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Research Article

Arch effect in silos on discrete supports - Is it a myth or reality?

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ABSTRACT

Steel silos are interesting, complicated facilities. In order to assure its complete emptying by gravity they are often placed on supporting frame structure above the ground. Values of stresses in joints between thin walled shell and supporting frame elements are very high. It can cause the local buckling in the shell. The simplest way to design steel silos is to divide hypothetically the cylindrical shell into two parts - ring beam, supported in some points and shell above, uniformly supported. This conception is accepted by European Standard EN 1993-4-1. The particular moment is that the ring beam and cylindrical body above it are separated. Actually the two elements are jointed and work together in the same time. Considering the last results of Zeybek, Topkaya and Rotter from 2019, and as well as his own research, the author asks the question if it is true that the transferring of discrete base reactions to the cylindrical body is done by bending work of the ring beam, which is the conception in EN 1993-4-1? Or the vertical reaction forces are actually redistributed on the height based on the work of the cylindrical shell under compression as an arch. Using the contemporary capabilities of the programs for spatial analysis of building structures the author will try to find the answer of this question.

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1. Introduction

Usually the steel silos are elevated above the ground facilities, placed on supporting structure. The purpose is to assure the easy and complete emptying of the stored product by gravity. The supporting structure is different for every facility because it depends on the real conditions of exploitation. The most used are the two types - built by horizontal girders and columns or only by columns. Both type of frame structures cause concentrated meridional forces in the cylindrical body of the silo. As a result the thin wall shell could buckles.

The simplest way to design steel silos is to divide hypothetically the cylindrical shell into two parts - discretely supported ring beam and uniformly supported shell above it, see Fig. 1. This conception is accepted by the European standard EN 1993-4-1. Obviously, to assure uniform supporting on the whole circumference of the circular shell, bending stiffness of the ring beam should be high. Unfortunately the EN 1993-4-1 does not say what should be the recommended stiffness of the ring beam.

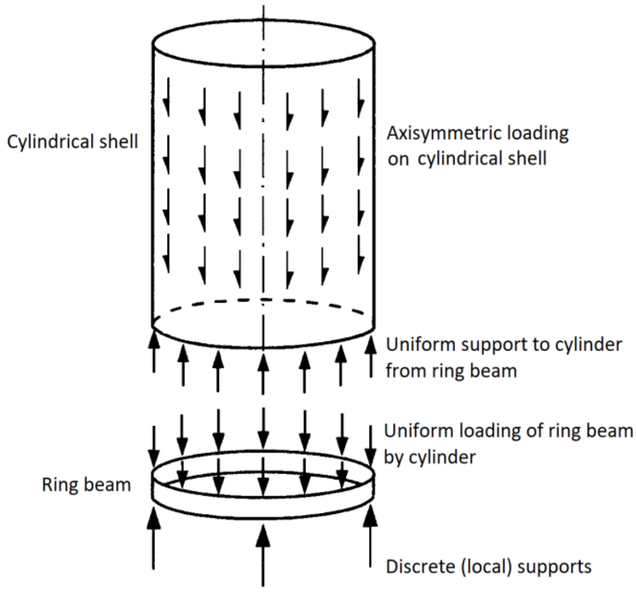

Fig. 1. Traditional design model for silos on discrete supports.

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Rotter (1985) suggested that a value of ratio ψ = 0.25 might be suitable for adoption in design, where:

$$\psi = \frac{K_{\text{shell}}}{K_{\text{ring}}} \tag{1}$$

in which:
K_{shell} is stiffness of cylindrical shell;
K_{ring} is stiffness of ring beam.

Based on English translation of study of Vlasov (1961) about of curved beams, stiffness of ring beam K_{ring} is expressed as:

$$K_{\text{ring}} = \frac{(n^2-1)^2 EI_r}{R^4}\frac{1}{f_r} \tag{2}$$

where:
n is number of uniformly spaced supports;
E is modulus of elasticity;
I_r is moment of inertia about a radial axis;
R is radius of ring beam centroid.

$$f_r = 1 + \frac{EI_r}{n^2 K_T} \tag{3}$$

in which:

$$K_T = GJ + n^2\frac{EC_w}{R^2} \tag{4}$$

where:
G is shear modulus;
J is torsional constant;
C_w is warping constant for an open sections.

