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Total potential energy minimization method in structural analysis considering material nonlinearity

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ABSTRACT

Minimum potential energy principle is the basis of the most of the well-known traditional techniques used in the structural analysis. This principle determines the equilibrium conditions of systems with reference to minimization of the sum of the total potential energy of the structure. In traditional applications, this methodology is formulized by using matrix operations. A methodology has been proposed in the last decades for structural analyses based on the idea of using metaheuristic algorithms to obtain minimum potential energy of the structural system instead of following this classical approach. This new method, called "Total Potential Optimization using Metaheuristic Algorithms (TPO/MA)", has been applied in this paper to truss structures considering linear and nonlinear behavior of the structural material. The metaheuristic method used in this process is teaching-learning based optimization (TLBO) algorithm. The proposed technique is applied on numerical examples and results are compared with other techniques in order to test the efficiency of the proposed method. According to results obtained, TPO/MA method with TLBO algorithm is a feasible technique for the investigated problem.

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1. Introduction

According to minimum potential energy principle, a structural system is in equilibrium if the total potential (TP) of the system is minimum. Classical methods do apply this principle in its pure form only in solving some demonstrative problems. The usual practice starts by writing down the TP in matrix form, as $TP = x^T Ax - Px$ where x is the vector of displacements, P is the vector of acting loads, and A is the flexibility matrix of the system. This is then followed by taking the derivative of TP with respect to x, and equating it to zero in the form $Ax = P$. This latter step is actually the application of the minimum energy principle which aims at finding x making TP stationary. Then the matrix equation $Ax = P$ is solved by using anyone of the well-established methods of matrix inversion or solving systems of linear equations. The method described here is valid for linear systems. For nonlinear systems there does not exist a common technique. The solutions for such systems vary according to type of nonlinearity (material nonlinearity, large deflections, nonlinear supports, under-constrained structures, missing or failing members, unstable structures, etc.) and a technique applicable for one type of nonlinearity is not applicable for another type except in some very special cases.

The method described in this paper, on the other hand, is valid for all types of linear and nonlinear structures, whatever the type of nonlinearity is. This technique, called "Total Potential Optimization using Metaheuristic Algorithms (TPO/MA)" is made possible thanks to advances in computer technology as to speed and also to emergence of and advances in metaheuristic algorithms for optimization problems. In this method, metaheuristic algorithms are employed for finding the displacements in a structure that makes the TP of the system a minimum. TP of the system is written as the sum of exact TP's of parts of the system. In this way the

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formulation does not necessitate use of matrices so that no big computer memory capacities become needed. Until recently the TPO/MA has been applied to a wide variety of problems such as truss, cables and tensegrity structures (Toklu 2004; Toklu et al., 2013; Temür et al., 2014; Toklu et al., 2015; Toklu and Uzun, 2016) using different types of metaheuristic algorithms.

In this paper, TPO/MA method is employed for structural analyses of trusses with nonlinear material properties. The metaheuristic algorithm used is a recently developed one, namely the teaching-learning based optimization (TLBO) algorithm. In order to evaluate the performance of TLBO algorithm, results are compared with those mentioned in the existing literature.

2. Methodology

Metaheuristic algorithms are developed from mathematical identification of natural phenomena. For example, Genetic algorithm (GA) mimics the process of natural selection (Holland, 1975 and Goldberg, 1989), particle swarm optimization (PSO) is inspired from the social behavior of animals (Kennedy and Eberhart, 1995), ant colony optimization (ACO) imitates the behavior of ants seeking a path between their colony and a source of food (Dorigo et al., 1996), harmony search (HS) is conceptualized on musician performance for seeking a harmony to admire the people (Geem et al., 2001).

The teaching-learning based optimization (TLBO) (Rao et al., 2011) is developed based on inspiration of teaching and learning procedure in a classroom. The method is not based on specific parameters and this is the most remarkable part of it. This property makes TLBO easily applicable and versatile among other metaheuristic algorithms.

The optimization process of the TLBO algorithm can be explained under two titles, namely First calculations and Iteration.

