed in detail with regard to each of the categories. I will be looking for trends or themes as I elaborate on each one.

I had originally planned to execute a textbook analysis using only one textbook, but it was suggested by my committee that I compare two textbooks. I have chosen one "discovery" textbook which embraces the constructivist theory of learning. A more complete description appears in chapter 3 of my dissertation. The other textbook is "standards-driven" which is more common according to Ginsberg et al., (2005). Its sequence of chapters remains mostly unchanged from the publisher's previous geometry textbooks. I have chosen two geometry textbooks because of the similarities in the scope of mathematical topics. Also, since geometry is an often-required course for ninth- and tenth-graders, the curricula are more likely to be similar in breadth and depth.

One of the differences I have noted about the two books is the sequence of topics. Most geometry begin with basic terms and deductive reasoning. The "discovery" book in my study begins with art-geometry connections and basic terms. This book moves through other topics and culminates with deductive reasoning and proof.

10/20/2016

The next step in analyzing the data will be to write a thick, rich description of each problem. This description will include my solution of the problem, followed by categorizing the problem using the codes in the qualitative codebook (Appendix A). I will be looking for relevant trends as I elaborate on the problems. This may result in creating new categories for the codebook.

I have already noticed that one book has several sections exclusively for word problems. This separation from general mathematics knowledge questions may prevent teachers from assigning them or using them with their classes (Verschaffel). When faced with a decision of whether to spend a day or two on these problems, rather than move on to a new concept or lesson, many teachers will favor moving on in the interests of time and covering the curriculum. These trends in mathematics education have been noted in my literature review.

11/13/16

One of the criticisms of traditional textbooks is that the word problems were often placed last or near the end of each set of Exercises. In collecting the data (reading through all of the problems in both textbooks), it is evident that word problems are not relegated to that last position. Problems with a real-life context appear in a variety of positions throughout the textbooks.

January 8, 2017

After I had categorized some of the problems from each textbook as having a real-life connection, I started looking for ways to pair problems from the two books for my analysis.

The Serra textbook's "A note to teachers from the author" asserts their claim that "when students are given the opportunity to be actively involved in their own discovery of mathematics, they become better problem solvers and develop a deeper understanding of the concepts" (p. xv). They further claim that the textbook's "concepts are connected to a story that explains how and why these geometry properties came to be".

I found this disconcerting as I read the beginning of section 1 in chapter 4 of the textbook. The section is titled "Triangle Sum Conjecture" and the second paragraph reads:

> Another application of triangles is a procedure used in surveying called triangulation. This process allows surveyors to locate points or positions on a map by measuring angles and distances and creating a network of triangles. Triangulation is based on an important property of plane geometry that you will discover in this lesson." (Serra, p. 200)

I found no further mention of triangulation or surveying in the lesson. Further, there were no notes to the teacher in the textbook's interleaf for the chapter or anywhere else in this lesson. I was disappointed in the lack of closure on this topic after the introduction to this lesson mentions a real-life connection to a particular career. The text does not tell how the measures of the interior angles sum of 180 degrees is used in surveying.

This issue has been noted over time as noted in the literature review:

Research shows that many teachers tend to use the textbook as their main curriculum resource (Chval, Chavez, Reys, & Tarr, 2009; Hiebert, et al., 2003; McClain, Zhao, Visnovska, & Bowen, 2009; Nicol & Crespo, 2006). They rely on the textbook for lesson cues and problems in the exercise sets for student work. The Freudenthal Institute for Realistic Mathematics Education (Utrecht University, 2016) maintains that textbooks should be regarded as valuable resources for teachers, especially when textbooks support teachers by providing realistic word problems that can be used within the mathematics classroom.

On another note, I have also found that *Discovering Geometry* has several sections throughout the book where the Exercises are exclusively word problems. This book’s purpose is to be different from other textbooks, to use discovery in cooperative learning situations rather than teacher-directed lecture and demonstration. With this purpose in mind, I believe that the curriculum of the textbook should provide support for teachers in creating a more student-centered approach to learning mathematics. However, when word problems are spotlighted as the only questions in a section, this can create several problems.

First, in the interest of time, a section which contains only word problems might be omitted by the classroom teacher. Second, a section of only word problems upholds a separation between mathematical concepts and their application to real-world situations. This reinforces the arguments of Verschaffel, Greer, and DeCorte (2000) that this way of organizing a curriculum maintains the separation of school mathematics and mathematics in the real world.

January 16, 2017

I am reading through all of the problems that were identified as real-world problems. In doing so, I have also found some questions through the Serra textbook that give the students an example of a geometric concept in a real-world context, and then ask them to provide a different one. For example, "An ice skater gliding in one direction creates several translation transformations. Give another real-world example of translation" (Serra, p. 373). Problems like these do not engage the students in truly considering the given real-world context, and directs students to find another example. The exercise does not prompt them to make connections other than to identify another context.

An excerpt from the movie, "The Wizard of Oz", is included in Serra's book on page 491. "In an isosceles triangle, the sum of the square roots of the two equal sides is equal to the square root of the third side". The validity of this statement is not discussed at all. Teachers are not directed to discuss or lead an investigation of this statement in the section or in the chapter interleaf. No exercise exists within this section for the students to investigate the validity of the Scarecrow's statement. Given that the students are cognizant of the rules for squares and square roots, this type of investigation is within their capabilities. Students should be aware that every statement in a movie, even a classic movie like The Wizard of Oz, can be questioned on its validity.

February 15, 2017

I spoke with a physics teacher in regard to the ADA Wheelchair Problem and the added science connection and question about the constant force needed to push a 200 lb wheelchair up a ramp. We discussed the necessity of knowing the constant force. The best reason we could muster would be if the wheelchair occupant had injured his arms and was told by a doctor not to lift more than x lbs or not to exert too much force. However, in this day and age of motorized wheelchairs, this scenario seems unlikely.

February 16, 2017

After defending my proposal, the discussion with my dissertation committee helped to shed some light on particulars of my research. I can more clearly define the process of analyzing the word problems. The steps that I used in choosing my sample:

Step 1: I located word problems in the Exercises of each regular section of the textbooks.

Step 2: I identified those word problems which are presented or set up using a real-world context, designed so that students will apply their mathematical knowledge to real-life situations.

Step 3: I narrowed the sample by choosing only those problems that required interaction with both the mathematics and the real-world context. Excluded from the sample are those problems that ask questions not necessary in real-life situations, word problems which include a pre-determined equation or a superimposed diagram, and those problems posed in a way inconsistent with real-world problem-solving, i.e., multiple choice questions.

In addition, our discussion helped me to focus the audience to include all practitioners, including teachers, curriculum developers, teacher educators, textbook publishers.

February 20, 2017

I am investigating the Iguana Pen problem from the Serra book today. The problem asks “What is the area of the largest rectangular pen that he can make with 100 meters of fencing?” The pen would turn out to be 82 feet on each side. I recall a property I once owned which was an 80-by-100 ft lot. This iguana enclosure would not have fit in the backyard of my home, yet that property was not considered to be small. Although the question asked about the largest rectangular pen possible, a smaller pen could be made, with fencing left over.

I now question how textbook questions are written. This particular problem appeared quite simply as “Ernesto plans to build a pen for his pet iguana. What is the area of the largest rectangular pen that he can make with 100 meters of fencing?” (Serra, 2008, p. 426). I wonder if it was hastily written with the intention of re-wording it or re-working it for the real-life situation it purportedly represents. I am thinking that 100 meters of fencing may have been a more appropriate measure for other pets, perhaps a large dog, leaving plenty of running area.

Did the author of the problem intentionally use an arbitrarily large number (100 meters) with the intention of investigating the issue and editing it at a later time? Was it overlooked in the process of publishing the textbook? Was it the author’s intent to elaborate on the problem itself? Or was it the author’s intent to leave the investigation for students? The problem is rather simply stated without much detail. Without more details of the real-life situation, the connection of the mathematics to the situation is superficial at best.

Label of application. In addition, I will need to examine the *label* of "Application" more closely. In at least one instance, the Iguana Pen problem in the Serra book, "Application" leaned toward a meaning of applying a geometric concept to an algebraic one, rather than connecting the mathematical content to the real-life context.

New Category: Location. I am considering a new code, *location*, to signify whether a problem is located within the textbook where the mathematical content that is expected to be applied is first introduced. I think that it is important in the consideration of the other codes. For example, if the problem is not located in a section that even mentions a particular mathematical concept, then it would encourage *alternative approaches*.

In addition, the issue of location is substantiated in the literature review: The activity of solving school mathematics word problems differs vastly from problem solving in real-life situations. Students often believe that the only way to solve word problems is to use the mathematics that they have learned in the classroom. This belief is first encouraged in elementary school students as they learn the basic operations, and then the word problems presented to them require the use of those operations (Balacheff, 1986; Verschaffel, Greer, & DeCorte, 2000). This practice continues through middle school and high school as word problems are placed within the textbook chapter or section containing the mathematics topics to be used in their solution (Bruer, 1993).

