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# Statistical Modelling and Prediction of Compressive Strength of Concrete

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Abstract

The matrix mixture of concrete can be made to have high compressive strength. In the present paper, statistical model was built-up to predict the compressive strength of concrete containing different matrix mixtures at fixed age or at different age of 1, 3, 7, 28, 56, 90 and 180 days. The model examines eight different parameters for the matrix mixture that includes: time, water, cement, metakaolin (MK), silica fume (SF), sand (S), aggregate (A) and superplasticizer (SP). This research addresses the effect of the matrix mixture of concrete on the compressive strength, where this information will help the cement industry in producing the required concrete strength. The results from the predicted model have high correlation to the experimental results for the concrete compressive strength.

*Keywords: Concrete; Compressive strength; Matrix Mixture; Prediction; Modelling.*

## 1. Introduction

Very few studies have investigated the effect of the matrix mixture and its effect on the concrete compressive strength. Most researchers present experimental results for the different matrix mixture without providing analytical procedures to determine the effect of the different parameters in the studies mixes. Demirboga et al. investigated experimentally the compressive strength of concrete made up of mixtures of expanded perlite (EPA) and pumice aggregate (PA) along with silica fume (SF) and class C fly ash (FA) on the compressive strength to produce lightweight aggregate concrete (LWAC) [1]. Ortiz et al. conducted experiments on different concrete mix and the influence of the environmental temperature on the concrete compressive strength after 7 and 28 days [2]. Colak provided empirical equation for calculating the compressive strength of Portland cement concrete compare to existing experimental data; however the equation focused only on the optimum water cement ratio to determine the optimum compressive strength [3]. On the other hand, Del Viso et al. investigated the influence of the shape and the size of the specimens on the compressive strength. They compared the performance of the cylinders and cube concrete prisms under the compressive strength [4].

Bhanja and Sengupta presented a mathematical model developed using statistical methods to predict 28-day compressive strength of silica fume concrete with water-to-cementitious material (w/cm) ranging from 0.3 to 0.42 and silica fume from 5 to 30% [5]. The cubic equation is simple and was with one parameter of (SF) only. Sahin et al. tested the effect of different pumice aggregate (PA) ratio with different cement dosage on the compressive strength. The tests results leads to decrease in the concrete density, and increase in its compressive strength [6]. Artificial neural network (ANN) was successfully used to predict the multiple variables and nonlinear behaviour of different parameters in the concrete mixture to obtain the compressive strength under different ages [7-11]. The drawback of ANN is that the model does not provide an equation to be used by others.

Other researchers investigated experimentally the strain-hardening of high-performance fiber-reinforced cement composites (HPFRCCs), and the strain-softening of fiber-reinforced cementitious (FRC) composites. Chao et al. experimentally tested five types of fibers within the concrete mix; 1. spiral reinforcement, 2. hooked steel fiber, 3. twisted steel fiber; either square to rectangular twisted steel fiber, 4. ultra-high molecular weight polyethylene fiber (UHM-PE), and 5. polyvinyl alcohol (PVA) fiber (PVA13, and PVA K-II) [12]. The experiment extended to obtain the relationship of the compressive strength and the pullout of the reinforced bars.

This research provides mathematical and mechanical model for the compressive strength of concrete ($f_c'$) in connection with experimental data set and the Levenberg-Marquardt (LM) method for nonlinear least squares analysis to estimate the initial parameters. The tested concrete matrix mixture for this study contains eight parameters; time, water, cement, metakaolin (MK), silica fume (SF), sand (S), aggregate (A) and superplasticizer (SP) [13-14].

## 2. Prediction Modelling

### *2.1. Mathematical model*

The proposed linear mathematical model can predict the compressive strength of concrete ($f_c'$) at the determined time. The weighted sums of the input components $\sum_{i=1}^{n} a_i x_i$ are plotted to the water-cement-ratio (w/c) as introduced in Equation (1) to obtain the optimum w/c content, where $a_i$: equation parameters, and $x_i$: matrix mixture including water, cement, cementitious materials, sand and aggregate). The LM analysis estimated the equation parameters to be 38.202, 54.261, 54.238, 128.235, 73.699, and -121.45for $a_1$ through $a_6$ respectively for the experimental data set shown in Table (1). The result shows that the coefficient of determinations $R^2 = 1$ showing true goodness of fit analysis.

