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Forecasting Short Term Trends in Prices of U.S. Stock Market

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Abstract

This thesis explores a cubic model to forecast short term trends in stock prices. Specifically, this model recognizes the limited applicability of instantaneous rate of change indications from the current stock price of an individual corporation. Discussed first is the nature of share price as a data vector and derivations of linear and non-linear mathematical operators. A proposed methodology demonstrates market entry and exit techniques that comprise a trading system and prediction range is evaluated with emphasis on error analysis.

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Chapter 1

Introduction

The U.S. stock market is a complex system. Within it, a population of global participants essentially plays a game. The playing field is the landscape of the world economies and every participating entity shares a common objective. Although participants may use different means, in the end, everyone is attempting to achieve financial success.

The growth of the Internet is the major cause of recently altered dynamics of the stock market. For individuals, barriers to market participation have never been fewer. A personal computer enables any participant to actively manage and monitor their portfolios, and in some circumstances, eliminate the services of a stock broker. The one factor every participant is most concerned with is the overall performance of a stock's respective share price. Measuring the share price performance of an individual stock can be shown by a relative comparison of today's share price with a past share price. A share price is assumed to be governed by supply and demand theory and determined through a bid/ask pricing mechanism. Because supply and demand continuously change, share prices fluctuate continuously.

Generally accepted as a fact in financial market science is the notion that throughout the life of an average stock, its share price performance will finish positive. Of course there are exceptions, but some market participants base their decision making process upon this idea while ignoring the inherent fluctuations in share price. Market participants who behave accordingly are more commonly referred to as stock market *investors* who employ a *buy and hold* strategy. Other participants share the belief of the investors that most stocks finish with positive share price performance, however, they attempt to maximize their success by timing the fluctuations of share price. Participants of this type are referred to as stock market *traders* who employ various mathematical models to capture the deterministic nature found within complexity. The renowned phrase *"buy low, sell high"* suitably characterizes the behavior of traders.

Many traders base their decision making process on a system of rules. Although a trader will readily adapt to new market information, most often they do not deviate from their system of trading rules. The collective use of all techniques available to a trader comprise part of a trading system to help determine the best time to enter and best time to exit the market. As news and new data arrive continuously, trading systems are updated to increase odds of success.

During the late 19^{th} century Charles Dow proclaimed prices move in distinguishable trend patterns, but later works suggested the advanced mathematics of Brownian motion governed price behavior. Among some of the current mathematical models used as tools in trading systems are applied probability models such as Markov chains, in particular, Martingale measures. Among the more pervasive tools used in trading systems is the Black-Scholes model which assumes stock returns are distributed lognormally. All of these mathematical models and others have proven to be useful given certain circumstances. The purpose of this research is to determine the extent of applicability for a trading system based on a piecewise third degree polynomial,

$$f(t) = at^3 + bt^2 + ct + d \qquad \text{for } t \in \Re$$

where a, b, c, and d are real constant coefficients and t represents time. Modeling market price data with a cubic function assumes price is time-dependent and fluctuates in a wave-like motion, much like the traversal of a sine wave curve which can be approximated by a cubic polynomial only for small values of t.

1.1 Present Status of the Proposed Problem

Analyzing the past movement of share prices and the volume of trading to forecast future price movement is important and referred to as technical analysis. The significance of the past is contradictory to the efficient market hypothesis. Efficient market theorists believe that share prices move randomly and instantly in response to new information, rendering useless all past information, such as past trading volume, price to earnings ratios, and earnings reports. Instead, a fundamental approach in which a critical analysis of a company, its industry, and current economic conditions, should mostly dictate investing and trading decisions.

Successful market participants use a combination of fundamental and technical analysis to locate a stock (fundamental) and time a trading decision (technical). Due to the growth of online trading, more market participants are trading and not investing. Thus, market volatility and the importance of short term trading has increased. Utilizing a piecewise cubic function as part of a technical analysis has been tested and researched by Gaur[8], Rehoblz[13], and Mak[11]. Mak simulated market price data as theoretical waveforms and tested the validity of a cubic function to predict trend reversals. Gaur and Rebohlz have applied catastrophe theory to the financial markets by attempting to comprehensively describe a dynamic system that exhibits discontinuous behavior under continuous stimuli. Rebohlz's model is an extension of Gaur's work and has been used to predict companies filing for Chapter 11 and the financial health of a company.

