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# Rigid or Bendable Manipulatives in Conceptualisation of 3-D Space from 2-D Nets in the Year 4 Classroom

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# RIGID OR BENDABLE MANIPULATIVES IN CONCEPTUALISATION OF 3-D SPACE FROM 2-D NETS IN THE YEAR 4 CLASSROOM

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## STATEMENT OF ORIGINAL AUTHORSHIP

*The work contained in this thesis has not been submitted previously for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.*

Signed J. Scott  Date 26/11/09

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Finally, my greatest thanks goes to God – He's the one who brought me here, got me through, and He's the one who will take me further than I could ever imagine.

# DEDICATION

My Grandfather was a primary teacher, and a very passionate one at that. I have always seen him as a good teaching role-model, and I know he would be proud to see that I have followed in his footsteps. So, Tubby, I dedicate this thesis to you. I know you will never see it, but I also know that you would have been proud of me. Thank you for the joy you brought to my life.

# ABSTRACT

Primary school students understand complex ideas on the basis of simpler concepts derived from practical experience. The usefulness of manipulatives (physical objects which may be safely handled) in the primary maths classroom has been frequently asserted. However, little has been reported concerning their use in Stage 2 geometry. The purpose of this study was to compare the effectiveness of two different types of manipulatives, bendable and rigid, as aids for the conceptualisation of 3-D solids from 2D nets (flat fold-outs of solid geometrical shapes) within the NSW Stage 2 Mathematics curriculum. Even though a lot of research has been performed on the use of manipulatives in the mathematics classroom, very few studies have been performed specifically relating to their use in the topic of 3-D space. Contrary to initial expectations, the bendable nets, although more attractive to pupils, did not prove superior to the rigid variety. By far the most significant advances in conceptualisation followed teaching experiences using the rigid nets. It is suggested that this may demonstrate that the greater mental engagement required to visualise 3-D solids from rigid nets may promote greater advances in conceptualisation. This research supports the idea that the use of a range of tactile experiences in the mathematics classroom not only diversifies assimilation pathways, but makes learning more enjoyable. They may even increase motivation to learn on the part of the student.

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# CHAPTER 1

# INTRODUCTION

## Background

Manipulatives are a mathematical resource that have been advocated for many years. However, there has not been a corresponding increase in their use in the mathematics classroom. This was demonstrated by a special edition of the Arithmetic Teacher published in 1986 which focused solely on manipulatives in the mathematics classroom. An article by Worth (1986, p.2) pointed out that although a great amount of support has been expressed for the use of manipulatives, there was evidence demonstrating their use could be more widespread. She cites a survey conducted by Fey (1979, p.12, cited in Worth, 1986, p.2) stating that "9 percent of elementary school classes (K-6) never used manipulatives and that 37 percent used them less than once a week".

It was from this evidence that it was decided that more research was needed in the area of manipulatives in the mathematics classroom. Following a wide search of the literature, it was discovered that very few studies have been performed on the use of manipulatives in the topic of space and geometry, specifically 3-D space. Therefore, this study investigated the use of two different types of nets and their impact on student conceptualisation in 3-D space.

## Significance of the Study

As noted previously, this study investigated a mathematical topic in which minimal research has been reported. This study also provides support for specific teaching strategies that are easily implemented into the Mathematical classroom.

## Research Questions

This study aimed to answer the specific question:

1. Are manipulatives that can be made into 3-D shapes more effective than rigid equivalents in the conceptualisation of 3-D solids from nets in the Stage 2 Curriculum?

## Research Methodology

This study was designed as a quantitative investigation into the relative effectiveness of two different types of geometrical manipulatives. After pre-testing student understanding of the concept, the class was split, with the help of the regular teacher, into two similarly sized groups as closely matched in mean ability as possible. It should be noted that the validity of the results was not significantly compromised by differences between these groups. The ordering of the two experiences with nets was different for each class group. A mid-test and a post-test for each group documented any improvement in concept understanding. It can be seen that at the conclusion of the research project both groups were enriched by exposure to both learning materials, thus eliminating any possible disadvantage to either group.

## Definitions

There are several terms used within this thesis that may require clarification. Their definitions are outlined below.

Manipulatives: Hynes (1986, p. 11) defines manipulatives as "concrete materials that incorporate mathematical concepts, appeal to several senses, and can be touched and moved by students". Young (1983, p.12, cited in Kennedy, 1986, p.6) supports this definition by defining manipulatives as "objects which represent mathematical ideas that can be abstracted through physical involvement with the objects". Therefore, a manipulative is any object used in the mathematics classroom that helps to support a concept.

Nets: one must be careful in the definition of nets in the geometrical sense of the term. One author identified them as a "plane diagram showing all faces of a 3-D shape, which can be cut out and folded to construct the solid" (Ainge, 1996, p.346). This definition only considers the bendable variety of net; however, any flat representation of a 3-D shape may also be classified as a net.

Geometry: this is a topic covered in the mathematics classroom, and is, as defined by Clements and Battista (1986, p.29), "the study of objects, motions, and relationships in a spatial environment".

## Assumptions

The following assumptions were made:

1. Student conceptualisation can be adequately measured by pencil and paper assessment.

## Limitations

The following limitations affected the conclusions of this study:

1. The study was conducted over a time period of just two weeks.
2. The study was done in only one school with one classroom teacher.
3. The sample size was very small.
4. The principal researcher was also responsible for the implementation of the study.

## Structure of the Thesis

This thesis is structured so as to clearly present the methodology, results and conclusions. Chapter 2 presents a review of the present literature relating to this study. Chapter 3 outlines the research method, discussing the instruments used and also the method of data collection. Chapter 4 presents the results obtained from the research, with Chapter 5 discussing these results and presenting conclusions.

# CHAPTER 2

# LITERATURE REVIEW

## Introduction

The teaching of Mathematics in the primary school occurs on a regular basis, with many topics continuing on throughout the primary school years. However, Mathematics for most students is simply "an endless sequence of memorising and forgetting facts and procedures that make little sense to them" (Battista & Larson, 1994, cited in Battista 1999). This may be because the most traditional form of teaching Mathematics is 'lecturing' (Baker & Beisel, 2001), with very few manipulatives used to enhance the teaching of Mathematics (Fey, 1979, cited in Worth, 1986).

## Traditional Teaching

Although lecturing may take on various forms, studies have found that the traditional forms of Mathematics teaching are not only ineffective, but also prevent the development of Mathematical reasoning and problem-solving (Battista & Larson, 1994, cited in Battista, 1999). Battista (1999) also asserts that although the teacher may give several examples of how to solve a problem, the in-class practice and homework set for students do not help them to develop a deep understanding of the concept. He states that, "instead of understanding what they are doing, students parrot what they have seen and heard" (Battista, 1999, p. 8). Many studies offer an alternative to the traditional methods of teaching, concentrating on the use of visual imagery, problem solving and hands-on experiences (Grouws, 1992; Kamii & Warrington, 1999; NCTM, 2000; Wheatley, 1991, cited in Baker & Beisel, 2001). Research undertaken in England, China, Japan and the United States supports the idea that the effectiveness of instruction on students' understanding will be more effective if manipulatives are used in the mathematics classroom (Canny, 1984; Clements & Battista, 1990; Dienes, 1960; Driscoll, 1981; Fennema, 1972, 1973; Skemp, 1987; Sugiyama, 1987; Suydam, 1984, cited in Heddens, 1997).

## Benefits of Manipulatives

Many studies have been performed on the use of manipulatives in the primary mathematics classroom, and the vast majority of researchers advocate their use (Kennedy, 1986, cited in Gresham, Sloan & Vinson, 1997). They enhance the learning of concepts, and students have a more tactile way of connecting Mathematical ideas (Texas Education Agency, 1994). This ability to connect Mathematical ideas enables students to gain confidence in their process of learning, thus increasing their enjoyment (Gresham, Sloan & Vinson, 1997). Additional to this, students enjoy learning through touch, and it is the sense of touch that is the most basic to all learning (Rowan & Bourne, 1994, cited in Gresham, Sloan & Vinson, 1997). There is a hierarchy outlined by Rowan and Bourne (1994, cited in Gresham, Sloan & Vinson, 1997, p. 20), the Texas Education Agency (1994), and Oberholzer (2008) that is associated with the teaching of Mathematics. This hierarchy includes three simple stages: "concrete (working with objects)...semi-concrete (pictorial or representational)...and abstract (mental or symbolic)". It is essential for students to move through the hierarchy in order, beginning each new concept with the concrete stage

(Oberholzer, 2008), enabling students to identify correlations between concepts, building on their prior knowledge.

"Props" as Gresham, Sloan and Vinson (1997) choose to label them, "invite reflection, stimulation, facilitate conversation, and assist explanation in which students can develop and apply abstract ideas, make hypotheses and test ideas".
As Oberholzer (2008) states, it is "through touching and moving objects [that] learning takes place". While using manipulatives to learn concepts, students employ the use of three senses: sight, sound and touch. The more senses that are employed in a learning experience, the more effective and long-lasting the learning is (Oberholzer, 2008). A deep understanding of concepts in Mathematics is important, since, as mentioned before, building upon concepts, and seeing relationships between them is vital to Mathematical learning. As Green, Piel and Flowers (2008) outline, the development of mental Mathematics, achievement and understanding is positively correlated with manipulative-assisted learning. This enables teachers to successfully lead their students through the three stages of Mathematical learning (concrete to semi-concrete to abstract), finishing with a high level of mental Mathematics, something that is important in the real world of Mathematics.

Many teachers may debate this stance by pointing out that students do not always require the use of manipulatives to grasp a concept. However, Jacobs (2008) points out that although this may be true, the use of manipulatives will increase a student's understanding of the concept. The use of manipulatives in the area of problem solving is also advocated (Clements and Battista, 1986).

Heddens (1997) outlines eight aspects of learning promoted by the use of manipulatives in the mathematics classroom. These include the ability to: relate to symbols in the real world; work cooperatively to solve problems; discuss Mathematical concepts and ideas; verbalise thinking; make presentations; identify different methods to solve one problem; identify different symbols for the same problem; and solve problems without only following teacher direction (Heddens, 1997). These eight aspects are vital to the learning of Mathematical concepts, as often students miss out on these, and many simply solve a problem following teacher direction, something they will be without when they enter the real world. In addition to these aspects being vital to Mathematics, they also add to the rest of the curriculum, building students to be more independent in their work, and able to think for themselves.

As Piaget (1970, 1974, cited in Green, Piel & Flowers, 2008) states, "to understand is to invent [and consequently,] conceptual knowledge originates in the inventive activities of the learner through actions on objects rather than from sensory impression or social transmission derived from teachers and parents". It can be affirmed therefore, that Mathematical conceptualisation is much greater when students are able to carry out ideas on objects, rather than just accepting the teacher's or parent's word.

Jacobs (2008) identifies another common use of manipulatives as resources for the teaching of remedial students. However, she states that although manipulatives are beneficial to the remedial student, they should not be used as a last resort (Jacobs, 2008).

## Classroom Use of Manipulatives

Contrary to what might be expected from the results of the research mentioned above, manipulatives are not typically used in the mathematics classroom (Green, Piel & Flowers, 2008). This may not only be due to the traditional style of teaching, but also to the belief that manipulatives are not essential to teaching and understanding, resulting in their use simply as an enjoyable diversion (Green, Piel & Flowers, 2008). Additional to this, research indicates that many teachers feel they are not appropriately trained in the proper use of manipulatives (Jacobs, 2008).

The temptation is then to revert to the traditional form of teaching, which may be superficially combined with manipulatives used as simple demonstrations. However, this form of teaching, as with the purely traditional way of teaching, is not sufficient (Heddens, 1997). The value of manipulatives in the mathematics classroom is maximised when each student uses them in order to explore the topic. As Heddens (1997) outlines, interest in Mathematics will be aroused when students are actively involved in manipulating the materials offered.

Interestingly, the Curriculum and Evaluation Standards document has a basic assumption about instruction in the mathematics classroom. This assumption is that "students will become active learners and that every classroom will be equipped with ample sets of manipulatives and materials" (NCTM, 1989, cited in Gresham, Sloan & Vinson, 1997, p. 19). This stance is supported by Dossey (1989, p. 20, cited in Gresham, Sloan & Vinson, 1997) who suggests that "the curriculum should provide for students' participation, across the grades".

Additional to this suggestion, Dossey (1989, cited in Gresham, Sloan & Vinson, 1997) states that the above structure must involve the students, encouraging them to 'do' Mathematics with manipulatives, then discuss the results of their investigations and conclude with writing up their results. He goes on to explain that the encouragement for students to apply Mathematics using manipulatives helps develop confidence in their ability to succeed in Mathematics (Dossey, 1989, cited in Gresham, Sloan & Vinson, 1997).

