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The Development and Tryout of a Program Designed to Increase Students' Multiplication Fact Fluency

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Dissertation Approval

This is to certify that the thesis presented to us by Linton Williams________

on the _6_ day of __June__, 2013, in partial fulfillment of the requirements for the degree of Doctor of Psychology, has been examined and is acceptable in both scholarship and literary quality.

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Abstract

Fluency skills are fundamental to the ability to complete relatively more complex problems using less mental energy. Most math curriculums do not stress the importance of automaticity of math facts. Variations in teaching styles and teaching modality seem to negate the true purpose for cementing and rapidly retrieving math facts. This study put to the test a program that was designed to increase students' ability to retrieve multiplication facts fluently. The program exercised cognitive structures and enabled students to retrieve multiplication facts quickly. The results of a matched-pairs t-test indicated that after the students completed the course of the program, there was a statistically significant different in the students' ability to retrieve multiplication facts accurately and speedily.

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Chapter 1

Introduction

According to the report of the National Math Advisory Panel (2008), math scores for American children who attend elementary and secondary school have declined. Although math is considered a universal language there appears to be a gap between males and females in the acquisition of math skills and in enrollment in accelerated courses that require higher math competencies (Reardon and Galindo, 2007). In addition, there appear to be cultural and environmental factors that significantly influence the development of math skills, with socioeconomic status being one of the most prominent of these factors (Reardon and Galindo, 2007 & You, 2010). Nevertheless, even within homogenous cultural and socioeconomic groups the gap between female and male levels of math competency during later school years appears to be widening. This widening of the math competency gender gap seems to be greatly influenced by a societal belief that males are better suited than females for learning math and holding math-related jobs (Cheryan, 2012). Additionally, teaching styles appear to have an influence on male and female levels of math skill development (Robinson, Lubienski, & Copur, 2011).

Statement of the Problem

Math curricula and preferred teaching modalities appear to favor students who already possess rudimentary numeracy skills as they enter formal schooling (Doabler, Cary, Jungjohann, Fien, Baker, Smolkowski & Chard (2012). However, for younger students with inadequate preparation, math instruction and the math curricula used in instruction may hinder progress in acquiring the needed prerequisite numeracy skills. Gaps in curriculum and ineffective teaching methods do not effectively engage or adequately strengthen the cognitive capacities needed to develop math skills (Weast, 2000). Mounting evidence indicates that multiple cognitive capacities including executive functions are needed for children to develop math competency (Osmon, Smerz, Braun, & Plambeck, 2006).

After young students acquire knowledge of basic math facts, the speed and accuracy with which they retrieve these facts needs to be developed. Typically, math curricula and instructional approaches can address the learning needs of most students. In some instances, however, math curricula and teaching methods are not varied enough to effectively reach students with diverse learning styles. As a result, the potential of these students for learning and developing math skills is not realized.

Purpose of the Study

This study reviewed the literature on math curricula and teaching methods, cognition and student learning styles, socio-economic status and gender in relation to the development of math skills in young children. Research related to the cognitive capacities underlying the development and improvement of math fluency, calculation, and problem-solving skills was used to develop an instructional program that was used to increase the storage and rapid retrieval of math facts. It was hypothesized that if students used the program on a regular basis, math fluency and accuracy would increase.

Chapter 2

Review of the Literature

Basic math skills are necessary to function effectively in most areas of employment today. Math is the cornerstone of innovation and development in technology and science and developing math proficiency is an essential component of all formal education programs (Geary & Hoard, 2001; Light & DeFries, 1995; Rivera-Batiz, 1992). Although the importance of mathematics skill acquisition is widely recognized, much debate remains over the most effective means of helping children acquire math skills. Several factors can hinder effective math instruction and learning and can contribute to delays in learning, understanding, retaining, and retrieving mathematics knowledge. These include ineffective use of teaching modalities, environmental conditions such as economic disadvantage, and the underdevelopment of learners' cognitive capacities. The focus of this literature review is on the early learning and instruction of elementary mathematics; the cognitive constructs involved with learning, retaining, and retrieving basic math facts; the usefulness of music in the teaching and learning of math, and the influence of environmental factors and student gender on math teaching and learning

Math Curriculum Instruction and Learning

Many school age students begin kindergarten with little or no knowledge of basic numeracy concepts critical to the development of math skills; this is especially true for students who attend public schools in impoverished areas (Lee, Gregg, & Dion, 2007; Rathbun & West, 2004). Despite efforts of the federal government to provide assistance to schools in impoverished areas, the gap between the math achievement of students from affluent communities and students from impoverished communities persists (Agodini, Harris, Atkins-Burnett, Heaviside, Novak, & Murphy, 2009).