Chapter 2

Operators and Indicators

If a share price was observed at equal moments in trading time as time progresses, each observation, ρ_t, would comprise an indexed sequence of numbers,

$$\vec{\rho} = \{\rho_{-(N-1)}, \rho_{-(N-2)}, \rho_{-(N-3)}, \ldots, \rho_{-2}, \rho_{-1}, \rho_0\},$$

where N is the number of observations in $\vec{\rho}$. When the present time is defined to be $t = 0$, ρ_0 is the last observed price since for $t > 0$, ρ_t represents future data that does not exist. Because each ρ_t is observed at equal moments in trading time, $\vec{\rho}$ is a sequence of share prices observed during the same time within each passing trading day, week, or month. As with many other empirical time series, $\vec{\rho}$ will never appear to have a fixed mean. However, $\vec{\rho}$ will exhibit homogeneity in the sense that one part of the time series looks much like any other part. This chapter illustrates the derivation and examples of four mathematical operators that map $\vec{\rho}$ as input data into a vector of output data by means of a finite set of operations.

2.1 Exponential Moving Averages

A common technique used to identify trends in $\vec{\rho}$ is the use of moving averages which show the average value of a share price over a period of time. The exponential moving average (EMA) at time t, δ_t, of input $\vec{x}$, can be defined as

$$\delta_t = \sum_{k=0}^{\infty} \alpha(1-\alpha)^k x_{t-k} \qquad \text{for } \alpha \in [0, 1].$$

α is referred to as the *filtering weight* and can be calculated as $\alpha = \frac{2}{L+1}$. L is a positive integer and often referred to as the *length* of the EMA. A recursive definition for the EMA is

$$\delta_t = \alpha x_t + (1-\alpha)\delta_{t-1}, \qquad \text{for } \alpha \in [0, 1].$$

Figure 2.1 shows a time series plot over the course of two years of $\vec{\delta}$ when $\vec{x} = \vec{\rho}$ for three different values of α. Figure 2.1 illustrates the fact that as $\alpha \to 0$, the