Location as a category is important because this can influence choice of mathematical content to apply to a problem. When a problem is not located in the section where the math content is introduced, it allows students to use their problem-solving skills of choice and recall. They will choose what to apply and incorporate prior knowledge. Problems such as these would foster problem-solving as it occurs in the real world: problem-solving in careers and everyday events is never labeled with a math content to apply.

February 24, 2017

> I find that students have been trained to find a single solution to a word problem. When they are faced with a problem which has multiple solutions, or a range of solutions, they skip that problem and wait for someone to give them “the correct answer”. For example, this problem was posed:
>
> “PUPPIES. Meredith’s new Pomeranian puppy is 7 inches tall and 9 inches long. She wants to make a drawing of her new Pomeranian to put in her locker. If the sheet of paper she is using is 3 inches by 5 inches, find an approximate scale factor for Meredith to use in her drawing” (Glencoe Geometry, p. 47).

I gave this problem on a worksheet to a group of Honors geometry students. I found that many of them left this problem blank.

The solution manual gives this solution: “1: 2.5 or 2:5 (answers may vary. But should be at least greater than 1:2.34)”.

February 28, 2017

Looking over the problems in section 1.7 Circles (Serra, 2008) regarding the Earth’s revolution about the Sun and Earth’s rotation on its axis, I wonder about real-life applications that require the information in the problems. I consider both problems to have a real-life context and each has a single numerical solution. But, how can this information be applied to other situations? Can this information be used in shadow problems, the kind we solve with proportions and similarity or trigonometric ratios? What other questions could be asked of high school students that relate circles to the solar system?

3/4/2017 In chapter 5 of the discovery book, one section leads the students through investigations of properties of special parallelograms. The investigations leave the theorems and postulates with blanks for the students to fill in. (This is a textbook, not a workbook.) As I was working through one of the problems and reading through the section again and again, I began to think about how teachers use, or don’t use, the textbook resources.

I have known teachers who use every resource that is supplied with the textbook. These teachers will photocopy every available worksheet and distribute them to the students. They fully rely on the textbook for support. I wonder if these teachers would use investigations of this kind if they were available. I am aware that some of them will continue to use the previously-used textbook if the current district-approved textbook does not suit them. (I have heard of science teachers who have used the same textbook curriculum for over 30 years, even though there have been advances in science that would make that old textbook obsolete.)

On the other hand, in my experience, few students will read their mathematics textbooks. Some students who do read it will be disappointed that the answers are not immediately available to them – and the conjectures are not filled in for their “memorization needs”! Also, when the blanks are not filled in, the textbook is not a resource that students can reference for information.

I am thinking that teachers who use the Serra book would have to “buy-in” for the entire curriculum. It would not be possible to use this book as a student resource with so many “holes” in the mathematical content. The book presents an approach to teaching and learning geometry which is a departure from the didactical contract that has been a tradition in many classrooms.

Thoughts on real world problems

As I work through many of the word problems and consider them with connections to the aspects if realistic-ness, I have come to question the treatment of word problems in the classroom:

Do teachers need to use so many word problems about the same mathematical topic, in a way that the students see them as exercises, rather than individual problem situations? If teachers would choose just one real-world problem, and encourage students to use realistic considerations, it is possible that the students would still learn how to apply the concept of area and perimeter, and, additionally, they would understand that these concepts can be applied to real-life situations.

How can teachers be given the supports they need to be able to answer student questions of: “what if . . .?” And “how do we know that . . .?” I have seen situations where a student’s inquisitive nature has been squelched in the interest of time or because of the possible complexity of the student’s inquiry. Is this serving the best interest of the students?

In looking at many of the problems that are offered in high school geometry textbooks, I believe that many of the problems could be solved by students in middle school or even elementary school. I find that many of the problems are not multi-step problems and do not require higher-order thinking or problem-solving skills. Yet, we wish for students to be "college- and career-ready". High school students are capable of doing more difficult work, solving multiple-step problems, and relating school mathematics to situations that they will encounter in the real-world.

3/5/2017

After analyzing several word problems, I am re-thinking the selection of the problems with a real-world context. I began by selecting (and counting) all problems with a real-world context – that is, any real-world connection at all. And shortly afterward, I realized I needed to exclude certain types of problems (as mentioned in chapter 3) – those with superficial connections that focused the work on the mathematical concept, those that had a superimposed diagram or equation, and those that ask students to pick a multiple-choice answer. At the time when I was selecting and enumerating these problems, I was not analyzing them for the depth of their connection to the real-life situations.

As I am analyzing the problems now, I realize that quite a few more of them only touch on the real-life situation, and that the focus is mainly on the mathematics. There are questions that simply ask the students to identify the mathematics that they see in a photo, for example, a line of symmetry. These problems will also be excluded from the real-life context word problems in my study.

I have also found several problems where the question that connects to real life is actually a recall question (DOK level 1 or Bloom's taxonomy level 1): asking a question whose simple answer could be found within the lesson's reading.

On teacher support.

There are questions that ask students to find instances of the mathematics in real-life. This type of assignment assumes that all teachers are able to see every possible connection to real world events. There is no limit to what students may claim fits the assignment's objective. The assignment itself, however, may ask the students to explain the connection, but the teacher support, the TE, only offers "Answers may vary."

Not only do teachers need to "buy-in" to this curriculum, I believe that they would need to be extensively trained through PD. PD would give them opportunities to work with other teachers and to discuss the kinds of real-life instances the students might bring in to prepare the teachers for the discourse that will arise when discussing these homework items.

This lack of teacher support for the problems supports a new category for my analysis: *teacher support*. Since textbooks provide Teacher's Editions and teacher resources in many forms, it seems to me that the TE should provide assistance to teachers for each of the word problems. Word problems are often omitted, and lack of teacher support for *each* problem could be the justification that teachers use in their decision to omit them.

On location of a problem within an Exercises set

From my literature review: Textbooks are often arranged with word problems placed as the last item introduced in a lesson or unit. This way of organizing a curriculum creates a separation between acquiring knowledge and applying it, between knowing the mathematics and doing the mathematics (Freudenthal, 1968; Verschaffel, Greer, & DeCorte, 2000). Additionally, this organization can relegate the word problem to a secondary status and may even facilitate a teacher’s decision to skip it entirely (Balacheff, 1986; Freudenthal, 1968).

In my experiences during my career, I have noticed this arrangement quite often: problems meant as practice exercises first, followed by applications and word problems. Often, there had been times when the number of practice problems was quite sizeable, and since teachers had been concerned with students learning procedures and algorithms, often the word problems would be omitted in assignments and class discussions.

In beginning my analysis of the word problems in the two textbooks, I have determined that the placement within the problem sets is no longer the same concern it once was. Practice problems, in these two books in particular, are interspersed with word problems and practical applications. This issue of relegating word problems to last place in a problem set seems to have already been resolved.

3-5-17 **Puzzle problems**

Again, on the selection of word problems with a real-life context, I have found several problems which appear to have a real-life context but then turn out to have more of a puzzle-like quality to them. For instance, the narrative of one such problem had a person zigzagging through the woods, say 1 km N, then 1 km W, then 1 km N, and so on.

The question asks if the person will make it out of the woods by sunset. In a case like this, I think the real problem-solver is the hiker in the woods: can they find the best path to get out of the woods quickly?

Repeated problems

In creating the word problem sample, I will also be eliminating repeated problems. For example, in the Serra book, the Handshake Problem is introduced in section 2.3. The Exercises section includes a word problem of that type. In the Exercises in section 2.5, there are two more Handshake problems, repeating the same problem, but with different numbers of people involved. It is not necessary for me to analyze the same problem three times. The first will be included, but subsequent repeated problems will not be included in the study.

March 6, 2017

On Facebook today – one of my friends posted about helping her daughter do her math homework last night. The story in the problem was about the Class of 2014 planning a field trip. Her daughter completed the multi-step problem, but she asked why she needed to find out how many students were going on the trip if they had already graduated!

Apparently, this problem was written and possibly used several years ago. The problem could have been written so that it read “Your class is planning a field trip” instead. The didactical contract is in play here: the students are expected to use the information in the statement of the problem to solve it, but they are not expected to question any of that information. My friend’s daughter is correct in her reasoning, why is she trying to figure out the budget for a trip that has already happened? This is another case of the problem not being relevant or not including enough realistic information to motivate and engage students to solve it. The entire text of the problem is as follows:

> The class of 2014 is planning a field trip to Great Adventure. The class has a budget of $800 for this trip. Fifty-five percent of the budget must be spent on food and beverages. The remainder of the money is to be used for transportation. If transportation costs $2.25 per person, how many students will be able to participate? (Source unknown).

March 8, 2017

My work today brings me to the Kite Design problem in the Serra text (p. 431). The problem is labeled as an Application, which are described as a way to "help students practice newly acquired skills in a real-world context" (Serra, 2008, p. xxi). It seems to me that this problem gives away too much of the information that the students are capable of calculating on their own. The numbers available in the diagram leave only the area calculations for the students to complete.

