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Nota Técnica:
Modelización por Cadenas de Markov de la evolución de la deformación en vigas de hormigón armado flexotraccionadas

Technical Note:
Markov chain modeling of evolution of strains in reinforced concrete flexural beams

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RESUMEN

A través del análisis de la evolución de la deformación superficial observada experimentalmente en vigas de hormigón armado al entrar en carga, se constata que dicho proceso debe considerarse estocástico. En este trabajo se estudia la utilización de cadenas de Markov para modelizar la evolución estocástica de la deformación de vigas flexotraccionadas. Se propone, para establecer el estado de deformación de estas, un modelo con distribución gaussiana tipo cadena de Markov homogénea de dos niveles (BLHGMC por sus siglas en inglés), cuyo empleo resulta sencillo y práctico. Se comprueba la utilidad del modelo BLHGMC para prever el comportamiento de estos elementos, lo que determina a su vez una mayor racionalidad a la hora de su cálculo y diseño.

Palabras clave: cadena de Markov; hormigón reforzado; modelado estocástico; tensión.

SUMMARY

From the analysis of experimentally observed variations in surface strains with loading in reinforced concrete beams, it is noted that there is a need to consider the evolution of strains (with loading) as a stochastic process. Use of Markov Chains for modeling stochastic evolution of strains with loading in reinforced concrete flexural beams is studied in this paper. A simple, yet practically useful, bi-level homogeneous Gaussian Markov Chain (BLHGMC) model is proposed for determining the state of strain in reinforced concrete beams. The BLHGMC model will be useful for predicting behavior/response of reinforced concrete beams leading to more rational design.

Keywords: *Markov chain; reinforced concrete; stochastic modeling; strain.*

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1. INTRODUCTION

Prediction of the behavior/response of reinforced concrete (RC) structures/structural members is required for design or for scheduling inspection/maintenance activities. A number of studies have been reported in literature on the application of stochastic models for planning of inspection and maintenance activities. For instance, Newby and Barker (1) proposed a bivariate process model for making decision about monitoring and maintenance of systems described by a general stochastic process, wherein the system is monitored and maintenance actions are carried out with respect to the *system state* observed. Recently, a survey of the application of gamma process in modeling *deterioration of systems* for making optimal inspection and maintenance decisions is given by Noortwijk (2). Kuniewski *et al*. (3) modeled the random initiation times and stochastic growth of defects in *deteriorating systems* using a non-homogeneous Poisson process and a non-decreasing time-dependent gamma process, respectively, and proposed a sampling-inspection strategy for the evaluation of time-dependent reliability of deteriorating systems. Nicolai *et al*. (4) presented a model for optimal maintenance of protective coatings on steel structures, wherein *the deterioration of coatings*, represented by the size of the area affected by corrosion, is modelled using a non-stationary gamma process. Barker and Newby (5) determined the optimal non-periodic inspection strategy for a system whose state is described by a multivariate stochastic process. These studies indicate that the application of stochastic processes for modeling the system response evolution is an active area of research for planning of inspection/ maintenance.

In order to ensure the satisfactory performance of the RC structural members against the serviceability limit states under service loads, stochastic modeling of response state evolution under these loads is important. It is well known that the serviceability limit states are closely related to the strain developed in the structural member at the level of reinforcement (for e.g. BS 8110 (6)). Hence, it is important to predict the strains in RC structural members under the service loads. Due to random variations in applied loads and variations in dimensions and material properties of the structural elements, there will be variations in the response of RC structure/structural element. Even at a given/specified load level, due to variations in system properties, the response should be considered as random. Hence, the evolution of response through different stages of applied loading will be a stochastic process. Thus, there is a need to develop suitable stochastic models for the prediction of response state of the structure/structural element at different stages of loading.

Markov chain (MC) models are the simplest stochastic models that are extensively applied in engineering (7-11). MC models have been successfully used for modeling crackwidths in RC flexural members under monotonic and fatigue loading (12, 13), for modeling load-deflection behavior of ferrocement elements (14), for condition assessment of RC bridge girders based on limited inspection data (15) and for condition assessment and remaining life assessment of RC flexural members subjected to corrosion of reinforcement (16). Prakash Desayi and Balaji Rao (12) proposed the use of *nonhomogeneous Markov chains* for modeling the cracking behavior of reinforced concrete beams subjected to monotonically increasing loads, wherein the elements of Transition Probability Matrix (TPM) are computed using Monte Carlo simulation. While in (12), the emphasis was more on understanding the cracking behaviour, use of Monte Carlo simulation may not be a viable option due to the computational efforts required. In the present study, a *bi-level homogeneous Gaussian Markov Chain (BLHGMC)* model is proposed for stochastic modeling of experimentally observed strain variations in three reinforced concrete flexural beams at different load steps. The elements of TPM are computed using the mean and standard deviation obtained from *first order approximation*, thus keeping the computations to a minimum. The predicted statistical properties of strains using BLHGMC model are compared with strains obtained from a probabilistic analysis and also with results from experimental investigations. The results indicate that proposed model shows promise for prediction of stochastic evolution of strains in RC flexural members with loading. *Since the scope of the paper is to evolve a simple methodology for prediction of strain, the application of fracture mechanics is not considered*.

The paper is organized as follows. The data on variation of strain with loading for RC beams used in this study are taken from results of experimental investigations reported in literature, and the salient details of these experimental investigations are given in Section 2.0. The need for stochastic modeling of evolution of strain with applied loading is demonstrated in Section 2.1, followed by the Markov chain modeling of evolution of strain. The step-by-step procedure for determining the evolution of strain in reinforced concrete beams at different load steps using the proposed BLHGMC model is given in the Section 2.3. The application of the methodology is considered in the Section 2.4. In Section 3.0 the results obtained using the proposed model is compared with the results from experimental investigations and the results obtained are critically discussed. Based on the results obtained conclusions are drawn in Section 4.0. Future direction of research is also presented in this section.