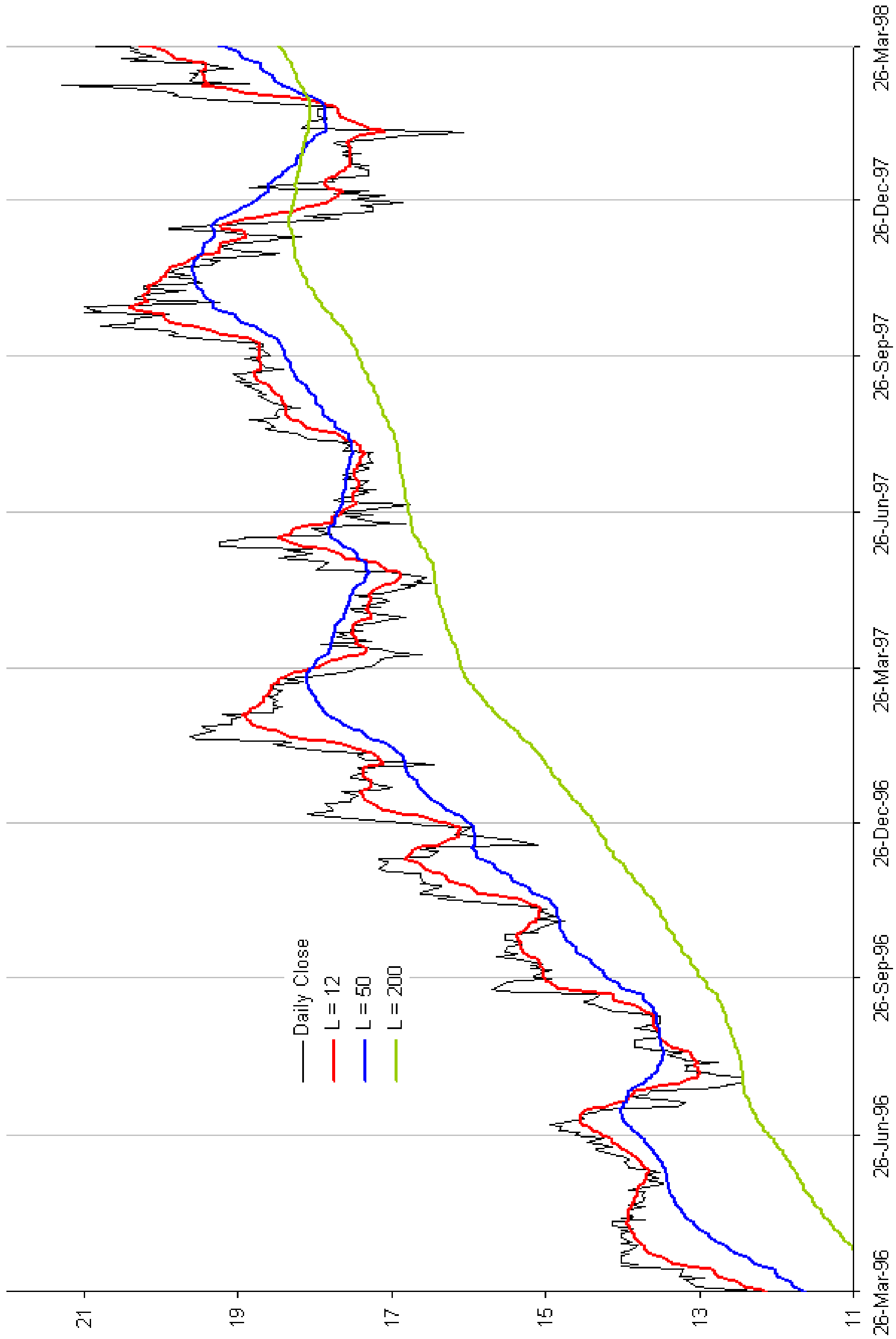


Figure 2.1: Three exponential moving averages of $\vec{\rho}$

smoother $\vec{\delta}$ becomes, and increasingly more information is filtered out of $\vec{\rho}$. Also evident from Figure 2.1 is the fact that $\vec{\delta}$ and $\vec{x}$ will always behave similarly even though $\vec{\delta}$ lags behind $\vec{x}$ in time. Thus, a smaller α will provide a smoother $\vec{\delta}$ but with greater time lag.

Time lag is significant since it ultimately affects a trading decision. To reduce time lag, the filtering weight of $\vec{\delta}$ can be automatically adjusted depending on recent volatility of $\vec{x}$. Automatically adjusting the filtering weight requires the recursive definition to be written as

$$\delta_t = \alpha\kappa_t x_t + (1 - \alpha\kappa_t)\delta_{t-1}, \qquad \text{for } \alpha\kappa_t \in [0, 1]$$

where κ_t is a ratio of volatility measures for $\vec{x}$ at time t. The preferred method used throughout this research calculates κ_t as a ratio of V_t to V_{t-b} where b is a positive integer. V_t is defined as

$$V_t = \frac{max(x_{t-(n-1)}, x_{t-(n-2)}, \ldots, x_{t-1}, x_t) - min(x_{t-(n-1)}, x_{t-(n-2)}, \ldots, x_{t-1}, x_t)}{\displaystyle\sum_{j=0}^{n-1} |x_{t-j} - x_{t-(j+1)}|}$$

where n is the number of periods over which volatility is being measured. EMA's of this form are referred to as adaptive exponential moving averages. When $\vec{x} = \vec{\rho}$, $\kappa_t > 1$ implies that price volatility is greater at time t than it was b periods ago and $\kappa < 1$ implies that price volatility is less at time t than it was b periods ago. Figure 2.2 illustrates how an adaptive EMA behaves differently than a non-adaptive EMA of equal length in times of high and low price volatility.

