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Adjunct Instructors' Opportunities for Learning Through Engagement with a Research-Based Mathematics Curriculum

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ADJUNCT INSTRUCTORS' OPPORTUNITIES FOR LEARNING THROUGH ENGAGEMENT WITH A RESEARCH-BASED MATHEMATICS CURRICULUM

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ABSTRACT

ADJUNCT INSTRUCTORS' OPPORTUNITIES FOR LEARNING THROUGH ENGAGEMENT WITH A RESEARCH-BASED MATHEMATICS CURRICULUM

by Zareen G. Rahman

There is a growing need to retain students in STEM fields and majors in the U.S. Improving students' experience in early mathematics courses like Precalculus can influence students' decisions to remain in STEM fields. Teachers can play an important role in providing effective learning experiences to the students. Supporting teachers and providing professional development can help the teachers in facilitating student learning. When it comes to implementing research-based mathematics curricula, teachers are key players in making the curriculum come alive inside their classrooms. The challenges that teachers face when implementing a research-based mathematics curriculum can provide opportunities for their own learning. As they engage with the curricular resources, the new curriculum challenges the teachers' current knowledge and teaching practice. In this dissertation I have explored three adjunct instructors' engagement with a research-based mathematics curriculum over the course of two semesters. Engagement with the curricular resources provided opportunities for their learning, as the instructors planned and enacted the curriculum, discussed it while collaborating with colleagues or reflecting. Some of these opportunities were availed and some were left unexplored. Findings of this study have implications for developing effective professional development programs for adjunct instructors.

Keywords: Adjunct instructors, research-based mathematics curriculum, teacher learning.

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Dedication

I dedicate this dissertation to Zanub, Faryal and Ammara.

Zanub for questioning everything, Faryal, for being wise beyond her years and Ammara for supporting me no matter what.

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Chapter 1

Introduction

Research in mathematics education has focused on the 'leaky pipeline' for STEM, and reasons why students drop out of courses in STEM disciplines. In the United States, despite the increasing demand for STEM majors, the number of students pursuing these majors has remained at a constant 30% (Carnevale, Smith & Melton, 2011; Hurtado, Eagan, & Chang, 2010). Of the students who do pursue STEM majors, fewer than 40% persist in receiving a STEM degree (President's Council of Advisors on Science and Technology [PCAST], 2012). Economic analyses indicate that the United States needs to produce around one million STEM college graduates in the next decade to remain competitive in science and technology (PCAST, 2012). In order for the United States to achieve this goal, the number of students receiving STEM degrees needs to increase by about 34% annually compared to current rates (PCAST, 2012).

Students' persistence in continuing to pursue STEM degrees is heavily influenced by their classroom experiences, especially in their first year of mathematics courses (Hutcheson, Pampaka, & Williams 2011; Pampaka, Williams, Hutcheson, Davis & Wake, 2011). University mathematics courses act as a filter and play a large role in why students refrain from pursuing STEM careers (Wake, 2011). In this regard, the quality of instruction can make a big difference in the retention of STEM students beyond beginning mathematics courses. Instructors' beliefs about the teaching and learning of

mathematics and their pedagogy can demotivate students and deter them from taking further mathematics courses (Pampaka et al., 2012). Hence, improved instruction may motivate students to learn more mathematics and consider pursuing a STEM degree (Ellis, 2014).

Keeping in mind the important role that teachers play in their students' learning, it should be noted that adjunct instructors teach many introductory mathematics courses as higher education institutions are increasingly employing more part-time, non-tenure track faculty (Curtis, 2014; Mason, 2009). It is important to understand adjunct instructors' needs and provide professional development that can influence their instructional quality (Leslie & Gappa, 2002). Many introductory mathematics courses are also taught by graduate teaching assistants (GTAs), and there already exists a growing body of research on best practices for professional development (PD) for them (DeLong & Winter, 2001). However, much needs to be learned in terms of providing PD for adjunct mathematics instructors.

Research indicates the need to support teachers as they implement research-based curricula to bring about sustainable change (Darling-Hammond, 2011). Curriculum has long been seen as a means to bring about educational change, as its design followed by distribution of curricular materials is one of the oldest strategies for influencing classroom instruction (Ball & Cohen, 1996). The changes in curricula are based on recommendations set forward to influence classroom practices through what content is taught in schools and how such content should be taught (Senk & Thompson, 2003). The goal of improving curriculum is to increase student learning, which has been shown to be

dependent upon improvements in teacher quality (Darling-Hammond, 2011). Rezat and Sträßer (2012) explain that using resources like textbooks and digital technologies is a complex process that involves interaction between user and resource. When implementing curriculum, teachers plan for instruction by interpreting the curriculum and deciding what resources to use (Remillard & Heck, 2014). These decisions can be aided by providing collaborative opportunities, such as professional learning communities. Such collaborations can act both as a means of support for the instructors as well as a form of PD (Cochran-Smith & Lytle, 1999; Jaworski, 2006; Nelson, Slavit, Perkins, & Hathorn, 2008; Rodgers, 2002).

In a professional learning community (PLC), instructors can work together to develop supportive conditions that promote collaboration and growth (DuFour & Eaker, 2005). Within education, PLCs are based on the idea that teachers have unique experiences and knowledge relevant to their practice. The members of the community go through similar experiences, which can make their participation in the PLC meetings beneficial to them in terms of increased content knowledge and attention to instructional practices. They can improve their knowledge by reflecting on their experiences together (Buysse, Sparkman, & Wesley, 2003). PLCs serve as a platform for discussing issues such as education reform, improving teaching as a profession, and teacher accountability (Lieberman & Miller, 2016). These PLCs may provide experiences in addition to the existing tools that aid teachers in curriculum implementation.

Significance

This research explores adjunct instructors' engagement with a research-based mathematics curriculum to analyze the opportunities for instructors' learning in terms of their knowledge and practice. The need for this research is rooted in the demand for improving STEM education and retention of students in STEM majors. One area of improvement is teachers' professional development to support them in implementing research-based curricula that are designed to better prepare students for success in STEM education (Ball & Cohen, 1999). These research-based curricula often suggest a shift from the traditional methods of teaching mathematics. For example, they may focus on conceptual understanding and require student centered pedagogical practices. Thompson and Carlson (2017) found that it is not easy for teachers who have had experiences teaching from a traditional curriculum to deal with the demands of a research-based curriculum and they themselves need support in guiding their students.

As mentioned earlier, my research focuses on the ways in which engagement with a research-based mathematics curriculum can provide opportunities for instructors' learning in terms of their knowledge and practice. I have explored adjunct instructors' engagement with a research-based curriculum over two semesters and I aim to answer the following research questions:

1. What is the nature of adjunct instructors' engagement with a research-based Precalculus curriculum?
2. How does engagement with a research-based Precalculus curriculum provide opportunities for adjunct instructors' learning?

Chapter 2

Review of the Literature

In this chapter, I present a review of research literature that is relevant to my work. Since the focus of my work is on adjunct instructors' engagement with research-based curricula, I present the literature on curriculum and its impact on instruction as well as teacher learning. I then present literature on teachers' role in implementing curriculum, as well as models of curriculum implementation and teachers' engagement with curriculum. I talk about professional learning communities as a form of teacher collaboration, then describe the population of adjunct instructors and their role in our current higher education system. I discuss main points from current research about what is known about how to help adjunct instructors be effective and successful, specifically research on job satisfaction and professional development.

Curriculum

Research-based curricula initially referred to the curriculum materials developed in the 1990's with support from the National Science Foundation (Senk & Thompson, 2003). These reform efforts were guided by the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), leading to mathematics curricula that differed from existing mathematics programs because of the focus on technology, multiple representations, real-world scenarios, and most importantly, mathematical processes (Stein, Remillard & Smith, 2007). I am using, *research-based curriculum* as referring to mathematics curriculum whose design has been guided by mathematics educational research and has been proven to improve student learning of mathematics. I am using

curriculum resources to include physical objects such as textbooks, student workbooks, and teacher guides as well as virtual materials such as digital presentations, apps, and online homework. I also include representations of mathematical concepts in the curriculum, such as diagrams, examples, scenarios, models, and phrasing; representation of tasks including, instructions and procedures like the use of investigations and suggested pedagogical practices; and sequencing and organization of mathematical concepts.

In order to explore teachers' engagement with research-based curriculum, I offer a theoretical review of curricular design models and how these models situate teachers as part of the curriculum design and implementation process. Teachers can influence the curriculum that is eventually implemented in the classroom and their involvement in the process can lead to their own learning. I use a sociocultural lens to explore the nature of teacher learning and situate teachers' experiences and learning within a wider network of actors and actions that are involved in the process of curriculum implementation. A sociocultural lens allows for the analysis of human action within its context. The goal of sociocultural analysis is to comprehend the ways in which mental functioning is related to social, institutional, and historical settings (Wertsch, 1998). When implementing a curriculum, teachers interact with various resources to make decisions regarding their students' learning. I analyze their engagement with the curriculum as they plan for instruction, as they enact their lessons inside their classrooms, as they use reflection as a source of guidance and interact with their colleagues. An in-depth look at the teachers' experiences as they implement a new curriculum provides an additional layer of

experiences to existing models that explain teacher's roles in curriculum implementation. In this regard, this work presents factors that influence teachers' experiences as they implement a research-based curriculum and provide opportunities for their learning.

Defining Curriculum

The term "curriculum" is used ubiquitously, and yet there is a lack of clarity in how it is used (National Research Council [NRC], 1999). A general use of curriculum provides an outline of the learning goals, but the term is used differently in various contexts (Confrey & Stohl, 2004). Confrey and Stohl themselves use *curriculum* and *curricular materials* interchangeably, and define these terms as comprising of resources designed by an author or a group of authors and published by one publisher. These resources, designed for specific grade levels, may include guides for teachers, assessments, hands-on activities, online components, materials for parents, or homework aids. They may also provide a list of curricular objectives that satisfy national, state, or local standards.

Remillard and Heck (2014) provide a broader definition of curriculum. According to them, *mathematical curriculum* is a learning plan that guides learners' experiences and helps them reach mathematical learning goals. Mathematical curriculum also includes resources that teachers use to support their students' learning. This definition encompasses more than a list of topics or objectives and includes plans for students' experiences that support their learning (Remillard & Heck, 2014). Remillard and Heck (2014) define various types of curricula based on their design and usage. To better understand how various curricula influence, and are influenced by teachers, I present

these various types of curricula in the section below. It is important to keep in mind that while there have been various definitions of the types of curricula, they mainly fall under two categories. The first category includes curricula designed by an educational organization or researchers with specific learning goals for the students. Remillard and Heck (2014) call this the *official curriculum*; it is this intended or formal curriculum that provides the authorized guidelines for student learning. The second category includes curricula that come alive inside the classrooms. These are the *operational curricula*, the teacher intended or enacted curricula that are influenced by teacher planning, classroom experiences and student learning outcomes (Remillard & Heck, 2014).

When implementing curriculum, teachers design instruction by interpreting the curriculum and deciding what resources to use (Remillard & Heck, 2014). They construct meaning of the instructional resources to guide their planning (Gueudet & Trouche, 2009). The plans that teachers design to use in their classroom are the teacher-intended curriculum. Teacher-intended curriculum is detailed and nuanced because it is designed for a specific group of students and becomes active in teachers' own classrooms (Remillard & Heck, 2014). Once the teacher actually conducts a lesson, the interactions that take place inside the classroom define the *enacted curriculum*, which cannot be completely pre-planned because the enactment is based on teachers' and students' responses to how events unfold in the classroom (Remillard & Taton, 2013). However, teacher-intended curriculum is different from the intended curriculum. According to Remillard and Heck (2014), the difference between the intended and teacher-intended curricula can be compared to the difference between a script of a play and each scene as

conceived by the director, while the enacted curriculum is the actual performance of a play. It is the enacted curriculum that leads to interactions in the classroom and has the biggest influence on student outcomes (Remillard & Heck, 2014, Stein, Grover & Henningsen, 1996). Remillard and Heck (2014) consider teacher-planned and teacher-enacted curricula as the same type of curriculum: enacted or implemented. Enacted or implemented curriculum is defined as the curriculum that teachers work with and that influences classroom experiences (Remillard & Heck, 2014). Similarly, Ball and Cohen (1996) explain that curriculum is often understood to include the textbook and curriculum materials, but the enacted curriculum is co-constructed by teachers, students, and the materials. As stated earlier, Remillard and Heck place the enacted mathematics curriculum within a wider context of the operational curriculum. According to their framework, the teachers transform the intended curriculum from curricular objectives to actual classroom activities. This transformed curriculum is the operational curriculum and includes the teacher-intended curriculum, the enacted curriculum, and student outcomes. Teachers strongly influence how the curriculum actually comes into play inside the classroom. I am interested in analyzing the ways in which teachers engage with the curriculum and how this engagement provides opportunities for their own learning.

Curriculum Impact on Instruction and Teacher Learning

Teachers influence the curriculum, and their role in shaping the curriculum has been well documented (Cohen & Ball, 1999; Ball & Cohen, 1996; Senk & Thompson, 2003; Darling-Hammond, 2011; Simmons, 2011). It is important to understand a teacher's role in curriculum implementation because one of the goals of curriculum

design and reform is to increase teacher's ability to facilitate their students' learning (Cohen & Ball, 1999). Curriculum has long been seen as a means to bring about educational change, as its design followed by distribution of curricular materials is one of the oldest strategies for influencing classroom instruction (Ball & Cohen, 1996). Curriculum reform consists of changes made in curricula based on a set of recommendations set forward to influence classroom practices by suggesting what is taught in schools and how it should be taught (Senk & Thompson, 2003). The goal of curriculum reform is to increase student learning and the changes are often connected to improving teacher quality (Darling-Hammond, 2011). According to Darling-Hammond (2011), teacher quality encompasses a variety of factors including personal abilities, content and pedagogical knowledge, and a desire to learn and improve teaching. When discussing curriculum reform, a term often used is *instructional capacity* (Cohen & Ball, 1999). A teacher's instructional capacity is the teacher's ability to support students in their learning, and increasing that capacity allows teachers to maximize their teaching efforts to benefit their students (Cohen & Ball, 1999). Teachers' knowledge of the subject matter, their knowledge of teaching and students, influence their instructional decisions. Teachers develop classroom environments and use curriculum materials based on their own knowledge and experience. According to the instructional capacity building model, the support that institutions provide teachers to develop instructional capacity can lead to increased student performance and decreased achievement gaps (Simmons, 2011).

Darling-Hammond (2011) explains that policies seeking to improve education often ignore the learning and development of teachers and focus instead on teacher

accountability by linking teachers' performance to students' test scores. The main assumption here is that teachers will be motivated to improve their performance because of a system of rewards and punishments. An alternate view of teacher development focuses on fostering teachers' capacity to support student learning as well as improving their own teaching. This view is based on the idea that teachers are motivated by their desire to engage in meaningful work and are interested in learning and improving their practice. This capacity building model is significant in promoting effective teaching. Educational reform policies that lead to outstanding instruction as well as deep student learning support teachers by providing learning and development opportunities for them. Cohen and Ball (1999) explain that the lack of learning opportunities for teachers is one of the main reasons why curriculum reform efforts are not able to bring about sustainable change.

According to Cohen and Ball (1999), the development of instructional capacity can be approached in various ways, with each approach influencing how teachers are supported. Improving instructional capacity can mean learning specific content and pedagogical knowledge, signifying a finite set of knowledge and skills that are to be acquired by teachers. Improving instructional capacity can also be perceived as an ongoing process of knowledge construction, with teachers learning from their practice as they use existing resources and create new ones. Teachers' own perception of instructional capacity either as a finite set of knowledge or as an ongoing process of knowledge construction can influence teachers' use of resources. Their perceptions of

their own capacity can also be influenced by how the teachers view their students' thinking and address classroom discourse.

Teachers' conceptions are not only influential in increasing their instructional capacity, but also influence how they use curriculum (Ball & Cohen, 1996). The ways in which teachers use their understanding of curricular content, and how this content impacts their students' learning, influences how teachers make instructional decisions (Cohen & Ball 1999). Teacher beliefs about what curricular material is important for students, guide teachers in selecting and adapting materials to match students' needs (Ball & Cohen, 1996). Teachers' decisions may lead to a discrepancy between the intended curriculum and what is implemented in the classrooms (Ball & Cohen, 1996). For this reason, it is important to understand the ways in which teachers influence curriculum, which I now address.

Teachers' Influence on Curriculum

Curriculum includes both the plans for student learning as well as the resources that teachers use to support their students' learning. Using the definitions of the different curricula used in the teaching and learning of mathematics, researchers have focused on developing models to unpack the role that teachers play in shaping curriculum. Below, I discuss some of these models to situate teachers as part of the design and implementation process for research-based mathematics curriculum.

Models of curriculum implementation. Teachers' influence on curriculum has led to the development of models that describe the role that teachers play in shaping the curriculum (Pepin, Gueudet & Trouche, 2013; Remillard, 2005). These models are

focused on the process by which teachers work and engage with curricular resources (Gueudet & Trouche's, 2009), a process by which teachers construct meaning of these curricular resources to guide their teaching practice (Pepin, Gueudet & Trouche, 2013). These models have implications for the need to focus on teacher learning and development because they view teachers as users and designers of curriculum materials rather than simply transmitters of knowledge (Remillard, 2005). That is, teachers play an active role in making curriculum come alive inside the classrooms, and their engagement with the curriculum not only has the potential to inform their use of curricular resources but to influence their practice, as well. Next, I discuss two mechanisms through which these changes occur.

Instrumental approach and *documentational genesis* are two models that provide a theory of teachers' use of materials, how these transactions shape the resources, and how these changes impact teachers (Remillard, 2005). The instrumental approach describes the use of curricular resources and has been linked to the *Theory of Didactical Situations*, which focuses on teaching and learning as well as interactions between teacher and students (Brousseau, 1997). The Theory of Didactical Situations (TDS) draws from the mathematical theory of games, to analyze and improve the teaching of mathematics. The term *situations* includes not only students, but also their interactions in the classrooms, the teachers and the educational system (Brousseau, 1997). According to TDS the teacher's role is not to instruct the students using direct teaching but to develop problems and situations that will allow their students to have effective learning experiences (Brousseau, 1997). The objective of this approach is to develop an inclusive

view of all resources that can be used in mathematics teaching (Trouche, 2004). The instrumental approach to curriculum implementation led to the development of the documentational approach of didactics (Pepin, 2014) and broadens the conception of resource to mean anything that is available for the teachers to use in supporting students' learning (Pepin et al., 2013). This approach links curricular resources to the ways in which they are used and includes content, support for teaching the content, the plan for using curricular resources, and how they are eventually used (Pepin et al., 2013).

Likewise, the documentational genesis theoretical perspective for curriculum implementation emphasizes how teachers shape resources and provides a model for understanding interactions between teachers and curricular resources (Pepin, et al., 2013). Gueudet and Trouche (2009) introduce documentational genesis as a process through which teachers interact with curricular resources. In the first step of the process, resources influence teachers' practice and develop teacher knowledge. In the second step, teachers' knowledge guides their choice of what resources to use and how they are modified (Pepin, et al., 2013). These models have implications for developing teachers' instructional capacity because they focus on teacher learning through engagement with curricular resources (Pepin, et al., 2013). Going deeper into how teachers engage with curricular resources and the ways in which they make instructional decisions can provide insights into how curricula can be designed to improve teachers' learning and practice.

When implementing curricula, teachers engage in various *instructional interactions*. Instructional interactions are the interactions between students, teacher, mathematical content, and instructional resources (Remillard & Heck, 2014). Cohen and

Ball (1999) present instructional interactions between three elements: teachers, students and educational materials (see Figure 1). These researchers place a heavy emphasis on the interactions between teachers and students around educational material and stress that teachers' instructional capacity is dependent upon the interaction between all elements. Further emphasizing the importance of interactions, they state that any instructional element can impact instructional capacity because of its interaction with other elements. Instructional capacity is mainly influenced by teachers' knowledge, their understanding of the curriculum materials and their expectations of their students. Teachers should be given opportunities to develop and increase this knowledge because it can determine how they shape curricula and interact with students (Cohen & Ball, 1999).

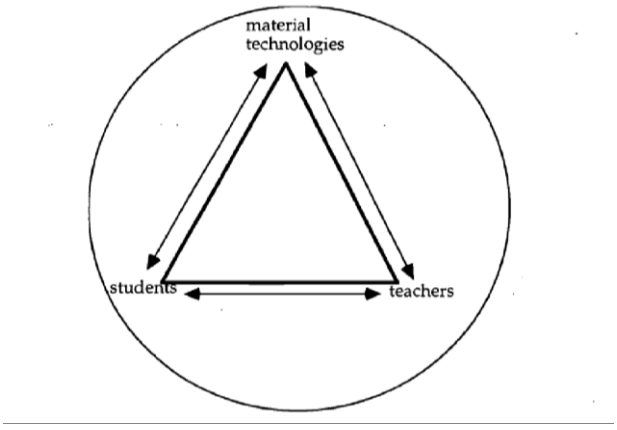


Figure 1. Cohen and Ball's triangular model (Cohen & Ball 1999).

Rezat and Sträßer (2012) present a similar model connecting teacher, students, mathematics content and resources such as mathematics textbooks, digital technology questions, problems, and specific language. This model is influenced by the assumption that mathematics education is heavily dependent on resources, or *artifacts*, as they're called in this model. Rezat and Sträßer's model starts with the didactical triangle that

focuses on the interaction between mathematics, students, and teachers and adds a fourth vertex of curricular resources or mediating artifacts (see Figure 2). Instead of expanding the triangle to a quadrilateral the authors argue that the interaction with artifacts impacts all other model components, resulting in a tetrahedron with each vertex interacting and influencing the others (Rezat & Sträßer, 2012).

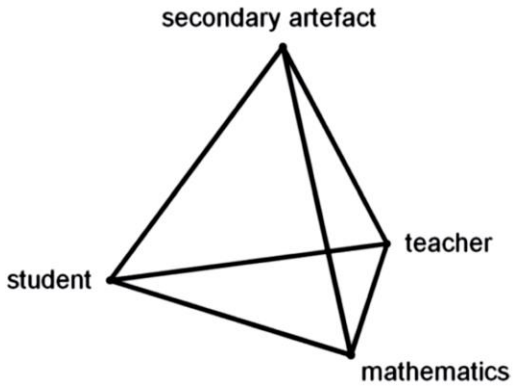


Figure 2. Rezat and Sträßers Didactical Tetrahedron (Rezat & Sträßer, 2012).

This work on the use of artifacts is driven by a desire to empower teachers by making them mindful of the relationship between the teachers themselves, their students, the content and the artifacts. This work seeks to identify the structure of teaching and learning situations through a model that includes mathematics, students, teachers, and artifacts (Rezat & Sträßer, 2012). The role of artifacts is emphasized because different users and groups of users share the same artifacts, and these artifacts impact how mathematics is taught and learned.

Rezat and Sträßer (2012) explain that using artifacts like textbooks and digital technologies is a complex process that involves interaction between user and artifact,

which is why artifacts are included as a new vertex to the didactical triangle of interactions. Understanding the relationship between teachers, students, and mathematics can be aided by understanding how artifacts are used and what role they play in the didactical triangle (Rezat & Sträßer, 2012). The researchers give examples of physical tools like mathematics textbooks, rulers, compasses, and log tables that have traditionally been used in mathematics education as well as non-physical tools like language, diagrams, and signs. These objects have allowed for representations of mathematical concepts and have been used to support the teaching and learning practice. Rezat and Sträßer draw upon Vygotsky's definitions of tools to distinguish between psychological and technical tools, with psychological tools aiming to alter mind and behavior, and technical tools aiming to change some object (Vygotsky, 1978). In a classroom setting the goal of tools is to aid in learning of mathematics, and all the tools used for teaching and learning of mathematics can be referred to as psychological tools. Rezat and Sträßer prefer to call these psychological tools as artifacts (Rezat & Sträßer, 2012).

The ways in which teachers use artifacts and their beliefs about the nature of teaching and learning mathematics can impact their students' learning but also their own behavior (Rezat & Sträßer, 2012). Rabardel (2002) called the process by which teachers use learning tools *instrumentalization*. According to Rabardel (2002) an instrument is made up of two parts, the first part is the artifact and the second part is one or more schemes to guide the actions. Tools guide the mental processes of a subject, these mental processes are focused on solving a problem posed by an object, and this whole process is called the instrumental act (Rezat & Sträßer, 2012). The use of tools can impact not only

students but teachers' own learning as well. Mathematics education research is beginning to recognize that the use of tools can also transform behavior. Not only do tools facilitate engagement with mathematics, they can also change the learner's experience of doing mathematics. Instrumentalization thus conceptualizes how the transformation of behavior takes place when a new tool is used (Rezat & Sträßer, 2012). Research on teachers' use of textbooks and digital technologies has shown that teachers' use of these artifacts has a great influence on the teaching and learning of mathematics in a classroom setting. Teachers often select the mediating artifacts (explained in detail below) for their students' learning and influence when and how they are used. It is therefore important to understand teachers' own use of artifacts.

Research on the use of curricular resources suggests that the use of resources is a process based on interaction between the teacher and the resource (Rezat & Sträßer, 2012). Further, research stresses that curriculum resources can provide learning opportunities for teachers in addition to their students (Ball & Cohen, 1996; Remillard & Bryans, 2004). For example, Doerr and Chandler-Olcott (2009) give an account of teachers implementing a standards-based mathematics curriculum and a shift in their teaching practice. As teachers coped to make sense of the demands of the curriculum, there was a shift in their ideas about the curriculum materials and their teaching practices involving the curriculum. As teachers became familiar with the curriculum, they gained curriculum knowledge that "involves the links between the development of communicative practices and the development of mathematical content" (p. 300). This knowledge aided the teachers' in helping their students develop mathematical writing

skills. The curriculum knowledge gained as a result of implementing the curriculum can inform teachers' practice as they implement a research-based mathematics curriculum. Drake and Sherin (2009), described case studies of three teachers, implementing a standards-based mathematics curriculum for two years. These teachers developed what the authors refer to as "curriculum vision," an understanding of the mathematical and pedagogical goals of the curriculum in terms of the 'big ideas' (p. 333); as well as trust in the curriculum. The big ideas were the mathematical ideas that the students were to learn. As teachers implemented the curriculum they understood the connections between the mathematical concepts presented in the curriculum and they started to use the curriculum materials without modifying them. This use of the curriculum materials without modification allowed them more time to plan how to engage the students in the classroom. Curriculum vision is a long-term plan for students' learning, and as teachers' become aware of this vision, their planning changes to incorporate each lesson into the bigger picture of students' learning goals; their focus moves from teaching to student learning.

Since the use of curriculum resources by teachers is a complicated process, it needs to be studied in action as it continues to develop (Lloyd, Remillard, & Herbel-Eisenmann, 2009; Rezat & Sträßer, 2012). I am interested in exploring teachers' engagement with the curriculum when implementing a research-based curriculum for the first time. I want to explore the nuances of teachers' engagement with the curriculum, the factors that mediate their interaction with curriculum, and the opportunities for their learning that arise as a result of these interactions.

Teachers' Engagement with Curriculum

To guide my research, I use a sociocultural framework focusing on "mediated action" (Wertsch, 1994). A sociocultural approach deals with the connections among human actions, which include mental actions like reasoning and remembering, as well as factors like culture, institutional or historical situations. The goal of a sociocultural perspective is to observe human action within context while considering the action and the setting as distinct but related (Wertsch, 1994). Here mediated action plays an important role in helping to understand sociocultural phenomena. As Wertsch explains, "Mediated action must be understood as involving an irreducible tension between the mediational means provided by the sociocultural setting, on the one hand, and the unique, contextualized use of these means in carrying out particular concrete actions, on the other" (p. 202). My goal is to use mediated action as a way to explore instructors' engagement with the curriculum and how it produces opportunities to learn. The mediating tools are instructors' ways of engagement that influence how they interact with the curriculum. I will be using interaction between teachers and the curriculum as the basic unit of analysis.

Mediated action stems from Vygotsky's (1978) account of mediation with the idea that mediational objects like language and technical tools, in addition to facilitating the action, can actually alter mental functions. Vygotsky tends to focus more on mastering the mediational means (mastering an existing meaning system and conforming to an existing sociocultural setting) and less on how engagement with these mediational means change meanings, change tools, and may lead to the creation of new mediational

means (Wertsch, 1994). The difference between mediation and mediated action stems from the important role that humans play in altering the cultural tools and their linked meaning systems (Wertsch, 1994).

Brown (2002) used Wertsch's theory of mediated action to investigate the teacher tool relationship. In his study Brown examined three urban middle school teachers' use of curriculum materials as they enacted a 10-week science project. He developed the Design Capacity for Enactment framework to examine the factors that affect teacher-tool interactions. He used a scale to describe teachers' dependence on curriculum materials to design their instruction. Brown defined three categories of teachers' use of curriculum: *offloaders (adopters), adapters* and *improvisers*. An offloader (adopter) transfers his or her agency as a designer of the classroom instruction to the curriculum materials. Offloaders (adopters) are usually inexperienced teachers. For an *improviser*, the agency shifts to the teacher, as he or she makes instructional decisions without relying too much on the curriculum. An *adapter* is somewhere in the middle, using both curricular resources and teacher resources. According to Brown, the use of curriculum as *offloader (adopter), adapter* or *improviser* brings into focus the extent to which curriculum materials can impact teaching practice. Understanding the ways in which teachers use curriculum materials can inform curriculum developers and professional development providers as they design materials and supports to fit the needs of the teachers.

His findings revealed that both curriculum design and teacher knowledge influence instructional outcomes. In addition, he introduced the idea of pedagogical

design capacity which refers to teachers' ability to use both teacher and curriculum resources to design instruction.

Professional Learning Communities

In a professional learning community (PLC), educators work together to develop supportive conditions that promote collaboration and growth (DuFour & Eaker, 2005). Although there is no single definition of a PLC, and a variety of definitions exist based on different contexts (Fulton, Doerr, & Britton, 2010), for this study I consider a PLC as "a group of people sharing and critically interrogating their practice in an ongoing, reflective, collaborative, inclusive, learning-oriented, growth-promoting way" (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006. p. 223). Within this definition, I include formal in-person or online interactions with colleagues guided by a facilitator; and informal interactions with colleagues during shared office hours as well as through text messages or emails.

Professional learning communities did not originate within education but serve as a platform for issues such as school reform, improving teaching as a profession, and teacher accountability (Lieberman & Miller, 2016). Within education, teacher PLCs are grounded in the idea that teachers have unique experiences and knowledge relevant to their practice. They can impact their knowledge by reflecting on their experiences together (Buysse, Sparkman, & Wesley, 2003). Research suggests that the use of curricular resources is a process based on interaction between the teacher and the resource (Rezat & Sträßer, 2012). Understanding the role of these interactions between resources and teachers in a learning environment is important to better understand the

dynamics of teaching and learning mathematics. In this regard, PLCs can influence teachers' engagement with curricular resources.

In PLCs, the foundational idea is that teachers can learn about best teaching practices by working with each other (Leiberman & Miller, 2008). Knowledge doesn't necessarily need to flow from expert to novice, and novices can also exchange knowledge among themselves (Schon, 1983). This exchange of ideas between teachers themselves can allow them to take ownership of their own learning and development. PLCs facilitate teacher collaboration by allowing them to reflect on their practice while sharing ideas and providing critical feedback to each other (Lieberman & Miller, 2016). This continuous reflection on teachers' practice is a major component of a PLC (DuFour, 2004). Leiberman and Miller (2008) explain that the focus of these collaborations between teachers is often less on the procedural skills and more so on what they find important in regards to their shared experiences. The learning that takes place through their collaboration is a result of their conversations and the relationships they build.

The structure of PLCs in the form of collaboration and support can allow teachers to bring about changes in their environments (Vescio, Ross, & Adams, 2008) and can impact the culture of schools (DuFour, 2004). Teacher engagement in PLCs benefits students and teachers by giving teachers increased authority over their instructional decisions and encouraging them to engage in their own learning and development (Lieberman & Miller, 2016). Within a PLC, teachers redefine their roles as collaborators in the PLC and develop the capacity to expand their sphere of influence beyond their classrooms (Leiberman & Miller, 2008). Working together to focus on issues allows them

to undertake tasks greater than what they could accomplish by working alone (DuFour & Eaker, 2005). Perhaps this is why the use of PLCs has been encouraged in curriculum development and education reform efforts (Stenhouse, 1975). Strong PLCs can be essential in supporting teachers to change their practice and pedagogy (McLaughlin & Talbert, 2001). PLCs can help to support institutional reform by analyzing teacher practice and refining it to help students (Seashore, Anderson, & Riedel, 2003).

Leiberman and Miller (2008) explain that within a PLC, teachers reflect individually on their practice. This reflective practice leads to the generation of a variety of ideas, activities and representations that can be used to guide them when facing new and challenging situations. This sharing of knowledge and experience allows them to navigate their professional lives. Working together in groups to collaboratively solve problems allows for the formation of shared knowledge. Implementing reform-based curricula may require a significant effort on the part of the teachers because these curricula often call for extraordinary shifts in teachers' practice (Spillane & Anderson, 1997). PLCs can be seen as a resource to support teachers in curricular implementation. Few researchers have analyzed the role of teacher collaboration in the curriculum enactment process (Remillard & Heck, 2014). Referring back to Wertsch's (1994) sociocultural framework focusing on mediated action, interaction with other teachers in a PLC is the mediated action that can support teachers in demanding situations where they are asked to implement a research-based curricula that calls on them to understand and apply new content.

Implementing curricula that challenge teachers' assumptions about mathematics may warrant significant support for teachers. In addition, while support can be beneficial for teachers, different institutions may require supports specific to the needs of teachers in that environment. Since the professional demands vary for teachers at different levels (K-12, undergraduate), it is important to understand the nuances of their realities, such as the ways in which they navigate the implementation of a research-based mathematics curriculum. One such group of teachers who require specific supports based on their unique work situations are adjunct instructors. In this project I have worked with adjunct instructors and in the next section, I present research about the teaching experiences and needs of this sub-group of teachers to help the reader understand their engagement with the curriculum.

Adjunct Instructors

Adjunct instructors are non-tenure track, part-time faculty teaching undergraduate courses. The teaching of college- and university-level mathematics has its own challenges and assumptions (Pepin, 2014). Faculty members in institutes of post-secondary education are diverse and can have different types of appointments (e.g., full-time, part-time, teaching responsibilities only, both teaching and research responsibilities) (Gappa, & Austin, 2010). According to Mason (2009), not all faculty members have positions that are tenured or can lead to tenure, and only about half of faculty members have a full-time position. In 1980, 55 percent of faculty were tenured or tenure-track full-timers; by 2003 that number declined to 41 percent (Mason, 2009). While the number of full-time faculty members decreased, the number of part-time faculty gradually increased. In 1975, part-

time instructors constituted 25 percent of the college and university faculty, and by 2011 that number reached 42 percent (Curtis, 2014). These numbers show a growing trend in the employment of adjunct instructors by higher education institutions. There are practical reasons for this trend, the first being budget constraints (Green, 2007). Part-time faculty benefit institutes of higher education because they save the institutions money. Institutes can have high enrollment, keep class sizes low and yet remain within their budgets. Part-time faculty receive less pay than full-time faculty, and if their classroom performance is lacking, it is easier to not rehire them (Gerhart, 2004). Regardless of the reasons for increased employment of adjunct instructors, their heavy presence in the undergraduate classrooms warrants attention.

Development programs for part-time faculty are important because regardless of their appointment type, all faculty members are important to an institution (Gappa, Austin, & Trice, 2007). With proper support, faculty can continue to be effective at what they do and strengthen the quality of their instruction, research, and outreach (Leslie & Gappa, 2002; Gappa et. al, 2007). These opportunities to expand faculty knowledge and skills are especially important, because the nature of faculty work demands an understanding of student learning and staying abreast of new technologies that can enhance teaching (Gappa et al., 2007).

When designing programs to support adjunct faculty, it is important to keep their particular needs in mind. Literature suggests that adjunct faculty need to feel like they are a part of the intellectual life of the institution (Gappa et al., 2007; Lyons, 2007). In addition, adjunct faculty require training in teaching and classroom management skills,

continued professional development, and recognition for good work (Lyons, 2007). According to Gappa and colleagues (2005), all faculty members, both full time and part time, should have access to resources needed to fulfill their responsibilities and opportunities for professional growth. The schedules of part-time instructors often do not allow them to interact with regular staff because they tend to teach evening classes, often in different buildings. They might also be teaching at various institutions at the same time. Thus, while many adjunct faculty members enjoy their work, they feel disconnected (Green, 2007).

The development of adjuncts should be an ongoing goal despite time and financial constraints (Bettinger & Long, 2005; Green, 2007). When designing programs for adjunct faculty, administrators should have regular meetings with them to discuss the goals of the institution so that adjuncts can learn about the evolving environment of higher education along with its demands and challenges (Green, 2007). It is also noteworthy that a faculty member who is on campus to teach just one course may have different professional growth interests than a full-time, tenure-track faculty member (Gappa et al., 2007).

To respond to the diverse interests and needs of individual faculty members, many institutions are taking innovative approaches to faculty development. For example, some universities and colleges, in recognition of the time pressures that faculty experience, are providing online and in-person professional development opportunities (Gappa, 2008). This opens access to professional development opportunities for faculty in all types of appointments both on and off campus and allows them to participate in professional development activities that best match their interests and their circumstances

(Gappa, 2008). Some institutions have introduced individual growth plans that open the door for productive discussions with department or institutional leaders about the relationship between individual and institutional aspirations, commitments, and goals. In addition, attention to mentoring is an area of special interest at a growing number of institutions (Gappa et al., 2007).

Lyons (2007) provided examples of several adjunct PD programs implemented at various institutions of post-secondary education. The professional development models were designed to fulfill the specific needs of these institutions. Several programs took a hybrid approach, combining series of in-person workshop sessions with an online program. The success of one such initiative led to the development of a faculty recognition program, which included a pay raise and a special title for the adjunct faculty in exchange for their commitment and effort to improve student learning. Another program took a similar approach to identify and reward outstanding adjunct instructors with appreciation through status and compensation.

Mentoring was an important component in many of these professional development efforts (Lyons, 2007). Some programs focused on matching a part-time instructor with an experienced instructor while others used mentoring opportunities in addition to features like online resources, brown bag PD luncheons, and recognition at institutional gatherings. This multi-pronged approach was also taken by another program that combined adjunct orientation, access to resources and long term mentoring that guided the adjunct faculty members in their career and personal development as well.

While the programs I mention proved effective in these specific institutions, there is no single model that can fit the needs of all the institutions. Institutions should develop programs that cater to their own needs (Austin & Sorcinelli, 2013). New approaches to adjunct faculty development need to be established to flourish within organizations (Austin & Sorcinelli, 2013). One way of providing support and development to the faculty that can prove effective despite the various organizational structures is a focus on collaboration both inside and outside the institution (Austin & Sorcinelli, 2013).

Teacher collaboration within a PLC can prove to be a beneficial model for adjunct support and learning. A situated community of adjunct teaching faculty is more knowledgeable about their circumstances and receptive to professional development that matches their needs (Bryk, Gomez, & Grunow, 2011). Participation in academic communities where colleagues are caring, respectful and appreciative is beneficial to all faculty members regardless of their appointment type (Gappa et al., 2007).

As mentioned earlier, changes in the faculty composition in colleges and universities in the U.S. with a trend towards an increasing adjunct workforce points towards the need to understand the connections between the experiences of adjunct faculty and student learning outcomes. It is important to understand how realities of adjunct faculty's working conditions including scheduling and lack of professional development opportunities can influence student learning (The Delphi Project, 2012). Research on mathematics adjunct instructors is scarce and there are currently no studies of adjunct instructors implementing a research-based mathematics curriculum.

Conclusion

The literature reviewed in this chapter helps to situate teachers' interactions with curriculum and how such interactions can influence their own learning and instructional practice. Through a sociocultural lens, research can provide an internal view of teachers' engagement with curriculum and aid in understanding how teachers' knowledge and practice is impacted by their engagement with curriculum. When implementing curriculum, teachers interact with various resources to make decisions regarding students' learning. Teachers need support for such interactions, especially when it comes to implementing a new curriculum. As institutes of higher education increasingly hire more adjunct instructors to teach undergraduate courses (Curtis, 2014), there is a need to find ways to support this particular group of teachers.

Chapter 3

Methods and Methodology

This chapter describes the design of the study, research site, participants, data sources, collection and analysis. The data were collected in the 2016-2017 academic year with three adjunct instructors teaching a research-based Precalculus curriculum for the first time.

Research Design

This research focuses on describing the ways in which adjunct instructors engage with a research-based Precalculus curriculum, and the ways in which their use of the curriculum influence their professional knowledge. Professional knowledge includes instructors' content knowledge, pedagogical content knowledge, and instructional practices. The current study is situated within a larger research project, described below, focused on course coordination and support of adjunct instructors implementing a research-based mathematics curriculum.

Teachers make various decisions regarding students' learning. Instructors' decisions not only influence the curriculum that is implemented in the classroom, but their involvement in the process can also lead to their own learning (Cohen & Ball, 1999). To better understand this process, this study aims to answer the following research questions:

1. What is the nature of adjunct instructors' engagement with a research-based Precalculus curriculum?

2. How does engagement with a research-based Precalculus curriculum provide opportunities for adjunct instructors' learning?

To answer these research questions, I focus on teachers' engagement with the curriculum; that is, the ways in which instructors interact with the curriculum materials and resources. I investigate what impact this has on the instructors' knowledge and practice, which I measure through the opportunities to learn that develop through this engagement. These learning opportunities could be in terms of content knowledge, pedagogical content knowledge, or classroom practice.

Since my aim is to capture teachers' engagement with the curriculum and analyze any opportunities to learn that ensued from this engagement, I use case study methodology. Case study lends itself well to studying phenomena when there is little to no possibility of controlling the events (Yin, 2009). A case study is a mode of empirical analysis that focuses on a phenomenon within a context; the boundaries between the phenomenon and the context are not clearly defined (Yin, 2009). It is an appropriate methodology for making sense of multifaceted social phenomena and is best suited for my research because of the nature of my research questions. Case studies are appropriate when answering questions like *how?* or *why?*, that aim to gain understanding of a phenomenon within its context (Yin, 2009). In this study a case consists of an adjunct instructor's engagement with the curriculum.

Context

In the following sections I provide information about the background and context of my study.