Development, Teaching and Learning Mathematic

Human infant brains seem to have a rudimentary concept of numbers (Antell & Keating 1983, Simon, Hespos, & Rochat 1995; Starkey & Cooper 1980; Starkey, Spelke, & Gelman 1983; Wynn, 1992). Seemingly, infants are aware of differences in quantity. Before they receive formal schooling, infants appear to develop a *number sense,* which is the "foundation for learning formal math concepts in elementary school" (Jordan, Kaplan, Locuniak, & Ramineni, 2007, p. 38). This concept of numbers may lie in the cognitive groundwork related to counting, number patterns, magnitude comparisons, and estimating and number transformation (Berch, 2005; Dowker, 2005; Lipton & Spelke, 2003). Careful exploration of how teachers exercise students' number sense may provide some insight on how math instruction should be facilitated. Elementary math instruction should encourage students to have a desire to solve more complex math efficiently and to gain math awareness as they move through the elementary school years. The National Council of Teachers of Mathematics (2000) stated, "Students become confident in their ability to tackle difficult problems, eager to figure things out on their own, flexible in exploring mathematical ideas in trying alternative paths, and willing to persevere" (para. 5). Thus, math instruction should not be limited to teaching mathematics using formulas, procedures, and tales about mathematics to solve problems in a "single specific way"; instead, a shift in teaching dynamics should focus on teaching math in a manner that draws attention to understanding mathematics.

Students could become active problem solvers and connect problem-solving abilities to their environment in ways of mathematical perceptions (Butler, Beckingham, & Lauscher, 2005; Herrera & Owens, 2002; Maccini & Hughes, 1997; National Council of Teachers of Mathematics [NCTM], 1989; Schifter & Fosnot, 1993). The NCTM (2000) suggested that mastery of number sense is needed before one can develop an understanding of mathematics and may be a prerequisite to numerical competencies such as automaticity, accurate recall of basic math facts, and visual estimate or recognition of the number of items without counting them (Grauberg, 1998; Kelly, 2006; McClain & Cobb, 1999; Salend & Hofstetter, 1996). An important primary goal of math instruction to early learners should include building upon students' number sense and helping the students to understand mathematics; this goal is achieved when instruction allows students to solve math problems using a variety of strategies (Gersten & Chard, 1999). Solving math problems using effective math strategies should increase the students' understanding of math and their ability to automatize math facts, which could lead to an increased ability to perform more complex math problems.

Klein and Bisanz (2000) implied the idea that for a youngster's mathematical knowledge to be learned and retained, it is imperative that he or she have a strong foundation of math-related skills. Thus, exploring the learning and retention of math skills might reveal the relationship between math skills and development. Dynamic instruction increases the speed and accuracy at which students perform basic math and automatizing math facts reflects a student's capacity for retention of visual information, retrieval of visual information and visual memory, and motor integration. Lack of fluent integration of cognitive processes that facilitate math skill acquisition may predict limitations in learning mathematical concepts.

Student Learning Styles and Instruction

Instruction in math should consider the fact that some students might learn and perform math concepts slowly or differently. For example, some students may need more time to complete math problems because of underdeveloped fluency skills, but the students may still answer math problems accurately (Ramos-Christian, Schleser, & Varn, 2008). Lovett's (1987) findings suggested that some children are able to demonstrate the accuracy component of fluency without the speed component. To strengthen math fluency skills, a combination of procedural math knowledge and a conceptual understanding are needed. These allow the student to go on to develop the basic calculation and problem-solving skills necessary to perform math tasks fluently (Ginsburg, 1998; Hiebert et al., 1997). Research suggests that students with increased skills in rapidly and accurately completing basic math problem experience less math anxiety and are more likely to attempt additional math lessons (Billington & Skinner, 2002; Cates & Rhymer, 2003).

The National Mathematics Advisory Panel (NMAP; 2008) stated that the break between instruction and learning styles in math could be minimized through explicit instruction focused on meaningful interactions, understood math instruction objectives, and a sequence of instruction that promotes an understanding of math. Students are more likely to learn and retain math concepts at a speed based on their needs and progress (Steedly, Dragoo, Arefeh, & Luke, 2008). This approach seems to consider prior learning, attention problems, executive dysfunction, or other cognitive processes that are involved in learning, retaining, and retrieving math.

A significant number of students in grades pre-K to eight continue to be instructed by teachers who may not have the appropriate certification to teach math effectively (Kilpatrick, Swafford, & Findell, 2001). If the effectiveness of learning mathematics is a result of the teacher’s knowledge and use of mathematical concepts, students taught by such a teacher are likely to be hindered in their acquisition of math skills. Conversely, if instruction blends mathematical content with effective teaching methods and relates to the student's personal experiences, students are more likely to be facilitated in their acquisition of math skills. Thus, effective learning depends on the mutual and interdependent interaction of three critical elements: appropriate mathematical content, a teacher with expertise, and engaged learners.

Considering the diversity of student learning styles, such differences in learning style could create a chasm between teaching and learning mathematics. One approach some teachers may take is show and tell, meaning that the teacher will show the students and will tell the students how to complete the math problem, and in turn, the students will show and will tell the teacher that the students can complete the math problem. This method has merit; however, the approach might not encourage teachers to instruct or encourage students to complete math calculation problems speedily and accurately. Some teachers are concerned with math procedures and performance and give little attention to whether or not a student understands and can generalize math concepts to relative meaning or mastery, as mentioned by critics of the U.S. educational system (Kubina & Morrison, 2000), thereby reducing the priority of automaticizing math facts.

Educators have multiple ways to teach mathematics to early learners, but they must consider the following questions: Do the diverse teaching methods take into account students who have significant difficulty with learning math? Does the diversity in teacher styles maximize student learning potential? Undoubtedly, teachers teach basic math algorithms: the question is whether or not the understanding of math and math concepts are taught equally. Often, students do not understand the reasons why they are learning addition, subtraction, multiplication, and division. They know only that they have to perform a learned or taught algorithm. Teachers who understand that teaching mathematics is a complicated task might not recognize that students sometimes need to understand the reasons why they need to perform mathematics.