One method that uses EMA's as trending indicators is referred to as the fast-slow crossover method as illustrated in Figure 2.3. The fast-slow crossover method employs two non-adaptive EMA's of different lengths. The EMA with the greater length (the smaller filtering weight) is referred to as the *slower* EMA and the EMA with the shorter length is the *faster* EMA.

The crossover technique implies a share price is trending up after its faster EMA becomes greater than its slower EMA. Thus, it is a considerable time to enter the market. Conversely, a considerable exit point occurs after the faster EMA becomes less than its slower EMA. The crossover points exhibited in Figure 2.3 are marked within several boxes. However, many false trend indications could be generated.

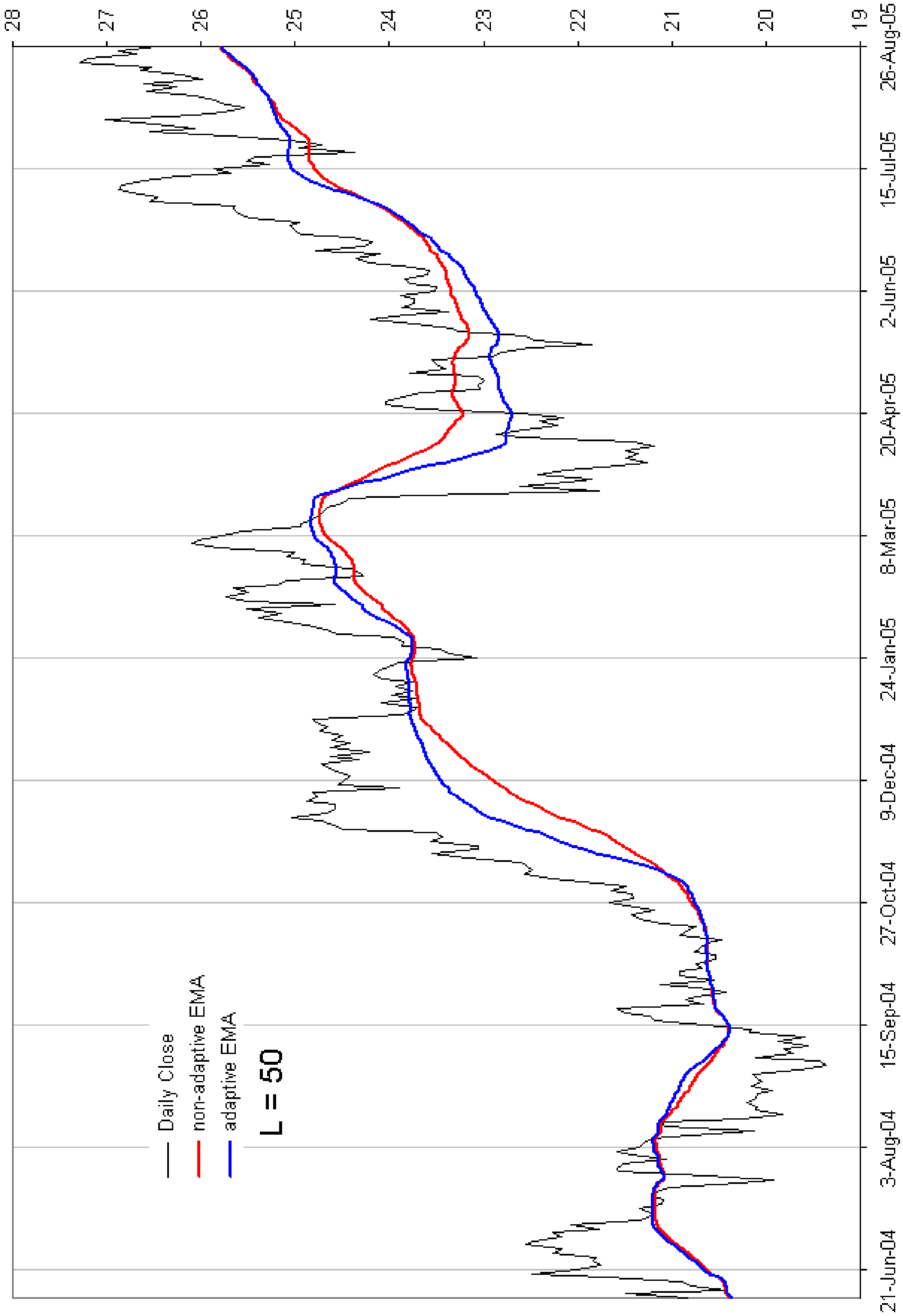


Figure 2.2: non-adaptive EMA versus adaptive EMA

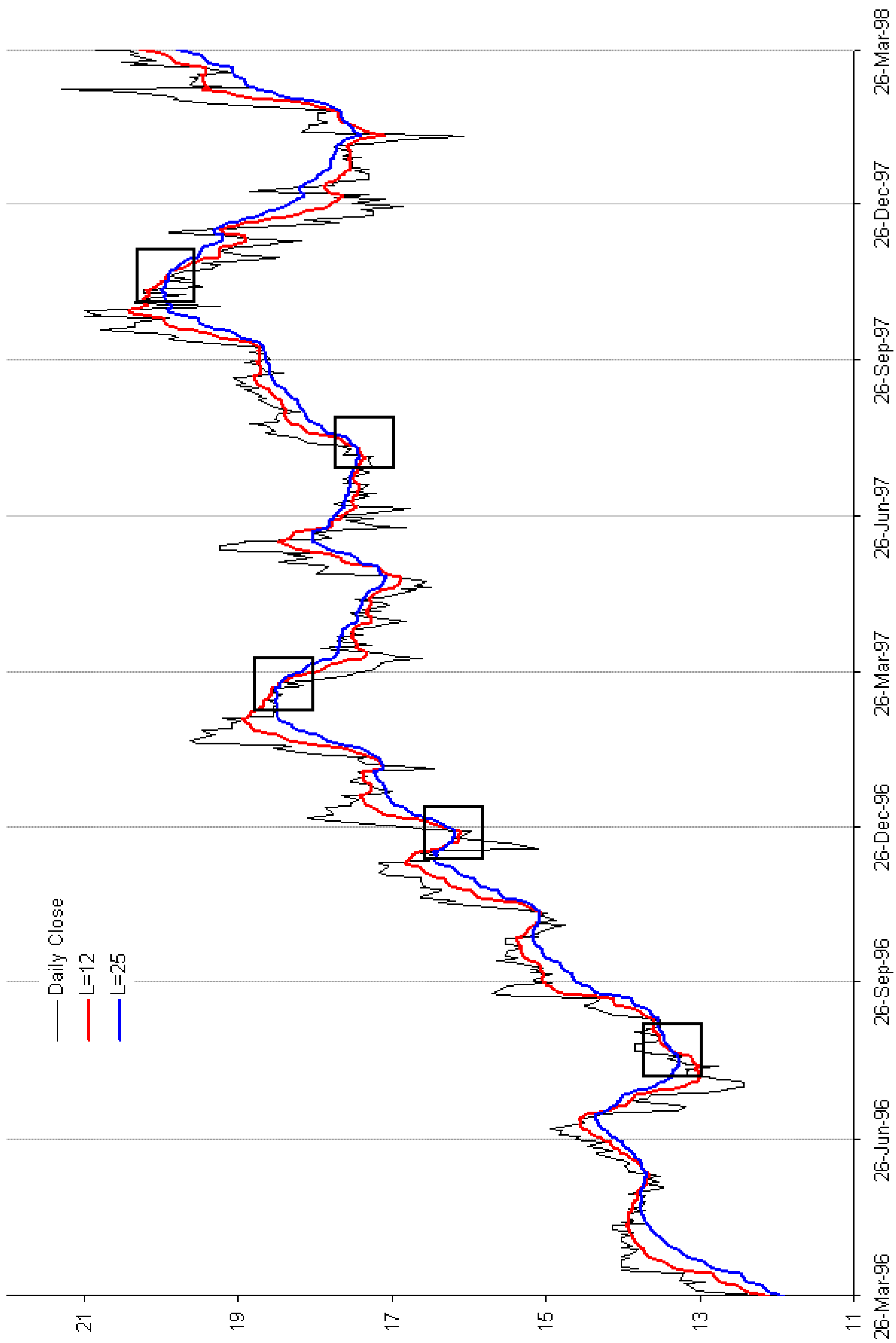


Figure 2.3: Fast-slow method

2.2 Velocity and Acceleration Indicators