School. This research takes place at Kara State University (KSU), a midsized state university in the northeastern United States. Specifically, the study is situated within a department of mathematical sciences, the largest department within the college of science and mathematics. Within this department, there are 31 full-time and approximately 22 part-time faculty members (depending on the semester). The part-time adjunct faculty members mainly teach introductory mathematics and physics courses, including Precalculus. Historically, adjunct faculty have been responsible for teaching over 75% of the Precalculus sections, serving approximately 600 students in STEM majors each semester. All of the Precalculus sections in the fall semester of 2016 and the spring semester of 2017 were taught by adjunct faculty.

Many of the adjunct instructors in this department have been teaching at KSU for over 10 years. In the past, no course coordination was in place for Precalculus. However, the department did provide the adjunct instructors with a syllabus for the course and suggested a textbook. This limited guidance led to inconsistency in how the course was taught. For example, there were discrepancies in terms of the content that was taught and the level to which conceptual understanding was emphasized by the instructors. In addition, there was wide variation in assessment and grading.

AMIRS Project. The data for this research was collected in the Fall 2016 and Spring 2017 semesters as part of a larger study focusing on Precalculus course coordination and supports provided for adjunct instructors at KSU. This project, *Adjunct Mathematics Instructor Resources and Support (AMIRS): Improving Undergraduate Precalculus Teaching and Learning Experience,* aims to build a model of Precalculus

course coordination and adjunct support to improve the teaching and learning of Precalculus in order to promote student academic success and retention in STEM majors. While some work has been done to understand the benefits of supports for part-time instructors at the undergraduate level, this work has focused mostly on graduate teaching assistants rather than adjunct instructors (Alvine et al., 2007; Barry & Dotger, 2011; Belnap & Allred, 2009; DeFranco & McGivney-Burelle, 2001; Ellis, 2014; Ellis, 2015; Gutmann, 2009; Hauk, Kung, Segalla, Speer, & Tsay, 2006; Hauk, Chamberlin, Cribari, Judd, Deon, Tisi, & Khakakhail, 2009; Kung & Speer, 2009; Kung, 2010; Luft, Kurdziel, Roehrig & Turner, 2004; Raychaudhuri & Hsu, 2012; Seymour, 2005; Speer, Gutmann, & Murphy, 2005). The AMIRS project received funding from the National Science Foundation to extend the work with graduate teaching assistants to adjunct instructors and to contribute to the less than robust research base regarding the adjunct instructor population. By building a model of adjunct instructor resources and support, AMIRS contributes to deeper understanding of how such efforts impact (1) adjunct instructor knowledge and instructional practices, (2) adjunct instructors' job satisfaction, and (3) student academic success and retention in STEM majors. This understanding may help other departments and institutions with similar instructor populations better support their adjunct faculty, thus improving student achievement and retention in STEM majors.

Through AMIRS, several supports, such as a summer workshop, course coordination, and an online PLC, were planned for the adjunct instructors. These supports were designed in conjunction with the adoption of *Precalculus: Pathways to Calculus* (or *Pathways*), a research-based Precalculus curriculum. This curriculum was developed at

Arizona State University (ASU) by building on over ten years of research to understand foundational concepts that are necessary for students to succeed in calculus (Carlson, 2014). In the following sections I discuss the curriculum and supports available for the Precalculus adjunct instructors.

Curriculum. Carlson and her colleagues developed *Pathways* to improve the teaching and learning of Precalculus by emphasizing exploratory engagement with mathematical concepts. The curriculum is divided into chapters called modules and within each module there are sections called investigations that organize the big ideas of the module into smaller parts that fit together. The problems in each investigation are designed to foster students' development of conceptual understanding of one or more big ideas. The core ideas of this curriculum are based on covariational reasoning, which has been shown to be foundational for understanding a variety of mathematical concepts (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002). Covariation can be viewed in terms of two or more quantities changing simultaneously. As Saldanha and Thompson (1998) describe:

> Our notion of covariation is of someone holding in mind a sustained image of two quantities' values (magnitudes) simultaneously. It entails coupling the two quantities, so that, in one's understanding, a multiplicative object is formed of the two. As a multiplicative object, one tracks either quantity's value with the immediate, explicit, and persistent realization that, at every moment, the other quantity also has a value. An operative image of covariation is one in which a

> person imagines both quantities having been tracked for some duration, with the entailing correspondence being an emergent property of the image. (p. 299)

The foundation of covariational reasoning is based in the theory of quantitative reasoning (Thompson, 1993, 1994, 2011). A main idea important to mathematics educators is that a quantity is a mental construction, which exists inside the mind of the perceiver (Thompson, 2011). A quantity consists of an object, a quality of the object, a unit, and a process that assigns a numerical value to the quantity (Thompson, 1994). It is a scheme, and therefore the conception of a quantity can vary from person to person depending on the *varying levels of development of the components of their schemes* (Thompson, 1994, p. 8). Educators need to be attentive to students' thinking and how to conceive situations because students' conceptions may be different from their own (Thompson, 2011).

Research has shown that the concepts of variation and covariation are necessary for explaining the reasoning of students who conceptualize a situation quantitatively and as a dynamically changing event. The ability to perceive quantities as varying simultaneously can help students and teachers develop useful and robust conceptions about important mathematical topics like functions (Thompson & Carlson, 2017). In contrast, an understanding that is based on symbolic manipulations and procedural techniques does not allow the student to see a general view of the mapping between sets of input and output values. Specifically, students do not develop the conceptual foundation needed for function relationships with continuously changing input and output variables (Carlson, 1998).

The goal of the *Pathways* curriculum is to ensure continued success in Calculus and to encourage more students to persist in studying mathematics and science by developing their problem-solving ability and confidence (Carlson, 2014). The *Pathways* curriculum was designed to help both students and teachers develop deeper understanding of key Precalculus ideas such as quantity, variable, function, rate of change, exponential growth, angle measurement, and trigonometric functions. It is based on in-depth research on learning processes, teacher knowledge, and teaching effectiveness. The *Pathways* model of effective teaching is based on teachers' understanding of the course's big ideas and how those ideas are connected to each other. The curriculum materials include a student workbook with investigations that students can engage in inside the classrooms. The student workbook includes access to an online textbook where students can access examples and read about the big ideas explored in the investigations. The online textbook also includes embedded videos and apps to enhance student learning. There is an online homework system created and maintained by the curriculum developers. For our Precalculus implementation we focused on modules 2 through 8. These modules included the following topics: Rate of Change, Functions, Exponents, Polynomials, Rational Functions, Trig Functions and Trigonometric Identities.

The curriculum developers provide guidance to the instructors in implementing the curriculum through professional development (see details of the workshop below) as well as Instructor Notes that accompany each module. The Instructor Notes, provide

instructional guidelines to the teachers. Some examples from the Instructor Notes are presented below:

- Asking the teachers to discuss the problems in class:
    - "We encourage you to let students discuss part c in a group or with another person in the class for a few minutes before calling on groups to report their thinking." (Instructor Notes, Module 2, Investigation 1, Problem 0)
    - "We encourage you to discuss how a specific expected resale value is determined given the previous years' expected resale value." (Instructor Notes, Module 4, Investigation 1, problem 1)
- Giving suggestions for phrasing to be used in class:
    - "It may be valuable to use the word 'represent' initially before using the equal sign as shorthand for defining variables. It is important that students come to 'see' these letters are standing for varying values of a quantity, so that expressions and formulas involving variables are meaningful to them throughout the course." (Instructor Notes, Module 2, Investigation 1, Problem 4)
- Stressing that the instructors allow students to develop concise mathematical meanings:
    - "For parts (a) through (f), take the time necessary to ensure that students see the numbers and letters in the expressions that they build *as*

representing values of different fixed and varying quantities." (Instructor Notes, Module 2, Investigation 1, Problem 7)

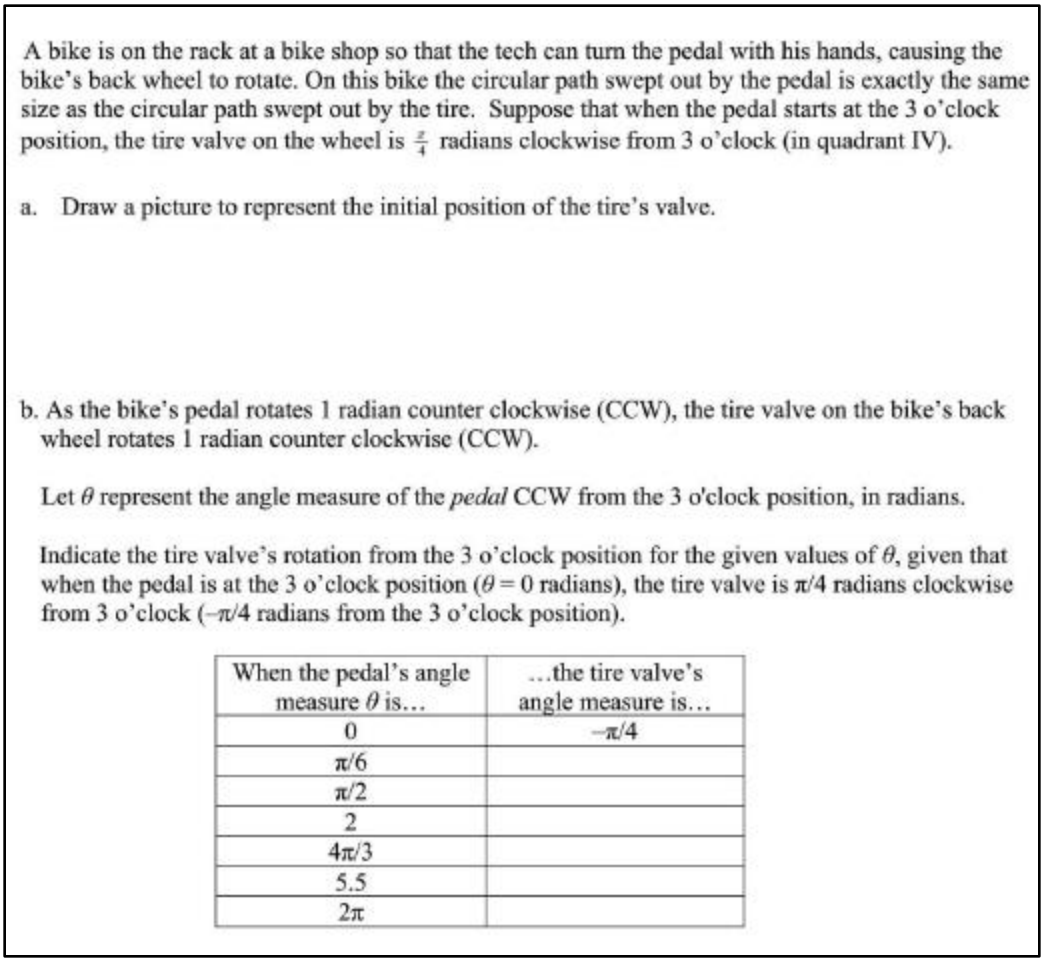
- "Make sure students can explain and represent in a diagram what distance *5t* represents and what difference *10t* represents." (Instructor Notes, Module 2, Investigation 1, Problem 7)
- "Encourage students to identify both the quantities and describe how they intend to measure each quantity. Be sure to require that they include units in their descriptions... Again, it is critical that you require students to speak meaningfully by being specific about what each quantity is measuring, the reference point from which the measurement is taken, and the units for the measurement. Pose questions such as "distance from what?", "distance measured in what units?" (Instructor Notes, Module 7, Investigation 3, Problem 1)
- "Have students describe how the length of the bug's vertical distance above the horizontal diameter changes." (Instructor Notes, Module 7, Investigation 3, Problem 2)
- "Also, continue to push students to identify distinct quantities on the diagram and how these are conveyed by the graph" (Instructor Notes, Module 7, Investigation 3, Problem 2)
- "Once students have completed the table for problem 4, prompt them to explain the range of values of the input quantity Θ for which *h(Θ)*

> completes a full cycle of values." (Instructor Notes, Module 7, Investigation 6, Problem 3).

These are some of the ways in which curriculum developers emphasized the goal of developing students' mathematical meanings. They suggested discussing the mathematical ideas in class as an effective teaching practice to achieve this goal.

The instructors who implemented the new curriculum were also supported through a pre-semester workshop focusing on the foundational concepts and philosophies of the *Pathways* curriculum; a weekly online professional learning community (PLC); and course coordination with access to a course coordinator for guidance.

Workshop. In order to aid instructors in implementing the Pathways curriculum, the curriculum developers provided a pre-semester workshop. The workshop provided instructors who were new to the curriculum, ideas for instructional practices that could be beneficial to their students. Through the workshop, the facilitators communicated the underlying philosophy of the curriculum and the main learning goals in each module. During the workshop, Precalculus instructors engaged in activities requiring analysis, conceptualization, and multiple representations of mathematical ideas. They were given a chance to put themselves in their students' shoes as problem solvers. They also had the opportunity to explain their thinking as they explored patterns of change between varying quantities and to demonstrate the logic behind their work. Instructors worked through investigations (see sample problem from an investigation, Figure 3) that are composed of a sequence of thought provoking questions.



A bike is on the rack at a bike shop so that the tech can turn the pedal with his hands, causing the bike's back wheel to rotate. On this bike the circular path swept out by the pedal is exactly the same size as the circular path swept out by the tire. Suppose that when the pedal starts at the 3 o'clock position, the tire valve on the wheel is $\frac{\pi}{4}$ radians clockwise from 3 o'clock (in quadrant IV).

a. Draw a picture to represent the initial position of the tire's valve.

b. As the bike's pedal rotates 1 radian counter clockwise (CCW), the tire valve on the bike's back wheel rotates 1 radian counter clockwise (CCW).

Let θ represent the angle measure of the *pedal* CCW from the 3 o'clock position, in radians.

Indicate the tire valve's rotation from the 3 o'clock position for the given values of θ, given that when the pedal is at the 3 o'clock position ($\theta = 0$ radians), the tire valve is $\pi/4$ radians clockwise from 3 o'clock ($-\pi/4$ radians from the 3 o'clock position).

When the pedal's angle measure θ is…	…the tire valve's angle measure is…
0	$-\pi/4$
$\pi/6$	
$\pi/2$	
2	
$4\pi/3$	
5.5	
2π	

Figure 3. Investigation 7, Module 7, Problem 2.

Facilitators helped instructors pay careful attention to the mathematical terminology to be used in the classrooms so that it could be accurate and convey the mathematical ideas meaningfully to the students. In addition, the instructors were guided on how to assess their students' thinking, pose meaningful questions and provide clear explanations.

The workshop took place in August of 2016 and included 2 full-day sessions and 1 half-day session. It was facilitated by one of the developers of the curriculum and her doctoral student. There were 9 participants in the workshop, 7 adjunct instructors and 2

tutors. The sessions were recorded and made available to those instructors who could not attend.

Professional learning community. While the pre-semester workshop aimed to help the instructors begin the semester, the AMIRS investigators understood the importance of providing on-going support to instructors to allay concerns that may arise during the semester. Research suggests that ongoing professional development that provides continued learning and growth opportunities is more effective than one-stop workshops that do not (Ball & Cohen, 1999; Guskey, 2002; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003; Putnam & Borko, 1997; Wilson & Berne, 1999). Professional development should also provide long-term growth opportunities and be immersed in the situation instructors are working in to be more aligned to their work (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010). Accordingly, the AMIRS team planned an online PLC to provide opportunities for growth and support to the instructors. The goal of the PLC was to continue conversations unpacking the curriculum content and philosophy of teaching and learning, and for instructors to bring relevant questions and scenarios to the meetings to have discussions about ideas connected to their classrooms. The online weekly PLC meetings were facilitated by a faculty member (Fall 2016 semester) and a doctoral student (Spring 2017 semester) who were familiar with the curriculum and part of the AMIRS research team. The meetings were conducted via an online learning management system that provided a virtual meeting platform. The platform offered the facilitator the ability to share the investigations on the screen as well as speak to the group. The group also had the ability to share their audio or type in their

responses in a chat. Each member of the group decided if he or she would prefer audio or typed responses. All instructors were invited to participate in the meetings, regardless of their participation in the research study. The PLC was designed to be responsive to the instructors' needs and it was anticipated that as the needs of the instructors changed, the focus of the PLC would change to meet their needs.

The meetings in the fall semester focused heavily on the content, where the facilitator provided the instructors with a list of suggested problems for each investigation ahead of time that he would solve on the screen. As the facilitator solved these problems, he would share effective ways to phrase various mathematical concepts that were related to the curriculum. During the spring semester, the new facilitator employed a similar model by providing problems to be discussed ahead of the scheduled meeting. However, the goal of the meetings during this semester was to discuss the problems in light of instructors' previous experiences.

Course coordination. Another method of support was that of course coordination. The coordination for the multi-section Precalculus course began with the adoption of a new curriculum and designation of a course coordinator. There was a need for Precalculus course coordination to provide consistent instruction, pacing, and learning objectives across all sections offered. Prior to course coordination, no textbook was assigned, only suggestions for textbooks were given. There was, however, a syllabus provided but without a pacing guide or a suggested content focus. Historically, the majority of the sections for this course were taught by part-time adjunct faculty with minimal departmental guidance. As a result, students' learning experiences in different

sections depended on the instructor. The Precalculus course coordination streamlined the information that instructors needed in providing a successful learning experience for their students. The coordination included clearly communicating course requirements and goals to the instructors, and providing common syllabi, pacing guides, assessments, and grading rubrics. Finally, a full-time faculty member volunteered to be the course coordinator to give continued guidance to instructors. The faculty member was also responsible for creating the pacing schedule for the course, creating exams, and was available to the instructors to help them with their issues and concerns (e.g., students having issues with the homework website, classroom issues with projector or whiteboard). The coordination also included access to tutors dedicated to Precalculus. Each semester, these tutors were trained in active learning processes, study skills, learning strategies and tutoring techniques through the tutoring center at KSU. Tutors who were part of the program in the first semester also attended the pre-semester *Pathways* summer workshop. This experience helped the tutors assist the adjunct instructors during class times. They were able to provide assistance during scheduled tutoring hours specific to *Pathways* curriculum in addition to general tutoring sessions available in the university's tutoring center.

The overall goal of providing supports for the Precalculus adjunct instructors was to provide students with a consistent experience as they enrolled in the course with various instructors and to remove some of the administrative responsibilities from the adjuncts so they could focus on teaching-related activities. The end goal for these

additional supports was the improvement of students' achievement and their retention in STEM.

The AMIRS project received support from the National Science Foundation (NSF) to cultivate a course coordination model that would eventually be disseminated to other departments in KSU and possibly other institutions. The AMIRS project aims to answer questions about the impact of supports provided to adjunct instructors. The impact includes any changes in the instructors' knowledge or practice, changes in their sense of belonging to the mathematics department, and impact on students' achievement or retention in STEM majors. Within this larger project, my research is situated in the question about changes in the instructors' knowledge and practice, and also looks at the specific impact through the curriculum. That is, my research explores the ways in which the instructors engaged with the new curriculum and how this engagement provided opportunities for their learning.

Participants

The participants in this study were three adjunct faculty members teaching Precalculus using the *Pathways to Calculus* curriculum. They were selected because of the similarities and contrasts in their past teaching experiences. All three participants had over ten years of experience teaching Precalculus at the high school or undergraduate levels. In order to maintain confidentiality of the information provided by the participants, I have reported the same gender (male) and used pseudonyms for all three instructors.

Caleb had taught Precalculus both at the high school and college levels. At the time of the study, he was also a doctoral student at KSU, working towards his degree in Mathematics Education. Caleb had an undergraduate degree in Accounting and a master's degree in Mathematics Education. He had about 18 years of experience teaching mathematics both at the undergraduate and K-12 levels and had taught Precalculus about 10 times previously.

Michael had only taught Precalculus at the college level with no experience teaching at the K-12 level. He was working as an accountant in addition to teaching classes at KSU. He had received both his undergraduate (B.S, Mathematics) and graduate (M.S. Mathematics, Education concentration) degrees from KSU and felt comfortable in the department.

Justin had experience teaching Precalculus only at the high school level, where he had been working as a full-time teacher for over ten years. He had an undergraduate degree in Mathematics. He had also received his master's degree (M.S. Mathematics, Pure and Applied concentration) from KSU. He had never taught at the college level before and the fall semester 2016 was his first time teaching at the college level. As a full time high school teacher he had taught Precalculus at the high school level several times but was unsure about the norms of teaching an undergraduate mathematics course. He was teaching accelerated Precalculus at his high school at the same time he was teaching Precalculus using the new curriculum at KSU.

Analytical Framework

In this work, I analyze teachers' engagement with curriculum through actions that include planning, reflecting, enacting and collaborating as the various forms of their engagement. I explore the ways in which teachers engage with curriculum and how this engagement provides opportunities for their learning. To understand teachers' use of curricular resources it is important to focus on teacher knowledge and the knowledge embedded in the resources. I'm using a sociocultural approach (Mediated action) to understand teachers' engagement with the curriculum and how it leads to opportunities for their learning. The study of mediated action focuses on how agents use mediational means when engaging in various forms of action (Wertsch, 1998). In this study teachers' engagement with the curriculum is the mediated action with the various forms of engagement, planning, enacting, collaborating, and reflecting acting as factors that mediate engagement. These forms of engagement are the mediational means that influence teachers' engagement with the curriculum. The foundational idea of mediated action is that there exists a *tension* between the mediational mean and the user within a context where the action is carried out (Wertsch, 1994). One of the results of this tension is that boundaries between the mediational mean and the user begin to wear down. This erosion of boundaries between the mediating mean and the user makes it important to observe their interaction within the context and as a whole. Further, the use of mediational means involves a level of uniqueness where any kind of mediated action will involve some level of variation and perhaps even innovation (Wertsch, 1994). My goal is

to showcase the various ways in which teachers interact with the curriculum and the ways in which their experiences influence their knowledge and their teaching practice.

In this section I present the frameworks I have used for analysis. I drew on Remillard and Bryans' (2004) work on teachers' opportunities to learn as they implement a curriculum, as well as Brown's (2002) Design Capacity for Enactment Framework. They provide the foundation and language used in my analysis. In addition I have used, Shulman's (1986) frameworks of content and pedagogical content knowledge (Shulman, 1986) and the decentering framework presented by Carlson and colleagues (Carlson, Moore, Bowling, & Ortiz, 2007).

For my analysis I am interested in instructors' engagement with the curriculum and how their various forms of engagement (Planning, enacting, collaborating and reflecting) are the mediating means in instructors' engagement with the curriculum. I have employed an amended version of Brown's Design Capacity for Enactment Framework (2002) to analyze instructors' engagement with the curriculum and the opportunities for instructors' learning that emerge through this engagement.

Design Capacity for Enactment Framework

Brown (2002) developed the Design Capacity for Enactment (DCE) Framework to understand the teacher-tool relationship. It focuses on the dynamics between teacher and curricular resources in the process of implementing curriculum as a teacher adapts, adopts (offloads) or improvises the curriculum by using such resources (see Figure 4).

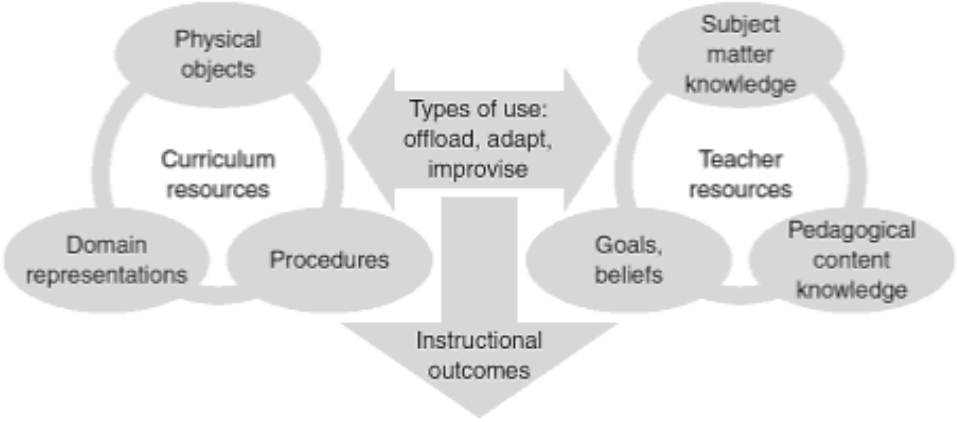


Figure 4. Design Capacity for Enactment Framework (Brown, 2009).

In Brown's framework, Curriculum resources include depiction and organization of domain concepts, curriculum materials and procedures. Teacher resources may include teachers' content knowledge, pedagogical content knowledge (Shulman, 1986), and their goals and beliefs (Brown, 2009). Brown distinguishes teacher knowledge from knowledge that is embedded in a curricular resource. For example, teacher knowledge includes, their subject matter knowledge and their pedagogical content knowledge. On the other hand if a book is a curricular resource, then knowledge in the form of concepts, examples, anecdotes are embedded in that resource. In order to understand the teacher-tool relationship, he also discusses knowledge that is produced through teachers' interaction with the curricular resources (Brown, 2002).

Engagement with Curriculum

To determine the ways in which adjunct instructors engage with a research-based curriculum and the impacts of this engagement, I use Remillard and Bryans' (2004) work on teachers' opportunities to learn as they implement a curriculum. Remillard and Bryans

(2004) found that teachers use curricula in different ways and the unique ways of engaging with curriculum provide different opportunities for student and teacher learning. They define *opportunities for learning* as arising from "events or activities that are likely to unsettle or expand teachers' existing ideas and practices by presenting them with new insights or experiences" (p. 12). Teachers can engage with curricular resources without asking any questions or without reservations about the new curriculum, or they can be selective in their use of curricular resources, drawing on their prior knowledge and experience to make instructional decisions. Their decisions are influenced by their orientation towards a particular curriculum, which in turn is influenced by their understanding of the teaching and learning of mathematics (Remillard & Bryans, 2004). For example, novice teachers may use the curriculum materials literally because they are open to guidance that a new curriculum can provide. An experienced teacher may use the curriculum materials by mixing them with their own knowledge and experience to design instruction.

The Design Capacity for Enactment (DCE) framework (Brown, 2002, 2009) frames teachers' use of curriculum materials as a design activity (see Figure 4). Teachers use curriculum materials to meet their needs as they map out their instruction. They focus on and use various features of curriculum materials when they plan their classroom instruction. This process goes through various stages, as the teachers select the materials to be used, interpret them, and change the materials to match their own goals for student learning. Teachers make additional changes to add or remove parts of the materials based on their knowledge of their own students, keeping in sight student strengths, weaknesses,

and specific needs (Brown, 2009). In addition, including teachers' orientation towards a particular curriculum (Remillard & Bryans, 2004) was outside the scope of the current study where the focus was on the ways in which teachers engage with a research-based curriculum and the learning opportunities they create for themselves through this engagement.

To address my analytical requirements, I used an amended version of Brown's (2002) DCE framework, focusing on teachers' engagement with the curriculum. Remillard and Bryans (2004) found that teachers' opportunities for learning emerged as they engaged with the curriculum while planning and then enacting their instruction inside their classrooms. They also suggested that engagement with the curriculum while collaborating with colleagues could lead to possible opportunities for teacher learning. Further, according to Shulman (1987), planning, enacting and reflecting encapsulate critical actions in his model of Pedagogical Reasoning and Action. In addition, research suggests that reflection plays a key role in improving practice (Schon, 1983, 1987; Calderhead and Gates, 1993; Korthagen, Kessels, Koster, Lagerwerf, & Wubbels, 2001, Korthagen & Vasalos, 2009). Guided by these findings I include planning, enacting, collaboration and reflecting in my definition of engagement. Below I expand on each of these forms of engagement.

Planning. Clarke and Yinger (1980) defined teacher planning as a teacher action that is taken to organize school related activities. These could be formal actions like developing a lesson plan, an instructional unit or an informal action like *invisible thinking* (p. 6) when commuting to and from school. They count planning as anything that guides

a teacher by providing a guideline for upcoming activities. It involves making judgements, weighing those judgements as successful or unsuccessful and making further decisions. In addition prediction is an important part of the planning process, in order to effectively select resources and design instruction. According to Yinger (1980), teachers go through an iterative process of making decisions about content and instructional practice. They try strategies and analyze the outcomes with the goal to reach intended student learning goals. Their unsuccessful trials can lead to developing new insights about their teaching practice.

Proper planning requires teachers to draw on their knowledge and teacher experiences to make decisions, weigh their decisions, take action and then revise their plans (Clarke & Yinger, 1980). It is important to note that planning does not take place inside a vacuum, it is influenced by factors like teachers' past experiences, their beliefs about teaching and learning, their students, the resources available to them, and institutional requirements to name a few. These factors play an important role in shaping teachers' decisions by either supporting or restricting their actions. Stein, Remillard, and Smith (2007) refer to teachers' instructional plans as the intended curriculum (as discussed in Chapter 2). The intended curriculum influences enacted curriculum which is the curriculum-based activity that unfolds inside the classroom.

Educational researchers at the post-secondary level have raised questions about the ways in which faculty plan their instruction (Stark, Lowther, Ryan, & Genthon, 1988). Joan Stark (2000) found that faculty members' prior beliefs and contextual factors of their institutions, influence their decisions about planning and instruction. He

explained that contextual factors include, student characteristics, departmental goals and resources available to the faculty. Negotiating the norms of their environments is a process that includes making sense of their situation and then making decisions (Hora & Ferrare, 2013). As challenges arise, instructors draw information from their environment and combine it with their existing knowledge to plan out solutions (Coburn, 2001; Weick, 1995). Sometimes the challenges they face, like student attitude and time constraints limits the scope of their instructional decisions (Henderson & Dancy, 2007). On the other hand the decision making process can over time lead to their own learning (Greeno, 1998). Research on post-secondary instructors' experiences with factors that influence their decisions (Hora, 2012; Lattuca & Stark, 2009) can help develop insights about the how they plan.

Enacting. Teachers play an important role in implementing the curriculum. It is through their enactment and planning that the curriculum comes alive inside the classroom. Research on enactment (Remillard & Bryans, 2004) suggests that novice and expert teachers enact the curriculum differently. Novice teachers are more likely to pilot a new curriculum without bringing in their past knowledge and experience. Given that all three instructors in the study had more than ten years of teaching experience I expected them to bring their own resources and experience as they planned their lessons and the experiences they encountered inside their classrooms. Research suggests that experiences that are encountered inside the classrooms can help teachers improve their practice (Remillard & Bryans, 2004). As they face challenges while enacting the curriculum, the

experienced teachers' knowledge and practice may be unsettled leading to opportunities for their learning.

Existing research on post-secondary instructors' teaching practice focuses on specific pedagogical techniques like lecturing (Dancy & Henderson, 2010; Deslauriers, Schelew, & Wieman, 2011) and peer instruction (Mazur, 1997). Although these techniques are useful in understanding the various teaching techniques, the view of classroom instruction provided by these studies is limited (Hora & Ferrare, 2013). Teaching as a practice is more complex in nature and in addition to specific teaching methods, it includes factors like instructors' enthusiasm and their preparation that can influence student learning (Feldman, 1989; Murray, 1983; Perry, 1997). There's a dearth of literature at the post-secondary level about faculty's instruction as it unfolds inside the classrooms, especially in mathematics and science classrooms (Hora & Ferrare, 2013).

Collaborating. Research suggests that teacher learning communities can be a source of collaboration with colleagues where they can share their experiences about their teaching practice (Cochran-Smith & Lytle, 1999). Teacher learning communities can be large, small, in-person or online, and can have a variety of goals (Barab, MaKinster, & Scheckler, 2003; Grossman, Wineburg, & Woolworth, 2001; Horn & Little, 2010; Little, 2002; McLaughlin & Talbert, 2001; Rosenholtz, 1989a; Wood, 2007; McLaughlin & Talbert, 2006). The main features of a learning community include (1) collegial and collaborative interactions, (2) participation and discourse norms for productive collaboration, and (3) focus on teaching and student learning (Cochran-Smith & Lytle, 1999). Collegial and collaborative interactions include group members sharing their own

knowledge and expertise for the group's benefit (Cochran-Smith & Lytle, 1999). It also involves teachers' recognizing individual differences in teaching approaches, knowledge, beliefs, and if everyone is given a chance to share their views, the learning opportunities for the group are increased (Grossman et al., 2001). Groups work towards developing norms and goals to grow as a community (Grossman et al., 2001). Participation and discourse norms are the practices for collaboration that the group develops (Cochran-Smith & Lytle, 1999). Group members give each other a chance to communicate their thoughts instead of just sharing their own views and develop a safe environment (Koellner-Clark & Borko, 2004; Grossman et al., 2001). The conversations focus on improving teaching and learning and supporting each other in doing so (Skerrett, 2010; Grossman et al., 2001; McLaughlin & Talbert, 2006). Teachers bring in ideas from their own teaching experiences and reflect on how these experiences can be used to improve their practice and their students' learning (Jaworski, 2006; Nelson, Slavit, Perkins, & Hathorn, 2008; Rodgers, 2002).

Collaboration and building a sense of community are relevant needs of post-secondary faculty (Uchiyama & Radin, 2009). Research suggests that collaboration between faculty members can be a form of support for them (Kelchtermans, 2006; Hindin, Morocco, Mott, & Aguilar, 2007). Lack of collaboration opportunities in higher education can influence the faculty to feel isolated which can impact their classroom performance (Briggs, 2007; Demir, Czerniak, & Hart, 2013; Lester & Kezar, 2012; Uchiyama & Radin, 2009). Providing collaboration opportunities for higher education

faculty like a collaboration space can lead to professional development and growth (Hutchings, Huber, & Ciccone, 2011).

Reflecting. Research suggests that reflection is a productive resource for fixing short term problems in classrooms (Korthagen & Vasalos, 2009). Given the time constraints they face, teachers are compelled to look for fast and easy solutions without giving greater consideration to the deeper causes of the issues at hand. As a result, some forms of reflection do not necessarily lead to learning opportunities for teachers. Schön (1987) explained the dangers of finding these quick fixes by saying that when practitioners develop strategies based on their personal view of a situation, the short-term solutions they find become a permanent part of their repertoire. Once permanent, the practitioner does not revisit or revise these practices, which hinders their professional development. The issue with such surface-level reflection is that a teacher might develop certain concepts to help understand their practice, but fail to structure their reflection so they can improve their practice (Korthagen & Vasalos, 2009). For example, a teacher might appreciate the conceptual nature of a research-based curriculum and understand that student discourse in classroom would help the students in their problem solving but fail to develop classroom norms that promote discourse among their students, they might use methods of direct instruction and give multiple examples, as a way to show their students problem solving strategies. In order for teachers to engage in ongoing professional learning, a deeper level of reflection that is structured and explores the underlying issues is required (Korthagen & Vasalos, 2009).

Educational research at the post-secondary level, especially on teacher preparation points to the need for preparing future teachers to become reflective about their practice. The reason is that student teachers can't be prepared for all the possible challenges that they will encounter while teaching and preparing them to become mindful about their practice will be beneficial for them (Calderhead, 1987; Clark & Peterson, 1986; Hellison & Templin, 1991; McNarnara, 1990; Shulman, 1987; Siedentop, 1991; Zeichner, 1987). In addition, research suggests that reflection is also beneficial for in-practice teachers as a way to provide professional development (Fendler, 2003; Hoffman, Artiles, & Lopez-Torres, 2003).When teachers reflect on their practice to solve instructional challenges they take charge of their own professional development (Zeichner & Liston, 1996). Reflecting on their practice allows teachers to be critical of their practice leading to the possibility of improvement (Calderhead, 1992; Cole, 1997; Bengtsson, 1995).

I included these modes of engagement with the curriculum, planning, enacting, collaborating and reflecting as the artifacts that facilitate the instructors' implementation of the new curriculum (see Figure 5). By adapting the DCE framework with these amendments (see Figure 5), I am able to explore the various ways in which a teacher interacts with curriculum and how the engagement generates learning opportunities.

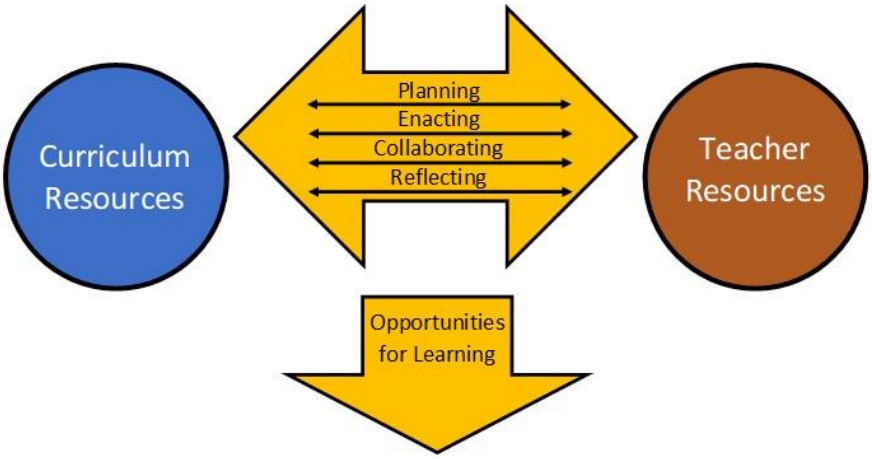


Figure 5. Model of teachers' opportunities to learn through engagement with the curriculum.

Opportunities to learn

In addition to the above framework, I draw upon the frameworks of content and pedagogical content knowledge (Shulman, 1986) and decentering (Carlson, Moore, Bowling, & Ortiz, 2007) to explore how engagement with a research-based Precalculus curriculum provides opportunities for adjunct instructors' learning (e.g., content and pedagogical content knowledge, and instructional practice).

Teacher knowledge. Shulman (1986) discusses knowledge that is important for a teacher. The first type of knowledge is content knowledge (CK), which refers to teachers' subject knowledge. The second type of knowledge is pedagogical content knowledge (PCK), which refers to the knowledge needed to teach a subject and includes knowledge about representing the concepts to a student and about student misconceptions and challenges in learning concepts. I use CK and PCK to guide me in understanding teachers' opportunities for learning that emerge through their engagement with the curriculum. I am interested in finding out the opportunities for learning that emerged as

the instructors planned and enacted the curriculum as they discussed it when collaborating with their colleagues and when reflecting on it. These opportunities may or may not have been availed of by the instructors.

According to Shulman (1987), content knowledge encompasses teachers' understanding of "the structures of subject matter, the principles of conceptual organization, and the principles of inquiry" (p. 9). This knowledge allows teachers to develop a broad understanding of their field by grasping main concepts and gaining expertise. In the case of Precalculus, the structures of subject matter (SOM) are the rules, procedures, definitions, and axioms such as trigonometric identities or rules of logarithms. Principles of conceptual organization (PCO) represent the Precalculus conceptual web. For example, the representation of a topic like rates of change, as a connection for the families of functions that students explore in Precalculus. Principles of inquiry (POI) are the mathematical habits of mind (Cuoco, Goldenberg, & Mark, 1996) associated with Precalculus (e.g., pattern recognition, problem solving, and ensuring the feasibility of solutions).

Pedagogical content knowledge includes knowledge about how to represent the subject effectively and convey ideas clearly so others can understand them. Examples of PCK include understanding the reasons why some topics are easy or difficult to learn, the pre-requisite knowledge students of different age groups bring with them to the classroom, or knowledge of common student misconceptions (Shulman, 1986). PCK became relevant in this study, when looking at how the instructors tried to help their students, for example by finding ways to help them develop their problem solving skills.

Many of their students struggled with word problems because they had to understand the problem and then apply mathematical concepts to various real-life situations. To develop students' problem solving, instructors needed knowledge of ways to support their students' learning so that they could become independent problem solvers.

Experienced teachers have greater knowledge, and their knowledge is structured differently than novice teachers (Krauss, Brunner, Kunter, Baumert, Blum, Neubrand, & Jordan, 2008). This may be because experienced teachers learn about effective means of representing ideas, either by studying the subject or because of their teaching experience (Shulman, 1986). As I analyzed teachers' engagement with the curriculum, the constructs of PCK and CK allowed me to describe the opportunities for their learning. As the teachers implemented the new curriculum, they found the novelty of some representations challenging, like circular motion. As stated earlier, Remillard and Bryans (2004) describe opportunities for learning as challenging experiences that allow the teachers to stretch their current knowledge with a possibility for learning new ideas and practices. Teachers' CK and PCK were unsettled, consequently leading to an opportunity to make new contributions to CK and PCK. Instructors not only learned the content themselves but also expanded their repertoire of teaching methods for their students' learning.

Instructional Practice. In order to understand the opportunities to learn afforded by engagement with the curriculum on the teachers' practice, I use the decentering framework (Carlson, Moore, Bowling, Ortiz, 2007). Building upon the work of Piaget (1955), Carlson and colleagues (2007) developed the decentering framework, which

helps in understanding how people try to make sense of a perspective that's not their own. Steffe and Thompson (2000) connected Piaget's idea of decentering to interactions between teachers and students. They described that decentering involves the ways in which people modify their own behavior to influence others in particular ways. Each person observes the other and creates models of their thinking. These interactions can take place in one of two ways, participating in the interactions reflectively by paying attention to others' contributions or participating unreflectively (Thompson, 2000). The observer can either assume that the other shares their thinking or that they have their own independent ways of thought. After this interaction, the observer tries to build models to understand the others' thinking (Steffe & Thompson, 2000). If the observer assumes that the others' thinking is identical to their own, they are thinking in a non-decentered way. Additionally, if they realize that the other may have different thinking than their own and do not try to build a model of that thinking, then they are also thinking in a non-decentered way. Finally, if one assumes that the other understands their statements exactly as they were intended, they are acting in a non-decentered way (Steffe & Thompson, 2000). For example, a teacher is decentering when a student asks a question and the teacher responds by probing the students' thinking through follow-up questions. A teacher is acting in a non-decentered way when he simply answers the question, having assumed that his and the student's thinking are congruent.

When applied to the classroom setting, this framework has the capacity to describe the ways in which teachers try to make sense of their students' understanding. Using a modified version of a decentering protocol has helped me in analyzing classroom

observation data (discussed below) of instructors to see the kind of information they elicit from students as they make models of students' thinking. In addition, a decentering perspective has illuminated the ways in which teachers try to make sense of students' conceptions or misconceptions by eliciting responses from them and paying attention to classroom discourse. Fortunately, the conceptual nature of the course allowed students to share their ideas and discuss problems in class, and this facet of classroom practice provided decentering opportunities for instructors. Observing the ways in which teachers encouraged classroom discourse also provided a window into challenges they faced as they enacted the curriculum. For instance, some of the instructors were challenged by their current practice when a student asked a question and they did not have an answer available, and when a student asked for an additional example and they were faced with having to create one on the spot. These insights into the challenges they faced while teaching are valuable to me as I try to understand the learning opportunities that emerge as instructors worked through them.

