CONCRETE RESEARCH LETTERS www.crl.issres.net Vol. 3(1) – March 2012

Behavior and Analysis of Cracked Self-Compacted Reinforced Concrete Beams

KH. M. Heiza *[1], A. T. Attwa [1], Y. B. Shaheen[1].

[1] *Civil Engineering Department, Faculty of Eng., Menoufiya University, EGYPT*

M. H. Seleem[2]

[2] *Engineering Materials Department, Zagazig University, Zagazig, EGYPT*

Abstract

The objectives of this paper are to compare between the fracture parameters of self compacting concrete (SCC) and normal vibrating concrete (NVC). The fracture behavior of both the plain and reinforced concrete beam specimens under three point bending (3PB) was investigated. It was found that the values of fracture toughness in reinforced concrete beams increased with increasing the notch– depth ratio, increasing the area of steel bars in cross section and with using dolomite as coarse aggregate in the mix. The self compacting concrete beams exhibit good fracture toughness than those of normal concrete at all the used variables. A model of Hillerborg was used to predict the fracture toughness of notched concrete beams.

Keywords: *Self compacting concrete; Fracture toughness; Linear & nonlinear fracture mechanics, closing stress intensity factor, Crack.*

1. Introduction

Fracture mechanics has been developed and applied for many decades. Its scope of application has been extended into numerous fields of materials, such as metal, ceramic, concrete, etc. Since Kaplan firstly introduced fracture mechanics into concrete beam to measure fracture toughness [1], more and more investigations have been performed for fracture mechanics of concrete. Moreover, the fundamental research and application in concrete structures have attracted increasing attention since early 1980s [2]. Some researches on the fracture of cement and concrete assumed that, linear elastic fracture mechanics (LEFM) could be applied to these systems. Thus, these studies have been devoted to measurement of K_c or G_c [3-4]. Due to the quasi-brittle manner of concrete, various fracture models have been developed to study the crack propagation in concrete structures, such as fictitious crack model [5], crack band model [6], two parameter fracture model [7], size effect model [8] and effective crack model [9].

1 [1*]Corresponding Author, Associate Professor, Civil Engineering Department, E-mail: khheiza@ yahoo.com, khheiza@ hotmail.com.

Self-compacting concrete (SCC) represents a major evolution in the building industry. Its positive features result from the elimination of mechanical compaction. Compared to ordinary vibrated concrete, SCC offers many advantages in terms of both technology and worker health.

The growing interest in research on the fracture mechanics of concrete eventually led to its application to study the fracture behavior of reinforced concrete [14]. In the work carried out by [15], it is found that the steel reaction force decreases but the moment of steel plastic flow increases with increasing crack-depth ratio (a/w). Also, the fracture process can become stable by increasing area of steel or specimen size. In the work performed by [16], it was found that the under-reinforced beams exhibit a stable crack growth or at least neutral equilibrium prior to the unstable softening mode of collapse. The objectives of this paper are to compare between the fracture parameters of SCC and NVC and explaining the effect of coarse aggregate types, crack-depth ratio (a/W) and area of steel reinforcement on the fracture toughness of SCC and NVC.

2. Experimentation

The experimental program was designed to investigate the effect of coarse aggregate types, crack- depth ratio (a/w) and area of tensile steel reinforcement (A_s) on the fracture toughness of SCC and NVC. The details of the experimental program which had been performed for both types of concretes are presented in Table 1.

All test specimens were fabricated using locally available materials. Type I ordinary Portland cement was utilized. Ordinary siliceous sand having 100% passing ASTM sieve No. 4 was used as fine aggregate. Three types of coarse aggregates (gravel, dolomite and air cooled slag) of 14 mm maximum aggregate size (MAS) were used. The properties of the used coarse aggregates are given in Table 2. Light gray silica fume with specific surface area of 18 m^2/gm and fly ash called Dura–Pozz which complies with the chemical and physical requirements of BS 3892 Part 1, ASTM C-618, EN 450 and all relevant international quality standards for fly ash were used as a powder. High range water reducing admixture that meets the requirements for super plasticizer according to ASTM C-494 Types G and F was used. Table 3 illustrates the mix constituents for both SCC and NVC. Reinforcing steel bars which used in this investigation were locally produced mild steel bars with diameters of Ø 6 and Ø 8mm. The mechanical properties of the used steel are given in Table 4.

TABLE 1: EXPERIMENTAL PROGRAM OF FRACTURE TOUGHNESS TEST FOR SCC AND NVC SPECIMENS

Coarse aggregate type	**a/w**	**Area of steel, A_s, cm^2**	**Symbols**	
			SCC	NVC
Dolomite	0.2	0.0	SD02	ND02
		0.565	SD12	ND12
		1.0	SD22	ND22
		1.5	SD32	ND32
	0.4	0.0	SD04	ND04
		0.565	SD14	ND14
		1.0	SD24	ND24
		1.5	SD34	ND34
	0.6	0.0	SD06	ND06
		0.565	SD16	ND16
		1.0	SD26	ND26
		1.5	SD36	ND36
Air coold slag	0.2	0.0	SS02	NS02
		0.565	SS12	NS12
		1.0	SS22	NS22
		1.5	SS32	NS32
	0.4	0.0	SS04	NS04
		0.565	SS14	NS14
		1.0	SS24	NS24
		1.5	SS34	NS34
	0.6	0.0	SS06	NS06
		0.565	SS16	NS16
		1.0	SS26	NS26
		1.5	SS36	NS36
Gravel	0.2	0.0	SG02	NG02
		0.565	SG12	NG12
		1.0	SG22	NG22
		1.5	SG32	NG32
	0.4	0.0	SG04	NG04
		0.565	SG14	NG14
		1.0	SG24	NG24
		1.5	SG34	NG34
	0.6	0.0	SG06	NG06
		0.565	SG16	NG16
		1.0	SG26	NG26
		1.5	SG36	NG36