$$f_c^{'} = [\sum_{i=1}^{n} a_i x_i] \tag{1}$$

TABLE1: RELATIVE COMPOSITION OF MATRIX MIXTURES [12]

| Matrix | Cement Type | Fly Ash | Sand* | Silica fume | High-range water-reducing admixture | Water | w/c | $f_c'$, MPa Exp. | **$f_c'$, MPa Pred.** |
|---|---|---|---|---|---|---|---|---|---|
| Inputs | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | | | |
| Mixture 1 | 0.8 | 0.2 | 1.0 | 0.07 | 0.04 | 0.26 | 0.325 | 76 | **76** |
| Mixture 2 | 1.0 | 0.15 | 1.0 | | | 0.40 | 0.240 | 52 | **52** |
| Mixture 3 | 0.8 | 0.2 | 1.0 | | | 0.45 | 0.563 | 41 | **41** |

*Flint sand ASTM 50-70

### *2.2. Mechanical model*

The proposed mechanical model in Equation (2) can predict the compressive strength of concrete ($f_c'$) at various ages. This rheological model is illustrated by Kelvin-Voigt [15] as a combination of spring and dashpot in parallel. The matrix mixture represented by the linear model as the spring, the time (T) represented by the viscous (η) dashpot, where both have the same strain.

$$f_c^{'} = \left[k\left(\frac{w}{c}\right) + \sum_{i=1}^{n} a_i x_i\right].\left[1 - \exp\left(-\frac{T}{\eta}\right)\right] \quad (2)$$

The spring part in the mechanical model that represent the matrix mixture can be rewritten as proposed in Equation (1) or as shown in Equation (2), where k: the constant for water-cement-ratio. The accuracy of this model would vary between 80 ~ 90% compared to the tested data. The model will overestimate or underestimate the compressive strength at earlier ages.

**Arrhenius equation** [15] gives the dependence of the rate constant which is the compressive strength ($f_c'$) in this research case of chemical reactions on the time (T) and activation energy which is the matrix mixture ($E_a$) in this case. The LM analysis will estimate the equation parameters; $k_1$, $k_2$ and $k_3$ for Equation (3).

$$f_c' = k_1 + k_2 e^{-E_a/k_3 T} \quad (3)$$

Matrix mixture ($E_a$) in Equation (4) is the main proposed part in the Arrhenius equation, and to be plugged into Equation (3), where k, $a_i$, and $b_i$: are constant obtained by LM analysis, w: is water content, $cm_i$: is cement & cemetitious components, and $A_i$: is the sand and aggregate components. The LM analysis estimated the equation parameters as shown in Table (2). The modified Arrhenius equation would have accuracy above 90% compared to the tested data, it also will underestimate or overestimate the compressive strength at earlier ages. Both models that depend on time need more data to increase its accuracy level.

$$E_a = \left[\frac{kw}{\sum_{i=1}^{n} a_i cm_i} + \sum_{i=1}^{n} b_i A_i\right] \quad (4)$$

Tables (3A and 3B) show 7 different matrix mixtures groups for total of 57 specimens. Compressive strength tests performed at different age of specimens (AS) of the 1, 3, 7, 28, 56, 90 and 180 days.

TABLE 2: LM ESTIMATED PARAMTERS

| | | | AS | C | MK | SF | SP | W | A | S |
|---|---|---|---|---|---|---|---|---|---|---|
| Parameters | $k_1$ | $k_2$ | $k_3$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $k$ | $b_1$ | $b_2$ |
| Values | 35 | 70.63 | 42.25 | 0.016487 | 8.4E-5 | 0.02712 | 0.02087 | 6.0645 | 0.002616 | 0.00534 |