Data Collection

To answer my research questions about the nature of adjunct instructors' engagement with a research-based Precalculus curriculum and how engagement with the curriculum provides opportunities to learn for adjunct instructors, I used semi-structured interviews (Merriam, 2002), audio recordings of PLC meetings, and classroom observations. I interviewed teachers at the beginning and end of each semester about their experiences implementing the new curriculum. Each weekly PLC meeting was recorded as the instructors shared their experiences. I transcribed these recordings and incorporated

text from the chat room conversations. Finally, I observed instructors' classrooms near the beginning and end of each semester to observe their practice in terms of classroom discourse and their use of decentering techniques. Rationales for these sources and clarity about them are provided next.

Semi-structured Interviews

A 'semi-structured' approach to the qualitative interview combines open-ended and structured questions (Merriam, 2002). Questions play an important role in providing access to quality data and should be selected carefully. The interviews focused on instructor experiences implementing the *Pathways* curriculum. I used semi-structured interviews to better understand the ways in which instructors engaged with the curriculum. I interviewed each instructor at the beginning and end of each semester (September 2016, November 2016, January 2017 and April 2017). Each interview lasted approximately 30 minutes. These interviews provided opportunities for teachers to reflect on their experiences, freely share their opinions, and make suggestions for improving the curriculum, its implementation, and the course coordination. I used an interview protocol to get a sense of teachers' experiences implementing the new curriculum, the challenges they faced, and what they learned (see Appendix A). The semi-structured nature of the interviews allowed me to ask follow-up questions and to also modify the protocol to optimally allow instructors to share their thoughts with me.

PLC Meetings Transcripts

Each semester that an adjunct instructor was actively teaching Precalculus, he or she was encouraged to attend weekly online meetings (60 minutes) led by a mathematics

educator in the department familiar with the curriculum (Fall 2016 semester) or myself (Spring 2017 semester). These meetings were scheduled at a time when most of the instructors were available. The meetings formed the foundation of a PLC consisting of members of the research team and adjunct faculty. The university's learning management system, Canvas Collaborate, was used to conduct the meetings online. The PLC used a similar format each week where the instructors would use a chat box to type in their comments or responses to questions, and the facilitator would share investigations through his or her screen as he or she verbally discussed the problems. I audio-recorded these meetings and saved the chat box conversations. I used the chat box comments to assess the instructors' participation in the PLC, including the questions, concerns, and ideas they shared. These sources of data provided insights into teachers' engagement with the curriculum as they collaborated with each other during the PLC meetings. This data also provided information about the challenges instructors faced when implementing the curriculum with respect to their knowledge and teaching practice. For example, the questions they asked and the topics they wished to discuss gave insight into what they found challenging with implementing the curriculum and how they used the PLC to help them implement the curriculum.

Classroom Observations

I observed each of the three adjunct instructors twice per semester, once near the beginning of the semester (September 2016 and January 2017) and the second near the end of the semester (November 2016 and April 2017). I audio-recorded each observation for the duration of the class (105 minutes) and took notes using an observation protocol

(see Appendix B). I listened to the audio recording of the observed classrooms in their entirety and selected relevant sections of the recording to transcribe. The observation rubric included notes on the questions asked by the students and the teacher's responses, as well as questions asked by the teacher. The observation protocol also included a rubric for observing instructors' decentering practice. The rubric provides a range of scores with five being a score for instruction most connected to decentering practice. That is, a score of five corresponds to a teaching practice where, "The teacher builds a model of a student's thinking and respects that it has a rationality of its own. Through interaction the teacher also builds a model of how he/she is being interpreted by the student. He/she then adjusts her/his actions (questions, drawings, statements) to take into account both the student's thinking and how the teacher might be interpreted by that student." A score of one corresponded to teaching practice where, "The teacher shows no interest in understanding the thinking or perspective of a student with which he/she is interacting." The transcriptions of the observations as well as scores from the rubric gave a sense of instructors' engagement with the curriculum as they enacted their plans inside the classroom. These data also provided insights into the challenges the instructors faced in terms of promoting discourse in the classroom and asking thought-provoking questions. Understanding these challenges and their causes was important in exploring opportunities for the instructors' learning.

Data Analysis

I analyzed the data using qualitative research methods in order to develop exploratory multiple case studies (Yin, 2009). Using multiple cases (three adjunct

instructors) allows me to show contrasting results and showcase the various ways in which the instructors engaged with the curriculum. The three data sources – interviews, PLC meetings, and class observations – provided me with information about differences in the instructors' use of curriculum. Teachers use curricula in various ways and their use is influenced by factors such as knowledge, beliefs, and teaching experience (Brown, 2002). Yin (1994) demands the use of a theoretical framework to guide the case studies. I have used the framework described in the previous section to guide me in identifying the ways in which the instructors' engagement with the curriculum leads to opportunities to learn for them. To answer my research questions, the analysis has two primary aims: (1) to analyze instructors' classroom practices, and (2) to analyze instructors' CK and PCK through opportunities to learn provided by engagement with the curriculum. As discussed in more detail below, I answer my research questions through the analysis of instructors' classroom practices gleaned through classroom observations and supported by interview and PLC meeting data. The analysis of opportunities to learn with respect to CK and PCK is supported by interviews and instructor conversations in the PLC meetings.

In order to avoid a common criticism of case studies that they lack an organized treatment of data (Yin, 2009), I report all the evidence in a systematic way. Further, in order to ensure construct validity, I use data collected from various sources (interview, observations and PLC meeting recordings). The use of multiple sources of evidence helps in developing *converging lines of inquiry*. This triangulation of data sources provides several measures of the same phenomenon, making the case study stronger.

My unit of analysis is the mediated action (Wertsch, 1998) between each of the adjunct instructors and the curriculum. The various forms of engagement, planning, enacting, collaboration and reflecting mediated their interaction with the curriculum. Using this unit of analysis, I employ the general analytic strategy of developing a case description by analyzing the characteristics of the relations between the instructors and the curriculum. A point to note is that case studies do not lead to scientific generalizations; the scope of a case study is to generalize to theoretical propositions and not to an entire population. I am using instructors' engagement with a research-based mathematics curriculum to modify Brown's DCE framework. I analyze instructors' engagement with the curriculum as they plan, enact, collaborate and reflect and report their opportunities for learning that emerge from their engagement. Using case studies can aid in expanding the scope of existing theories or provide possible links between events.

The case study approach lends itself to a number of analytic techniques that fall under my general analytic strategy. I developed descriptive case studies and pattern matching is a suitable data analysis technique used for explanatory or descriptive case studies (Yin, 2009). Using this technique, I compare emerging patterns in instructors' engagement with the curriculum with the predicted ones. As teachers engage with the curriculum during implementation, their engagement can lead to opportunities for their learning (Remillard & Bryans, 2004). As recommended by Yin (1994), overall my analysis is based on relevant evidence provided by the data and is guided by the frameworks I've employed.

Codes were developed through an iterative process and were revised several times to lead to final codes (see Appendix D). I analyzed all the transcripts of interview data and online PLC meetings, chat text and field notes (Merriam, 2002) from the meetings to learn about instructor's engagement with the curriculum and any opportunities for learning this engagement provided them. In this section, I provide details of the analysis of the data collected from each of the aforementioned sources.

Semi-structured Interviews

To analyze the interview data, I focused on each instructor's engagement and opportunities to learn individually before conducting a cross-case analysis. In the first cycle of coding I used elemental methods, and specifically, descriptive coding, where I used phrasing that described parts of the transcribed data for an initial set of codes (Saldaña, 2009). I then grouped these initial codes together for the second round of coding. In the second round of coding, I employed pattern coding (Saldaña, 2009) to further categorize the data for analysis. Pattern codes are explanatory or inferential in nature and they help in classifying emergent themes. They are an efficient way to sort and categorize data into meaningful groups (Miles & Huberman, 1994). The second round of coding aided me in bringing together themes for each case study.

I coded each interview for evidence of instructors' CK, PCK and teaching practice as connected to their engagement with the curriculum. For CK, I identified instances where instructors mentioned any perceived gains in their own CK, shared anecdotes about classrooms or about their lesson planning that shed light on challenges or learning. For example, an instructor sharing that they learned a new way of representing

trigonometric functions. For PCK, I focused on instances of conversation about student knowledge, misconceptions and difficulties, types of questions asked by the instructor or student, mentions of classroom discourse, as well as any mention of instructors' use of representations or examples to make the content understandable for the students.

In addition, I recorded in field notes the ways in which instructors generally engaged with the curriculum. For example, I noted any mention of the ways in which they collaborated with their colleagues, planned for their upcoming lessons, enacted the curriculum inside the classrooms, or reflected about their experiences with the curriculum. I also looked for any patterns or changes in their engagement with the curriculum over the two semesters.

Finally, I analyzed the interview data for evidence of opportunities for learning that resulted from engagement in the PLC. I used the definition of *opportunities for learning* as events or activities that can unsettle the teachers and expand their existing ideas and practices by presenting them with new insights or experiences (Remillard & Bryans, 2004). I coded any and all references that the adjuncts made to perceived changes in their teaching practices, new understandings, and insights they may have gained by using the curriculum. I also noted perceived changes in the instructors' practice or understanding reflected in their conversations during the interviews.

Classroom Observations

Similar to the interview data, the analysis of the observational data also began with a focus on each instructor individually. My goals were to understand the ways in

which the instructors engaged with the curriculum and to characterize the teaching practices that the instructors emphasized in their classrooms.

To analyze the audio recordings and field notes from observation, I focused on the ways in which the instructors presented mathematical concepts to the class, whether they used multiple representations, the phrasing they employed, and whether they used language from the *Pathways* curriculum. In addition, I noted the frequency of questions asked by the instructor, the frequency of questions asked by students, the kind of questions asked by the instructor (e.g., if the questions made the students think, if they elicited responses from students, etc.), and the answers provided by instructors.

The decentering rubric included in the observation protocol focused on two main themes: 1) if students were given opportunities for reflection, such as when an instructor encouraged the students to reflect on the reasonableness of their responses; 2) if the instructor's actions exhibited any evidence of them, developing models of their students' thinking and using those models to guide the instructor's own instruction.

The observation data, along with instructors' scores from the decentering protocol, contributed to assessments of the ways in which the instructors engaged with the curriculum and how this engagement led to opportunities for learning. These data served as supporting evidence for the findings from the interview and PLC data.

Professional Learning Community

I analyzed the transcripts of the PLC recordings to look for evidence of instructors' CK, PCK, and classroom practice. I also looked for instructors' engagement with the curriculum as they collaborated with each other and the facilitator in the PLC.

For CK and PCK, I focused on the questions that the instructors asked. The questions and the way that instructors phrased their questions and comments when describing a mathematical concept helped to provide evidence of the opportunities to learn afforded by engagement with the curriculum. For example, if an instructor asked if there was an error in the book that he recognized, that quote was coded as CK, because the instructor was able to recognize a mathematical error in the textbook. Using the analytical methods described in the interview section, I used the PLC data to analyze the instructors' engagement with the curriculum when collaborating. I examined the instructors' participation, the curricular focus of the PLC, and the sharing of classroom experiences, challenges, and ideas for planning ahead. I also used the PLC data to get a sense of the impact curricular engagement has on the instructors' teaching practices.

Finally, after analyzing the data for the three instructors individually, I conducted a cross-case analysis. My goal was to compare and contrast the ways in which they engaged with the curriculum in relation to their knowledge and practice to identify learning opportunities. I used data collected from various sources (interview, observations, PLC meeting recordings) to interpret, explain, and check for discrepancies in the patterns that emerged as the data were compiled and analyzed. The tables below give a breakdown of the data sources used.

Table 1
Evidence of Engagement – Data sources used

I. Planning	II. Enacting	III. Collaborating	IV. Reflecting
PLC recordings: Conversations & questions about content, teaching practice in the context of planning.		PLC recordings: Conversations & questions about content, teaching practice in the context of collaborating.	
Interviews: Conversations & questions about experiences implementing the curriculum, coded as engagement: planning.	Interviews: Conversations & questions about experiences implementing the curriculum, coded as engagement: enacting,	Interviews: Conversations & questions about experiences implementing the curriculum, coded as engagement: collaborating,	Interviews: Conversations & questions about experiences implementing the curriculum, coded as engagement: reflecting.
	Classroom Observations: Questions asked by the instructor in class, questions asked by the students and instructor responses to those questions, conversations from the classroom in the context of enacting.		

Table 2
Evidence of Opportunities to Learn – Data sources used

I. Knowledge	II Classroom Practice
PLC recordings: Change in phrasing, types of questions asked about content and teaching practice, focus of conversation about content and teaching practice.	PLC recordings: Conversation expressing challenges and/or differences in teaching practice.
Interviews: Conversation expressing learning content or changes in teaching practice.	Interviews: Conversation expressing challenges and/or differences in teaching practice.
	Classroom Observations: Practices exhibiting instructors' interest in fostering students thinking. Specifically, the questions they ask the students, the questions that students ask and their responses to the students' questions.

Codes

After several iterations of revising the codes they became effective in describing the data. I developed the following codes for theorizing about instructors' engagement with curriculum. Engagement fell into four major categories: planning, enacting,

reflecting and collaborating. I define each of these forms of instructors' engagement with the curriculum below (see Appendix D).

Planning. I define planning as instructors' engagement with the curriculum as they actively select instructional resources and make pedagogical decisions for their upcoming lessons or upcoming assessments. For example, thinking about what problems in an investigation to select to discuss the big ideas with their students, ways to phrase their questions to challenge their students thinking or providing resources for students to help them study for an upcoming exam.

Enacting. I define enacting as instructors' experiences as they implement their instructional plans inside their classrooms with their students. I conceptualize *enacting* as instructors' engagement with the curriculum as they implemented the curriculum (e.g., their interaction with their students and their own experiences inside the classroom).

Collaborating. I conceptualize *collaborating* as all forms of instructor interaction with their colleagues with the shared goal of helping them implement the curriculum. This includes collaboration that takes place during online meetings or in unstructured settings (e.g., email, text, or in-person conversations). That is, collaborating refers to instructors' engagement with the curriculum as they share experiences and build on each other's experiences. When collaborating, the instructors may be planning, reflecting, reading or exploring the curriculum. Instructors collaborate in two ways (1) interacting with their colleagues but with no meaningful conversation based on their instructional experiences, the challenges they faced, the questions they asked, and so forth, and (2) while bringing their ideas to the table and sharing their thoughts with the others. The

participants considered and commented on each other's perspectives, and reflected on their practice based on new ideas.

Reflecting. My definition of reflecting includes reflection that focuses on specific issues related to teaching practice and structured reflection that dives deeper to learn about root causes of issues to help me learn about instructors' various ways of reflecting. My definition also makes connections between conceptual ideas and actions to improve teaching practice. That is, reflecting is instructors' engagement with the curriculum as they think about their experiences, examine their practice and make connections between conceptual ideas and their teaching practice. Reflection can take place when instructors plan, enact, collaborate or on their (e.g., when driving).

The analysis yielded data for the construction of case studies for Caleb, Michael and Justin that describe their engagement with the curriculum. It also allowed me to determine how their experience of implementing the new curriculum, including the challenges they faced, provided opportunities for their learning.

Chapter 4

Case Studies

Caleb's Engagement – A Case of Self Professional Development

"My challenges are part of my own professional development. How do I make it [teaching practice] better?" (Interview 1, Spring 2017)

Caleb's engagement with the curriculum was marked with a desire to improve his teaching practice. He saw his role as a teacher to be highly influential in his students' learning. He wanted his students to have effective learning experiences in his classroom. As he engaged with the curriculum, he tried to improve his own teaching practice to help his students.

His experiences as a teacher, as well as a doctoral student in Mathematics Education, had given him insights about teaching and learning mathematics and he was excited about implementing the new curriculum. He actively participated in the summer workshop which provided a preview of the curriculum itself and what was expected of him as an instructor implementing the new curriculum. During the summer workshop, Caleb engaged with the curricular materials by solving problems in the investigations and discussing pedagogical techniques recommended as beneficial for student learning. Caleb appreciated the preparation that the summer workshop provided him as a teacher. However, he was still concerned about what the experience of implementing a new curriculum would entail. He went into his first semester with a positive outlook.

Contextualizing Caleb's Engagement

In the next few paragraphs I provide information that helps situate Caleb's engagement within the context of his experience as an adjunct instructor and his overall experience with the curriculum.

Supports

This was Caleb's first time as an adjunct instructor receiving supports to implement a curriculum. He appreciated the course coordination and the resources in the form of a pacing guide, syllabus, common exams, and most importantly a course coordinator as a go to person to ask his questions. In his past experience working as an adjunct instructor, he had never received any support other than being assigned a course to teach and provided a course syllabus. Due to the summer workshop that took place prior to his first semester of implementation, Caleb shared that he went into the semester with an overview of the goals of the curriculum and a sense of his responsibilities as he implemented it.

Perception of the New Curriculum

Caleb seemed excited by the new curriculum and willing to put in the effort to implement it well. He also seemed onboard with the choice of the curriculum and the reason for its selection by the department. That is, to better prepare the students for calculus and to retain them in STEM majors. He shared that in the past "We found students lacking when they went to Calculus," (Interview 2, Fall 2016), and gave the example of students' weak understanding of function notation. He expressed a positive attitude towards the benefits of the new curriculum for his students. However, he still had

many questions about the curriculum and shared that he was anxious to enact the curriculum in his classroom. Talking about the summer workshop he mentioned, "It's only a short three day training, you still have a lot of questions about certain things, but now I'm getting a better picture of where we need our students to be at, to be successful, not only in my class but in all their STEM classes." (Interview 1, Fall 2016)

As early as the first semester of implementation, Caleb shared that he started to draw upon the curricular resources as a guide for his own pedagogy, like noticing how the curriculum resources introduced ideas to the students. For example, observing the online homework portal and how it led the students through different parts of the problems. The portal provided one piece of the problem first, then guided the students through other parts to allow students to build on their own responses. Caleb shared that he recognized what the curriculum developers were trying to achieve by having the students struggle through the problems and guiding their learning. He said that the homework problems, "Take them through, take them through, take them through and then the function comes last. As opposed to us giving them the function and having them understand!" (Interview 2, Fall 2016) Caleb explained that he was able to see a contrast between how the curriculum approached student learning and the ways in which traditional teaching approached student learning. He shared that allowing students to develop their knowledge by working through different parts of a problem, "Make[s] them understand, you know, what does this mean!" (Interview 2, Fall 2016) He appreciated the curriculum's approach to student learning and shared, "That's what I like about it, not giving information up front." (Interview 2, Fall 2016) Being mindful of the difference

between curricula he had experienced in the past and the new curriculum, Caleb exhibited a curiosity towards the new curriculum and how it presented learning opportunities for his students.

Developing a Vision of the Curriculum

As the semester progressed, Caleb shared that his vision of the curriculum itself broadened. During an interview at the end of his second semester of implementation, Caleb talked about the curriculum and said, "I still like it, I think it's a very good curriculum and we keep shaping it to best instruction for the students... it gives a different approach to teaching and learning as well." (Interview 2, Spring 2017) After implementing the curriculum for two semesters he shared:

> The content is necessary for the students to engage in richer conversation in the course as well as in their calculus course when they go on... students have to take the investigative approach, I don't know if they appreciate it but it's a good way to get them to deal with the bigger picture especially when they get to [an] upper level course. (Interview 2, Spring 2017)

Caleb explained that he had a better sense of the curriculum's goals, the big ideas to be discussed in class, as well as the recommended pedagogy and that he found the curriculum beneficial for the students.

Developing as a Teacher

Caleb described that he continuously tried to incorporate changes into his own teaching practice to implement the curriculum well. He shared that this constant effort to improve his own teaching practice was challenging for him but he embraced it.

Explaining this challenge, he stated, "My challenges are part of my own professional development. How do I make it [teaching practice] better?" (Interview 1, Spring 2017) His engagement with the curriculum was a way for him to improve his teaching practices. He had taught Precalculus about ten times at the college level prior to teaching the Pathways curriculum but he shared that he found this curriculum challenging. Caleb explained that the difficulty came from delivering instruction in ways that emphasized student engagement and developing their thinking. He said, "I know when I got my Master's degree it was all about, a lot of progressive education and changing student's habits to make them think more, and I would say that the curriculums in the past, all called for it but tend to move away from it; whereas I think this one is pushing for the thinking from the jump, and trying to be consistent to emphasize student thinking throughout." (Interview 1, Fall 2016) Caleb explained that he began to develop his own perception of what it meant for him to implement the curriculum well. He said, "It has pushed me as an educator to do a lot of thinking or providing questions, almost like Bloom's taxonomy delivery. So we can get the students to be more comfortable with widening their thinking." (Interview 1, Fall 2016) He was developing a sense of what his own role as a teacher would be in helping his students learn from the new curriculum.

Caleb's Engagement

As Caleb engaged with the curriculum through planning, enacting, collaborating and reflecting, the challenges he faced provided opportunities for his learning. In the following section, I describe Caleb's engagement with the curriculum.

Planning

In this section, I describe the ways in which Caleb planned, the factors that influenced his planning, and the challenges that he faced. These challenges emerged as Caleb encountered his students' experiences in the classroom and had to plan out new ways to help them. In addition, Caleb's planning was influenced by the novelty of the new curriculum in terms of both content and pedagogy.

Challenges when planning. One of the ways the curriculum challenged Caleb was when he planned his instruction. He shared that while in the past he could depend on his previous Precalculus teaching and planning experience, the new curriculum demanded additional planning time. According to Caleb, previously it took him forty-five or fifty minutes to plan a lesson, however, planning one lesson during the first semester of implementation took four hours. He added that this planning time decreased as the semester progressed and he became more confident about his teaching practice; however, it still took comparatively longer time and more effort than his prior experience teaching Precalculus.

Reasons for challenge when planning. Caleb explained that one of the reasons for spending more time when planning the lessons was that he was intimidated by the new curriculum and wanted to be better prepared. He said, "I want to make sure I have my bases covered." (Interview 1, Fall 2016) Giving an example of what he found challenging he said, "I'll tell you something! I was scared with trigonometry in this book... when I got to it, I was like wow! I was like, how am I gonna go into my class not knowing what's going on!" (Interview 2, Fall 2016) He continued, "We're so used to also

in trigonometry talking in degrees, they're all about radians, and I never ever did a lot with radians like this!" (Interview 2, Fall 2016) Initially, he shared feeling uncomfortable with the content and the way it was presented in the new curriculum, and that he spent time being prepared before going into class.

Another one of Caleb's concerns throughout all the semesters, with respect to planning, was his pacing of the lessons. He shared that he wanted to keep up with the scheduled pacing guide provided by the course coordinator. As the semester progressed, he became less anxious about the curriculum but pacing remained a concern when he was planning. He described his concern about the pacing as follows:

> At first it (*Pathways*) was challenging because it was something new so timing was a problem, not finishing the investigations I wanted to finish in one class setting that I wanted to finish but once I got hang of the language, the mentality of how the program is set-up it became much stronger for me to really get through the content as the course kept going, you still fall behind but not as much as I used to fall behind in the opening of the course. (Interview 2, Fall 2016)

He shared that his pacing concern impacted his planning because he tried to develop efficient lessons. He wanted these lessons to allow him to reach his goals for his students' learning in a given time period. Caleb explained that familiarity with the big ideas in each investigation, as well as the whole curriculum, allowed him to align his teaching goals to the big ideas and develop effective lessons.

Caleb's plans. Caleb shared that when he planned, he would read through the textbook, use the presentations provided by the curriculum as well as draw from his own

experience and knowledge to design his lessons. He gave one example of his planning: "I look at the problem I have in the book and I try to mimic it in a way, so they talk about the Ferris wheel. I might be talking about a tire or change the numbers around just to give them other perspectives about the problems." (Interview 2, Fall 2016) He drew from the problems in the curriculum to design his own lesson. Caleb explained that he wanted his students to have the examples from his lesson as a reference. He said, "They can use the textbook and my examples as a ways to study for their problems." He planned a mini lesson for the beginning of each class and then planned to have the students investigate the problems from the workbook. Caleb shared that he wanted to be well prepared to go into class in order to, "provide them the environment so they can do their thinking on their own." He also realized that no amount of planning could allow him to prepare for all the possible questions that the students could ask. He said, "I can't necessarily prepare for every question that's going to be asked of me but I want to make sure that I cover every basis." (Interview 1, Fall 2016) He wanted to plan for a classroom experience where students would be able to think and he would be able to guide their learning.

Caleb shared that his careful attention to planning was to ensure he himself as a teacher went into class with a design of what would unfold inside the class. He wanted to be prepared for his students' questions and to help them learn.

Influences on Caleb's planning. Caleb' reflections from his classroom experiences as well as his conversations with his colleagues influenced his lesson planning. He shared that he would reach out to his colleagues when he had questions about how to deliver instruction or if he felt 'stumped'. He also paid close attention to the

conversations during the online meetings and used the ideas discussed in those meetings to guide his planning. In addition, Caleb shared that it was important for him to get a sense of the bigger picture and a clarity about the main ideas of a lesson. He described that when he planned his lessons, he tried to break down the big ideas and the thinking behind any situation in order to "Deliver that and structure the instruction in the proper way." (Interview 1, Fall 2016) He said that his reflection on the main ideas that he read in the textbook or that were discussed in the online meeting allowed him to better plan his lesson.

> Like right now we're teaching sine and cosine and I'll always say well y is sine and x is cosine but now I understand about the height in terms of the Ferris wheel… it was just ringing in my head when I was planning the lesson! (Interview 2, Fall 2016)

Caleb's planning was influenced by the ideas discussed with colleagues and from reading the textbook. In addition, he shared that reflecting on his past experiences inside the classroom also aided his planning. He would draw on his students' mistakes or the challenges that they faced to plan learning experiences for his students in the following semesters. He shared:

> I try to recall some of the pitfalls that students went through in the first semester and try to prepare the students to follow those pitfall or try to make sure that they don't go through those pitfalls. Some pitfalls you want the students to go through because you want them to figure out a way to get out of it. (Interview 1, Spring 2017)

This was an important insight into Caleb's planning. He wanted his students to struggle through the problems and shared that he mindfully tried to incorporate challenges into his plans. According to Caleb, his experiences in the classroom were a source of guidance for him as he planned his lessons. Especially what he learned about his own students and their learning experiences. Reflecting on his students' experiences allowed him to incorporate perceived challenged into his plans. His reflection on his classroom experience also allowed him to be an effective teacher. Caleb cited pacing of the course as a concern but with experience he was able to foresee discussions that would be less productive for student learning. He planned to navigate the student responses for productive learning experiences for his students.

Due to his experience with planning and enacting the curriculum, Caleb reported that planning became less daunting for him. Pacing was still a concern for him and he shared his concerns in the online meetings through the end of the second semester. He reported that the experience he gained from implementing the curriculum guided his planning.

Enacting

In this section, I describe Caleb's engagement with the curriculum through enacting it inside the classroom. The instructional plans that he designed unfolded in class when he, as the teacher, interacted with both the curriculum and his students.

Challenges when enacting. Caleb was already experienced at teaching Precalculus but the novelty of the new curriculum made him feel unsure about his teaching practice. He expressed concern regarding his own ability to teach the problems

when the curriculum represented concepts in a new way. In addition, student buy-in was a concern for Caleb. He shared that he recognized the benefits of the new curriculum and found its investigative approach beneficial for the students. However, he was also concerned about how his students would feel about the new curriculum, specifically the investigations. He expressed that his goal was to ensure that his students would adjust to a classroom where he wanted them to be engaged in their own learning. Caleb explained that many of his students were used to the type of instruction where teachers present problems on the board instead of students "[h]aving to critically think about things and model things." (Interview 2, Fall 2016) He shared that in his experience, the students found an investigative approach to learning, difficult. He had to be patient as the students resisted his efforts to engage them, and shared that eventually his students came on-board (Interview 2, Fall 2016). Caleb explained that he tried to be patient with his students, as well as attentive and responsive to his students' concerns. In the end, he tried to address concerns about the investigative nature of the curriculum in class to help his students have a successful learning experience.

Response to challenges. Caleb tried to approach the challenge of getting the students on board with the investigations, by creating a classroom environment where his students felt safe. He shared that he wanted to create an equitable classroom where all his students would have a chance to succeed. His goal was to make students feel encouraged in that space. He wanted his students to know that he was "not here to make you feel like you're beneath anybody else." (Interview 1, Fall 2016) In his observed classes, students worked in groups and were encouraged to discuss their ideas with partners before

presenting them to the class. To facilitate this, Caleb gave students ample time to think. According to Caleb, the reason why students do not share their thoughts with the class is because they might be afraid to fail in front of their peers. He stressed that it was important to encourage students to feel safe by giving them opportunities to ask questions and discuss their thoughts in smaller groups before sharing them with the class. Caleb explained that creating a safe classroom environment where students felt comfortable to discuss their ideas, was beneficial not just for the students but also for himself. First, if the students' felt safe they would be more likely to share their thoughts without fear of being judged. Secondly, by teaching in an environment where everyone felt encouraged to discuss their thoughts, he shared feeling comfortable himself in trying out the new curriculum.

Caleb wanted to model mathematical practices for his students and he was observed asking the students thought provoking questions to engage them in discussions. His shared that his goal was to allow the students to discover the concepts instead of him providing them with all the information through direct instruction. Caleb explained that his goal to stay away from direct instruction was influenced by the curriculum and its investigative nature. He said that it was easy to "revert back to traditional learning" (Interview 1, Fall 2016), which in his view is teacher-centered, and he had to put in the effort to maintain active discussion in his classroom. He tried hard to "push them [students] back to, no we're not trying to ever get back to that in this course!" (Interview 1, Fall 2016) He shared that his efforts to enact the curriculum well were influenced by his students' learning and the benefits of the new curriculum for them.

Becoming comfortable with the new curriculum. Caleb shared that he initially had concerns about how he would implement the new curriculum. As he started enacting the curriculum, he reported that he was able to connect the new curriculum to his past experiences. He was able to make connections between the new representations of concepts to the ones he already knew. Speaking about his first impression of the curriculum, Caleb said, "I opened it up and I said okay this is brand new. It wasn't brand new!" (Interview 1, Fall 2016) Initially, he was apprehensive about the way in which the new curriculum would impact his own teaching experience, he reported that he was able to find a connection between the new and the old. He shared, "I felt very very anxious, nervous, but once I got in the classroom I was like, you are a good teacher so just go with it!" (Interview 1, Fall 2016) He continued to share, "I think as time goes on you have to make yourself comfortable with this because, it's brand new for us too. You figure things out." (Interview 1, Fall 2016) Caleb allowed himself time to get used to the new curriculum.

Influences on Caleb's enactment. Caleb shared that he was open to change and eager to learn from the new curriculum. He was able to experience "Aha moments!" (Interview 2, Spring 2017) even while he was teaching. As he explained, "Sometimes that even happens while you're teaching and you're like I didn't see it this way." (Interview 2, Spring 2017) He found, teaching from the new curriculum beneficial not only for his students but also for his own growth as a teacher. In his own words, "I think it [*Pathways*] made me a better teacher! Because I just saw math in a different way, like sometimes I'm like oh now I get it! Oh wow!" (Interview 2, Fall 2016) He was mindful of

the new representations in the curriculum and the ways in which they influenced his practice. Caleb shared that he would reflect on his lessons after class and plan his lessons meticulously based on his classroom experiences. His planning and his reflection on ideas gleaned from reading the text or from interaction with his colleagues, fueled his classroom experiences. They afforded him flexibility inside the classroom. Caleb explained that he would walk into his classroom with a plan, knowing what direction the lesson would go, but based on student interaction he would change plans during class. He shared that having a clear goal for his lessons and knowing how concepts learned in one class connected to other concepts, supported his enactment of the curriculum

Caleb's engagement with the curriculum in the form of enactment was influenced by his desire to improve his practice. In order to improve the enactment of his plans inside his classroom, Caleb reflected on his classroom experiences and collaborated with his colleagues. He paid attention to his students' struggles, and tried to adjust his practice to help them.

Collaborating

In trying to figure out how to best implement the new curriculum Caleb drew on his colleagues' expertise for both guidance and support. All the instructors in the first semester were implementing Pathways for the first time and the experience was new to them even if they had prior experience teaching Precalculus. Caleb shared that the curriculum provided "the same common language" (Interview 2, Fall 2016) to aid communication, as the instructors tried to navigate the new curriculum together. The

instructors discussed their challenges with the curriculum, asked each other questions and shared ideas and classroom experiences that helped them implement the curriculum.

Caleb's Collaboration. Caleb reported that he informally collaborated with his colleagues in the adjunct faculty lounge if their schedules allowed it. These informal conversations took place during their office hours or in between scheduled classes. He described that during these meetings, the instructors would inquire about pacing in their classes, any new instructional techniques they were using, share classroom concerns and seek advice. Sharing their instructional experiences provided an opportunity for the instructors to plan their lessons based on others' experiences and try new techniques in their classroom. Caleb shared that the informal collaboration allowed him to improve his instruction. Beyond informal collaboration, Caleb also found the formal PLC meetings beneficial. The hour long, online PLC meetings took place every week at a time that was convenient for most instructors. As gleaned from the PLC data from the two semesters, Caleb actively participated in the meetings. He shared that he found the PLC meetings beneficial and said that even in the second semester, during the meetings he had, "those aha moments, it's not like oh wow, oh this is boring, this is not useful." (Interview 1, Spring 2017) It was a chance for him to learn from his colleagues.

Sharing experiences and concerns. Caleb used his participation in the PLC meetings as an opportunity to ask questions and share his concerns with his colleagues. He was often observed bringing in examples from his own classroom to share and get feedback from the instructors because he valued their opinion. Speaking about the PLC Caleb shared, "It's ongoing conversation on a weekly basis, that keeps us thinking about

our teaching, thinking about our student's learning." (Interview 1, Spring 2017) Caleb shared that he felt comfortable asking questions and sharing his experiences in the PLC meetings. For example, he would ask clarifying questions about the content. "So are we looking at tangent, as it is touching the circle at one point (your blue line)? I'm a bit lost, sorry guys!" Until he finally understood, "Got it!" (PLC meeting, 11-29-2017) He also shared his own classroom experiences with this students. For example, during a PLC meeting he shared:

> This is when we clarify horizontal versus vertical... sin and cos or 6.28 divided into 4s... finding the radian measure... 1.25 radians... bc we discovered 1.57 is the highest in quad 1... remember when I said I did 6.28 divided by 4s... making that connection... and they realize it makes sense. (PLC meeting 3-27-2017)

For Caleb, this exchange of support was beneficial. He said, "When they [other instructors] stumbled, we support them in getting that; coming up with a common thread of us, as adjuncts." (Interview 1, Fall 2016) He found this experience helpful.

In terms of pedagogy, Caleb shared that collaboration with colleagues, allowed him to keep challenging himself to improve his practice. He said the collaborative discussion, "keeps you away from going back to the drill and kill way of how things go." (Interview 1, Spring 2017) He described that he recalled the discussions during the meetings when he would plan his lessons and they helped him to plan his classroom activities from a "problem solving/modeling perspective." (Interview 2, Fall 2016) He would share his classroom experiences with the group and sometimes he would express his concern about student discomfort with him not giving them the information in a

lecture. He shared, "They [students] were frustrated that I was not using the slope formula outright and when I asked, what does slope mean to them? Crickets!!!!" (PLC meeting, 9-20-2016) During the meetings, he was often concerned about his students and would share specific incidents to get feedback from the group ("They are quick to write all changing equation with regular variables as oppose to the delta variables." (PLC meeting, 9-20-2016) or "I spent a lot of time total on the differences between t and delta t." (PLC meeting, 9-20-2016)). He expressed concern about his teaching practice and asked if other instructors had similar concerns. One particular concern, where students were confused about the notation for change in quantities, was shared by many of the other instructors in the group as well. The comparison allowed Caleb to assess his own teaching.

He also compared the pacing of his class in terms of the schedule with his colleagues' pacing in their classes. He would share a lot of concern about the pacing of his lessons. He mentioned in a meeting, "I get through 3-4 [problems] and I take the 'mini-lesson, and they work on it' approach." (PLC meeting, 9-20-2016) For Caleb, his pacing concern was also connected to his concern about discourse. For example, in one PLC meeting he shared, "The thinking part, and me delaying to allow them to think and tell me, requires patience and time." (PLC meeting, 9-20-2016) He wanted his students to have the time to develop their thinking but was also struggling to stay on track with regards to the pacing. He would express his concern about the pacing in the group, "I gotta catch up... Gheesh I am the lacky here lol" (PLC meeting, 9-27-2016) or share, "I was, happy I spent a lot of time on 7.3 because the concept was new in terms of looking

at it from a moving object." (PLC meeting, 11-22-2017) The pacing concern continued to be a topic of his comments in the PLC meetings, during both the semesters.

Getting a different perspective. For Caleb, both his formal and informal conversations with his colleagues were beneficial. He shared that he reached out to them when he needed help and found that his colleagues were ready to provide guidance and to discuss ideas. Caleb shared that his collaboration with colleagues allowed him to learn from their experiences. He explained that he was able to get a different perspective from them in terms of "how they see things, how they deliver things... and just seeing their talents." (Interview 1, Fall 2016) The various instructors had different backgrounds. Some were more focused on mathematics education, others had a greater interest in theoretical mathematics, and some had a high school mathematics inclination. Caleb shared that, "because of the their backgrounds as well, I got to see a lot of that transition from understanding the curriculum to understanding the path, the actual pathway of the curriculum." (Interview 2, Spring 2017) The conversations with colleagues provided a trajectory of mathematics that allowed him to see what the students needed. He added that, "the colleagues... connected back a lot to what they [students] need for calculus." (Interview 2, Spring 2017) These meeting offered him the opportunity to, "see a lot of people's backgrounds" (Interview 1, Fall 2016) so he could get ideas from their experiences to help his students.

Interaction with other instructors had an impact on Caleb's phrasing of mathematical concepts. He shared that he was able to pick up precise language through discussions with other instructors. Speaking about his interactions with a specific

instructor he said, "He's a pure and applied person, and then we are the Ed persons. So... just to hear sometimes, oh he makes my math language better!" (Interview 1, Spring 2017) In this regard the online meetings proved helpful by listening to the way, concepts were phrased using precise mathematical language. In addition to learning from his colleagues, the PLC space also allowed Caleb to learn from the facilitator. Caleb gave an example of one specific instance during his second semester of implementation:

> It was a high moment for me I was like oh I can make it better by saying that, oh I see now where [facilitator] was trying to take this last semester, so I might have struggled with it in the classroom but now I have a good idea how to move forward there and it seems cleaner. (Interview 1, Spring 2017)

Caleb shared that through collaboration, he was able to learn new ways to phrase ideas during his own instruction.

Form of Support. Caleb expressed that he would have liked to have more direct interaction with his fellow teachers outside of the PLC. He was aware that as adjunct instructors it was not possible for the group to meet outside of the PLC due to the group's individual time constraints and schedules. He mentioned that while teaching in school he would have common planning time and sometimes he would grade exams together with his colleagues, this was not possible to recreate in the college environment, especially with adjuncts. He appreciated the opportunity to interact with his colleagues whether in a formal PLC setting or informally. In his own words, "You don't feel like you're on your own, you don't feel like Tom Hanks with Wilson in the corner." (Interview 1, Spring 2017) He explained that taking the time to attend the online meeting was not always easy

for him but he made an effort to do so because he found the support really beneficial in helping him implement the curriculum.

Reflecting

In addition to collaborating with colleagues, Caleb shared that reflection was an important form of engagement with the curriculum. He described that he reflected on his classroom experience while planning, collaborating with his colleagues, and also on his own. When describing his reflection he said that he reflected about the conversations in the formal and informal meetings with his colleagues. He reflected while planning his lessons and he also spent a great deal of time reflecting about improving his teaching practice. He explained the role that reflection played in his teaching:

> It's ongoing, so it's changing your mind, especially when you're like teaching the content. You're in your head. You're thinking about the phrasing of questions. The way you want to deliver this instruction. The way you want them to take the initiative to learn the instruction and then you know, giving hint or feedback to your team, other adjuncts, and then hearing what they have to say. What happened in their classrooms, and that all helps right there. (Interview 1, Spring 2017)

Caleb shared that his biggest reflection time was in his car, when he would commute to and from the institutions where he taught. During this reflection time he would think about his classroom experiences for that day and ask himself general questions, such as "Did I accomplish something? Did they get it? Did I learn something today? Did I learn something about the students today?" (Interview 2, Spring 2017), or specific questions such as "There's ways to improve that language and what's that? Percent change! You

figured they should know percent change from high school or even business class, they don't! They still mess it up so how to make that real for them?" (Interview 1, Spring 2017) Caleb said that he wanted to improve his teaching practice and provide a better teaching experience for his students. He shared that reflection afforded him the chance to learn from his classroom experience, to learn from this collaboration with colleagues and to improve his planning and enacting of the curriculum.

Caleb's engagement with the curriculum through planning, enacting, collaborating or reflecting was guided by his desire to improve his teaching practice. Caleb reported that he was open to the idea of improving his own practice and learning himself. He collaborated with his colleagues, which was apparent in the online meetings where he participated actively. He summed up his experience with implementing the new curriculum by stating:

> What I learned is that, I am open to change in teaching. I learned that problem solving mathematics is difficult and difficult to even create... a curriculum like this is difficult to create... it's so meaningful and I see why people have gone to drill and kill because it's easy, this is difficult, but yet rewarding. (Interview 1, Spring 2017)

Regardless of the challenge posed by the new curriculum, Caleb shared that he was motivated to do a good job because he found the curriculum to be beneficial for his students and for improving his own instructional practice. He shared, "The curriculum opened our minds, for us to become, to be stronger math teachers I think that we, it kind of did our own professional development as it stuck with us [to] teach math in a different

way." (Interview 2, Fall 2016) Implementing the new curriculum did bring up challenges for Caleb. His approach to engaging with the curriculum was of improving his own teaching practice as he faced the challenges.