Students who enter school with a foundation of math have an advantage. Understanding how a student as young as 4years of age can learn basic math concepts is one key to instructing students on how to become efficient at math calculations, math fluency, and solving math word problems. The need to exercise particular cognitive processes that correlate with learning and retaining math is essential. The National Council of Teachers of Mathematics (2000) stated, “The most direct route to improving mathematics achievement for all students is through better mathematics teaching” (p. 7). Students’ success in math reflects the efficacy of the instruction and can be negatively influenced by poor teaching, the design and materials of the curriculum, or both (Wendling & Mather, 2009). Research conducted by Maccini and Gagnon in 2006 concluded that teachers are inadequately prepared to teach math skills because they take few courses and learn few methods for teaching math.

Mathematics, Learning, and Curricula

Early learners of math develop a number sense, and if taught well, they might increase the understanding they have. However, early learners of math concepts must automatically and accurately recall numerical skills in order to move to more complicated math. After exploring the prerequisites for proficiently learning math, students’ learning styles, and educators’ teaching techniques, careful examination should be directed at curricula and implementation. Researchers sponsored by the U.S. Department of Education stated, “Seven math curricula make up 91 percent of the curricula used by K-2 educators and the curricula are based on different theories for developing math skills” (Agodini et al., 2009, p. xvii).

After carefully examining curricula such as Investigation in Numbers; Data and Space; Math Expressions; Saxon Math; Scott Foresman-Addison Wesley Mathematics, and Harcourt Math, the research showed of these all provided a thorough curriculum, although none appeared to focus on increasing math fluency directly or on strategies to increase the speed and recall of math facts. Wendling and Mather (2009) stated, “Some curriculum designs can be especially troublesome for students who struggle with math, including spiraling curriculums, teaching to mastery, and focusing on procedures versus understanding” (p. 168). Because of this phenomenon, supplemental instruction in math could greatly benefit teachers in instructing students in math and could increase the students’ knowledge, skill, and desire to learn math.

Environmental Factors in Learning Math Concepts

Infants seem to have an awareness of magnitude and nearly all seem to develop or have an awareness of numbers (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Wynn, 1992). As with reading literacy, early exposure to numerical competencies may increase math ability; however, several external factors may compromise mastery of math skill. For example, variables within the home environment and school environment could hinder growth in math skills (Carnine, 1991; Mullis, Dossey, Owen, & Phillips, 1991; Russell & Ginsburg, 1984).

During the preschool years, children should develop a sense of the meaning and constancy (i.e., one-to-one correspondence) of numerals. In assessing readiness, tasks eliciting familiarity with number concepts should be taught and children should show the beginnings of counting skills (Levine & Reed, 2001). The cornerstone and foundation of the mathematics domain is the concept of numbers. As the brain integrates with the world, it constructs additional concepts that it represents as mental models in distributed neural networks (Berninger & Richards, 2002).

When considering how a preschooler internalizes math concepts and the mental processes in which the child engages, environmental factors that may increase or decrease the likelihood of learning math are important considerations. Although little or no research has been conducted to support this claim as it relates to math concepts, children seem to have the ability to distinguish, aurally, a number of voices in a room, in contrast to a single voice. The recognition is evident in the child visually scanning the environment and trying to determine the direction from which the voices originate. When only one voice is heard, the child looks in the direction of that one voice. Children also have the ability to recognize, visually, more objects versus fewer objects. The ability to recognize more than and/or less than could be related to brain structures connected to or activating an emotional response; therefore, a child may have an emotional reaction to many voices versus one voice.

Children have the ability to notice increases and decreases in sound. For example, a child may not notice music at a lower volume simply because the lower volume does not cross the auditory threshold. After the music crosses the child's auditory threshold, the child notices and recognizes the music playing. If the music volume increases, the child may react. A relationship appears to be present between an increase and decrease of sound and a baby's response.

Brain systems that prepare children to learn math concepts and brain systems related to emotional responses and the environment also appear to have a relationship. As a result, math concepts such as more than and less than, and an increase versus decrease may develop in the early stages of life, which suggests the environment may have a significant influence on the ability of children to learn math concepts. Hale and Fiorello (2004) determined that some children with mental disorders have significant psychosocial concerns, yet others seem to be well adjusted. The reason for their differential presentation may be environmental, biological, or most likely, some combination of both.

Cognitive Processes and Math Concepts

Ramos-Christian et al. (2008) stated, “Cognitive abilities as well as math fluency play an important role in mathematical skills, and understanding the relationship between cognitive abilities and mathematical skills is imperative to teaching effective arithmetic skills” (Abstract). Deficits in cognitive processes such as attention and working memory may hinder fluency. Because of the deficits, increased thinking resources are concentrated on attention and working memory for simple processing, which leaves relatively fewer resources for completing or learning more complex tasks. With limited cognitive capacity comes more difficulty with attending, simultaneously multitasking, and encoding information into short-term memory and working memory. Assigning tasks that are short and less complex could allow committing more mental energy to attention and working memory, which could increase fluency (Dahaene, 1997; Delazer et al., 2003; Pellegrino & Goldman, 1987).