Semi-membrane theory of shells, proposed by Vlasov (1964), gives an expression of stiffness of cylindrical shell, as follow:

$$K_{\text{shell}} = n\sqrt{(n^2-1)}\frac{E}{\sqrt[4]{3}}\left(\frac{t}{R}\right)^{3/2}\frac{1}{f_s} \tag{5}$$

where t is thickness of the cylindrical shell.

$$f_s = \frac{(e^{\eta})^2 - 2e^{\eta}\sin(\eta) - 1}{(e^{\eta})^2 - 2e^{\eta}\cos(\eta) + 1} \tag{6}$$

in which:

$$\eta = \frac{2\pi H}{\mu} \tag{7}$$

where:
H is height of cylindrical shell;
μ is expressed by Calladine (1983) long wave bending half-wavelength:

$$\mu = \frac{2\pi\sqrt[4]{3}}{n\sqrt{(n^2-1)}}\sqrt{\frac{R}{t}}R \tag{8}$$

Based on Eqs. (2) and (5), stiffness ratio ψ will look like as:

$$\psi = \frac{K_{\text{shell}}}{K_{\text{ring}}} = \frac{0.76\,(Rt)^2}{I_r}\sqrt{\frac{R}{t}}\sqrt{\frac{n^2}{(n^2-1)^3}}\frac{f_r}{f_s} \tag{9}$$

For simplification, the Eq. (6) could be represented by two simple relations:

$$f_s = \begin{cases} \frac{\eta}{3}, \text{when } H \le H_{cr} \\ 1.0, \text{when } H > H_{cr} \end{cases} \tag{10}$$

where H_{cr} is critical height of cylindrical shell. It could be determined by formula:

$$H_{cr} = \frac{3\sqrt[4]{3}}{n\sqrt{(n^2-1)}}\sqrt{\frac{R}{t}}R \tag{11}$$

H_{cr} represents the height of shell which is effective of redistributing of discrete forces from supports and equalizing of axial normal stresses. When height of shell $H \le H_{cr}$, entire shell resists axial loads from supports. When $H > H_{cr}$, only that part between bottom of shell and critical height H_{cr} is effective in redistributing of vertical reactions from discrete columns.

In their researches Topkaya and Rotter (2011a) (2011b) conducted extensive finite element analyses for verification of Rotter's criterion about stiffness of ring beam. With 1,280 separate finite-element analyses (FEA), covering two different types of ring sections, various heights and radii of cylindrical shells, the authors checked validity of suggested by Rotter (1985) ratio ψ = 0.25. On basis of done FEA they concluded, when a stiffness ratio $\psi \le 0.1$, axial stresses will not deviate more than 25% from the uniform support assumption.

Research of Zeybek, Topkaya and Rotter (2019) shows that the equations, based on the theory of Vlasov (1961) for a curved beam provide results with acceptable accuracy when the girder is separated from the cylindrical body. When the ring beam and cylindrical shell are jointed, the received through finite elements analysis values are considerably different from the analytical results in closed form. The differences going high with increase of the thickness of the cylindrical shell.

It should be noted that all above mentioned researches are conducted on the smooth steel shells without vertical stiffeners on them. On other side, common practice in design of steel structures is to place stiffening elements on the point, where are applied concentrated loads. In our case, the vertical stiffeners should be placed above the discrete supports, see Fig. 2.

Fig. 2. Stiffening elements above discrete supports of the shell.

In his research Zdravkov (2017a, 2018) shows that vertical stiffening elements increase the height of the critical zone, where the vertical reactions of discrete supports are redistributed. Considering the last results of Zeybek, Topkaya and Rotter (2019), as well as his own research, the author put the question if it is true that redistributing of the separate reactions of supports is done by the bending of ring beam which is the conception in EN 1993-4-1, see Fig. 1. Or vertical forces in discrete supports are transferred on the height by work of the cylindrical shell above the supports as a compressed arch, see Fig.3.

In this article the author will try to find answer of these questions.

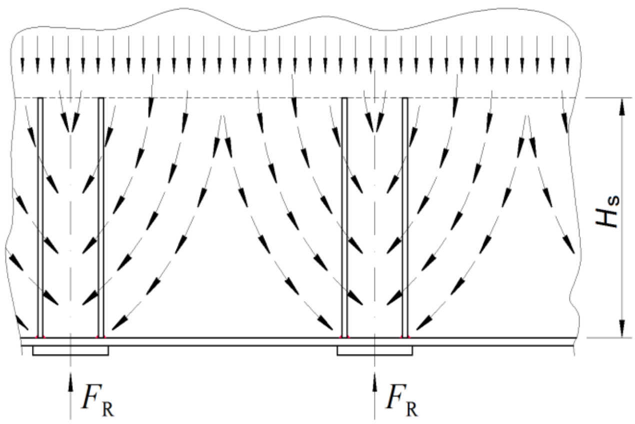


Fig. 3. Compression forces in the cylindrical shells above the supports.