2. DETAILS OF EXPERIMENTAL INVESTIGATIONS

The data on variation of strain with loading for reinforced concrete beams used in this study is taken from results of experimental investigations reported by Prakash Desayi and Balaji Rao (17, 18). This data is used in the present study, since strain measurements over the entire constant bending moment region along the span at eight different positions for different loading stages (upto ultimate) for three reinforced concrete beams have been taken and reported, which will be useful for studying the characteristics of strain evolution for developing the models and for studying the usefulness of the proposed model. *Availability of such extensive data is scanty in literature.* Salient information regarding the experimental investigations is given below.

Three beams of similar cross-sectional dimensions of 250 mm x 350 mm and 4.8 m long were cast and tested in two-point bending over an effective span of 4.2 m. Stirrups of 6 mm diameter were provided in the combined bending and shear zone to avoid shear failure, and no stirrups were provided in the constant bending moment zone. Details of the beams are given in Table 1. The constant bending moment zone of the beams (1.4 m) was divided into eight sections (denoted as D′, C′, B′, A′, A, B, C and D on the west face and, D_1, C_1, B_1, A_1, A_1', B_1', C_1' and D_1' on the east face) , with each section having a gauge length 200 mm (see Figure 1). In each section, demec points were fixed at eight different positions on both faces of the beam (east face and west face). As can be seen from Figure 1, position 1 corresponds almost to the extreme compression fibre for all beams; and position 7 in case of beam KB1 and positions 7 and 8 in case of beams KB2 and KB3 correspond to position of steel bars. The beams were tested in two-point loading in a 25 ton (245.25 kN) capacity testing frame. To measure the surface strains at different positions, a demec gauge with least count $1{\times}10^{-5}$ and gauge length of 200.1 mm was used. The loads applied on the beams at different loading stages are given in Table 2.

Table 1. Details of beams (17, 18).

Beam	Effective depth (*d*) (mm)	A_{st} (mm²)	150 mm concrete cube compressive strength (MPa)	Modulus of rupture (MPa)*	Cracking load (kN)*	Ultimate load (kN)*
KB1	311.0	402.123	33.078	4.036	23.549	95.389
KB2	305.4	437.929	40.417	3.578	14.014	104.653
KB3	303.5	529.327	22.508	2.950	8.899	84.291

Note: span (*l*) = 4200 mm, breadth (*b*) =200 mm and depth (*D*) = 350 mm for all three beams
* - obtained from experimental investigations.

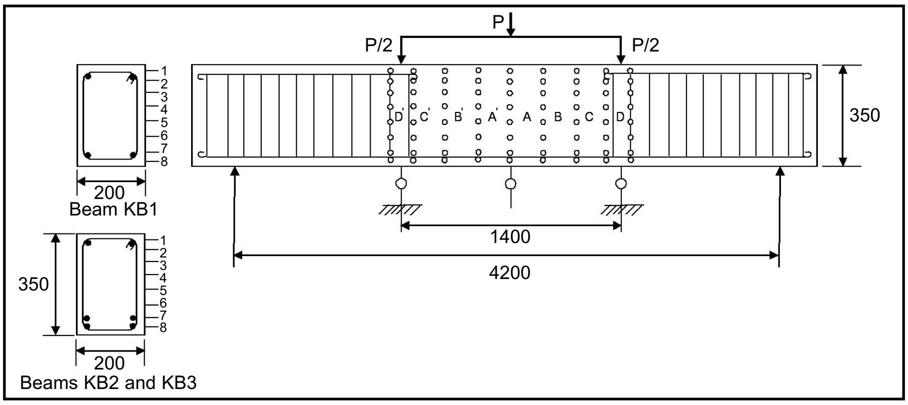

Figure 1. Test specimen (dimensions in mm) (17, 18).

Table 2. Loads applied on the beams at different stages of loading.

Loading Stage	Load Applied (kN)		
	KB1	KB2	KB3
1	10.52	4.48	8.90
2	16.61	8.90	17.80
3	28.99	14.24	26.67
4	38.28	17.80	39.78
5	48.95	22.25	52.97
6	57.37	26.67	70.90
7	65.66	35.42	-
8	-	44.16	-
9	-	57.44	-

2.1. Need for stochastic modeling of strains

The strains at positions 1 and 8 on different sections of both faces of beam KB1 for loading stage 6 are shown in Figure 2. From this figure, it is observed that strains at a given depth and at a given loading stage show considerable scatter. Same trend has been observed for strains at other positions and also for strains at different positions of beams KB1 and KB3, for different loading stages. Generally, analysis and design of reinforced concrete members is based on cross-section analysis (of critical section). The experimentally observed variations in strain, along the length, correspond to possible variations at different depths at a cross-section (as all the sections are located in constant bending moment zone and can be considered to be nominally similar). To take into account this scatter, the strain (ε) at a given depth at a given loading stage should be considered as a random variable. Figure 3 shows strains at different sections (average of strains for the two faces) at positions 1 and 8 of beam KB2 for different stages of loading. From this figure, it is noted that the variations of strains at different sections, at a given depth and for a given loading stage is different for different stages of loading. Treating strain at a given depth as a random variable will not account for random characteristics of strain at previous loading stages. In order to model evolution of strain with loading, taking into account its past history, it has to be modeled as a stochastic process, i.e., when nominally similar reinforced concrete beams are subjected to same loading, evolution of strain at a given depth with loading should be treated as a stochastic process. In the strict sense, the process should be treated as stochastic along the length of beam also. But in the present study, only evolution of strains with loading is considered as stochastic.

To study correlation between strains (at a given depth) at two successive stages of loading, corresponding strains at all the sixteen sections are paired together. These sixteen pairs of data are enhanced to 1600 data points using bootstrap method. It is found that there is a strong correlation ($\rho \approx 0.99$) between strains at two successive stages of loading. This indicates that the stochastic process should be described at least as a one-step memory process. Considering the stochastic process to be a one-step memory process (i.e., present state of system can be completely determined by the immediate past state), the process can be considered as one-step Markov.