Applying a piecewise cubic function to a time series like $\vec{\rho}$ or an EMA of $\vec{\rho}$, employs four adjacent data observations. If $\vec{x}$ is the data on which the piecewise cubic function is being fit, then

$$x_t = at^3 + bt^2 + ct + d \qquad \text{for } t \in \Re.$$

Evaluating x_t at $t = 0,\ -1,\ -2,$ and -3 yields

$$\begin{aligned} x_0 &= d, \\ x_{-1} &= -a + b - c + d, \\ x_{-2} &= -8a + 4b - 2c + d, \text{ and} \\ x_{-3} &= -27a + 9b - 3c + d. \end{aligned}$$

Since $x_0,\ x_{-1},\ x_{-2}$ and x_{-3} are four known observations, the four unknown coefficients $a,\ b,\ c$ and d can be found and calculated as linear combinations of $x_0,\ x_{-1},\ x_{-2}$ and x_{-3}:

$$\begin{aligned} a &= \tfrac{1}{6}x_0 - \tfrac{1}{2}x_{-1} + \tfrac{1}{2}x_{-2} - \tfrac{1}{6}x_{-3} \\ b &= x_0 - \tfrac{5}{2}x_{-1} + 2x_{-2} - \tfrac{1}{2}x_{-3} \\ c &= \tfrac{11}{6}x_0 - 3x_{-1} + \tfrac{3}{2}x_{-2} - \tfrac{1}{3}x_{-3} \\ d &= x_0. \end{aligned}$$

The cubic function can now be used to calculate a rate of change of $\vec{x}$ with respect to time by evaluating the first derivative of x_t,

$$\tfrac{d}{dt}x_t = 3at^2 + 2bt + c,$$