2. Finite Element Analysis

For the purpose of research, three steel cylindrical shells are modelled, using software ANSYS. Their parameters are as follow:

a) Dimensions:
- shell 1 – diameter D = 3 m, height H = 6 m;
- shell 2 – diameter D = 4 m, height H = 8 m;
- shell 3 – diameter D = 5 m, height H = 10m.

b) All shells are with constant thickness t = 5 mm;

c) All shells are supported by six immovable supports with dimensions in plane 125×125 mm, see Fig. 4.

d) In Fig. 4 every support are placed two vertical steel plates with section 8x100 mm and with different height. On their upper end exists an intermediate ring with a section L100x8 mm, see Figs. 4 and 5.

e) Cylindrical body in three models of the shells is continuous, see Fig. 5(a). In the other three models are made openings between the vertical stiffeners, see Fig. 5(b). On this way the “skirt” of the silo cannot works as a ring beam. The support’s forces will be transferred up on the cylindrical body only through work of the cylindrical shell above the supports on compression, as an arch, see Fig. 3.

f) The heights of the openings h_o are different, depending on the height of the stiffeners. They are calculated according to the simple formula:

$$h_o = H_s - 100 \text{ mm} \quad (12)$$

where H_s is the height of vertical stiffeners, see Fig. 6.

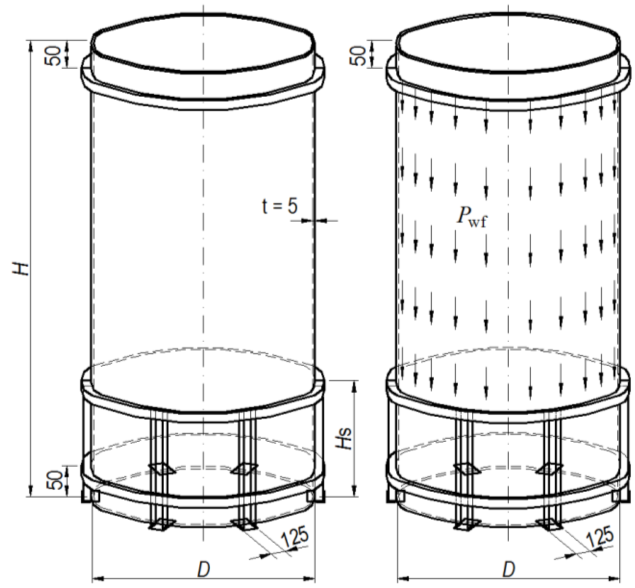
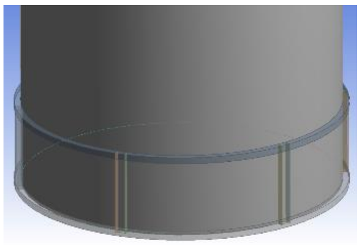
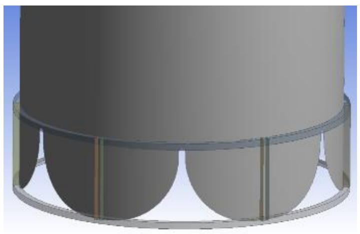


Fig. 4. Numerical models – dimensions and loading.



a) shell without opening



b) shell with openings

Fig. 5. Vertical stiffeners on the cylindrical shell.

g) In order to strengthen the shells in radial direction, on 50mm above the lower edge and on 50mm below the upper edge are placed rings with section L100x8 mm, welded as is shown on Fig. 7.

h) The stored in the facilities product varies. For each shell it is as follow:
- shell 1 – cement;
- shell 2 – lime;
- shell 3 – sand.