First Calculations: In the first calculations step, data of the structural system are defined. This data contains information about supports, start and end points of structural members, cross sectional areas of members, parameters about material properties of members, etc. In addition to the data, the upper and lower limits of the design variables, population number and maximum iteration number (as stopping criterion) are also defined in this section. Coordinates of the joints of the deformed system are the design variables of the problem.

Then, by randomizing the design variables between their defined limits, a group of the structures is obtained. This group is defined as initial solution matrix and number of structures (or solution vectors) in the group is equal to population size (pn). At the end of this step, the strain energy (Eq. (1)), work done by external loads (Eq. (2)) and total potential energy (Eq. (3)) for each generated system (or objective function) are calculated for future comparisons.

$$U = \frac{1}{2}\int_{VOLUME} \varepsilon^T \, \sigma dV \,, \qquad (1)$$

$$W = \int_{S_1}\left(T_x u + T_y u + T_z u\right) dS \,, \qquad (2)$$

$$\Pi = U - W = \frac{1}{2}\int_{VOLUME} \varepsilon^T \, \sigma dV - \int_{S_1}\left(T_x u + T_y u + T_z u\right) dS . \qquad (3)$$

In Eqs. (1-3), ε^T is the strain vector, σ is the stress vector and V is the volume of the body, u, v, and w are displacements in the x, y, and z directions, and T_x, T_y and T_z are the components of external forces in x, y, and z directions. The objective function of the problem is to minimize the total potential energy of the system (Eq. (3)).

Iterations: This process contains two phases: teacher (tp) and learner (lp) phases. In the teacher phase, new solution vectors are generated according to the best solution vector ($X_{teacher}$) which is the vector currently having the minimum value for TP of the system. Formulation of the generation of new values can be written as

$$X_{new,i}^{ip} = X_{old,i} + rnd(0,1) \cdot (X_{teacher} - T_F \cdot X_{mean}) \,, \qquad (4)$$

where $X_{old,i}$ is previous values of the design variables, X_{mean} is the mean value of the design variables, rnd is a uniformly distributed random numbers within the range of [0, 1] and T_F is an integer number that takes a value 1 or 2 (Eq. (5)).

$$T_F = round[1 + rnd(0.1)] \rightarrow \{1 - 2\} \,. \qquad (5)$$

In the learner phase, the value of new solution are generated from the two existing vectors that are randomly chosen from the solution matrix. The expression of the learner phase is defined as

$$X_{new,i}^{lp} = \begin{cases} X_{old,i} + rnd \cdot \left(X_i - X_j\right); & f(X_i) > f(X_j) \\ X_{old,i} + rnd \cdot \left(X_j - X_i\right); & f(X_i) < f(X_j) \end{cases} , \qquad (6)$$

in which $f(X_i)$ and $f(X_j)$ are objectives of selected vectors. After application of both phases, the objective functions of new vectors are calculated and if it is better than the old one, it is replaced with the old one. Iteration process is repeated until the maximum iteration number is satisfied. The optimization process is summarized in the pseudo code given in Fig. 1.

3. Numerical Examples

Numerical examples are presented in this section considering a 6-bar plane truss (Fig. 2) (Toklu, 2004) and three type of materials. In order to show the efficiency of the proposed method the results are compared with HS algorithm. The cross sectional areas of members 2-4 are 100 mm^2 and cross-sectional areas of other members are 200 mm^2. A concentrated load with 150 kN density is applied to system at joint 4. The first material considered (MAT 1) is a linear one with elasticity modulus of $2{\cdot}10^5$ N/mm^2. The second material (MAT 2) is a bilinear one, the third one (MAT 3) represents a highly nonlinear material. Stress-strain diagrams for all these materials are presented in Fig. 2(b).

```
Randomly generate the initial students
Calculate objective function
While stopping criteria
    (Teacher Phase)
    Calculate the mean of each design variable
    Identify the best student as teacher
    For i=1:N_variable
        Calculate teaching factor Eq. (5)
        Create a new solution based on teacher Eq. (4)
        Calculate objective functions for the new solutions Eq. (3)
        If X_new is better than X_old
            X_old = X_new
        End If
    End For
    (Learner Phase)
    For i=1:N_variable
        Select any two solution randomly [i, j]
        Create a new solution based on selected solutions Eq. (6)
        Calculate objective function for the new solution
        If X_new is better than X_old
            X_old = X_new
        End If
    End For
End While
```

Fig. 1. Pseudo code of optimization process with TLBO.