TABLE 3A: COMPRESSIVE STRENGTH DATA SETS AND RESULTS [13] [14]

| AS | C | MK | SF | W | A | S | SP | $f_c'$(MPa) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (days) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (l/m$^3$) | Exp. | **Pred.** | **Variance** |
| 1 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 35 | **41.56783** | **-18.7652** |
| 1 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 35 | **41.67337** | **-19.0668** |
| 1 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 48 | **40.01697** | **16.6313** |
| 1 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 38 | **38.32347** | **-0.85125** |
| 1 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 38 | **38.58331** | **-1.53501** |
| 1 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 34 | **37.7708** | **-11.0906** |
| 1 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 32 | **37.92039** | **-18.5012** |
| 3 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 67 | **67** | **1.76E-06** |
| 3 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 63 | **67.1705** | **-6.61983** |
| 3 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 63.5 | **64.25208** | **-1.18437** |
| 3 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 60.5 | **60.5** | **1.05E-06** |
| 3 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 57.5 | **61.14793** | **-6.34423** |
| 3 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 59 | **59** | **9.78E-07** |
| 3 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 53 | **59.42435** | **-12.1214** |
| 3 | 475 | 25 | 0 | 150 | 1087 | 721 | 0.6 | 73 | **61.84351** | **15.28287** |
| 3 | 475 | 0 | 25 | 150 | 1087 | 716 | 0.6 | 67 | **62.05385** | **7.382313** |
| 7 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 76.5 | **85.30878** | **-11.5147** |
| 7 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 75.5 | **85.42348** | **-13.1437** |
| 7 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 72 | **83.4097** | **-15.8468** |
| 7 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 80 | **80.64386** | **-0.80483** |
| 7 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 74.5 | **81.13735** | **-8.90919** |
| 7 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 74 | **79.47322** | **-7.39624** |
| 7 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 70.5 | **79.80854** | **-13.2036** |
| 7 | 475 | 25 | 0 | 150 | 1087 | 721 | 0.6 | 88.2 | **81.6594** | **7.415644** |
| 7 | 475 | 0 | 25 | 150 | 1087 | 716 | 0.6 | 79.3 | **81.81575** | **-3.17244** |
| 28 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 89 | **99.88906** | **-12.2349** |
| 28 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 88.5 | **99.92602** | **-12.9108** |
| 28 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 83.5 | **99.26783** | **-18.8836** |
| 28 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 94.5 | **98.32952** | **-4.0524** |
| 28 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 98.5 | **98.5** | **1.38E-07** |
| 28 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 84.5 | **97.91949** | **-15.8811** |
| 28 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 89.5 | **98.03776** | **-9.5394** |
| 28 | 475 | 25 | 0 | 150 | 1087 | 721 | 0.6 | 103.6 | **98.67887** | **4.750124** |
| 28 | 475 | 0 | 25 | 150 | 1087 | 716 | 0.6 | 106.5 | **98.73215** | **7.293758** |
| 56 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 95 | **102.7006** | **-8.10594** |
| 56 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 93 | **102.7199** | **-10.4515** |
| 56 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 84.5 | **102.3758** | **-21.1548** |
| 56 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 96.5 | **101.8821** | **-5.57734** |
| 56 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 101.5 | **101.9721** | **-0.46512** |
| 56 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 87 | **101.6653** | **-16.8566** |
| 56 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 90.5 | **101.7279** | **-12.4065** |
| 90 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 98 | **103.7942** | **-5.91247** |
| 90 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 96.5 | **103.8064** | **-7.57141** |
| 90 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 85.5 | **103.5886** | **-21.1563** |
| 90 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 97.5 | **103.2755** | **-5.9236** |
| 90 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 104 | **103.3326** | **0.641692** |
| 90 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 89 | **103.1377** | **-15.885** |
| 90 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 92 | **103.1775** | **-12.1495** |
| 90 | 475 | 25 | 0 | 150 | 1087 | 721 | 0.6 | 112.9 | **103.3925** | **8.421199** |
| 90 | 475 | 0 | 25 | 150 | 1087 | 716 | 0.6 | 110.2 | **103.4103** | **6.161286** |