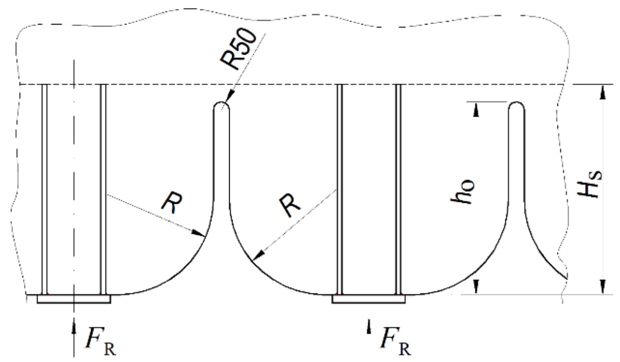


Fig. 6. Opening in the base of the cylindrical shell.

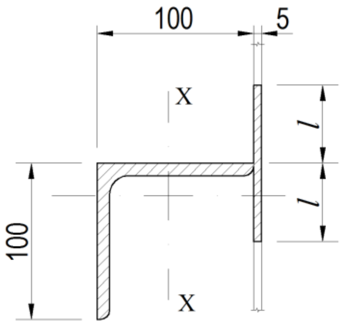


Fig. 7. Shape of the intermediate stiffening ring.

Every product causes vertical load P_{wf} due to the friction between the stored material and the shell. Its values are determined for every particular product according to standard EN 1991-4. All loads are uniformly distributed and applied as a surface pressure on the shell. They are applied to internal surface of the shells, see Fig. 4.

i) Shells 1, 2 and 3 are analysed for four different heights H_s of vertical stiffeners above supports.

The heights reached by the stiffening plates are determined as follows:

- using an average value of distribution of discrete forces F_R from supports $\alpha = 45°$, see Fig. 8. The height H_{45} is determined with the expression:

$$H_{45} = \frac{\pi R}{n} \tag{13}$$

where R is radius of cylindrical shell.

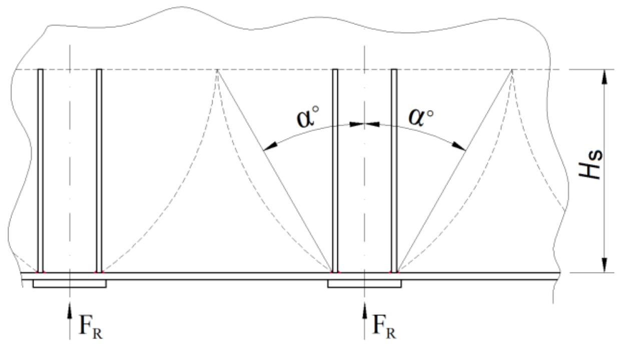


Fig. 8. Average angle α of distribution of the compressive forces on height.

- at ideal position of the intermediate stiffening ring on the shell.

Topkaya and Rotter (2014) determined the ideal position of intermediate stiffening rings on the shell. They expect a ring, placed at this ideal position, can effectively remove all circumferential nonuniformity in the axial membrane stress above it. The simple expression of ideal location H_I is:

$$H_I = \sqrt{12(1+\nu)}\ \frac{R}{n} \tag{14}$$

where ν is a coefficient of *Poisson*.

- using an average value of distribution of discrete forces F_R from supports $\alpha = 30°$. The height H_{30} should be calculated by the formula:

$$H_{30} = \frac{\pi R}{n} \tan(90° - \alpha°) \tag{15}$$

- the length of the stiffeners H_L is equal to distance between the supports. It is calculated according to the formula:

$$H_L = \frac{2\pi R}{n} \tag{16}$$

j) Material of elements is steel S235, with a properties according to European standard EN 10025-2:2004.

Necessary stiffness of intermediate stiffening rings is determined by Zeybek et al. (2015). Stiffness ratio χ could be expressed as:

$$\chi = \frac{K_{\text{shell}}}{K_{\text{stiffener}}} = \frac{Rt\left(AR^2 + I_x n^2(n^2-1)\right)}{12\sqrt{3}(1+\nu)^{3/2} A I_x n (n^2-1)^2} \tag{17}$$

where:

K_{shell} is circumferential stiffness of the shell;
$K_{stiffener}$ is circumferential stiffness of circular ring;
A is cross sectional area of the stiffening ring;
I_x is moment of inertia of the stiffening ring about vertical axis "x-x".

The results in research of Zeybek et al. (2015) indicate that ratios below about $\chi < 0.2$ provide a satisfactorily uniform axial membrane stress distribution above the intermediate ring stiffener, so this limit is recommended for practical design. In his later research Zeybek et al. (2017) confirmed, that correlation smaller than $\chi < 0.2$ are sufficient even when the rings are placed under their ideal position.