Moreover, the problem describes the Mylar will be folded over string and glued in place, but the questions in this problem do not call for this information. I believe that these calculations are easily within the capabilities of high school students. The focus of the problem may be on the area of triangles, trapezoids, and kites (as the title of the section suggests), but I strongly believe that high school students are competent enough to comprehend the entire kite building process.

High school students should be able to take Eduardo's design from the problem and not only calculate the lengths of the diagonals for the balsa wood and the area of the Mylar, but they can calculate the total length of the string necessary. In fact, students at this level should also be able to calculate the cost of building the kite. In other problems, they are given the unit cost for materials and asked to find the total cost. So, it is not unreasonable to ask them to complete this for a problem like this with a real-life context.

The PD session I attended today at work also brought up this issue – that often we underestimate the abilities of the students. If we are able to challenge students at the level of their potential, rather than underestimating it, eventually the range of their potential can be broadened. This will make them college- and career-ready, so that they can become more productive members of society.

Problem solvers

How can we help students to become problem-solvers? What are some of the challenges? Again, I will mention our PD session today. A conversation among some of the mathematics teachers was about students and how they will skip word problems and wait for someone else to provide the solution.

I have experienced this with some students in my own classes. I assigned a word problem within their capability and asked them to work in groups to decipher the problem, come up with a plan for solving, and discuss. One group just sat and waited. When I asked them why they weren't discussing it, one student replied that they don't need to put in the effort, they will wait for me to tell them how to do it. My students were invoking the didactical contract – there is only one way to do the problem, and I was the source of the solution to the problem: I would tell them how I wanted them to complete it.

The truth is – I wanted them to use their knowledge, and I wanted to find out what they know about the problem situation. I wanted them to try different methods and to discuss different possibilities. I was trying to break the didactical contract in which the students have been indoctrinated over the years.

It is not a simple task to break the contract. Change will have to come over a period of time. Mathematical discourse and "math talks" are emerging as activities to be used in classrooms. (Student-to-student talk and discussions are the "latest trend" and the topic of our PD session today.) But, in order for these to become the norm, the traditional didactical contract must be broken.

Perhaps instead of assigning a group of word problems for homework or classwork, teachers could assign just ONE word problem. The students can persist in problem-solving, spending more quality time immersed in one problem, rather than rushing through a series of problems because there are so many and because they have other homework to do. In using just one problem, the students can investigate the problem to greater depths – looking up information in their textbook or other sources or on the Internet. They can bring their experiences with the real-life context to share with others. They can discuss this one problem with their parents, their peers, and others to get a fuller understanding. The students can discover options for solving, discuss assumptions that should and should not be made. They can discuss the ambiguities that may exist in the problem, or discuss how the wording of the problem can be altered to be more coherent.

The problems that adults will encounter in real-life situations will not necessarily fit the three-component structure (Gerofsky, 2004b). I think of work projects that involve architectural design of a building or event planning, and problems which are not necessarily mathematical in nature. Often in these cases, skeletal details are given at the onset, but then other details emerge that must be considered in the plan. This type of problem will not be presented in three sentences, but rather an aggregation of data, perhaps over time. Students need to have this type of experience in order to make them college- and career-ready.

In real life, problems are not given with prescribed methods for solving them – that’s why they are problems in the first place. If a prescribed method could be given, then the problem is not a problem – it is easily resolved. In real-life situations, alternative approaches are encouraged and expected. Problem solvers are those who can see alternative approaches, as well as alternative solutions.

Some of my most memorable students are those who have challenged the word problems they have been given to solve. My friend’s daughter who wants to know why the students who graduated 3 years ago are planning a class trip; students in my classes who realize that a 30-square-foot area is a good size for a table, but not for a deck to put a table on; those who question why they would need to calculate the height of their friend instead of just asking them. They are aware that some problems ask ridiculous questions, and some use unreasonable values to practice mathematical algorithms and skills. Some problems are posed as puzzles or just as recreational value. When students are accustomed to being faced with problems that just don't make sense, they find no value in spending their time to find the answer to a puzzle or a problem with ridiculous values.

Recreational problem in another textbook: a bear traveled 1 km south, then 1 km due west, then turned and traveled 1 km due north and returned to his starting point. What color is the bear? This certainly is an amusing problem (who expects that question? ☺), and makes the solver consider locations other than the Euclidean plane. This problem certainly can be used to motivate creative student thinking, and in turn, alternative approaches and solutions.

I have also begun the school year with problems such as this: "How much dirt is there in a hole that is 6 ½ feet long, 5 ⅓ feet wide and 3⅛ feet deep?" Some students will grab their calculators and start pushing buttons, others begin to panic because I have included fractions on the very first day. None of them have read the problem. They assume that they are in a math class, this is a math problem, they need to add or multiply, and that is it! But, the problem does not ask how much dirt will fit in the hole, it asks how much *is in the hole.* The answer is: there is no dirt in the hole.

I have several other problems that I often use in the beginning of the year. I want the students to consider the problems in different ways.

Another is this: "If the number of strawberries in a basket doubles every five minutes, and the basket is full at 10:00 AM, at what time is the basket ¼ full?"

In this instance, the students will say that they do not have enough information. They want numbers! How many strawberries did you start with? Or how many strawberries are in the basket at 10:00 AM? They don’t realize that those pieces of data are not necessary. Working backward, the basket would be half full at 9:55 and one-quarter full at 9:50 AM.

The last in this series of problems is usually the Sand Pile Problem: “A child is sitting on the beach playing in the sand. He has 2 ½ sand piles on his right, 3 sand piles on his left and 4 ⅓ sand piles directly in front of him. If he puts them all together, how many sand piles does the child have?” Usually, by this point, most students have caught on to my “game”. Nonetheless, I will still have some students grabbing their calculators and grumbling about fractions. The others are more than eager to shout “One!”

March 9, 2017

The latest buzzwords “student-centered”. How can we keep students engaged in student-to-student talk and mathematical discourse? How can we keep students engaged with the context and the mathematics throughout the process of problem-solving?

I firmly believe that the real-life context must be fully embedded in the mathematical problem when we tell students that they are solving real-world problems. The connection between the two should not be superficial. The problem should be presented with enough facts for it to be believable, and the facts should be verifiable through reliable sources. These sources are literally available at our fingertips, as well as our students, through their laptops and cell phones. There is no reason that students should be given a problem that only grazes the surface of a real-life context. Students are capable of more than we give them credit for.

"Notice and wonder" is another current trend in math education. This activity presents the "bones" of a problem, the background information for the problem situation. This could be in the form of a picture or diagram, or the "set-up" part of the problem. The students are then asked what they "notice" about the given information. This promotes mathematical discourse and gives students the opportunity to immerse themselves in the situation through a group discussion. This is followed by the question "What do you wonder about this situation?" This should elicit a conversation that further probes the problem's real-world context as well as the connections to any mathematical concepts.

March 9, 2017

Grace Hopper is credited with having said, "The most dangerous phrase in our language is 'We have always done it this way'." This statement speaks volumes, and can easily have been said about mathematics education and the didactical contract. Why do we expect students to do only the simplest word problems? "We have always done it this way." Why do we give students several practice exercises to complete followed by several word problems? "We have always done it this way."

We have seen that from the earliest recorded instances of word problems (ancient Babylonia between 2000 to 1600 BC), "we have always" used ridiculous values to be substituted into word problems. If we delve further into those same word problems, we may find that the real-life situations are only mentioned, and not fully part of the reasoning during the problem-solving process.

3/10/2017

Today I was re-assessing the sample of word problems from the two books to complete round two of my selection. One of the two books has an extensive chapter on probability, while the other does not. I question whether those problems should be included in the analysis. Also, are these problems truly embedded in a real-world context?

Second, the standards-based book has six sections of real-world connections, located at the end of the chapters. I have not these problems in the count of problems for this analysis, since these problems are not included in the Exercises of each section. I have also omitted the problems in the chapter reviews. All of these problems were intentionally left out of my study because they are often intentionally left out by teachers. Their location of these problems can influence teacher treatment of them, as evidenced by the literature (page 17 of proposal):

Textbooks are often arranged with word problems placed as the last item introduced in a lesson or unit. This way of organizing a curriculum creates a separation between acquiring knowledge and applying it, between knowing the mathematics and doing the mathematics (Freudenthal, 1968; Verschaffel, Greer, & DeCorte, 2000). Additionally, this organization can relegate the word problem to a secondary status and may even facilitate a teacher's decision to skip it entirely (Balacheff, 1986; Freudenthal, 1968).

The counts of the word problems for analysis now stand at 259 from the Burger et al., book and 245 from the Serra book. The change in counts resulted from eliminating problems using several considerations: problems with multiple choice solutions (not a realistic real-world occurrence); problems with diagrams overlaid upon pictures; and problems for which the real-world context was not carried through the entire problem. These eliminations will be justified after a reliability check with a peer.

3/11/17

The focus of the realistic-ness of word problems is not necessarily on college-readiness, but on career-readiness and real-life-readiness. The didactical contract, as it has been conceived, prepares students for school mathematics, including most college and university level mathematics. Under the didactical contract, the 3-component structure is often employed. One solution is expected. Students are directed to: "Show all work."