The variations in statistical properties (namely, mean and standard deviation) of strain, obtained from

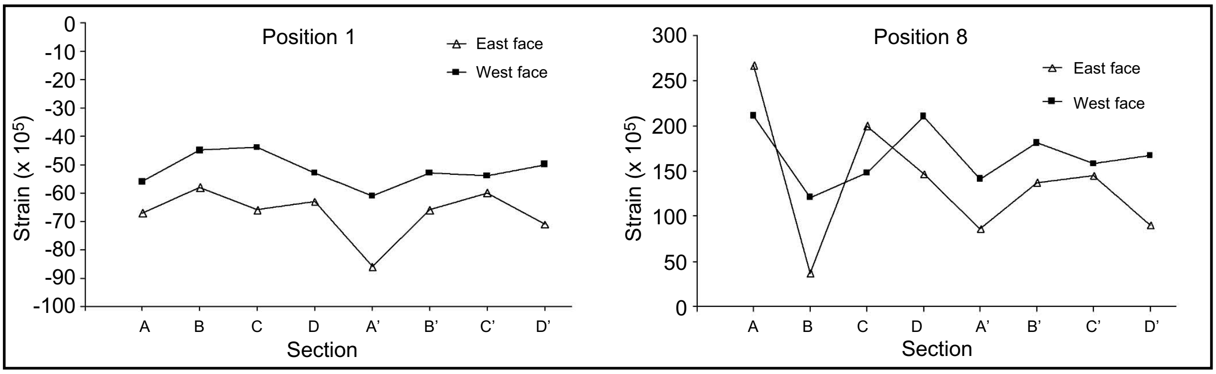

Figure 2. Variation of strain along the span in the constant bending moment region at positions 1 and 8 for beam KB1 (loading stage considered = 6, see Table 2).

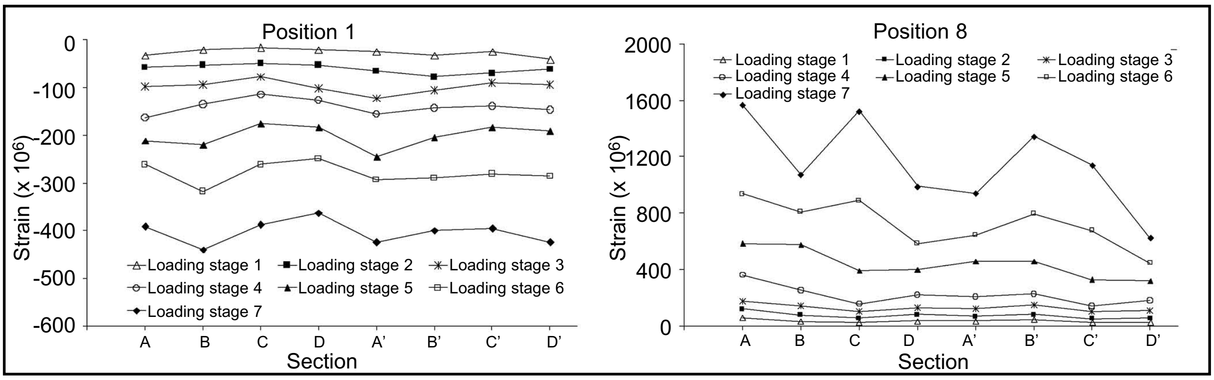

Figure 3. Variation of average strain at different sections along the span in the constant bending moment region at positions 1 and 8 for beam KB2 (refer to Table 2 for details of loading stages).

experimental investigations, at the extreme compression fibre (position 1) and at the level of reinforcement with loading for the beam KB3 are shown in Figure 4. Normally, a good design will not allow tensile reinforcement to undergo yielding under the service loads. Assuming the service load to be ultimate load/1.5, it is noted from Figure 4 that a bilinear approximation (with the lines representing the strain evolution before and after cracking) for evolution of statistical properties of strain with loading at a given depth holds good within the service loads. Similar behaviour has been observed for the beams KB1 and KB2 also. A trilinear curve, with the third line representing the evolution of strain after the yielding of tensile reinforcement, would be more appropriate for representing the variations in the statistical properties (see Figure 4). Since in the present study, the interest is in modeling the evolution of strain under the service loads, a bi-level Markov Chain (first level for modeling the behavior before cracking and second level for modeling the behavior after cracking) is used for modeling the strains at different stages of loading.

2.2. Markov chain modeling of evolution of strain

From the above discussion, it is noted that evolution of strains at a given depth with loading should be treated as a stochastic process. The index space of this stochastic process is the load step, which can be considered as discrete, $\{P_1, P_2, \ldots, P_n\}$. The state space of the stochastic process represents strain range at a given depth at any load step. By dividing the state space into finite number of discrete states (each state represents a finite range of strain at the depth considered in the reinforced concrete beam), the strain can treated as a stochastic process with discrete state space and discrete index space. Hereafter the reinforced concrete beam is referred to as the system. The probabilistic evolution of the process, in general, can be described by the transition probabilities [1].

$$TP = P\{\varepsilon(P_i) = i \mid \varepsilon(P_{i-1}) = i-1, \varepsilon(P_{i-1}) = i-2, \ldots, \varepsilon(P_1) = 1\} \quad [1]$$

In this study, the probabilistic evolution of strain is obtained by making the following assumptions.