The steel angle section L100x8 and a part of the cylindrical shell form an intermediate stiffening ring with a shape as is shown on Fig. 7.

Effective width l of the steel sheets over and below the joint is calculated according to the standard API 650, by the expression:

$$l \leq 13.4\sqrt{Dt} \tag{18}$$

where:

D is a diameter of the cylindrical shell, m;
t is thickness of the cylindrical shell, mm.

Effective width l for the shells with the smallest diameter, D = 3 m, is l = 51.9 mm. The author accepts to have effective width l = 50 mm for all shells. It is on way of safety.

The geometric characteristics of the obtained stiffening ring are:
a) Area - A = 20.5 cm^2;
b) Moment of inertia about vertical axis „x-x"- I_x=358.4 cm^4.

For different shells, the ratio of the stiffness's χ, calculated according to the Eq. (17), has the values as follow:
- shell 1 – χ = 0.042;
- shell 2 – χ = 0.0764;
- shell 3 – χ = 0.130.

The maximum value of the ratio χ = 0.130 < 0.2, so it could be expected that the stiffness of the intermediate ring will be sufficient to equalize the meridional stresses in the shell above it.

The shells are modeled by 2D quad elements shell181 with a maximum length of side 50 mm. The method of their creation is "All quad". Element's midside nodes are controlled by the program.

Thin shell structures are sensitive for effect of changes of geometry during loading. On that reason geometrically nonlinear analyses (GNA) are used, according to the recommendations of EN 1993-1-6.

ANSYS's option "symmetry" is activated to reduce a calculation time. In analysis is used a quarter of silo only, see Fig. 9.

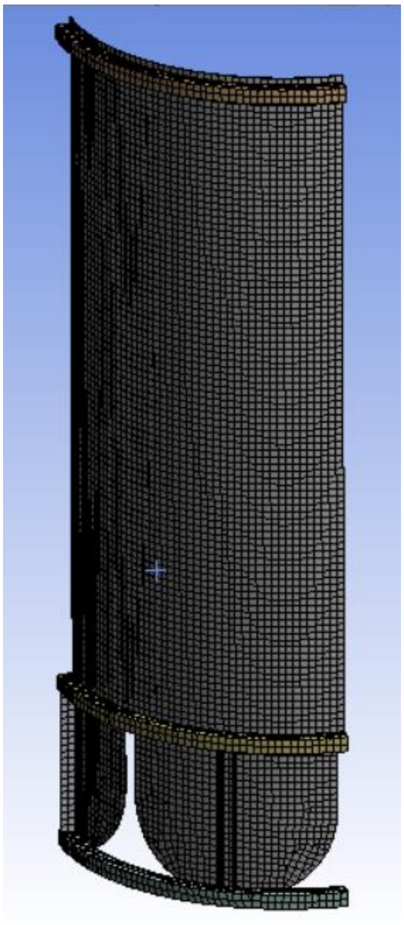


Fig. 9. Quarter of silo, used in numerical analysis.

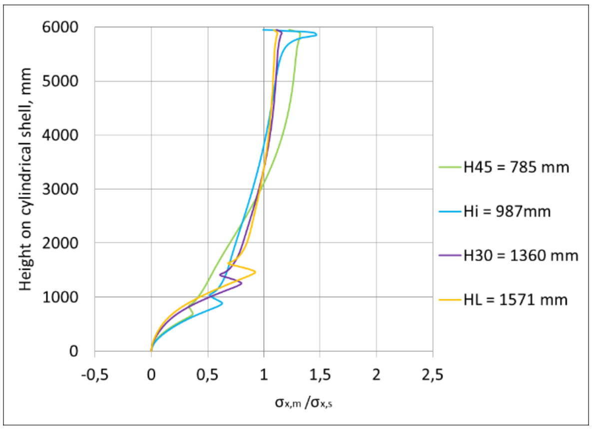
Axial normal stresses are accounted by the height of shell, in the middle between two supports and above the supports. After that are determined the values of ratio $\sigma_{x,m}/\sigma_{x,s}$, where:
$\sigma_{x,m}$ is meridional normal stress by height of the cylinder, in the middle between two supports;
$\sigma_{x,s}$ is meridional normal stress by height, above the supports.

The idea is that where the ratio $\sigma_{x,m}/\sigma_{x,s}$ =1.0 , is the upper border of the critical zone in the shell, in which are redistributed vertical reactions of supports. Above that border circumferential nonuniformity in the axial membrane stresses does not exists and the shell is continuously supported.