Although it is strongly advocated that manipulatives be used throughout the entire primary school (Oberholzer, 2008; Texas Education Agency, 1994; Gresham, Sloan & Vinson, 1997), Jacobs (2008) notes that manipulatives are often only used within the lower primary school for counting purposes. Although this is an important step in relating to the three stages of Mathematical learning, the significance of manipulatives in the middle and upper grades should not be discounted.

## Requirements for Effective Use of Manipulatives

While there are many studies advocating the use of manipulatives in the mathematics classroom, there are also some specific 'requirements' that must be kept in mind when implementing manipulatives in the primary classroom. Firstly, as Gresham, Sloan and Vinson (1997, p. 20) note, using a manipulative approach in the mathematics classroom requires the "knowledge, skills and experiences necessary to respond to students who are learning mathematics". In other words, teachers who employ the use of manipulatives in their classroom must have the ability to encourage appropriate use by their students. Additional to the ability to encourage appropriate use of manipulatives, a careful selection of materials must ensue in order to promote optimal use. The materials that are chosen must be suited not only for the concept being developed, but also for the developmental stage of the students (Heddens, 1997). In addition to manipulatives needing to meet the developmental stage of students, they must also be, as stated by Hamilton (1966) and Reys (1971), "bright, precisely constructed, and aesthetically pleasing" (cited in Hynes, 1986, p. 12).

Another trap into which one may be tempted to fall, is that of giving students a relevant manipulative and expecting them, in the absence of guidance, to discover its purpose and relevancy to the topic (Jacobs, 2008). Jacobs (2008) affirms the value of manipulatives in the mathematics classroom, but suggests that their full worth is realised as teaching accompanies student experience with them. This requires teachers to provide specific activities for the students, so they are aware of the proper use of the manipulatives and are able to use them themselves (Jacobs, 2008).

A third trap into which teachers may fall is using manipulatives as the only tool in the mathematics classroom. As the Texas Education Agency (1994) outlines, not only should the manipulation of concrete materials closely match the concept being developed, but the actions should also be coupled with appropriate questioning by the teacher and reflection by the student. This enables students to identify the purpose of the manipulatives, and be able to relate the process to other concepts.

## Examples of Manipulative Use

Since the benefits of manipulatives in the mathematics classroom have been outlined, some examples of use are now presented. There are many examples of uses in the topic of numeration, and just a few will be outlined here. Fractions, generally recognised as a complex concept to acquire, is an ideal topic in which to use manipulatives. The Texas Education Agency (1994) suggests that the use of pattern blocks, fraction circles, fraction bars and Cuisenaire rods assist students in the understanding of fractions independent of the physical representation. Teachers are able to implement the use of these materials to teach the concept in various ways, encouraging students to develop different processes to approach a problem involving fractions.

The topic of place value is another one which is very suitable for implementation using manipulatives. Base-10 blocks, identified by Green, Piel and Flowers (2008) as containing small units that represent ones, thin rods to represent tens, ten-by-ten flats that represent hundreds, and large ten-by-ten-by-ten blocks that represent thousands, are the most common form of manipulative associated with teaching place value. When used to their greatest potential, and preventing students' dependency on them, base-10 blocks are able to improve students' conceptual understanding of this topic, specifically arithmetic operations (Carpenter, et al., 1999; Fusion & Briars, 1990, cited in Green, Piel & Flowers, 2008).

## Manipulatives in Space and Geometry

Due to this research study focusing on the type of manipulative most beneficial to the conceptualisation of 3-D space from 2-D nets, the use of manipulatives in the general topic of Space and Geometry was investigated. What was found was very minimal. However, it provides some guidance as to the purpose of manipulatives in geometry. Chassapis (1999, cited in Green, Piel & Flowers, 2008) identifies compasses as tools that better aid in the understanding of centre and radius concepts than does simply tracing circles and templates.

In addition to the concept of geometry, Jacobs (2008) discusses the use of cardboard and foam shapes to teach the concept of geometric shapes. She expands on this idea and continues the development of understanding by encouraging students to make their own shapes by gluing toothpicks onto paper (Jacobs, 2008). These two activities help develop the concept of shape by building from the concrete stage to the semi-concrete stage (Gresham, Sloan & Vinson, 1997; Oberholzer, 2008; Texas Education Agency, 1994), while encouraging students to personally create the semi-concrete stage. Once again, the hierarchy outlined earlier is imperative to the conceptualisation of Mathematics.

## Conclusion

As can be seen from the discussion above, there has been a lot of research performed on the use of manipulatives in the mathematics classroom, and many of the researchers strongly advocate their use (Kennedy, 1986, cited in Gresham, Sloan & Vinson, 1997). These researchers also discuss the stages in which Mathematics should be taught, and the ages at which manipulatives are beneficial.

There are however, very few studies performed specifically relating to the use of manipulatives in the area of 3-D space. Therefore, this particular study should significantly add to the literature.

# CHAPTER 3

# RESEARCH METHOD

## Introduction

This chapter will present the research methodology used in this study: the design chosen for the study, the form of data analysis adopted and the instruments employed. Ethical considerations and information on the participants involved in the study are then discussed, followed by details of the data collection process employed.

## Designs

Research designs are traditionally classified as qualitative or quantitative. The former is "research in which the researcher relies on the views of participants; asks broad, general questions; collects data consisting largely of words (or text) from participants; describes and analyses these words for themes; and conducts the inquiry in a subjective, biased manner" (Creswell, 2008, p.46). On the other hand, quantitative research is a design "in which the researcher decides what to study, asks specific, narrow questions; collects quantifiable data from participants; analyses these numbers using statistics; and conducts the inquiry in an unbiased, objective manner" (Creswell, 2008, p.46).

### Quantitative Research

Since this study sought to identify the most effective variety of net for enhancing students' conceptual development, a quantitative research design was chosen.

This allowed for unbiased, numerical analysis of data, and the identification of statistical evidence for conceptual improvement.

## Analysis

Due to the quantitative nature of this study, two related forms of statistical analysis were employed: hypothesis testing and confidence intervals. The hypothesis testing was used to indicate whether there was any statistically sustainable benefit to students' conceptualisation resulting from the introduction of bendable and rigid nets. The confidence interval analysis enables predictions for the upper and lower bounds of the population mean, thus providing some indication of the extent of this benefit, if any.

## Instruments

The instruments used in this study can be divided into two groups: instructional instruments (nets) and evaluative instruments (tests and worksheets). Each of these instruments is discussed below.

### Nets

A variety of products was considered when choosing a material from which to construct the nets. Taken into consideration were cost, range of colours and ease of construction. This comparison is detailed in Table 3.1.

Flute-board, a safe, plastic sheet, available from office supply stores, was chosen as the most appropriate material.

When creating the nets, handling and storage were considered also. Following some experimentation, the most appropriate sizes for the nets were decided upon. The sizing for each net is detailed in Table 3.2.

**Table 3.1 Material Comparison**

| Material | Hinges required? | Rigidity | Colour | Cost/m$^2$ |
|---|---|---|---|---|
| MDF | Yes | Excellent | Wooden | $6.75 |
| Perspex sheet | Yes | Excellent | Varied | $78.84 |
| Flute board | No | Good | Varied | $15.19 |

**Table 3.2 Net Sizes**

| Net | Face/s | Side Length |
|---|---|---|
| *Cube* | Square | 50mm |
| *Square-based pyramid* | Square | 75mm |
| | Triangle | 75mm |
| *Triangle-based pyramid* | Triangle | 75mm |
| *Hexagonal prism* | Rectangle | 100x35mm |
| | Hexagon | 35mm |
| *Triangular prism* | Triangle | 75mm |
| | Rectangle | 150x75mm |

The bendable nets were made by systematically cutting bend lines into the sheeting with a 'V' cut using an angled picture-framing trimmer, while the rigid nets had the bend lines drawn on the top surface. There were four colours used randomly: red, green, yellow and blue. Note that none of the nets were colour coded. A selection of the nets is shown in Figure 3.1.

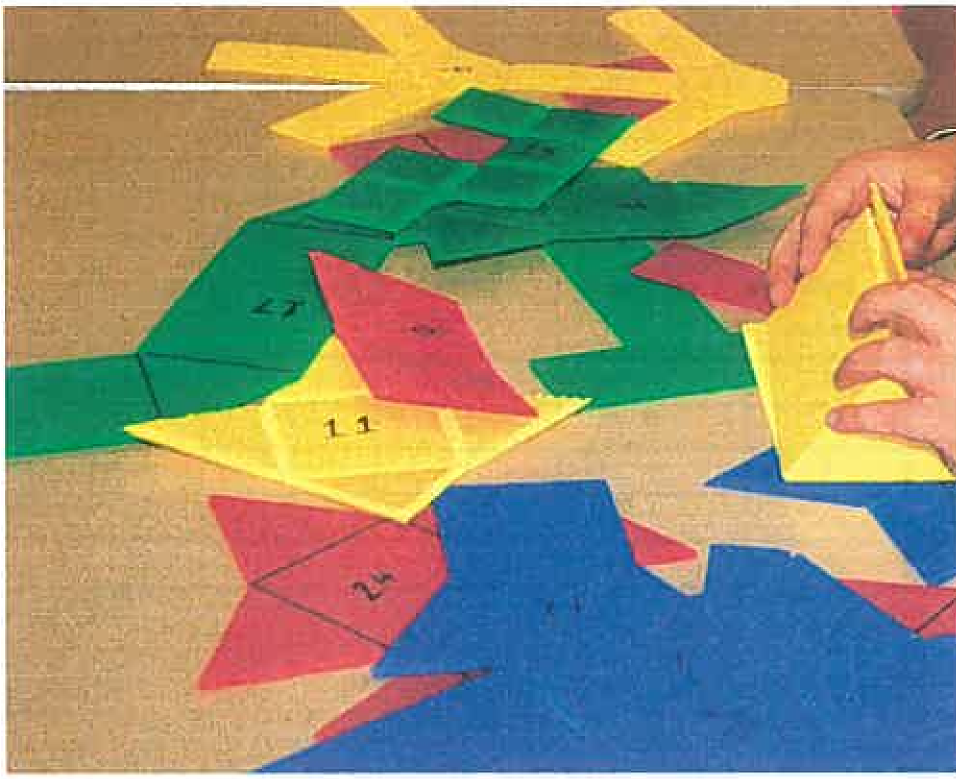
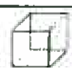
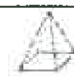


**Figure 3.1 Flute Board Nets**

Two different sets of 2D nets were designed and constructed, each including examples which did correspond to 3-D solids, and others which did not. Five solids were represented in these sets: cube, rectangular prism, hexagonal prism, square-based pyramid and triangular-based pyramid. One set consisted of rigid nets, the other of bendable nets. Each set was comprised of 15 different nets, creating a total of 30 nets. The breakdown of these sets is detailed in Table 3.3. Note that a '✓' implies that the net did correspond to a 3-D solid, while a 'x' implies that this was not the case.

**Table 3.3 Net Division**

| |  Cube |  Triangular prism |  Hexagonal prism |  Square-based pyramid |  Triangle-based pyramid | |
|---|---|---|---|---|---|---|
| Bend ✓ | 3 | 2 | 2 | 2 | 1 | 10 |
| Rigid ✓ | 3 | 2 | 2 | 2 | 1 | 10 |
| Bend × | 1 | 1 | 1 | 1 | 1 | 5 |
| Rigid × | 2 | 1 | 1 | 1 | | 5 |
| | 9 | 6 | 6 | 6 | 3 | **30** |

**Tests**

Three 45-minute tests of similar difficulty were designed, and identified as T1, T2 and T3 (see Appendix 1). Each test involved a variety of polyhedra, including: triangular, rectangular, pentagonal and hexagonal prisms; triangular, square, hexagonal and pentagonal pyramids; cubes; cylinders; and cones. It should be noted that many of these polyhedra were not included in the manipulative sets mentioned above. The purpose of this was to test student understanding rather than just knowledge and skills.

Each test contained a total of five questions divided into a number of sections. These questions focused on students' understanding of 3-D objects, their number and type of faces and appropriate nets.

T1 represented the pre-test completed by all students at the commencement of the study; T2 represented the post-test completed by students after their experience with bendable nets and T3 the post-test completed after experience with rigid nets.