Although additional, unknown cognitive processes likely exist that allow individuals to effectively and efficiently complete math calculation, math speed and accuracy, and math word problems, three cognitive processes have been highly researched: (a) information retrieval, (b) working memory, and (c) speed of processing (Bull & Johnston, 1997; Geary, 1990, 1994; Geary, Brown, & Samaranayake, 1991; Hitch & McAuley, 1991; Shafir & Siegel, 1994; Swanson, 1994). In addition, Floyd, Shaver, and McGrew (2003) and McGrew and Hessler (1995) conducted research strongly suggesting a consistent relationship with cognitive ability clusters of comprehension-knowledge, fluid reasoning, and processing speed. Floyd et al. (2003) also noted short-term memory as a strong contributor to math achievement. Likewise, research has linked math achievement with comprehension-knowledge, fluid reasoning, processing speed and short-term memory (Hale, Fiorello, Kavanaugh, Hoeppner, & Gaitherer, 2001; Keith, T. Z. 1999; McGrew, Flanagan, Keith, & Vanderwood, 1997; Williams, McCallum, & Reed, 1996).

Children have to commit counting and one-to-one correspondence to long-term associative memory. Basic math calculations (e.g., subtraction, addition, and multiplication), counting, and answers are paired in working memory. Once the pairing cements quickly and accurately, information is then sent to long-term associative memory. Students have to rely on the retrieval of that information in order to answer basic math problems. Efficiency with counting helps develop this process (Geary et al., 1991; Lemaire & Siegler, 1995). The development of processing speed may contribute to whether or not a student counts quickly or slowly and may determine the rate at which the student can retrieve information; the impact of long- and short-term memory and reading performance are further considerations (Bull & Johnston, 1997; Torgesen, Wagner, & Rashotte, 2001).

When teaching basic mathematical concepts, educators expect students to be fluent and accurate when performing basic mathematical computations. The basic neurological demands required to perform basic mathematics algorithms are complex. Berninger and Richards (2002) stated:

> The Computing Brain [math brain] also borrows from the reading brain, for example, in solving word problems in math, and from the writing brain, for example, in using its transcription module (grapho-motor component) to transcribe visual notations for numbers during paper and pencil computation. The Computing Brain also borrows from many non-language brain systems. In the case of the computing brain, however, the construction process carries over to the completely new domain of knowledge, the quantitative domain, in which numbers or quantity is coded. In contrast to the verbal

> domain that begins by attaching names to objects, the quantitative domain begins by assigning magnitude or amounts to objects. From this simple beginning, the computing brain develops into an eloquent representational system that can be used to describe, explain, and operate upon the structure of the physical world and solve problems in daily living. . . . Thus, the computing brain is multilingual, in this representational format, drawing on quantitative, visual, motor, verbal, and imaginary colds. Likewise, it draws on many streams of thinking, including quantitative, verbal, and visual spatial. (pp.193-194)

More specifically, psychological processes that are involved with the acquisition of mathematical calculations, word problem, and math fluency skill require a combination of the following Cattell-Horn-Carroll processes, as described by Hale, B. (personal communication, February 2009), in order to perform math algorithms: (a) visual-spatial processing, (b) sequential/short term memory, (c) visual-spatial motor integration, (d) cognitive processing speed, (e) working memory, and (f) executive functions. Mathematical word problems also require a combination of the preceding processes because they includes crystallized intelligence/knowledge and quantitative knowledge.

The interaction between the cognitive processes when completing math computation and math reasoning are as follows: The dorsal stream pathway is primarily responsible for gathering sufficient external sensory information for interpretation. This information allows the somatosensory cortex to make an informed decision on the body movement that is required. The ventral stream, automaticity timing and object processing is activated.

The ventral system carries encoded motor programs in nerve impulses to respond to the interpreted sensory information (Snider, Arthur, Thompson, & LeSage, n.d.); the dorsolateral prefrontal cortex and direction left, and spatial right are activated. Motor coordination (supplementary motor) and Exner's areas for writing numbers are activated, and primary motor for handwriting number and primary and secondary somatosensory for sensory feedback is active. Word problems that involve a degree of reasoning are carried out first through auditory processing and Wernick's Area for language comprehension; then information travels across the Arcuate Fasciculus, which is a large bundle of nerve fibers that connects Wernick's Area to the Broca's Area (Gorelick & Bowler, 2008) for language expression. Substantial executive demands are put into play (prefrontal cortex), and Exner's and Broca's area are taxed for writing numbers and speaking.

The slow process of algorithms, difficulty in working memory, and flexibility difficulty when sequencing and maintaining information in immediate awareness could be the cause of students having difficulty reading numbers aloud and performing poorly on writing numbers from dictation. Difficulties in the inferior frontal gyrus, which breaks down large numbers into smaller numbers, may be responsible for difficulties in that area. In addition, difficulty with learning and remembering basic math facts and the speed and accuracy of completing basic math facts may be related to challenges in recognizing numbers—symbol association and math computational speed. Presented challenges may be related to difficulties in the typical occipital-temporal region of the brain, which might affect speed and accuracy and the automatic recognition of numbers in digits.