The realistic-ness of word problems that I am pursuing is the foundation for creating problem-solvers – not just mathematics problems – but real-world problems, everyday living problems, home improvement/repair problems, designing, planning, etc. Students need to learn how to approach a task and use their intuition and real-world knowledge along with school learning to be competent and successful in their careers and everyday life situations. Realistic word problems are a means to prepare them for these occurrences in their futures.

Many college graduates (and others) go on to work in the building or industrial trades. It is my belief that their mathematics classes in high school and college do not prepare them for the kind of problem-solving and decision-making inherent in those trades. It has been my experience that teachers tend to move on, skipping many word problems, to "get through the curriculum" and prepare the students for the next math class in succession. Time is not to be "wasted" by delving into a problem situation or spending more than minimal time on each problem.

And, teachers need supports to make this happen. This is a departure from the traditional didactical contract.

For example, most adults can calculate the area of a floor that they wish to cover with laminate flooring. But when they shop for the flooring, it is sold in boxes of x square feet. Now, they need to calculate how many boxes of laminate they need to purchase for their room. Furthermore, the instructions for installing the laminate floor suggest that you buy 10% more flooring than you have calculated as your need. (This extra is due to cutting the planks and remainder pieces that may not fit how and where you need them.) In most cases, of course, the amount of flooring you have calculated is not going to work out to 8 boxes exactly. Adding 10% most probably won't make it a whole number of boxes either. At what point is it allowable to round up or down? For how many adults is this a troublesome problem that they have difficulty with?

Buying the laminate is only one item that is needed to complete the flooring project. There is the underlayment: the cushioning between the floor and the laminate; the molding, for the perimeter of the room which also holds the laminate in place; room transitions: the pieces between the rooms where the laminate may meet tile, carpet, or another floor surface.

These other components of the flooring project involve calculations that can be easily done by students at the high school and middle school levels. With a little bit of description or a short video clip, the components of the project can be provided for them.

A more comprehensive problem could ask the students to compute the total cost of the flooring project. After all, isn’t this the kind of information a homeowner needs? Isn’t this within the capabilities of most middle or high school students? Why aren’t there more problems that would allow them to make the connections between school mathematics and the real world?

The trouble with many textbook word problems is that the focus is on the mathematics and little attention is given to the details of the real-world situation. This is an example of the separation between school mathematics and mathematics in the real world. But, which is the driving force? Is this separation an effect of the didactical contract? Or is it one of the causes?

Can word problems transcend time? Yes! Of course, there are still some word problems that will not be relevant for most students: those about record players or long distance phone service of yesteryear. But, with the internet, there is so much information available and it is constantly updated: the most recent guidelines from OSHA and the ADA, the cost of tickets for a field trip, the cost of materials for a home improvement project, all literally at your fingertips. The amount of real-time data is infinite and using the internet as a resource makes the practice of problem-solving relevant and realistic. I believe that students and teachers alike will see more value in spending time on problem-solving if the results are realistic and relevant.

Chapter	section	prob #	Label	Location	Variation	Alternative Approaches	Realistic Considerations	Support
0	1	8	N	Y	Y	Y	Y	Y

Coding: I will use this row above to illustrate how I have coded the problems.

8. Shah Jahan, Mughal emperor of India from 1628 to 1658, had the beautiful Taj Mahal built in memory of his wife, Mumtaz Mahal. Its architect, Ustad Ahmad Lahori, designed it with perfect symmetry. Describe two lines of symmetry in this photo. How does the design of the building's grounds give this view of the Taj Mahal even more symmetry than the building itself has? (Serra, 2008, p. 6)

Analysis. This problem is not *labeled* at the beginning of the problem. It is *located* in a section titled "Geometry in Nature and Art" where the focus is on symmetry. The notes to the teacher for "Using This Chapter" (page 1B) explain that "students are likely to have enough previous experience with and intuitive grasp of symmetry to answer relevant review questions". This section is the very first in the textbook; it is likely that students will use their intuition about symmetry to formulate an answer for this question.

This problem *varies* from the three-component structure of the set-up, information, and the question. There are no numerical values for calculations, and no calculations are necessary. The problem demands a description of two lines of symmetry, and then asks how the design affects the view of the building. The question of "How?" is a departure from most word problems which usually ask for answer to a mathematics calculation.

Since this question appears in the first section of Exercises for the students, if they are not given much direction for expectations when completing assignments, they may conform to the didactical contract and provide what they think the teacher expects. Usually, this will be a very brief response to the question, and sometimes, no response at all, since there is no calculation involved. This problem is one that invites an *alternative approach*, asking "how" rather than "what". Its inclusion so early in the textbook curriculum should encourge students and teachers alike to consider alternative approaches throughout.

Using a real photograph of an existing building should inspire the use of *realistic considerations*. To promote connections to this example of historically-important architecture, this problem could ask other questions regarding the architectural design: Was the symmetry a requirement given by Shah Jahan, or was it the architect's idea? What other symmetry is present in the Taj Mahal? Are there any intentionally asymmetrical elements in the design? What aesthetic purpose does symmetry have in architecture? These questions would further promote both realistic considerations and alternative approaches throughout the textbook curriculum. Furthermore, high school students are capable of investigating these kinds of questions in detail with the Internet so easily accessible.

The TE gives a detailed response to the question. The only mention of the "building's grounds" is the reflecting pool, but the other landscape elements can also be included as part of a student's response to this question. There are also other side notes within the section that provide *teacher support* and may inspire more questions for the teacher during a discussion of the student responses to this question.

3/11/17

The van Hiele levels. I have noticed the van Hiele levels mentioned in the TE side notes of the Serra book. After some searching, I found the Levels of Understanding on page xxxii of the TE. If teachers do not receive PD when they begin using this textbook, some of the supports that are available to them may be overlooked, or confusing. If I was a teacher using this book and I had first noticed the sidenote on page 85, I wouldn't have known where to find these Levels of Understanding. Van Hiele is not an entry in the index of this book, since the index matches that of the Student Edition.

It is important that teachers receive the training necessary to begin a new curriculum. It is not enough to have PD when the new book is introduced in a school. It should be offered each time another teacher begins using the book. Often, teachers are teaching up to five different subjects, and it is difficult to devote the time necessary to each subject to locate this kind of important information

March 14, 2017

I am in the process of coding word problems. I have already been through the data three times, and this is my fourth time through. The first time, I identifed problems with a real-world context, and the second time, I eliminated problems that only had a superficial connection to a real-world context (as mentioned in my proposal and earlier in this journal). The third time, I made a list of the problems that I need to code, checking again for those that needed to be eliminated. During this fourth time, I am tentatively coding the prolems.

I am having some thoughts about the *realistic considerations* category. It is a major category, given the topic of my dissertation. Perhaps I need to split this into two parts: *Do students need to use the real-life context in their solution/answer?* And *Does the problem use the real-life context throughout the problem, without assumptions that alter the "realistic-ness" of the problem?* It seems that some problems rely on the context more than others. I will continue to think about this while I code.

March 15, 2017

Label. The literature review showed that one of the past concerns with word problems was that problems were often labeled with a mathematical content – an algorithm or theorem to be applied in solving the word problem. For instance, the label Pythagorean Theorem, would direct the solver to use the theorem in solving the problem, even if using special right triangles or trigonometry could be an option.

In this study, I found that each book took a different approach to labeling the problems. The discovery book was less likely to label the problems at all, but sometimes the label was simply "Application". Given that the author defines an application as a way to "help students practice newly acquired skills in a real-world context" (Serra, 2008, p. xxi), I find this curious. By my guidelines for choosing word problems with a real-world context, I found that there are 236 such problems in this textbook, yet only 68 of these problems are labeled as Applications.

On the other hand, the standards-driven book labeled 162 word problems with a real-world context with their connection to the real world: "Business" or "Astronomy" for examples. Fifty-five of the problems in this book had no label at all; the others were labeled with a generic label: Estimation, Short Response, or Critical Thinking. None of these generic labels gave any clue as to how to solve the problem.

Location. In both books, the problems were usually located within the section where the mathematical content intended for solution was introduced. In the Discovery book, 169 out of the 236 problems met these criteria while all but one of the problems in the standards-driven book did. Again, this was a concern as noted in the literature review.

In addition, the Discovery book has several sections which exclusively contain word problems in the Exercises.

Section	Section Title
1.9	A Picture is Worth a Thousand Words
6.6	Around the World
8.3	Area Problems
9.4	Story Problems
10.4	Volume Problems
12.5	Problem Solving with Trigonometry

The Standards-driven book has several "real-world connections" sections. The sections are located at the very end of a chapter. In chapter 2, for example, the Real-World Connections appear as the last feature – after the Study Guide and Review, the Chapter Test, College Entrance Exam Practice, Test Tackler, Standardized Test Prep, a full fifteen pages after the end of the last lesson section in the chapter. The Real-World Connections pages appear at the end of chapters 2, 4, 6, 8, 9, and 11.