TABLE 3B: COMPRESSIVE STRENGTH DATA SETS AND RESULTS [13] [14]

| AS | C | MK | SF | W | A | S | SP | $f_c'$(MPa) | | |
|---|---|---|---|---|---|---|---|---|---|---|
| (days) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (kg/m$^3$) | (l/m$^3$) | Exp. | **Pred.** | **Variance** |
| 180 | 475 | 25 | 0 | 135 | 1050 | 720 | 43 | 99 | **104.7081** | **-5.76572** |
| 180 | 475 | 0 | 25 | 135 | 1050 | 725 | 43 | 97.5 | **104.7142** | **-7.39922** |
| 180 | 500 | 0 | 0 | 150 | 1050 | 695 | 19 | 87.5 | **104.6038** | **-19.5472** |
| 180 | 425 | 75 | 0 | 150 | 1050 | 680 | 19 | 99.5 | **104.4448** | **-4.96962** |
| 180 | 425 | 0 | 75 | 150 | 1050 | 680 | 19 | 106.5 | **104.4738** | **1.902521** |
| 180 | 450 | 50 | 0 | 165 | 1050 | 690 | 12 | 92.5 | **104.3746** | **-12.8374** |
| 180 | 450 | 0 | 50 | 165 | 1050 | 685 | 12 | 93.5 | **104.3949** | **-11.6523** |

Fig. (1) shows the values obtained from the predicted model and the experimental testing. The values show the coefficient of determination ($R^2$) equals to 90.26% of confidence. The proposed model shows its capability of generalizing between input and output variables with reasonable good predictions.

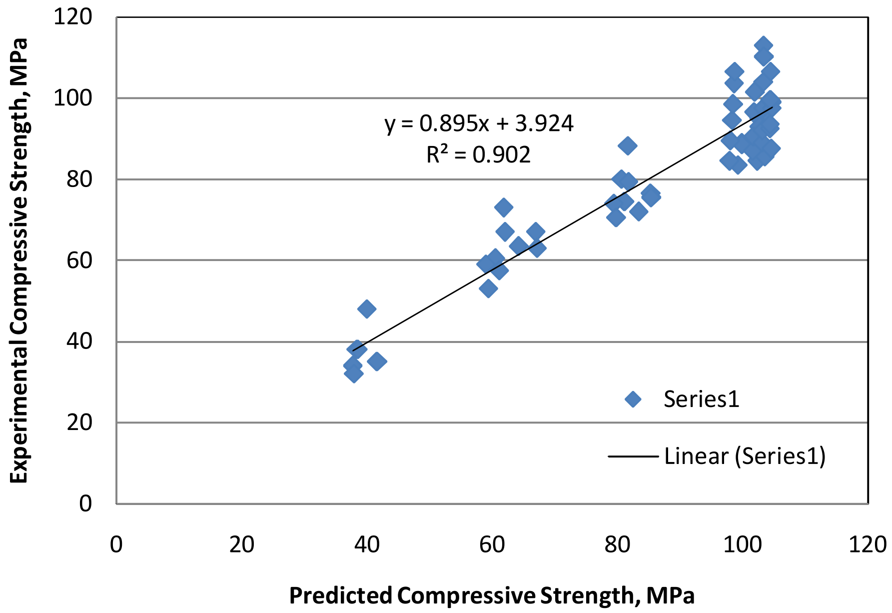


FIG. 1COMPARISON OF $f_c'$ EXPERIMENTAL RESULTS WITH PREDICTED RESULTS

## 3. Results and discussion

In this study, the error arose during the modelling was optimized by the **Unbiased Nonlinear Least-Squares** (UNLS) curve fitting with Microsoft Excel solver which was used in connection with the experimental observed data. As a measure of how well the model y = $f$(x) fits the collection of points {($x_1$, $y_1$), ($x_2$, $y_2$), ($x_3$, $y_3$) … ($x_n$,$y_n$)}, where the squares of the differences between the actual y-values and the values given by the model to obtain the sum of the squared errors (SSE).

$$SSE = \min \sum_{i=1}^{n} [f(x_i) - y_i]^2 \tag{5}$$

Graphically, SSE can be interpreted as the sum of the squares of the vertical distances between the graph of $f$ and the given points in the plane. If the model is perfect, then SSE = 0. However, perfection is not feasible, and model can be settled for minimized SSE.

**Coefficient of determination** $R^2$ (r-squared) is defined as the proportion of the total variation in Y "explained" by the regression of Y on X. The coefficient of determination ranges from 0 (when the estimated regression model explains none of the variation in Y) to 1 (when all points lie on the regression line). Also it can be interpreted as the fraction of uncertainty explained by the fitted model. Normal $R^2$ is a widely good-of-fit measure; however in sometimes it doesn't have its usual meaning for nonlinear curves.

$$R^2 = 1 - \frac{SSE}{SST} \tag{6}$$

$$SST = \sum_{i=1}^{n} (y_i - \tilde{y})^2 \tag{7}$$

$$\tilde{y} = \frac{1}{n} \sum_{i=1}^{n} y_i \tag{8}$$

Where SST = the total sum of square; $\tilde{y}$ = the mean of the observed data.