### Worksheets

Two worksheets were designed: W1, relating to the bendable nets and W2, relating to the rigid nets (see Appendix 2). Each worksheet had a total of 15 questions, each question relating to a specific net, identified by number. Students were required to complete the worksheet: deciding whether the net would make a 3-D shape, naming the 3-D shape, and drawing a corrected net for those they identified as not creating a 3-D shape.

## Ethical Clearance

Permission was obtained from the Avondale Human Research Ethics Committee before the study was carried out (see Appendix 3 for Ethics Clearance Form). In order to preserve anonymity, each student was allocated a letter of the alphabet to use in place of their name. This enabled the data analysis to track the conceptualisation of individual students if required. Permission to run the study was obtained from the authorities of the school system and the school itself. Student participation was voluntary and a letter of consent was sent to the parents of the students. No parent withheld their child from the study and the identity of all participants has been protected (see Appendix 4 for permission letters).

## Participants

This study involved students within a single grade 4 class, divided into two groups of 12 students and covering a wide range of mathematical abilities. It was originally intended that both groups would be of equal ability. However, as it transpired, Group A was a much stronger group mathematically. As will be discussed in Chapter 4, this difference in ability ultimately proved an advantage to the study.

## Data Collection

The data for this study were collected using a pre-test (T1) post-tests (T2 & T3) and worksheets administered to the students over the seven 50-minute mathematics lessons allocated for this study. Details of data collection are discussed below.

### Period 1

To begin the study, the researcher explained the process to the students, giving a brief outline of what was expected of them, and the purpose of the study. Following this introduction, letters to be used in place of student names on the pre- and post-tests as well as the worksheets were distributed to each student. This allowed for anonymity, in keeping with ethical clearance requirements earlier discussed. Each student was then given the pre-test, T1, to complete in the remaining time of the mathematics lesson.

### Period 2

During this period, the students were randomly divided into two groups, Group A and Group B. The researcher worked with Group A first, introducing the Stage 2 Mathematical topic of Geometry as outlined in the Board of Studies Mathematics Syllabus (Board of Studies, 2002) using bendable nets. For this 25 minute session, students were encouraged to simply manipulate the nets, familiarise themselves with them, and discuss their properties and the 3-D shapes they made. For the nets that did not correctly represent a 3-D shape, students were encouraged to discuss how the net could be altered to create one that would correspond to a 3-D shape. To further the conceptualisation of nets and their relationship to 3-D shapes, students were encouraged to verbally describe a net that had the correct number of faces for a particular 3-D shape but did not actually form up into a 3-D shape.

During this exploratory session, the members of Group B were participating in a regular mathematics lesson taken by the supervising teacher.

Following the allocated 25 minutes, the two groups swapped, with Group A moving to the regular mathematics lesson, and Group B participating in the exploratory session with the researcher. The session for Group B mirrored that of Group A, except that experience was provided with rigid nets rather than bendable nets. Similar questions were asked of Group B as for Group A, thus ensuring the validity of the process.

**Period 3**

Similar to Period 2, this session had the class divided into the respective groups, again rotating after 25 minutes. Group A worked first with the researcher while Group B participated in a regular mathematics lesson with the supervising teacher. During this period, Group A again worked with the bendable nets, individually completing W1. Each worksheet question corresponded to one of the fifteen bendable nets. Students were required to identify the 3-D shape the net should make from the faces, identify whether the net actually made a 3-D shape, and draw a correct net if this was not the case. Each student was given 90 seconds with each net before passing them to the next student. Students were not allowed to communicate with each other during this exercise, in order to ensure the validity of the results.

Following this activity, the researcher collected all worksheets and nets, putting them in a place inaccessible to the students in the class. Then, the two groups swapped, Group B moving to work with the researcher, completing W2, the worksheet corresponding to the rigid nets in a similar manner to that described for Group A.

**Period 4**

During this period, the groups were not split. Each student completed the appropriate 50-minute test. Group A completed T2 whilst Group B completed T3.

**Period 5**

This period was a complete reflection of Period 2, with Group A being exposed to the rigid nets and Group B the bendable nets. The purpose of this exchange was to ensure that each student was exposed to the different kinds of nets, and allow for a more detailed analysis of conceptualisation. Similar questions and experiences were carried out in that of Period 2. Groups again swapped after 25 minutes.

**Period 6**

Similarly, this period was a reflection of Period 3, with each student completing the worksheet relevant to the kind of nets they were using.

**Period 7**

Finally, this period once again combined the two groups, each student completing the post-test relevant to type of net most recently used.

# CHAPTER 4

# RESULTS

## Introduction

Previous chapters have outlined the literature relevant to the topic of study, and the methodology used in the data collection. This chapter presents the results obtained from the data.

It was anticipated that T2 (following experience with bendable nets) would show a greater increase in marks, compared to the pre-test, than T3 (following experience with rigid nets). However, analysis of the data demonstrated otherwise. A number of statistical factors confirmed that it was in fact T3 that indicated greater improvement.

These unexpected results raised concern over the equivalent difficulty of T2 and T3. It was realised that the results obtained could be explained by T3 being easier than T2, despite all attempts made in their preparation to ensure equality. Accordingly, both T2 and T3 were submitted to four academic peers with mathematical experience in order to compare the relative difficulty of the tests. However, all four of the academic peers rated both the tests as very close, there being no predominant judgement of one being more difficult than the other. This implied that the results obtained with the students were not an artefact of uneven test difficulty.

The data from the three tests were standardised by excluding those students who did not complete all three tests. This reduced the sample size down to nine students in each group.

## Raw Data

The raw data collected for this study, shown in Table 4.1 and Table 4.2 as percentages, were used in statistical analysis. It was quickly realised that it was not possible to make consistent claims from simply analysing means and standard deviations.

If only the raw data were considered, one could conclude that, for Group A, the best results were obtained from T3, following the rigid nets, identifying them as the most effective in developing student conceptualisation. On the other hand, the best results from Group B were that of T2, following the bendable nets. From these results, it could be concluded that it is not the variety of net that influences the conceptualisation of 3-D space, but rather the amount of experience students have had with manipulatives.

**Table 4.1 Raw Data, Group A**

| | T1 | T2 (First) | T3 (Second) |
|---|---|---|---|
| | 68 | 55 | 75 |
| | 75 | 82 | 77 |
| | 57 | 70 | 93 |
| | 57 | 84 | 70 |
| | 66 | 98 | 86 |
| | 59 | 23 | 82 |
| | 57 | 55 | 45 |
| | 80 | 93 | 89 |
| | 73 | 61 | 100 |
| $\overline{X} =$ | **65.66** | **68.94** | **79.80** |
| $s =$ | **8.76** | **23.51** | **15.85** |

**Table 4.2 Raw Data, Group B**

| | T1 | T2 (Second) | T3 (First) |
|---|---|---|---|
| | 68 | 98 | 89 |
| | 61 | 86 | 77 |
| | 52 | 55 | 41 |
| | 32 | 95 | 43 |
| | 52 | 32 | 70 |
| | 45 | 52 | 55 |
| | 77 | 84 | 93 |
| | 36 | 34 | 39 |
| | 77 | 89 | 82 |
| $\overline{X} =$ | **55.81** | **69.44** | **65.40** |
| $s =$ | **16.55** | **26.29** | **21.43** |

where: $\overline{X}$ = sample mean

$s =$ sample standard deviation

## Worksheets

Two worksheets were employed in the collection of data for this study. These worksheets were used in order to consolidate what had been learnt in the exploratory sessions previous to the lesson involving the worksheets. Each worksheet corresponded to the type of net being used by the students. The results of the worksheets for both groups are detailed in Table 4.3 in the form of average percentages.

**Table 4.3 Worksheet Data**

| **Group A** | |
|---|---|
| **Result for W1** (following learning experience with bendable nets) | **Result for W2** (following learning experience with rigid nets) |
| 75.3% | 71.5% |
| **Group B** | |
| **Result for W2** (following learning experience with rigid nets) | **Result for W1** (following learning experience with bendable nets) |
| 55.7% | 77.9% |

From these results, it can be seen that the worksheet involving bendable nets produced the highest results for both groups. This may be due to the opportunity for students to physically manipulate the bendable nets in order to identify whether the net created a 3-D solid or not. This provided an easy way out for students, one which was not offered with the rigid nets.

## Hypothesis Testing

In order to analyse the effectiveness of both types of nets on the conceptualisation of 3-D space for Year 4 students, a series of hypothesis testing was carried out on the data collected. These tests indicated whether some improvement in students' conceptualisation could be statistically confirmed.

A confidence interval of 98% was selected, producing a critical *t*-value $t_{critical} = 2.896$ obtained from the *t*-distribution table. In order for the introduction of either rigid or bendable nets to be judged effective in developing the conceptualisation of students to this level of confidence, the resulting test value needed to exceed the critical *t*-value, based on a right-tailed test.

This test was carried out for both Group A and Group B, between the pre-test T1 and post-test T2; and the pre-test T1 and post-test T3 (see Appendix 5 for details of the hypothesis testing).

### Group A

Although an increase in means from the raw data in Table 4.1 was identified, the hypothesis testing showed that a comparison between the pre-test, T1, and the post-test, T2, produced a test value $t_{12} = 0.4567$ (see Table 4.4). This value is much smaller than $t_{critical} = 2.896$, from which the hypothesis testing confirmed that the introduction of rigid nets for Group A resulted in no significant improvement in students' conceptualisation. This is demonstrated in Figure 4.1.

In contrast, the test value of $t_{13}$ = 2.9535, exceeded $t_{critical}$ = 2.896, providing enough evidence in the hypothesis testing to support the claim that the introduction of bendable nets significantly increased the conceptualisation of students in Group A. This is demonstrated in Figure 4.2.

**Table 4.4 Hypothesis Testing, Group A**

| | Number of Subjects | Sum of Differences | Sum of Differences Squared | Mean of Difference | Standard Deviation of Difference | Critical *t*-value | Test Value |
|---|---|---|---|---|---|---|---|
| $t_{12}$ | 9 | 29.55 | 3817.15 | 3.28 | 21.56 | 2.896 | 0.4567 |
| $t_{13}$ | 9 | 127.27 | 3450.41 | 14.14 | 14.36 | 2.896 | 2.9535 |

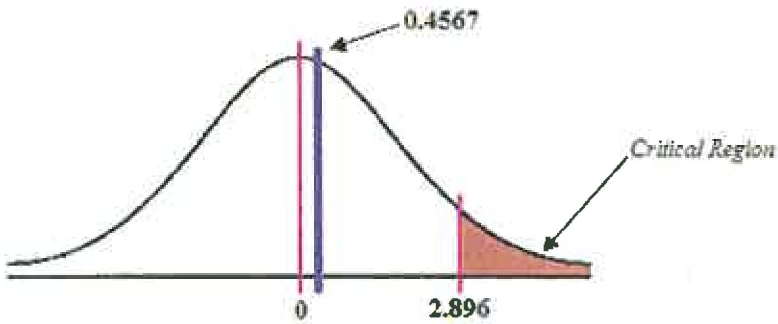



**Figure 4.1 Test Value $t_{12}$ for Group A**

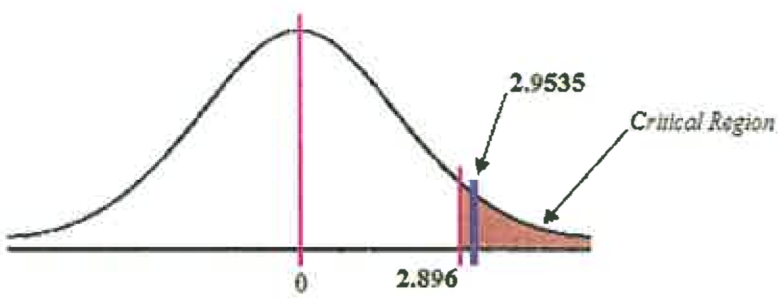



**Figure 4.2 Test Value $t_{13}$ for Group A**

**Group B**

The same hypothesis testing was carried out for Group B, where the order of net experience was opposite to Group A. It was anticipated that this different sequencing may have had an impact on the ensuing results. These results are detailed in Table 4.5.

Again, as identified with Group A, the test value $t_{12}$ was significantly less than $t_{\text{critical}} = 2.896$, in this case, 1.7201. This value lies well outside the critical region, as demonstrated in Figure 4.3, indicating that there is not enough evidence to support the claim that the introduction of bendable nets significantly increased the conceptualisation of students in Group B.

In contrast to $t_{12}$ for Group B, the test value $t_{13} = 2.8912$, is approximately equal to $t_{\text{critical}} = 2.896$, indicating that there is enough evidence to support the claim in the hypothesis testing, that the introduction of rigid nets significantly increased the conceptualisation of students in Group B. This is demonstrated in Figure 4.4.