Students who have difficulty with visual-spatial ability might demonstrate difficulty with visualizing math problems. Visualizing math algorithms is necessary for completing math word problems; however, difficulties may exist in organization, concept formation, attending to math signs for operation, and lining the problem correctly; these might indicate difficulties in the angular gyrus and thus difficulties with visual pattern recognition of mathematical facts and symbols (Feifer & Defina, 2005; Geary, 1993; Hale & Fiorello, 2004; Von Aster 2000).

Executive Functions and Math Concepts

For these preceding thinking processes to function effectively when performing math calculation, math fluency, math reasoning, or a combination of all math domains, executive functions have to be engaged. Many of the tasks that children who are entering school for the first time face are completely novel, and as such, may place particularly heavy demands on cognitive processes such as short-term memory, working memory, and executive functioning. The need for supporting cognitive competencies may change as children become more skilled in numerical understanding (Ball, Thames, & Phelps, 2008). Findings of moderate to strong relationships between executive function and mathematics ability in developmental literature are consistent with neurological scientific work, demonstrating an overlap in the neural substrates supporting executive process and numerical ability and quantitative reasoning (Blair & Peters, 1997). The student has to be able to use self-regulatory executive functions in order to complete and retain basic math calculation skills and math reasoning skills.

Self-regulation of executive functions could drive what Piaget (1965) suggested is the foundation of understanding numbers; that is, classification, ordering, and one-to-one correspondence. Classification can be defined as

> "The capacity to perceive categorical relationships. . . . It involves the ability to group objects according to specific properties within consistent categories. . . . The capacity to classify develops rapidly between the ages of five and seven and becomes an important contributor to mathematical reasoning. Ordering is the capacity to organize materials in a

> logical sequence. . . . This is a requisite for the assimilation of numerical concepts and counting."

One-to-one correspondence means that a particular number of objects has fixed values despite the size or nature of the object (Levine, 1999). Executive functions help the brain communicate with different structures and draw on qualitative codes, numerical codes, motor codes, and visual-spatial codes.

To increase automatic and fluent retrieval of multiplication facts is an apparently simple concept. For a student with difficulty in this area, the seemingly simple task of automatic retrieval presses heavily on other cognitive processes that work in concert with processing speed. To retrieve information from long-term storage, the information would have to be crystallized. Thus, the information has to exist before it is recalled. Scholars suspect this may occur in students who have difficulty cementing multiplication facts in long-term memory. For information to be hardened in long-term memory, other cognitive processes have to align with each other, such as attention, short-term memory, working memory, and long-term storage. The information can then be easily and fluently retrieved using fewer cognitive resources, meaning information could be recalled from long-term storage. Berg and Hutchinson (2010) explained that processing speed functions as a resource to access basic facts from long-term memory. Therefore, information could be automatically recalled; this could free up mental energy to complete higher-level math. Prifitera, Saklofske, and Weiss (2005) stated the following concerning processing speed:

Although little research exists on this topic, it can be hypothesized that children with processing speed deficits may learn less material in the same amount of time, or take longer to learn the same amount of material, compared to those without processing speed deficits. These children may also mentally be tired more easily because of the additional cognitive effort required to perform a routine task. (p. 27)

Students who process information more slowly might take longer to complete multiplication problems. Geary (1993) implied that processing speed may represent a unique source of difficulty for children with a math disability because these children tend to perform mathematics problems more slowly than do typically developing children. Vukovic and Siegel (2010) suggested that processing speed may interfere with the ability to store and retrieve numerical information from long-term memory, likely through a connection with working memory. Processing speed plays an important role in later grades because the mathematics deficits in the group become more severe.

Some researchers suspect that children with math difficulty have less fluency and retrieval. In this impairment, as implied by Berg and Hutchinson (2010), processing speed challenges relate to accessing information quickly from long-term memory. Findings by Bull and Johnston (1997) suggested that number-fact knowledge is related to processing speed. Slower processing speed might cause problems for students in learning math facts and also for students in retrieving math facts once solidified or encoded in long-term memory. If so, the antithesis might be true. Therefore, increasing processing speed when learning and retrieving multiplication facts could become less effortful and could increase the efficiency in other related cognitive processes, such as increasing the capacity of short-term memory (e.g., Case, 1985).

Math Calculation Errors

Deficits in cognitive functioning could lead to slower and more inaccurate calculations in simple math problems for early learners. Inaccuracy and/or slow retrieval of math facts, operational procedure errors, and visual-spatial difficulties have been noted within neurological studies (van Lehn, 1982). When looking at slower or inaccurate retrieval of math facts, a student's ability to attend must be a consideration. For example, a student must be able to register, encode, and temporarily store auditory and visual information; rehearse that information, and send it to long-term memory. After the information is stored in long-term memory, the student engages executive function such as retrieval of stored math facts in order to recall math facts accurately and speedily (McCloskey, 1992). Typically, most students who suffer from a specific learning disability in math have difficulty with math fact retrieval (Geary & Brown, 1991). Research suggests that math fact retrieval may also be difficult for young students who do not have a specific learning disability in math (Jordan, Hanich, & Kaplan, 2003). In that case, improving or maintaining math fluency for simple calculation may benefit all early learners.