The performance of the mathematical model or the mechanical model to predict the compressive strength of concrete using variable inputs in the mixture matrix performed well and confirmed by the testing the statistical values with the coefficient of determination ($R^2$). The mathematical model resulted in $R^2$ of 100%, showing true goodness of fit analysis, and the mechanical model resulted into $R^2$ of 90.26% and considering the effect of time.

## 4. Conclusion

The proposed mathematical and mechanical models are capable of predicting the effect of the mixture matrix to produce the required concrete compressive strength. The models are validated with true goodness of fit analysis that can help the cement industry to provide design parameters for required compressive strength.

### References

[1] R. Demirboga, I. Orung and R. Gul, "Effects of expanded perlite aggregate and mineral admixtures on the compressive strength of low-density concrete," *Cement and Concrete Research,* vol. 31, no. 11, pp. 1627-1632, 2001.

[2] J. Ortiz, A. Aguado, L. Agullo and T. Garcia, "Influence of environmental temperatures on the concrete compressive strength: Simulation of hot and cold weather conditions," *Cement and Concrete Research,* vol. 35, no. 10, pp. 1970-1979, 2005.

[3] A. Colak, "A new model for the estimation of compressive strength of Portland cement concrete," *Cement and Concrete Research,* vol. 36, no. 7, pp. 1409-1413, 2006.

[4] J. del Viso, J. Carmona and G. Ruiz, "Shape and size effects on the compressive strength of high-strength concrete," *Cement and Concrete Research,* vol. 38, no. 3, pp. 386-395, 2008.

[5] S. Bhanja and B. Sengupta, "Investigations on the compressive strength of silica fume concrete using statistical methods," *Cement and Concrete Research,* vol. 32, no. 9, pp. 1391-1394, 2002.

[6] R. Sahin, R. Demirboga, H. Uysal and R. Gul, "The effects of different cement dosages, slumps and pumice aggregate ratios on the compressive strength and densities of c," *Cement and Concrete Research,* vol. 33, no. 8, pp. 1245-1249, 2003.

[7] N. Hong-Guang and J.-Z. Wang, "Prediction of compressive strength of concrete by neural networks," *Cement and Concrete Research,* vol. 30, no. 8, pp. 1245-1250, 2000.

[8] M. Saridemir, "Prediction of compressive strength of concretes containing metakaolin and silica fume by artificial neural networks," *Advances in Engineering Softwares,* vol. 40, no. 5, pp. 350-355, 2009.

[9] M. Sarıdemir, "Predicting the compressive strength of mortars containing metakaolin by artificial neural networks and fuzzy logic," *Advances in Engineering Software,* vol. 40, no. 9, pp. 920-927, 2009.

[10] Z. H. Duan, S. C. Kou and C. S. Poon, "Prediction of compressive strength of recycled aggregate concrete using artificial neural networks," *Construction and Building Materials,* 2012.

[11] J.-S. C. Chou and C.-F. Tsai, "Concrete compressive strength analysis using a combined classification and regression technique," *Automation in Construction,* vol. 24, pp. 52-60, 2012.

[12] S.-H. Chao, A. E. Naaman and G. J. Parra-Montesions, "Bond behavior of reinforcing bars in tensile strain-hardening fiber-reinforced cement composite," *ACI Structural Journal,* vol. 106, no. 6, pp. 897-906, 2009.

[13] C. Poon, S. Kou and L. Lam, "Compressive strength, chloride diffusivity and pore structure of high performance metakaolin and silica fume concrete," *Construction and Building Materials,* vol. 20, no. 10, pp. 858-865, 2006.

[14] H. Wong and H. Abdul Razzak, "Efficiency of calcined kaolin and silica fume as cement replacement material for strength performance," *Cement Concrete Research,* vol. 35, no. 4, pp. 696-702, 2005.

[15] A. M. Neville, Creep of concrete: plain, reinforced and prestressed, Amsterdam: North-Holland Publishing Company, 1970.