It can therefore be concluded, with 98% confidence, that improvement in the conceptualisation of 3-D space can be identified with the introduction of rigid nets for both groups.

**Table 4.5 Hypothesis Testing, Group B**

| | Number of Subjects | Sum of Differences | Sum of Differences Squared | Mean of Difference | Standard Deviation of Difference | Critical $t$-value | Test Value |
|---|---|---|---|---|---|---|---|
| $t_{12}$ | 9 | 93.18 | 5325.41 | 13.64 | 23.78 | 2.896 | 1.7201 |
| $t_{13}$ | 9 | 65.91 | 1203.51 | 9.60 | 10.00 | 2.896 | 2.8912 |

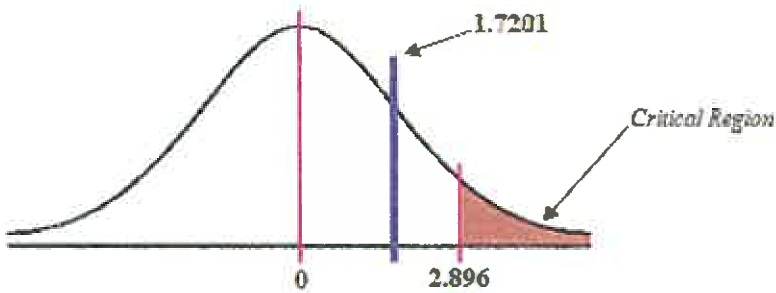



**Figure 4.3 Test Value $t_{12}$ for Group B**

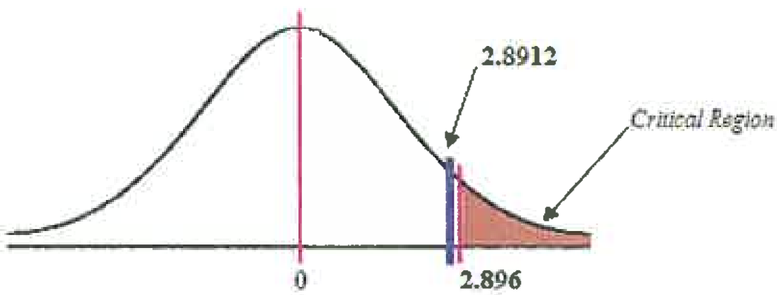



**Figure 4.4 Test Value $t_{13}$ for Group B**

## Confidence Intervals

The statistical analysis tool, confidence intervals, relates the data collected from a limited sample, to the associated population (see Appendix 6 for the complete analysis). This form of analysis also provides a more quantified representation of the statistical improvement identified by the hypothesis testing.

In order to maintain continuity with earlier analysis, a 98% confidence interval was used again for this procedure. The corresponding $t_{\alpha/2}$ value is 2.896. The resulting confidence intervals are displayed in the following table, Table 4.6.

**Table 4.6 Confidence Intervals**

| | Group A | Group B |
|---|---|---|
| **T1** | $57.20 < \mu < 74.12$ | $39.83 < \mu < 71.79$ |
| **T2** | $46.24 < \mu < 91.64$ | $44.06 < \mu < 94.82$ |
| **T3** | $64.50 < \mu < 95.10$ | $44.71 < \mu < 86.10$ |

where: $\mu$ = population mean

Hence, from the Group A results, based on a sample size of 9, one can be 98% confident that the population means for T1, T2 and T3 are between 57.20% and 74.12%; 46.24% and 91.64%; 64.50% and 95.10% respectively. Additionally, from the Group B results, one can be 98% confident that the population means for

T1, T2 and T3 are between 39.83% and 71.79%; 44.06% and 94.82%; 44.71% and 86.10% respectively.

It can be noted therefore, the progressive increase of the population averages for both groups, indicating the student conceptualisation has progressively increased.

The following figure, Figure 4.5, provides a visual representation of these confidence intervals. The overlap of both groups has been used to show the allowed intervals for each test. As can be seen from Figure 4.5, the estimated T1 interval for the population is between 57.20% and 71.79%; that for T2 is between 46.24% and 91.64%; and that for T3 is between 64.50% and 86.10%.

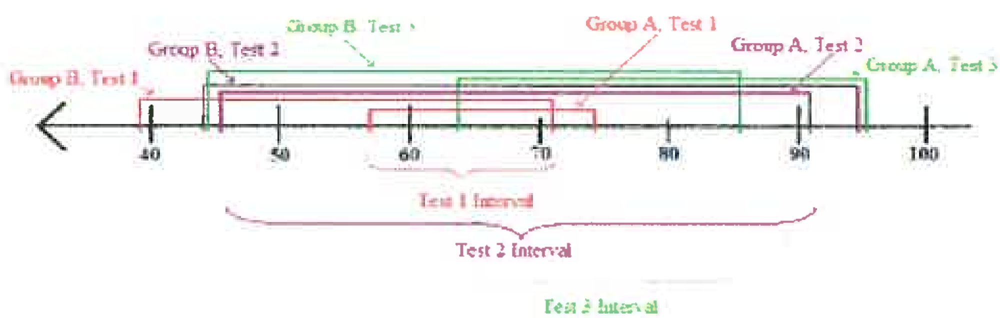



**Figure 4.5 Confidence Intervals**

As may be seen, T2, the test following experience with bendable nets, has a very wide range, showing that it cannot be argued that the conceptualisation of students has increased, compared to the pre-test, T1. However, the results from T3, the test following the rigid nets, have moved up significantly from T1. This demonstrates that the conceptualisation has increased for the population.

# CHAPTER 5

# DISCUSSION AND CONCLUSION

## Introduction

This chapter discusses the results obtained and presents some tentative conclusions. It also explores the implications of this study for today's teachers, and suggests some areas in which further investigation might be conducted.

## Discussion of Results

As outlined in Chapter 1, this study set out to answer the following question:

1. Are manipulatives that can be made into 3-D shapes more effective than rigid equivalents in the conceptualising of 3-D solids from nets in the Stage 2 Curriculum?

### Hypothesis Testing

For Group A, the hypothesis testing, to a confidence interval of 98%, confirmed a significant improvement in conceptualisation following class experience with the rigid nets, whereas this was not the case for the bendable variety. This was contrary to initial expectations. It was thus recognised that some of this improvement may be attributed to accumulating experience, since Group A experienced the rigid nets last. However, Group B showed a similar pattern of results, although having a reversed order of experience. This suggested that the experience factor was not significant.

**Confidence Intervals**

The confidence interval analysis subsequently enabled some quantitative estimate of the amount of conceptual improvement between T1 and T3. As can be seen from Figure 4.5, this was an improvement of approximately 7%. No such improvement can be identified for a comparison between T1 and T2 due to the wide range of possible results in the case of T2.

**Worksheets**

Students using the bendable nets with the worksheets were able to identify their 3-D shape and whether or not it resulted in a correct solid by physically manipulating the net. There is therefore no surprise that these worksheets showed higher levels of conceptualisation when using the bendable nets than for the rigid variety. However, these tactile and direct experiences were not available during the paper tests which followed. Rather, students were required to simply identify nets and 3-D shapes by observation and abstract thought.

## Conclusions

It can be hypothesised that the differences in thought processing patterns between the problem solving challenges resulted in students performing to a lower level on the written test following experience with the bendable nets. On the other hand, students learning with the rigid nets were required to identify the corresponding 3-D shape and decide whether they created a correct solid from their flat configuration. They were therefore forced to manipulate the shapes in their minds rather than their hands, focusing on mental rather than physical processing. Thus, although not performing as well on the worksheet, they developed higher levels of spatial awareness in identifying 3-D shapes from flat nets, and therefore performed at a much higher level on the written test.

Statistical analysis of the data collected has shown that: manipulatives that can be made into 3-D shapes are **not** more effective than rigid equivalents in the conceptualising of 3-D solids from nets in the Stage 2 Curriculum. In fact the rigid nets proved to be the more effective manipulatives. The findings and conclusions from this study have been submitted for publication to the Australian Mathematics Teacher journal. See Appendix 7 for the full article and letter of receipt.

## Implications for Teachers

Whilst performing this study, it was observed that the students thoroughly enjoyed working with the manipulatives, the bendable variety being the most popular. This supports the idea that the use of a range of tactile experiences in the mathematics classroom not only diversifies assimilation pathways, but makes learning more enjoyable. Such teaching techniques may even increase motivation to learn on the part of the student.

Although the construction and assembly of manipulatives represents a considerable investment in planning and time, this effort is definitely worthwhile. It may also provide the opportunity to utilise the constructional expertise of parents and carers, strengthening relational links between home and school.

## Recommendations for Further Research

Clearly there are many other ways in which this study could be developed. These include:

- An extension of the study to include a wider data sample;
- An extension of the study to include a wider range of grade levels;
- A comparison between groups of students of the same ability, one group taught **without** any manipulatives and the other **with** manipulatives;
- A comparison between three groups of the same ability, one group taught without any manipulatives, one group taught with bendable nets only, and the other group taught with rigid nets only;
- A comparison of the conceptualisation of 3-D shapes between boys and girls;
- An extension of the study to include a wider range of geometrical concepts.

# REFERENCES


Ainge, D.J. (1996). Upper primary students constructing and exploring three dimensional shapes: A comparison with virtual reality with card nets. *Journal of Educational Computing Research; 14*(4), 346.

Baker, J. D., & Beisel, R. W. (2001). An experiment in three approaches to teaching average to elementary school children. *School Science and Mathematics; 101*(1), 23-30.

Battista, M. T. (1999). The mathematical miseducation of America's youth. Phi Delta Kappa International. Retrieved October 26, 2007 from http://www.pdkintl.org/kappan/kbat9902.htm

Board of Studies. (2002). *Mathematics K-6 Syllabus*. Board of Studies: Sydney.

Clemments, D. C. & Battista, M. (1986). Geometry and geometric measurement. *Arithmetic Teacher; 33*(6), 29-32.

Dossey, J. A. (1989). Transforming mathematics education. *Educational Leadership; 47*(3), 22-24.

Green, M. G., Piel, J. A., & Flowers, C. (2008). Reversing education majors' arithmetic misconceptions with short-term instruction using manipulatives. *The Journal of Educational Research, 101*(4), 234-244. Retrieved October 23, 2008 from ProQuest database.

Gresham, G., Sloan, T., & Vinson, B. (1997). Reducing mathematics anxiety in fourth grade "at-risk" students. Retrieved October 23, 2008 from ERIC Database.

Heddens, J. W. (1997). Improving mathematics teaching by using manipulatives. Kent State University. Retrieved October 26, 2008, from www.fed.cuhk.edu.hk/~fllee/mathfor/edumath/9706/13hedden.html

Hynes, M. C. (1986). Selection Criteria. *Arithmetic Teacher; 33*(6), 11-13.

Jacobs, T. (2008). The proper use of manipulatives in the math classroom. Balance. Retrieved October 26, 2007, from http://www.bjupress.com/resources/articles/balance/tips/t1003.php

Kennedy, L. M. (1986). A rationale. *Arithmetic Teacher; 33*(6), 6-7.

Oberholzer, D. (2008). Making math come alive with manipulatives. Balance. Retrieved October 26, 2007, from http://www.bjupress.com/resources/articles/balance/tips/t1505.php

Texas Education Agency. (1994). Guidelines for teaching grade 4 mathematics. Division of Curriculum Development. Retrieved October 23, 2008 from ERIC Database.

Worth, J. (1986). By way of introduction. *Arithmetic Teacher; 33*(6), 2-3.