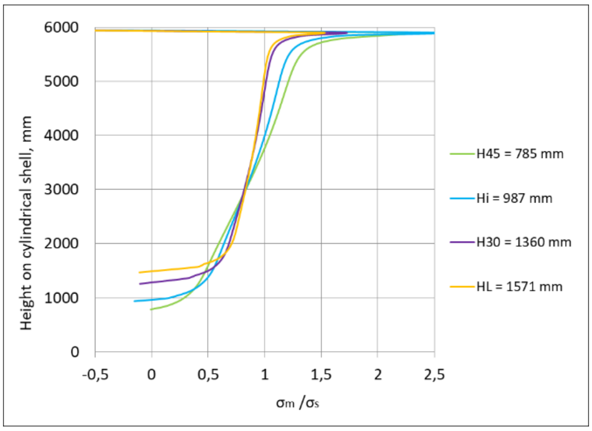
On the second stage, in the used program ANSYS is activated the option "Buckling Analysis". Through this option it is possible to calculate the reserve of bearing capacity k of the cylindrical shell before that it losses stability, completely or partially. The reserve k gives a quantity assessment of the influence of the made openings on the bearing capacity of the shell.

3. Results and Discussion

The graphics below, see Figs. 10-12, show the changes of ratio $\sigma_{x,m}/\sigma_{x,s}$. by height, calculated using numerical methods for analysis.

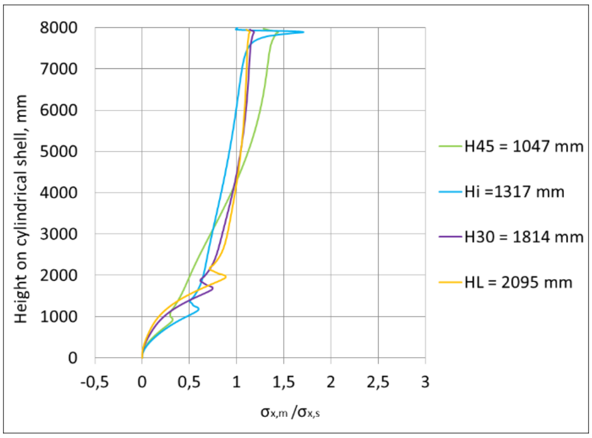


a) without openings in the ring beam

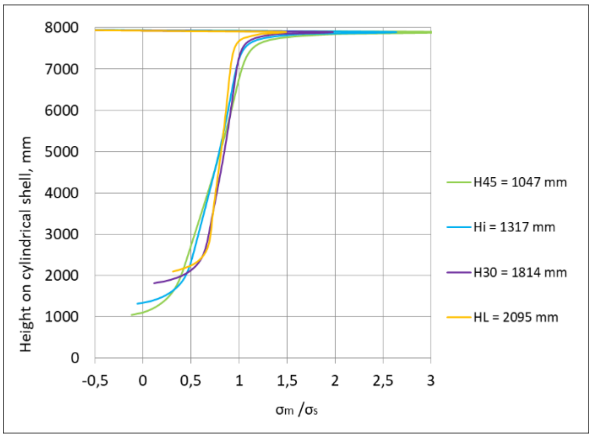


b) with openings in the ring beam

Fig. 10. Change of ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of the cylindrical shell 1 (D = 3 m, H = 6 m).