Variation. Many of the problems in both books are written in three sentences or less; they fit the profile of the three-component structure (Gerofsky, 2004b) of traditional mathematics word problems. This is most likely due to space constraints, especially given the number of exercises throughout each textbook, and the number of pages per book.

	Discovery book	Standards-driven book
Number of RWP in sample	236	250
Total number of exercises	2176	3795
Total number of pages in student edition	858	926

Other categories to consider are: Alternative Approaches, Realistic Considerations, Support.

March 17, 2017

I contend that word problems in high school textbooks are not realistic enough to prepare students for problem-solving as they will encounter it in the real world – both in careers and everyday occurrences.

Current textbooks tend to simplify word problems. Textbook problems rarely go beyond a few basic steps; in doing this, students do not become fully engaged in the connection between the mathematics and the real-world situation. I also contend that given appropriate time and opportunities, students are capable of tackling more complicated, multi-step tasks. They have proven themselves to be able to undertake major projects in other subject areas, but the work they are given in mathematics classes is often quite simple by comparison. The tasks given to them by other teachers often involves research, sorting facts, creating an outline, and working within a timeframe to meet a deadline for the final product. The process is not called problem-solving, but that is exactly what the students are doing to complete a major project with several components within the required timeframe. Problem-solving in the mathematics classrooms looks much different from this.

The standards and current trends in education call for making students better problem-solvers. The Standards for Mathematical Practice include initiatives for students to: "make sense of problems and persevere in solving them". Standard #4 for Mathematical Practice on modeling with mathematics states "By high school, a student might use geometry to solve a design problem . . . Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. . .They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Further, standard #5 states: “Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. . .” and “Mathematically proficient students are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

I believe that if we want to help students become better at problem-solving, then we need to give them better problems to solve. The word problems in many mathematics textbooks are exercises in applying prescribed mathematical concepts in a limited way. Using a textbook curriculum exclusively does not give students opportunities to delve into solving more complex problems. Textbooks do not offer teachers the support they need to make this possible.

The textbooks in this study have shown some progress from what was studied in my literature review. Very few problems are labeled with mathematical content, and if they are, the label is more generic. The label does not prescribe a formula or method; rather, it describes the problem as “estimation” or “application”, both of which can have several connotations.

Most of the problems are located within the section where the mathematical content intended for use in its solution is introduced. This directs students to use particular mathematical content and also discourages alternative approaches to solving that particular problem.

Most problems in real world situations cannot be addressed in a concise manner as are many of the textbook problems. Some real-world problems, such as design or architecture problems, are often not seen as mathematical problems at all. And, issues that arise during a design problem can change many factors in the problem. For instance, a family is designing a new home and they hire an architect. Their description of their new home probably cannot be made in just two or three sentences. In many cases, new ideas or changing needs for the family will arise, necessitating a change to the home’s design. The architect’s job is to problem-solve – to include as many of the features as the family desires, keeping size and budget within the guidelines of the family and town ordinances. This is the kind of problem-solving I would like to see high school students doing.

March 18, 2017

I have finished the preliminary coding of the word problems in both textbooks and have assembled the results in an Excel spreadsheet. I used formulas (COUNTIF) to count the number of occurrences of the codes in each category. I used conditional formatting and right- and left- justifications to check for errors in the spreadsheet. I have coded 250 problems in the standards-driven book and 236 in the discovery book.

3/22/2017

The construction of word problems in the sample is not consistent throughout the two textbooks. Most problems follow the 3-component structure. Some go beyond that, and the problem is almost a project – several parts with several steps. Still these problems may be seen as a challenge to do, but all of the steps are within the capabilities of high school students.

Pair	(Burger, et al., 2012)	(Serra, 2008)
Table problems	More detail, more realistic	Table dimensions too large
Quilting	Area of all fabric colors lumped together	Find area of each fabric color; calculate how many squares for a king-size bed, add border to create bedspread

When the problem statement is lengthy, the task may seem overwhelming. If there are too many problems in an exercise set, how does a teacher choose one? Will one problem be efficient for students to learn the concepts?

Another question about word problems ---

What happened to **story problems**? Where is the story? I remember a Funky Winkerbean comic where the student got "too involved in the story" and never finished solving the problem on his test. The shadow problem that does not tell why we need to know the height of a lamppost. The problem that asks for the weight of the kite but doesn't tell why it's important?

(The discovery book has a section of word problems labeled story problems.)

Is the context of the problem well-defined?

3/24/2017 - Realistic Considerations – subcategories

The category of realistic categories needs to be broken down in to subcategories. Some problems have been written with good intentions of connections to real-life contexts, but then they fizzle out. For instance, this Critical Thinking problem appears in chapter 1, section 1 in the Standards-driven book: "Explain how rescue teams could use two of the postulates from this lesson to locate a distress signal" (Burger, et al., 2012, p. 11). The idea of connecting geometry to rescue teams is intriguing, but then the problem uses the word "postulate" which is a school mathematics term. The intended connection is that a distress signal may be received by two rescue teams, each team is now connected to the distress signal because two points determine a line. Then, two lines are created by the three points, and the intersection of the two lines will be the location of the distress signal.

Had the problem read "Explain how rescue teams could use ideas from this lesson to locate a distress signal", it would be in a format which might be used in real-world events. The solution of the problem would be the same, but it reads more like a realistic problem. I am coding problems that use mathematics terms in their presentation as TMC (for text of the problem using mathematical content).

Additionally, some problems present a real-world context but finish up with a question that asks about the mathematics rather than the real-life context. In another problem from the standards-driven book, the Flatiron Building in NYC is described as a triangular shaped building. There are details about its location and two of side lengths are described in relation to the length of the third side. The questions then ask: “Find the two unknown side lengths” and “Classify the triangle by its side lengths” (Burger, et al., 2012, p. 228). These questions revert the problem from real-life context back to school mathematics. Word problems like this one will be coded as QM (for question is about mathematics content).

Some problems appear to have the realistic elements throughout the problem. One such problem, “Tom is buying tile for a 12 ft by 18ft rectangular kitchen floor. He needs to buy 15% extra in case some of the tiles break. The tiles are squares with 4 in. sides that come in cases of 100. How many cases should he buy?” (Burger, et al., 2012, p. 684). The problem does not mention a mathematical content to use, nor does it ask for the area of the kitchen. From start to finish, the problem is worded in a way that seems realisitic, almost a question you could ask at the home improvement center. Problems like this will be coded as PRW (for problem being stated as if from the real world context).

Some of the problems also seem to be missing some vital information - in the story, they do not tell *why* the problem is important. These are the questions that students often have – Why do I need to calculate this? Or why would someone need to know the answer to this question? Or why is this important in the real-world context? The answers to these questions may be obvious to textbook writers and publishers, but these questions can put a teacher “on the spot” if they are not familiar with the context. This is important because teachers are often asked “When are we going to have to use this?” and “Why do we have to learn this?” Word problems are an opportunity to answer these questions.

The last example of Tom the tiler answers these questions: he's tiling the kitchen floor, and he needs extra in case some break. Those problems which tell their purpose or usefulness will be coded USE.

The problems will be coded using all four of these subcategory codes. Each problem could have any number of these codes, depending on the wording of the problem. These subcategory codes for realistic considerations are not meant to be mutualy exclusive.

4/3/2017

All this thinking about word problems especially in a real-world context. I was listening to a colleague who said he need to make himself a better schedule for his after-school tutoring. He travels to each student's house and tutors them in math and/or science. I told him it was like the Traveling Salesperson Problem, although it is an NP-complete problem, he could find his own optimal solution. And, it hit me, I am stuck in "The Math Curse".

The Math Curse is a children's book by Jon Scieszka and Lane Smith (also the authors of The Stinky Cheese Man). The book begins with the child narrator explaining, "On Monday in math class, Mrs. Fibonacci says, 'You know, you can think of almost everything as a math problem.' On Tuesday I start having problems" (1995).

No spoiler alerts necessary here, but the narrator finds him/herself thinking of *everything* as a math problem. What the child calls a curse is resolved, as one would expect in a children's book.

I see this less as a curse but as a way of interpreting real-life events. If students can be encouraged to continue the natural wondering and curiosity with which they begin school at age 5 or 6, they might be more interested in making connections between mathematics and the real world. Somewhere along the line, mathermatics is now taught for the sake of calculations and it becomes a subject separate from all others. But are there not connections to science: formulas, charts, calculations, data analysis, and graphs? Connections to social studies involving timelines, politics, apportionment, maps, and again, graphs and charts. Connections to music and art through tessellations, fractals, perspective drawing, etc.

I have always found it curious as a mathematics educator that we have been directed over the years to include "writing across the curriculum". We have been required to have students write about mathematics. Writing in all subjects has become a school or district goal at several times throughout my career (in different districts and school settings).

In only one of the districts did they attempt "math across the curriculum". This initiative was short-lived – only one year, There was limited support – two hours of professional development delivered by the math teachers, two assigned to each of the other departments. Much of this "math across the curriculum" was reduced to simple counting and shapes. For instance, the band director told me he uses math with his students everyday to keep time (1-2-3, 1-2-3) or to count steps in marching formations (1-2-3-4-5-6-7-8, etc). Their marching formation included a rectangle! Unfortunately, without the proper training, he was leaving out the other things he does in his music classes – fractions for the lengths of notes and the fractal composition styles of some of the pieces.