(a/w): is the crack – depth ratio
A_s: is the cross section area of steel reinforcement

TABLE 2: PROPERTIES OF THE USED AGGREGATES

Type of aggregate	Loose density, t/m^3	Compacted density, t/m^3	Impact value	Crushing value	Specific gravity
Sand	1.63	1.72	-	-	2.47
Dolomite	1.40	1.57	12.5%	14.5%	2.73
Air cooled slag	1.59	1.64	14.6%	16.5%	2.66
Gravel	1.60	1.68	16.5%	18%	2.65

TABLE 3: MIX PROPORTIONS FOR BOTH SCC AND NVC

Mix Proportion		**C**	**W/C**	**F.A**	**C.A**	**S.F.**	**f.A.**	**VEA**
Mix Type	SCC	1	0.4	2.132	1.745	0.037	0.4	0.02
	NVC	1	0.4	1.89	2.84	-	-	-

Where: C = Cement with content 400 Kg for 1 m^3 of the mix, W/C = Water cement ratio, F.A. = Fine aggregate (sand), C.A. = Coarse aggregate (crushed dolomite or air cooled slag or gravel), S.F. = Silica fume, f.A. = fly ash, VEA = Viscosity enhancing agent.

TABLE 4: MECHANICAL PROPERTIES OF THE USED STEEL BARS

Bar diameter, mm	Yield strength, MPa	Ultimate strength, MPa	% elongation
6	175	343	28
8	298.4	397.9	26

The slump flow, U – box and V-funnel tests were carried out on the SCC mix to find its fresh properties and the results are given in Table. 5. To determine the mechanical properties of both SCC and NVC mixes, compression test on cubes 150 mm side length, indirect tension test on cylinders 100 mm in diameter and 200mm height and flexure test on prisms 100×100 cross section and 400mm loaded span were performed. The results of these tests are listed in Table 6.

TABLE 5: FRESH PROPERTIES OF SCC

Type of test	Unit	Specifications*	Type of coarse aggregate		
			Dolomite	Slag	Gravel
Slump Flow (Cone test)	(mm)	600 – 650	760	650	650
Flow Time until 500 mm	(sec.)	3 - 15	5	4.5	6.5
U- type filling capacity	(mm)	Min. 300	323	309	295
V- type Funnel flow time	(sec.)	8 - 15	7	4	4

*Requirements for the SCC are shown according to the recommendations of JSCE and Japan Highway public corporation [21].

TABLE 6: MECHANICAL PROPERTIES IN (MPA) FOR SCC AND NVC

Type of coarse aggregate	Compressive strength		Tensile strength		Flexural strength	
	SCC	NVC	SCC	NVC	SCC	NVC
Dolomite	59.5	41.5	3.87	3.35	4.02	3.73
Air cooled slag	58.2	45.8	3.71	3.25	3.92	3.56
Gravel	51.3	38.55	3.63	2.94	3.81	3.32

The fracture toughness test was carried out on notched plain and reinforced concrete (RC) beams. The nominal dimensions of the test specimens for both plain and reinforced concrete were (100 × 150 × 900 mm) (B × d × L) with 80 mm loaded span. A steel plate of 0.5 mm thickness was used to create a notch at the mid span of the tensile surface of fracture toughness test specimens before casting.

Materials of the specified mix were weighted first, and then mixed in dry state for a time. The required amount of water was then added. The contents were left to agitate in the mixer until a homogenous cement paste covers all coarse aggregate particles. The mixed materials are then transported to the specimen wooden moulds in such a way to prevent any risk of segregation or setting before placing. The compaction for normal concrete specimens was carried out on a mechanical vibrating table with care to avoid segregation. The beams were removed from the moulds after 24 hrs from casting. After removing the beams from the moulds, they were placed immediately in clean water. The specimens were kept in water for 28 days for curing. All the specimens were cast and treated under the same environmental conditions. A universal hydraulic testing machine of capacity 1000 kN was used to test all specimens. All beams were tested under three point bending 3PB configuration. A digital gauge with accuracy of 0.001 mm was used for measuring the mid span vertical deflection in notched plain and reinforced concrete beam specimens. The maximum load in plain notched specimen and that corresponding to the start of crack propagation in RC notched specimens were recorded.

3. Results and Discussion

The fracture toughness (K_{IC}) based on LEFM for single edge crack specimen loaded in 3PB was calculated using ASTM E399 expression:

$$K_{IC} = [6\, M \sqrt{a} / Bw^2]\, Y(a/w) \quad (1)$$

$$Y(a/w) = 1.99 - 2.47(a/w) + 12.97(a/w)^2 - 23.17(a/w)^3 + 24.8(a/w)^4 \quad (2)$$

Where M is the moment calculated at the crack initiation load.

