View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by Collin College: DigitalCommons@Collin

Collin College
DigitalCommons@Collin

Faculty publications | Mathematics

2005

The Harmony of the World

Chris Arthur
Collin College, CArthur@collin.edu

Follow this and additional works at: https://digitalcommons.collin.edu/mathfaculty

Part of the Applied Mathematics Commons

Recommended Citation

Arthur, Chris, "The Harmony of the World" (2005). *Faculty publications*. 1.
https://digitalcommons.collin.edu/mathfaculty/1

This Article is brought to you for free and open access by the Mathematics at DigitalCommons@Collin. It has been accepted for inclusion in Faculty publications by an authorized administrator of DigitalCommons@Collin. For more information, please contact mtomlin@collin.edu.

The Harmony of the World

Christopher Arthur
American Mathematical Society Member
536 Cumberland Drive
Allen, TX 75002, USA
timaeus@users.sourceforge.net

Abstract

Experimental music with mathematics and astronomy is discussed. Chord-like pitch arrangements are determined with geometric proportions arising in planetary movements. Rudimentary digital audio with Fourier series and the JPL on-line ephemeris is developed as a software solution. Computer musicians may listen to and select harmonies by specifying a date in time. A study and application of the ideas in *The Harmony of the World* by Johannes Kepler is presented with a software demonstration.

Musical Intervals in Geometry

The many musical intervals are traditionally ratios which may be expressed geometrically using arcs delimited by constructible polygons. For example, the semitone, 15:16, is derivable from arcs on the equilateral triangle and the octagon.

$$\frac{\text{Arc CGD}}{\text{Arc AFB}} = \frac{\frac{5}{8}}{\frac{2}{3}} = \frac{15}{16}$$

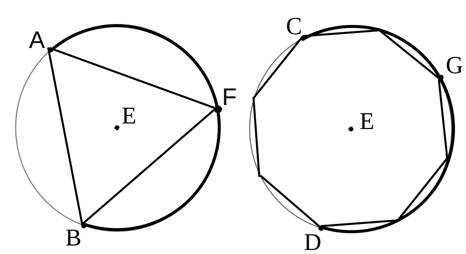

Figure 1. Geometrical derivation of the semitone.

A geometric approach to musical harmonics dates back as early as Euclid [2], with the idea of line segments interpreted as strings on an instrument. Equally ancient is the idea of using astronomy as a compositional determinant in music with the work of Claudius Ptolemy [3], also well-known for his geometric model of planetary motion.

Musical Intervals in Astronomy

The motions of the planets are periodic and may in some aspects be described with intervals, which in turn may be approximated to musical intervals. In particular, angular velocity, ω, due to the elliptical orbits of the planets, has a minimum at perihelion and a maximum at aphelion, as was observed by the astronomer Kepler. The ratio of these two ω–bounds may be analogously interpreted as a distance between two notes in a scale. To find such a distance, the following facts are observed: first, musical intervals are ratios of frequencies; second, smaller intervals combine into larger intervals by multiplication of ratios; and third, as the interval size increases, the ratio decreases (and is always between 0 and 1).

For example, the Martian angular velocity min/max ratio $\omega_m{:}\,\omega_M$ is about 0.69, as computed from the JPL HORIZONS ephemeris system [4]. On the other hand, the musical interval of a perfect fifth is 2:3 = 0.666..., which exceeds $\omega_m{:}\,\omega_M$ by very nearly a triple comma (27:28),

$$0.69 \simeq \frac{\omega_m}{\omega_M} \simeq \frac{2}{3}\left(\frac{27}{28}\right)^{-1} = \text{Perfect 5th} - \text{Triple Comma}$$

where the triple comma is inverted in the equation since in terms of note distances, it is here subtracted from the perfect fifth. In this manner, for each planet is estimated a pitch range, which begins with perihelion and ascends to aphelion.

Digital Audio from Planetary Data

For any given moment it is possible to ask what harmony or musical chord the planets express. Instantaneous angular velocity of a planet may be computed from the ephemeris and mapped to a scale starting (somewhat arbitrarily) at a low G-note or a frequency of 98.12 Hz. Since the slowest of all motions considered here is the perihelion motion of Saturn, ω_S , it is set to this value, and a frequency map, $f\,(\omega)$, is defined as:

$$f(\omega) = 98.12\,\frac{\omega}{\omega_S}$$

An audio signal may be derived from $f\,(\omega)$ by means of a Fourier series $\sigma(t)$ with coefficients a_i and b_i chosen to produce a desired waveform such as a square wave. For digital audio, the number of partials, N, present in the resulting waveform is, however, finitely limited by half the sampling frequency or by the Nyquist rule, which is essentially due to the fact that at least two samples are required per period of any oscillating signal. Since digital CD-quality samples at 44.1 KHz, the general signal is expressed as:

$$\sigma(t) = \sum_{i=0}^{N} (\cos(2\,i\pi t\; f(\omega))\,a_i + \sin(2\,i\pi t\; f(\omega))\,b_i) \quad \ni \quad N \leq \frac{441000}{2\; f(\omega)}$$

The Software

In order to hear the harmonies among the planets, there is developed in Java a computer program (*http://harmony.sourceforge.net/*). Users specify a date, and the program acquires data from the on-line ephemeris, which it uses to compute an audio file in a (WAV) format. A selection of basic waveforms and sound mixing tools are offered so that musicians may experiment with many possibilities.

References

[1] Kepler, Johannes, *Harmonice Mundi*, English translation from Latin by E.J. Aiton, A.M. Duncan, J.V. Field, American Philosophical Society, V. 209, 1997.
[2] Ptolemy, Claudius, *Harmonicorum Libri Tres*, translation by John Wallis, Broude Brothers, 1977.
[3] Euclid, *Greek Musical Writings*, edited by Andrew Barker, Cambridge University Press, 1984.
[4] HORIZONS Ephemeris, *telnet://ssd.jpl.nasa.gov 6775*, JPL Solar System Dynamics Group.