# APPENDICES

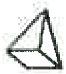
# Appendix 1 Tests 1, 2 and 3

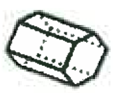
# TEST 1

Letter: ______________

1. Name these 3D shapes:

a)  ______________________________

b)  ______________________________

c)  ______________________________

d)  ______________________________

2. Name the faces of the 3D shapes above:

a) ____________________________________________

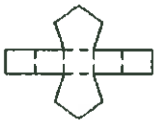
b) ____________________________________________

c) ____________________________________________

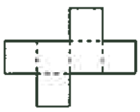
d) ____________________________________________

**A net can be described as a 2D shape that can be folded on the lines to make a 3D shape.**

3. Using the nets below, identify the 3D shapes that can be made.

a)  ______________________

b)  ______________________

c)  ______________________

d)  ______________________

4. Draw a net that works for the following shapes:

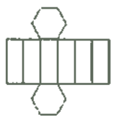
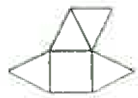
a) 

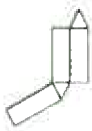
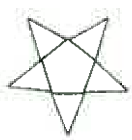
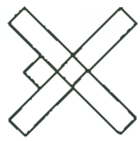
b) 

c) 

d) 

5. Circle the nets that do **not** make 3D shapes.

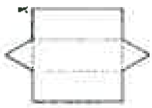
a)   

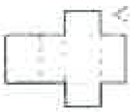
b)   

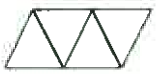
# TEST 2

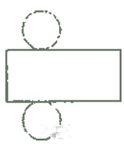
Letter: ______________

1. Name these 3D shapes:

a)  ______________________________

b)  ______________________________

c)  ______________________________

d)  ______________________________

2. Draw the 3D shapes of the nets above:

a)

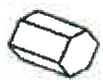
b)

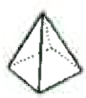
c)

d)

3. Name all the faces for the following 3D shapes.

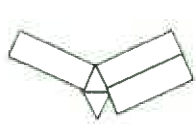
a)  ______________________________

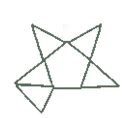
b)  ______________________________

c)  ______________________________

d)  ______________________________

4. Circle the nets that do **not** make 3D shapes.

a)   

b)   

5. Draw a correct net for the following shapes:

a) Pentagonal prism

b) Hexagon-based pyramid

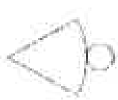
c) Square-based pyramid

d) Hexagonal prism

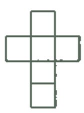
## TEST 3

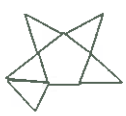
Letter: ______________

1. Name these 3D shapes:

a)  ______________________________

b)  ______________________________

c)  ______________________________

d)  ______________________________

2. Draw the 3D shapes of the nets above:

a)

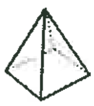
b)

c)

d)

3. Name all the faces for the following 3D shapes.

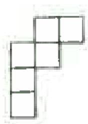
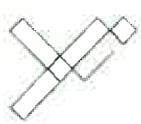
a) 

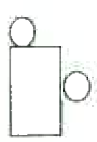
b) 

c) 

d) 

4. Circle the nets that do **<u>not</u>** make 3D shapes.

a)   

b)   

5. Draw a correct net for the following shapes:

a) Square-based pyramid

b) Rectangular prism

c) Cylinder

d) Pentagonal prism

## Appendix 2 Worksheets 1 and 2

**Worksheet 1**          Letter: ____________

| NET NUMBER | DOES THIS NET MAKE A 3D SHAPE? | | NAME OF 3D SHAPE | CORRECTED NET |
|---|---|---|---|---|
| 1 | Yes | No | | |
| 2 | Yes | No | | |
| 3 | Yes | No | | |
| 4 | Yes | No | | |
| 5 | Yes | No | | |
| 6 | Yes | No | | |
| 7 | Yes | No | | |

Letter: ______________

| NET NUMBER | DOES THIS NET MAKE A 3D SHAPE? | | NAME OF 3D SHAPE | CORRECTED NET |
|---|---|---|---|---|
| 8 | Yes | No | | |
| 9 | Yes | No | | |
| 10 | Yes | No | | |
| 11 | Yes | No | | |
| 12 | Yes | No | | |
| 13 | Yes | No | | |
| 14 | Yes | No | | |
| 15 | Yes | No | | |

**Worksheet 2** Letter: ____________

| NET NUMBER | DOES THIS NET MAKE A 3D SHAPE? | | NAME OF 3D SHAPE | CORRECTED NET |
|---|---|---|---|---|
| 16 | Yes | No | | |
| 17 | Yes | No | | |
| 18 | Yes | No | | |
| 19 | Yes | No | | |
| 20 | Yes | No | | |
| 21 | Yes | No | | |
| 22 | Yes | No | | |
| 23 | Yes | No | | |

Letter: ____________

| NET NUMBER | DOES THIS NET MAKE A 3D SHAPE? | | NAME OF 3D SHAPE | CORRECTED NET |
|---|---|---|---|---|
| 24 | Yes | No | | |
| 25 | Yes | No | | |
| 26 | Yes | No | | |
| 27 | Yes | No | | |
| 28 | Yes | No | | |
| 29 | Yes | No | | |
| 30 | Yes | No | | |

## Appendix 3 Ethics Clearance Form

AVONDALE COLLEGE

HUMAN RESEARCH ETHICS (HRE) COMMITTEE
(Form updated December 2006)

# APPLICATION FOR APPROVAL OF RESEARCH USING HUMAN SUBJECTS

*HRE Committee use only*

☐ Approved

☐ Not approved

☐ Returned for modification

Signed.......................................

Date.........................................

## 1. INSTRUCTIONS

Have you read the guidelines on the disk for completing this form?

☒ Yes – Proceed to point 2.

☐ No – Please read them, then proceed to point 2.

## 2. TITLE OF RESEARCH PROJECT (Complete all answers in *italics*)

| |
|---|
| *A Comparison – Rigid or Bendable Manipulatives: Which is Most Beneficial to the Conceptualisation of 3D* |
| *Space from 2D Nets?* |
| |

## 3. PRINCIPAL INVESTIGATOR

| |
|---|
| NAME *Jacqueline Scott* |
| QUALIFICATIONS *4th year Bachelor of Education (Hons) student* |
| FACULTY *Education* |

## SUPERVISOR

| |
|---|
| NAME *Anton Selvaratnam* |
| QUALIFICATIONS *BSc (Hons), GradDipTch (Maths), MSc (Thesis)* |
| FACULTY *Science and Mathematics* |

| |
|---|
| NAME *Lynden Rogers* |
| QUALIFICATIONS *BSc (Hons), MSc, PhD* |
| FACULTY *Science and Mathematics* |

## 4. CO-INVESTIGATOR(S)

| |
|---|
| *N/A* |
| |
| |

## 5. INITIAL, CONTINUING OR AMENDED APPLICATION

Please tick one of the following:

(a) New project [•]

(b) Continuing project [ ]

(c) Amended project [ ]

## 6. DURATION OF THE RESEARCH PROJECT

| | |
|---|---|
| PROPOSED COMMENCEMENT DATE | *February 2009* |
| PROPOSED DURATION OF THE PROJECT | *Ten months. The thesis must be submitted late October/early Nov 2009* |

## 7. FUNDING

Do you have funding? [ ] Yes

[•] No

If yes, identify the source:

| |
|---|
| |
| |
| |

## 8. COMMONWEALTH PRIVACY LEGISLATION

Does your project require access to data governed by Commonwealth Privacy Legislation?

[ ] Yes

[•] No

If yes, please give details:

| |
|---|
| |
| |
| |

## 9. SCIENTIFIC OR EDUCATIONAL AIMS OF THE PROJECT

*Primary school students understand complex ideas on the basis of simpler concepts derived from practical experience. The purpose of this study is to investigate the relative effectiveness of two types of manipulatives (physical objects which may be safely handled) in a Stage 2 (Grade 3 or 4) Mathematics classroom for the conceptualisation of 3D solids from 2D nets (flat fold-outs of solid geometrical shapes).*

## 10. REPLICATION STUDIES

Has the same or similar research been conducted in Australia or overseas?

- [ ] Yes
- [x] No

If yes, please give details:

*A reasonably comprehensive literature search revealed little work done in this area and none on the specific topic of this research project.*

Give reasons why replicative studies are required:

## 11. DETAILS OF PROJECT

11.1 RESEARCH QUESTION OR HYPOTHESIS/SES

*Are manipulatives which can actually be deformed into 3D shapes more effective than rigid equivalents in the conceptualising of 3D solids from nets in the Stage 2 Curriculum?*

11.2 TARGET POPULATION

*A single Grade 3 or 4 class taught by the researcher during the coming practicum.*

11.3 RESEARCH DESIGN

*This investigation has been designed as a quantitative investigation into the relative effectiveness of two already-existing classroom techniques. After pre-testing the concept understanding the class will be split, with the help of the regular teacher, into two similarly sized groups as closely matched in mean ability as possible. It should be noted that the robustness of the results will not be significantly compromised by minor differences between these groups. The actual research project involves two learning/teaching experiences of the sort commonly employed by professional teachers in normal classroom delivery. The ordering of these two experiences will be different for each class group. A mid-test and a post-test for each group will document any improvement in concept understanding. It can be seen that at the conclusion of the research project both groups will have been enriched by exposure to both teaching strategies, thus eliminating any possible disadvantage to either group.*

11.4 SAMPLING

11.4.1. How will the subjects be selected?

*The participants will be members of the practicum class allocated to the researcher and will be divided into two groups at the discretion of the supervising teacher.*

11.4.2 Number of Subjects

*All class members where parents/guardians give permissions (estimate = 25)*

11.4.3 Informed Consent: What procedures do you plan to follow to gain informed consent from your subjects? Please attach a copy of the consent form to be used. If you do not intend to gain informed consent explain why.

*An information letter requesting consent will be sent to the school headmaster, the supervising teacher and the parents/guardians of pupils. A letter to the individual pupils in the participating class is also included. These letters are attached. However, due to the content of this research being a compulsory aspect of student learning in the Stage 2 New South Wales Board of Studies Mathematics Syllabus Document, and since it is taught as a normal classroom activity, advice from a number of Grade 4 teachers suggests that written consent is not required for pupils involved. Further, it was stated that such a letter may adversely impact student participation. Some guidance from the Committee on this point would be appreciated.*

11.5 DATA COLLECTION PROCEDURES

Please attach a copy of your subjects' information letter and instrument, if applicable.

11.5.1. Briefly describe all data collection methods (eg. blood taking, ingestion of chemicals, interview, survey
instruments, educational testing, observation, etc) to be used with human subjects.

*Quantitative data will be collected through the use of pre-, mid- and post-testing of both class groups. These evaluative exercises will be paper tests of conceptual knowledge.*

11.5.2 Outline the possible dangers, risks or ill effects of these procedures and the precautions to be taken to prevent or minimise them.

*There is no risk associated with this research project as the proposed teaching strategies are both within school policy and typical of those in legitimate and widespread use by professional primary teachers. The manipulatives to be employed have been designed to be safely handled by Stage 2 students and will be constructed from non-toxic materials.*

11.5.3 In the case of interviews or surveys, how will confidentiality of data be maintained?

*No interviews or surveys are being used. Furthermore, no student names will be recorded on evaluation instruments. In addition, while the researcher will obviously know the identity of those involved, no subsequent report or publication will contain the name of the school, the supervising teacher, or any participating student.*

11.5.4 Where will the procedures involving human subjects be undertaken?

*The research will take place in the natural context of the child's classroom, in regular teaching periods involving some four hours per week over two weeks.*

| 11.5.5 What facilities are there for dealing with emergencies? (If applicable) |
|---|
| *The usual procedures available at all primary schools should suffice for any foreseeable emergency.* |

| 11.6 DATA ANALYSIS PROCEDURES |
|---|
| *The descriptive and inferential conclusions from this study will emerge from quantitative data obtained through the pre-, mid- and post-testing sequence.* |

## 12. PROPOSED STORAGE OF AND ACCESS TO DATA AND RESULTS

| 12.1 Where will the data be stored? |
|---|
| *Electronic data will be retained in a password protected file on a notebook computer usually stored in a locked room. Hard data will be stored in a locked cupboard in the locked Research room of the Science Department.* |
| 12.2 When and how will the data be disposed of? |
| *Electronic data will be destroyed after five years by being permanently deleted from the computer. At the same time all hard data will be destroyed by shredding.* |

## 13. ADDITIONAL APPROVAL FROM OTHER ETHICS COMMITTEES, ORGANISATIONS, INDIVIDUAL/S

| 13.1. To which other ethics committee, organisation, or individual/s have you or do you intend to submit this proposal? |
|---|
| *Permission will be obtained by the Faculty of Education, Avondale from Dr John Hammond, the Education Director of the Australian Union Conference, to approach Dr Peter Kilgour, the Education Director for the Greater Sydney Conference. Upon project approval from the latter, subsequent letters will be sent to the school principal, supervising teacher and later to the parents/guardians of pupils. Consent letters will also be sent to the class pupils if required.* |
| 13.2 If a decision has been made by one of these authorities, what was the decision? (Please attach documented evidence) |
| *The researcher currently awaits acceptance by a school and will notify the Chair of the Ethics Committee and also the Honours Coordinator when this acceptance is received.* |

## Appendix 4 Permission Letters

Avondale College

Jacqueline Scott
PO Box 306
Budgewoi,
NSW 2262

19th November, 2008

Dear (School Principal),

My name is Jacqueline Scott and I understand that I have been accepted at your school for three weeks practicum beginning in February. As a senior student I am currently undertaking an Honours program at Avondale College. My associated research project asks whether manipulatives which can actually be deformed into 3D shapes are more effective than rigid equivalents in the conceptualising of 3D solids from nets in the Stage 2 Mathematics Curriculum.