Opportunities for Caleb's Learning

In the following sections, I provide examples of how Caleb's engagement with the curriculum provided opportunities for his learning. Opportunities to learn emerge as teachers face challenges when implementing a curriculum. For example, the online homework was a cause of concern for his students so Caleb had to address the problems in the class which impacted his planning and classroom experiences. He asked questions during the PLC meetings specific to the online homework and reflected about it. While he tried to help his students tackle the homework problems, he shared noticing that the homework problems were presenting learning opportunities for his students. He paid more attention to them and picked up on how they were presenting the ideas to the students by allowing the students to build on their own learning.

As mentioned earlier, Caleb's engagement with the curriculum was driven by his own goals to improve his teaching practice and to provide better teaching experiences for his students. In his own words he shared, "My challenges are part of my own professional development. How do I make it better?" (Interview 1, Spring 2017) He was excited by the idea of learning new things and was not afraid of change. Caleb shared that in order for him "not to stay stale in this environment" (Interview 1, Spring 2017) he had to "continue to change, and change for the betterment of the students." (Interview 1, Spring 2017) He saw the new curriculum as an opportunity for his own learning and said that it

was important for him to "keep an open mind to the material and be ready to learn every single day." (Interview 2, Spring 2017) He realized that he had learned the content when he was in high school but he was open to getting a deeper understanding of the same content. He shared, "Even though you've learned the content since, I don't know, like 9th, 10th grade, but be ready to learn because there's always something new coming up all the time and you're like oh aha! You even get aha moments." (Interview 2, Spring 2017) Caleb took pride in his teaching practice and wanted to help his students be successful. He was willing to learn from the challenges that came up during implementation of the new curriculum. His motivation allowed him to avail of the opportunities to learn and to improve his teaching practice.

Opportunity for Learning: Student Discovery

Caleb faced many challenges in the process of implementing the curriculum. In dealing with his challenges, he was afforded an opportunity to learn and to improve his instructional practice. For example, Caleb often shared his concern about his students' experiences with online homework. During the first semester the set-up of the online homework was a cause of concern for Caleb because his students struggled with the problems. As mentioned earlier, Caleb had taught Precalculus in the past. He shared that due to his experience, he was familiar with a different platform for online homework for the students. The new platform aligned with *Pathways* was different from the previous homework platform in several ways. First, it had a new user interface that required some getting used to for the new instructors, but the instructors received training during the summer workshop to get them started. The second way in which the online homework

was different was the content itself. The type of problems that were posed to the students were thought provoking and challenged students to think. It was not the case that students could refer to the sample problems they did in class and use the procedures as a guide for their homework. Instead, the students had to transfer the big ideas that they learned in class to aid them in solving the homework problems.

Caleb saw the homework platform as a challenge but also as an opportunity for his own learning. During the online meetings he often shared concerns about his students' difficulties with the homework. Initially, he sided with his students and believed that the homework problems were not very useful. During one of the earlier meetings in the first semester he shared, "Some of my students said, they had to guess what variables to use" (PLC meeting, 9-27-2016) and "I agree. All I hear [is] that these problems are very hard and are not connected to the lessons or the workbook." (PLC meeting, 9-27-2016) He was himself beginning to familiarize himself with the new system. He shared, "I do not know what to cut out of the HW" (PLC meeting, 10-18-2016) as one of his concerns. During one of the PLC meetings in the same semester another instructor showed him how to create his own assignments online, which allowed him to select the problems he considered more appropriate for his students. Learning how to select problems to be assigned afforded him some control over the content of the homework assignments. However, even by the end of the second semester, students' difficulties with the online homework proved to be a source of concern for Caleb. During the beginning of the second semester he shared in a PLC meeting:

> As we are doing more and more calculation problems in Logs/exps in the HW, the program is not telling the students where to round their answers to and the students are losing points for that... It is stressing the students out and also stressing me out. (PLC meeting, 10-25-2016)

Caleb was aware of his students' difficulties with the homework and he shared that he paid attention to the homework problems; the way they were presented and the format in which answers were to be submitted. Caleb explained that he began to realize that the problems in the online homework were asking the students to be more precise with their answers. In a PLC meeting during the second semester he shared:

> My students all weekend still do not understand the difference between the function notation and the whole equation, I got 5 emails about the homework being wrong, they only wanted *h(5*) and not the whole equation, *h(5)* is how far you drove from the start after 5 minutes. (PLC meeting, 2-20-2017)

After becoming familiar with the online homework system, he tried to help his students in the classroom by preparing them to be more precise with their homework answers. This change happened as early as the first semester. In the beginning of the first semester, he asked the students to be mindful of the ideas they learned in the class (e.g., "Our domain becomes our range, our range becomes our domain" (Observation 1, Fall 2016)) when doing their homework. By the end of the first semester he prompted them more frequently. For example, in one observed lesson Caleb advised the students to make sure all the problems they had done in class were completely finished before they attempted

their homework. He continued to guide them during the rest of the class period by asking the students to be more precise in how they answer the questions. He said:

> Pay attention to what they want you to put in your homework assignment. The answer will be cosine of 0.8, sine of 0.8. As we discussed there... and we want them in decimals... remember, four decimal places is always safe in these problems. So in your homework assignment, I am pretty sure, it will probably nag you over the weekend if you didn't put the decimals in it, at least 4 decimal places will work. (Observation 2, Fall 2016)

In addition to being precise, Caleb also suggested that the students pay attention to what the questions were asking them to think about and figure out how to represent their answers so they made sense. For example, he told them that if a problem was asking for measurements, to be mindful of the context, he said, "So, I am pretty sure your homework, you are going to put in a negative there, because you can't write, *below!* Okay!" (Observation 2, Fall 2016) He wanted them to think about the questions, the answers and the format in which they would submit their answers.

By the end of the two semesters, he shared that the students, "Hate the iMath (homework) program... I know and you know that's been their challenge, the iMath program has been a challenge for them!" (Interview 2, Spring 2017) He also shared that he could see the benefit of their approach. He shared that he appreciated that the online homework problems guided the students to develop their thinking even as they struggle through the problems. Talking about the homework assignments he said that they, "make them [students] understand, you know what does this mean!" (Interview 1, Fall 2016) He

continued to explain, "That's what I like about it not giving information up front." (Interview 1, Fall 2016) Caleb shared that inspired by the curriculum, he used this approach in his own teaching when inside the classroom.

Caleb expressed that he was open to learning from his experience of implementing the new curriculum. He found opportunities for his own learning, in many of the challenges he faced. He shared:

> Because the curriculum opened our minds for us to become…to be stronger math teachers I think that we…it kind of did our own professional development as it stuck with us to teach math in a different way. I think it (Pathways) made me a better teacher! Because I just saw math in a different way like sometimes I'm like oh now I get it! Oh wow! NOW I got professionally developed to teach it a better way, not different but a better ways. (Interview 2, Fall 2016)

Caleb had a personal goal to improve his own teaching practice. His engagement with the curriculum provided him with opportunities for learning. He was able to recognize those opportunities and tried to avail of them. He found his engagement with the curriculum beneficial because he was mindful of the learning opportunities the challenges provided and was willing to put in the work to improve his own teaching practice.

Opportunity for Learning: Fostering Discourse in the Classroom

During the summer workshop, the facilitators had suggested the investigative approach as being beneficial for students. Caleb tried to incorporate this approach in his own lessons. Right from the beginning of the first semester he tried to develop an interactive environment in his classroom. He said that he tried to "stay away from giving

them [students] the information at first" (Interview 2, Fall 2016) and allowed them to work on the problems themselves. He shared that he was influenced by the curriculum, especially the online homework to adopt this approach. Caleb shared that the curriculum's approach to modeling and problem solving was beneficial for him. Comparing it to his past experience teaching Precalculus he said that the new curriculum provided "a problem solving/modeling perspective as opposed to drill and kill type of environment." (Interview 2, Fall 2016) According to Caleb, it was beneficial for him as a teacher to engage with the curriculum because, in his words, "When you're doing the modeling you're saying oh that's what that meant all of those years I was teaching it." (Interview 2, Fall 2016) Through his engagement with the new curriculum he was able to observe examples of how modeling and problem solving could be supported in his own classroom.

As observed in his classrooms Caleb started his class with a mini lesson, using a presentation to go over the main concepts of investigations for that day. He would develop examples similar to the problems in the book and used phrasing that the students would encounter in the investigations. The mini lessons took about 15 minutes and then he would ask the students to dive into problems he had selected. Right from his first classroom observation at the beginning of the first semester, Caleb's classroom exhibited conversations between himself and the students as well as between students themselves. The discussions were teacher led but Caleb tried to mimic what he saw as the curriculum's approach to allow the students to have their own aha moments. He wanted to refrain from giving the students all the information in the form of a lecture. Through

observation data and classroom notes, it was apparent that Caleb routinely encouraged his students to participate in investigating the concepts. However, Caleb also engaged in direct instruction and seemed to take charge of the discussion to convey big ideas to the students. For example, in the following episode from Caleb's classroom (Observation 1, Fall 2016) his students were working on a problem exploring inverse functions. [Module 3: Investigation 4, Problem 5 (see Figure 6)]:

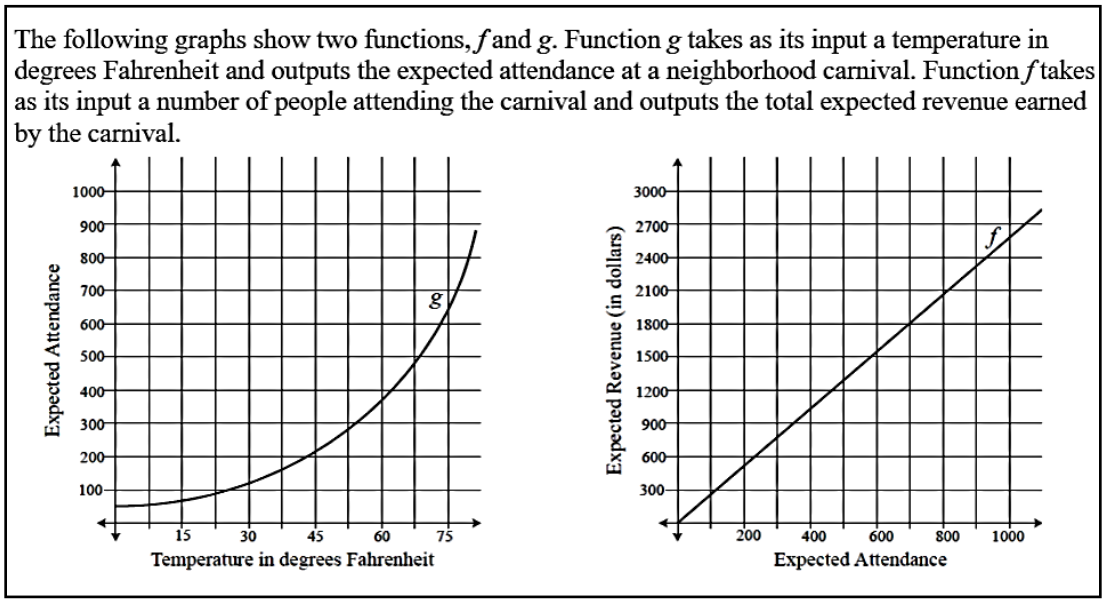


Figure 6. Module 3: Investigation 4, Problem 5, Student Workbook.

Caleb: So basically, the output of g becomes the input of *f*. Does everybody see that? The output of *g* becomes the input of *f*. This one what happens is it will be done first in order for me to jump here. All right. Now, we begin the questions. I look at the questions here that you just went through right now**. It says does this question *f* of *g* of 70 [*f(g(70))*] have a real-world meaning**

		in this context? Let's go back to the graph, and explain to me what's happening. Go ahead, Brittany.
	Student:	That the temperature determines how many people attend, therefore how much revenue the park will make.
	Caleb:	**Can you come up here and show me that? I want to know what you're talking about.**
	Student:	I think with 70, if it's 70 degrees, which 70 is the input for the 70 degrees, you go up. I just rounded and put five...
	Caleb:	That's fine, but we said I don't really care about the numbers. I care about the learning, right? Good. Go ahead.
	Student:	Here $500 ... Not $500, 500 expected attendants as we go up, it's one. It's not there. It's here. It's then about $1,300 in revenue.
	Caleb:	Okay. **As she said, when the temperature ... We're going to say it in words, right? We're going to say what she said in words. Don't move. The temperature, you have 70 degrees, the expected attendance is going to generate 500 people roughly, she said, right?**
	Student:	About. Yeah.
	Caleb:	Okay. **How do I show that here? How do I show that here? That's the work, how I show it here?** Add for 500. Does everybody agree with that?

28	Class:	Yes.
29	Caleb:	Okay. Then she said when the expected attendance is 500, the
30		expected revenue should be about...
31	Student:	1,300.
32	Caleb:	Make sense?
33	Student:	Yes.

Caleb allowed the students to work on their own and had a student come up to the board to share her work. As Caleb reported, his goal was to work towards a student-centered classroom, but he wasn't accomplishing it based on the observation - this episode still shows a teacher led conversation. He used student work to demonstrate the main points to the class and while he asked some open-ended questions (Lines 5, 11 & 25), Caleb was quick to answer them himself. The other questions he asked were more directed towards a specific response (Line 19). During this episode, Caleb prompted the class to check if their answers made sense. This is an important part of the modeling process. In particular, Caleb was ensuring that the students checked their answers for reasonableness and to see if they made sense for the given context. He asked them, "Here is my question to you guys. Is this real-world? Does it make sense?" and students agreed. The class continued to other parts of the same problem. At another time during discussion about the same problem he asked the students to check if the problem made sense:

1	Caleb:	So this is in terms of revenue. There is no existence of revenue on
2		that graph over there. Does everybody see that? That graph is with

		respect to temperature in degrees, based off of what happens with the expected attendance. Does that have a real-world context? No! If you were to look at it to say that was temperature, 150 degrees temperature, in reality can you have 150 degrees temperature outside?
	Student:	Yeah.
	Caleb:	Really? I don't think people really have that. Therefore, part B there is no real-world context using those words there.

Here, the first two parts of the problem themselves allowed Caleb to help his students check their responses for real world feasibility. The first two parts stated whether *f(g(70))* and *g(f(70))* had a real-world meaning, and asked the students to explain their reasoning. Caleb himself wanted to implement the curriculum to help his students develop their conceptual understanding, and the way the problems were set up sometimes aided him in achieving his goal. Based on Caleb's observed classroom lessons, as the semester progressed, he maintained his interaction with the students during lessons. As exhibited by the observation notes and recordings of his lessons, Caleb's students were comfortable in asking him questions and to ask for better explanations. The discussions were still teacher led with him trying to draw the information out from the students as seen in this example (Observation 2, Fall 2016):

	Caleb:	They're showing you that this Ferris wheel made two full rotations. Can everybody see that? Meaning it went around each time. So my

3		question to you is: If I asked you to estimate the equation of this
4		graph here, what would the equation of this graph be?
5	Student:	Y equals sine of x.
6	Caleb:	He said Y equals sine of x. Does everyone agree with him on that?
7	Student:	Yes.
8	Caleb:	Right. Cool.

Caleb shared that his goal was to foster discourse in his classroom, while he tried to achieve this goal, his classroom exhibited lessons where the discussions were mostly teacher was led. He provided ample opportunities for students to share their thoughts and used their responses to guide the lesson. He shared that he carefully planned his lessons, keeping students' challenges in mind and providing opportunities for them to struggle through problems.

Opportunity for Learning: Phrasing and Precision

His engagement with the curriculum over the course of two semesters provided Caleb with multiple opportunities to learn about the content and instructional practice. Caleb shared that he noticed an improvement in his phrasing of mathematical concepts as well as asking the students to be precise in their work. Caleb shared that he reflected on the concepts that he had presented in class and about the upcoming investigations. He shared that he wondered about possible ways to "improve that language!" (Interview 1, Spring 2017) in terms of making the concepts more accessible to the students, to help them understand. Caleb reported that engagement with the curriculum when collaborating with

colleagues played a big role in helping him improve his knowledge and teaching practice. The discussions that he engaged in during the online PLC meetings made him mindful of how to phrase concepts. During an interview, Caleb shared that the PLC meetings "opened up my eyes to a lot of things that I know about math but never math in this language." (Interview 2, Fall 2016) He shared his goal for his students to be precise when discussing ideas in class and to guide them in how to phrase their thoughts. For example, this excerpt below (Observation 1, Fall 2016) demonstrates some of the terminology Caleb picked up during the PLC meetings.

1	Caleb:	If I switch F and C, what will happen to my inputs and outputs if I
2		was to switch them up?
3	Student:	**They also switch.**
4	Caleb:	**What do you mean they also switch? Tell me in terms of inputs**
5		**and outputs. What happens?**... If I was to switch them around,
6		my inputs now become my outputs, and my outputs become the
7		inputs. Am I changing the numbers?
8	Student:	No.
9	Caleb:	No. I'm just reversing the process. Make sense to everybody?
10		Okay. Cool. Let's go through that. If *h* reverses the process of *g*,
11		then we have *h* is ... It will become the inverse function of *g*, or the
12		inverse of *g*. If *g* reverses the process of *h*, *g* is now becoming the
13		inverse function of *h*, or the inverse of *h*.

14	Caleb:	As Sally said, hopefully everybody wrote that down. If we're
15		reversing the process, our output become our inputs and our inputs
16		become the outputs.
17	Caleb:	**How about if I share in terms of domain and range? How**
18		**about I go back to the work we went through last class? What**
19		**will happen there to be reversing the process?**
20	Student:	**Won't they switch?**
21	Caleb:	What do you mean they switch?
22	Student:	Our domains will become a range, and our range will become a
23		domain.
24	Caleb:	Does everybody agree with Brittany?
25	Student:	Yes.

Caleb wanted his students to be precise in their use of mathematical language so they could communicate effectively. Caleb shared that through his engagement with the curriculum (the summer workshop, reading the textbook and collaborating with his colleagues), influenced him to pay attention to precise mathematical language in his classroom. For example, when the students said, "They also switch" (Line 3), Caleb asked, "What do you mean they also switch? Tell me in terms of inputs and outputs." (Lines 4) He guided the students to answer in terms of inputs and outputs first. Later in the episode he asked them to respond in terms of domain and range (Line 17), when a student asked, "Won't they switch?" (Line 18) Caleb again asked the student to explain

the response and the student responded in terms of domain and range. Caleb was using the language of the curriculum and he was guiding his students to clearly communicate their mathematical ideas. As the semester progressed, Caleb added more phrases to his repertoire of presenting the concepts to the students. As seen in this excerpt from later in the first semester.

1	Caleb:	All right, so last class, last Tuesday, guys, when we were dealing
2		with the Ferris wheel problems, we worked with two quantities
3		that were very important to us figuring out how did the Ferris
4		wheel move around, right? What were the two quantities that we
5		worked with a lot last class?
6	Student:	Radius?
7	Caleb:	Not the radius.
8	Student:	Vertical distance.
9	Caleb:	**The vertical distance from the horizontal diameter and**
10		**worked with the horizontal distance from the vertical**
11		**diameter**, right? So what's added today is, we also can work with
12		another co-variant quantity, we can talk about the rotation of the
13		Ferris wheel. We didn't really talk about the rotation as much of
14		the Ferris wheel. As the Ferris wheel rotates, remember it sweeps
15		out the angle that it's making either in radians or degrees, okay?
16	Caleb:	So, those are the co-variant quantities that we can work with.

Caleb's used "The vertical distance from the horizontal diameter" and "The horizontal distance from the vertical diameter" (Line 9) to guide the student to be more precise in their description. He picked up this language from the summer workshop and when reading the textbook, and it was also reinforced in the PLC meetings. He continued to use phrasing picked up from the curriculum and the PLC meetings during both the semesters.

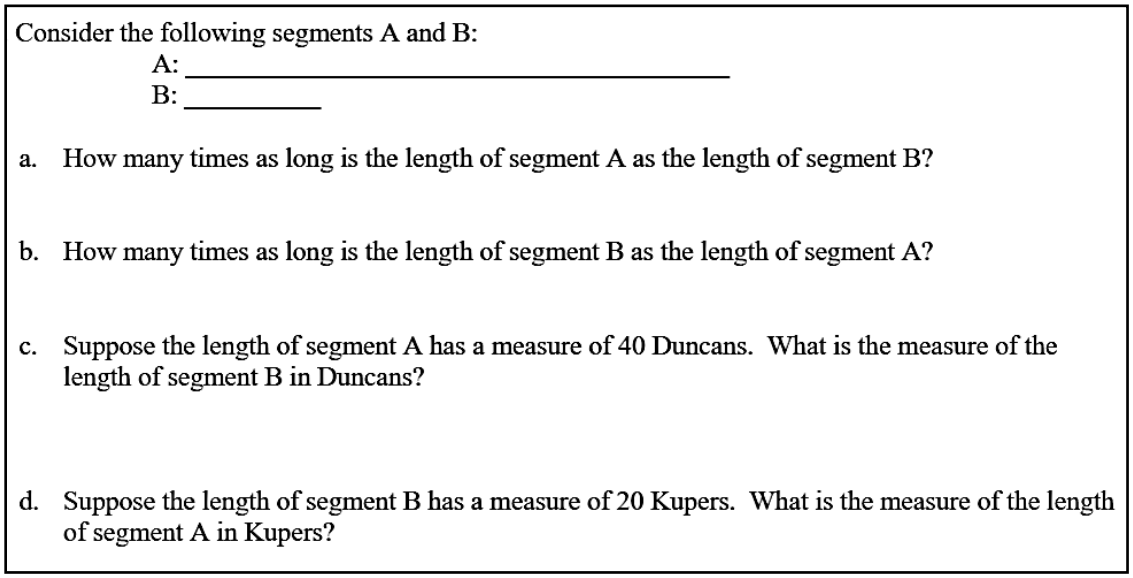
The problems in the investigations also helped Caleb in modeling precision for his students. For example, the following excerpt during the second semester of implementation semester shows his students working on a problem exploring percent change.

1	Caleb:	So guys, what happens now is, new language, the new price is, 114
2		percent of the original price cause remember it's still a multiplier...
3		as many times as... new price is 114 percent of the old price.
4		Alright?
5	Student:	This is not 114!
6	Caleb:	Percent!
7	Student:	How did you get that?
8	Caleb:	Think about it! I want you to look at this number and try to think
9		about how I got that. I want you to think about that, how did I get
10		that number? I didn't make it up.

He gave the students some time to think about it and then asked, "So I did, I get the 114?" Another student responded, "You added the 14% to the original 100%?" He built on that response by stating, "The idea you're building on that original 14% is because

you're looking at the old price as the 100%, when you go to the new price, it 114% of the old price." He then proceeded to ask the class, "How do I mathematically check that?" He wanted the students to check their work to see if it made sense.

The above example was part of Caleb's mini-lesson before he had the students start the investigation. Afterwards, he had the students work on a problem that compared the lengths of two given line segments [Module 4: Investigation 1, Problem 1 (see Figure 7)]:



Consider the following segments A and B:

A: _____

B: _____

a. How many times as long is the length of segment A as the length of segment B?

b. How many times as long is the length of segment B as the length of segment A?

c. Suppose the length of segment A has a measure of 40 Duncans. What is the measure of the length of segment B in Duncans?

d. Suppose the length of segment B has a measure of 20 Kupers. What is the measure of the length of segment A in Kupers?

Figure 7. Module 4: Investigation 1, Problem 1, Student Workbook.

He wanted the students to build on the examples they had already done so he said:

> All right, come back to me! So walking around, so one observation, right, I want to go back to the premise of the course, the idea of the course is so whatever context I put you in you should be able to adapt, right! Not every problem is the same! I saw a lot of 35 and I'm wondering why? Because the question never said, something went from increase to decrease! What the question said was, **how**

> **many times! As large, is one segment to the other?** So in reality what did we just learn? What am I attacking there from what we just learned? (Observation 1, Spring 2017)

This prompt allowed the students to go back to the examples they had done earlier and the class discussion led to him asking, "What was the 114?" A student answered, "The percent of the new price to the old price!" He then stated, "As many times as! It was as large as!" This response prompted the students to pay attention to the language so they could compare two quantities where one was as large as the other. Saying "Right, $57 is 114% as large as the original" and "Or as many times as the original!" explicitly allowed the students to make the connection.

Caleb was mindful of asking students to pay attention to precision when solving problems. During an interview Caleb shared, "So today we spent a lot of first 15-20 minutes about when they would defining their variables, how they were incomplete" (Interview 1, Fall 2016) He shared that his students would give him several responses and he kept encouraging them to be more precise. During the beginning of the first semester of implementation he prompted the students by saying, "I see you guys make drawings, which is good. I don't see labels. You got to label your variables. We learned it from day one, right? Label the variables." (Observation 1, Fall 2016). He would walk around his classroom to check student work and ask them to make sure their work reflected precise definitions and diagrams. He would make quick remarks such as, "I saw the length in feet. The length of what?" He made such remarks to remind students to make sure their answers were complete. During one interview he shared that he worked hard to develop

good mathematical practices in his students and they responded to the effort he put in. He would ask them, "What was the object of the problem? What's the problem telling you to do? And that's to fill in! How's the water filling in?" His goal would be to ensure the students knew that they need to clearly define their variables and he shared that after a while the students began to understand why he was asking them to be precise.

His engagement with the curriculum as he planned and enacted his lessons, participated in the online meetings and reflected, allowed Caleb to be mindful of the phrasing that could be used to precisely describe mathematical concepts. He reported learning new ways to phrase concepts first through the summer workshop then as he planned his lessons by reading the textbook and going through problems in the investigations. The new terminology was reinforced as he discussed the curriculum with his colleagues both in the formal PLC meetings as well as the informal conversations in the adjunct faculty lounge. More important than himself being aware of the phrasing and the need for precision in describing mathematical concepts was Caleb's desire to guide his students to be precise. Caleb shared that he wanted his students to use the new terminology and to describe mathematical concepts with precision. He reflected about ways to guide his students in the classroom. Caleb also shared ideas and challenges with his colleagues and asked them for advice. All these experiences provided Caleb with opportunities to develop his own professional knowledge.

Opportunity for Learning: Time Management and Pacing

Over the two semesters of implementing the curriculum, pacing remained a concern for Caleb as he planned his lessons. He shared this concern with his colleagues

and constantly compared his pacing with his peers. During an interview in the second semester he shared that staying on track was important to him. He said, "It's always timing. You're always worried about how long you're gonna let the mini lesson go, how safe you feel that they can get the pieces, that you're not covering in the lesson, in the investigation problems." (Interview 2, Fall 2016) He also shared that during the second semester he felt like he had more control on the pacing. In his own words:

> Time! Just making sure I can cover the material in a timely fashion and not feel so rushed. I was catching up a lot last semester, now I feel like okay I have a handle on this. This is where they want you to be at, yeah you're gonna fall behind but don't, you're gonna cover it don't worry about it. I think last semester... towards the middle and end I felt more comfortable. (Interview 1, Spring 2017)

His PLC comments in the first semester focused more on his inability to stay on track and him sharing his concerns. For example, "I'm up to 4.3 on Thursday. I could not fit it in today." (PLC meeting, 10-18-2016). "I'm still in 6.1." (PLC meeting, 11-8-2016). "I got through 7.4 and 7.5 today." (PLC meeting, 11-29-2017). By the end of the first semester and during the second semester, still concerned with pacing, Caleb would compare his progress with his colleagues by sharing, "I am finishing 4.8, 4.9, and 5.1 tomorrow." (PLC meeting, 2-27-2017) or "the week of the 27th I am giving the exam." (PLC meeting, 3-13-2017). However, by this time Caleb had a sense of estimating the pacing of his class. For example he shared, "I won't teach mod 7 until April." (PLC meeting, 3-13-2017). Caleb also felt more in control of the pacing of his class and how to use the curriculum to better inform his planning and enactment. For example, during the second

semester, in the PLC meetings he would ask questions like, "Where's the tortoise and the hare problem?" (PLC meeting, 1-23-2017) or "What's good to cover in 3.1-3.3 because I am up to that on Thursday." (PLC meeting, 1-23-2017)

By the end of the second semester, Caleb explained that pacing remained a concern for him because he wanted to balance his responsibility as a teacher to ensure his students were prepared for Calculus. He said, "I feel like, just no matter what happens, each semester, it's just so much that has to get done." (Interview 2, Spring 2017) Caleb wanted to ensure that his students were well prepared to succeed not only in his class but also in the following math courses that they would take. He said, "Time has always been a challenge, to make sure you cover all the material that is necessary for students to have before they go to Calculus." (Interview 2, Spring 2017) As early as the end of the first semester, he shared that he had learned how to better select the problems and improved how he planned his lessons:

> At first it (*Pathways*) was challenging because it was something new so timing was a problem. Not finishing the investigations I wanted to finish in one class setting that I wanted to finish. But once I got hang of the language, the mentality of the, of how the program is set-up it became much stronger for me to really get through the content as the course kept going. You still fall behind but not as much as I used to fall behind in the opening of the course. (Interview 2, Fall 2016)

His comfort with his planning also appeared in some of his comments in the second semester PLC meetings. Caleb was still concerned about his students but was also willing to give some of the responsibility for their own learning to them. For example,

when sharing his plans for review before an upcoming plan he stated, "I am still behind because of snow... I am finishing 6.2 and 6.3 on Tuesday... I am thinking about Thursday is review... I am afraid they will not have enough trig due to the loss of 2 days." (PLC meeting, 3-27-2017) or "finish 6.2 and 6.3 tomorrow, review Thursday, and exam next Tuesday... idk tho... yea I am going in prepared for Thursday as the exam... sometimes they gotta get it on their own... I would teach everything by tomorrow tho... so they have the content." (PLC meeting, 3-27-2017)

Balancing pacing of the course with allowing ample time for the students to investigate concepts in class was representative of challenges that Caleb faced when implementing the new curriculum. Caleb shared that he understood the importance of having his students investigate the ideas in class and have meaningful discussions that would lead to deeper conceptual understanding. He also wanted to make sure they were well prepared to succeed in Calculus. To Caleb this meant, having the foundational knowledge they would need to build higher level concepts on. According to Caleb, figuring out how to achieve a balance between the two goals was the challenge.

Through collaboration with colleagues, Caleb reported being able to seek advice and learn from their experiences. Caleb reported that this engagement helped him to be mindful when planning his lessons and when enacting them in his classrooms.

Opportunity for Learning: Developing a Vision of the Curriculum

Through his engagement with the curriculum Caleb reported that he began to develop a vision of the curriculum. Caleb explained that this vision included a larger picture of the goals of the curriculum, the reasoning behind representation and

organization of concepts and the benefits of the curriculum for his students. He shared that as he started to engage with the curriculum this vision continued to develop. This development started from his participation in the summer workshop, through teaching Precalculus using the new curriculum over two semesters. The summer workshop had given the instructors an opportunity to engage with the investigations and get a sense of the problems. The workshop facilitators guided the instructors about their students' needs, the challenges that might arise during implementation, and the big ideas to be mindful about during the planning and enacting of the curriculum. The summer workshop gave them a foundation, which the instructors could further develop through their own experience and knowledge. They had the opportunity to develop their own vision of the curriculum and the best way to implement it in order to benefit their students. Caleb shared that he maintained some concerns about the curriculum in that it did not "spend enough time on things" that he thought were needed for Calculus. By things he meant preparing students for the procedural fluency that he perceived as necessary for students to be successful in future mathematics courses. He also expressed his concern that many of the problems required students to have a solid pre-requisite knowledge of mathematical concepts. He shared that overall he did recognize the benefit of the curriculum for his students. First of all, he appreciated that his engagement with the curriculum allowed him, as a teacher, to see a connection between Precalculus concepts and those studied in Calculus classes. He stated:

> I do like the fact that it opened up our eyes about things that are coming up in Calculus because I felt a lot of time, Precalc is a complicated class because you're

> covering a lot of content and... I always saw it as a prelude to Calculus but not this semester. I saw it really as being a serious prelude to Calculus. (Interview 2, Fall 2016)

Caleb shared that he was able to see a bigger picture of how the concepts were connected. He had taught Precalculus before and saw the goals of a Precalculus course to provide pre-requisite knowledge for Calculus. Caleb shared that engagement with Pathways, allowed him to see how the students were conceptually being prepared with the foundational knowledge that they would need to understand the concepts in Calculus. Experiencing a new perspective allowed Caleb to appreciate the benefits of the new curriculum for his students in preparing them for calculus. Comparing the new curriculum to traditional curricula he had used before Caleb said:

> [Before Pathways] We never talked about concavity, we never talked about point of inflection, that kind of stuff, and we [Him and the students] do a lot of the conversation here, not necessarily all the calculus behind it but all the conversations so when the students get to calculus they can connect with it (Interview 2, Fall 2016).

As can be seen in the statement above, Caleb was attending to the importance of a Precalculus concept, such as concavity, for his students' eventual success in calculus.

Caleb, was especially impressed by the approach the curriculum took in introducing the concepts. The curriculum provided a structure for gradual development of new ideas in connection to concepts the students had learned. As Caleb engaged with the curriculum, especially through planning, he reported that he was able to see how the ideas

were connected to each other and how the problems in the curriculum supported students' understanding of simpler to more complicated mathematical concepts. He said that instead of telling the students to "do one thing because they get to calculus they need to use this in the quotient rule" (Interview 2, Spring 2017), the curriculum leads them towards those concepts, "not necessarily saying that, but getting us there to do that!" (Interview 2, Spring 2017)

In addition, Caleb found the representation of concepts novel. For example, he talked about the problems on transformation of functions that defined circular motion and said, "Oh! The pedal! The bike pedal and the wheel, understanding what the argument was, that was, I was like that's what that means! Never saw it in that context ever!" (Interview 2, Fall 2016) He shared that he could notice how the concepts were connected to higher order mathematical concepts, he saw the novelty in how some of the concepts were represented and also paid attention to the curriculum's approach to introducing concepts to the students where the ideas emerged as students investigated the problems.

In conclusion, Caleb's opportunities to learn occurred as he faced challenges that unsettled his existing knowledge and practice. As he planned his lessons and enacted them inside the classroom, the challenges provided opportunities for his learning. He discussed his challenges with his colleagues, drawing on them as a resource to get ideas and to learn from their experiences. The challenges also provided a space for Caleb to reflect on his knowledge of the content and teaching. That is, Caleb attended to precision of language, facilitation of classroom discourse, pacing and a vision of the curriculum. Caleb was able to avail these opportunities to learn, as evident from accounts of his own

experiences, his observed classrooms and his conversations with his colleagues during the online meetings.

Michael's Engagement – Searching for Ways to Help the Students!

"I taught in the way that [Summer workshop facilitator] was talking about when I'm up in front of the classroom, I do engage my students all the time." (Interview 1, Fall 2016)

Michael's engagement with the curriculum was influenced by a desire to help his students. He wanted his students to have an effective learning experience in his class and for them to be prepared for future Mathematics courses. As he engaged with the curriculum, he tried to find ways to fix the immediate problems his students were experiencing.

He had prior experience teaching Precalculus at the college level but it was his first time implementing a research-based mathematics curriculum. He actively participated in the summer workshop and shared that he felt confident about implementing the new curriculum. He appreciated the summer workshop because it provided an overview of the curriculum. As an adjunct instructor, he felt supported as he started the new curriculum, this support was contrary to his past experience teaching as an adjunct instructor.

Contextualizing Michael's Engagement

In this section, I will provide some information about Michael's experiences that will help set the stage for reading about his engagement with the curriculum.

Supports

Participating in the summer workshop allowed Michael to become familiar with the goals of the new curriculum for student learning and to receive suggestions for his teaching practice. In the past he had never experienced any type of support for

implementing a curriculum. This was his first time receiving a pacing schedule and having access to a course coordinator to ask questions about the course. He shared, "So what was beneficial, was having the workshops there to anticipate what was coming... this is what you're supposed to cover, this is how you're gonna teach it... rather than being thrown into something... knowing what's coming was nice." (Interview 1, Fall 2016) He shared that he appreciated the supports that he received especially the summer workshop.

Perception of the New Curriculum

According to Michael, the new curriculum was different from curricula that he had used before, and implementing it would be a new experience for him as well as for his students. He explained that he liked the new curriculum and shared that he found the focus on conceptual understanding of mathematical meanings, to be beneficial for his students. He said that this focus was, "Good for them [students] for the future because in Calculus and in later courses they're gonna need to have that conceptual way of thinking rather than well plug and chug... you have to actually think about your answers." (Interview 2, Fall 2016) Engagement with the curriculum during summer workshop gave him an overview of the new curriculum and the weekly online meetings provided guidance and support in implementing the curriculum.

Michael shared that he was aware of the benefits of the curriculum but he also had some concerns about the emphasis on conceptual understanding. At the beginning of the first semester he shared that his students were struggling with the investigations. Michael attributed this difficulty to his students' prior experiences in a mathematics classroom.

His perception was that students were used to a classroom that required them to master procedures instead of developing conceptual understanding. As he started to implement the curriculum he shared that he had some idea about the challenges he would face. For Michael his collaboration with his colleagues played a big role in supporting him. Speaking about the online meetings, Michael said:

> I loved the weekly meetings with [facilitator], it really helped solidify what we would teach and it gave us more of a set well, we're gonna go through this, we're gonna go through this, be sure to mention this, be sure to mention that. So it made it feel like, a lot of sense to get everybody on the same page so the weekly meetings were my favorite I have to say. (Interview 1, Spring 2017)

According to Michael, his collaboration with colleagues continued to be a form of support for him over the course of the two semesters.

Challenges

In terms of challenges implementing the curriculum, Michael's initial challenge was learning the language used in it and paying attention to the precision of the terms used. He shared that in the past he had taught using traditional textbooks and the language used in those textbooks, influenced the mathematical language he used in class. His engagement with the curriculum during the summer workshop had given him the opportunity to get an overview of the new curriculum. He shared learning about the ways in which the new curriculum was different from the other Precalculus texts, he had used in the past. During the workshop he experienced how the facilitator defined variables and suggested that the instructors encourage their students to develop clear mathematical

meanings. The summer workshop also presented an opportunity for him to learn the language used in the new curriculum. It emphasized clarity in mathematical meanings. According to Michael, he realized that he would have to spend some time to get used to the new curriculum.

During the first semester, Michael reported that his participation in the PLC meetings allowed him to learn the phrasing used to describe mathematical concepts and becoming confident about using the language of the curriculum. He shared that one of his challenges was, "Getting the kids onboard!" (Interview 1, Fall 2016) He reported realizing early on in the first semester that his students were not used to investigating concepts in the classroom, or being attentive to the use of precise language. He shared his thoughts:

> 'Cause they're not used to it, they're not used to this so even while I was talking during my first class they were just looking at me like I'm crazy... Why do I have to define that? I mean it's like understood but I said no it is not understood so just getting them to be onboard at first is going to be a difficult task. (Interview 1, Fall 2016)

According to Michael, familiarizing himself with the new curriculum and getting his students on-board were his main challenges when implementing the new curriculum.

Guiding the Students

He shared his game plan for taking on this challenge, by saying that he would guide the students through the shift from procedural to conceptual based focus of a classroom. In his own words, he explained, "Just baby steps at first is going to make them

more aware of defining variables in word problems later, which leads to solving problems in real life." (Interview 1, Fall 2016) He shared that in his experience with his students, having a guided approach worked in allowing them to understand concepts and develop problem solving practices. He gave an example of his students' lack of precision when asking questions, "Sometimes all they can say to me is I do not understand. What they're asking me! So I'm hoping that by the end of this semester they can understand what we're asking them at least." (Interview 1, Spring 2017) Michael shared that he wanted to help the students by taking them through their difficulties in solving problems and in understanding concepts. He also wanted to help his students in improving their study habits and for them to participate more in class.

Michael's Engagement

Michael's engagement with the curriculum in the form of planning, enacting, collaborating and reflecting, were influenced by his students' learning needs. As he faced the challenges that arose during the two semesters, these challenges provided opportunities for his learning as he tried to come up with solutions. In the following sections I will present Michael's engagement with the curriculum.

Planning

In terms of his planning, Michael shared that he drew upon the curricular resources, like the textbook, the workbook and the presentations, and also reached out to the course coordinator with his questions. Most importantly, he collaborated with his colleagues and drew on them as a resource. He actively participated in the PLC meetings, asking questions, sharing experiences, and assisting other instructors with their questions

and concerns. After over 10 years of working in the same department, he had developed rapport with many of the adjunct instructors and he was comfortable asking them questions.

Planning goals. Michael explained that his goal when planning his lessons was to be prepared before going into his classroom. He shared that he wanted to provide an effective learning experience for his students. This included providing detailed explanations to clearly present mathematical concepts to his students, thoroughly answering their questions and providing additional examples to facilitate their understanding. He reported searching for practice problems for his students to work on during and outside of class. In addition, Michael shared, that when planning, he thought about ways to keep his students motivated so that they continue to work on difficult problems. For example, since solving modeling problems was a concern for his students, he searched for ways to aid them so they would be successful. Like his peers, pacing was a concern for Michael as well. He wanted to plan his lessons so that his schedule stayed on track.

Michael's plans. Michael drew upon several resources to help his planning. During the first semester of implementation, Michael shared that he found the resources provided by the course coordinator, beneficial in his planning. He reported using the pacing guide as a guideline for his planning, in terms of the topics that should be covered in each class and how much time he should spend on each topic. In addition, he shared that he found the course coordinator to be really beneficial because for the first time he now had a go-to person to ask his questions. He had never received anything more than a

syllabus and a suggested textbook to guide him. He went to the course coordinator with his questions and concerns about the course. He also called upon his colleagues to aid his planning. Michael actively participated in the PLC meetings and reached out to his colleagues with his questions. Each week, the PLC meeting previewed problems in upcoming investigations. He shared that participation in the meetings, set the stage for his planning. He was able to plan his lessons based on the main ideas in each investigation, by selecting problems that best suited his students' needs. He also created his own assignments for the online homework, selecting problems that he found fitting for his students' learning. He brought his concerns about pacing, student engagement, upcoming assessments to the PLC meetings as well as shared them with his colleagues in the adjunct faculty lounge.