- Since all sections located in constant flexure zone (Figure 1) are nominally similar, evolution of strain in one section only is considered in modeling.
- The stochastic process can be described as a one-step memory process. This implies that the process is Markov and present state of the system can be completely determined by its immediate past state. This assumption is justified since strains at i^{th} step (i.e. at load step P_i) more or less depends on strains at $(i\text{-}1)^{th}$ step (i.e. at load step P_{i-1}), as indicated by

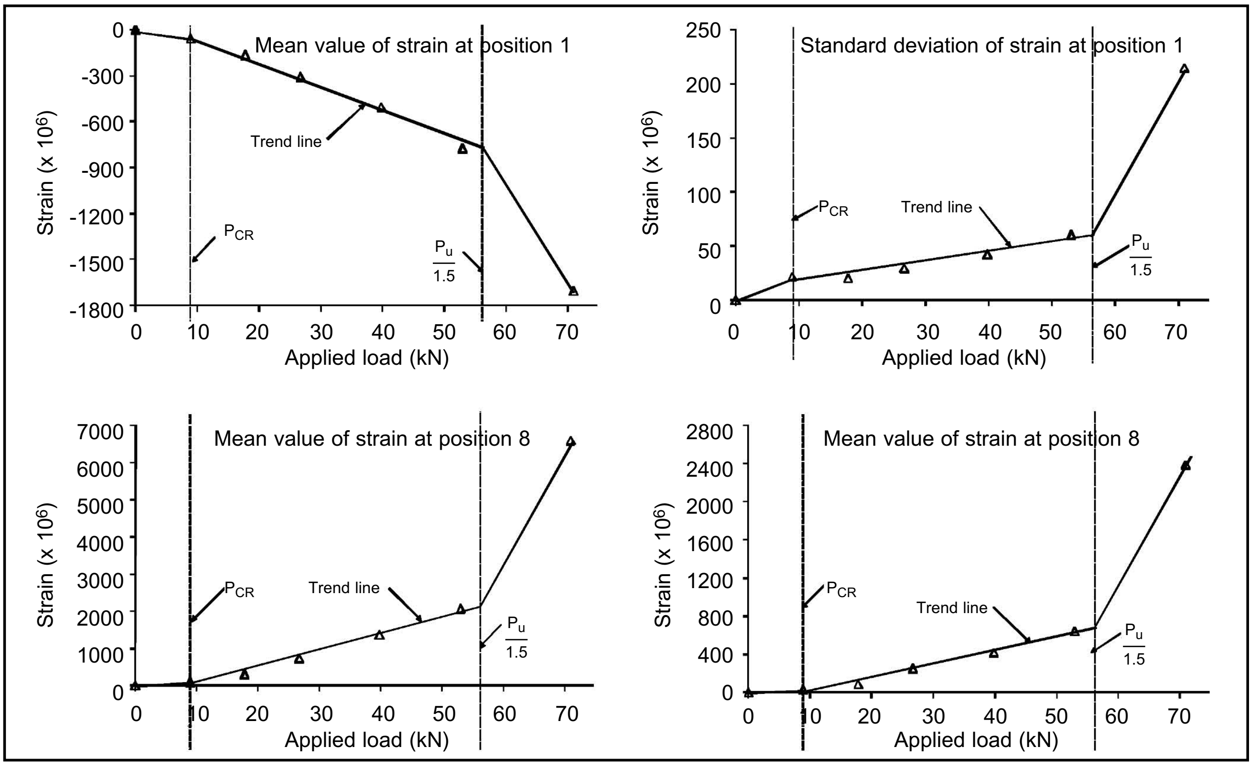

Figure 4. Values of mean and standard deviation of strains at positions 1 and 8 of the beam KB3 (strain data obtained from experiments, P_{cr} and P_u are the experimentally observed cracking load and ultimate load, respectively).

the strong correlation between strains in two successive load steps.

- Since variations in statistical properties (namely, mean and standard deviation) of experimentally observed strains with loading at a given depth can be approximated by a bilinear curve (Figure 4), a bi-level homogeneous Markov Chain can be used for modeling evolution of strains (i.e., first level corresponding to evolution of strains before cracking and second level corresponding to evolution of strains after cracking).
- The stochastic process has a discrete, finite state space {1, 2, …, m}, and, a discrete index space {1, 2, …, n}, where index 1 is interpreted as load step = P_1, index 2 is interpreted as load step = P_2, and so on. The system can make transitions from a given state to all higher and/or lower states. This implies that state of strain can increase or decrease with increase in loading. This assumption is justified due to the following: In tension zone, strain in concrete at a section depends not only on loading but also on the degree of cracking. While the average strain across the beam at a given depth increases with loading, strain in a particular section can increase or decrease due to the formation of new cracks in the tension zone and the state of cracking in the adjacent nominally similar sections. This is consistent with the first assumption. Accordingly, magnitude of strain in compression zone can also increase or decrease.

Using these assumptions, transition probability for the system is given by [2].

$$p_{ij}(P_k, P_{k+1}) = P\{\varepsilon(P_{k+1}) = j \mid \varepsilon(P_k) = i\}; \quad 1 \le i \le m, \ \ 1 \le j \le m, \ \ 1 \le k \le n-1 \qquad [2]$$

The probabilistic evolution of the system is given by the transition probability matrix (TPM) [3].

$$P(P_k, P_{k+1}) = \left[p_{ij}(P_k, P_{k+1})\right]_{1 \le i \le m, 1 \le j \le m}, \quad for \ 1 \le k \le n-1 \qquad [3]$$

Since the state space considered is such that the states are mutually exclusive and collectively exhaustive [4].

$$\sum_{j=1}^{m} p_{ij}(N_k, N_{k+1}) = 1, for \ 1 \le i \le m \qquad [4]$$

2.2.1. Determination of k-step TPM

The probabilistic description of the state of strain after k-load steps (within a given segment of evolution) is given by (Chapman Kolmogorov equation [5]).

$$P(P_1, P_k) = P(P_1, P_2) \times P(P_2, P_3) \times P(P_3, P_4) \times x \ P(P_{k-1}, P_k) \qquad [5]$$

It is noted that the elements of 1-step TPM [3] and *k*-step TPM [5] are conditional probabilities [2]. Since a homogeneous Markov Chain is considered in this study, $P(P_i, P_{i+1}) = P(P_{i-1}, P_i)$. Hence, the *k*-step TPM is given by $P(P_1, P_k) = P^k(P_1, P_2)$. The unconditional probability vector of the state of strain, after *k*-load steps can be determined from [6].

$$\left(P^U(P_1, P_k)\right)_{1\times m} = (P(0))_{1\times m} \times \left[P(P_1, P_k)\right]_{m\times m} \qquad [6]$$

where is the vector representing the probabilities of initial states of the system.