at different values of t, and in particular, at the present time, $t = 0$.

$$\tfrac{dx_t}{dt}|_{t=0} = c = \tfrac{11}{6}x_0 - 3x_{-1} + \tfrac{3}{2}x_{-2} - \tfrac{1}{3}x_{-3}$$

or

$$\tfrac{dx_t}{dt}|_{t=0} = \nu_0$$

where ν_t is defined by the convolution sum

$$\nu_t = \frac{11}{6}x_t - 3x_{t-1} + \frac{3}{2}x_{t-2} - \frac{1}{3}x_{t-3}.$$

ν_t is interpreted as a nominal measure for the current *velocity* of $\vec{x}$ at time t based on the last four observations in $\vec{x}$. If $\vec{x} = \vec{\rho}$, then for $\nu_t > 0$, the trend in prices at time t is generally an upward trend. For a $\nu_t < 0$, the trend in prices at time t is generally a downward trend.

When $\nu_t = 0$ the current trend in the input data experiences some kind of trend reversal. In order to differentiate what type of trend reversal may be occurring, a second indicator is employed. Evaluating the second derivative of x_t,

$\frac{d^2}{dt^2}x_t = 6at + 2b$

at $t = 0$ calculates the current rate of change of current velocity with respect to time, or the current acceleration indication, ω_0.