Michael shared that he started each semester with an overall goal for his students. An example of his overarching goal for the semester was to be, "more prepared, to go in with a lot more examples and a little bit more depth for them." (Interview 2, Spring 2017) He shared that he was better able to achieve this goal during his second semester of implementation because he had taught the course earlier and was aware of the resources that were available to him. He described his experience by saying that, "This semester I think I worked on that, only because like I taught it once before so I knew like where to look for, key ideas where to look for maybe you need another example here, there." (Interview 2, Spring 2017) He also got a better grasp of the reasons behind some of the representations in the new curriculum and the ideology behind it. Michael shared that being aware of all the resources that were available to him when planning allowed him to

feel confident. According to Michael, this confidence with his lesson planning paralleled his increased comfort enacting the curriculum inside him classroom, he shared, "In the classroom I knew how to put in those extra examples for them which was also nice. Teaching it the second time around was definitely a lot easier, and a lot more pleasurable than teaching the first time around." (Interview 2, Spring 2017)

His collaboration with colleagues and his own experiencing implementing the curriculum supported Michael in planning his instruction.

Enacting

In this section I describe Michael's engagement with the curriculum in the form of enactment. His engagement in the form of enacting was influenced by the summer workshop, collaboration with his colleagues but most importantly the problems faced by his students and his efforts to help them.

Influence of the summer workshop. Michael had experience teaching Precalculus but shared that enacting the curriculum initially required some getting used to. During the summer workshop, he engaged with the curriculum as a student as well as a teacher. He went through the problems in the investigations, solving them and discussing them with other instructors. The workshop facilitator modeled the investigative approach that is conducive to student learning when using the curriculum. The facilitator suggested that the instructors probe their students to share their meanings. In addition, they provided suggestions on how to foster discourse in the classroom, and suggested phrasing to clearly represent mathematical concepts. Engagement with the summer workshop provided an opportunity for Michael to see that the curriculum

emphasized students' conceptual understanding and that it presented mathematical ideas in a way that was different from his past experience teaching Precalculus.

During the summer workshop, the facilitators also emphasized the importance of student discourse to understand mathematical concepts. The idea behind promoting classroom discourse was that when students explained their thinking to each other, it not only gave the students an insight into each other's thinking but also allowed the teacher to get a better idea of what their students were learning.

Influence of collaboration. According to Michael, during both the semesters, collaborating with his colleagues helped him in enacting the curriculum. During the PLC meetings, the facilitator emphasized the big ideas of the curriculum and used the preferred phrasing for presenting mathematical ideas. In addition, the instructors had a chance to share the teaching techniques they used in their classes and the difficulties they faced as they implemented the curriculum in their classrooms. They also shared their students' responses, their own concerns with pacing and also asked questions that the group would discuss. Michael shared that he would recall the discussions from the online meetings while he was teaching in class. (Interview 1, Spring 2017)

He shared that discussing problems to investigate in the next class, suggestions from instructors and the facilitator, and instructors' classroom experiences were beneficial for him when he was in his classroom with his students.

Michael's perception of his own teaching. According to Michael, the pedagogical suggestions by the summer workshop facilitator, were already aligned with his practice. Describing his own teaching, Michael shared that his teaching style was

engaging for the students. He shared being able to connect to the summer workshop and saw that his teaching style was similar to what was suggested in the workshop. Speaking about his teaching he said:

> I mean I taught in the way that [facilitator] was talking about when I'm up in front of the classroom, I do engage my students all the time. It's not something that's new to me, just because I think that when they're engaged they learn the material better. (Interview 1, Fall 2016)

According to Michael his teaching practice was engaging for the students and that he encouraged student discourse in the classroom. Michael perceived his students' engagement, as them talking to other students in the class or with the teacher. This interaction included, students responding to the teachers' direct questions or teacher responding to a student's question. According to Michael his teaching practice already encouraged his students to communicate in class.

Contrast between perceived and observed teaching. Describing his own teaching Michael said, "The way [summer workshop facilitator] was talking about, is not very peculiar to me it's very normal in my classroom." (Interview 1, Fall 2016) However, in his classroom, student responses were often a result of prompts from the teacher. Even when students shared ideas, Michael guided their conversations. For example, he shared an example of discourse in his classroom, when teaching about translating the sine wave in the context of turning wheels of a bicycle. One of his students said, "Well it's delayed" and another said, "No it came before!", Michael interjected, "No when you're delayed

what happens do you come before or after?" and the students said , "After." (Interview 2, Fall 2016) According to Michael this was an example of a student led conversation.

Michael used direct instruction as his mode of teaching and in his classrooms, he was observed to be asking questions to guide the students towards the main ideas or to be more specific. For example, here's a segment from his classroom (Observation 2, Fall 2016):

1	Michael:	So, like in section 7.5, you look at number 4. It says the balls
2		travels 2 radians per second. The ball was traveling faster than
3		when it was in example 3 where it was traveling at 1 radian per
4		second. **What is faster, one radian per second or two radians**
5		**per second?**
6	Student:	2 radians per second.
7	Michael:	So, **how many sine waves you expect to see between 0 and 2π**
8		**going twice as fast?**
9	Student:	It's going twice as fast.
10	Michael:	So, **you are going to see how many sine waves?**
11	Student:	Two sine waves.

This excerpts presents a typical interaction that took place in Michael's class. He used questions to have his students identify information that he could use to take the lesson forward (Lines 4, 7, 10).

Overall, having the students engage in discussions was a challenge for Michael. As observed in his classrooms, he had a good rapport with the students and they seemed

comfortable talking to him and asking him questions. When it came to having the class engage in discussions, that was often a difficult task. In Michael's observed lessons, he first introduced the main ideas for the lesson and then encouraged the students to talk to each other and work on problems on their own or with a partner. In order to encourage them to talk he would walk around to see student work, hear their conversations and talk to them. Sometimes the size of the classroom or the set-up of the classroom would be a challenge for him to walk around and he would not be able to get to all the students.

Michael reasoned that the students did not engage in discussions because of the conceptual nature of the course. According to Michael, his experience in his other classes was different when compared to his Pathways classes. He shared that his students would talk more in other classes compared to his students in the Pathways classrooms. Michael conjectured, that for his students the focus on understanding the concepts instead of learning procedures was demotivating. Talking about his experiences he shared, "I'm more used to a vocal classroom, we talk, we're a little rowdy, I sometimes have to calm them down when we're in class because you know they get a little bit too outspoken but this didn't happen." (Interview 2, Fall 2016) He explained that his students' reluctance to share their thoughts was because they were not confident about their own ideas. They were hesitant to share their thoughts with the class because they did not want their answers to be incorrect.

He introduced the procedural component of teaching mathematics by adding side notes on the board that his students could refer to while solving problems. As observed in his classes, the side notes included big ideas from the lesson, mathematical formulas, and

any pre-requisite knowledge he deemed important for the students. Michael explained that he did this to help his students gain some confidence. However he shared that when it came to solving problems on their own, even setting up the problem was a challenge for his students. He said:

> It's still the same challenge of them being able to understand word problems and then take away like an equation or a model for it... on their own. When I do it, they're like oh that was easy! But on their own I feel like it's so hard for them to see that. (Interview 1, Spring 2017)

Michael shared his confusion about why his students were not becoming the independent problem solvers he wanted them to become.

Collaborating

Staying in touch with his colleagues was a form of support for Michael throughout the semester. Michael had built a rapport with many of the returning instructors so he could reach out to them if needed. He shared that he would meet informally with the instructors during their shared office hours. Given the changing schedules of the instructors it was not possible to work with the same instructor every semester. Since shared office hours were the most convenient meeting time, there were semesters where Michael did not have the opportunity to meet any of his fellow instructors face-to-face. In this regard, he welcomed the online PLC meetings. Michael shared that these meetings were his first experience of formally meeting his colleagues. He engaged in the online meetings actively and contributed insightful comments to the conversations.

Informal collaboration. During the informal meetings with his fellow instructors, the discussions focused on their classroom experiences, asking questions about specific problems or investigations and techniques for delivering instruction. In the past he had shared, office hours with several of the instructors (like Caleb), who were teaching Precalculus and had developed a rapport with them. He felt comfortable reaching out to these instructors via email, text messages or phone calls to ask them questions. Michael explained that if he and a colleague were teaching the same course, it was valuable to share their experiences and draw on them to improve their own teaching. Describing these informal meetings he shared that, "It gives us a chance to talk about classes that we have in common and the do's and don'ts that they did during the semester." (Interview 1, Spring 2017) Michael shared that as a result of these conversations, he was able to change the pacing of his classes, if needed, get ideas to improve his students' learning experience inside the classroom by stressing certain ideas and avoiding situations that were not fruitful for the class.

Michael also observed a colleague while they taught the new Precalculus curriculum. He shared that he was open to the idea of classroom observations but that he wanted these observations to be informal, so it didn't feel like he was obligated to do them. He described that when he observed a colleague it was only for a short duration, just to get an idea of what their class looks like. He said that it was beneficial for him because observing his colleagues helped him build confidence. For example, he explained that if his students were quiet during class it wasn't just him. Sometimes other instructors also struggled with having their students engage in conversations. He shared

his experience when observing another instructor's class, and said that, "I saw that his [class] was a little bit on the quieter side also and I was like okay maybe it's not me!" (Interview 1, Spring 2017). Michael shared that while he found them useful, he wanted the observations to be informal so no one feels stressed to prepare a special lesson. He wanted to maintain the camaraderie with his colleagues.

Formal collaboration. The online PLC meetings provided another venue for collaboration. In the online PLC meetings, guidance from the facilitator was combined with a chance for Michael to collaborate with his colleagues, even the ones he did not have similar office hours with. This virtual collaboration with colleagues offered him the support to implement the new curriculum. He shared his past experience when teaching Precalculus:

> I already know everything so you just go into your classroom and you would teach whatever was supposed to be taught that day so you're doing you know sines, cosines and tangents in one day and you just go about your lesson and teaching sines, cosines and tangents without much thought about the process. (Interview 2, Spring 2017)

Michael shared that in the past he did not feel the need to discuss the actual teaching with his colleagues but with the new curriculum there was a need to do so. He explained that in the past, teaching Precalculus was a matter of teaching mathematics to his students as he had taught many times before. He shared that he felt comfortable with the content and knew how to present it to the students, provide examples and answer their questions. He reasoned that with the new curriculum, things were different because the curriculum did

not present mathematical ideas the same way as the textbooks that he had used in the past. He explained that even coming up with a new example in class required some additional thinking instead of just pulling an example off the top of their heads. There was a need for the instructors to discuss their experiences and share ideas about teaching the new curriculum, but with this need also came the opportunity through the PLC to collaborate.

Benefits of collaboration with colleagues. According to Michael, his collaboration with his colleagues allowed him to plan his lessons and he supplemented the facilitator's recommendations with his colleagues' reflections about their classroom experiences. Michael explained that, of the problems discussed in the online meetings, if some colleagues found them to be not that effective, he would select different problems for his own class based on the colleagues' suggestions (Interview 2, Spring 2017). He shared that one time during a PLC session, "Somebody suggested doing a table or a graph instead... of what they had in the book... for them [students] to visualize it better." (Interview 1, Spring 2017) Michael reported integrating these suggestions into his planning and enacting of the curriculum. He shared:

> When I'm in class I remember those moments during the PLC, I'll be like oh they [other instructors in the PLC] said that, they did this here, or they said that there or that helped them here you know, so it does! It does ring with me when I'm teaching the class. (Interview 1, Spring 2017)

Michael shared that he would recall the conversations from the weekly meetings and incorporate ideas as he saw fit for his own students. During the PLC meetings the

instructors shared the benefits of certain teaching practices and how they impacted their students, Michael shared that he would try these practices in his own classroom and often found them beneficial.

Challenges of virtual collaboration. Even though Michael shared that he found the PLC meetings supportive, he also reported facing some challenges. In particular, this mode of collaboration was not easy to adapt to because of the online platform. The online platform required the instructors as participants of the PLC, to type their questions and comments in a chat window. Michael reflected that typing was a challenge when communicating during a PLC meeting because, "Sometimes the thought goes away before you finish typing your sentence." (Interview 1, Spring 2017) While it was nice that they were collaborating as a group, it was an adjustment to read, think and type at the same time. As he shared, "You know the reading part of course is quick... but it's like getting your point across sometimes, trying to type quickly before the, before the conversation goes away, moves on." (Interview 1, Spring 2017) This was specifically more challenging when typing out equations. Regardless of the challenges with the online platform Michael was overall the most active participant in the PLC meetings.

The two types of collaboration, formal and informal influenced each other so that there was an exchange of ideas that continued from one format to the other. The conversations during PLC meetings also fueled conversations outside of the meetings in the informal settings. For example, Michael shared that he would speak with the instructors about upcoming exams, to get a sense of agreement about what needs to be changed, what can be improved to ensure the assessments were fair and efficient. When

comparing his experience in the online meetings versus his informal interactions with the colleagues, he shared that:

> When I'm in the PLC it covers conceptual things so we'll go over the problems and you know this part of the problem would be nice but when I'm talking to a colleague we don't have the books in front of us we're not talking about specific problems we're talking about broader ideas like what was covered, what didn't work in class. (Interview 2, Spring 2017)

He was active in helping his colleagues and was often seen in the online meetings, willing to offer help. During an interview in the second semester, he mentioned that he was able to help a colleague set up online homework. He shared:

> I was helping Caleb with the online homework... So actually he was home and I was home but we were able to connect computers through mutual software [Free software Michael was familiar with, not part of the PLC platform]... I was able to log into his computer and I was able to move the mouse on his computer... so he could follow along and take notes and stuff and he was able to see what I was doing. (Interview 2, Fall 2016)

Michael shared that he found ways to collaborate with his colleagues and his network of instructors supported him through implementing the new curriculum.

Reflecting

In addition to getting support from his colleagues, Michael shared that he reflected on his practice in order to learn from his own mistakes. He explained that once he went through one semester of implementation he was able to gauge the problems as

being beneficial for his students or not, and this reflection helped his planning for the next semester. He was able to be more selective and design his instruction better by deciding which questions would work best for his students, which questions he could combine or omit to better get the main ideas across to the students.

He also shared that he reflected after class to contemplate if he provided a good learning experience for his students. He explained this experience by saying that, "when the lesson is over, some days I feel like a rock star, other days I feel like a complete failure and I think every teacher had gone through this point where ah that really didn't work." (Interview 2, Spring 2017) Michael shared that he took notes in his workbook as he planned, during and after class to guide his teaching the following semester. According to Michael, improving his teaching practice was a long-term process and he changed his lesson from one semester to the next to improve his instruction. Talking about reflecting on his practice he said, "That's why I have notes all over my book, I don't know if you want to see but, don't do this and do this, cover this don't cover that, so reflection is helpful for semesters to come." (Interview 2, Spring 2017)

In the next sections I provide examples of how Michael's engagement with the curriculum provided opportunities for his learning. As mentioned earlier, opportunities to learn emerge as teachers face challenges when implementing a curriculum. Michael shared that his goal was to ensure that his students were successful in his class and were prepared to be successful in their subsequent courses like Calculus. Majority of the concerns he expressed, were about his students' success in his class and their readiness for Calculus.

Opportunities for Michael's Learning

Teachers use curricula in different ways and the unique ways of engaging with curriculum provides different opportunities for their learning. As teachers implement a new curriculum they may face experiences that challenge their existing practice and knowledge, these are the opportunities for the teachers own learning (Remillard & Bryans, 2004). The ways in which they engage with the curriculum and the decisions they make determine how they learn from these opportunities.

Michael shared that he cared deeply about his students' learning and wanted them to do well in his class. He actively participated in the summer workshop as well as the weekly online meetings. The summer workshop facilitators communicated the importance of questions as a pedagogical tool and their potential to influence student learning. The facilitators of the summer workshop also emphasized the value of discourse in the classroom and the role that teachers can play in asking challenging questions to foster meaningful discussions. According to Michael his teaching was aligned with the curriculum developers' vision and he encouraged student discourse. He shared:

> I think that when they're engaged they learn the material better and I even told them that, you guys are gonna be talking a lot in my class, I don't want you being quiet the way the class is gonna go and I want you to be able to communicate with each other as well as me when there is a problem. You know sometimes I don't explain something so well as the neighbor in your class does. So just having that open communication in the class and having them be more involved in the class is beneficial to the class. (Interview 1, Fall 2016)

Michael started his first semester of implementation with a perception that his teaching practice already supported student discourse in the classroom. As Michael engaged with the curriculum, he shared that he initially struggled through the new representations in it. According to Michael, several challenges appeared when he planned his lessons or during discussions with his colleagues, however his main source of concern was the challenges his students faced.

Michael's Challenges Emerged from his Students' Struggles

Michael's biggest challenges emerged from his students' experiences inside the classroom, especially the challenges that they faced. Enacting the curriculum inside the classroom allowed him to experience not only his own struggles but also his students' struggles with the curriculum. Student challenges that he experienced while enacting the curriculum, provided opportunities for his learning as he tried to resolve them. He expressed his concern about the pushback from students as he asked them to investigate the problems themselves. Facing these challenges provided opportunities for Michael's learning.

The focus on precise mathematical language in the curriculum and the emphasis on carefully defining variables was a challenge for Michael inside the classroom. Especially when trying to get the students onboard. Michael shared, "It's difficult to get my language used to it at first, because we're so used to talking about the terms that the books give us when we're teaching a Precalc course." (Interview1, Fall 2016) Michael was talking about his past experience teaching Precalculus and how he was not used to putting the emphasis on defining the variables as precisely as the new curriculum

suggested. He said, "When we start talking in terms of finding variables and being able to define them specifically, like from the ground to whatever point we're looking at, it's difficult to put my language around it." (Interview 1, Fall 2016) He explained that he was learning how to convey the importance of precision in defining variables to his students. This challenge came through in his teaching as he used the language specific to the curriculum to ask students questions, to show them what the important concepts were, and to be specific in their language. For example, this excerpt from earlier in the first semester (Observation 1, Fall 2016), shows an exchange between Michael and a student regarding defining variables [Module 3: Investigation 5, Problem 2 (see Figure 8)]:

Suppose water is being added to an empty swimming pool at the rate of 7 gallons per minute.

a. Define a function f that determines the number of gallons in the swimming pool in terms of the number of minutes, t, since the water started running into the empty pool.

b. Evaluate $f(6)$ and say what your answer represents.

c. Define the rule for the function f^{-1} that defines the volume (or number of gallons) of water in the pool in terms of the time (or number of minutes) since the pool started filling.

d. Solve $35 = 7t$ and say what your answer represents. Evaluate $f^{-1}(35)$ and say what your answer represents. What do you notice about how "solving an equation for some number b" is related to evaluating $f^{-1}(b)$?

e. Evaluate $f^{-1}(f(9))$ and explain the process you carried out to determine your answer?

f. Evaluate $f(f^{-1}(63))$ and explain the process you carried out to determine your answer—be sure to reference the quantities referenced by the notation?

Figure 8. Module 3: Investigation 5, Problem 2, Student Workbook.

1	Michael:	How am I going to undo $7t$?
2	Student:	with 1 over 7.

Michael:	Not 1 over 7, but *t* over 7. But is it *t*? You can't put *t* there because *t* is time. Our input is now volume, so it gotta be *v* over 7.
Michael:	*v* is volume. I can name it anything. It could be *n* for the number of gallons…we changed the representation of our variables. You have to inverse them.
Student 1:	**I'm confused! Where did we get *v* from?**
Michael:	*v* is volume. I can name it anything. It could be *n* for the number of gallons!
Student 2:	**It was supposed to be *t* over 7, why did we change it?**
Michael:	Because here input is time, here input is volume of water. So you can't put *t* there, you have to put *v*, your input is volume of water not time any more. We changed our representation of our variables. You have to inverse them.
Student 3:	**So how would I say, what's the inverse of new one that we got?**
Michael:	*f* inverse of *v*, is *v* over 7, that's the inverse.
Student 3:	**But what's the inverse of the original, how would I do that? You know how we were going back and forth between *f* of *t* and *f* of *s* or whatever?**
Michael:	Oh what would I name it? What would I name *f* inverse? They didn't give us any, they just said to name *f* inverse. So we just have our function we have our inverse function. Here we also had

		a function and an inverse function but they named the inverse function something else first.
Student 2:		**So would it be incorrect to, for c the inverse, would it be wrong if I did, *f* of *t* as well?**
Michael:		Yes because you can't use *t* anymore. *t* is only representative of time.
Student 2:		Ohhhhh!
Michael:		When you use *t* for time, be consistent with your variable. So if *t* is time, use something else for your volume. Even if you were to state, now *t* is volume it just very confusing. Always use another variable just to minimize that confusion and to make things more explicit.
Michael:		So they could have used *n* here instead of *v*. If they don't give you something to use, you can use the first letter of your name, just make sure it is explicit, and write down that I will now, if I use *m* for first initial of my name then I would use *m* for volume. As long as you are explicit with your variable usage, any variable is okay!

Students struggled with defining variables (Lines 8, 11, 16, 18, 26), and Michael explained to them that it was okay to select any letter as a variable as long as they were careful that it made sense in the context of the problem. As mentioned earlier, Michael's concerns and challenges mostly stemmed from his students' experiences. Sharing his

concerns about his students he said that, "getting the kids onboard" was a challenge for him. (Interview 1, Fall 2016) He explained that he was concerned that the students were not used to defining the variables the way he was asking them to, he said, "While I was talking during my first class they were just looking at me like I'm crazy... Why do I have to define that? I mean it's like understood but I said no it is not understood." (Interview 1, Fall 2016) Michael conjectured that the, "conceptual nature of the course" (Interview 2, Fall 2016) was discouraging the students. He mentioned that students would question, "Why do we have to learn it this way?" (Interview 2, Fall 2016), and he found it challenging to motivate them. He said, "I'm hoping that they understand a little bit more of the modeling... how to come to a conclusion or an answer ... because sometimes all they can say to me is I do not understand what they're asking me." (Interview 1, Spring 2017) Michael shared that he wanted to help his students become proficient at solving conceptually demanding problems. He explained further, that he wanted his students to gradually improve their problem solving skills and wanted to guide them through the process. He said:

> I tell them sometimes, read through it slowly, try to understand 'cause sometimes like it slips me up too, so you have to go back and you have to read it again. Don't just give up and [say] I don't know what they're asking me. Read it again, see what they want from you. Working with them, it's working with them. (Interview 1, Spring 2017)

Michael shared that he was aware of his students' struggles with solving problems on their own and he wanted them to be successful. As mentioned earlier, Michael's

engagement with the curriculum was driven by a search for ways to help his students succeed. His student's challenges with solving problems on their own was a concern for Michael. In the following section I provide Michael's response to this concern.

Opportunity for Learning: Through the Challenge of Developing Independent Problem Solvers

According to Michael, students' challenges with word problems stemmed from their past experience in mathematics, he felt that the students lacked pre-requisite knowledge. He shared, "I don't even feel like... that's from us! I feel like that's just carried with them throughout their whole mathematics education through their lives so... we're gonna struggle with it and I don't think it's gonna be easy." (Interview 1, Spring 2017) He continued to explain that students' experience of learning of mathematics for conceptual understanding for just one semester was not enough to undo the way they had learned mathematics all their lives. He said, "You can't really teach that logic in one semester, you know some kids see it and some kids don't." (Interview 1, Spring 2017) Michael perceived his students' struggles with problem solving as a problem that developed over a long time and would require more time than just a semester to be fixed.

Use of direct instruction and questioning. Michael used direct instruction to ensure that his students saw all the nuances of a problem. He shared that he was mindful of his students' difficulties with understanding what the problems were asking and would structure his instruction to make them see what they were missing. For example, the following excerpt from his classroom (Observation 2, Fall 2016) shows Michael breaking down the problem for his students [Module 7, Investigation 6, Problem 1 (see Figure 9)]:

A Ferris wheel has a radius of 52 feet and the horizontal diameter is located 58 feet above the ground.

a. If the Ferris wheel rotates at a constant rate of ¼ radian per minute, how many minutes does it take for the Ferris wheel to make one full revolution?

b. Define a function g to represent the distance (in feet) of a Ferris wheel bucket above the *ground* as a function of the number of seconds, t, since the Ferris wheel began rotating from the 3 o'clock position. Assume the Ferris wheel rotates at a constant rate of ¼ radian per minute.

c. Suppose the Ferris wheel rotates at a constant rate of 3 radians per minute. How many minutes will it take for the Ferris wheel to make one full revolution?

d. What might the function $h(t) = \sin(2t)$ represent in this situation? (Hint: Consider the meaning of the value of $2t$.)

Figure 9. Module 7, Investigation 6, Problem 1, Student Workbook.

Michael: **What is the amplitude going to be? What did I say the amplitude corresponded to? How high the sine wave was, right? But off our circle, what value was that?**

Student: 52.

Michael: **52 Right. The radius. So, 52 and we are looking at the Ferris wheel above the ground. So, is that the horizontal or vertical distance?**

Student: Vertical.

Michael: **Vertical distance and which function did we use to measure vertical distance?**

Student: Sine.

12 Michael: **Sine! So, we have sine. We are looking at the function in terms**
13 **of time, are we going to use ¼ t here or are we gonna use**
14 **theta?**
15 Student: ¼ t.
16 Michael: We going to use ¼ t and we are looking at a function in terms of t.

This was a common occurrence in all of Michael's observed classrooms, where Michael broke down the problems for his students (Lines 1, 5, 9 & 12). He said, "You have to explain it to them the first time hoping that the next time they see it, they're gonna get it." (Interview 1, Spring 2017) Michael shared that he wanted to explain all the main ideas as well as go over the language of the problems in the classroom because seeing all the pieces once would allow the students to better understand it when they do the problems on their own.

Michael used the questioning as a way to try and guide his students through the big ideas that he intended for them to learn in that class. For example, in a classroom observation (Observation 1, Fall 2016), the following episode took place [Module: 3, Investigation: 5, Problem: 1 (see Figure 10)]:

The *formula* that determines the perimeter of a square p (in inches) in terms of the length of the side of a square s (in inches) is $p = 4s$.

a. Define a *function f* that determines the perimeter of a square $f(s)$ in terms of the squares side length (or when the square's side length s is known).

b. Write a *formula* that determines a square's side length (in inches) in terms of the square's perimeter (in inches) (or when the square's perimeter is known).

c. Define a function h that determines a square's side length $h(p)$ in terms of the square's perimeter, (or when the square's perimeter is known).

d. Use function composition to evaluate
i) $h(f(2.5)$ ii) $f(h(10))$ iii) $h(f(7))$ iv) $f(h(28))$

e. How are the functions, f and h related? How are the input (independent) and output (dependent) quantities of f and g related?

Figure 10. Module: 3, Investigation: 5, Problem: 1, Student Workbook.

1	Michael:	Ok. Let's look at Module 3, Investigation 5. We start talking about
2		inverse functions and reversing the process. **Now, the idea of**
3		**function inverse is basically undoing what you have. So, if you**
4		**have x plus 1, to undo x plus 1, you are going to do x minus 1.**
5		**Right? That is the undoing of our function.** So, let's read
6		through number 1. Define a formula that determines the perimeter
7		of a square in terms of the square's side length or when the
8		square's side length is known.
9	Michael:	**How do we find the perimeter of a square? We said the**
10		**perimeter of a square equals...?**
11	Student:	4s.

Michael: 4 times s. The perimeter of a square is 4 times s. So, part A wants you to define the function**. Perimeter equals 4 times s is your formula. That's your formula. The reason why is it called a formula because it doesn't explicitly state what your output is.** When you are defining a function [writes on the board]…did they tell you what to name the function? What did they tell you to name the function? ***f*, so what am I going to put for my function? *f(s)* = 4s. *f(s)* = 4s. What is your input?**

Student: *f(s).*

Michael: **No, your input is s. Input is goes in…that's your input. What is your output?**

Student: *f(s).*

Michael: *f(s)* is your output. Your input in this case is your…? Is it your side length? Is it the perimeter? **What is your input? What do you put in?**

Student: Side length.

Michael: The sides, s. Your input is length of the sides. **Your output is going to be the...?**

Student: Perimeter.

In this excerpt we see several examples of Michael's use of questioning to guide his students. Michael wanted to ensure that students were aware of the main ideas that he

wanted them to be mindful of that. For example, he pointed out the difference between a function and a formula to them (Line 13) and discussed the big idea of an inverse function (Line 2). He also wanted the students to participate in the lesson as he wanted to make it a discourse based lesson. However, his use of questions in this regard was mostly to drive the lesson, making his lessons teacher led for the most part. He would ask leading questions (Lines, 9, 18, 21, 25, 28) and take any student's response to move forward with his explanation.

As observed in his classrooms, Michael took his students answers as a que to move on with the lesson. He also seldom asked clarifying questions to get a sense of his students' understanding of the concepts. Given his use of questioning as a pedagogical technique and his stated interest in improving his students' problem solving skills, it may have helped him to dive further into improving his own questioning technique. He wondered why his students had a difficult time in solving problems independently. When asking his students questions in class, Michael had the opportunity to actually use his questions to get a sense of his students' understanding. However, he only used his questions and student responses to continue with the lesson. This practice continued by the second semester of implementation. In the following excerpt from his classroom (Observation 1, he guided the class through a problem [Module: 4, Investigation: 1, Problem 15 (see Figure 11)]:

Imagine that Jerry weighed 180 pounds and his weight increased by 5% each year for 4 years.

a. Complete the missing entries in the table.

	product notation	exponential notation	decimal notation
starting weight (lbs)	180	180	180
weight after 1 year (lbs)	180(1.05)		189
weight after 2 years (lbs)		$180(1.05)^2$	
weight after 3 years (lbs)	180(1.05)(1.05)(1.05)		
weight after 4 years (lbs)			218.79

b. During this four-year period, did Jerry's weight change by the same number of pounds per year? Why or why not?

c. If n represents the number of years since Jerry weighed 180 pounds and w represents his weight in pounds, write a formula that represents w in terms of n.

d. After 4 years, what was Jerry's ending weight gain in pounds? What is this change as a percentage of his starting weight?

e. After 4 years, Jerry's weight is what percent of his starting weight?

Figure 11. Module: 4, Investigation: 1, Problem 15, Student Workbook.

Michael: So it says imagine that Jerry weighed 180 pounds and his weight increased by 5 percent each year for four years... He is gaining 5 percent each year. So his starting weight is 180. Exponent notation is... **if you look on year 2, you have that exponent because... 1.05 times 1.05 will give you 1.05 squared.** So why don't you go through the table on your own and we'll discuss it when you finish. Go through the table on your own and we'll discuss it when you finish. **Why is it times 1.05? Why isn't it times 0.5?**

Student: Initial weight plus 5 percent.

Michael: **Plus 5 percent of his initial weight right which is 180 times 1 plus 0.5 [180 * 1 + 0.5] right, and that's 180 times 1.05 [180 ***

1.05]. So that one accounts for his initial right, because he's gaining weight, it's 1.05. If he was losing 5 percent per year what would happen here? Would you have 1.05, would you have 1 point something or would you have less than 1?

Student: Less than one.

Michael: **It's gonna be less than one because you're subtracting the 0.05, it will be 0.95, if you were losing that 5 percent but since he's gaining that 5 percent you need to account for your initial by using that one there. When you call that the 180 when you're doing the algebraic process.** So go ahead and do that table and I'll stop talking. [Michael walked around and talked to various students while they worked on the problem. After about 3 minutes he asked the students to move on to the next part of the problem.]

Michael: Part b wants you to think about the changes between the years. So what do you notice about the changes between the years? If you can see the graph nice right. What do you observe about the changes between the years? Yeah?

Student: Progressively as the years start to increase the weight level also expands.

Michael: **As time goes on the changes increase. As time increases, the changes between two consecutive years increase as well.** [He

took this opportunity to expand on the idea of varying rate of change and provided an explanation to the class.]

Michael: **So as the years go on, his weight increases right but taking 5% of his new weight every year, and if his weight increases, so if you take 5 % of 180 and 5% of 189, those 5%s are going to be different. 5% of 189 is going to be larger than 5% of 180. 5% of 198 is larger than 5% of 189. So your changes will also increase. So for part b, the answer is no! Because if it was constant, what would that tell you? What would it tell you if it was constant? It would tell you that it increased by a certain number of pounds every year not a percentage, but since the increase is by a percentage every year then that means that you're taking the percentage of the new number which is always going to be larger because it's an increase. So the answer is no! 5% change is always measuring an increase relative to the weight one year in the past.**

Michael had the opportunity to check for his students' understanding but his focus was on guiding his students through the investigation. He asked questions to check if the students were understanding the different parts of the problem correctly (Line 8) but used one student's response as a representative response for the class' understanding or answer the questions himself (Lines 10 & 17). He also made sure he provided relevant information

about the problem at hand so the students would be able to solve it (Lines 4 & 17). There were times where Michael assumed his student's meanings when they answered his question. (Line 31). He had the opportunity to ask a clarifying question to ensure there was a connection between what he thought the student said and what the students actually said. The structure of the problems provided opportunity for students' exploration, but Michael opted for guiding them and providing explanations (Line 35) instead of allowing them to explore the ideas themselves. He led them through the problems, finally giving them the final answer before moving on to the next part of the problem. This episode gives an example of an opportunity for learning that was missed. As Michael enacted the curriculum inside the classroom he had several opportunities to improve his own teaching practice but sometimes he was not able to recognize or avail of them.

Michael shared that he wanted his students to become independent problem solvers but despite his efforts, his students' problem solving remained a challenge for him. Michael provided guidance for his students by providing direct instruction and going over the main ideas of the problems solved in class. He solved the problems on the board, wrote main ideas and formulas on the side of the board and encouraged his students to take notes, but he realized that his students still struggled to solve the problems on their own. He shared, that while the students were using the side notes when solving word problems, it was not as effective a tool as he had hoped. He explained, "So lately what I've been doing is those side notes to leave the procedural stuff on the side and then for them to refer back to it when they're doing the word problems but when they set up the word problems is where I come across the problem." (Interview 1, Spring

2017) They were not becoming the independent problem solvers he wanted them to become. This was a missed opportunity for Michael's learning. He did not allow his students to engage in independent problem solving. His perception about his teaching practice, combined with what he diagnosed as his students' need (side notes for pre-requisite knowledge and more direct instruction) proved to be a missed opportunity for improving his teaching practice. He shared that perhaps his students needed to struggle with the material in the classroom to experience the challenge of solving problems. He said:

> It's still the same challenge of them being able to understand word problems and then take away like an equation or a model for it... on their own. When I do it, they're like oh that was easy! But on their own I feel like it's so hard for them to see that. (Interview 1, Spring 2017)

He shared his dilemma regarding his students' difficulty with solving problems but did not connect it to any changes in his own teaching practice.

Opportunity for Learning: Through Facing the Challenge of Fostering Classroom Discourse

As mentioned earlier, Michael perceived his teaching practice to be aligned with what the facilitator of the summer workshop had suggested. While enacting the curriculum, Michael shared that he recognized his students' struggles with the curriculum. He explained that when he tried to grasp his students' challenges with the curriculum, he would often come up with reasons that held the students' responsible for their challenges. For example, he said that his students did not study for assessments and

came to class unprepared. That they were not able to gauge their own understanding of the concepts to be able to perform well on assessments. He shared that they did not study for assessments because, "they over estimate what they know. They didn't study! That's probably a factor of why they didn't do so great!" (Interview 1, Spring 2017) Michael's comments during the interview and observation of his teaching practice indicated that he sometimes perceived his students' challenges as being disconnected from his own teaching practice.

Michael described his classes as normally being very talkative and shared his concern about his Precalculus students not being as interactive as students in the other courses that he taught. Michael shared that he was not used to his classes being quiet. He shared:

> I always get my kids involved, I always get the students involved, so I'll go through the material first and then I will have them talk to each other. They were a very quiet bunch. They didn't really want to talk to each other... this class, the 111 [Precalculus], they didn't really want to talk to each other so I had to push them a little bit. (Interview 1, Fall 2016)

He shared his concern about lack of student participation starting from earlier in the first semester and continued till the second semester. Michael hypothesized that the novelty of the new curriculum might be the reason for his students' lack of participation, "I'm more used to a vocal classroom, we talk, we're a little rowdy... but this didn't happen this Precalc class and I feel like maybe because students were a little bit hesitant to give answers they were unsure of themselves." (Interview 2, Fall 2016) According to Michael

one of the challenges that the new curriculum posed was asking the students to think. He explained, "We're making them think and they hate that!... Maybe that's where the quietness come from... They lost their confidence." (Interview 1, Spring 2017) Michael expressed that he wanted greater student participation in the classroom, but figuring how to develop classroom discussions was a challenge for him. He described that when in class, the students would often not respond to his questions with the exception of one or two students. He said:

> Sometimes I'll stand in front of them, they'll laugh at me and I'll just stare at them. I'm like well? Any takers on what to do? And then somebody will finally like raise their hand. I always have at least a few people that are following with me but sometimes I'll tell them no no give somebody else a chance. So I don't know I just struggle with that a little bit. (Interview 1, Spring 2017)

Michael explained that he wanted his students to get involved in the classroom discussions. As observed in his classrooms, Michael's students would sit in groups depending on the set up of the classroom. They would either move their chairs around or stay in their fixed seating, with a chance to have discussions with their neighbors. When he asked them to work on the problems in the student workbook, he gave them the choice to work with a partner or individually, but did not influence their decision if they chose to work individually. In order to get a sense of his students' learning, Michael would walk around the classroom when students would work on the problems. He would look at their work and listen to their conversations while asking questions and guiding them.

In order to encourage student discourse, Michael mentioned that he tried to incorporate student feedback and questions into his lessons. He explained:

> A lot of the time when you are in class, students even bring in suggestions, for example, like with translating the sine wave, and one of my students said well it's delayed and one of my other students said no it came before and I was like no when you're delayed what happens do you come before or after? And they said after after... so things come from students as well. (Interview 2, Fall 2016)

The episode Michael was discussing came from a class (Observation 2, Fall 2016) at the end of the first semester [Module: 7, Investigation 7, Problem 2 (see Figure 12)]:

A bike is on the rack at a bike shop so that the tech can turn the pedal with his hands, causing the bike's back wheel to rotate. On this bike the circular path swept out by the pedal is exactly the same size as the circular path swept out by the tire. Suppose that when the pedal starts at the 3 o'clock position, the tire valve on the wheel is $\frac{\pi}{4}$ radians clockwise from 3 o'clock (in quadrant IV).

a. Draw a picture to represent the initial position of the tire's valve.

b. As the bike's pedal rotates 1 radian counter clockwise (CCW), the tire valve on the bike's back wheel rotates 1 radian counter clockwise (CCW).

Let θ represent the angle measure of the *pedal* CCW from the 3 o'clock position, in radians.

Indicate the tire valve's rotation from the 3 o'clock position for the given values of θ, given that when the pedal is at the 3 o'clock position ($\theta = 0$ radians), the tire valve is $\pi/4$ radians clockwise from 3 o'clock ($-\pi/4$ radians from the 3 o'clock position).

Figure 12. Module: 7, Investigation 7, Problem 2, Student Workbook.

1 Michael: So, it says to sketch a graph next. How do you think the graph is
2 going to look?

3	Student:	One of them is going to be delayed.
4	Michael:	**One of them is going to be delayed. Delayed means the graph**
5		**is going to go...?**
6	Student:	Slower.
7	Michael:	Slower, right! **Slower means that, is it going to come before or**
8		**after on your axes? So if this is, P we said was sin (θ). If this is**
9		**P and the other one is delayed, does that means it is going to**
10		**come before or after? If you are delayed going to the train**
11		**station, do you go before or after everybody else?**
12	Student:	After.
13	Michael:	You go after. So, if you go after, the graph is going to look like
14		this for the second one. And the difference is going to be $\frac{\pi}{4}$. This
15		$T(\theta) = \sin\left(\theta - \frac{\pi}{4}\right)$. It is delayed.
16	Michael:	So, it moved $\frac{\pi}{4}$ units to the right. $T(\theta)$ is $\frac{\pi}{4}$ units to the right.

Michael had shared this episode as an example of incorporating student feedback into his lessons. However, he was asking direct questions and using student responses to continue with the direct instruction. Michael asked his students questions with specific answers in mind that would allow him to make explicit connections between ideas (Lines 4 & 7). Michael shared that asking questions and responding to students' questions was challenging for Michael because he worried about his class getting side-tracked if the

ideas discussed were too broad or not directly connected to the lesson. He explained that it was important for him as a teacher to make decisions about how much time to spend on answering student questions. If a short yes or no answer would suffice, he would make the decision to move on with the lesson without "bogging down the students with too much information... you skip over, you move on a little bit." (Interview 2, Fall 2016) This was another example of an opportunity for improving his own practice that was missed. Michael perceived his teaching practice to be aligned with the effective teaching practices suggested during the summer workshop. Michael did notice that his students in the Precalculus classrooms were not as talkative as in his other classrooms. He attributed the students' quietness to the new curriculum. He made the effort to observe a colleague's classroom who was teaching another section of the same course. He shared that he observed his class for only twenty minutes and found the same level of student interaction and discourse as in his own class. This observation gave him some assurance as he shared:

> When I saw that his class was a little quiet, cause I'm on the quiet side too... and I wasn't sure if it was me or if it was the material or it was just that class... and then I saw that his [class] was a little bit on the quieter side also and I was like okay maybe it's not me. (Interview 1, Spring 2017)

The observation allowed him to compare his own classroom to another instructor's classroom who was teaching the same course. Michael had a rapport with this instructor as both had collaborated in the past while teaching the same courses. Even after the observation, Michael shared that he wondered about the reason for his own students' lack

of participation. His concern for his students' lack of engagement and his desire to help them provided an opportunity for Michael to improve his teaching practice.

Opportunity for Learning: Developing a Vision of the Curriculum

After implementing the curriculum for two semesters, Michael reported that his engagement with the summer workshop, the PLC meetings as well as reading the textbook allowed him to become familiar with the new curriculum. In the beginning, he perceived the teaching and his own learning of the representations to be a challenge. In his words, "I had to learn some of it. So that's one of the challenges." (Interview 2, Semester 2) However, this challenge turned out to be a learning opportunity for him. He shared by the end of the first semester:

> In terms of content, one of the things I learned, about the... quadratic [formula] was that the first part of it was the vertex and the second part was finding the distance between the vertex and the... zeros. Even though I knew that one piece was the vertex and the whole things were the zeros, just that connection between okay that second part is the distance between the vertex and the zeros was mind blowing. So that's one thing that I learned, that really like blew my mind. (Interview 2, Spring 2017)

When he first started implementing the curriculum, he reported being unsure about how different the content in the new curriculum was from what he already knew. As he implemented the curriculum, he shared that he grew confident about his own knowledge and began to make connections between what he already knew and the content presented in the curriculum. He explained that the novelty of the new curriculum made him lose

some confidence about his content knowledge. Initially, during the first semester if he found some errors in the book or the presentation slides when planning, he thought of himself as being wrong. He shared that, "A lot of slides... had mistakes... that made me doubt myself sometimes." (Interview 2, Fall 2016) After gaining back his confidence, he identified the typos in the book as errors on the part of the curriculum and not an error in his content knowledge. He started sharing the typos in his class to guide the students. For example, during an observed lesson he said:

> Ok. So, for part C in your books, it says define the rule for the function f inverse that defines the volume for the number of gallons of water in the pool, in terms of the time. Define the volume in terms of the time. We did that already so that cannot be f inverse. That's a typo! (Observation 1, Fall 2016)

After he had implemented the curriculum for the first semester, he became confident about his content knowledge. During the second semester, he started to bring up these mistakes in the PLC meetings, for example he shared, "Number 2 had the typo in part d I think" (PLC meeting, 1-23-2017) or "Part c was the one with the typo in the book right?" (PLC meeting, 2-6-2017) Over time he shared gaining more confidence about his own content knowledge to be sure that he was correct if he caught an error in the book, instead of doubting himself.