2.2.2. Determination of elements of TPM

A typical element of 1-step TPM [2], can be written as [7],

$$p_{ij}(P_k, P_{k+1}) = \frac{P\{\varepsilon(P_{k+1}) = j \ n \ \varepsilon(P_k) = i\}}{P\{\varepsilon(P_k) = i\}} \qquad [7]$$

which gives the probability of strain being in state '*j*' at load step P_{k+1} given that the strain was in state '*i*' at load step P_k. Computation of these probabilities requires information regarding joint probability density function (jpdf) of state of strain at any two successive load steps, (P_k, P_{k+1}) and pdf of state of strain at load step, P_k. Since it is difficult to generate this information from test data, in the present investigation, it is assumed that strain states at successive load steps follow bivariate normal distributions and at any load step, strain state follows a normal distribution. The choice of bi-variate normal distribution is also supported by the large values of correlation coefficient ($\rho \approx 0.99$) observed between strains in two successive load steps. It is also noted that when values of mean, variance and correlation coefficient between strain states at two successive load steps ($\rho_{k,k+1}$) are known, the maximum entropy distribution would be bivariate normal distribution (19). Similarly, the mean and variance are the only information available with respect to the state of strain at any load step, and in such a case, the maximum entropy distribution is the normal distribution (19). Hence, it is assumed that the state of strain at any load step follows normal distribution. Knowing the jpdf and pdf, and using [7], the elements of TPM can be computed. A typical element of the conditional 1-step TPM is given by [8].

$$p_{ij}(P_k, P_{k+1}) = \frac{\int_{\varepsilon_{i-1}}^{\varepsilon_i}\int_{\varepsilon_{j-1}}^{\varepsilon_j} f_{k,k+1}(\varepsilon_k, \varepsilon_{k+1}) d\varepsilon_k d\varepsilon_{k+1}}{\int_{\varepsilon_{i-1}}^{\varepsilon_i} f_k(\varepsilon_k) d\varepsilon_k} \qquad [8]$$

where $f_{k,k+1}(\varepsilon_k, \varepsilon_{k+1})$ is the bivariate normal distribution and is given by [9].

$$f_{k,k+1}(\varepsilon_k,\varepsilon_{k+1})=\frac{1}{2\pi\sigma_k\sigma_{k+1}\sqrt{1-\rho_{k,k+1}^2}}\exp\left\{-\frac{1}{2(1-\rho_{k,k+1}^2)}\left[\left(\frac{\varepsilon_k-\mu_k}{\sigma_k}\right)^2-2\rho_{k,k+1}\left(\frac{\varepsilon_k-\mu_k}{\sigma_k}\right)\left(\frac{\varepsilon_{k+1}-\mu_{k+1}}{\sigma_{k+1}}\right)+\left(\frac{\varepsilon_{k+1}-\mu_{k+1}}{\sigma_{k+1}}\right)^2\right]\right\}$$
$$-\infty\le\varepsilon_k\le\infty;\ -\infty\le\varepsilon_{k+1}\le\infty;\ \ -1\le\rho_{k,k+1}\le 1; \quad [9]$$

and $f_k(\varepsilon_k)$ is the univariate normal distribution, given by [10].

$$f_k(\varepsilon_k)=\frac{1}{\sqrt{2\pi}\sigma_k}\exp\left\{-\frac{1}{2}\left(\frac{\varepsilon_k-\mu_k}{\sigma_k}\right)^2\right\};\quad -\infty\le\varepsilon_k\le\infty \quad [10]$$

[8] is general and can also be used to formulate TPM for cases wherein the jpdf is other than bivariate normal.

2.2.3. Determination of mean and standard deviation of unconditional strain state at any load step

The statistical properties of state of strain of the system at any load step are computed using the following procedure proposed by Balaji Rao and Appa Rao (13).

- Divide state space into mutually exclusive and collectively exhaustive event sets and compute the central value of each event set, namely, (Δs_1, Δs_2, ..., Δs_{m-1}, Δs_m)
- Compute unconditional probability vector of states of the system at any load step, that is, (p_1, p_2, p_3, ..., p_m) using [6].
- Compute mean and standard deviation of strain in the beam at the position considered using [11].

Mean = $\langle S_{1,k}\rangle=\sum_{i=1}^{m}(\Delta s_i)p_i(P_1,P_k)$ [11]

Standard deviation [12] =

$$SD_{1,k}=\sqrt{\left(\sum_{i=1}^{m}(\Delta s_i)^2 p_i(P_1,P_k)\right)-\langle S_{1,k}\rangle^2} \quad [12]$$

2.3. Proposed BLHGMC model

The step-by-step procedure for determining the state of stain in reinforced concrete beams at different load steps using the proposed BLHGMC model is given below.