Domahs and Delazer (2005) essentially echoed this same notion, suggesting that difficulty in fluently recalling math facts is not exclusive to students who have a specific learning disability in math fact retrieval. Examining errors in simple calculation could reveal *cognitive strategies*. When learning math facts, younger students rely on manipulatives: for example, students are encouraged to use individual physical objects such as toys, fingers, flash cards, pencil/paper activities, and so on. As younger students develop math fluency, they rely on several strategies to increase rapid and accurate recall of math facts; however, the continued use of particular cognitive strategies may suggest difficulty with speedily and accurately recalling math facts (Geary, Hoard, Byrd-Craven, & DeSoto, 2004). Although the concept is not thoroughly researched, the use of manipulatives likely reduces the demand for the use of visual and auditory short-term/working memory. Using repetition of math facts and using manipulatives should crystallize math facts for quick and accurate recall. Most curricula introduce manipulatives and do not focus on specific or direct strategies to increase the chances for students who learn differently to speedily retrieve math facts.

Geary, Hoard, Nugent, and Byrd-Craven (2007) revealed that the underdeveloped ability to manipulate mental math facts, operational performance, and speedy and accurate recall of math facts errors exist with most students who suffer from a specific learning disability in math. Examining errors by observing self-correction might give an indication of a student's ability to inhibit impulsive responses. When completing problems, math students can self-monitor their performances and rethink over-learned automatic responses or strategies that may hinder accuracy of math fact retrieval. The ability to inhibit automatic responses or ineffective strategies usually increases for early learners (Espy, 1997; Espy, Kaufmann, McDiarmid, & Glisky, 1999; Levin et al., 1991); however, most curricula do not offer aids to increase self-monitoring, which in turn could improve the ability of a student who has retrieval difficulties to increase the accuracy of his or her math fact recall.

Visual-spatial difficulty may hinder the speed of written math fact retrieval. Students who have challenges with attention and impulsive behavior may have challenges with visual motor integration (Stevens, Stover, & Backus, 1970; Zentall &Kruczek, 1988). An assignment in which students have to write a math calculation problem and then perform the operation might misalign math problems, which may slow up the student's ability to recall math facts. Students with agitated visual motor skills may benefit from reducing the amount of writing by providing the written calculation and requiring the student to provide only a written answer. Reducing the demand for writing also may increase the speed of processing visual information, thus increasing accuracy. Younger students who are easily distracted, overly energetic, and sub-vocal while performing simple math calculations might be under-aroused. Students may benefit from increased levels of stimulation while completing basic math calculations, such as music directly related to the math lesson (Radosh & Gittelman, 1981; Zentall, 1993; Zentall & Zentall, 1983).

Sex, Cultural Issues, and Mathematics

No concrete information definitively concludes that significant cultural or gender differences negatively affect math performance. Investigation might address the question of how environmental and internal factors such as anxiety affect executive functions, which in turn may have an adverse effect on math performance. For example, students who experience the influence of environmental variables that may create anxiety might suffer from limitations in a taxed executive system when performing math problems. When environmental variables overuse the executive functioning system, students may have difficulty with math calculation and math reasoning. Failure to consider such circumstance could lead to observations based on limited knowledge, creating an over-generalization or an underestimation of different cultures and genders. Technically, researchers have recorded no differences between boys' and girls' math achievements (Woolfolk, 1998).

The specific events in the natural world in which children are raised may significantly influence their roles according to their sex, although math ability does not seems to be influenced by biological makeup (Carr, Steiner, Kyser, & Biddlecomb, 2008; Lachance & Mazzocco, 2006). Some researchers have suggested that as students become older, differences in biological make-up and math performance are readily noticeable (Baenninger & Newcombe, 1995; Hyde, Lindberg, Linn, Ellis, & Williams, 2008). Other researchers have found that Black American girls perform better than Black American boys in high school; however, no difference was found between Asian American boys and girls. Girls performed better on math calculation, reasoning, and conceptual problems, but boys performed more successfully on spatial relation problems (WoolFolk, 1998).

All too familiar is the notion that low-income students, particularly students who attend inner-city schools, are relatively unsuccessful in all academics, specifically math. Several theories and studies have offered every possible reason why inner-city schools perform poorly in math, and most research hovers under the cloud of poverty, race, limited English proficiency, under educated parents, and single-parent families (Natriello, McDill, & Pallas, 1990). The fact that inner-city schools do not have the many needed resources has not been thoroughly researched, although discussion by the academic world seems to consider the situation as a hopeless matter. Academia has, at best, a loosely knit relationship with the inner-city.

Conclusion

After young students acquire knowledge of basic math facts, the speed and accuracy with which they retrieve these facts needs to be developed. Typically, math curricula and instructional approaches can address the learning needs of most students. In some instances however, math curricula and teaching methods are not varied enough to reach students with diverse learning styles effectively. As a result, the potential of these students for learning and developing math skills is not realized.

The research related to the cognitive capacities underlying the development and improvement of math fluency, calculation, and problem-solving skills was used to develop an instructional program that was used to increase the storage and rapid retrieval of math multiplication facts.

Research Questions

It was hypothesized that if students used the program on a regular basis, math multiplication fact fluency and accuracy would increase. The steps in the program development process are described and the following research question was addressed through an analysis of archived data collected with students who used the program to work on multiplication fact fluency: Did students who used the multiplication fluency program improve their performances on a timed assessment of multiplication fact fluency?