The normalized fracture toughness, $K_{IC}/\sigma_f.MAS^{0.5}$, against a/w for SCC and NVC made with different types of coarse aggregates are respectively shown in Figs. 1 and 2. It is clear that K_{IC} decreased with increasing a/w for either SCC or NVC. This is a typical behavior of concrete materials [3]. On the other hand, SCC recorded higher normalized K_{IC} values as compared to NVC. This may be attributed to the enhancement in the matrix properties and the decrease in micro-voids and the internal defects. The figures show also the role of coarse aggregates in controlling the fracture resistance of concrete. The SCC or NVC made with dolomite as coarse aggregates recorded the highest normalized K_{IC} relative to the other two types of coarse aggregates. Gravel concrete recorded the least values of normalized K_{IC}. The enhancement in the K_{IC} for dolomite concrete is attributed to the strong aggregate-matrix interface strength due to surface roughness of dolomite aggregate. In the case of gravel concrete, although the crack path avoids the coarse aggregates, the fracture energy not increased but decreased. This may be due to the weak strength of the interface as a result of the smooth aggregate surface. This makes the interface an appropriate site for crack initiation. This is in agreement with the results obtained by [17].

In the case of pure mode I, the cracks have been initiated and propagated in the same direction and plane regardless the depth of the pre-notch. Based on this fact, the strain energy release rate, G, was suggested to be calculated by subtracting the energies absorbed by the two beams which had different a/w and then divided by the difference between the un-cracked area in each beam, i.e. G was calculated in each interval (a/w = 0.0- 0.2, 0.2-0.4, and 0.4-0.6). This method considers only the energy absorbed by the beam to the point of instability, i.e. the maximum load (ascending curve). Therefore, the mean values from load– deflection curves for SCC beams and NVC beams for different types of coarse aggregate and different a/w were drawn and the area under these curves were calculated as the energy absorbed up to complete failure.

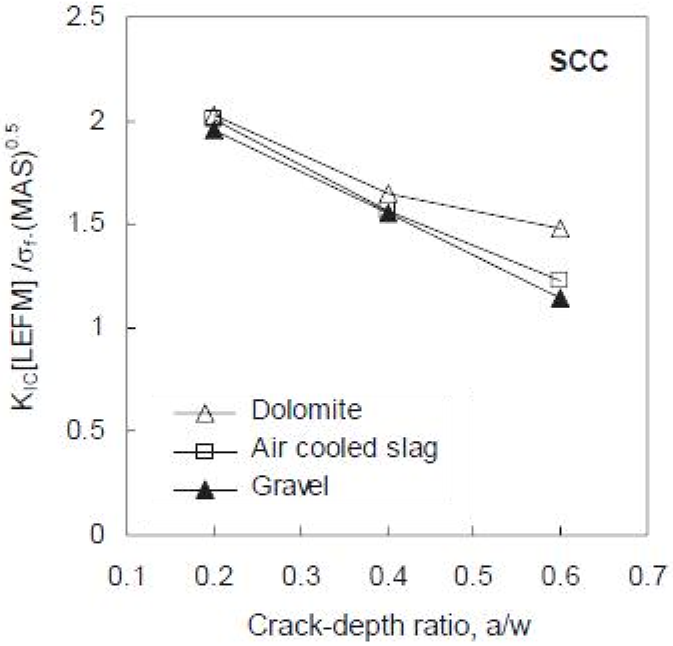

Fig. 1 Variation of K_{IC} based on LEFM concept with a/w for SCC

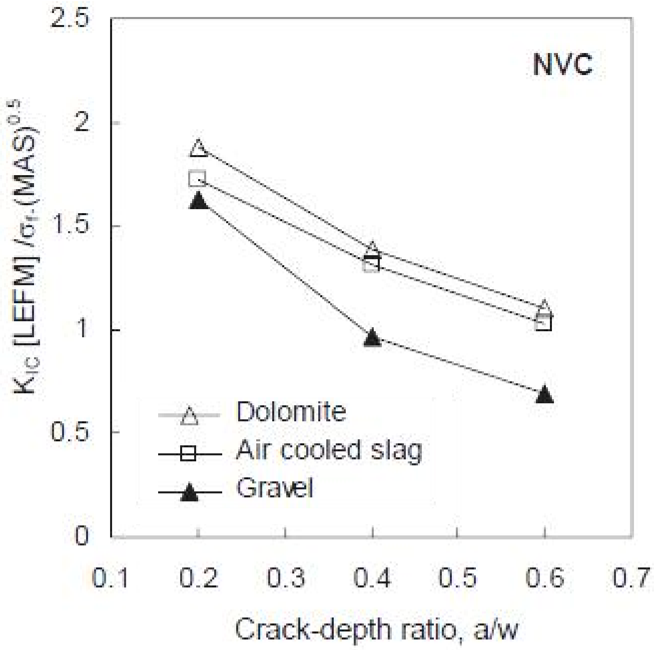

Fig. 2 Variation of K_{IC} based on LEFM concept with a/w for NVC

To determine the accuracy of this approach, Table 6 contains the values of strain energy release rate of SCC and NVC for different intervals of a/w. It is clear that, there is a wide discrepancy between the values of G in each material for the range "0.0-0.2". As known, the fracture toughness can be expressed by G, $K_{IC} = \sqrt{(GE)}$. According to the Egyptian code of practice, the modulus of elasticity $E = 4400\sqrt{(\sigma_{cu})}$ MPa, where σ_{cu} is the 28 days cube compressive strength. Therefore, the mean value of K_{IC} is calculated in each material based on the mean values of G excluding the values for the range "0.0-0.2", as shown in the Table 7.