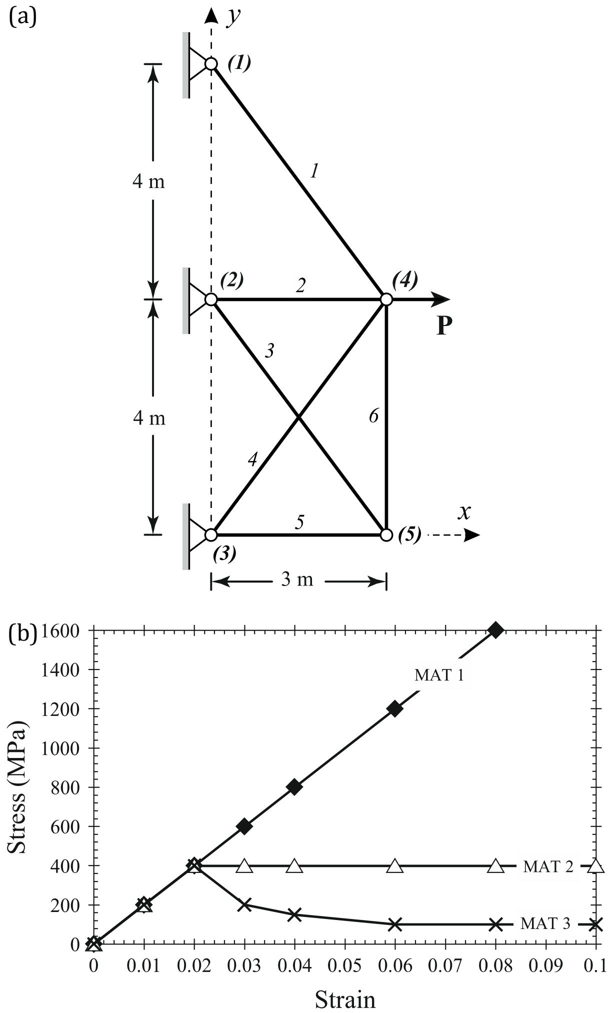

Fig. 2. (a) 6-bar plane system; (b) Material properties of problem.

In Figs. 3-5, for all these three materials, plots about the convergence to optimum result of TLBO and HS is given where the population number is equal to 10. On the TLBO approach, optimum results are found after about 100, 400 and 800 cycles for MAT 1, MAT 2 and MAT 3, respectively. For the HS based analyses, these numbers are about 2500, 500000 and 500000, respectively. It can be concluded here that convergence for linear material is much better than other materials and, for all cases, convergence performance of TLBO is better than HS approach.

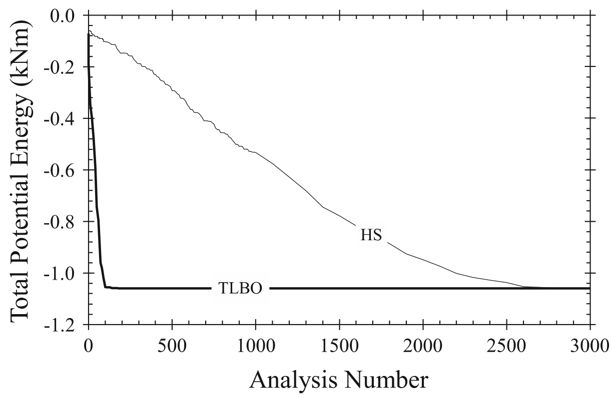

Fig. 3. Convergence to optimum results for MAT 1.