After implementing the curriculum, the first semester, Michael shared that he had gained a broader picture of how the concepts within Precalculus connected to each other, as well as to those concepts covered in Calculus. According to Michael, as a teacher he had an easier time adjusting to the new curriculum than his students, he found that

implementing the curriculum was beneficial for him as a teacher as well. He shared, "We pick up on it [new curriculum] much faster than the students... but getting a lot of the background now, kind of just brings and ties everything together for us as professors as well." (Interview 2, Spring 2017) By tying things together Michael was referring to the mathematical concepts and how they were connected to each other. He reported developing a clearer vision of the curriculum in terms of the big ideas.

As defined earlier, teachers' opportunities for learning occur as they face challenges that unsettle their existing knowledge about content and practice. These challenges give teachers a chance for reflection and improvement. For Michael, these opportunities occurred as he tried to help his struggling students. He wanted to help them become better problem solvers and also to improve student discourse in his classroom. Michael was not able to avail of many of these opportunities because he used his existing repertoire of teaching tools to face the challenges. He did mention developing a vision of the curriculum and learning about the goals of the modules and how the concepts were connected but in terms of improving his pedagogy many of the opportunities for learning were missed.

Justin's Engagement – Grappling with the Problem Solving Perspective

> *"I think it's important that we always, in any type of course, create new material, just because then it will never get stale. I do that with all of my classes. I mean, tests and everything. Everything always is brand new because there is never a shortage of math problems."* (Interview 2, Spring 2017)

Justin's engagement with the curriculum, exhibited a focus on developing his own instructional materials. Justin explained that he cared about his students' learning and wanted them to understand the mathematical concepts they were learning. He perceived the instructional materials themselves as possible tools to facilitate his students' learning. The problems in the curriculum resonated with Justin as he had an inclination towards creating new learning resources for his students.

He was concerned about his lack of experience teaching at the college level. His participation in the summer workshop allowed Justin to get an overview of the new curriculum, what the problems in the investigations looked like, and pedagogical suggestions from the workshop facilitators. As he started to implement the curriculum, new challenges emerged for him. Some were issues like encouraging student discourse, others were concerns about actually implementing the curriculum and engaging his students.

Contextualizing Justin's Engagement

In this section I am providing information that will place Justin's engagement within the context of implementing the new curriculum, his expectations, and his challenges.

Justin's Concern about his Experience

Speaking about his concern towards his lack of experience teaching at the college level, Justin shared, "The first two classes, part of the issue that I was trying to deal with was my nervousness working at this specific level with them you know at KSU." (Interview 1, Fall 2016) Prompted by his inexperience, Justin shared that during the first semester, he talked to one of the other instructors regularly. This instructor had over ten years of experience teaching as an adjunct instructor and like Justin he also taught at a high school. Justin knew him from outside of the university setting and reached out to him for advice. Justin shared that he wanted to get a sense of his responsibilities as an adjunct instructor.

Perception of the New Curriculum

Justin was receptive to the new curriculum and appreciated its focus on developing students' conceptual understanding of Mathematics. According to Justin, the new curriculum allowed the students to explore mathematics and to learn it by investigating the concepts. Justin first experienced the curriculum during the summer workshop and he shared that he was excited about implementing it. He appreciated the problems themselves as well as the investigative nature of the curriculum. He shared, "It [Curriculum] goes immediately to the problem that students have, you know word problems, it doesn't avoid them at all! It embraces them! I'm finding that very useful." (Interview 1, Fall 2016) Justin saw the instructional materials specifically the context based word problems, as effective learning tools for his students.

Justin shared that he even saw the online homework, as a useful resource for their learning, when many other instructors found it challenging for their students. Talking about the online homework system he said that, "the problems were really related to what they were learning and what they were going to see on the exams and stuff, and the fact that the numbers changed and they [students] had access to them for every student who had [made] the initiative. It was an excellent tool and it really just saved a significant amount of time." (Interview 2, Fall 2016) Justin perceived the online homework as another useful resource for the students that they themselves could draw on to aid their own learning.

Supports

Justin was introduced to the investigations in the new curriculum through his participation in the summer workshop. He shared that the summer workshop provided him with an opportunity to learn about the new curriculum and guidance on teaching practices that would help him implement the curriculum. He shared the benefits of the workshop by stating:

> I thought that workshop was essential! I would never have picked up the value that the curriculum was placing on these very simple concepts. So simple concepts you just glance over really quick and don't give it the justice that you should be when you're teaching, and [the workshop facilitator] made it very clear that we should be focusing on the relationship between numbers and comparisons and units and stuff like that, and the real thing is much more than just saying well it's important but really giving an actual reason why. (Interview 1, Fall 2016)

He appreciated the curriculum's focus on helping students develop clear mathematical meanings. Justin shared that he valued having experienced the curriculum during the summer workshop before teaching it to the students so he had a chance to understand what the investigations entailed. He realized that he would have to dive deeper into the mathematical concepts and explain to his students what they meant and why they were important.

Challenges

Justin's experience with the summer workshop also allowed him to become aware of some of the challenges he might face. For example, he shared that teaching a course using a traditional textbook would allow him to make up a problem on the spot and there were many similar problems to choose from in the book as well. In this course however, it would be difficult for him to do so. According to Justin, inside his classroom he found it challenging to get his students engaged. The facilitator of the summer workshop suggested developing student discourse in the classroom. Justin found it challenging to get his students involved and shared, "I think there is a little resistance... we're having kids do work together in a group that really aren't used to that so they naturally fight it!" (Interview 2, Fall 2016) Justin blamed his lack of experience teaching at the college level for his students' lack of engagement in the classroom. He shared that he was spending a section of each class to motivate the students, which was challenging for him, he stated, "They [Students] need to be reminded that if this is challenging then you need to be working on this course more than your other courses." (Interview 1, Spring 2017)

According to Justin, his students were demotivated if they thought the problems were too difficult and he wanted to motivate his students to be persistent.

In addition to motivating students, Justin expressed his concern about the pacing of the course. He shared that he found it challenging to follow the pacing guide and often fell behind schedule. According to Justin, it was a balancing act to decide when it was acceptable to move on to the next problem or investigation and hold the students responsible for their own learning. He shared that he struggled with this challenge, by saying:

> We have so much information... we don't get to necessarily do it to the depth so that these students could use [the information]. You know they see it and they can use it for a bit but they can't you know, they don't own it! (Interview 1, Spring 2017)

Justin expressed that he wanted the students to investigate the mathematical concepts in class. He explained that this goal included, allowing them the time to actually understand the mathematics well enough to apply the big ideas to problems in a variety of contexts. However timing was a huge concern for him because of the list of concepts to be covered in class according to the pacing guide. Justin shared, that he wondered about possible ways to improve the pacing of the curriculum by being selective with the topics to be covered. He described that he wanted to balance out student need to be prepared for Calculus while allowing them time to investigate the Precalculus concepts in depth.

Justin expressed these concerns throughout the two semesters as he engaged with the curriculum. In the following sections, I provide an account of the various ways in which Justin engaged with the curriculum.

Justin's Engagement

Justin's engagement with the curriculum through planning, enacting, reflecting and collaborating allowed him to appreciate the curriculum even more. In the next section, I describe Justin's engagement with the curriculum as he planned his instruction.

Planning

Justin's engagement with the curriculum through planning involved developing instructional plans for his classes as well as developing learning resources like worksheets, assignments and projects for his students.

According to Justin, when he planned his instruction, his main goal was to develop efficient lessons. Such lessons would provide learning opportunities for his students while allowing him to stay on track with the pacing guide. He shared that initially, during the first semester, he started his planning with the online PLC meetings where investigations for the upcoming week were discussed. Justin explained that when thinking about a topic to be discussed in class, Justin reflected on the ideas that were discussed in the meetings, and thought about possible ways to design his lesson for the next class. Describing his planning process, Justin shared that he spent time planning lessons and reflecting on how his plans unfolded in the classroom. He reflected on his classroom experience immediately after class, often when driving home. Justin explained that he would replay his classroom experience for that day and think about the things that

worked, things that didn't work, and taking mental notes for future reference. He would make adjustments to his existing lesson plans or develop new ones for the upcoming classes.

Justin shared that he was mindful of strategies that would allow him to develop effective lessons in the long run. For example, he shared that when implementing Pathways, Justin recognized that the curriculum emphasized one big idea at the beginning of the course and developed that idea throughout the semester. As students investigate problems and progressed through the modules, their understanding of the various concepts developed in a multifaceted way. He explained that sometimes, the curriculum presented different problems using the same context so the students could see the development of a concept within a familiar context. Justin expressed that he saw this strategy as beneficial for his own planning because being aware of the main ideas of each investigation and knowing how they were connected allowed him to develop specific learning goals for his own students.

According to Justin, when he presented an idea in class, he wanted it discussed thoroughly. Since Justin had expressed staying on track with the pacing schedule as a goal, he wanted to plan his lessons to investigate the concepts only once. He shared that he did not want to spend additional class time on topics after they had been discussed. He said, "Honestly, I just try and hope that they see it in the first time through because with the limited amount of time, you really only have one shot going through these problems." (Interview 1, Spring 2017) He wanted to plan effective lessons being mindful of the limited instructional time.

Another concern that Justin shared, was to have enough examples to show his students, so they understood the concepts. Justin shared that from his past experience teaching Precalculus, he could have made up an example to answer his students questions, he was not sure of his abilities to do so with the new curriculum. For example, Justin wanted his students to work on an additional problem after his students had solved all the parts of a certain problem, since there were no more problems, he had to make a problem up on the spot for his students. Justin explained that it was challenging for him to come up with new examples on the spot because these were not problems he was used to. In one observed class, as he tried to come up with an example (Observation 1, Fall 2016), he said, "Let me give you one more [example]. There is none in the book so you can come up with this. Let's see. What am I gonna work with here?" He continued to say, "I'm gonna say this and then we'll see if it works or not. If it doesn't work, you have to tell me it doesn't work. You'll still get it if you're able to tell me it doesn't work." He posed the following question to the class, "$g(f(x))$ is equal to zero. Is there an x that does this? It may or may not. I don't know. How do we come up with that answer? $g(f(x))$ equals zero?" He then continued to explain to the class how to find the value of x that would satisfy the given equation. Justin explained that some problems were novel in their representation of ideas and he would have to spend time thinking about the problem to come up with an example. Since pacing was a concern, he wanted to plan well so the class time was spent effectively and he had examples that he could provide his students.

According to Justin, he recognized that the problems in the investigations were not all the same. Many problems had several parts to them and they explored different

mathematical ideas. He shared that he found it important to select the questions carefully to ensure that the big ideas of each investigation were discussed in class. The PLC facilitator suggested problems ahead of time so the instructors could use that list of problems as a guide for their planning. In the spring semester, Justin shared being able to select problems he thought were a good fit for his students' needs. He explained that if the problems were too easy or repeated the same idea, he would select different ones to investigate in class, thus allowing students to go over other (similar) problems on their own. Being aware of the big ideas of the curriculum, modules and investigations provided him the necessary knowledge to do this. He shared, "Well having seen the course once you just reflect on it and you notice, you just know the objectives of the task better now. You know like... just having seen and experienced the task and seeing how students responded to these tasks and understanding what some of their weaknesses are." (Interview 1, Spring 2017) Justin shared that he became more adept at finding the resources he needed to help his students.

In addition to selecting problems to help his students' learning and their performance on assessments Justin was also interested in developing learning materials for his students. Right from the first semester of implementing the curriculum, Justin was keen on developing his own investigations and assignments based on the big ideas from the curriculum. Justin perceived developing learning materials as a way to match his students' needs. Speaking about his preference for developing his own material he said:

> I think it's important that we always, in any type of course, create new material, just because then it will never get stale. I do that with all of my classes. I mean,

> tests and everything. Everything always is brand new because there is never a shortage of math problems. (Interview 2, Spring 2017)

He continued to do this each semester he worked as a Precalculus adjunct instructor. Sometimes these were small ideas like changing the city names from the state where the curriculum was developed to the state where it was being implemented that his students might be more familiar with. He developed his own assignments for the students and shared his ideas about developing comprehensive projects that could replace the actual exams.

Due to his perception of learning materials as effective tools for student learning, Justin's accounts of his planning emphasized both planning for his classroom instruction and developing learning materials for his students.

Enacting

Enacting the curriculum meant, Justin's implementation of the curriculum inside of his classroom. It was while enacting the curriculum inside his classroom, that Justin's plan came alive as both him and his students engaged with the curriculum together. As observed in his classrooms, Justin started his lessons by lecturing about the big ideas for that day, these were the mathematical concepts that he wanted his students to learn in that lesson. He introduced these ideas and solved example problems on the board. He spent a considerable amount of class time engaging in direct instruction, but did provide students time to work on problems in groups or individually. As the semester progressed, and even after two semesters of implementation, his mode of instruction remained mostly lecture based.

As gleaned from the four classroom observations, during class Justin often used the presentations from the curriculum that accompanied the textbook. The slides on these presentations included the questions from the workbook as well as figures. Justin used the presentations, along with his own notes on the board to lecture about the main ideas. Throughout the lecture, Justin would periodically ask the students to work on the investigations in their workbooks. He encouraged them to share their work with class, for example he would say, "What are the changing quantities in this problem? I was able to come up with ... I'm not gonna say. I'm gonna let you guys answer. I came up with one, two, three ... three changing quantities to use." (Observation 1, Fall 2016) He suggested that his students work on the investigations in groups, but allowed them to work individually if they chose to do so, he also encouraged them to share their work with the class by either explaining it or putting their work on the board. For example he would say:

> We're gonna talk about individual parts of this problem and then I'm gonna assign you another problem and we'll talk about the different parts of that. At some point, too, I'd be able to have some of you guys actually come up and give some of the answers to some of these. (Observation 1, Fall 2016)

This excerpt shows an effort by Justin to engage his students. In his observed classroom, the mode of instruction was teacher centered, where he would provide examples and solve problems for the class. There was a contrast between what Justin shared as his preferred pedagogy and his observed teaching practice.

According to Justin, discussing the problems would be beneficial for his students. He shared that he wanted his students to engage in problem solving as a group, so they could learn from each other. In order to encourage his students to communicate with each other as well as with him, he decided to have his students build a rapport with him as a teacher. He described struggling with this goal during the first semester, which motivated him to work on building a better connection with students during the second semester. He explained that he encouraged his student to seek help if they had any questions and made himself available to answer their questions. He asked the students to email him their questions, being specific about the parts of the problems that they found challenging. He would then answer them by email or by calling them on the phone. Justin reported that his students made use of this opportunity and reached out to him for help.

Over the two semesters, Justin consistently reported that he appreciated the investigations in the curriculum and how they could be used to develop students' conceptual understanding. He shared his excitement about developing problem solving skills in his students and to have his students investigate the problems. However, his classroom practice emphasized the procedural aspects of teaching mathematics. For example, talking about the concept of function, Justin said (Interview 2, Spring 2017), "The concept of the function is probably the most important in all of our program, right? I mean we use it constantly." To define function during Precalculus instruction, Justin took an idea out of programming. He explained, "[Programming is] very syntax heavy, but also structurally heavy. You must do this! You must indent at certain times or this program won't work!" Using this idea, Justin had his students practice techniques like

defining a variable so it would become an automated response for them when they saw a function. He explained that his goal was to develop a procedure to first, define the function, then specify the inputs and the outputs. Having a procedure in place for every function would allow the students to follow these steps every time they encountered a function. They would become proficient at working with functions. He said, "We do this over and over and over again, they're gonna be pros at functions!" He wanted his students to become proficient at working with functions. His approach was to provide a procedure that the students would become proficient at. This approach was fundamentally different from what he shared he liked about the curriculum.

Justin appreciated that the curriculum was designed to develop students' understanding of the mathematical concepts so if they developed a foundational understanding of the functional relationship the students could then use this foundation to develop other idea as they move on to mathematics courses after Precalculus. Justin's engagement with the curriculum as he enacted the curriculum in class showed a contradiction between his appreciation for allowing students to investigate the problems and a teacher-centered approach to teaching.

Collaborating

Justin's engagement with the curriculum through collaborating with his colleagues happened mostly through the PLC meetings. During the first semester, Justin reported that he found the weekly online PLC meetings beneficial, not just for planning but also for getting a bigger picture of how concepts in the new curriculum were connected. He explained, "I really enjoyed the Tuesday night group sessions where we

previewed what was coming up over the next week. It forced me to just be aware of, you know, where we're heading." (Interview 2, Fall 2016)

Justin appreciated the PLC because of the collaborative space that it provided. He shared:

> In the most fundamental way, you feel appreciated as an actual teacher, where your opinions matter. That you're supposed to actually meet and you actually get funded to meet on a weekly basis. You as a teacher feel much more important than you do, maybe at other schools. So that alone encourages me to always continue to work on this, you know what I mean. Like this is actually valuable time to me this is not a matter of me just showing up and doing this you know there is real value. I feel like this is important because we are being paid additionally to have these meetings. This, making a teacher feel valuable is a really big thing. (Interview 2, Spring 2017)

Justin shared his appreciation for the collaboration space that the online meetings provided. When asked if he would prefer to meet in person for the weekly meetings, he responded, "I don't know if it would work. It would be hard, I would imagine, for all of us because of our role as adjuncts. This isn't our primary job. That's why the online is great. It truly is, and we all can be available." The online format of the PLC was feasible for him because of his schedule as an adjunct instructor. As mentioned earlier, Justin was teaching at a high school while also teaching Precalculus at KSU.

Justin's participation in the PLC meetings. Even though Justin expressed how the PLC meetings were helpful for him, his participation was minimal. One reason might

have been the online format of the PLC meetings. He shared that he found it difficult to type his thoughts instead of explaining them to the group in person. Justin described this challenge, "There was a time when I wanted to type [something] out and I was like, just to even try and explain what I'm saying? I'm gonna need examples." (Interview 2, Spring 2017) It took him time to collect his thoughts and to explain them well with examples, he did not want to intrude on the group as they moved forward with their discussion. While the online format of the PLC allowed him to attend the meeting on a regular basis, the same format was also a challenge to his full participation in the group discussions.

The difficulty Justin had in typing his thoughts in real time was also tied to his hesitancy to disrupt the flow of the group's conversation. He explained (Interview 2, Spring 2017), "You know what it is too... a lot of times I don't get, I want to say something a little lengthier but by the time I actually get to type it in it's going to be more of a drawing back in a, not in a good way necessarily you know what I mean and I think it's going to be impossible for the person running the meeting to try and keep up with these things." He recalled later that sometimes his participation was only in terms of listening to the problems, but that it was still beneficial for him. "I definitely found it [PLC] useful because I don't know if I'm doing it right, I don't know how it's worded that type of stuff." (Interview 2, Fall 2016) He also shared that he learned from the comments posted by other instructors on the chat board. He read through them and adjusted his instruction in various ways, such as what homework to assign.

Justin's changing need. After Justin taught the curriculum once during the first semester, he had a different need from the PLC. He stated:

> For the first time through it was really really useful, but I do have a very good memory, you know what I mean? So, I don't need to see things that often, so once I've done this style of problem in this curriculum I recall almost immediately the stuff I've done, you know? (Interview 2, Spring 2017)

He explained that during the first semester, he found the PLC meetings useful because he wanted to familiarize himself with the new curriculum. He continued to share, "Eventually, when it becomes rote, I am like a computer program! I will set my mind to just listen you know what I mean, and I don't need to be physically present." (Interview 2, Spring 2017). He explained what he would like from the PLC, "I like stuff where we're just always creating new stuff!" (Interview 2, Spring 2017) While Justin appreciated the PLC meetings, he shared that he wanted the group to actively engage in collaboration. Due to his preference for developing learning resources, he wanted the focus of the PLC meetings to change. During the end of the second semester, Justin shared his definition of collaboration to include working together towards solving a shared problem or develop shared resources. While during the first semester his needs were being met by the PLC, he explained that during the second semester he needed the group to work towards being more creative together, for example, by developing lessons or projects for students. Justin saw developing resources for his students as part of his teaching practice, he wanted to engage in this practice with his colleagues. Even during the first semester he developed an extra credit project for his own students that he shared with the group.

Ideas for collaboration. Justin valued the meeting time and had ideas about how it could be used to benefit the group. He saw the PLC meetings and the shared online page on the university's classroom management platform, as ways to communicate with other instructors and to ask questions. He had his own ideas about how these platforms could be used to collaborate further. He shared, "We haven't all come together yet and you know, try to create something that would be external to what's in the course itself." (Interview 2, Spring 2017) Justin wanted to be an active participant in the PLC meetings where the group brought both their challenges and their knowledge to work together. He described this need as, "What I would like to do more of is actually collaborate and say hey you and I let's grade together" (Interview 2, Spring 2017) or develop, "Extra credit assignment once a week or something like that." (Interview 2, Spring 2017) He also suggested that a good use of the group's time would be to select a discussion topic ahead of time, share resources and then work together. He elaborated, "The meeting runs as long as it needs to run. Like a real business meeting, where your objectives are to get things done." (Interview 2, Spring 2017) Justin had several ideas about how to make the common meeting time more effective to meet his needs as a teacher.

Justin appreciated the collaboration opportunity that the PLC provided but he wanted more active engagement inside the PLC. He did not interact much with other instructors outside of the PLC. Initially, Justin asked questions from one of the senior instructors who was also a high school teacher. He asked for his guidance to familiarize himself with the norms of teaching at the college level. Other than that interaction, the online PLC meetings were his only means of collaborating with the group of instructors

implementing *Pathways*. After the first semester, because of his changing needs and what he expected from the PLC, even this mode of collaboration became limited.

Reflecting

Justin was open to the idea of improving his teaching practice and shared that he often reflected on his classroom experiences. He shared that he often reflected on his lessons immediately after class and would use this reflection to try and improve his instruction. He was aware of his own reflective practice and shared that he would often overthink experiences. He explained, "In terms of education that works out very very well because you need to really reflect on the small details of how things go." (Interview 2, Spring 2017) He would go over details such as if he had full control over the classroom or if he addressed specific issues. He considered such details because, "I'm always trying something different to readjust." (Interview 1, Fall 2016) He expanded on this thought by saying:

> So I just try and be aware of those feelings immediately when I'm leaving a class you know. We're aware of them during but you try to push them aside but you know just try and remember those and just think about what brought those feelings on and then just analyze that data. (Interview 1, Spring 2017)

Justin shared that, in order to be a good teacher one needed to reflect and be open to both good and bad experiences. He was mindful of both his success and failure inside the classroom and focused on understanding his actions that caused the success or failure. For example, he explained that if his goal was to motivate his students or engage his students in classroom discourse, after a class he would reflect to see if that goal was met.

Reflection on engagement with the curriculum. Justin had many ideas for the PLC and how to take it forward. Many of these ideas were a result of Justin's reflection about the course. Justin shared that he often reflected about his teaching, picking an idea during the PLC meeting and then thinking about it, before or after teaching his class. For example, he said, "The thing with the units has been haunting me the most wherever I go around... Thinking about units and converting!" (Interview 2, Fall 2016) He shared that he often reflected about the lessons while driving, and thought about the possible ways he could conduct a lesson. He would continue to reflect until the plans would unfold inside the classroom. Describing his reflecting, Justin shared that after a lesson, he would recall the classroom experience and reflect on the different components of the lessons in terms of what worked and what did not work. He would then adjust his strategies, anticipating the lesson that will take place in the next class. This reflection was about the content as well as his pedagogy, in Justin's words, "You know like, did I have full control of the curriculum? Did I have full control over the classroom itself? If there are specific issues that I feel, you know I didn't do something... it continuously changes my class on a daily bases, I'm always trying something different to readjust." (Interview 1, Fall 2016) According to Justin, as he reflected, he thought about how to improve his practice and provide a better learning experience for his students.

Justin's short-term and long-term reflection. He classified his reflection into two different kinds, long-term reflection that focused on bigger challenges and short-term reflection that focused on smaller issues. He explained that the short-term reflection included instances during planning a lesson where Justin would consider what questions

to select (or delete) in an investigation. This short-term reflection helped him improve his lessons by recalling problems he faced inside the classroom, problems in terms of questions his students asked, presenting ideas to the class, students experiences etc. In talking about short-term reflection, Justin shared that it is, "really beneficial in the day to day, from semester to semester." (Interview 2, Spring 2017) For the longer term reflection, he shared that it focused on "how we can improve certain things that they have longer reaching impacts too." (Interview 2, Spring 2017) He realized that these were long term goals and shared that, "it takes longer to even flesh those ideas to make them even valuable or how we can change them as a group and make them happen is really a longer term process." (Interview 2, Spring 2017) This long-term reflection was his vision of change that he foresaw in his students' learning, his own teaching, his engagement with the curriculum, and in his collaboration with his colleagues.

Justin's engagement with the curriculum as he collaborated, planned, enacted and reflected provided certain opportunities for his learning. As he faced challenges implementing the new curriculum his efforts in trying to overcome these challenges provided a chance for him to increase his knowledge and improve his teaching. In the next sections, I provide examples of the learning opportunities that emerged as Justin engaged with the curriculum

Opportunities for Justin's Learning

As described earlier, opportunities for learning arise through engagement with a curriculum, as teachers experience challenges when implementing it.

Justin began his first semester of teaching the new curriculum with a curiosity about the new curriculum and teaching Precalculus at the college level, in terms of pedagogy and student needs. He shared, "This semester is really just learning experience to see... what is Precalc at the college level. What's the most important pieces that someone needs to learn calculus? What is most important for them [students]?" (Interview 1, Fall 2016) He shared that he was curious about the new curriculum and how it would unfold as the semester proceeded. Engagement with the curriculum through the summer workshop, allowed Justin to get an introduction to the new curriculum. He had a chance to learn about the curriculum in terms of its focus on covariational reasoning. He shared that even though he found the summer workshop beneficial for him, he still had more questions and he was curious to see how the concepts in the curriculum were connected throughout the curriculum. (Interview 1, Fall 2016)

Justin shared his excitement about implementing the new curriculum. Over the two semesters he participated in this research, he reported, developing a deeper understanding of the goals of the curriculum and its approach to student learning. Justin appreciated the new curriculum, and shared, "I love it [Pathways] I really do, I like it a lot." (Interview 2, Fall 2016) One of the reasons he liked it was because of the modeling based approach to problem solving. The majority of the problems in the investigations were context based, and appealed to him as a teacher. He said, "You know we always say, *do word problems! Word problems!* and then it just doesn't seem to work out. They seem to have gotten the word problems down well." (Interview 2, Fall 2016) Justin's

engagement with the curriculum gave him an opportunity to improve something he was already inclined towards, developing effective learning resources for his students.

Opportunity for Learning: Representations Provoked Thinking

Experiencing the new representations of the mathematical concepts in the curriculum allowed Justin to rethink the concepts themselves. He gave an example of how the topic of units and converting earlier on in the curriculum pushed him to think (Interview 2, Fall 2016) Justin was talking about the first section he explored in the course, which laid the foundation of covariation of quantities. He shared that he found the time spent on this section to be beneficial for him as a teacher and said, "I really enjoyed covering it in the beginning... it was so long but I also learned a lot because it was so long." (Interview 2, Fall 2016) Sharing his fascination with the way that the units were presented in the curriculum he said, "I love the way we look at units... every number is relative to some other number when we're stating them as ratios... and everything needs to be compared, and I think that's something interesting 'cause that's something we can grasp." (Interview 1, Fall 2016) Justin shared that he was impressed by the way that concepts were connected and how they were initially represented as a simple concept while gradually developing into a recurring theme in the curriculum. He shared:

> It's entirely something almost like a deeper thought and it's like, wow! And here with students we can take on a philosophical thought about how this stuff works, and it makes me feel a little bit successful because... they get this! It doesn't involve big formulas and numbers and all that stuff, but it really makes them think like a mathematician. (Interview 1, Fall 2016)

Justin appreciated the way that the ideas were connected because it allowed for a discussion about the connections between mathematical concepts. He explained that he would think about the concepts even when he was not in the class or planning for his lessons. For example, when driving he would see "miles per hour" (Interview 1, Fall 2016) on the road and connect it back to the content in the book. He said, "I'm hoping that the same thing happens with the students, that when they're just driving somewhere down the road and they see a number they start to think what that number refers." (Interview 1, Fall 2016) Justin shared that he wanted his students to appreciate the concepts as much as he did.

Opportunity for Learning: Problem Solving Approach – Curriculum or Pedagogy?

Justin maintained his initial appreciation for the problem solving approach and its benefits for developing his students' conceptual understanding. Throughout the two semesters he spoke about the problem solving approach as being beneficial for his students' learning. However, he remained ambiguous about how it was helpful for the students, was it beneficial as the curriculum or as a pedagogical approach? He stressed that it was beneficial for his students, but without specifying problem solving as a function of the tasks in the investigations or his own teaching practice.

On a broader level, he was able to connect problem solving using a real world context to prepare his Precalculus students for Calculus. He said, "Often a student may have taken Precalculus in high school and then do they know anything about math, no! They probably don't, they can answer some problems but they can't actually put it into any real world sense at all." (Interview 1, Spring 2017) According to Justin, it was

important that his students understood the big ideas learned in class and be able to apply them as needed. He said that people often hold a misconception about mathematics and don't realize how prevalent it is in the world around them. He explained, "You see this all the time, you see people in the real world using math and then they say they have no idea about math and they have no clue that they're using it." (Interview 1, Spring 2017) For him the new curriculum was useful to discredit that idea because students were learning mathematical concepts within context. He shared:

> I think this curriculum kind of just destroys the way that math has been presented in the past. I can't imagine anyone saying that they don't know some math walking out of this class and you know in later on in life too they're going to encounter it and they're gonna say that is math. (Interview 1, Spring 2017)

Justin shared that while he found the investigations to be closer to what a science major would experience in their labs, it would be an adjustment for himself as the teacher to teach using the investigations in the curriculum. He explained:

> We're teaching this to hopefully future scientists, so I think it's presented in a scientific way that they're used to seeing. I think it's probably more strange for the teacher than the student themselves, so it's keeping it in a format that they're more accustomed to because of where they're going with this. (Interview 1, Spring 2017)

As Justin engaged with the curriculum, his appreciation for the problem solving approach, and for the problems in the curriculum provided an opportunity for learning.

His own inclination towards developing learning resources allowed him to avail the opportunity.

Opportunity for Learning: Learning from the Word Problems to Develop Better Word Problems

As presented in the engagement section, Justin was interested in developing his own instructional materials for his students. Beyond the novelty of representations and connections among mathematical concepts, Justin found the word problems and investigations in the curriculum beneficial for his students. He explained that in this curriculum, the word problems were more effective than a traditional curriculum and said, "They're broken down. There is real meaning to the word problems. They just last longer in your head." (Interview 2, Fall 2016) Justin gave a specific example of a problem that employed a Ferris wheel as its context. He said that using the word "Ferris wheel" allowed the students to better understand the concepts. Justin found these problems not only beneficial for his students, but also thought provoking for himself.

His own interest in developing instructional resources allowed Justin to appreciate the problems in the curriculum but he also critiqued them as he found necessary. He shared that some of the problems spent too much time on basic concepts. He struggled with the way the investigations stressed the basic ideas first and then moved onto connecting the basic ideas to more complex ones. He found some of problems to be very simplistic, and shared, "Sometimes the question is so easy that you can't answer because you're like it couldn't be that answer... you end up saying, it's gotta be harder than this and I can't get it." (Interview 2, Fall 2016) He wanted the problems to be more

challenging for the students. He said that sometimes he skipped certain problems. In his words, "I thought some of the questions were a little bit too easy at times where we had to go through them... there have been times where I have cut some problems out because of the quality of the problems." (Interview 2, Fall 2016) He wanted the curriculum developers to improve these questions in the future editions and said, "I wanna have a little bit more trust in those problems! You know I wanna may be cut some of those out because of time but not because of quality." (Interview 2, Fall 2016). Justin did develop his own examples and projects influenced by the problems in the curriculum. Engaging with the problems in the investigations, afforded him the opportunity to learn from the problems and develop his own.

Opportunity for Learning: Learning from Problem Solving Approach to Improve Pedagogy

While Justin shared that he found the problem solving approach to be advantageous for his students, he still had to face challenges in adjusting his teaching practice. As mentioned earlier, facing challenges provide learning opportunities for teachers.

Beyond the general benefit of word problems to increase student conceptual understanding, Justin was excited about the context based word problems in the curriculum because he thought they would be beneficial for his students who were science majors and had worked in labs. He shared, "They're getting to experience a curriculum that's more valuable to what they're going to do in their career. You know, it's math with a science flavor to it, so you know it doesn't seem like basic math."

(Interview 2, Spring 2017) He expressed a preference for having the students work on investigating a problem with various parts that connected mathematical ideas. Students built on their knowledge as they progressed through the various parts of the problem. He shared that he found the investigative approach to be more scientific in nature. He explained, "It's more than just here's section A, and there are ten problems, you do this method, and the next section you do this method, it's a real world problem ... they have the same kind of experience from the field." (Interview 1, Fall 2016)

Justin explained that he realized, his experience teaching Precalculus in the past was different from what the new curriculum required him to do as a teacher. He was excited about the problem-solving approach of the curriculum and the emphasis on modeling but he still had some reservations about teaching from the curriculum. In particular, because he taught Precalculus before, he had preconceived notions about what should be taught and how. According to Justin, in order to implement the curriculum to best help his students he would have to adjust his teaching practices in many ways. The way the curriculum presented the content was different from how Justin has taught it in the past. He shared, "I know trig very well already. I didn't think I liked the radians treatment because I've always been more of the traditional one where we're doing all the nice angles and stuff that always come out and they did not shy away from un-nice angles which was very cool." (Interview 2, Fall 2016) Justin shared that it was an adjustment for him to understand the way the curriculum was approaching various mathematical concepts that he had taught in a different way.

Justin compared his experience implementing the new Precalculus curriculum to his past experience with a traditional curriculum and preferred the new curriculum. Speaking about his past experience teaching Precalculus he said that previously his goal was to develop, "specific tools [procedures] that they [students] are going to use in general Calculus type problems." He said, "Yeah at the high school level I taught it to the accelerated group... I mean there's word problems and stuff but it's not word problem driven" (Interview 1, Spring 2017). He explained that a course that stresses procedural fluency, limits what the students can learn. In such a case, his students would be proficient in the procedural skills that were needed for Calculus, but when it came to applying those skills in a certain context, they would not be able to. He added that when curricula are not investigative in nature, students do not get a chance to solve problems. As a result, students' problem solving skills are not developed. He said, "[Students] can sit down and, given an equation, can perform something on it. But, if you give them a real world problem on it, they may not even know to apply those." (Interview 1, Spring 2017) Justin shared that he found the curriculum to be investigative in nature and that engaging in investigations would allow his students to develop problem solving skills that will stay with them long after the course was over. What Justin didn't mention was how the investigation driven curriculum would come alive inside his own classroom.

Justin shared that he felt responsible for his students' experiences inside the classroom and for their learning. He wanted to improve his own teaching practice to provide effective learning experiences for his students. By the end of the first semester he said, "Right now, all the issues that I have, I blame myself... All my issues are classroom

management issues, trying to make the students more involved, maybe try to make the problems a little more fun make them actually fun." (Interview 2, Fall 2016) Earlier in the second semester, Justin expressed his motivation to learn from his experience in the previous semester because he had a better grasp of what the big ideas were. He explained his goals by saying, "I want to improve my delivery of this content to them [students], knowing now what really is going to be asked of them on these tests and where my bigger focus, my specific focuses need to be." (Interview 1, Spring 2017) Justin wanted to help his struggling students succeed and wanted to do his part to aid them in their learning. He did not want them to struggle because of his teaching. He shared, "Students who do poorly it's not because of my weak delivery it's just because of you know the choices on their end." (Interview 1, Spring 2017) He shared that he wanted his students to be successful, and wanted to take responsibility for improving his own teaching practice.

Fostering student discourse. According to Justin the problems in the investigations were designed to be discussed. He said, "The way that it's worded they [students] have to discuss the problem... that's a real world problem and you just don't start solving it, you need to discuss it and think about it and I think it models that well." (Interview 2, Fall 2016) Justin shared that he found having the students discuss the problems in class challenging. Over the two semesters, his classrooms mostly exhibited instruction where Justin did most of the talking. His students were observed asking him question but there was not much student discourse. He lectured at the beginning of the class, gave his students a chance to work on problems individually or in groups and answered their questions.

Justin explained his struggle to have his students investigate the problems themselves by saying that he experienced opposition from the students as he asked them to discuss the problems in class. He said, "We're having kids to work together in a group that really aren't used to that so they naturally fight it... That's not necessarily even the curriculum." (Interview 2, Fall 2016) To Justin, students' lack of engagement inside the classroom was a result of their past experiences, they did not want to investigate the problems and were used to direct instruction that delivered procedural information. He explained:

> I don't know if it's specific to this curriculum itself either, I think it's the class itself. Teaching more advanced mathematics material to students who you know tend to struggle with mathematics but need to be able to understand this, you know understand it conceptually, you know not just plug and chug type stuff. (Interview 2, Spring 2017)

According to Justin, he had to bring the students on board and wanted to encourage them to participate in class. For example in this following passage from his classroom (Observation 1, Fall 2016), Justin wanted his class to identify varying quantities. He explained to the class what the question was asking (Line 1), he then asked the class to identify three varying quantities. When a student responded, he used questions to guide the student to be more precise (Lines 12 & 14):

1	Justin:	**We want to identify the constant quantities, the values that**
2		**aren't going to change in this problem. What we're talking**
3		**about is making running at a specific greater speed for a**

		specific amount of time for a certain amount of miles. That's what we're discussing. So, with that being discussed ... with that being what we're discussing, what are the changing quantities in this problem? What are the changing quantities?
		I was able to come up with ... I'm not gonna say. I'm gonna let guys answer. I came up with one, two, three ... three changing quantities to use.
	Student:	d, seconds, and calories.
	Justin:	**d, seconds, and calories. When you say d, what do you mean?**
	Student:	Distance.
	Justin:	**The distance that...?**
	Student:	Distance in miles.

This excerpt exhibits Justin's challenge with engaging his students. According to Justin, he had to ease them into the conceptual way of learning mathematics rather than the emphasis on procedure. Justin shared that because he was aware that the curriculum used multi-step problems in investigations, he intentionally grouped his students so they could build on their peers' work. For example, Justin divided his class into groups and assigned numbers to each group. He shared his expectation with the class that each group had to take a turn to respond to a problem or part of a problem. He explained, "and I'll present one [investigation] and then I'll have them work on one as a group and then each row in the class will you know be responsible for a specific answer." (Interview 1, Spring 2017)

According to Justin, this technique encouraged his students to pay attention to what the groups before them had said because they would also be asked to share their thinking with the class. Justin said, "It kind of takes some pressure off because they know they're going to be called but they know exactly in the order they're going to be called." (Interview 1, Spring 2017) The following passage from his class early in the first semester, shows how he would ask the students to work in groups:

> What I want you to do is to answer the rest of the parts in this problem. So what I want you to work on and then we're gonna talk about individual parts of this problem and then I'm gonna assign you another problem and we'll talk about the different parts of that - At some point, too, I'd like to have some of you guys actually come up and give some of the answers to some of these, but **what I'd like you to do, working in groups, is to calculate**, So on this problem, I want you to work on part C, D, E, and F - Then at the end, for Part F, I want to go over a bigger discussion with you on that part as well. So **work on the Part C, D, E and F - We're only gonna do partners on this about five minutes. So, work with someone. If you don't find someone now, you're going to need to in a little bit.** (Observation 1, Fall 2016)

Justin struggled with promoting discourse in the classroom, the passage above shows an attempt to encourage his students to engage in discussions. This did not seem to be an effective way to promote discourse in the classroom especially with students who were reluctant to collaborate in the first place.

Another instructional practice Justin employed to increase participation and help students learn to listen to each other was to call upon the students to explain their reasoning. Justin shared that he wanted his students to be mindful of other students as they asked questions or shared a comment about a problem. He explained:

> The nice things about the investigations are everything builds up so you can't ignore what's been happening and just give us an answer you know. For the ones in the past where it was just five of the same functions used for all the tasks no one had to pay any attention you know to everyone else's performance. (Interview 1, Spring 2017)

This was an attempt by Justin to address the challenge in the classroom of lack of student participation. Despite his efforts to employ strategies to generate greater student participation, his mode of instruction remained mostly teacher centered.

Motivating students. According to Justin, encouraging his students would allow them to work hard and persist through challenging problem. He shared that it did not come naturally to the students to struggle through the problems or to ask for help. He shared:

> So far my biggest learning experience with these students, because I'm new to this school and teaching math at the college level, is seeing how much I need to

> be on top of these students and be giving you know 90 percent of the [time] you need to be talking content but there's gotta be a 10 percent of just pep talk. Every single class. (Interview 1, Spring 2017)

Justin explained that sometimes the students became demotivated and he took it upon himself to motivate them. He said that he found it a challenge to have his students focus on the problems, especially when the problems became demanding. In the following excerpt from his classroom in the first semester (Observation 1, Fall 2016) Justin tried to motivate the students as they struggled to do a problem focusing on inverse functions.

1	Justin:	**You all have the ability to solve this problem, it's just a little**
2		**logical problem that you're not getting.**
3	Student:	Let's use trial and error.
4	Justin:	Well, you can do trial and error, but there's much easier way to...
5	Student:	How do you do it? I don't know how.
6	Justin:	**Let's think about this problem first. What do you get?**
7	Student:	x equals two.