Chapter 3

Methods

Program Review

The primary focus of this action-oriented research project was the development of a program that could be utilized as part of math instruction for the purpose of supporting students' development of multiplication math fact fluency. The rationale for program development and the steps in the program development process are described in detail in the result section of this study.

In addition to the development of the program, archived data were accessed and analyzed to determine if students who used the program for six months increased their performances on a standardized multiplication fact math fluency test. The remainder of this chapter describes the data source, the fluency measure used, the study design, and the statistical analysis applied with the archived data,

Data Source

Archived data gathered on students from a classroom that used the multiplication fluency program as part of math instruction were analyzed for this study. The multiplication fluency program was used as part of math instruction in a 3^{th} grade classroom in a suburban public elementary school in the Mid-Atlantic Region of the United States.

Materials Used to Generate the Archived Data

Students' math fluency was assessed using the Math Fluency – Multiplication Subtest of the Wechsler Individual Achievement Test – Third Edition (WIAT-III). The WIAT-III Math Fluency – Multiplication Subtest is an individually administered, timed test of basic multiplication fact fluency. The test comprises 40 multiplication math fact items printed in a

Response Booklet that is provided to the student. The student is allowed 60 seconds to complete as many of the math fact items as possible.

The WIAT-III Math Fluency – Multiplication Subtest was nationally standardized on a sample of 1,400 children in grades kindergarten through 12. The standardization sample was stratified, based on age, grade, gender, ethnic background, parent educational level, and geographic region; its basis is determined by the U.S. Census data in order to reflect the composition of the U.S. population of school age children. Reliability and validity data are provided in the WIAT-III test manual.

Design

The data source was archival data in the form of math multiplication fluency test scores collected prior to the use of the program in math instruction and math multiplication fluency test scores collected four months after initiating the program. The design utilized for analysis of the archived data was a pre-post analysis design.

Data Analyses

A t-test for matched-pairs was conducted to determine if students' math fluency scores improved after using the program for six months as part of math instruction.

Chapter 4

Results

Description of the Program

A workbook and CD developed by the writer of this dissertation was used in this study. The math fluency workbook is 153 pages of math calculation problems ranging from the one times table to the twelve times table. It also contains colorful artwork, arrays, instructions, simple multiplication word problems, fluency speed drills, and uncomplicated multiplication calculation problems, with answers provided for self-monitoring. The workbook is designed to increase the fluent and accurate retrieval of multiplication facts and to support the development of math problem-solving skills.

Exercises in the workbook put demands on psychological processes involved in the rapid retrieval of information from long-term storage, visual-spatial processing, short-term memory, auditory-visual-spatial motor integration, cognitive processing speed, self-monitoring, working memory, and executive functions. The program makes use of both auditory and visual input, basic reading comprehension, math reasoning, and self-monitoring. The workbook attempts to improve the fluent, automatic retrieval of answers to simple multiplication problems through repeated visual, auditory, and kinesthetic presentation of basic multiplication facts, including student practice of retrieving these math facts, completing basic multiplication word problems, and self-monitoring of work for accuracy.

The program CD presents multiplication facts spoken rhythmically to beats that are influenced by the hip-hop culture. The CD is designed to heighten student interest, motivation and engagement and to provide instruction through math fact repetition, motivation. The CD was developed to be used in conjunction with the workbook exercises. Although, the workbook and the CD should be used in concert, both can serve as independent instruction tools. The program was designed to enable differentiated instruction; student can be instructed in small or large groups or can works individually on specific facts with which they may be having difficulty.

Results

A group of 15 third grade students was assessed with the WIAT-III Math Facts – Multiplication Subtest before the start of the program as a daily supplement to the standard math instruction. After four months of daily use of the program, students were re-tested with the WIAT-III Math Facts – Multiplication Subtest. The pre- and post-test scores were analyzed, using a repeated measure t-test. Table 1 shows the pre and post test ages of the students, pre and post test norm-referenced percentile ranks, and pre and post test Growth Value Scores (GVS). See table 1 on p.27.

Table 1

Pre and Post Test Data for Students Included in the Program Tryout

Student	Age at Pre-test	Age at Post-test	Pre-test Growth Scale Values	Post-test Growth Scale Values	Growth Value Differences	Pre-test Percentile Rank	Post-test Percentile Rank
1	8-10	9-4	373	611	238	4	75
2	9-11	10-5	373	496	123	1	5
3	8-4	8-10	479	749	270	68	99
4	8-7	9-1	415	719	304	18	98
5	9-2	9-8	415	584	169	5	53
6	9-2	9-8	373	471	98	2	10
7	8-3	8-9	415	663	248	34	97
8	8-5	8-11	373	676	303	8	96
9	9-9	10-3	394	597	203	1	42
10	8-6	9-0	394	690	296	13	97
11	8-8	9-2	373	529	156	4	50
12	9-2	9-8	373	650	277	2	81
13	8-5	8-11	442	548	106	27	73
14	8-4	8-10	373	650	277	8	96
15	9-0	9-6	373	584	211	2	63

A matched-pairs t-test was completed using the pre and post-test growth scaled values to determine whether or not the post-test mean GVS score was statistically different from the pre-test mean GVS. The pre-test mean GSV for the 15 students was 395.87 and the post-test mean GSV was 614.47. The mean difference of 218.6 between pre and post GSVs was statistically significant ($t = 11.633$, $df = 14$, $p < .001$.