1. Determine the cracking moment (M_{cr}) for the beam and the corresponding cracking load (P_{cr})
2. Divide the loading into discrete load steps as {P_1, P_2, ..., P_n} with P_n < P_{cr} for the first level (i.e., before cracking) and {P_{n+1}, P_{n+2}, ...} with P_{n+1} > P_{cr} for the second level (i.e., after cracking).
3. Determine the mean and standard deviation of strain at the initial two load steps before cracking (i.e., at applied loadings of P_1 and P_2) and also at first two load steps after cracking (i.e., at applied loadings of P_{n+1} and P_{n+2}). In the present study, first order approximation is used for determining the mean and standard deviation of strains at different positions for different load steps.
From the results of experimental investigations, it is noted that the average strain variation across depth of the beam can be considered to be linear almost up to ultimate load. It is also noted that after cracking, variation in depth of neutral axis with increase in loading is not significant even at higher stages of loading (the maximum variation in neutral axis depth with increase in loading are 3.27%, 7.11% and 6.57% of total depth of beam for KB1, KB2 and KB3, respectively). Hence, strain in the steel (ε_s) at any given load step is computed assuming a linear strain variation across the depth of beam as [13]:
$$\varepsilon_s=\frac{M(d-x)}{IE_c} \quad [13]$$
where d is effective depth of steel from extreme compression fiber, x is depth of neutral axis from the extreme compression fiber, E_c is modulus of elasticity of concrete and, I is moment of inertia of the cross-section ($I = I_g$ if the cross-section is uncracked and $I = I_{cr}$ if the cross-section is cracked). The mean and standard deviation of strain at different load steps are determined using first order approximation (FOA). *Based on results of simulation, a value of $\rho_{x,I_{CR}}$ = 0.90 is recommended for futuristic applications* (for flexural members).
4. Determine the elements of 1-step TPMs for both levels (i.e., before cracking and after cracking) using [8]. (Based on the detailed studies presented in this paper, *a correlation coefficient ($\rho_{k,k+1}$) of 0.90 is recommended* for evaluation of the elements of TPM).
5. Determine the k-step TPMs for different load steps and compute the unconditional probability vector for strain state at any load step using [5] and [6].
6. Determine the mean and standard deviation of unconditioned strain state at any load step using [11] and [12].

From the above, it is noted that the proposed methodology is simple because it makes use of first order approximations to determine the statistical properties of strain used in the computation of the elements of TPM, thereby not requiring any computationally intensive simulations. The application of the proposed model is illustrated in the next section.

2.4. Application

The proposed BLHGMC model is used to determine stochastic evolution of strains with applied loading for the three reinforced concrete beams considered

(17, 18). The predicted statistical properties of strains using BLHGMC model are compared with results from experimental investigations and also with strains obtained from a probabilistic analysis (FOA). The variables considered as random in the analysis together with their statistical properties are given in Table 3.

3. RESULTS AND DISCUSSIONS

3.1. Prediction of mean strain

The values of mean strain at position 1 (near extreme compression fiber) and at position of reinforcement

Table 3. Statistical properties of random variables considered in the probabilistic analysis.

Random Variable	Mean	Coefficient of variation
Depth of neutral axis (X)	from analysis	0.10
Gross moment of inertia of the gross section (I_G) in mm^4	$\frac{bD^3}{12}$	0.05
Moment of inertia of the cracked section (I_{CR}) in mm^4	$\frac{bX^3}{3}+\frac{E_s}{E_C}A_{st}(d-X)^2$	0.15
Modulus of elasticity of concrete (E_C) in MPa	$5015(f_c')^{1/2}$	0.15

Note: m - modular ratio; f_c' - the compression strength of concrete cylinder in MPa.