$\frac{d^2x_t}{dt^2}|_{t=0} = \omega_0 = 2b = 2x_0 - 5x_{-1} + 4x_{-2} - x_{-3}$

where ω_t is defined by the convolution sum

$$\omega_t = 2x_t - 5x_{t-1} + 4x_{t-2} - x_{t-3}.$$

As in calculus, the second derivative represents the concavity of a curve. Thus, $\omega_t > 0$ imply the input data at time t is concave up and $\omega_t < 0$ imply the input data at time t is concave down. Various interpretations of ω_t can be made in conjunction with v_t. Mak proposes that when $v_t = 0$ and ω_t remains approximately unchanged from ω_{t-1}, then the input data at time t is experiencing a major trend reversal. Otherwise, if ω_t does not remain approximately unchanged from ω_{t-1}, then the trend in the input data at time t will remain the same.

The top graph in Figure 2.4 displays $\vec{\rho}$ and its non-adaptive EMA of length 200, $\vec{\delta}$. The middle plot displays $\vec{\nu}$, the output of the velocity indicator operated on $\vec{\delta}$. After May 20, $\nu_t < 0$, thus implying the trend in prices at time t is a downward trend according to a relatively *slow* EMA. The change in velocity from positive to negative can be further analyzed by comparing $\vec{\omega}$, the corresponding acceleration vector generated by applying the acceleration indicator to $\vec{\delta}$. For several days prior to the date of investigation, ω_t mostly remains negative suggesting price velocity will remain negative after having approached and crossed zero from the positive side. Two significant variables considered in the interpretation of charts like Figure 2.4 are the length of the EMA employed to first filter $\vec{\rho}$ and the time unit of the share price data (i.e. seconds, minutes, hours, days, weeks, or years). Both will be further discussed in Chapter 3.