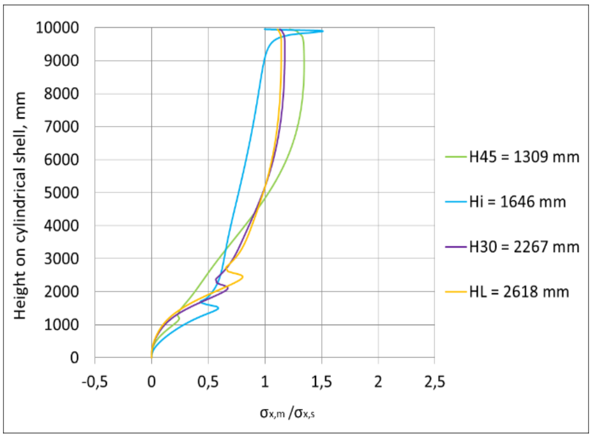


a) without openings in the ring beam

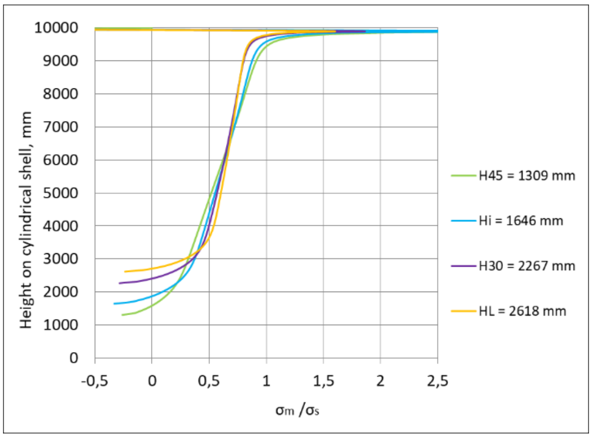


b) with openings in the ring beam

Fig. 11. Change of ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of the cylindrical shell 2 (D = 4 m, H = 8 m).



a) without openings in the ring beam



b) with openings in the ring beam

Fig. 12. Change of ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of the cylindrical shell 3 (D = 5 m, H = 10 m).

Obviously removing of material from the space between the supports, see Fig. 5 and 6, has its influence. At least the diagrams showing the changes of the ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of shell have different shape. More important is that the values of the stresses $\sigma_{x,m}$ and $\sigma_{x,s}$ in models with openings become equal on the bigger height. It means that the ways of transfer of compression stresses by the height are different for the continual bodies and for the bodies with openings.

Moreover, if on the models shown on Figs. 5(b) and 6 the base reactions obviously are redistributed on the height only through the work on compression of the cylindrical shell as an arch, this is not the same for the shell without openings. From where it can be concluded that the concept shown in EN 1993-4-1 has a grain rationality.

On the Figs. 10-12 we can see ratio $\sigma_{x,m}/\sigma_{x,s}$ > 1.0. It means that in the part of the shell meridional stresses in the middle, between supports, are bigger than the stresses above the supports. The similar phenomena was observed in the previous research of Zdravkov (2017a) and (2017b).

The results of carried out Buckling Analysis, reporting buckling above the vertical stiffeners and stiffening ring above them, see Fig. 13(a), for every one shell are as follow:

a) shell 1 – diameter D = 3 m, height H = 6 m

Height of stiffeners	Buckling reserve capacity k	
	shell without openings	shell with openings
H_{45} = 785 mm	31.416	30.675
H_{I} = 987 mm	36.381	34.703
H_{30} = 1360mm	42.644	41.416
H_{L} = 1 571 mm	46.33	44.558

b) shell 2 – diameter D = 4 m, height H = 8 m

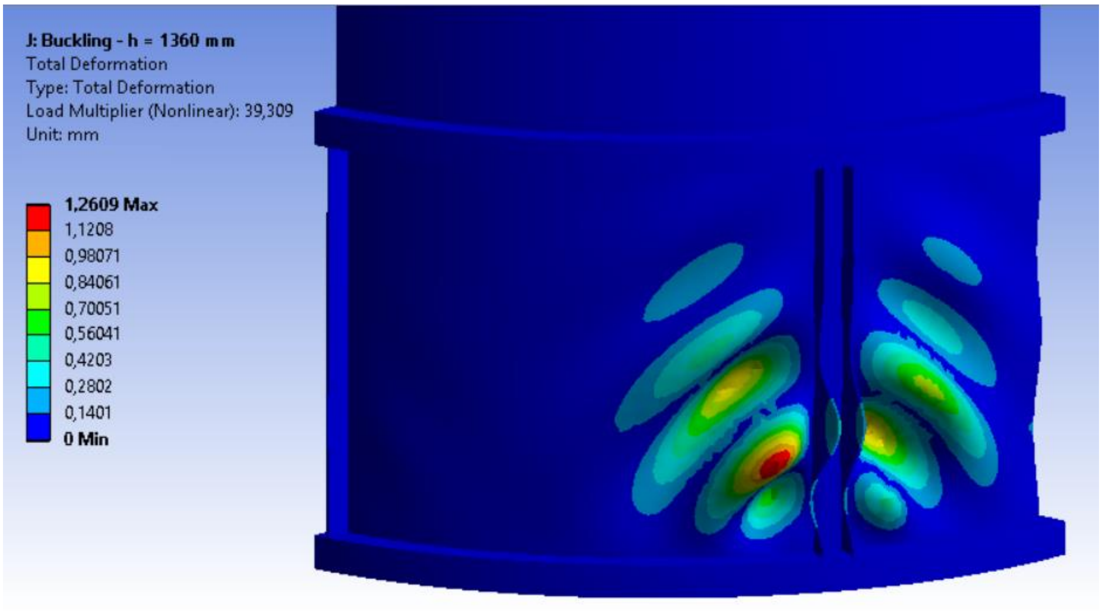
Height of stiffeners	Buckling reserve capacity k	
	shell without openings	shell with openings
H_{45} = 1047mm	24.943	23.468
H_{I} = 1317 mm	28.809	26.897
H_{30} = 1814mm	34.478	34.021
H_{L} = 2095mm	38.277	36.243