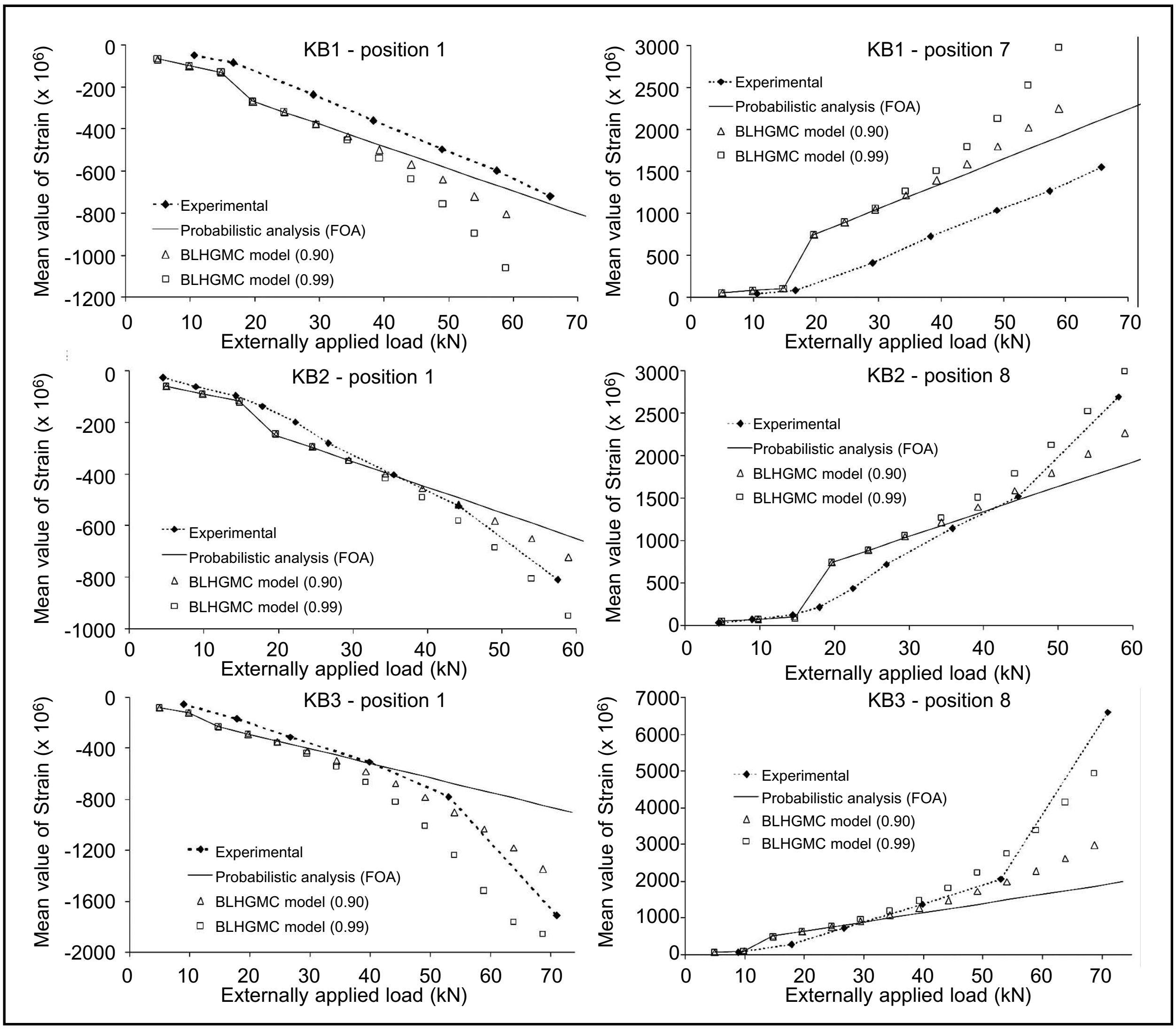

Figure 5. Comparison of mean strains obtained from probabilistic analysis (FOA) and BLHGMC model (values of r used in the model are given in brackets) with experimental results.

(position 7 for KB1 and position 8 for KB2 and KB3) from experimental study are compared with that obtained from the probabilistic analysis (FOA) and from the BLHGMC model (see Figure 5). From this figure, it is noted that BLHGMC model gives better results for beams KB2 and KB3 (even at higher stages of loading), compared to mean strain predicted using FOA. However, for beam KB1, values of mean strain predicted using FOA are closer to experimental values. This suggests that when cracking is dense and distributed well in the tension zone, it is important to consider one-step dependency in prediction of strains.

It is noted that the strains predicted by both the BLHGMC model and the FOA are not in good agreement with the experimental values at higher stages of loading. It should be noted that the assumption of linear strain variation across the depth may not be valid at higher stages of loading. This may be because once the stress in the steel crosses the proportionality limit, the strain in steel varies nonlinearly with stress, i.e., the rate of increase of strain is higher compared to the rate of increase of strain (17). Hence, the strain variation will no longer be linear with stress. A tri-linear HGMC model would have predicted strain values which are in better agreement with the experimental values. But, as already mentioned, since the interest is on stochastic modeling of evolution of strain under the service loads, the assumption of linear strain variations across the depth can be made, and hence a bi-level HGMC model will suffice. It is also noted from Figure 5 that the strains predicted by the BLHGMC model with a correlation coefficient ($\rho_{k,k+1}$) of 0.90 are in better agreement with the experimental results under the service load range. Hence, a correlation coefficient ($\rho_{k,k+1}$) of 0.90 is recommended for futuristic applications.

3.2. Prediction of strain range

Ranges for strains obtained from the experimental studies together with the lower and upper bounds (computed as mean ± 1.64 × standard deviation) obtained from FOA and BLHGMC model are shown in Figure 6. From this figure, it is noted that range obtained from experimental studies are better covered by the bounds predicted using BLHGMC model compared to bounds predicted using FOA for beams KB2 and KB3 (especially the experimental upper bounds which are of more importance). But, for beam KB1, ranges predicted using FOA are closer to the experimental values. For all three beams, upper bound of strains predicted using BLHGMC model are higher than experimentally observed strains (except at very high applied loads), and hence are conservative. These results indicate that the proposed BLHGMC model can be used for determining the stochastic evolution of strains with loading in a reinforced concrete beam.

3.3. Prediction of strain evolution at different depths

The usefulness of BLHGMC model for predicting strain evolution at different depths of reinforced concrete beam is also studied. In Figure 7, mean values and bounds of strain along depth of beam at an applied load of 44.15 kN for beam KB2 predicted using BLHGMC model are given along with mean values and range obtained from experimental study. It is noted from Figure 7 that the proposed BLHGMC model can predict upper bound for strains at different positions in compression zone, and at all positions considered except one in the tension zone. The values of coefficient of variation (cov) of strain across the depth for different values of applied load (greater than the cracking load), for beam KB2 using BLHGMC modeling along with those obtained from results of experimental study are shown in Figure 8. From this figure, it is noted that the trend of variation of cov along the depth obtained using BLHGMC model is similar to that obtained from experimental study. It is also noted that after cracking, values of cov is almost a constant at a given depth for different load steps, but changes with distance from neutral axis, the maximum value of cov being nearer to neutral axis. This is because due to cracking in the tension zone (which is a random phenomenon), there will be variations in position of neutral axis at different section along the span in the constant bending moment region. Hence, at a depth which is close to neutral axis, stresses (and hence strains) will alter between compressive and tensile states as one move along the span. This results in low values of mean strain and higher values of cov of strain near the neutral axis.

4. CONCLUSIONS

Analysis of experimental data of strain evolution with loading of three flexural members suggest that the proposed BLHGMC model shows promise for modeling the stochastic evolution of strains with loading. The methodology developed is simple since it *makes use of mean and standard deviation computed using first order approximation* of strain. Based on the analysis of data, a value of *correlation coefficient of 0.90 is suggested for prediction of strains at different stages of loading*. The BLHGMC model will be useful for predicting the behavior/response of reinforced concrete beams that would lead to more rational design.