Chapter 5

Discussion

This research and development project attempted to develop a program to help students increase multiplication math fact fluency. The program CD that was developed made use of music specifically composed and performed for this program and of visual presentations of math exercises similar to those used during classroom instruction to develop math fact fluency. A workbook accompanied the audiovisual program CD and was used to provide additional math fact fluency practice. The program was designed to improve math fluency, enhance self-monitoring of production, and increase basic math problem-solving.

The program was provided to a school district and a 3rd grade classroom teacher used the program for six months with 15 students as a supplement to the standard classroom math instruction. Students' math fluency was assessed prior to the start of the program and student math fluency was assessed again after 6 months of weekly use of the program.

Statistical analysis of pre and post program multiplication math fact fluency scores indicated a statistically significant difference, with post program mean performance significantly better than pre program mean performance. Visual inspection of the data provided in Table 1 in Chapter 4 shows that all but one of the students pre-test math fluency scores were below the 35th percentile, whereas all but three of the students post-test math fluency scores were above the 50th percentile. All students' post-program scores were higher than their pre-program scores. For 12 of the students, norm-referenced score gains were extremely large. Of the 8 students that started the program with performance levels at or below the 5th percentile, all but 2 were able to raise their post-program scores to the 42nd percentile or higher. Even the two students that continued to perform below the 42nd percentile showed some growth in their performances at the end of the program trial period.

Conclusion

The purpose of this project was to develop a program that could be used to engage students in an entertaining and interesting way to help them store and quickly retrieve multiplication math facts. The literature on the cognitive constructs underlying the development of math fluency was used as the basis for developing the program. After development of this program, it was used by a classroom teacher as part of her math instructional routine for six months. Pre and post-program multiplication fluency tests were administered and the results were archived. Permission was obtained to use the archived pre and post-program math fluency scores in an analysis to test for statistical difference between pre and post-program math fluency performance.

The results of the analysis of the archived data from a tryout of the program support the hypothesis that when cognitive capacities underlying the development and improvement of learning math are exercised frequently, using a combination of auditory and visual stimuli, storage of, and retrieval of multiplication facts increases. In this study, all students who used the program improved in the fluent, accurate retrieval of multiplication facts.

Contributions to the Field of Education and School Psychology

The present outcomes have practical implications for school-based intervention using math fluency instruction. Results of a comparison of pre and post program math fluency skills support the idea that students' math fluency can be increased through practice with a supplemental program. The program developed during this research project may prove to be a very effective tool for increasing students' multiplication math fact fluency skills. Results of this study suggest that school based interventions for special-education students and for students who are not classified with a specific learning disability but who learn math at a slower rate than same-grade peers may benefit from a math curriculum based on a balanced approach to numeracy that includes teaching math facts. Math fluency instruction may increase the speed and efficiency of retention and retrieval of math facts and free up cognitive resources in order to facilitate the completion of more complex math problems. This might prepare students for completing more advanced, complex math problems.

Limitations

The present study has several important limitations. First, because of the small sample size and the homogenous nature of the sample demographics, the results may not generalize to larger, more diverse samples. Second, the effect of teacher involvement and enthusiasm when administering the program was not directly quantified. A teacher's positive involvement, encouragement and expectations for students who learn differently can foster a very positive atmosphere for learning. The teacher's enthusiasm for and competence with delivering this program was likely to have influenced the results considerably.

Third, the program was designed to be administered in various ways; providing the CD and book to individual students is acceptable, but combining the CD and the book as an interactive unit is considered to be a more effective approach. Combining the book and CD, actively encouraging involvement during group practice, and providing an opportunity for students to take the program home and practice are considered the ideal conditions for maximum efficiency. This study did not examine these different methods for program delivery, so it is not possible to know at this time if differences in the method of program delivery would be factors that differentially influence math fluency performance.

Fourth, this study did not make use of a matched-control study design that could demonstrate that using the program with standard math instruction can increase math fluency more successfully than standard math instruction without using the program. It is possible that students' math fluency skills will have increased with standard math instruction and without using the program.

Finally, some aspects of program development may have influenced the program's overall effectiveness. It is possible that the wording of some of the word problems made it difficult for students to understand what they were to do. During development tryouts, the wording and structure of two specific word problems was reported to be confusing to students. An attempt was made to revise these items in order to reduce the complexity of these two word problems. It is possible that other problems contained in the program were also difficult to understand and may need to be revised. Also, because the CD is influenced by the hip-hop culture, teachers may be reluctant to send the program home with students because they may not know how the parents will respond to this fact.

Future Research

Further research might evaluate the effect of implementing this program in the second and/or third tier of Response to Intervention for students who are specifically identified with fluency deficits. Future research that examines this topic likely would benefit from directly studying whether or not increasing math facts fluency improves problem solving, reduces the use of working memory load, and increases processing speed.

Further research might evaluate the effect of implementing this program in the second and/or third tier of Response to Intervention for students who are specifically identified with fluency deficits. Furthermore, additional research is needed to compare this program with other math fluency based instructional strategies.

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