TABLE 7: STRAIN ENERGY RELEASE RATE, G, AND K_{IC} FOR SCC AND NVC

Type of coarse aggregate	Type of concrete	a/w			Mean	
		0-0.2	0.2-0.4	0.4-0.6	G, N/mm	K_{IC},MPa.m$^{0.5}$
Dolomite	SCC	0.25	0.07	0.03	0.05	41.8
Air cooled slag	SCC	0.19	0.06	0.02	0.04	36.2
Gravel	SCC	0.17	0.07	0.02	0.04	35.8
Dolomite	NVC	0.2	0.07	0.03	0.05	36.5
Air cooled slag	NVC	0.178	0.05	0.02	0.03	31.2
Gravel	NVC	0.09	0.04	0.01	0.02	23.7

(a/w): is the crack – depth ratio
K_{IC} : is the Critical Stress Intensity Factor, Fracture Toughness
G: is shear modulus

Based on Hillerborg concept (fictitious crack model adopted by RILEM), i.e. total energy, [18], the K_{IC} for different notched plain concrete beams was calculated from the relation $K_{IC} = \sqrt{G_f E}$. The normalized fracture toughness based on Hillerborg concept against a/w for SCC and NVC are shown respectively in Figs. 3 and 4. These figs. showed similar behavior as that for K_{IC} (LEFM) which shown in Figs. 1 and 2.

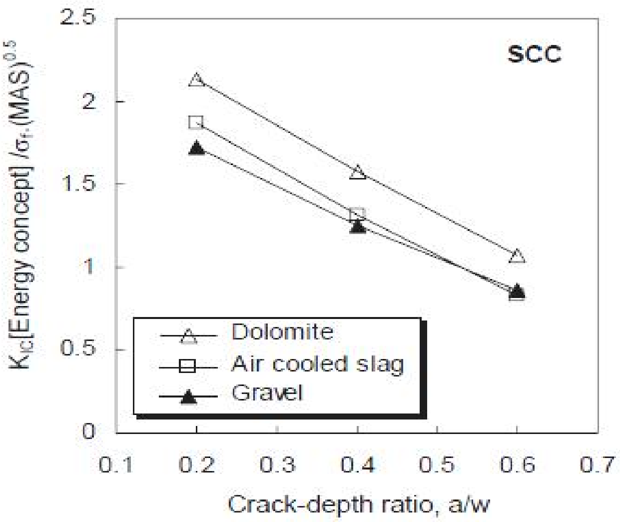

Fig. 3 Variation of K_{IC} based on energy concept with a/w ratio for SCC

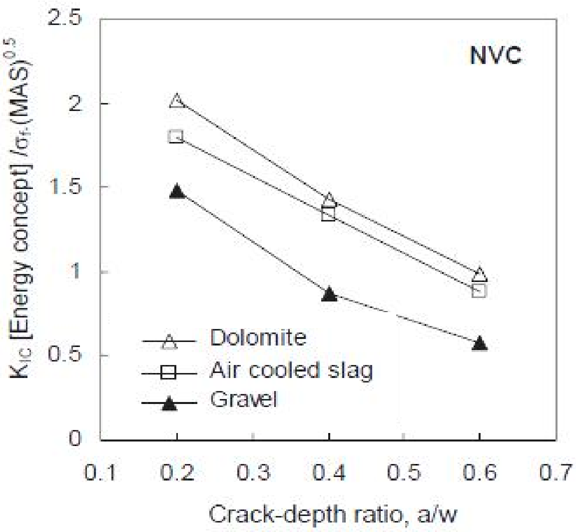

Fig. 4 Variation of K_{IC} based on energy concept with a/w for NVC

The average values for K_{IC} based on Hillerborg concept are listed in Table 8. The K_{IC} which calculated based on the LEFM and the suggested subtracted energies for the different mixes of SCC and NVC are also presented in this Table. The K_{IC} calculated from Hillerborg model showed similar trend to that calculated from LEFM. However, the K_{IC} values which calculated based on subtracted energies method are much higher than those calculated from the other two methods. The largest size of undamaged internal defect (d_{max}) in the smooth specimen, i.e. after that size the strength of smooth specimen decreases with increasing the size of this defect, can be predicted using the equation of SIF in this form:

$$d_{max} = (K_{IC}/ 1.12\ \sigma f)^2 / \pi \quad (3)$$

Where σ_f is the flexural strength of concrete in MPa. The values of d_{max} based on LEFM and Hillerborg concept were compared to the MAS (14 mm) and the results are listed in Table 9. It is clear that, the predicted values of undamaged defect based on LEFM and those based on Hillerborg concept are comparable to the maximum aggregate size. Therefore, the values of K_{IC} which calculated based on LEFM and those based on Hillerborg concept are reasonable.

TABLE 8: AVERAGE FRACTURE TOUGHNESS BASED ON LEFM AND ENERGY METHODS

Type of Coarse aggregate	K_{IC}(subtracted energies) MPa. mm$^{0.5}$		K_{IC} (LEFM) MPa. mm$^{0.5}$		K_{IC}(Hillerborg) MPa. mm$^{0.5}$	
	SCC	NVC	SCC	NVC	SCC	NVC
Dolomite	41.8	36.5	25.93	20.3	23.4	20.3
Air cooled slag	36.2	31.2	23.5	18.03	19.3	17.52
Gravel	35.8	23.7	22.07	13.58	17.93	11.95