Justin continued to explain the procedure to the class. He tried to motivate them by saying, "You all have the ability to solve this problem, it's just a little logical problem that you're not getting", then continued to guide them along. This was Justin's effort to motivate the students to solve the problem on their own.

Developing a rapport with his students. As mentioned earlier, the challenges Justin faced provided opportunities for his learning. Justin shared that he wanted his

students to stay motivated and to persist in trying to be successful in the course. He attempted to solve this problem by having them communicate their coursework related issues with him. Justin conjectured that if he built a rapport with his students they would feel comfortable enough to ask him questions and to come to him with their concerns about the curriculum. He shared, "The biggest thing that I learned which was the struggle for me last semester was getting students to communicate with me. I learned that it wasn't something that students assumed from the beginning." (Interview 2, Spring 2017) According to Justin, he needed to encourage his students to ask him for help. So during the second semester he emphasized the importance of them seeking help and asking him questions. He explained that he tried to make it convenient for them to ask him questions and told the students, "Please email me if you have a question, all you need to do is include the actual problem." (Interview 2, Spring 2017) Justin shared, "Instead of them just going to someone and getting the answer or looking it up online they're actually reaching out to me asking, how do I do this problem and I'm able to answer real quickly through my phone." (Interview 2, Spring 2017) Justin expressed that he was pleased to see this change in his students' communication with him. He compared student communication with him in the second semester to the previous semester and said, "Just today I received four emails from students, where last year, I think, uh last semester I got a total of three emails." (Interview 2, Spring 2017) According to Justin, he noticed a change in his students, in that they were seeking help earlier on in the semester. He said, "I'm seeing the students' behavior, they're not all waiting till the last minute to do these

questions because they know they actually can ask me the question." (Interview 2, Spring 2017).

This was another example of Justin responding to a challenge he faced while engaging with the curriculum. It was his attempt to provide a support to his struggling students by providing access to the teacher as a resource outside the classroom.

Opportunity for Learning: Pacing

Another major concern for Justin as he engaged with the curriculum was the pacing of the course. As he struggled to stay on track with the pacing, this experience offered him an opportunity to learn about the content as well as about instructional practice. One indication of these opportunities to learn was that according to Justin, pacing influenced many decisions Justin made when planning and teaching during both the fall and spring semesters. Justin shared that struggling with the pacing issue allowed him to think about the reasons for this concern. This pacing concern offered him an opportunity to think about the curriculum itself as well as his own teaching practice. He explained that the pacing concern was not specific to the new curriculum, but was, in fact, a "common challenge in Precalculus. There is a lot to cover in a short period of time." (Interview 1, Spring 2017) Justin stressed that it was important for students to study fewer concepts in depth instead of being introduced to a long list of concepts that they may not fully grasp. Justin explained that by rushing through the concepts, the students did not understand the concepts well enough to actually apply them when needed.

Struggling with the pacing provided the opportunity for Justin to reflect on the reasons for his concerns. Justin wondered about the goals of Precalculus and what it meant to prepare the students for calculus. When it came to pacing, there was a common concern in the PLC meetings about the curriculum not spending enough time on trigonometry, specifically trigonometric identities. Justin shared that one way to fix this problem would be to introduce the trigonometric identities earlier on and change the pacing to allow more time for trigonometry. While this was a quick fix, Justin also reported analyzing the deeper issue of what key concepts were necessary for his students to be successful in Calculus. He shared that in order to restructure the Precalculus curriculum and remove some of the concepts from the syllabus, it was important to understand the goals of Precalculus and what students actually needed to be successful in Calculus. In considering this challenge, Justin pondered, "What can we cut from the curriculum? I guess that really depends on where they [students] are going and how important all that stuff is as well." (Interview 2, Fall 2016) Justin expressed that he liked the idea of investigating fewer concepts in depth instead of focusing on all the concepts that are traditionally taught in a Precalculus course. Through PLC meetings, he was aware of the concerns of his colleagues about certain concepts not given enough class time. Speaking about his colleagues' concerns he said, "We've heard concerns, they do so little on trig IDs, how are they going to do this in Calc and I'm like why is it actually that important in Calc?" (Interview 2, Fall 2016) He described his mindset that students need to be proficient problem solvers and use all the resources available to them when solving problems. He shared:

> If you need a trig ID use a computer and get it! You know what I mean but if you can set that problem up and solve it and your only issue was the trig ID, then well you know, do what everyone else is doing anyway, we're using a computer, we do it. (Interview 2, Fall 2016)

Justin reasoned that there were ways to edit the curriculum so that some of the concepts were removed from the long list of topics to be covered in Precalculus. He shared that it was possible to "weed some of that stuff out." (Interview 2, Fall 2016) allowing students, to learn and be prepared for Calculus. However, Justin shared that he was cautious about what the students actually needed to be prepared for Calculus. He explained that he was aware that his students would experience the investigations based curriculum only in the Precalculus course, and not at the Calculus level. He acknowledged his colleagues' concerns about students needing procedural knowledge to succeed in a traditional Calculus classroom. He shared that if both Calculus and Precalculus curricula were conceptually oriented, it would be an easier transition for the students to go from one class to the next. He said, "If Calculus goes this route the way that this (Precalculus) goes, I mean not now they're not! But it if they do, that would be all word problem based then the way to think about it would not change." (Interview 2, Fall 2016)

Experiencing the pacing concern, provided Justin with an opportunity to reflect on a longer term change in the teaching and learning of mathematics and what it meant to prepare the students to succeed in mathematics.

The challenge also gave him an opportunity to improve his teaching practice. Justin explained that in response to the pacing concern, he paid careful attention to his

selection of problems. For example, by the end of the first semester he was selecting problems based on what suited his students best. He described that selecting the problems was important because students had to investigate the various parts of the problem in a short amount of time. This left only enough time in the class to focus on a few problems in depth. Justin explained his concern, "I think the challenge I've been having so far is that you go through a lot less problems because the problems are lot more developed." (Interview 1, Fall 2016) According to Justin, he had to plan ahead and be prepared to have the students do those problems well.

He shared that selecting questions was important for him in order to maintain the pacing schedule for the course. He explained that he tried to figure out which problems to select based on the concepts they explored. According to Justin, selecting which problems to do in class allowed him to improve his pacing and he was able to help his students by suggesting additional problems for their own practice or if they wanted to explore them on their own. He explained, "You know if there are four problems that are really doing the same thing do one or two of them really really well and let the students have the opportunity to do those other ones on their own." (Interview 2, Spring 2017) According to Justin, his experience implementing the curriculum, allowed him to better plan his classroom activities. Since pacing was a concern, he realized that he needed to select effective examples in order to save time. He explained:

> Honestly, I just try and hope that they see it in the first time through because with the limited amount of time you really only have one shot going through these problems. You know you don't have where you can do two or three examples of

> the same type of problem so just try and perfect that the first time through and keep on making sure that they're getting that the first time through knowing that you're not gonna be presenting it again to them. You know try and focus on the delivery. (Interview 1, Spring 2017)

Pacing was a challenge that emerged from all forms of Justin's engagement with the curriculum. In addressing the pacing concern, Justin had the opportunity to draw on his knowledge of the curriculum and knowledge of mathematics taught before and after Precalculus. He also had a chance to reflect on his teaching practice to find efficient ways to maintain the pacing of the course.

Opportunity for Learning: Planning

In terms of his planning, Justin had short-term plans that included his planning for the classes every week, and long-terms plans that focused on a larger vision of improving the implementation of the new curriculum. According to Justin developing new materials, like worksheets, projects, or quizzes, was an important part of his planning. He explained that he spent a considerable amount of time thinking about how to develop these resources for his students.

Justin's short-term planning was influenced by his reflections on his in-class experiences. He shared that he reflected on all his lessons regardless of whether he perceived them as successful or not. According to Justin, reflecting on his lessons provided a chance for him to learn to improve his future lessons. He reported making changes to his lessons as a result of those findings. Justin shared that after having implemented the curriculum for a semester, he gained some knowledge about the

curriculum, which gave him confidence to make decisions about the classroom activities he designed. He shared, "Well having seen the course once, you just reflect on it and you notice, you just know the objectives of the task better now." (Interview 1, Spring 2017) Justin was talking about the goal of each investigation, which allowed him to plan his instruction so he could meet the specific goal for his students' learning. In addition to the goals of the investigations, he shared recalling his students' experiences and using them to improve the design of his instruction. He said, "It's just having seen and experienced the task and seeing how students responded to these tasks, and understanding what some of their weaknesses are... and take those past [experiences], how students struggled and adjusted, you know this semester." (Interview 1, Spring 2017) In addition to the specific goals of the investigations, Justin also reported, developing a broader view of the curriculum itself and becoming aware of the vision of the curriculum. He shared that this knowledge made him more proficient at planning his lessons, selecting problems he knew to be more effective and anticipating the challenges his students might face. He shared:

> Now with some oversight on what the information is gonna be, and how it's going to run, and you know what the focus on the tests are going to be just trying to get a better, trying to, just picking and choosing the things that are truly necessary for what we're trying to improve, what the real objectives are. (Interview 2, Spring 2017)

According to Justin, implementing the curriculum gave him new ideas to improve the material. Justin shared that he was excited by the new curriculum and had many ideas

about how to improve the implementation of the curriculum to make it become more beneficial for the students. He shared:

> In some sense because we are already doing the hard part, making them think differently than what they were doing so why not just follow through and you know like we got a lot of like Lego nowadays they use and stuff like that they're having a blast, like you know in all those engineering classes where they have Legos and all that other stuff, it's amazing projects and they learn so much. We've already made it into a project now it just needs to be a fun project. So you give them one large project per class and their homework is another project just like that and it hits on every single topic cause we kind of took the idea of 50 problems and broke it down into 6 investigations per class why not go down from 50 problems to just 1 investigation. (Interview 2, Fall 2016)

Justin explained that he found the current investigations comprehensive, but also felt that they could be improved in a way that students could work on a single investigation and be able to understand several concepts while working on a single theme. In his own words:

> One investigation that really brings the entire section through, all with one problem so you can measure yourself and say I can do this. I really do get everything that we just did and it's the last thing you do to test yourself. (Interview 2, Spring 2017)

These suggestions for improving the curriculum were part of Justin's long-term plans to improve the teaching and learning of mathematics.

After two semesters of engaging with the curriculum, Justin shared that he remained excited about implementing. His perceived challenges were either student based, like resistance from the students. At other times, his perceived challenged stemmed from him adjusting his own teaching practice to try a new approach. Justin took the approach, "I adapt, I try!" (Interview 2, Fall 2016), to implementing the new curriculum. This approach led him to come up with new strategies, which he would then reflect on to assess them. He shared feeling optimistic that he was off to a good start and had many ideas for improving the implementation of the curriculum.

Justin's engagement with the curriculum supported his penchant for creating new learning materials for his students. His engagement was marked with an appreciation of the curriculum and how the investigations were beneficial for the students. His appreciation of the curriculum was more in terms of the problems themselves instead of the pedagogy and he saw his own role as the facilitator who provides effective learning tools for his students. The problems in the curriculum provided an opportunity for him to create and impove learning materials for his students. It was only after he had implemented the curriculum for the first semester that he began to see the important role his own pedagogy could play in facilitating his students' learning. His engagement with the curriculum allowed him the opporuntiy to start improving his own teaching practice. He recognized this improvement to be a part of a long term process.

Chapter 5

Cross Case Study Analysis

In order to answer my first research question, [What is the nature of adjunct instructors' engagement with a research-based Precalculus curriculum?] I looked across the three adjunct instructors' engagement with the curriculum. Here I provide the main themes that emerged from studying their engagement.

The instructors went into their first semester of implementation with initial plans that would help them face their anticipated challenges. All three instructors searched for possible ways to motivate their students while also maintaining the pacing of the course. The instructors wanted to achieve a balance between the pacing of the course, preparing students for Calculus and also allowing them time to investigate the Precalculus concepts in depth. Caleb and Michael went into the first semester with a loosely developed strategy for dealing with the challenges of implementing the new curriculum. Justin did not have a plan for dealing with any challenges he would face while implementing the curriculum. During the first semester, he wanted to get a better grasp of the new curriculum while also understanding what it meant to teach at the college level.

When engaging with the curriculum, Caleb was motivated to improve his own teaching practice. From the beginning of the first semester of implementation he focused on developing his own teaching skills that would help provide better learning experiences for his students. Caleb engaged with the curriculum with hopes of finding ways to improve his teaching practice. He drew upon curricular resources as a guide for improving his own pedagogy. For example, he paid attention to the ways in which the

curricular resources introduced ideas to the students and allowed them to explore mathematical concepts. Caleb realized that in the curricular resources (e.g., investigations in the student workbook, online homework), ideas were gradually presented to the students as they worked through the problems. He also paid careful attention to the language used in the curriculum to incorporate that into his own lessons.

Michael's engagement with the curriculum was motivated by a focus on helping his students succeed in Precalculus. He perceived his students' struggles with problem solving to result from a lack of pre-requisite knowledge. His plan was to use direct instruction to provide the pre-requisite knowledge he thought the students were missing. He also relied on direct instruction to show students how to solve problems. He shared that he found a guided approach to be successful in helping students understand concepts and develop problem solving practices. As Michael engaged with the curriculum, he was looking for resources that would allow him to guide his students and to help them succeed.

These initial ideas provided some guidance for the instructors as they began to implement the new curriculum. They continued to face challenges as they engaged with the curriculum through planning, enacting, collaborating and reflecting. In the next section, I will describe the various ways in which the instructors engaged with the curriculum over the course of the two semesters.

Planning

One of the ways in which the instructors engaged with the curriculum was planning their instruction. All three instructors wanted to help their students be successful

in Precalculus but they had different ideas about how they would achieve this goal. Caleb focused on improving his teaching practice, Michael focused on guiding his students and using direct instruction to help them learn, and Justin was open to exploring and learning from his new experience of teaching Precalculus at the undergraduate level. They all shared similar goals when planning their lessons.

Goals for Planning

When the instructors first started to implement the new curriculum, their motivations for planning were similar. All three instructors wanted to ensure that they went into the classroom well prepared. Michael wanted to go into class with several examples to help the students understand the concepts so he spent time searching for examples and practice problems for his students. Justin shared that in the past he could come up with examples on his own when answering a student's question but he was unsure if he would be able to do that with the new curriculum. He was not sure if the example he would give would be sufficient enough to convey all the specific ideas that questions in the investigations would include. Being prepared to provide examples to students meant spending longer time on planning. Caleb shared that while in the past it took him 45 minutes to plan a lesson, it initially took him 4 hours to plan a lesson for the new curriculum. One reason for this extended planning time could have been his feeling intimidated by the new curriculum. For example, he was unfamiliar with how concepts in trigonometry were represented in *Pathways*. The intimidation forced him to spend extra time to plan his lessons so he would feel confident when teaching in class.

Challenges when Planning

The three instructors faced similar challenges when it came to planning their lessons. When planning lessons they used the curricular resources available to them as they tried to find ways to address their concerns. For example, pacing was a major concern for all three instructors. They had to find a balance between allowing students to spend enough time to investigate the problems and staying on track with the pacing schedule. Their planning was influenced by their pacing concern. Justin shared that he tried to plan effective lessons that would allow the students to understand the concepts after investigating the problems once. This would end the need to spend additional class time on topics already discussed. When Caleb planned his lessons, he structured them so that students understood the main ideas and were able to investigate the content while allowing him to stay on track with the pacing of the courses. For Michael, planning included spending time to find ways of presenting concepts so his students had a successful learning experience yet the class stayed on schedule.

What Planning Looked Like

When planning lessons, all three instructors wanted to design effective classroom experiences for their students and to stay on track with the pacing. The instructors used various curricular resources like the textbook, student workbook, presentations, applications, and the homework website. In addition to the curriculum, the pacing guide provided a structure for planning for all three instructors. When planning, they thought about the upcoming classes by referring to the pacing guide as well as discussions in the online PLC meetings.

Their plans included time for a lecture and student group work. They used the main ideas in each investigation to develop a lecture for their class. The facilitators of the summer workshop had suggested teaching practices that would be helpful for students. For example, it was suggested that instructors allow their students to discuss the problems so that the students develop their understanding gradually by working through various parts of the problems. The instructors planned their lessons to allow the students time to work through the investigations but included mini lessons to provide initial guidance for them.

Each instructor would adjust their plan after each class based on how much of the plan actually unfolded inside the classroom. For example, if the instructors had planned for the students to work on 6 problems from 2 investigations, and the students were only able to do 3 or 4 problems, the instructors went back and reassessed their plans and changed them. In addition to the pacing, if the students struggled with mathematical concepts and needed more time than the instructors estimated, they revised their plans to help their students.

When planning, all three instructors reported paying attention to the language of the curriculum and the challenges their students faced in previous classes. Caleb and Michael shared that they used the discussions that took place in the online PLC meetings to guide their planning. Their experience engaging with the curriculum for one semester, afforded the instructors a better grasp of the possible challenges the students would face. Their experiences from the fall semester guided their planning in the spring semester in terms of deciding on the big ideas to focus on during class. The instructors used this

knowledge for planning effective lessons for their students. They were also able to develop their own problems and examples, guided by the problems provided by the curriculum.

Collaborating, Enacting and Reflecting - Influence on Planning

During the first semester of implementation, all three instructors participated in the online PLC meetings. These meetings provided a structure for their planning by providing suggested questions and the facilitator explaining the big ideas for each investigation. They all reported picking up how the facilitators phrased the mathematical concepts and taking notes on the phrasing, with plans to use them in their own classrooms.

During the first semester, all three instructors reported using the online meetings to start their planning. They would listen to the facilitator as he discussed the investigations for the upcoming week, taking notes on the ideas discussed in the meetings. Outside of the online PLC meetings, Caleb and Michael also worked together with their colleagues informally, sharing their classroom experiences and asking questions about upcoming lessons or assessments. They shared that these informal conversations also guided their planning. Caleb explained that his lesson planning was influenced by his conversations with his colleagues. He asked his colleagues questions about content and pedagogy. The ideas discussed during the online meetings also guided him in his planning. Both Michael and Caleb had worked in the department for several semesters and had developed rapport with many of the adjunct instructors in the department. They were comfortable asking them questions and did so.

Rather than collaboration, Justin's accounts of his planning revealed an influence of his engagement with the curriculum through reflection. This engagement took the form of reflection about his students' classroom experiences after each class, which he did not share with colleagues either informally or in the formal online PLC. More than collaboration with colleagues, Justin's reflection on his lessons guided his planning. He shared that he spent time planning out the lessons and revised them based on his classroom experiences to improve them for his students. He elaborated that he reflected on his classroom experience immediately after class, often when driving home, thinking about the things that worked, things that didn't work, and taking mental notes for future reference. He used these mental notes to improve his instructional plans. Similar to Justin, Caleb also shared that he reflected about his coursework while driving. He was working at other institutions while teaching at KSU and the long commute provided an opportunity for him to reflect on his plans and his classroom experiences.

For all three instructors, planning became easier as they gained experience implementing the curriculum. For example, Caleb mentioned that his planning time decreased by the end of the first semester. Michael shared that knowledge of key ideas as well as being aware of all the curricular resources helped him find better examples for his students. Justin's initial concern was that he would not be able to provide impromptu examples for his students but he was observed in class, making up examples out of his head, even as early as the first semester of implementation.

One thing that they all commented on was being aware of how the mathematical concepts were connected in the curriculum; they shared finding this knowledge to be

influential on their planning. According to Justin, being aware of the main ideas of each investigation allowed him to develop specific learning goals for his own students. The larger picture also allowed him to develop effective lessons in the long run by setting the foundation in earlier modules for concepts that would develop in later modules. He shared that he became adept at selecting problems that he thought were a good fit for his students' needs. For example, favoring a problem with key ideas over others and altering the problems as needed. In Caleb's view, seeing the bigger picture helped him get a clear vision about the goals of each of his lessons. He shared that being aware of the goals allowed him to plan effectively for his students' learning. Michael shared that his experience with the curriculum allowed him to gain confidence when planning his lessons. He said that this experience allowed him to navigate the resources to search for the perfect examples for his students.

All three instructors shared that their knowledge of the curricular resources and their experience engaging with the curriculum allowed them to become more proficient at planning their lessons.

Enacting

Participation in the summer program provided all three instructors with a sense of what their engagement with the curriculum would look like. They started their first semester aware of the novelty of the new curriculum in terms of its focus on covariational reasoning as well as the workshop facilitators' recommendations for effective teaching practices. They reported gaining an awareness about the investigations in the curriculum and their goal for the students to develop clear mathematical meanings. They also

perceived the new curriculum to be beneficial for their students. All three of them had taught Precalculus before, at the high school or college level, but the novelty of the new curriculum made them anxious about enacting it in their classrooms.

Caleb wanted to improve his teaching practice and expressed concern about his own ability to teach problems where concepts were being represented in a new way. He also shared a concern about how his students would react to the investigations in the curriculum. Michael shared that he was worried about getting his students acquainted with the language of the new curriculum as well as having to work through the problems in the investigations. Justin reported that his initial concerns were about the dynamics of a college classroom and about getting the students engaged in his classroom.

Student Engagement

Student engagement in the classroom was a problem faced by all three instructors. During the summer workshop, the facilitators suggested that encouraging students to discuss the problems and working through them to gradually develop their understanding would be beneficial for them. The problems in the curriculum were designed to support the teachers in asking their students questions that would challenge their thinking. The instructor notes in the curriculum also recommended that the teachers foster discourse in the classroom which would benefit the students in understanding mathematical meanings. The workshop facilitators suggested that promoting discourse in the classroom was also beneficial for the teachers to get a sense of their students' understanding. Research shows that teachers can make sense of their students' understanding by paying attention to their ongoing conversations and also by asking the students questions. These questions can

help the teacher clarify their students' meanings that may be different from the teachers' meanings (Carlson, Moore, Bowling, & Ortiz, 2007). Having a better understanding of students' thinking can allow the teachers to modify their own teaching practice to align it with their students' needs. For example, in response to a student question, a teacher may ask follow up questions to get a better understanding of the students' understanding of mathematical concepts.

Fostering discourse in the classroom was a challenge for the instructors. In addition, the instructors reported a common challenge of holding the students responsible for their own learning inside the classrooms. Instead of using direct instruction and giving students the answers, the instructors were encouraged by the workshop facilitators to allow the students to struggle through the problems themselves. The instructors shared that they found it challenging to bring the students on-board in having them investigate the problems. Caleb shared that his students found critically thinking about mathematical concepts and modeling real life situations difficult. According to Michael, his students were challenged by, and uncomfortable with discussing ideas with each other. Both Michael and Caleb explained that their students' challenges stemmed from a focus on conceptual understanding of mathematical ideas instead of procedural fluency. They explained that this contrast in their experiences inside a mathematics classroom made the students feel less confident about their answers and therefore less inclined to share their responses with each other. Justin also reported, facing a similar challenge, of fostering discourse inside the classroom.

One way to encourage student participation was to develop a safe environment where students would feel comfortable asking questions or responding to challenging questions. Both Caleb and Justin used a similar approach, in that they wanted to make their students feel comfortable to share their thoughts and ask questions in class. Caleb applied this idea by creating a safe environment for his students where he established classroom norms that encouraged group discussions. Caleb shared that he found having the students investigate the problems themselves beneficial for them. In his classroom he was observed providing opportunities for student investigation but he also used direct instruction. He shared that he did not want to revert back to a model of teaching where he used only direct instruction to give them all the information. During classroom observations, students were instructed to work in groups, share their ideas, questions and findings with their groups before sharing them with the classroom.

Justin also shared that he wanted his students to gain confidence and share their thoughts inside the classroom. According to Justin, the solution to this problem was to improve communication between himself and his students. He wanted his students to approach him with their questions even when outside the classroom. He encouraged them to email him questions when they were doing their homework and he would be quick to respond via email or by phone. His goal was for his students to feel comfortable coming to him with their problems with the hope that they would feel comfortable asking questions in class as well.

Similar to Caleb and Justin, Michael shared his goal of making his students feel comfortable. His approach was to make the students feel confident about their content

knowledge. According to Michael, the students needed to feel confident about their responses in order to share their thoughts with their classmates. It was their lack of confidence about their conceptual understanding that held up their classroom engagement. Michael shared that his students struggled with solving the problems in the investigations and therefore did not want to share their responses. His focus was on helping his students improve their problem solving skills. He shared that it was his students' lack of background knowledge that was hindering them from becoming successful problem solvers. He introduced side notes on the board, where he would write formulas and main ideas from the lesson for the day that students could use as a reference. He explained that his goal was for his students to use this information to solve problems on their own and hoped that it would help them gain some confidence in their problem solving skills.

All three instructors had teacher centered classrooms where they led most of the discussions. After the summer workshop they had developed some ideas about how they would implement the curriculum to help their students' learning. They incorporated strategies like developing a classroom environment where students felt safe, providing support to the students outside of the classroom so they felt comfortable asking questions, providing a list of formulas and background mathematical knowledge as a reference to help the students feel confident. As gleaned through their classroom observations, the main mode of instruction for all three teachers remained direct instruction. They would start with a mini lesson, either using a presentation to describe the big ideas for that lesson or walked the class through a problem from an investigation. After this initial

lesson they would ask students to work in groups and ask them questions afterwards to draw conclusions. For the most part these questions were direct in nature leading towards guided instruction. They would allow the teacher to receive a specific answer that they would then use to connect to a big idea. Michael was observed almost always asking leading questions and seeking specific responses but incorporating student responses into his teaching. From their observed classrooms, Caleb and Justin, in addition to asking direct questions also asked some open ended questions allowing students a chance to explain their thinking. Both Caleb and Justin also asked their students follow up questions to their responses in order to get a sense of their students thinking.

In terms of their practice, I was interested in their use of decentering, or the instructors trying to make sense of their students' thinking. There was not much change in their use of decentering over the two semesters. The instructors often did build a rudimentary model of a student's thinking when the student asked a question but their follow up was not consistent. They also were able to recognize when student responses were different from their own way of thinking. However, they often did not incorporate their alternate responses into teaching. It was mostly responding to the student questions so that the responses aligned with their instructional plans. The problems in the curriculum provided an opportunity to challenge the students. When the students were working on a problem from investigations, the questions that the teachers asked were often leading in nature. They directed the students towards a specific answer that the teacher then used to continue with the lesson. When a student asked a question, the teacher either answered it or followed up with another question to make sense of their

students' thinking. As mentioned earlier, all three instructors' lessons were teacher led and they asked leading questions to draw the students towards a specific idea.

In terms of their classroom environments, the instructors shared that they were working towards greater student discourse and less teacher centered classrooms. Over the course of the two semesters this shift in classroom from teacher centered to student centered was not apparent. All the instructors shared that they agreed with the benefits of their students' investigation of mathematical ideas. They shared, that they put in the effort to develop classrooms that would allow the students to have successful learning experiences. Their classrooms however did not mirror what they said, they were trying to achieve.

The instructors reported that engagement with the curriculum in the form of enactment was beneficial for them. In implementing the curriculum over the course of the two semesters, the three instructors shared gaining confidence about the language of the curriculum and were observed incorporating phrasing specific to the curriculum in their teaching. They shared becoming confident about the mathematical representations in the new curriculum and also tackling student questions when discussing problems from the new curriculum. In terms of their classrooms, they mostly remained teacher centered with the teachers, lecturing and leading discussions.

Collaborating

In a PLC, educators work together to develop supportive conditions that promote collaboration and growth (DuFour & Eaker, 2005). My definition of a PLC for this study was "a group of people sharing and critically interrogating their practice in an ongoing,

reflective, collaborative, inclusive, learning-oriented, growth-promoting way" (Stoll et. al, 2006. p. 223). I included within this definition both formal in-person or online interactions with colleagues guided by a facilitator and informal interactions with colleagues during shared office hours as well as through emails, text messages or emails.

Engagement with the curriculum through collaborating with colleagues took the form of a support for the instructors to help them in implementing the curriculum. Instructors collaborated with their colleagues in both formal and informal settings. Right from the beginning of the first semester, Caleb and Michael actively participated in the online meetings but Justin's engagement as well as attendance was sporadic. In addition, Caleb and Michael had built a rapport with their other colleagues who were teaching the same course because they had taught at KSU before. Justin did not have this rapport with his colleagues because he was new to teaching at the college level and at KSU.

Formal Collaboration

During the first semester of implementation as the three instructors were trying to figure out strategies to implement the new curriculum, any guidance was welcomed. The online meetings offered a platform where all three instructors could get ongoing support as they implemented the curriculum. Justin shared that he found the online meetings beneficial during the first semester. They guided his planning and also provided a larger picture of the goals of the curriculum. He shared learning about the big ideas for each investigation and the concepts to stress in the classroom. Even though he was a silent observer for the most part during the meetings, he shared that he found them useful during the first semester. By the end of the first semester Justin shared various ideas

during his interviews and an in-person, end-of-semester meeting, about how to use the online PLC meetings to collaborate with his colleagues. He wanted to collaboratively develop instructional resources for students that would build on the investigations in the curriculum but be more aligned to his own students' needs at KSU. His participation in the meetings was minimal in the second semester. One reason could be that Justin did not find the online meetings as useful as the first semester. He shared that once he understood the content he did not have a need to go back and relearn it. He preferred spending the online PLC time to actively work towards a specific goal, like developing a lesson or an assignment for students.

Caleb also shared that he found the formal online PLC meetings beneficial and actively participated in them. He would share his classroom experiences and his concerns about his students' learning. He said that he appreciated the experiences shared by other instructors and the guidance provided by the facilitator. He continued to actively participate in the online PLCs during both the semesters. He shared that the online PLC provided a form of continued support for his teaching and an opportunity for him to think about his teaching practice and his students' learning.

Michael was the most active participant in the online PLC meetings. He shared that he enjoyed collaborating with colleagues even outside the online PLC and felt comfortable sharing thoughts and asking questions during the online PLC. It provided him with a virtual platform for collaboration with colleagues to support him in implementing the new curriculum.

Informal Collaboration

For both Caleb and Michael, their informal interactions with their colleagues provided a valuable opportunity for their learning. As they tried to figure out the curriculum for the first time, they shared that their colleagues provided an effective form of support. They could draw on the knowledge and experience of other instructors who were also implementing the new curriculum for the first time. They explained that they met with the instructors during their shared office hours. During these informal meetings they would ask questions about content and pedagogy, share classroom experiences and seek advice. These informal meetings were dependent upon the instructors' schedules and if their schedules allowed them to share office hours. As a result, they did not have access to the same support each semester. Since both Michael and Caleb had taught at KSU before, they had built a rapport with many of the instructors who taught Precalculus and were able to contact them via email, text messages, or phone calls to seek advice. During the first semester of implementation, Michael also went to observe one of his colleagues. He shared that he wanted to compare the students' engagement in his colleague's class with that in his own class, and was satisfied to see that his class followed a similar format as the other instructor's class.

Overlap between Formal and Informal Collaboration

There was an overlap in the exchange of ideas across the formal and informal settings. The ideas discussed during online PLC meetings would also lead to conversations outside of the meetings in the informal settings. For example, phrasing of certain concepts and pacing of the lessons to name a few. Sometimes the instructors

stayed online after the online PLC meetings to help each other. For example, after one meeting, Michael virtually helped Caleb in setting up his online homework and answered his questions about selecting problems for his online homework assignments. In a similar way, the informal discussions would also influence the conversations during the formal online PLC meetings. Instructors would sometimes have discussions about questions on the exams or their students' learning needs outside of the online PLC and get a sense of agreement before bringing them up to the online PLC group.

Collaborating as a Form of Support

Collaboration with colleagues was beneficial to the instructors in different ways. Both in the formal and informal settings, the instructors brought in their own knowledge and experience. Working with colleagues instead of working in isolation allowed the instructors to draw upon each other as resources. They shared their classroom experiences and the various pedagogical techniques that worked for their students. Sometimes the questions asked, allowed the instructors to think about ideas that they would not have thought about on their own. These could be ideas about goals for student learning, students' learning needs, or conceptual knowledge of mathematics. It was up to the instructors to avail of the opportunities that the collaboration offered.

The online PLC meetings provided a form of support for the instructors as they implemented the new curriculum. This platform was available for them but, as mentioned earlier, collaboration was dependent upon the instructor's own effort to actively seek out ways to develop their knowledge and teaching practice. For example, Caleb shared that he used collaboration with colleagues to improve his teaching practice and used

discussions with them to plan his lessons. He shared that his conversations with colleagues challenged him to improve his pedagogy. These conversations allowed him to remain motivated in his efforts to foster student discourse in his classrooms and improve their problem solving skills. He used the formal and informal conversations as a guide for his planning. He actively asked questions and shared his concerns with his colleagues during the online PLC meetings as well as outside the online PLC meetings. For example, since pacing was a concern for him, Caleb often asked where the rest of the teachers were in terms of the scheduled investigations for the week and how many problems their students were able to do in one class. This allowed Caleb to adjust his own pacing so he was on track with the others. Caleb shared that collaboration with colleagues gave him a sense of camaraderie. He enjoyed sharing his experiences with others, especially as they tried to figure out how to implement the new curriculum. He expressed his appreciation for the fact that there were other instructors who were in a similar situation as himself and drew on them as a form of backing. Despite his own busy schedule he put in the effort to attend the online PLC meetings and collaborated with his colleagues informally because these interactions provided him with a sense of camaraderie.

Like Caleb, Michael also shared that he used collaboration with colleagues to guide his instruction. Michael said that he used the ideas discussed during formal and informal conversations to plan his lessons. According to Michael, he would pay careful attention to his colleagues' suggestions about teaching practices and their classroom experiences in terms of what teaching techniques they found successful or unsuccessful. These suggestions influenced his planning and enactment of the curriculum. He shared

that he found the online platform of the PLC meetings challenging in terms of typing out his responses to reflect his thoughts. It was challenging for him to balance between reading the comments, listening to the facilitator and typing out his responses. Despite these challenges, Michael was the most active participant in the online PLC meetings, where he shared ideas and actively engaged in discussions.

As mentioned earlier, Justin shared that he found the online PLC beneficial during the first semester. It helped him in his planning of the lessons as well as learning the language used in the new curriculum. His participation in the online PLC meetings was minimal but during the first semester he attended the weekly meetings fairly regularly. By the second semester, he rarely attended the online PLC meetings and shared that he did not find them as useful. One reason for his lack of participation in the online PLC meetings was the online format of the meetings. He shared that he found it challenging to type out his thoughts as fast as the rest of the group. The time it took for him to gather his thoughts and come up with relevant examples, the group would have moved on and he did not feel comfortable disrupting the flow of the groups' discussion. He shared that during the first semester, he still found the online PLC meetings valuable. They provided a sense of guidance in terms of the goals of the curriculum, language used in the curriculum, and the main ideas to be focused on in each investigation.

After he had taught the curriculum once, Justin explained that his needs from the online PLC evolved. He understood the big ideas of the curriculum and wanted the group to work together to develop teaching material for their students. For example, he wanted the group to develop lessons together and also projects for their students. At the end of

the first semester, he shared that he was looking forward to working with others to develop lessons because he needed the group to be creative together. He appreciated the opportunity that the online PLC meetings provided as they made him feel appreciated. Justin expressed an appreciation for the PLC meetings, sharing that he valued the allocated online PLC time, and wished for it to be used towards active collaboration. According to Justin, his personal goal was to develop new material for the students that was relevant to their needs. He expressed many ideas for collaboration during the online PLC meetings. Specifically, he wanted the group to select a topic of discussion ahead of time, share resources and work together to achieve a goal that the group came up with. He was leaning towards a more authentic community where the instructors as participants would lead the meetings to fulfil their own professional needs.

Like planning and enacting, collaborating was another form of instructors' engagement with the curriculum. PLCs provide opportunities for exchange of ideas between teachers, allow them to reflect on their practice and to provide critical feedback to each other (Lieberman & Miller, 2016). In a PLC, teachers have an opportunity to discuss what they find important with regards to their shared experiences. The learning that takes place through their collaboration is a result of their conversations and the relationships they build (Leiberman & Miller, 2008). For the three instructors, collaborating took the form of support. The instructors were able to benefit from this support based on their need, their ability to collaborate and the relationships they built.

Reflecting

Engagement with the curriculum when reflecting supported the instructors in their implementation of the curriculum. Just as the instructors collaborated in various ways, there were different ways of reflecting, depending upon their individual needs. For Caleb, reflection was an important part of his teaching practice. He shared that he reflected on his classroom experiences after class, when collaborating with colleagues and when planning his lessons. According to Caleb, reflecting on his teaching practice was a way for him to constantly improve it, since improving his teaching practice was an ongoing goal for Caleb. He reported that he reflected about his teaching the most when driving. As an adjunct instructor working at several institutions, he had a long commute and he used this time to reflect on his teaching. He shared that he thought about his classroom experiences for that day, with questions about his goals for the lesson, his students' learning, and his own learning about his students' needs, his practice and his knowledge.

Michael shared that he reflected after every class as well, thinking about his students' learning experience, taking note of what worked and what did not in terms of his teaching practice. He shared that overall his reflection had a broader focus, to improve his teaching in the following semesters. Michael explained that he actively took notes during the first semester and used those notes to guide his instruction in the following semesters. According to Michael, reflecting on practice allowed him to learn from his mistakes in the past semesters, and reflecting on his past experiences also allowed him to be more effective in his planning. He shared that he was able to select problems that were beneficial for his students, and would help his students understand the big ideas because

he knew how they were connected. Michael shared that in his view, improving his teaching practice was a long term process and he tried to improve his lessons from one semester to the next.

Like Caleb, Justin reported that he often reflected about his teaching when driving. He taught at a high school while working as an adjunct instructor at KSU, therefore he often drove to KSU right after his school day ended. Justin shared that he would select an idea discussed in the online PLC meeting or one that came up while planning a lesson and then think about it, trying to figure out ways to present it to his class. He explained that he continued to reflect until the plans would unfold inside his classroom. After he enacted the lesson, he would reflect on both the positive and negative aspects of the lesson and adjust his lessons for the next class. He elaborated that these adjustments would include details about classroom management, the curriculum, and any specific issues with the classroom.

Justin expressed that he enjoyed reflecting about his teaching practice and did so in various ways. According to Justin his reflection was categorized into two main kinds: (1) He focused on issues that would take a longer time to resolve, which he thought of as long-term changes in the teaching and learning of mathematics; and (2) He reflected on immediate issues like selecting appropriate questions for planning a lesson, or improving his lessons by recalling student experiences. To him the reflection that focused on immediate issues was beneficial as he planned his lessons for each class and from one semester to the next. The long-term reflection was his goal to bring about larger changes in the teaching and learning of mathematics. The long term-plans were about laying the

foundations of change for his student learning, his teaching, his engagement with the curriculum, and his collaboration with his colleagues. Justin shared that he realized that achieving these long-term goals was a slow and time consuming process but believed that it was valuable to have these goals. He was aware of his own reflective practice and shared that when it came to education, it was important to pay attention to small details to improve his teaching practice in the long run.

As a form of engagement, reflecting on their experiences supported the instructors in implementing the curriculum. The instructors reflected on the challenges that they faced inside the classroom, their students' questions or a problem in an investigation that challenged them when planning their lessons. In addition, they thought about the comments posted by other instructors during the online PLC meeting, and many more ideas that were connected to their implementation of the new curriculum.

Each instructor's engagement was influenced by their students' experiences. They all shared a common goal, to help their students have effective learning experiences. Caleb's perception of helping his students was to improve his own teaching practice. Caleb's engagement with the curriculum was marked with a desire to develop pedagogical techniques that would add to his repertoire. Somewhat similar to Caleb, Michael also perceived facilitating student learning as connected to his own teaching practice. Michael's engagement with the curriculum exhibited his efforts to try teaching techniques he already knew to help his students. Justin's view of helping his students was different from Caleb's and Michael's in that he found curricular resources as a tool to

guide their learning. Justin's engagement with the curriculum revealed his preference for developing learning resources for his students.

Instructors' engagement with the curriculum in the form of planning, enacting, collaborating and reflecting were connected and influenced each other. The challenges they faced while enacting the curriculum provided an opportunity to improve their instructional plans, the ideas that were discussed in the online meetings had the ability to unsettle their current knowledge and practice. These challenges that the instructors faced while implementing the curriculum, presented opportunities for their learning as they tried to overcome them. In the next section I will describe the opportunities for teacher learning that emerged through their engagement with the curriculum.

Opportunities for Learning

In order to answer my second research question [How does engagement with a research-based Precalculus curriculum provide opportunities for adjunct instructors' learning?], I examined the challenges that the three instructors faced while implementing the research-based Precalculus curriculum. The challenges that the instructors faced provided opportunities for their learning.

Since teachers engage with curricula in different ways, their unique ways of engagement provide different opportunities for their learning. As mentioned earlier, opportunities for learning are "events or activities that are likely to unsettle or expand teachers' existing ideas and practices by presenting them with new insights or experiences" (Remillard & Bryans, 2004, p. 12). These opportunities arise as the teachers engage with the curriculum and while making instructional decisions to provide effective

learning experiences for their students. Based on the DCE framework, teacher's engagement with the curriculum can be seen as a design process. Teachers use curriculum material in their own unique ways to meet their students' learning needs. The design process goes through various stages, as each teacher selects the curricular materials to use, interprets them, and changes them to match their students' needs. As teachers plan and enact the curriculum, they go through cycles of designing and enacting instructional plans that provide opportunities for their learning (Remillard & Bryans, 2004). In this section, I zoom in on the opportunities for learning that emerged for the instructors as they engaged with the curriculum while planning, enacting, collaborating and reflecting.

In order to avail of those opportunities, it is important that teachers are mindful of the challenges, that they are able to explore in depth what those challenges entail and also are also willing to take the necessary steps for facing the challenges. Caleb was willing to learn from the challenges that came up during implementation of the new curriculum. It was his goal to learn from the new curriculum and to find ways to improve his teaching practice. He actively looked for opportunities for his own learning and professional development. As he faced challenges implementing the new curriculum, his efforts in trying to overcome those challenges provided a chance for him to increase his professional knowledge and improve his teaching. For example, the online homework that was part of the curriculum posed a challenge for Caleb's students. They found it difficult to enter their responses into the online system and would often come to Caleb for help. The students were convinced that they were entering the correct responses which

the system marked as incorrect. Initially, Caleb was concerned because previously he had experienced working with another online homework system that he found more user-friendly. He compared the two online homework systems and agreed with his students that the system was challenging and not useful.