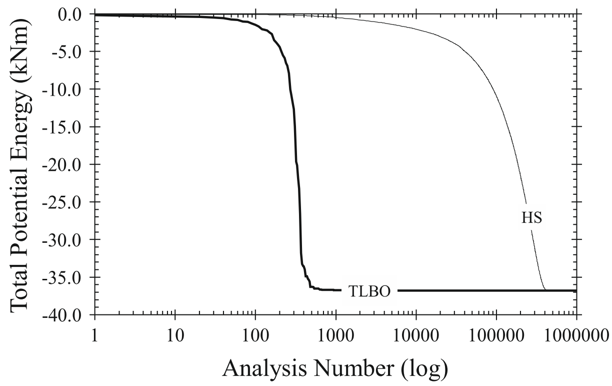

Fig. 4. Convergence to optimum results for MAT 2.

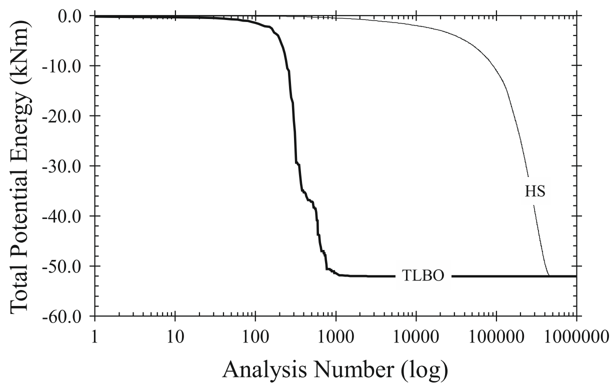

Fig. 5. Convergence to optimum results for MAT 3.

In order to investigate the effect of population number when applying TLBO on number of cycles, computations are repeated for seven different population numbers (Fig. 6). According to results, for all cases, analyses numbers needed for obtaining optimum results increase almost linearly as the population number increases. Statistical treatment of results for different population number is summarized in Table 1. According to these tests, results are all acceptable except for population number as 5. For case where population number is 10 or greater, the difference between the upper and lower bounds obtained for minimum potential energy is negligible so that the standard deviations are very small.

Table 1. Optimum result for different population number.

		5V	*10V*	*15V*	*20V*	*25V*	*30V*	*35V*	*40V*
	Min	-1059735	-1059735	-1059735	-1059735	-1059735	-1059735	-1059735	-1059735
MAT1	*Max*	**-1035346**	-1059735	-1059735	-1059735	-1059735	-1059735	-1059735	-1059735
	St Dev	**3192.258**	0.024875	1.86×10^{-9}	1.86×10^{-9}	1.86×10^{-9}	1.86×10^{-9}	1.86×10^{-9}	1.86×10^{-9}
	Min	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}
MAT2	*Max*	$\mathbf{-3.62 \times 10^{7}}$	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}	-3.68×10^{7}
	St Dev	**55387.39**	0.011541	8.2×10^{-8}	8.2×10^{-8}	8.2×10^{-8}	8.2×10^{-8}	8.2×10^{-8}	8.2×10^{-8}
	Min	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}
MAT3	*Max*	$\mathbf{-5.02 \times 10^{7}}$	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}	-5.21×10^{7}
	St Dev	**186907.6**	0.001228	7.45×10^{-9}	7.45×10^{-9}	7.45×10^{-9}	7.45×10^{-9}	7.45×10^{-9}	7.45×10^{-9}

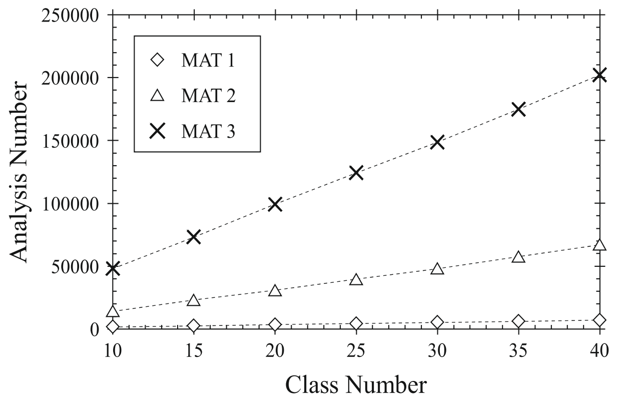

Fig. 6. Analysis number vs. population number plot.