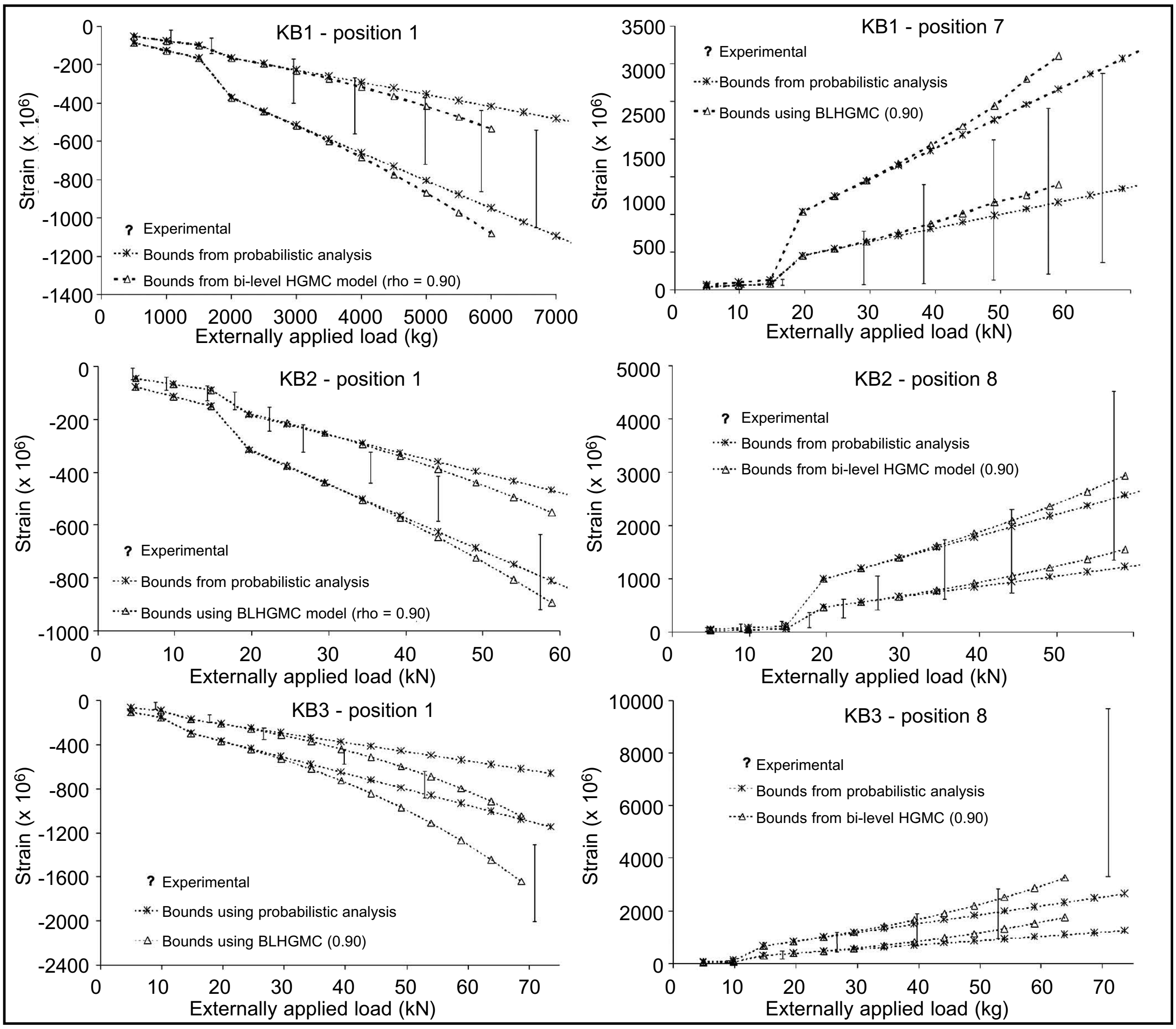

Figure 6. Comparison of ranges of strains obtained from experimental studies with bounds (mean ± 1.64 * standard deviation) obtained from probabilistic analysis and BLHGMC model (values of ρ used in the model are given in brackets).

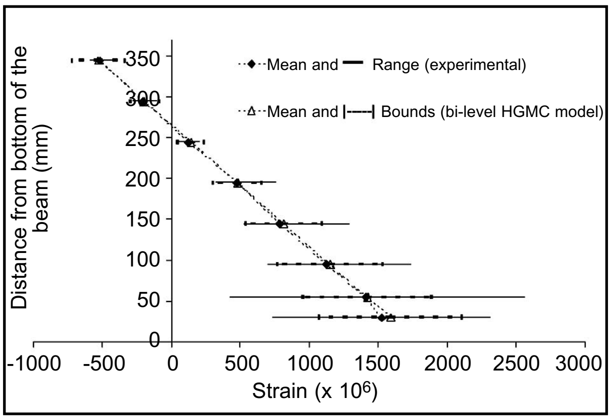

Figure 7. Variation of strain along the depth for beam KB2 (applied load = 44.15 kN, ρ for BLHGMC model = 0.90).

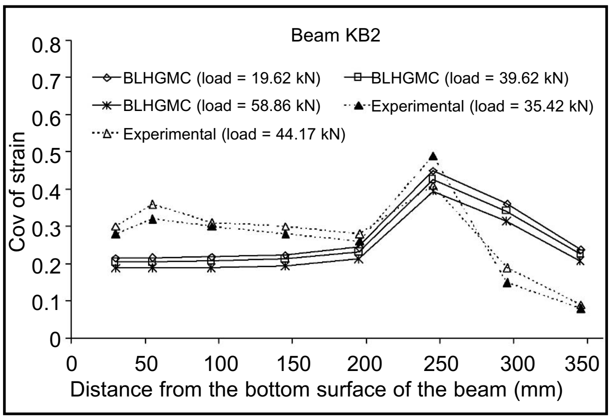

Figure 8. Variation of cov of strain across the depth for beam KB2 (ρ for BLHGMC model = 0.90).

Future direction: For assessment of state of condition/ health of reinforced concrete structures, strain measurements are generally used. In general, during the field investigations, strains are measured using embedded electrical resistance strain gauges or strain gauges with very small gauge length. While they can provide relatively accurate information based on which local condition assessment can be carried out, the effect distributed damage (such as cracking in RC beams) is difficult to account for in overall condition assessment. The BLHGMC model is based on surface strains measured over a longer gauge length of 200 mm. Thus, this model has a potential for predicting the overall condition of flexural member. Also, in this study, only flexural members are considered. The applicability of the present model for development of strains in the presence of combined load effects can be examined.

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