K_{IC} : is the Critical Stress Intensity Factor, Fracture Toughness

TABLE 9: MAXIMUM SIZE OF NON- DAMAGED DEFECT BASED ON LEFM AND HILLERBORG CONCEPT

Mix Type		LEFM				Hillerborg concept			
		d_{max}, mm		d_{max}/MAS		d_{max}, mm		d_{max}/MAS	
SCC	NVC	SCC	NVC	SCC	NVC	SCC	NVC	SCC	NVC
SD02	ND02	3.33	2.97	0.238	0.212	3.42	3.08	0.244	0.220
SD04	ND04	3.00	2.55	0.214	0.182	2.89	2.53	0.204	0.180
SD06	ND06	2.84	2.28	0.203	0.162	2.42	2.16	0.173	0.154
SS02	NS02	3.23	2.71	0.231	0.194	3.12	2.78	0.223	0.198
SS04	NS04	2.85	2.37	0.204	0.169	2.55	2.33	0.182	0.166
SS06	NS06	2.52	2.10	0.180	0.150	2.07	1.95	0.148	0.139
SG02	NG02	3.09	2.46	0.221	0.176	2.91	2.35	0.208	0.168
SG04	NG04	2.76	1.89	0.197	0.135	2.42	1.76	0.173	0.126
SG06	NG06	2.36	1.60	0.169	0.114	2.05	1.47	0.147	0.105

d_{max}: is the largest size of undamaged internal defect

The apparent fracture toughness of RC beams (K_Q) due to applied moment was calculated at the onset of crack initiation in those beams and the results are demonstrated in Fig. 5 for SCC beams and Fig. 6 for NVC beams. The figs. clearly indicated an increase in K_Q with increasing a/w and area of steel reinforcement, A_S. The crack was started to propagate in these beams when the SIF at the crack tip reached K_{IC} of the plain concrete beams. The presence of reinforcing steel bars located within the tensile stress layer of the RC beams offers a closing effect on the tensile (opening) crack. The corresponding closing stress intensity factor due to such steel reinforcement (K_S) is equal to the difference between the apparent fracture toughness due to the applied moment in RC beams and the fracture toughness of plain concrete beams, i.e.

$$K_S = K_Q - K_{IC\ plain\ concrete}. \qquad (4)$$

After that, the crack does not propagate until the applied load increases, i.e. the K_Q increases with increasing the crack length. This behavior is the main advantages of composite materials like RC. The effect of area of steel reinforcement and a/w on the closing SIF is shown in Figs 7 and 8 for self compacting and normal vibrated concretes respectively.

It is clear that K_s increases with increasing A_s. This can be explained as follows: the presence of steel bars creates a closing effect on the crack. This closing effect due to the local compressive stress field around steel bars inhibits the ability for crack propagation. This local compressive field increases with increasing A_s. Also K_S increases with increasing a/w because the tensile forces are carried out by steel reinforcement due to cracking of concrete below the neutral axis. Recently references [19-21] stated that, "As known, fracture toughness of fiber reinforced composites such as reinforced concrete increases as the crack length increases due to crack bridging.