The “math across the curriculum” initiative was superficial, especially given that this was in a high school setting. There was no depth to the math that high school students can use for Excel spreadsheets in business classes, scale factor for maps and models, or persepctive drawing and tessellations in art class.

We often talk about cross-disciplinary lessons, but we still have many barriers preventing this from happening on the scale at which it is absolutely possible!

April 11, 2017

Thinking back over my career during the past week, I am reminded of several interesting events regarding my treatment of word problems.

Back in the 1990s, I taught in a different district. I was called by a parent for a teacher-parent conference. I was told by this parent that he and other parents were dissatified with their children’s geometry grades. He summed it up by saying that he knows that they need “drill-and-kill”. I cannot give them different problems, they need to practice-practice-practice. He had been their seventh-grade math teacher and had just retired. What he didn’t realize is that the “drill-and-kill” method made them good memorizers and not good thinkers. They did not *understand* the math, they knew the process without thinking.

Another parent was incensed that I would include word problems on an algebra quiz. “You know that’s unfair because they can’t do word problems!” This statement came from high school math teacher from another district.

Fast forward to the Fall of 2015. A first year teacher in my department asked her mentor for advice on teaching word problems. She intended to spend at least a day or two helping students with word problems for the particular section she was teaching in algebra 1. Two mathematics teachers from our department started badgering her right away about the waste of time that would be and how they do not teach word problems. I’ve since had several discussions with her to help her through the process and to support her in her convictions.

I have many concerns about word problems, but the biggest is probably that these instances that I relate here are only the tip of the iceberg. These are the incidents that I have witnessed (or been a participant of) in my career. It is very difficult to stand your ground when you are alone in the fight. This shouldn’t be a fight. But I see that my department, for the most part, wants to teach mathematics for mathematics sake, and leave out any connections to real life. If we don’t teach students how to use word problems, then how can they be prepared for their futures – for everyday life?

May 21, 2017

After careful consideration of the support for teacehrs category and several conversations with Dr. Fernandez, I have decided that a subcategory is needed for support to determine how many of the problems have support for the real-world context for teachers. It is possible that the teacher is unfamiliar with a real-life context, putting them at a disadvantage, or putting them in a position to skip the problem. A little support could go a long way to familiarize an adult with a little more information than what may be presented in the set-up and information in the statement of the problem. One more round through the data!

May 25, 2017

There was not a lot of support for teachers in the books. One only had real-world support for a handful of the problems, the other had about fifty. Another aspect that needs some work.

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Appendix D: Audit Trail

The audit trail is a detailed record of my interaction with the data in the sample. This documents the codes and categories and how they emerged.

4/7/2017	design reliability test for coder	design reliability test for coder	text message from Dr. Fernandez	professional contact	
4/7/2017	plan for interrater reliability tests	directions for grad student coder	email to Dr. Fernandez for advice	professional contact	
4/8/2017	intercoder reliability table created	Excel spreadsheet created	Excel worksheet	methodology	
4/9/2017	updated chapter 4 results	data analysis	updated chapter 4 results	data analysis	
4/9/2017	phone call with Dr. Fernandez	re: intercoder reliability checks	send email to Dr. Herr	professional contact	
4/9/2017	email to Dr. Herr	question re:intercoder reliability checks	audit trail v interrater reliability	professional contact	
4/9/2017	updated chapter 5		word document Chapter 5 Discussion		
4/10/2017	email from Dr. Herr	not necessary to do interrater reliability checks, can do audit trail	audit trail v interrater reliability	professional contact	
4/11/2017	reflection on word problems in my career	How other teachers perceive word problems and help to keep the didactical contract in play	journal entry		
4/12/2017	read article (Lincoln and Guba)	audit trail components	components of audit trail		
5/21/2017	communication with Dr. Fernandez	adding a subcategory for Support to reflect real-world context support	new subcategory R	added new subcategory	coding, recording data, writing.

4/3/2017	re-write chapter 4	additions to qualitative interpretive analysis	chapter 4 re-organized		
4/3/2017	Everything as a math problem	Math Curse, Traveling Salesperson Problem,	journal entry		
4/4/2017	re-write chapter 5	discussion, addition of subcategories	chapter 5 reorganized		
4/5/2017	implications	for textbook stakeholders	chapter 6 reorganized		methodology
4/6/2017	Qualitative codebook updated with subcategories and definitions	Label codes (8 codes)	Excel worksheet	data analysis	
		Altern. Approaches: LOC or DMS			
		Variation: 3CS or VAR			
		Support: SUP or NSUP			
		Realistic Considerations: 4 subcategories: TMC, QM, PRW, and USE.			methodology
4/6/2017	phone call with Dr. Fernandez	coding guidelines for coder	written directions for coder	professional contact	
4/7/2017	directions for coder written	directions for grad student coder	word document Chapter 4 results	methodology	methodology

3/24/2017	subcategories for realistic considerations	for advice	email to Dr. Fernandez on subcategories	professional contact	
3/25/2017	Realistic considerations subcategories	new subcategories: TMC, QM, PRW, USE; definitions and codes	new subcategories for realistic considerations		
3/25/2017	subcategories for realistic considerations	Word document	to add into chapter 3 methodology		
3/26/2017	updated qualitative codebook	show subcategories with rich, thick description	updated Appendix A Qual Codebook	data analysis	
3/26/2017	problems analyzed	problems analyzed	problems analyzed	data analysis	
3/27/2017	Shoring up qualitative codebook	refining definitions of codes	Qualitative codebook in good order	data analysis	
3/27/2017	More updates to qual codebook	More updates to qual codebook	appendix A	methodology	
3/28/2017	phone call with Dr. Fernandez	coding guidelines for coder	coding guidelines for coder	professional contact	
3/30/2017	counts of real-life word problems updated	counts of real-life word problems updated	counts of real-life word problems updated	data analysis	
3/30/2017	coding guidelines for coder	coding guidelines for coder	coding guidelines for coder	methodology	
4/2/2017	correlation to discussion and implications	CCSS Standards of mathematical practice	story problem, real-life problems, journal entry		

3/17/2017	updated counts of real life word problems	Burger 250; Serra 236	journal entry/ preliminary code counts	data analysis	
3/18/2017	Describe checks of coding	preliminary coding finished; description of spreadsheet checks		data analysis	
3/18/2017	work on chapter 5 discussion of results		journal entry; discussion of variation category	methodology	
3/22/2017	Story problems	Where's the story?	journal entry		
3/23/2017	Analysis of a word problem	Ohio Star Quilt Problem (so much that was missing from the previous quilt problem is evident here)	Word problem analysis	data analysis	
3/23/2017	Selection of word problems for sample	clarifying the selection for the sample	new subcategories for realistic considerations	data analysis	
3/23/2017	phone call with Dr. Fernandez	ways to characterize realistic considerations	new categories:TMC, QM, WHY, RBE	professional contact	
3/24/2017	Realistic considerations subcategories	new subcategories: TMC, QM, PRW, USE; definitions and codes	new subcategories for realistic considerations	data analysis	
3/24/2017	subcategories for realistic considerations	Word document	to add into chapter 3 methodology	data analysis	

3/11/2017	teacher supports	vanHiele levels discussed in beginning of Serra book (I did not find too much that refers back to them)	organization		
3/12/2017	proposal updated	reflects new RQ	proposal updated		
3/13/2017	Analysis of a word problem	Perimeter fence	Word problem analysis		
3/14/2017	analysis of a word problem	Molecule Problem	word problem analysis		
3/14/2017	subcategories for realistic considerations category?	Do students need to use the context in their answer to the problem? Does the problem use realistic-ness throughout?	journal entry, discussion of results	data analysis	
3/15/2017	categories for coding described and discussed in further detail	Label, Location, variation in detail;	journal entry	data analysis	
3/16/2017	Analyses of word problems	Squaring the garden (pair with rectangular room); kite weight (balsa/fabric)	Word problem analysis	data analysis	
3/17/2017	problems are oversimplified	Standards ask for more than what we currently do	journal entry	data analysis	
3/17/2017	Three-component structure	Does not work or exist in real life	journal entry	data analysis	

3/9/2017	How the didactical contract came into being (?)	Grace Hopper: "The most dangerous phrase in our language is 'We have always done it this way'." She could have been speaking of the didactical contract	journal entry		
3/10/2017	reduction of sample	One book has a whole chapter on Probability: not part of geometry curriculum; separate Real World Connections sections in Burger book	journal entry/question for Dr. Fernandez/ counts at 259 in Burger and 245 in Serra	data analysis	
3/11/2017	Analysis of a word problem	Taj Mahal problem	Journal entry; word problem analysis		
3/11/2017	new subcategories	word problem analysis	journal entry/ thoughts on possible subcategorization	data analysis	
3/11/2017	coding scheme	use of Excel to record coding	journal entry	methodology	
3/11/2017	reflection on realistic-ness	substance of the problems, creating problem-solvers, not necessarily just mathematics word problem solvers	journal entry/discussion/implications		
3/11/2017	transcending time	use internet to get up-to-date info	coding and analysis of one problem		