c) shell 3 – diameter D = 5 m, height H = 10m

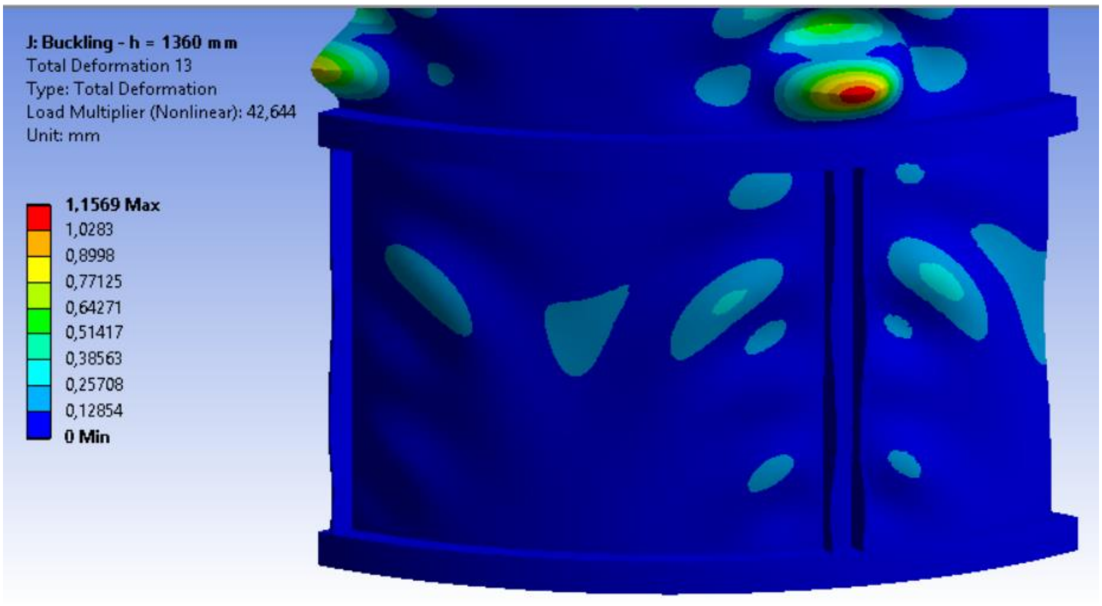
Height of stiffeners	Buckling reserve capacity k	
	shell without openings	shell with openings
H_{45} = 1309mm	5.434	5.363
H_{I} = 1646 mm	6.463	6.306
H_{30} = 2267mm	7.951	7.671
H_{L} = 2618 mm	8.857	8.374

It gives impression that the reserve of the bearing capacity k in the tight shell is always bigger. Which is another proof that the redistributing of discrete base reactions on the height is not only achieved by compression forces as it is shown on Fig. 3.

It is important to notice that when the cylindrical shells without openings are researched, the first form of buckling is always caused by shearing. The area of buckling is in both sides of the vertical stiffeners, see Fig. 13(a).



a) buckling due to shearing



b) buckling due to compression

Fig. 13. Modes of buckling in the cylindrical shells.

4. Conclusions

The current research, made for six cylindrical shells on discrete supports, shows that the continual shells have different behaviour than the shells with openings between the supports.

From where could be concluded that the reactions of discrete supports are not distributed on the height only by work of the cylindrical shell on compression above them, as an arch. In the conception of EN 1993-4-1 for dividing the cylindrical body of the silo into discretely supported ring beam and cylindrical body there is some truth. For that reason the body of silo above the supports should be checked for:

- buckling in the area above vertical stiffeners, caused by meridional (axial) forces;
- buckling to left or right of vertical stiffeners, due to shearing forces.

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