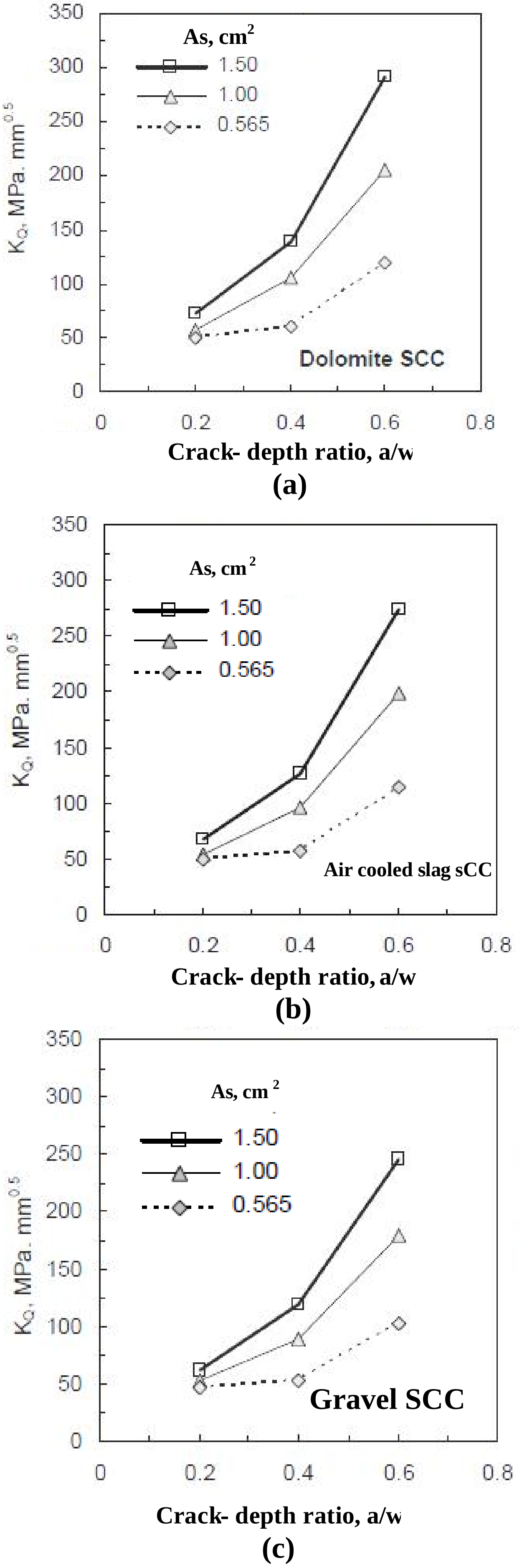

Fig. 5 Variation of K_{IC} with a/w for SCC reinforced concrete beams

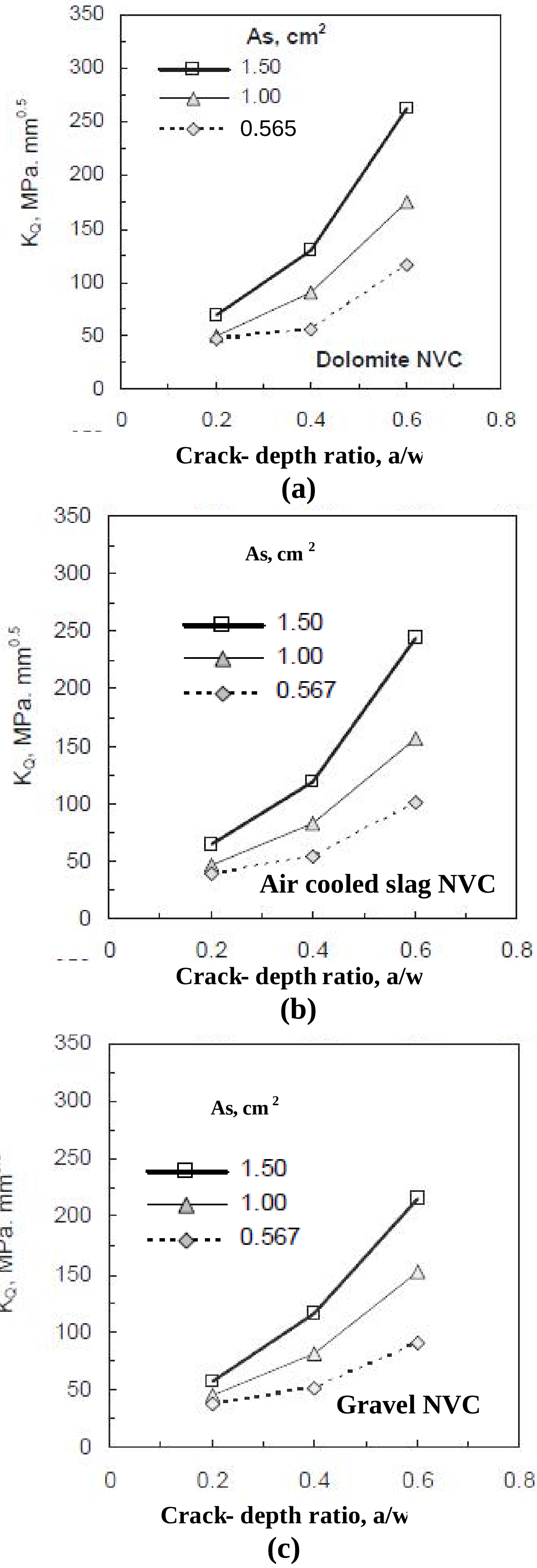

Fig. 6 Variation of K_{IC} with a/w for NVC reinforced concrete beams

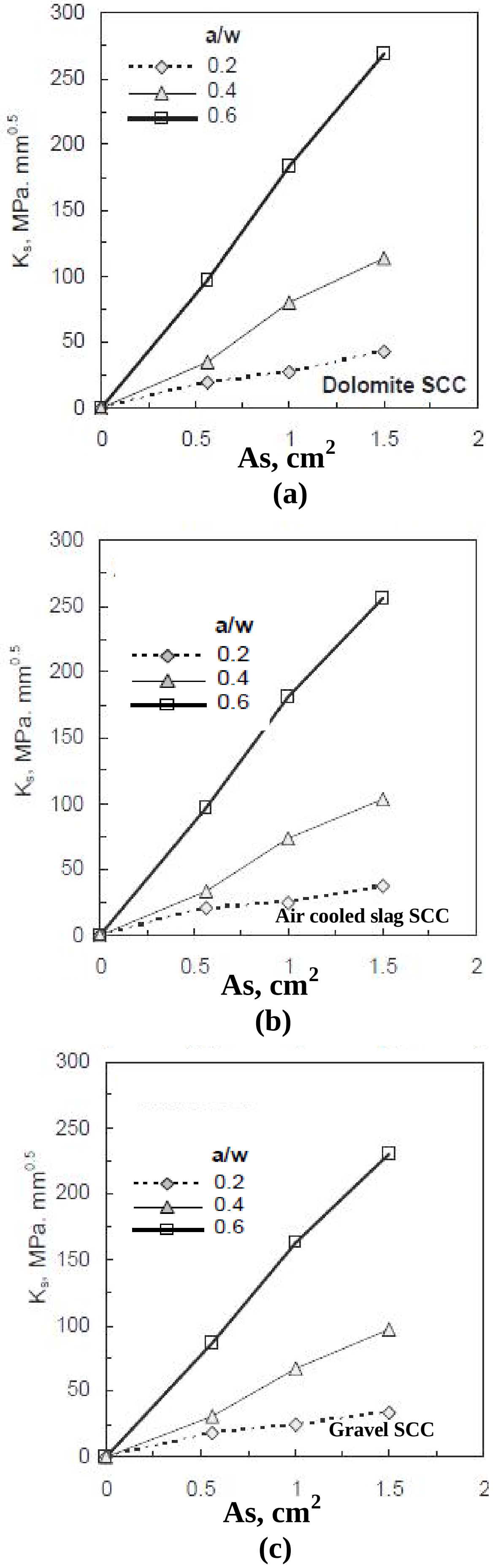

Fig. 7 Effect of area of steel on the closing SIF of SCC

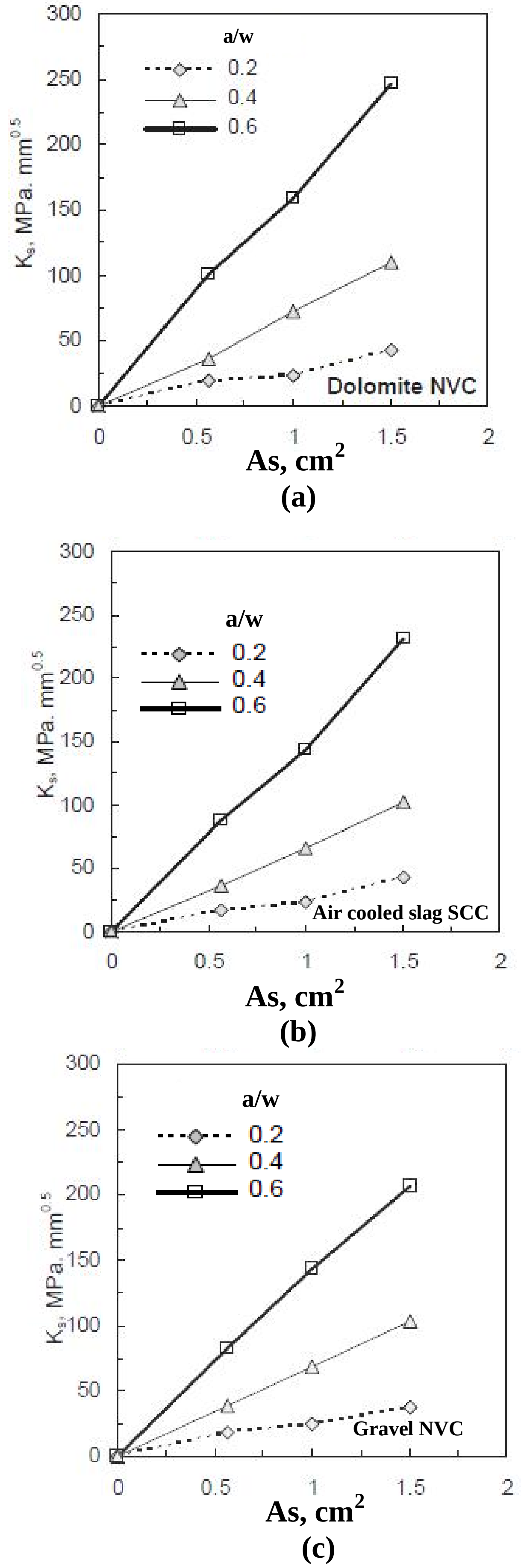

Fig. 8 Effect of area of steel on the closing SIF of NVC

The closing SIF due to steel bars was calculated based on the compact tension model analogy by assuming a compressive load equals to T_S trying to close the crack. The following formula of the K_s is used according to the standard specification ASTM E399 as follows:

$$K_s = -T_s \;/ (B^* \sqrt{w}) * g\,(a_1/d) \tag{5}$$

$$g\,(a_1/w) = [\,2 + a_1/d\,]\,[\,0.886 + 4.64\,(a_1/d) - 13.32\,(a_1/d)^2 + 14.72\,(a_1/d)^3 - 5.6\,(a_1/d)^4]\,/\,[1-(a_1/d)]^{3/2} \tag{6}$$

Where T_s is the force in steel bars calculated from the following equation:

$$T_S = [nB\sigma_t\,(d-z)(d-z-a_1)A_s]/[(Bz^2-2nA_s(d-z)] \tag{7}$$