3/6/2017	relevance of word problems	Class of 2014 is planning a class trip. Student wants to know why a class that has already graduated is planning a field trip. Assigned as HW	journal entry/problem analysis results		
3/8/2017	analysis of a word problem	Kite design - problem mentions glue and string but never uses them in the question or solution of the problem	journal entry/discussion section	data analysis	
3/8/2017	underestimating the abilities of students	Higher Expectations - they can do it!	journal entry		
3/8/2017	analysis of a word problem	Bring geometry to class (2 problems)	word problem analysis		
3/8/2017	reflection on problem-solving	how the didactical contract holds students back, how teachers allow the didactical contract to stand between them and their students	journal entry		
3/9/2017	phone call with Dr. Fernandez			professional contact	
3/9/2017	student-centered classrooms	flipped classroom, Notice and Wonder, math talks	journal entry		

3/4/2017	reflection on real world problems	Are there supports for teachers to ask the difficult questions: Why do I need to know this? What if . . .? And How do we know that . .?	journal entry		
3/4/2017	PowerPoint presentation revised	to show changes in research question and outline of validity and reliability	revised PowerPoint presentation		
3/5/2017	problem analysis	Island post offices, Hyacinth crossing street	word problem analyses		
3/5/2017	more reductions to sample	recall questions, DOK 1 or Bloom's level 1	reductions to number of problems in the sample	data analysis	
3/5/2017	New category	teacher supports available in TE	New category: SUP	data analysis	
3/5/2017	puzzle problems with no real value	zigzagging through the woods	further reductions to the sample	data analysis	
3/5/2017	repeated problems	Handshake problem appears several times	journal entry on relevance		
3/6/2017	proposal		mailed to Dr. Fernandez via USPS		

2/25/2017	study results	Shipping the baseball bat problem	word problem analysis	data analysis	
2/26/2017	study results	rectangular room	word problem analysis		
2/26/2017	study results	ISS Problem - view to the horizon from the ISS	word problem analysis	data analysis	
2/27/2017	study results	hang glider problem	word problem analysis		
2/27/2017	study results	OSHA ladder problem	word problem analysis	data analysis	
2/28/2017	study results	Quilt patterns (2 related problems)	word problem analysis	data analysis	
2/28/2017	proposal revised with new research question	removed comparison of the two textbooks; analyzing the aspects of realistic-ness in the WP	results to be reported in results, discussion or in appendix	methodology	
2/28/2017	reflection on solar system questions	Why do we need to know this?	revised proposal		
3/1/2017	study results	Pet Iguana Pen Problem	word problem analysis	data analysis	
3/2/2017	study results	Parallel parking	word problem analysis	data analysis	
3/3/2017	study results	Quilt triangles Problem	word problem analysis	data analysis	
3/4/2017	pizza pair problems analyzed	analysis of pizza problems (better buy) using categories and codes	word problem analyses	data analysis	
3/4/2017	study results	Perimeter Fence Problem	word problem analysis	data analysis	
3/4/2017	textbook intended curriculum	How do teachers use these resources?			

		3. Narrowed sample by keeping those problems that required interaction between mathematics and the RW context			
2/18/2017	study results	Table/determine-a-plane problems	word problem analysis	data analysis	
2/19/2017	study results	Earth revolution/rotation problems	word problem analysis	data analysis	
2/20/2017	Journal entry	questioning how problems are written. Perhaps hastily with intent to fix later?	new code	data analysis	
2/20/2017	New Label code	Application (A)	new code (A)	data analysis	
2/22/2017	proposal updated	revised research question aprroved by committee	Revised research question in proposal		
2/24/2017	study results	Target with laser light problem (Poolroom math)	word problem analysis	data analysis	
2/24/2017	study results	pizza diameter problem	word problem analysis	data analysis	
2/24/2017	reflection on student treatment of word problems	Why students skip problems and wait for guidance (didactical contract)	results to be reported in results, discussion or in appendix		
2/25/2017	study results	TV screen diagonal problem	word problem analysis	data analysis	

2/15/2017	Constant force	spoke with Physics teacher (J. Santonacita) about the need to know the constant force for ADA wheelchair problem		professional contact	
2/16/2017	Defense of Dissertation proposal			professional contact	
2/16/2017	meeting with committee	Audience	Focus audience to practitioners, teachers, curriculum developers, teacher educators, and textbook publishers	professional contact	
2/16/2017	proposal defense presentation	proposal information	PowerPoint presentation completed for review	professional contact	
2/16/2017	Refine research question	remove comparison of the two textbooks	Looking for aspects of realistic-ness in word problems in sample, not comparing the textbooks	professional contact	
2/17/2017	better defining steps of data sampling	1. Found all word problems in both books	results to be reported in results, discussion or in appendix	data analysis	
		2. Identified problems presented with real-world context			

1/24/2017	for proposal defense	pairings of problems	email from Dr. Fernandez: need my solutions of the two problems	professional contact	
1/28/2017	table seating problems	analyzed two table seating problems	Pairing of problems for proposal hearing	data analysis	
2/2/2017	phone call with Dr. Fernandez	work on PowerPoint; discussed possible implications of study; developing a framework: what to look for; findings: my solutions and research on internet	fixed slides: segue from JFK problem to research question; alternative approaches definition	professional contact	
2/10/2017	Rethinking category	alternative approaches/location	short paper: analysis of two table problems for proposal defense	data analysis	
2/11/2017	problems for presentation reviewed	analysis of table problems revised	word problem analysis	data analysis	
2/11/2017	Study results	ADA Wheelchair problem	Word problem analysis		
2/12/2017	Study results	ADA Wheelchair problem	Word problem analysis		
2/13/2017	Study results	ADA Wheelchair problem	Word problem analysis		

1/8/2017		Pairing problems	journal entry		
1/8/2017	Journal entry	lack of follow-through in textbook; purpose of the problem section was to discuss triangulation, yet that topic is missing from that section	journal entry		
1/8/2017	sections of word problems exclusively	substantiates prior research in lit	journal entry		
1/16/2017	re-reading through all real-world problems	some problems do not engage students in a real-world context; superficial connections	Possible reduction to sample	data analysis	
1/20/2017	phone call with Dr. Fernandez			professional contact	
1/22/2017	for proposal defense	pairings of problems	Word document of two table problems	data analysis	
1/23/2017	for proposal defense	PowerPoint for proposal presentation	sent to Dr. Fernandez via email	professional contact	
1/24/2017	phone call with Dr. Fernandez	what is needed for proposal defense		professional contact	

11/29/2016	need more detail in responses	answers to Dr. Krupa's questions on proposal	email from Dr. Fernandez	professional contact	
11/29/2016	reliability checks for coding	new work that must be completed for reliability	email from Dr. Fernandez	professional contact	
12/1/2016	respond to questions on proposal	answers to Dr. Krupa's questions on proposal	more in-depth responses to Dr. Krupa's questions	professional contact	
12/2/2016	phone call with Dr. Fernandez	Chapter 1 new version		professional contact	
12/4/2016	proposal to Dr. Fernandez			professional contact	
12/6/2016	dissertation work	work deleted from chapter 2	moved to chapters 5 and 6		
12/8/2016	responses to Dr. Krupa's questions			professional contact	
12/8/2016	email to Dr. Fernandez			professional contact	
1/5/2017	phone call with Dr. Fernandez	solve each problem and refer to my solutions;carefully consider wording of each problem; absence of diagrams worth remarking on; books' constraints: omission of diagrams; how content is delivered; RC I think of. . .		professional contact	

10/29/2016	proposal work	study of textbooks, US & Singapore elementary schools	Possible framework for analyzing textbook word problems	data analysis	
10/29/2016	proposal work	reliability and validity piece			
10/30/2016	proposal work	Ginsberg categorizes 10 problems, bullet points, 3 categories, short paragraph for each	response to last email from Dr. Fernandez	professional contact	data analysis
11/1/2016	phone call with Dr. Fernandez			professional contact	
11/4/2016	proposal work	validity and reliability for chapter 3 written		professional contact	
11/4/2016	proposal work	in choosing problems noticed that wp are scattered throughout exercises set	Problems are no longer relegated to last place in the section		
11/11/2016	proposal sent to committee	proposal sent to committee	Dr. Fernandez sent my proposal to committee	professional contact	
11/13/2016	proposal work	placement of word problems within a section's exercises	journal entry		
11/28/2016	respond to questions on proposal	answers to Dr. Krupa's questions on proposal	emailed to Dr. Fernandez	professional contact	