My interest in this topic arises from my belief that students should be given the best opportunity for learning, and that competing instructional strategies should be tested. This research is being conducted under the joint supervision of Mr Anton Selvaratnam, Mathematics lecturer at Avondale College, and Dr Lynden Rogers, Dean of the Faculty of Science and Mathematics at Avondale College. This letter requests your permission to conduct this research during my professional experience at your school, within the Grade 4 class to which I have been assigned.

The project would commence by pre-testing the concept understanding, using a paper test that will be designed with the assistance of the class teacher. I propose that the class then be split, with the help of the regular teacher, into two similarly sized groups as closely matched in mean ability as possible. The actual research project involves two learning/teaching experiences, one using deformable manipulatives, the other using non-deformable equivalents. The ordering of these two experiences will be different for each class group. A mid-test and a post-test for each group will document any improvement in concept understanding.

It is hoped that at the conclusion of the research project both groups will have been enriched by exposure to both teaching strategies, thus eliminating any possible disadvantage to either group. My research will not only benefit the students concerned but be of benefit to the teaching profession.

Data collection will include the results of the paper tests, with all students remaining anonymous. There will be no video taping or voice recording in this study, as all results will come from the tests described above. Details of participants will at all times remain confidential. The results will be converted into statistics which can then be compared across the study. This data will be safely stored in a password protected file on a password protected computer, and will be held for the mandated time. Results gained from this study may be used in a thesis or journal publication. However, care will be taken to ensure total anonymity of both students and the school.

Please note that participation is voluntary and the participants may withdraw at any time. Further, if at any time during the study parents/guardians request that their child not be further involved, this request will be honoured. In this case, it is anticipated that an alternative program will be available with the regular classroom teacher. I will take every precaution possible not to negatively impact on the students taking part.

This project has been approved by the Avondale College Human Research Ethics Committee (HREC). Avondale College requires that all participants are informed that if they have any complaint concerning the manner in which a research project is conducted it may be given to the researcher, or if an independent person is preferred, to the College's HREC Secretary, Avondale College, PO Box 19, Cooranbong, NSW, 2265 or phone (02) 4980 2121 or fax (02) 4980 2117. If you have any further questions, do not hesitate to contact me on 0431 279744 or Dr Rogers on (02) 4980 2213.

I have enclosed a postage paid envelope for your reply.
Thank you for your consideration,

Jacqueline Scott

Jacqueline Scott
PO Box 306
Budgewoi,
NSW 2262

19th November, 2008

Dear (supervising teacher),

My name is Jacqueline Scott and I understand that I have been accepted into your classroom for three weeks practicum beginning in February. As a senior student I am currently undertaking an Honours program at Avondale College. My associated research project asks whether manipulatives which can actually be deformed into 3D shapes are more effective than rigid equivalents in the conceptualising of 3D solids from nets in the Stage 2 Mathematics Curriculum.

My interest in this topic arises from my belief that students should be given the best opportunity for learning, and that competing instructional strategies should be tested. This research is being conducted under the joint supervision of Mr Anton Selvaratnam, Mathematics lecturer at Avondale College, and Dr Lynden Rogers, Dean of the Faculty of Science and Mathematics at Avondale College. This letter requests your permission to conduct this research during my professional experience within your Grade 4 classroom.

The project would commence by pre-testing the concept understanding with a paper test designed with your assistance. I propose that the class then be split, with your help, into two similarly sized groups as closely matched in mean ability as possible. The actual research project involves two learning/teaching experiences, one using deformable manipulatives, the other using non-deformable equivalents. The ordering of these two experiences will be different for each class group. A paper mid-test and post-test for each group will document any improvement in concept understanding. Again, these tests will be designed with your assistance.

It is hoped that at the conclusion of the research project both groups will have been enriched by exposure to both teaching strategies, thus eliminating any possible disadvantage to either group. My research will not only benefit the students concerned but be of benefit to the teaching profession.

Data collection will include the results of the paper tests, with all students remaining anonymous. There will be no video taping or voice recording in this study, as all results will come from the tests described above. Details of participants will at all times remain confidential. The results will be converted into statistics which can then be compared across the study. This data will be safely stored in a password protected file on a password protected computer, and will be held for the mandated time. Results gained from this study may be used in a thesis or journal publication. However, care will be taken to ensure total anonymity of both students and the school.

Please note that participation is voluntary and the participants may withdraw at any time. Further, if at any time during the study parents/guardians request that their child not be further involved, this request will be honoured. In this unlikely event I hope that you might assist by offering an alternative program. I will, however, take every precaution possible not to negatively impact on the students taking part.

This project has been approved by the Avondale College Human Research Ethics Committee (HREC). Avondale College requires that all participants are informed that if they have any complaint concerning the manner in which a research project is conducted it may be given to the researcher, or if an independent person is preferred, to the College's HREC Secretary, Avondale College, PO Box 19, Cooranbong, NSW, 2265 or phone (02) 4980 2121 or fax (02) 4980 2117. If you have any further questions, do not hesitate to contact me on 0431 279744 or Dr Rogers on (02) 4980 2213.

I have enclosed a postage paid envelope for your reply.
Thank you for your consideration,

Jacqueline Scott

Avondale COLLEGE

Dear Parents/Guardians,

My name is Jacqueline Scott and I am currently studying a Bachelor of Education Primary (Honours) degree. As part of my Honours program I am proposing to conduct research within your child's class. The aim of the research is to compare two commonly employed teaching strategies for teaching the relationship between solid geometrical shapes and their associated two dimensional fold-out patterns (nets). Both instructional strategies utilise hands-on materials. This topic is part of the regular maths curriculum for your child. This research is being conducted under the joint supervision of Mr Anton Selvaratnam, Mathematics lecturer at Avondale College, and Dr Lynden Rogers, Dean of the Faculty of Science and Mathematics at Avondale College.

The study will involve the splitting of the class into two similar groups, with the help of the supervising teacher. Each of these groups will get the benefit of both instructional strategies. However, the order of presentation will be different for the two groups. Data collection will be through three similar tests, conducted at the beginning, the middle and the end of the classroom activities. Pupil's names will not be recorded on these tests and no interviews or surveys are being used.

As this study will occur during my practicum, I will be involved in teaching students the content required for the study. My research should in no way negatively impact on the class or your child. If any aspect of this project were to be judged by the teacher as negatively impacting on the class the project would be suspended and problem addressed.

Data will be collected through three paper tests, designed with the assistance of the classroom teacher, and will be converted into statistics for subsequent use. This data will be safely stored in a password protected file on a password protected computer, and held for the mandate time. The results from this study may be used in a thesis or journal publication. However, great care will be taken to ensure the anonymity of both the students and the school.

Details of your child's participation will at all times remain confidential. If at any time during the study you request that your child not be further involved, this request will be honoured. In this case, the student will still be involved in the teaching, but the data from children without consent from parents will not be included in the final results.

This project has been approved by the Avondale College Human Research Ethics Committee (HREC). Avondale College requires that all participants are informed that if they have any complaint concerning the manner in which a research project is conducted it may be given to the researcher, or if an independent person is preferred, to the College's HREC Secretary, Avondale College, PO Box 19, Cooranbong, NSW, 2265 or phone (02) 4980 2121 or fax (02) 4980 2117.

If you would like more information please do not hesitate to contact me on 0431 279744, or Dr Rogers on (02) 4980 2213.

..........................................................................................................

I have read and understood the accompanying letter by Jacqueline Scott describing her proposed project and give consent for my child's involvement as part of this research. I have discussed this with my child who is also happy to consent.

Signed: ______________________________ Date: ____________
*(Parent/guardian)*

Signed: ______________________________ Date: ____________

Avondale COLLEGE.

Dear (pupil),

My name is Miss Scott and I am nearly finished my training at Avondale College as a primary teacher. While teaching in your class I would like to compare two interesting ways of teaching you all about shapes. The study of shapes is one of the usual Grade 4 Maths topics which you will be studying anyway. Your parents/guardians are happy for you to take part. Please remember that if you don't want to be involved anymore, just let me know. Would you like to help me? If so, please sign your name on the line below.

.............................................................................................

I have read and understood the letter from Miss Scott and agree to help her.

Signed: ________________________________ Date: ____________

# Appendix 5 Hypothesis Testing

## Hypothesis Testing

In order to analyse the effectiveness of both types of nets on the conceptualisation of 3-D space in Year 4 students, hypothesis testing was carried out on the data collected. The statistical quantities used are shown below with their generating formulae.

The following formulas are used to determine the various statistics:

Mean of Difference: $\overline{D} = \frac{\sum D}{n}$

Standard Deviation of Difference: $s_D = \sqrt{\frac{\sum D^2 - \frac{\left(\sum D\right)^2}{n}}{n-1}}$

Test value: $t_{1j} = \frac{\overline{D} - \mu_D}{s_D / \sqrt{n}}, \quad j = 2, 3$

where: $D$ = Differences of the values of the pairs of data

$\overline{D}$ = The mean of the differences

$n$ = Number of pairs

$s_D$ = The standard deviation of the differences

$t_{1j}$ = The test values for the differences of means with (n – 1) degrees of freedom

$\mu_D$ = The mean of the differences for the whole population

**Table 1: Group A:**

| T1 | T2 | T3 |
|---|---|---|
| 68.18 | 54.55 | 75.00 |
| 75.00 | 81.82 | 77.27 |
| 56.82 | 70.45 | 93.18 |
| 56.82 | 84.09 | 70.45 |
| 65.91 | 97.73 | 86.36 |
| 59.09 | 22.73 | 81.82 |
| 56.82 | 54.55 | 45.45 |
| 79.55 | 93.18 | 88.64 |
| 72.73 | 61.36 | 100.00 |

| | | | |
|---|---|---|---|
| $\overline{X} =$ | 65.66 | 68.94 | 79.80 |
| $s =$ | 8.76 | 23.51 | 15.85 |

**Procedure:** Testing the difference between the means of T1 and T2.

Step 1: State the hypothesis and identify the claim.

In order for the introduction of the bendable nets to be effective, T1 marks must be significantly less than the T2 marks: hence the mean of the differences (T2-T1) must be greater than 0.

Letting $H_0$ denote the null hypothesis, and $H_1$ denote the alternative hypothesis, that is:

$$H_0: \ \mu_D \leq 0 \qquad H_1: \ \mu_D > 0 \quad \text{(claim)}$$

where, if we gave the entire population of Grade 4 students these two tests,

$H_0$ indicates that the mean of the differences would be $\leq 0$;

$H_1$ indicates that the mean of the differences would be $> 0$.

Step 2: Find the critical value.

Degrees of freedom = d.f. = $n-1=9-1=8$

Hence for the right tailed-test at $\alpha=0.02$ (98% confidence interval), we have $t_{critical}$ as:

$t_{critical}=$ **2.896**

(obtained from the $t$-distribution table).

Step 3: Compute the test value.

The following table is considered to determine the test value $t_{12}$:

| T1 ($X_1$) | T2 ($X_2$) | $D=X_2-X_1$ | $D^2$ |
|---|---|---|---|
| 68.18 | 54.55 | -13.64 | 185.95 |
| 75.00 | 81.82 | 6.82 | 46.49 |
| 56.82 | 70.45 | 13.64 | 185.95 |
| 56.82 | 84.09 | 27.27 | 743.80 |
| 65.91 | 97.73 | 31.82 | 1012.40 |
| 59.09 | 22.73 | -36.36 | 1322.31 |
| 56.82 | 54.55 | -2.27 | 5.17 |
| 79.55 | 93.18 | 13.64 | 185.95 |
| 72.73 | 61.36 | -11.36 | 129.13 |
| | | $\sum D=29.55$ | $\sum D^2=3817.15$ |

| | |
|---|---|
| Mean of Difference = $\overline{D}$ = | 3.28 |
| Standard Deviation of Difference = $s_D$ = | 21.56 |

| | |
|---|---|
| Test Value $t_{12}$ = | 0.4567 |

**Procedure:** Testing the difference between the means of T1 and T3.

Step 1: State the hypothesis and identify the claim.

In order for the introduction of the rigid nets to be effective, T1 marks must be significantly less than the T3 marks: hence the mean of the differences (T3 -T1) must be greater than 0.