Where n is the modular ratio, z is the height of the compression block, a_1 is the distance between the steel bar and the crack tip and d = (w – concrete cover). The experimental values of closing SIF due to steel reinforcement for SCC beams and NVC beams were compared to those predicted from the compact tension model analogy and good agreement was found between them as shown in Figs. 9 and 10. The results in the two figures show a good agreement between the experimental results and the predicted values.

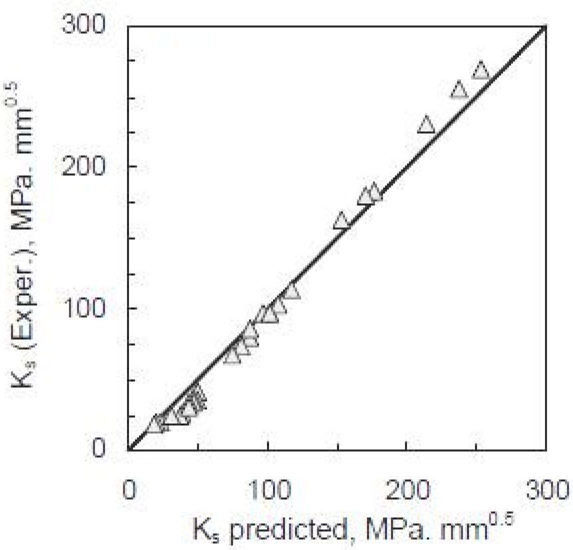

Fig. 9 Comparison between the experimental results and the predicted values of Ks for SCC beams.

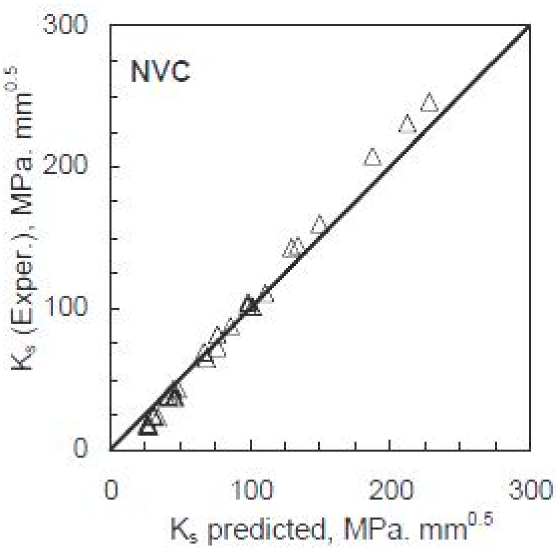

Fig. 10 Comparison between the experimental results and the predicted values of Ks for NVC beams

4. Conclusions

From the experimental results that obtained in this study, the following conclusions can be drawn:

- The fracture toughness for either self compaction or normal vibrated concrete decreases with increasing crack–depth ratio.
- Self compacting concrete recorded higher resistance to crack propagation compared to normal vibrated concrete irrespective of the type of coarse aggregate.
- The use of dolomite as a coarse aggregate gave the highest value for the fracture toughness in both self compacting and normal vibrated concrete while gravel aggregate recorded the lowest values.