10/19/2016	textbook choice discussed	discovery v standards-driven	Appendix for Qualitative codebook	methodology	
10/20/2016	chapter 3 revision	categorization of textbooks	email to Dr. Fernandez	professional contact	
10/20/2016	thick rich description	Qualitative Codebook			
10/21/2016	phone call with Dr. Fernandez	read-through of chapter 3 with comments		professional contact	
10/21/2016	proposal work	chapter 3 revision			
10/23/2016	proposal work	Qualitative Codebook	Qualitative codebook	data analysis	
10/23/2016	phone call with Dr. Fernandez			professional contact	
10/25/2016	proposal work	looking for a well-defined construct for analysis of word problems;connection between didactical contract and textbook wp weak; reliability and validity sections lacking	email from Dr. Fernandez	professional contact	
10/27/2016	proposal work	Qualitative Codebook	Codebook table written with initial codes	data analysis	

9/28/2016	codes and categories	codes and categories	phone call with Dr. Fernandez	professional contact	
10/1/2016	second round of data analysis	reduction in sample due to trivial connections to real-life and other counting issues		data analysis	
10/1/2016	initial codes	Realistic Considerations, Variation, Alternative Approaches, Procedural Complexity, Label, Location	Initial codes	data analysis	
10/2/2016	Ch. 3 sent to Dr. Fernandez		Initial codes	professional contact	
10/4/2016	categories for coding	realistic considerations, alternative approaches, procedural complexity, label and variation	Possible categories for coding	data analysis	
10/13/2016	comments from Dr. Fernandez			professional contact	
10/18/2016	collection of images for paper		textbook sources chosen	data analysis	

8/16/2016	proposal	Chapter 1 to Dr. Fernandez	email to Dr. Fernandez		
8/16/2016	proposal	Chapter 1 work	ch 1 changed to introduction with definitions and research questions		
8/17/2016	proposal	Chapter 1 comments from Dr. Fernandez		professional contact	
8/21/2016	proposal	link textbook word problems to didactical contract	email to Dr. Fernandez (short Word document)	professional contact	
8/21/2016	proposal	work back from Dr. Fernandez		professional contact	
8/23/2016	work to Dr Fernandez		303 in Serra and 552 in Burger et al.	data analysis	
8/25/2016	First round data analysis	find all problems in data sources with a real-life context		data analysis	
9/5/2016	work back to Dr. Fernandez			professional contact	
9/12/2016	proposal	direction for chapters 1, 2, and 3	phone call with Dr. Fernandez	professional contact	
9/17/2016	proposal work	from Green and Emerson			
9/19/2016	proposal work	pieces to put into Ch. 2 and Ch. 3	wrote new pieces to include to clarify certain points		
9/25/2016	chapter 2 revisions		Serra 262; Burger 345	data analysis	

6/27/2016	writing proposal	chapter 2 lit review			
6/29/2016	writing proposal	definitions in Chapter 1	definitions in Chapter 1		
7/4/2016	writing proposal	revisions to chapter 2	email to Dr. Fernandez		
7/10/2016	writing proposal	chapter 2 lit review	Word document		
7/11/2016	writing proposal	Chapter 2 submitted	Word document		
7/17/2016	writing proposal	work rec'd from Dr. Fernandez	Word document		
7/21/2016	writing proposal	coding	Word document	data analysis	
7/24/2016	writing proposal	more work on Ch. 2	Word document		
7/29/2016	writing proposal	piece for dissertation	Word document		
7/30/2016	writing proposal	aspects definitions for ch3	Word document	data analysis	
7/31/2016	writing proposal	chapter 3	Word document	methodology	
7/31/2016	writing proposal	chapter 1	Word document		
7/31/2016	writing proposal	chapter 2	Word document		
8/2/2016	writing proposal	work on chapter 3 methodology	Word document	methodology	
8/8/2016	writing proposal	chapter 3	Word document	methodology	
8/10/2016	writing proposal	chapter 1	Word document		
8/10/2016	writing proposal	chapter 1	Word document		
8/10/2016	writing proposal	chapter 2	Word document		
8/14/2016	writing proposal	chapter 3	Word document	methodology	
8/14/2016	appendix B	analysis of problems from Serra Book	word document		

1/16/2016	request meeting with Dr. Fernandez	discussion of plan for dissertation	email to Dr. Fernandez	professional contact	
2/18/2016	request for meeting with Dr. Krupa	request for meeting with Dr.Krupa	email to Dr. Krupa	professional contact	
3/15/2016	meeting with Dr. Krupa	paring down lit review; reflect on didactical contract and textbook word problems; aspect table from Green and Emerson (2010)	discussion on substance topics	professional contact	
3/16/2016	framework	selection of word problems before using framework	email exchange with Dr. Krupa		
4/5/2016	writing proposal	lit review and proposal			
4/10/2016	Poster presentation for Student Research Symposium	PowerPoint for poster presentation	PowerPoint for poster presentation		
4/19/2016	presentation	poster presentation	Student Research Symposium		
5/1/2016	writing proposal	appendix	analysis of word problems		
5/25/2016	writing proposal	aspects table	choosing aspects and categories for word problem analysis	methodology	
6/19/2016	writing proposal	part of problem statement			

7/2/2015	writing preliminary lit review	what is a word problem	chapter 1 definition		
7/4/2015	writing preliminary lit review	lit review	chapter 2		
7/4/2015	writing preliminary lit review	recommendations for further research	chapter 2 conclusion		
7/10/2015	writing preliminary lit review	the roles of teachers and students in the didactical contract	chapter 2		
7/11/2015	writing preliminary lit review	lit review	chapter 2		
7/11/2015	writing preliminary lit review	the roles of teachers and students in the didactical contract	chapter 2		
7/13/2015	writing preliminary lit review	lit review	chapter 2		
8/3/2015	writing preliminary lit review	lit review	chapter 2		
9/14/2015	writing preliminary lit review	review comments from Dr. Krupa		professional contact	
12/16/2015	lit review and comps presentation	power point for lit review presentation (comps)	PowerPoint for poster presentation		
12/19/2015	mailed comps and lit review to MSU	completed revisions to comps and lit review	completed copy sent to MSU via USPS		
12/30/2015	request Dr. Fernandez to continue as committee chair	request Dr. Fernandez to continue as committee chair	email to Dr. Fernandez	professional contact	

1/3/2015	writing preliminary lit review	what an authentic problem is and isn't	Word document		
1/10/2015	writing preliminary lit review	outline of topic and lit review	Word document		
1/17/2015	writing preliminary lit review	lit review bibliography	Word document		
1/17/2015	writing preliminary lit review	outline topics for lit review	Word document		
2/9/2015	writing preliminary lit review	outline with topics	Word document		
2/17/2015	writing preliminary lit review	summary of lit review	Word document		
3/21/2015	writing preliminary lit review	intro for lit review	Word document		
3/21/2015	writing preliminary lit review	deleted info from lit review	Word document		
4/25/2015	writing preliminary lit review	outline for lit review	Word document		
4/28/2015	writing preliminary lit review	draft of lit review	Word document		
5/30/2015	writing preliminary lit review	lit review edited			
6/14/2015	writing preliminary lit review	edited lit review			
6/15/2015	writing preliminary lit review	lit review	chapter 2		
6/15/2015	writing preliminary lit review	students and their role in the didactical contract	chapter 2		

9/20/2014	writing preliminary lit review	notes for use in lit review	Word document		
9/20/2014	writing preliminary lit review	suspension of sense-making	Word document		
9/20/2014	writing preliminary lit review	teacher knowledge	Word document		
10/19/2014	writing preliminary lit review	notes for use in lit review	Word document		
10/21/2014	writing preliminary lit review	code definition memo	Word document	data analysis	
10/25/2014	writing preliminary lit review	beginning of lit review for dissertation	Word document		
12/30/2014	writing preliminary lit review	engaging students in "doing math"	Word document		
1/2/2015	writing preliminary lit review	sorting through notes and topics	Word document		
1/3/2015	writing preliminary lit review	beginning outline	Word document		
1/3/2015	writing preliminary lit review	defining terms	Word document		
1/3/2015	writing preliminary lit review	engaging students in "doing math"	Word document		
1/3/2015	writing preliminary lit review	examples of authentic problems	Word document		
1/3/2015	writing preliminary lit review	suspension of sense-making	Word document		
1/3/2015	writing preliminary lit review	teacher knowledge	Word document		

DATE	PURPOSE	SUBSTANCE	OUTCOME	Activity	
4/23/2014	research topic	beginning thoughts on word problems - student view of realistic situations in word problems	journal entry		
4/29/2014	read MT article on authenticity	framework for authenticity	journal entry		
4/29/2014	literature review	reflection on student work	lit review work		
4/30/2014	connect math to real life	When are we going to have to use this?	journal entry		
5/1/2014	possible outcomes of research	possible outcomes of research	journal entry		
5/4/2014	list of application word problems	which are realistic?	journal entry	data analysis	
5/12/2014	discussion of possible research options	talked about options: action research, curricular analysis	meeting with Dr. Fernandez	methodology	
7/6/2014	reading literature	bibliography, beginning reading	email to Dr. Fernandez	professional contact	
8/20/2014	categories to begin writing	authentic problems, what they are or aren't engaging students in "doing math"; suspension of sense-making; teacher knowledge; and ways to characterize word problems.	email to Dr. Fernandez	professional contact	data analysis