Here, using the same $H_0$ and $H_1$ for T1 and T3, the null and alternative hypotheses are:

$H_0: \ \mu_D \leq 0$ $\qquad$ $H_1: \ \mu_D > 0$ $\qquad$ (claim)

Step 2: Find the critical value.

Degrees of freedom = d.f. = $n-1=9-1=8$

Hence for the right tailed-test at $\alpha = 0.02$ (98% confidence interval), $t_{critical}$ is:

$t_{critical} = \mathbf{2.896}$

(obtained from the $t$-distribution table).

Step 3: Compute the test value.

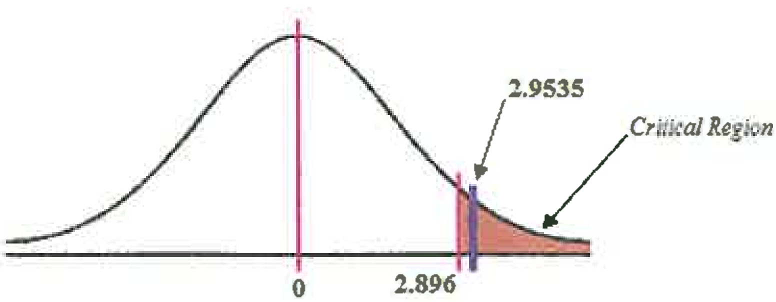
We consider the following table to determine the test value $t_{13}$:

| T1 ($X_1$) | T3 ($X_3$) | $D=X_3-X_1$ | $D^2$ |
|---|---|---|---|
| 68.18 | 75.00 | 6.82 | 46.49 |
| 75.00 | 77.27 | 2.27 | 5.17 |
| 56.82 | 93.18 | 36.36 | 1322.31 |
| 56.82 | 70.45 | 13.64 | 185.95 |
| 65.91 | 86.36 | 20.45 | 418.39 |
| 59.09 | 81.82 | 22.73 | 516.53 |
| 56.82 | 45.45 | -11.36 | 129.13 |
| 79.55 | 88.64 | 9.09 | 82.64 |
| 72.73 | 100.00 | 27.27 | 743.80 |
| | | $\sum D = 127.27$ | $\sum D^2 = 3450.41$ |

| | |
|---|---|
| Mean of Difference = $\overline{D}$ = | 14.14 |
| Standard Deviation of Difference = $s_D$ = | 14.36 |

| | |
|---|---|
| Test Value $t_{13}$ = | 2.9535 |

Step 4: Make the decision.




Since $t_{13} = 2.9535 > t_{\text{critical}} = 2.896$, and is in the critical region, the decision is, "Reject the null hypothesis." i.e., accept the alternative hypothesis.

Step 5: Summarise the results.

With 98% confidence, there is enough evidence to support the alternative hypothesis, $H_1$, indicating that the introduction of the rigid nets has significantly increased the marks of these students in Group A.

**Table 2: Group B:**

| Test 1 | Test 2 | Test 3 |
|---|---|---|
| 68.18 | 97.73 | 88.64 |
| 61.36 | 86.36 | 77.27 |
| 52.27 | 54.55 | 40.91 |
| 31.82 | 95.45 | 43.18 |
| 52.27 | 31.82 | 70.45 |
| 45.45 | 52.27 | 54.55 |
| 77.27 | 84.09 | 93.18 |
| 36.36 | 34.09 | 38.64 |
| 77.27 | 88.64 | 81.82 |

| | | | |
|---|---|---|---|
| $\overline{X} =$ | 55.81 | 69.44 | 65.40 |
| $s =$ | 16.55 | 26.29 | 21.43 |

**Procedure:** Testing the difference between the means of T1 and T2.

Step 1: State the hypothesis and identify the claim.

In order for the introduction of the bendable nets to be effective, T1 marks must be significantly less than the T2 marks: hence the mean of the differences must be (T2-T1) greater than 0.

For T1 and T2 from Group A, the null and alternative hypotheses are:

$H_0: \ \mu_D \leq 0$ $\qquad H_1: \ \mu_D > 0$ (claim)

Step 2: Find the critical value.

Degrees of freedom = d.f. = $n-1=9-1=8$

Hence for the right tailed-test at $\alpha=0.02$ (98% confidence interval), we have the $t_{\text{critical}}$ as:

$t_{\text{critical}} = \mathbf{2.896}$

(obtained from the $t$-distribution table).

Step 3: Compute the test value.

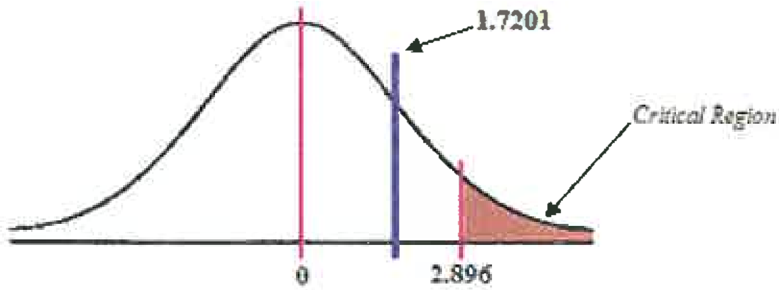
We consider the following table to determine the test value $t_{12}$:

| T1 ($X_1$) | T2 ($X_2$) | $D=X_2-X_1$ | $D^2$ |
|---|---|---|---|
| 68.18 | 97.73 | 29.55 | 872.93 |
| 61.36 | 86.36 | 25.00 | 625.00 |
| 52.27 | 54.55 | 2.27 | 5.17 |
| 31.82 | 95.45 | 63.64 | 4049.59 |
| 52.27 | 31.82 | -20.45 | 418.39 |
| 45.45 | 52.27 | 6.82 | 46.49 |
| 77.27 | 84.09 | 6.82 | 46.49 |
| 36.36 | 34.09 | -2.27 | 5.17 |
| 77.27 | 88.64 | 11.36 | 129.13 |
| | | $\sum D = 93.18$ | $\sum D^2 = 5325.41$ |

| | |
|---|---|
| Mean of Difference = $\bar{D}$ = | 13.64 |
| Standard Deviation of Difference = $s_D$ = | 23.78 |

| | |
|---|---|
| Test Value $t_{12}$ = | **1.7201** |

Step 4: Make the decision.




Since $t_{12}$ = 1.7201< $t_{\text{critical}}$ = 2.896, and is not in the critical region, the decision is, "Do not reject the null hypothesis." i.e., accept the null hypothesis $H_0$.

Step 5: Summarise the results.

With 98% confidence, there is not enough evidence to support the alternative hypothesis, $H_1$, indicating that the introduction of the bendable nets has significantly increased the marks of these students in Group B (even though an increase in the averages can be observed).

**Procedure:** Testing the difference between the means of T1 and T3.

Step 1: State the hypothesis and identify the claim.

In order for the introduction of the rigid nets to be effective, T1 marks must be significantly less than the T3 marks: hence the mean of the differences (T3 -T1) must be greater than 0.

Using the same notations as the above procedures:

$H_0: \ \mu_D \leq 0$ $\qquad$ $H_1: \ \mu_D > 0$ $\qquad$ (claim)

Step 2: Find the critical value.

Degrees of freedom = d.f. = $n-1=9-1=8$

Hence for the right tailed-test at $\alpha = 0.02$ (98% confidence interval), $t_{\text{critical}}$ is:

$t_{\text{critical}} = 2.896$

(obtained from the $t$-distribution table).

Step 3: Compute the test value.

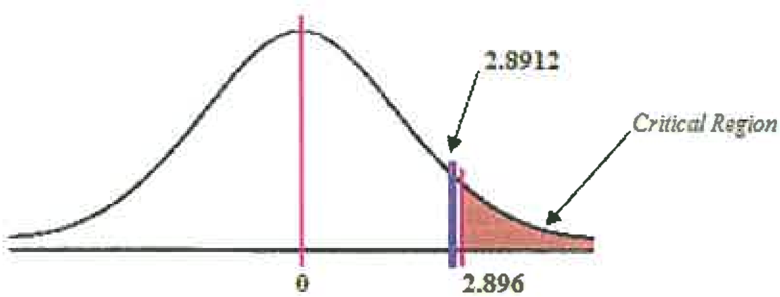
We consider the following table to determine the test value $t_{13}$:

| T1 (X1) | T3 (X3) | D=X3-X1 | D^2 |
|---|---|---|---|
| 68.18 | 88.64 | 20.45 | 418.39 |
| 61.36 | 77.27 | 15.91 | 253.10 |
| 52.27 | 40.91 | -11.36 | 129.13 |
| 31.82 | 43.18 | 11.36 | 129.13 |
| 52.27 | 70.45 | 18.18 | 330.58 |
| 45.45 | 54.55 | 9.09 | 82.64 |
| 77.27 | 93.18 | 15.91 | 253.10 |
| 36.36 | 38.64 | 2.27 | 5.17 |
| 77.27 | 81.82 | 4.55 | 20.66 |
| | | $\sum D = 65.91$ | $\sum D^2 = 1203.51$ |

| | |
|---|---|
| Mean of Difference = $\overline{D}$ = | 9.60 |
| Standard Deviation of Difference = $s_D$ = | 10.0 |

| | |
|---|---|
| Test Value $t_{13}$ = | 2.8912 |

Step 4: Make the decision.




Since $t_{12}$ = 2.8912 ≈ $t_{\text{critical}}$ = 2.896, the decision is, "Reject the null hypothesis." i.e., accept the alternative hypothesis.

Step 5: Summarise the results.

With 98% confidence, there is enough evidence to support the alternative hypothesis, $H_1$, indicating that the introduction of the rigid nets has significantly increased the marks of these students in Group B.

# Appendix 6 Confidence Intervals

## Confidence Intervals for Groups A and B

A 98% confidence interval with the sample size of 9 (degrees of freedom = 8) for the data obtained will be considered. This indicates that the corresponding $t_{\alpha/2}$ value will be 2.896.

i.e., $n = 9$, $d.f = 8$, Confidence Interval = 98% $\Rightarrow$ $t_{\alpha/2} = 2.896$

To determine the confidence intervals in each case, the following formula will be used:

$$\overline{X} - t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) < \mu < \overline{X} + t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right) \quad \text{-----------} \quad \textbf{(1)}$$

where:

$\overline{X} =$ Sample Mean $\mu =$ Population Mean

$s =$ Sample Standard Deviation $n =$ Number of samples

**Table 1: Group A:**

| | T1 | T2 | T3 |
|---|---|---|---|
| | 68.18 | 54.55 | 75.00 |
| | 75.00 | 81.82 | 77.27 |
| | 56.82 | 70.45 | 93.18 |
| | 56.82 | 84.09 | 70.45 |
| | 65.91 | 97.73 | 86.36 |
| | 59.09 | 22.73 | 81.82 |
| | 56.82 | 54.55 | 45.45 |
| | 79.55 | 93.18 | 88.64 |
| | 72.73 | 61.36 | 100.00 |
| $\overline{X} =$ | 65.66 | 68.94 | 79.80 |
| $s =$ | 8.76 | 23.51 | 15.85 |

**Table 2: Group B:**

| | T1 | T2 | T3 |
|---|---|---|---|
| | 68.18 | 97.73 | 88.64 |
| | 61.36 | 86.36 | 77.27 |
| | 52.27 | 54.55 | 40.91 |
| | 31.82 | 95.45 | 43.18 |
| | 52.27 | 31.82 | 70.45 |
| | 45.45 | 52.27 | 54.55 |
| | 77.27 | 84.09 | 93.18 |
| | 36.36 | 34.09 | 38.64 |
| | 77.27 | 88.64 | 81.82 |
| $\overline{X}$ = | **55.81** | **69.44** | **65.40** |
| $s$ = | **16.55** | **26.29** | **21.43** |

Using equation (1), and the values for $\overline{X}$ and $s$ from tables 1 and 2, the following confidence intervals for Tests 1 – 3 for groups A and B are obtained:

| | **Group A** | **Group B** |
|---|---|---|
| **T1** | $57.20 < \mu < 74.12$ | $39.83 < \mu < 71.79$ |
| **T2** | $46.24 < \mu < 91.64$ | $44.06 < \mu < 94.82$ |
| **T3** | $64.50 < \mu < 95.10$ | $44.71 < \mu < 86.10$ |

Hence, one can be 98% confident that the population means for T1, T2 and T3 for **Group A** are between 57.20% and 74.12%; 46.24% and 91.64%; 64.50% and 95.10%, respectively.