- The predicted values for the maximum size of undamaged defect based on LEFM or Hillerborg model are comparable to the maximum aggregate size. Therefore, the values of K_{IC} calculated based on LEFM or energy concept are reasonable.
- The presence of steel reinforcement in the cracked section created a closing effect to the crack propagation. This means that, the resistance of RC beams to crack propagation for long cracks is higher than those for short cracks.
- The closing SIF due to steel reinforcement increases with increasing the area of steel and crack- depth ratio. The estimated closing SIF from compact tension model analogy was in a good agreement with the experimental values.

References

[1] Kaplan, M. F. Crack Propagation and the Fracture of Concrete. ACI Journal, 1961.58(5), pp.591– 610.

[2] Wittmann F. H, Fracture Mechanics of Concrete. Amsterdam: Elsevier publication. 1983.

[3] Nallathambi, P., Karihaloo, B. L. and Heaton, B. S. Coarse Aggregate Texture upon Fracture Toughness of Concrete. Magazine of concrete research. 1984. 36(129), pp. 227-236.

[4] Petersson, P.E. Fracture Energy of Concrete, Practial Performance and Experimental Results. Cement and concrete research, 1980. 10(1), pp. 91-101.

[5] Zhimin Wu, Shutong Yang , Xiaozhi Hu, Jianjun Zheng. An analytical Model to Predict the Effective Fracture Toughness of Concrete for Three-Point Bending Notched Beams. Engineering Fracture Mechanics, 2006. 73, pp. 2166–2191.

[6] Bazˇant, Z. P, Oh B. H. Crack Band Theory for Fracture of Concrete. Mater Struct, 1983.16(93), pp. 155–77.

[7] Jenq, Y. S., Shah S. P. Two Parameter Fracture Model for Concrete. J Engng Mech, 1985. 111(10), pp. 1227–1241.

[8] Bazˇant, Z. P., Kazemi, M. T. Determination of Fracture Energy, Process Zone Length and Brittleness Number from Size Effect. With Application to Rock and Concrete. Int J Fracture, 1990. 44, pp. 111–131.

[9] Karihaloo, B. L., Nallathambi P. An Improved Effective Crack Model for the Determination of Fracture Toughness of Concrete. Cement Concrete Research, 1989.19, pp. 603–610.

[10] Erhan Güneyisi. Mehmet Geosoǧlu Properties of self-compacting Portland pozzolana and limestone blended cement concretes containing different replacement levels of slag. Materials and Structures, 2011. 44:1399-1410.

[11] Fava, C., Bergol L., Fornasia G. Giangrasso F., Rocco C. Fracture Behavior of Self Compacting Concrete. Proceedings of third RILEM international symposium on self compacting concrete Reykjavik Iceland. Bagneux, France: RILEM Publications, 2003. PRO 33, pp. 628–636.

[12] Zhao, Y., Ma J., Wu Z., Xu S., Gao H. Study of Fracture Properties of Self-Compacting Concrete Using Wedge Splitting Test. 1st international symposium on design, performance and use of SCC, China 2005, Hunan China, RILEM Publications, 2005. PRO 42, pp. 421–428.

[13] Rozie`re, E, Granger, S., Turcry, Ph., Loukili, A,. Influence of Paste Volume on Shrinkage Cracking and Fracture Properties of Self-Compacting Concrete. Cement & Concrete Composites 2007. 29, pp. 626–636.

[14] Trisha Sain. J. M Chandra Kishen Fracture stability and residual strength assessment of reinforced concrete beams. Materials and Structures, 2008. 41:1451-1463.

[15] Carpenteri, A. Stability of Fracturing Process in RC Beams. Journal of Structural Division, ASCE, 1984. 110(3).

[16] Azad, A. K., Mirza, M. S. and Chan, P. Fracture Energy of Weakly Reinforced Concrete Beams. Fatigue & Fracture of Engineering Materials and Structures, 1989. 12 (1), pp. 9-18.

[17] Rosselló, C., Elices, M. and Guinea, G. V. Fracture of Model Concrete: 2. Fracture Energy and Characteristic Length. Cement and Concrete Research, 2006. 36, pp. 1345–1353.
[18] Hillerborg, A. Analysis of fracture by means of the fictitious crack model, particularly for fiber reinforced concrete. The International Journal of Cement Composites, 2, 1980, p 177-190.
[19] Nabil, A. B. Yehia, Noran M. Wahab. Fracture Mechanics of Flanged Reinforced Concrete Sections. Engineering Structures, 2007. 29, pp. 2334–2343.
[20] Seleem, M. H., Sallah H. E. M., Heiza Kh. M., Shahean. Y. B, and Attwa A.T. Fracture Toughness of Self Compacting concrete. Mesomechanics, HBRC, Giza, Egypt. 2008. January 28 – February 1.
[21] Attwa, A. T. Fracture Toughness of Self Compacting concrete. Ph.D Thesis Civil Engineering Department. Menofia University. Egypt. 2008.