Since the online homework was a cause of concern for Caleb, he focused on understanding his students' concerns and worked to figure out what was causing their problems. Inside the classroom he paid attention to his students' concerns about the homework. He also asked questions about the online homework during the online PLC meetings and reflected on it. His focus on solving his students' homework concerns allowed him to realize that while the new online homework system had a user interface that required some adjustment time for the students, the format of the problems as well as the problems themselves were actually beneficial for the students. The problems in the online homework were meant to challenge the students to think so they could apply the big ideas that they learned in class. When doing the homework problems, students were not able to refer back to a sample problem and copy a similar procedure to get desired answers. Instead they had to think about the question itself and pay attention to the phrasing of the problem, the key pieces of information that the problem provided, as well as understand what the question was asking of them. Caleb was observed helping his students in the classroom by giving them reminders about being precise when answering the online questions. During the lessons he would give examples of the types of big ideas the homework problems might focus on and the types of mistakes they might be able to avoid.

Paying attention to the homework problems provided an opportunity for Caleb to help his students with their homework struggles. It also allowed Caleb to notice the way in which the homework problems were allowing the students to develop their conceptual understanding of mathematical ideas. According to Caleb, the problems guided the students to develop their thinking even as they struggled through the problems. The problems were context based and the students had to apply mathematical knowledge to a scenario that might be different from what they saw in class. The problems required the students to apply the big ideas they had learned in class to a new context that they may not have seen in class. Caleb shared that he liked this approach to teaching and tried to use it inside the classroom. By engaging with the curricular resources and being mindful of the challenges he faced, Caleb was able to avail of opportunities for improving his teaching practice. His students' experiences with the homework unsettled his current teaching practice. As he tried to help his students, he paid more attention to the homework problems and this experience provided an opportunity for improving his own teaching.

As previously mentioned, it is important for teachers to be mindful of challenges they face, as possible opportunities for their own learning. The opportunities that presented themselves as they engaged with the curriculum required effort on the part of the instructors to actually learn from them. They had to actively seek support and find ways to deal with the challenges they faced. Michael's concerns were mostly student related but he did not perceive his students' struggles as connected to his own teaching

practice. According to Michael, his teaching practice was aligned with what the facilitator of the summer workshop had recommended.

Being aware of his students' challenges could have provide an opportunity for Michael to seek guidance and improve his own practice. Opportunities for learning emerge as teachers face challenges and these challenges unsettle their existing knowledge and practice. Noticing his students' struggles was not enough, he needed guidance to understand their challenges and to connect them to his own teaching practice. For example, Michael shared that he wanted his students to become proficient problem solvers. He wanted to help them solve the problems in the investigations independently and was aware that it was challenging for the students.

He attributed his students' challenges with word problems, to their past experience in mathematics. He shared that his students lacked the pre-requisite knowledge needed to solve the problems on their own. If the students had access to the concepts and formulas that they needed, it would give them the confidence to solve the problems on their own. In order to help them, Michael used direct instruction and provided important definitions and formulas on one side of the board. He asked the students to use those notes as reference when solving problems.

In his observed classes, Michael guided his students when solving problems. He gave them tips to improve their problem solving skills, like carefully reading the problem statement to gather important information and to figure out what the question was asking. Michael also led students through each problem by highlighting the main ideas and directing them towards information that they may have missed. Michael explained that he

wanted to explain all the main ideas as well as go over the language of the problems in the classroom so they have all the pieces to solve the problems on their own.

These efforts to develop independent problem solving skills were unsuccessful. Michael found that when he read the problems in class and guided the students towards important information they were able to solve the problems. That is, once he helped them unpack each problem the students referred to the side notes and were able to solve them. However, the students were not able to make sense of the problems or break them down on their own. This made it seem to Michael that the students were not becoming the independent problem solvers he wanted them to become. Michael was aware of this challenge and had the motivation to help his students, but he needed guidance to better help them. Michael shared that when he read through the questions in class, the students had an easier time solving them when compared to solving them on their own. His solution to this problem was to provide direct instruction and to explain the problems so that the students could use the examples as a reference. He also provided notes on pre-requisite mathematical knowledge that the students could refer to when solving problems.

Even though Michael utilized direct instruction, he shared that he was aware of the benefits of classroom discourse. All of the Precalculus instructors had experienced working through the investigations during the summer workshop. They had discussed the types of questions teachers could ask to make sense of their students' mathematical meanings. Michael's efforts towards making his students independent problems solvers might have been more successful if he had focused on seeking support to improve his own teaching practice. In class Michael had the opportunity to check for his students'

understanding to figure out why they were struggling with solving problems. His own focus was on guiding his students through problems in the investigations. Michael chose to guide them instead of allowing them to productively struggle with the ideas themselves. He led them through the problems and would ultimately provide them the answers. His students' struggles with solving problems independently provided an opportunity for Michael to improve his teaching practice.

Being mindful of the challenges faced when implementing the curriculum allowed the instructors to perceive challenges as possible opportunities for their own learning. They could then draw on the supports that were available to them to face the challenges and in turn influence their knowledge and practice. For example, Caleb shared that pacing was a concern for him, and he regularly compared the pacing of his course with that of his peers, during the online PLC meetings. According to Caleb, this comparison allowed him to gauge if he was on track or falling behind in terms of the pacing schedule for the course. For Michael, his students' lack of classroom engagement was a concern. Thus, he observed a colleague because he thought observing another instructor who was implementing the same curriculum would prove beneficial for his own class. In order to achieve their goals, the instructors drew on resources that they believed would help them.

During the first semester, all three instructors attended the online PLC meetings and stated that they found them beneficial. Participating in the meetings was beneficial for the instructors to catch the specific phrases used and the big ideas that were emphasized in the curriculum. During the first semester Caleb and Michael participated actively in the meetings while Justin attended as a silent observer, commenting here and

there. Justin later explained that typing during the online PLC meetings was difficult for him because it took him time to gather his thoughts and by the time he was ready to type, the group's conversation had progressed on to other topics. This was one reason why Justin's participation in the online PLC meetings dwindled during the second semester when he rarely attended the meetings.

During the second semester, the meetings continued to be useful for Michael and Caleb but for Justin they no longer served a need. After the first semester of implementation, Justin shared that he felt confident about the content itself. He also shared that he had developed a bigger picture of the goals of the curriculum and how the concepts were connected across the sections in the curriculum. He explained that he no longer wanted to attend the online meetings that he perceived as only discussing content. His stated need was to use meeting time to work with other instructors either planning lessons or developing learning resources for students. He elaborated that he wanted the instructors to go into the meetings with an agenda of their own that they would decide upon. This agenda could include discussing a certain concept or ideas for developing instructional plans and learning resources for students. He then wanted the instructors as a group to work towards their agreed upon goals.

Even as the nature of discussions during the second semester changed from the first semester, Justin's participation continued to be minimal. During the second semester, the instructors who had already implemented the curriculum started to share their experiences. They would actively engage in sharing their thoughts and concerns and asked questions anticipating potential challenges. Even so, Justin did not engage in the

online PLC as an active participant. However, he felt comfortable sharing his ideas during in-person interviews and even mentioned that the interviews to him were a form of support because he was able to share his thoughts with ease.

Similar to the other two instructors, Justin expressed his concern about the pacing of the course. Pacing remained a concern for him during both the semesters. He shared that the challenge was to balance the need to prepare his students to succeed in Calculus, yet provide enough time for them to investigate and stay on track with the pacing schedule. According to Justin, it was important for students to study fewer concepts but understand them well enough so they could apply the ideas learned to problems in any context. One way of achieving this balance, he shared, was to restructure the Precalculus curriculum and remove some of the concepts from the syllabus.

Instead of engaging with the curriculum while collaborating with his colleagues he opted to reflect about it. Justin shared that he spent a lot of time reflecting about his classroom experiences. According to Justin, this reflection was helpful in allowing him to become aware of the effectiveness of his lessons. Gleaned from Justin's accounts of his reflection, his engagement with the curriculum through reflection happened while he thought about challenges, planning lessons, student needs and overall teaching and learning of mathematics. Justin explained that he reflected often about his lessons and how to improve them. It was just him thinking about his students' learning experiences and how to improve them. Guidance on effective reflective practice that can allow professionals to improve their practice could have been beneficial for Justin.

All three instructors had their own ways of engaging with the curriculum and they were presented with different learning opportunities based on their engagement. From their first engagement with the curriculum during the summer workshop, the instructors began to develop a sense of what the new curriculum entailed. They were introduced to the investigative nature of the problems in the curriculum; its focus on allowing students to develop concise mathematical meanings, especially about covariational reasoning; and the pedagogical techniques that would allow the instructors to aid their students' learning. During the workshop the instructors engaged with the investigations and received guidance from the workshop facilitators about their students' needs, possible challenges during implementation of the curriculum, and the big ideas to focus on. These experiences provided a foundation for them to implement the curriculum. Over the two semesters, the instructors continued to engage with the curriculum while they planned their instruction, enacted their plans inside their classrooms, shared their thoughts with their colleagues while collaborating, or reflected on their own. Their experience with the summer workshop, combined with their ongoing engagement with the curriculum allowed them to develop a familiarity with the curriculum. They developed a clear view of its goals and gained a broader perspective of how the concepts in the curriculum were connected. The instructors continued to work towards their goal of designing effective learning experiences for their students.

Chapter 6

Discussion, Implications and Conclusion

In this study, I examined the ways in which adjunct instructors engage with a research-based Precalculus curriculum, and the ways in which their use of the curriculum influenced their professional knowledge. Professional knowledge includes instructors' content knowledge, pedagogical content knowledge, and instructional practices. I employed, a case study methodology because it lends itself well to my research. In this study each case consists of an adjunct instructor's engagement with the curriculum. I leveraged the DCE (Brown, 2002) framework to analyze instructors' engagement with the curriculum and their learning opportunities. In this chapter I discuss my findings, implications of my study as well as areas for future research.

Discussion

The case studies allowed me to zoom into the framework and look at the ways in which adjunct instructors engaged with a research-based mathematics curriculum. A point to note is that case studies do not lead to scientific generalizations. Instead, the goal is to generalize to theoretical propositions and not to an entire population. I am using instructors' engagement with a research-based mathematics curriculum to modify Brown's (2002) DCE framework. I analyzed instructors' engagement with the curriculum as they planned, enacted, collaborated and reflected. Here I report their opportunities for learning that emerge from their engagement.

The need for this research is grounded in the efforts to improve STEM education and retention of students in STEM fields. Teacher professional development is one such

effort to support teachers in implementing research-based curricula (Ball & Cohen, 1999). Implementing research-based curricula may prove to be challenging for teachers especially if they have experience using traditional curricula in the past, and they may need support and guidance to help their students (Thompson & Carlson, 2017). In order to develop support for these teachers, it is important to understand their experiences.

I have analyzed the experience of adjunct instructors, a population of teachers that is of special importance within the teaching and learning of mathematics. There is a growing trend of employing adjunct instructors by higher education institutions (Mason, 2009; Curtis, 2014), and research on adjunct instructor professional development is needed (Green, 2007; The Delphi Project, 2012; Austin & Sorcinelli, 2013). Research on mathematics adjunct instructors is scarce and there are currently no studies of adjunct instructors implementing a research-based mathematics curriculum. Findings from this study add to this limited body of research within mathematics education.

Generalizing the Results of Instructors' Engagement to Expand the DCE Framework

I employed the case study methodology to analyze instructors' engagement with the new curriculum as it unfolded during two semesters. The strength of this research methodology lies in generalizing results to a theory (Eisenhardt, 1991; Vaughan 1992; Yin, 2009; Ridder, 2017). When comparing multiple cases, the similarities and differences across cases lead to theoretical conclusions (Vaughan, 1992), making case study research a means of advancing theories (Ridder, 2017). Case studies rely on analytic generalization, where the investigator aims to *generalize a particular set of*

results to some broader theory (Yin, 2009). Yin explains that identifying a theoretical framework to generalize the case study results provides a *blueprint* for the study. A detailed theory design provides the researcher with the ability to interpret the data and generalize the results to a broader framework. While theories already exist for some work, other cases may need some effort in developing a framework so that the research design embodies a theory of what is studied. For this project the blue print was provided by the modified DCE framework (Brown, 2002) (see Figure 5).

Brown's (2002) DCE framework represents teachers' use of curriculum materials as a design activity as they use these materials to plan their instruction. A teacher's process of designing instruction is iterative in nature as they select the materials to be used, interpret them and change them as needed. The DCE framework provided a starting point for my case studies, but as explained earlier, I amended this framework to focus specifically on the instructors' engagement with the curriculum. I included in the framework a teacher's engagement with the curriculum as the mediator between instructors and the curriculum (see Figure 5).

The three instructors engaged with the curriculum as they planned their lessons, enacted these plans inside their classrooms, collaborated with their colleagues both formally and informally, and reflected about their experiences. The nature of teachers' engagement with the curriculum provided affordances and constraints to the teachers (Wertsch, 1998). Each form of engagement influenced the instructors' implementation of the curriculum. These forms of engagement (planning, enacting, collaborating and reflecting) were the mediating artifacts that the instructors used as they implemented the

curriculum (Wertsch, 1998). An important characteristic of artifacts is that they aid people in achieving goals they could not have achieved on their own. Furthermore, the nature of the artifact determines the nature of the tasks that can be accomplished with it, and the artifact provides certain constraints and affordances for the task at hand (Wertsch 1991; Wertsch 1998). Instructors' engagement with the curriculum as they planned, enacted, collaborated and reflected helped them in implementing the new curriculum.

Each form of engagement not only acted as a mediator between the teacher and the curriculum but also influenced other forms of engagement. For example, engagement with the curriculum when planning did not take place in a vacuum. When planning lessons, the instructors incorporated the curriculum resources with their own knowledge and experience to design effective learning experiences for their students. However, planning, as a form of engagement with the curriculum, was also influenced by other forms of engagement, like enacting, reflecting, and collaborating (see Figure 13).

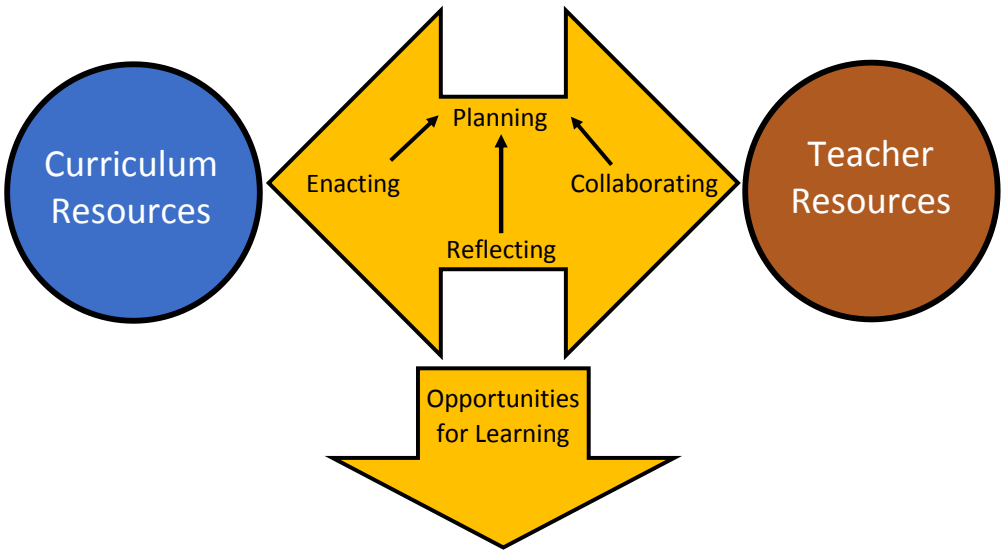


Figure 13. Planning was influenced by other forms of engagement with the curriculum.

When planning, the instructors used ideas they had learned from collaborating with their colleagues during the online PLC meetings or informally in the adjunct office. They reflected on their classroom experiences, thinking about the challenges they faced, as well as their students' concerns. As the instructors engaged with the curriculum, they drew from their various forms of engagement to support their implementation of the curriculum.

Similar to planning, the instructor's, enactment, collaboration and reflection, were also influenced by the other forms of engagement (see Figures 14, 15 & 16 in Appendix C). By enacting their designed instructional plans inside their classroom, the instructors saw the curriculum come alive. They recalled their classroom experiences when planning, shared them with their colleagues and reflected upon them to improve their enactment in the future. Likewise, the discussions they had while collaborating formally and informally were influenced by their experiences when planning their lessons, their classroom experiences as well as their reflections outside of the classroom. Figure 17 shows that various modes of engagement influence each other, as teachers navigate implementing a research-based curriculum.

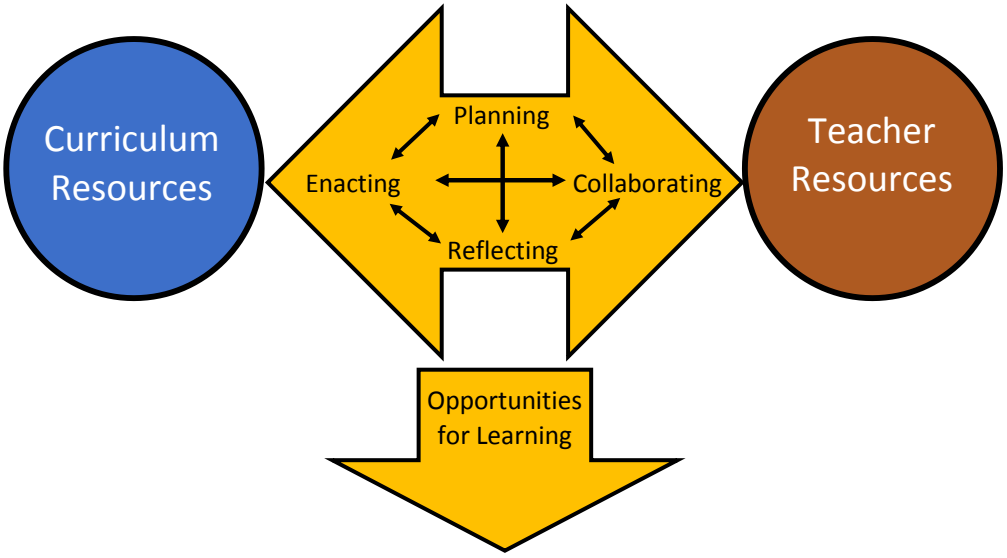


Figure 17. Various forms of engagement with curricular resources influencing each other.

Instructors' engagement with the curriculum took the form of planning, enacting, collaborating and reflecting. While these four forms of engagement influenced each other as the instructors implemented the curriculum, it was when planning and enacting that the instructors often experienced challenges. Collaborating with colleagues and reflecting were forms of supports for the instructors. More than planning and enacting, collaborating provided an opportunity for them to reflect on their teaching practice as they implemented the curriculum. While researchers have noted that PLCs are effective with high school teachers, these cases provide evidence supporting PLCs as a means of professional development for adjunct instructors as well. The instructors also reflected on their own about their classroom experiences, ideas from the curriculum and challenges they faced when planning and enacting it. Reflecting allowed them to think about the curriculum and ways to implement it. Like collaborating, reflecting was a form of support that the instructors could draw on to help them implement the curriculum.

Each instructor's engagement was influenced by their students' experiences. They all had different perceptions of what it meant to help their students have effective learning experiences. As they planned, enacted, reflected and collaborated, these perceptions impacted their engagement. For Caleb, helping his students was embodied by his efforts to improve his own teaching practice. His engagement with the curriculum showcased these efforts, for example as he tried to help his students with the online homework. Likewise, Michael's view of facilitating student learning was associated with his teaching practice. He fell back on techniques he already knew to face the challenges that emerged as he engaged with the curriculum. In contrast to both Caleb and Michael, Justin's engagement revealed a link between developing resources and his students' learning.

Frequently, challenges would appear with planning lessons or enacting designed instructional plans inside their classrooms. The instructors would reflect about these challenges and discuss them with their colleagues in formal or informal settings, with hopes of finding some support in facing the challenges. It was important for the instructors to be aware of their challenges and to determine an area where they needed support. It was also important for the instructors to perceive reflecting and collaborating as being beneficial in helping them with their concerns. The instructors reflected or collaborated in different ways, and their individual way of reflecting or collaborating determined how useful these forms of engagement were as a form of support for their implementation of the curriculum.

New challenges emerged during planning and enacting the curriculum. These challenges were possible learning opportunities for the instructors. In addition, their engagement in the form of collaboration and reflection had the potential to be a support for the instructors. However in order to draw on these resources for support, the instructors had to (1) be aware of the challenge, and (2) be able to effectively engage in collaboration and reflection. For example, Michael communicated an awareness of the challenge that his students faced in terms of their independent problem solving. He had already built a rapport with another colleague to observe his classroom. Here, guidance in the form of effectively observing a classroom was a potential area for his professional development. Justin identified his students' lack of engagement as a challenge, however his did not access collaboration with colleagues as a form of support. He relied instead on reflecting about his students' challenges and his classroom experiences. For Justin, guidance on using reflection to improve his teaching practice would have been beneficial. I have identified areas for future research associated with these experiences in a later section.

Implications

Instructional interactions between teachers, students, curriculum materials and content take place when a new curriculum is implemented (Remillard & Heck, 2014). Cohen and Ball (1999) stress that teachers' instructional capacity is dependent upon the interaction between teachers, students and materials (see Figure 1). The support that institutions provide teachers to develop instructional capacity can lead to increased

student performance, decreased achievement gaps (Simmons, 2011) as well as sustainable reform (Cohen & Ball, 1999).

Teachers' conceptions of their students' thinking and how they address classroom discourse influence their instructional capacity (Ball & Cohen, 1996). Their perception of their own instructional capacity can influence teachers' use of resources. The ways in which teachers use their understanding of curricular content, and how this content impacts their students' learning, impacts how teachers make instructional decisions (Cohen & Ball 1999).

Analysis of the three instructor's experiences, revealed that their engagement with the curriculum took the form of planning, enacting, collaborating, and reflecting. They faced many challenges when implementing the curriculum, and overcoming these challenges provided opportunities for their learning. Often, the challenges were experienced by the instructors when they planned their lessons or when they enacted the curriculum while working with their students. The instructors made these challenges the focus of their discussions when collaborating with colleagues, or the instructors would reflect about possible ways to overcome these challenges. In order for these challenges to become learning opportunities, the instructors had to be aware of them as possible areas of improvement that they could work on. Being aware of a challenge and connecting it to their own professional development provided motivation to seek support. These findings have implications for developing professional development programs for adjunct instructors.

Research on developing effective professional development programs for teachers suggests that continuous professional development efforts are more effective than a single, one-time workshop to provide learning and growth opportunities (Ball & Cohen, 1999; Guskey, 2002; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003; Putnam & Borko, 1997; Wilson & Berne, 1999). Professional development programs should aim to provide opportunities for long-term growth and be relevant to the instructors in terms of their work (Loucks-Horsley, Stiles, Mundry, Love & Hewson, 2010). For adjunct instructors, their working situations provide constraints on the professional development programs that they can engage in (The Delphi Project, 2012). However, in this study, I provide some ways adjunct instructors can benefit from ongoing professional development.

Instructors' participation in the summer workshop introduced them to the investigative nature of the curriculum. Over the course of the two and a half day workshop, the instructors received a brief overview of pedagogical techniques to engage their students in solving problems. For example, from the summer workshop they learned about the importance of classroom discourse in students' learning. However, they needed ongoing guidance in how to foster discourse in their classrooms. The PLC meetings during both the semesters focused more on the content than pedagogy. During the second semester, instructors discussed their experiences in the classroom, shared their instructional strategies and asked questions about teaching techniques, but the overall focus of the meetings was still more on content than pedagogy. Shifting the focus of the PLCs to address the instructors' concerns would make the PLCs more relevant to them.

For example, pacing was a common concern for all the instructors. Drawing from research in K-12 education can help allay this concern for adjunct instructors. In a later section I provide suggestions for further research to improve PLCs and make them more relevant for adjunct instructors.

In addition, while pacing guides communicate information about the expectations of an institution in terms of what should be taught (David, 2008) they can be improved to facilitate their use. Teachers often find it hard to balance between covering the list of topics included in the guide and spending time in class to allow for learning (David, 2008; David & Greene, 2007). In order to cover all the topics listed in the pacing guide, teachers favor direct instruction that is more predictable and seems more efficient (David, 2008). Pacing guides themselves are beneficial in guiding the teachers as they plan their instruction (Kauffman, Johnson, Kardos, Liu, & Peske, 2002) but they can be made more effective to support the teachers. Instead of focusing on a list of topics, the guides can include the big ideas to be focused on, provide links to sample lessons and instructional strategies (David, 2008). Providing such supports is especially important for adjunct instructors due to the time constraints on their schedules. The pacing guide can be supplemented with suggestions for effective pacing in their ongoing professional development.

Limitations and Areas for Future Research

I selected the case study methodology to examine instructors' engagement with the curriculum and their emergent opportunities for learning as a result of their engagement. A research design based on case studies is appropriate for answering my

research questions because case studies have allowed me to analyze instructors' engagement with the curriculum as it happened during the first two semesters of implementing it. According to Yin (2009), case studies are most suitable for answering *how?* or *why?* questions when the investigator is examining a phenomenon that is contemporary and has little control over it. In order to develop my case studies I used data collected from various sources, interviews, observations and PLC meeting recordings. My goal in using multiple sources of evidence was triangulation of data sources to view instructors' engagement through these data sources. This triangulation of data sources was not feasible for all the themes. While the semi-structured interviews allow a researcher to ask specific questions of interest and also follow up questions, the conversations in the PLC meetings and the classroom observations did not allow access to all the themes. Future research in the area of adjunct instructor engagement with research-based mathematics curricula can be guided by the findings from the current research.

In order to support the instructors in promoting classroom discourse, it would be beneficial to specifically add facilitating classroom discourse as a goal of PLC meetings. With careful planning to incorporate both content and pedagogy in PLC discussions during each meeting, instructors would get a chance to focus on both. In this study, the online PLC meetings already provided a collaborative space for the instructors. Within that space, allotting specific times for content as well as pedagogy would encourage the participants to think about both. In terms of pedagogy, a facilitator can provide suggestions on how to improve student discourse in the classroom or ask instructors to

share their own success stories. Having different sections within the PLC meetings would encourage more participants to join. For instance, if an instructor felt confident about the content but had questions and specific concerns about motivating students, then they would have a space where their needs would be met. More research is needed to understand adjunct instructors' engagement with the PLCs.

In addition to providing specific allotted time for content and pedagogy, the participants might also benefit from receiving feedback on their own teaching. As mentioned earlier, in order for the challenges to become learning opportunities, instructors need to be aware of the challenges they experience in the classroom as somehow connected to their own teaching practice. While being mindful of instructors' comfort with receiving critique on their own teaching practice, it would be beneficial for the facilitator to develop opportunities where nuances of effective teaching practice can be discussed. During interviews, PLC meetings, formal and informal discussions with the instructors, and end-of-semester meetings, many such suggestions were shared by the instructors and were welcomed warmly by the group. Some of the suggested approaches were actually observing the other instructors' classrooms to get a sense of their teaching practice, their classroom environment and their students; encouraging the instructors to use the PLC meetings as a collaborative space to develop a lesson together; for the facilitator to share pedagogical techniques that were successful in some of the instructors' classrooms during an observation; for the instructors to watch a video (either from one of their classrooms, or from a video repository) and to critique it as a group. Future research

may be conducted to explore the benefits of guidance on specific pedagogical techniques for adjunct instructors.

Current research on collaboration at the post-secondary level supports the benefits of collaboration and community building for faculty (Uchiyama & Radin, 2009; Kelchtermans, 2006; Hindin, Morocco, Mott, and Aguilar, 2007; Briggs, 2007; Demir, Czerniak, & Hart, 2013; Lester & Kezar, 2012; Hutchings, Huber, & Ciccone, 2011). However there is a need to conduct research on designing professional development and support opportunities specifically for adjunct instructors (Bettinger & Long, 2005; Green, 2007; Gappa, Austin, & Trice, 2005, 2007; Lyons, 2007; The Delphi Project, 2012). Research on developing collaboration opportunities and guidance on improving their teaching practice may help to fill this gap.

Lastly, reflection was commonly employed by instructors as a means to assess their teaching practice. Two of the adjuncts mentioned that they reflected while driving as they commuted between the institutions where they taught. As adjunct instructors, KSU was not the only place where they were employed and driving provided reflection time for them. Reflecting while driving, allowed instructors to think about the curriculum and ways to implement it. Their reflection impacted their engagement with the curriculum while planning, enacting and collaborating. They reflected on the challenges that they faced in their classrooms while enacting their plans. Examples of experiences that provided seeds for reflection included, students' questions, pedagogical challenges like trying to keep the students engaged, comments posted by other instructors during the

PLC meeting, and a problem in an investigation that challenged them when planning their lessons.

Since reflection influenced every aspect of their engagement with the curriculum, guidance on effective reflective strategies could be beneficial to the instructors. Korthagen and Vasalos (2009) suggest that mentoring teachers in ways of effective reflecting can lead to their learning and professional development. Teachers often reflect about specific problems that they seek instant solutions to, and as a result they often select the first solution that comes to mind for a problem without allowing enough time to think about the problem itself. Such quick solutions, while helpful in the moment, can be ineffective in the long run and may even impede a teachers' professional development. Korthagen and Vasalos (2009) suggested a model of reflective cycle where teachers go through stages of reflection where they (1) experience an action that took place, (2) look back at the action, (3) become mindful of the essential aspects of that occurrence, (4) develop an alternate plan and (5) then try it (see Figure 18).

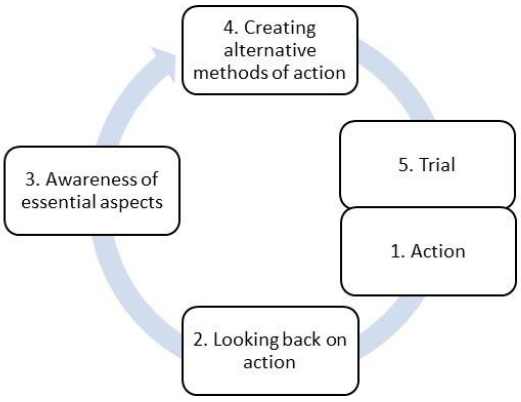


Figure 18. The ALACT model of reflection (Korthagen & Vasalos, 2009).

Instead of leaping from an action to a plan that can be put into action, guiding the instructors to follow the intermediate steps where they take the time to reflect on a challenge, seek advice to gather essential aspects of a situation that they might be unaware of, and then carefully formulate a plan, could improve their reflective practice. Empowering the instructors to become aware of their own reflective practice and then guiding them to challenge themselves even as they reflect, would allow them to use reflection for their own continued learning and professional development.

Current research on reflection at the post-secondary level supports the benefits of reflection for preparing future teachers (Calderhead, 1987; Clark & Peterson, 1986; Hellison & Templin, 1991; McNarnara, 1990; Shulman, 1987; Siedentop, 1991; Zeichner, 1987) as well as for in-service teachers (Fendler, 2003; Hoffman, Artiles, & Lopez-Torres, 2003). Reflecting on practice can be a form of teacher professional development (Zeichner & Liston, 1996; Calderhead, 1992; Cole, 1997; Bengtsson, 1995). As mentioned earlier adjunct instructors' schedules place unique demands on the professional development that they can engage in and there is a need to develop professional development programs that are specific to their needs (Bettinger & Long, 2005; Green, 2007; Gappa, Austin, & Trice, 2005, 2007; Lyons, 2007; The Delphi Project, 2012). Two of the adjunct instructors mentioned reflecting while driving, professional development that utilizes this reflection time may be a unique area to explore for future research.

Conclusion

The three instructors' engagement with the curriculum offered them opportunities for their own learning. The instructors experienced challenges as they engaged with the curriculum while enacting, planning, collaborating, and reflecting. It was the efforts they made in overcoming these challenges that provided opportunities for their learning. Through their engagement with the curriculum, the instructors had the opportunity to learn precise language and specific terminology that the curriculum used to describe concepts. In addition, the instructors' challenges allowed them to explore ways to improve their teaching practice. Professional development programs for adjunct instructors implementing research-based curricula should focus on supporting instructors in first recognizing challenges and then supporting the instructors' efforts to overcome their challenges. This support will allow the instructors to become mindful of the opportunities present for their own learning and encourage them to benefit from these opportunities.

Developing sustainable reform efforts to improve STEM education is a continuing area of concern. Successfully bringing research efforts inside the classroom to influence the students' learning requires providing supports to the teachers. Teachers play an important role in bringing curriculum materials alive inside their classrooms. In order to support them it is important to understand their experiences and the challenges that they face. In addition, research on adjunct instructor professional development is needed because of their increased employment in higher education institutions. In this study I have provided experiences of three adjunct instructors as they implemented a research-

based mathematics curriculum. I have provided steps towards improving professional development for adjunct instructors to support implementation of research-based curricula in terms of content and pedagogy. More research is needed to better understand this complex issue.

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Appendix A

Interview Protocol

Welcome, my name is – I am interested in learning about your experiences with any kind of support that you received this semester. All the answers you provide will be kept strictly confidential and you can ask me to stop at any point during the interview.

1. Please state your name.
2. How many years of experience have you had teaching? At what level?
3. How many years of experience do you have teaching Precalculus?
4. How do you feel about the Pathways curriculum? (End of semester and subsequent semesters: Now that you have taught using the Pathways curriculum, how do you feel...)
5. What challenges do you foresee in implementing the Pathways curriculum? (End of semester and subsequent semesters: Now that you have taught using the Pathways curriculum, what challenges did you face...)
6. What do you think might be some of the benefits of the Pathways curriculum?
7. Have you ever collaborated with your colleagues while teaching? Please elaborate on that experience.
8. Has collaboration with colleagues been beneficial to you in the past?
9. (End of semester and subsequent semesters) Now that you have taught using the Pathways curriculum what do you think you have learned from this experience?
10. What is/was your goal this semester?

11. Please speak about the role of reflection in improving your teaching practice?

12. How have you been planning your lessons?

13. Please speak about the role of reflection in improving your teaching practice?

Appendix B

Classroom Observation Protocol

Teachers will submit their goals for student learning before observation, either via email or state them.

1. Student reflection opportunities: The teacher encourages students to reflect on the reasonableness of their responses.
2. Decentering: Teachers actions that exhibit how they develop models of their students' thinking and use those models to aid instruction.

Student Reflection Opportunities

Student Reflection on answers	1. The teacher asked students if they checked whether their answers were reasonable but did not promote discussion that emphasized conceptual understanding, or
	2. The teacher encouraged students to reflect on the reasonableness of their answers, and the discussion involved emphasis on conceptual understanding.

Teacher Decentering Actions

Decentering Codes	Description
TDM1	The teacher shows no interest in understanding the thinking or perspective of a student with which he/she is interacting.
TDM2	The teacher appears to build a partial model of a student's thinking, but does not use the model in communication with the student. The teacher appears to listen and/or ask questions that suggest interest in the student's thinking; however, the teacher does not use this knowledge in communication.
TDM3	The teacher builds a model of a student's thinking and recognized that it is different from her/his way of thinking.
TDM4	The teacher builds a model of a student's thinking and acts in ways that respect and build on the rationality of this student's thinking and/or understanding.
TDM5	The teacher builds a model of a student's thinking and respects that it has a rationality of its own. Through interaction the teacher also builds a model of how he/she is being interpreted by the student. He/she then

	adjusts her/his actions (questions, drawings, statements) to take into account both the student's thinking and how the teacher might be interpreted by that student.

Notes on classroom discourse

Time that the teacher lectured:

Time students spend working together:

Questions asked by instructors:

Questions asked by the students:

Teacher's responses to students' questions:

(These responses as well as the questions that the instructor asks reflect the instructor's understanding of the material and reveal any perturbations that may lead to their own learning.)

Appendix C

Figures

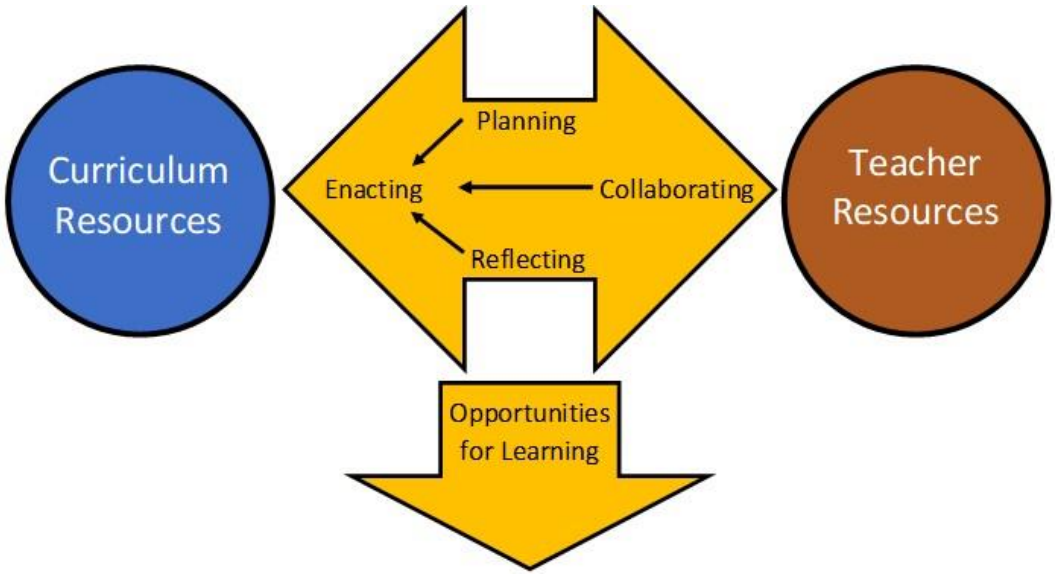

Figure 14. Enacting was influenced by other forms of engagement with the curriculum.

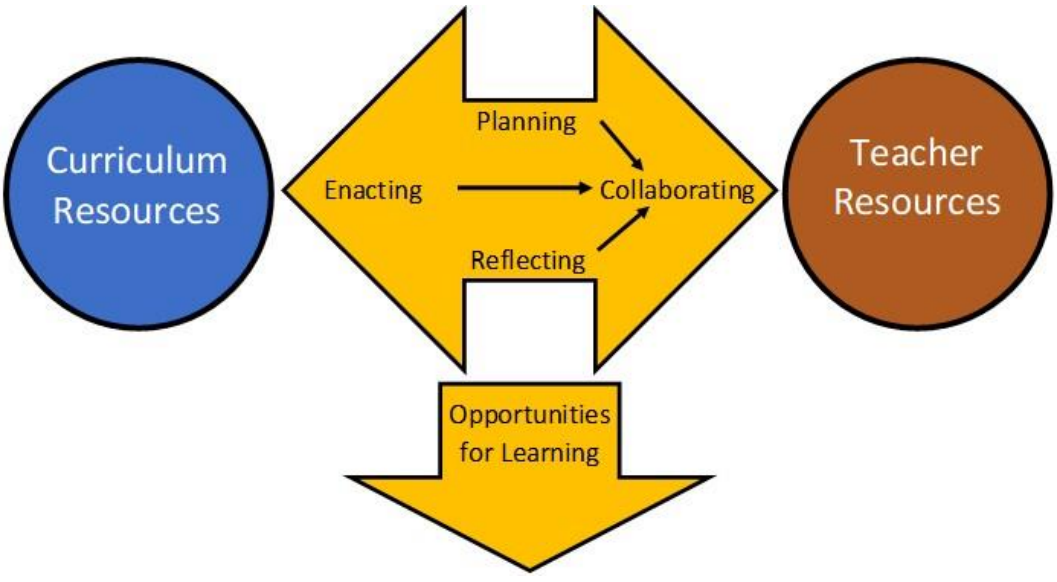

Figure 15. Collaborating was influenced by other forms of engagement with the curriculum.

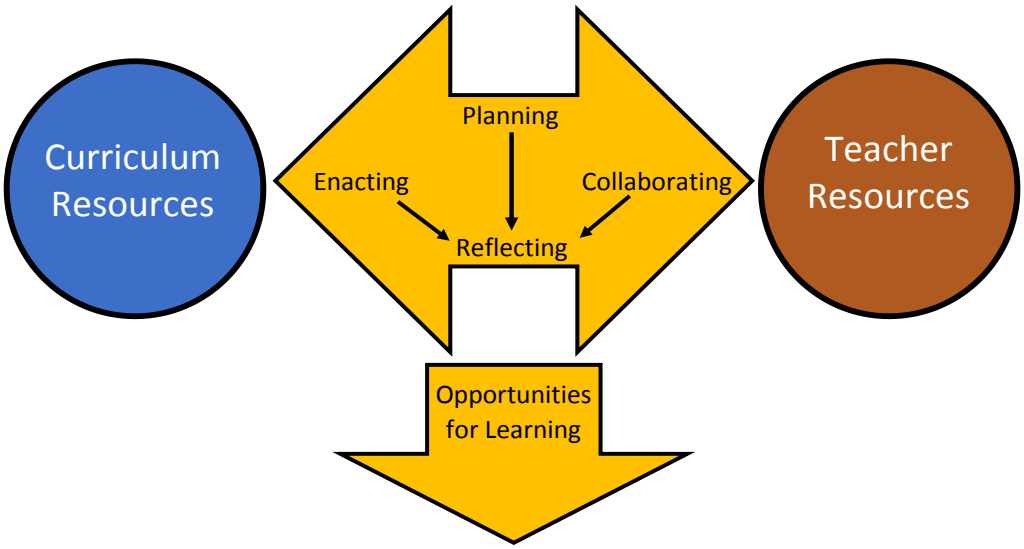

Figure 16. Reflecting was influenced by other forms of engagement with the curriculum.

Appendix D

Tables

Table 3
Engagement Codes

Level 1	Level 2	Level 3
Engagement	Planning Enacting Collaborating Reflection	Specific topic/ Specific problem/ Student

Table 4
Knowledge Codes

Level 1	Level 2	Level 3	Level 4
Teacher Knowledge	General	CK/PCK	Statement/Question/Phrasing used/Explicit change
	Focused on Covariational Reasoning	CK/PCK	Statement/Question/Phrasing used/Explicit change

Table 5
Practice Codes

Level 1	Level 2	Level 3	Level 4
Instructional Practice	Decentering	Phrasing/Questions/Discourse	Statement/Question/Phrasing used/Explicit change/Techniques used/Collaboration/Concern/Student experience
	General Practice	Pacing/In-class experience/Homework	Statement/Question/Phrasing used/Explicit change/Techniques used/Collaboration/Concern/Student experience

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