Similarly, one can be 98% confident that the population means for T1, T2 and T3 for **Group B** are between 39.83% and 71.79%; 44.06% and 94.82%; 44.71% and 86.10%, respectively.

Clearly the above results indicate that the upper limits of the 98% confidence interval for both groups of students have progressively increased according to the order of test completion, indicating the overall increase in conceptualisation. Further, the standard deviation decreased in T3 for both groups, demonstrating that the spread of marks has been condensed. This shows that the conceptualisation of students at the lower end of abilities has increased.

# Appendix 7 Journal Article

**Title:**

# Bendable Manipulatives Prove no Advantage in Conceptualising 3D Objects from their 2D Nets

**Authors:**

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## ABSTRACT

The usefulness of manipulatives (physical objects which may be safely handled) in the primary maths classroom has been frequently asserted. The purpose of this study was to compare the effectiveness of two different types of manipulatives, bendable and rigid, as aids for the conceptualisation of 3D solids from 2D nets (fold-outs of solid geometrical shapes) within the NSW Stage 2 Mathematics curriculum.

Contrary to initial expectations, the bendable nets, although more attractive to pupils, did not prove superior to the rigid variety, except possibly in the case of weaker students. By far the most significant advances in conceptualisation followed teaching experiences using the rigid nets. Although this was a preliminary study and the sample sizes were insufficient to support solid conclusions, it is posited that the data were sufficiently robust to support tentative observations.

We suggest that the poorer than expected results for the bendable nets may be partially explained by the reduced conceptual demands made by these more "obvious" shapes. Correspondingly, it is thought that the greater mental visualisation required when working with the rigid nets may have produced better student conceptualisation.


## Introduction

It has long been recognised that the careful use of manipulatives enhances maths learning among primary and secondary students (Martinie and Stramel 2004; Reys et al. 2007; Shaw 2002.) and some attention has been given to their geometrical applications (Obara 2009). Barger and McCoy (2009) have even argued the value of manipulatives for teaching geometry at tertiary level. However, little appears to have been done on the use of manipulatives in relating 2D nets to their corresponding 3D solids at the Stage 2 level. Further, although manipulatives may be constructed which allow differing degrees of "manipulation" by the student, and which thus display different levels of correspondence to the concept under investigation, there have been no reports of the relative effectiveness of such different types of manipulatives. This study presents preliminary results from such an investigation.

Geometry is one of the oldest branches of mathematics, with important connections to most other mathematical disciplines and to so much of life's experience, although recent decades have seen its substantial displacement by other topics in the maths classroom. In fact, if one Googles "the importance of geometry in our daily life", over 600,000 sites invite investigation. Such considerations suggest the importance of those geometrical topics retained in the primary curriculum and of instructional strategies which enhance their assimilation.

## The manipulatives

A rigid 2D net corresponding to any regular 3D solid, such as a prism, cube or pyramid, may be cut out of any flat medium. Similar 2D nets can be made which do **not** correspond to any 3D solid, due to the transposition of one or more sides. Some materials allow the construction of such nets with the additional capability of bending up into their 3D solid, thus providing a more obvious correspondence between the two forms. (Care must be taken with terminology when discussing nets. For example, Ainge (1996, p. 346), defines a net just in terms of the bendable variety, being a "plane diagram showing all faces of a 3D shape, which can be cut out and folded to construct the solid".)

The medium chosen for this study was flute board – a safe, plastic sheet product available from office supply stores in a variety of bright colours. Cost, ease of handling and storage considerations suggested sizes for the 3D solids in the order of 3-6 cm side length. The bendable examples were made by systematically cutting away one side of the sheeting with a "V" cut using an angled picture-framing trimmer.

Two different sets of 2D nets were designed and constructed, each including examples which did correspond to 3D solids, and others which did not. Five solids were represented in these sets: cube, rectangular prism, hexagonal prism, square-based pyramid and triangular-based pyramid. One set consisted of rigid nets, the other of bendable nets. Each set consisted of 15 different nets.

## Methodology

The Grade 4 class was split into two groups of 12 students with the aid of the supervising class teacher. Originally it was intended that these two groups be approximately equal in ability. As it transpired, Group A was markedly stronger than Group B but this may have ultimately proved an advantage to the study, since it provided results for groups of different ability. A maximum of just seven 50-minute class periods was allocated to this investigation by the class teacher, so the study was configured within these constraints. In fact, the equivalent of two full maths classes for each group were conducted by the class teacher within these seven periods, thus the study fitted well within the allotted time.

Three 45-minute tests of identical structure and similar difficulty were constructed and designated T1, T2 and T3. Appropriate to Grade 4, the 3D polyhedra chosen were:

- Prisms: triangular, rectangular, pentagonal, square and hexagonal;
- Pyramids: triangular, square, hexagonal, and pentagonal; and
- Cylinders and cones.

It should be noted that some of these shapes were not included in the manipulative sets mentioned above, with which pupils would have experience. This was done deliberately so as to test understanding rather than knowledge and skills. The principal features of these tests were questions relating to whether or not a given 2D net accurately corresponded to any 3D solid, and if so, which one. These were paper tests, for which the pupils did not have access to the nets.

Two worksheets were also constructed: W1 corresponding to the bendable nets and W2 corresponding to the rigid nets. Both worksheets consisted of 15 questions where each question related to a particular net. Students were allowed 90 seconds with each net during which to answer the appropriate worksheet question, after which the nets were rotated. These worksheets comprised the second part of the teaching experience. The tests and worksheets were deployed according to the schedule shown below.

<u>Period 1</u>

Following a general introduction to the topic, all students completed the pre-test, T1.

<u>Period 2</u>

For the first half-period, Group A participated in a familiarisation and learning experience involving the bendable nets with the principal researcher, while the class teacher conducted a regular maths class with Group B. Groups were then swapped and during the second half-period, Group A moved to the regular maths lesson, and Group B participated in the familiarisation and learning experience involving rigid nets with the principal researcher.

Period 3
For the first half-period, Group A completed worksheet W1, involving the bendable nets, while the class teacher conducted a regular maths class with Group B.
During the second half-period, Group B completed worksheet W2, involving the rigid nets, while the class teacher conducted a regular maths class with Group A.

Period 4
Both groups completed tests: Group A completed T2, relevant to bendable nets, while Group B completed T3, the test relevant to rigid nets.

Period 5
This was carried out as a reflection of Period 2, with Group A using rigid nets, and Group B using bendable nets.

Period 6
Similarly, this was carried out as a reflection of Period 3, with Group A completing worksheet W2 and Group B completing worksheet W1.

Period 7
This was a reflection of Period 4, with Group A completing T3 and Group B completing T2.

## Results

The data from T1, T2 and T3 were standardised by excluding those students who did not complete all three tests. This reduced the sample size down to nine students in each group. Table 1 shows the overall percentages and *t*-test scores for Tests 1, 2 and 3 for both groups. Table 2 shows the overall percentages for the worksheets W1 and W2.

**Group A**

| **Result for T1** (Pre-test) | **Result for T2** (following learning experience with bendable nets) | **Result for T3** (following learning experience with rigid nets) |
|---|---|---|
| 65.7% | 68.9% | 79.8% |
| ← $t_{21} = 0.46$ → | | |
| ← $t_{31} = 2.95$ → | | |

**Group B**

| **Result for T1** (Pre-test) | **Result for T3** (following learning experience with rigid nets) | **Result for T2** (following learning experience with bendable nets) |
|---|---|---|
| 55.8% | 65.4% | 69.4% |
| ← $t_{31} = 2.89$ → | | |
| ← $t_{21} = 1.72$ → | | |

**Table 1.** Test results for both groups.

**Group A**

| **Result for W1** (following learning experience with bendable nets) | **Result for W2** (following learning experience with rigid nets) |
|---|---|
| 75.3% | 71.5% |

**Group B**

| **Result for W2** (following learning experience with rigid nets) | **Result for W1** (following learning experience with bendable nets) |
|---|---|
| 55.7% | 77.9% |

**Table 2.** Worksheet results for both groups.

As may be seen from Table 1, the largest increase in test mean followed the teaching experience involving the rigid nets. This was true for both groups and to a very similar degree. For Group A (the stronger group) there was a 3.2% mean increase in test mean following experience with the bendable nets and a further improvement of 10.9% after the rigid nets. Group B showed a 9.6% increase in test mean following experience with the rigid nets. However, when Group B was exposed to the bendable nets, a small increase of 4.0% was observed.

The difference between T2 and T1, and that between T3 and T1 was also compared using *t*-test analysis. These results are also shown in Table 1. For our small sample size the critical *t*-value corresponding to a 98% confidence factor was 2.90. As may be seen, the T3/T1 comparisons for both groups gave *t*-values very close to, or exceeding, this critical value. However, the T2/T1 comparisons gave results much less than the critical value for both groups. Interestingly, the different ordering of the tests appeared not to have affected these results. Clearly, this data shows that the teaching experience using rigid nets produced a statistically significant improvement in test score, whereas this can not be said of that involving the bendable variety.

Clearly, one possible explanation of these disparities is that T3, which followed the learning experience involving rigid nets for both groups, was easier than T2. As earlier mentioned, the reason that T2 and T3 were each made specific to a different learning experience was to standardise results across the two groups. It was recognised, however, that different levels of test difficulty would inevitably compromise comparisons between the two learning tools, thus every attempt had been made to produce two equivalent tests. Both were identically structured, with the same number and type of question in each section. Despite these precautions, however, it was recognised that some disparity might exist. In order to further investigate this possibility, T2 and T3 were submitted to four academic peers with mathematical experience, all of whom were asked to complete them and compare their difficulty. All four rated the tests as very close, there being no predominant judgement of one being more difficult than the other. This implies that the results obtained were not an artefact of uneven test difficulty.

## Conclusions

For Group A there was clearly a much bigger improvement in conceptualisation following class experience with the rigid nets than with the bendable variety. This was contrary to our initial expectations. It was recognised that some of this improvement may be attributed to accumulating experience, since this group experienced the rigid nets last. However, Group B showed a similar pattern with a reversed order of contact, suggesting that the experience factor was not significant.

Students using the bendable nets could identify their 3D shape and whether or not they "worked" by actually bending and seeing. It is then no surprise that the worksheet results showed better levels of completion when using the bendable nets than for the rigid variety. This was particularly true of the weaker group, which might be expected. However, these tactile and direct experiences were not available during the paper tests which followed. Rather, students were required to simply identify nets and 3D shapes by observation and abstract thought. It may be that the differences in thought processing between these different problem solving challenges resulted in students performing to a lower level on the written test following experience with the bendable nets.

On the other hand, students learning with rigid nets were required to identify the corresponding 3D shape and decide whether they worked or not from their flat configuration. They were therefore forced to manipulate the shapes in their minds rather than with their hands, focusing on mental rather than physical processing. Thus, although not performing as well on the worksheet, they developed superior abstract skills in identifying 3D shapes from flat nets, and so performed at a much higher level on the written test.

Whilst performing this study, it was observed that the pupils thoroughly enjoyed working with the manipulatives, the bendable variety being definitely the most popular. This supports the idea that the use of a range of tactile experiences in the classroom not only diversifies assimilation pathways but makes learning more enjoyable.

Clearly there are many other ways in which this study could be developed, including a comparison of the conceptualisation of 3D shapes between boys and girls.

## References


Ainge, D.J. (1996). Upper primary students constructing and exploring three dimensional shapes: A comparison with virtual reality with card nets. *Journal of Educational Computing Research. 14*(4), 346.

Barger, R. H. & McCoy, A. C. (2009). Sacred Cows Make the Best Hamburgers. *Mathematics Teacher, 102*(6), 414-418.

Martinie, S. and Stramel J. (2004). Manipulatives in the Middle School. *Mathematics Teaching in the Middle School. 9*(6), 332-33.

Obara, S. (2009). Where does the formula come from? *Australian Mathematics Teacher 65*(1), 25-33.

Reys, R.E., Lindquist, M.M., Lambdin D.V. and Smith N.L. (2007). *Helping Children Learn Mathematics* (8th ed.). Hoboken, NJ: John Wiley and Sons.

Shaw, J.M. (2002). *Manipulatives Enhance the Learning of Mathematics.* Retrieved 14 September 2009 from http:www.eduplace.com/state/authpor/shaw.pdf.

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|---|---|
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| **Sent:** | Monday, 28 September 2009 9:43 AM |
| **To:** | Lynden Rogers |
| **Subject:** | Article for AMT |

Dear Lynden,
Thank you for the article submitted to AMT. This email acknowledges receipt of article by the journal editor. It will be sent for review in the near future.
Regards,
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--
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