In Figs. 7-9, total potential energy values for increased loading from 0 kN to 150 kN are given. The effects of material properties on total energy values can be clearly seen on the figures. The energy graph is quite monotonic for linear MAT 1 material, but sudden changes become observed for the other nonlinear materials MAT 2 and MAT 3.

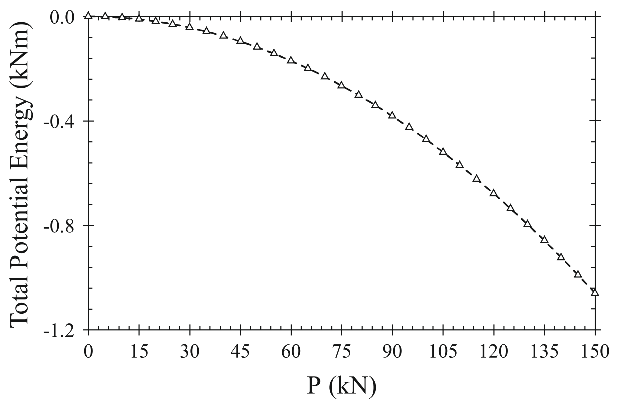

Fig. 7. Potential energy value of increasing loading for MAT1 material.

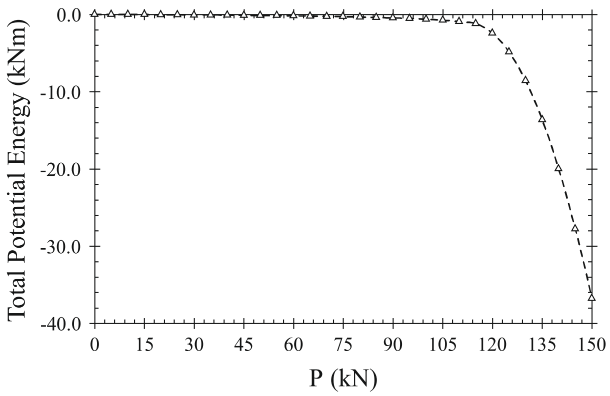

Fig. 8. Potential energy value of increasing loading for MAT2 material.

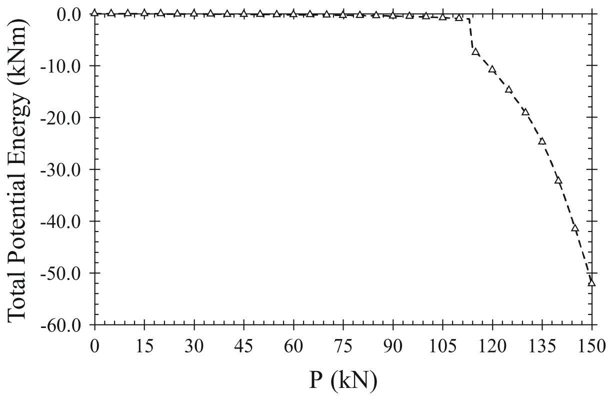

Fig. 9. Potential energy value of increasing loading for MAT3 material.

4. Conclusions

The analyses of truss systems are investigated by using TPO/MA method for different material properties such as linear, bilinear and nonlinear. The efficiency of the proposed TLBO method is checked by comparing with results obtained by HS approach. According to results, both TLBO and HS algorithms gave the same minimum potential energy for all material cases. Comparing the computational times (analyses numbers for optimum results) of the methods, the TLBO approach is 20% to 80% shorter than HS algorithm. According to observation of the effect of population numbers on analyses number, if the population number of TLBO is equal to or more than 10, the minimum energy value can be successfully found. But comparing the statistical suitability and analyses number, using a population number defined as 10 can be the best selection.

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