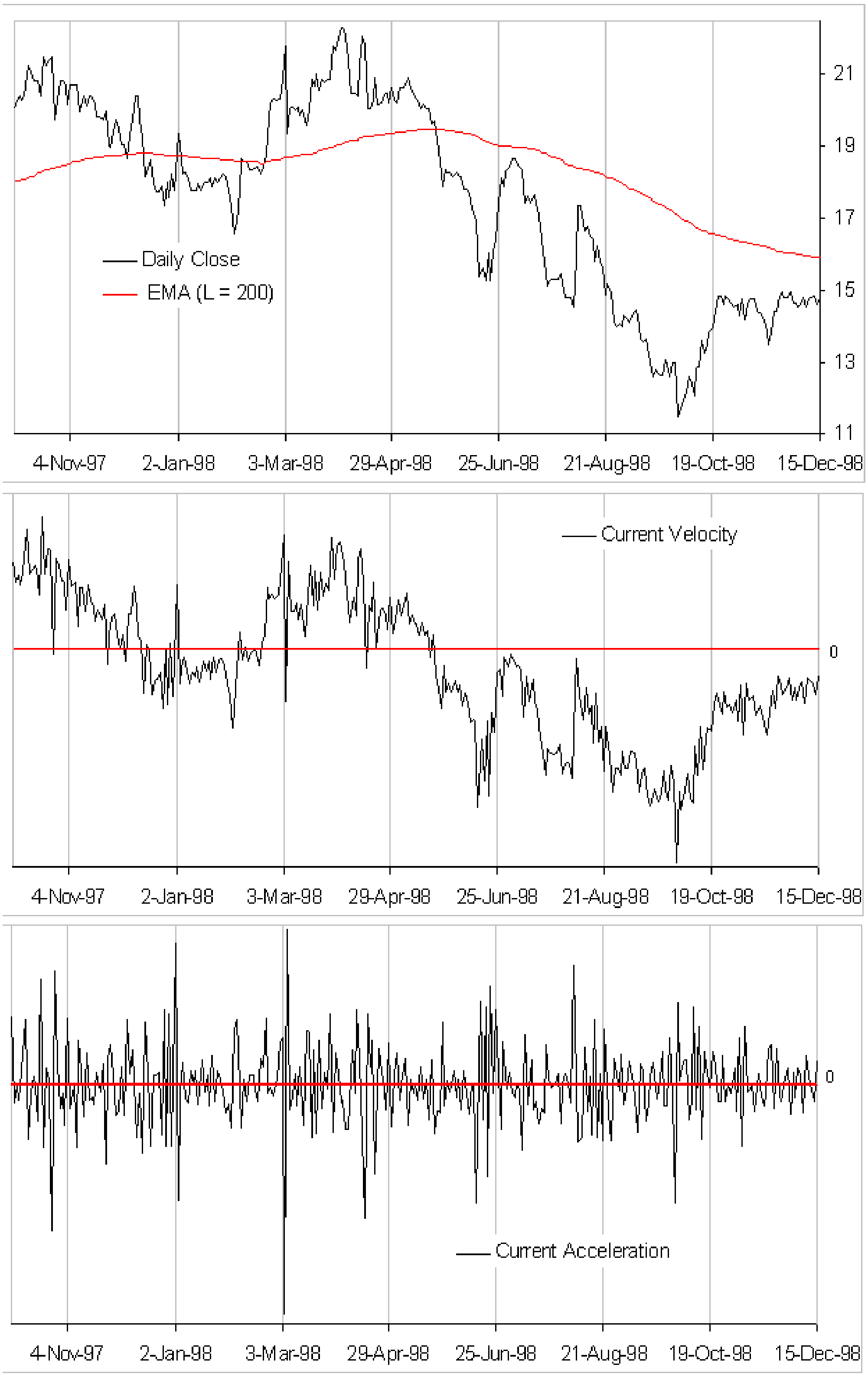


Figure 2.4: Current velocity and acceleration output

Because velocity and acceleration indicators generate oscillatory output, one or more EMA's will be used to reduce *noise* in the output. Consider an EMA of $\vec{\rho}$ with filtering weight α,

$$\delta_t = \sum_{k=0}^{N-1} \alpha(1-\alpha)^k \rho_{t-k}.$$

Suppose a velocity indicator was applied to $\vec{\delta}$,

$$\nu_t = \frac{11}{6}\delta_t - 3\delta_{t-1} + \frac{3}{2}\delta_{t-2} - \frac{1}{3}\delta_{t-3}.$$

Factoring the summation yields,

$$\nu_t = \sum_{k=0}^{N-1} \alpha(1-\alpha)^k (\frac{11}{6}\rho_{t-k} - 3\rho_{t-1-k} + \frac{3}{2}\rho_{t-2-k} - \frac{1}{3}\rho_{t-3-k})$$

which equals an EMA of weight α applied to data that has been operated on by the velocity indicator. Therefore, the non-adaptive EMA, velocity, and acceleration operators are linear operators. For example, suppose $\vec{\rho}$ was first filtered by a non-adaptive EMA with $\alpha = \alpha_1$ to generate $\vec{\delta}$. Then apply the velocity indicator to $\vec{\delta}$ to generate $\vec{\nu}$. Finally, filter $\vec{\nu}$ with an EMA with $\alpha = \alpha_2$. Identical output can be computed by simply reversing the order of use of α_1 and α_2.

2.3 Vertex Indicator

The non-linear indicator utilized in the trading system is the vertex indicator. A cubic curve of the form

$$x_t = at^3 + bt^2 + ct + d$$

that is fit to a sequence of four share price observations or a sequence of four *smoothed* share price observations usually yields two points at which velocity equals zero. The two turning points, or vertices, can be found by equating the first derivative of the cubic function to zero,

$$\frac{d}{dt}x_t = 3at^2 + 2bt + c = 0,$$

and solving for the unknown using the quadratic equation,

$$t = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}.$$

Because $t = 0$ represents present time, positive values of t represent future turning points and are of more interest to a trader than negative values. If $b^2 - 3ac < 0$, then no vertices exist. Thus, the vertex indicator determines at what time a trend could have reversed or at what time in the future a trend could reverse. Each plot in Figure 2.5 geometrically illustrates the three indicators. Each plot in Figure 2.5 contains a cubic curve and its velocity and acceleration indications represented as a parabola and straight line respectively. The y-intercepts of the parabola and line, and the x-intercepts, if any at all, of the parabola are the current velocity, acceleration, and vertex indications respectively.

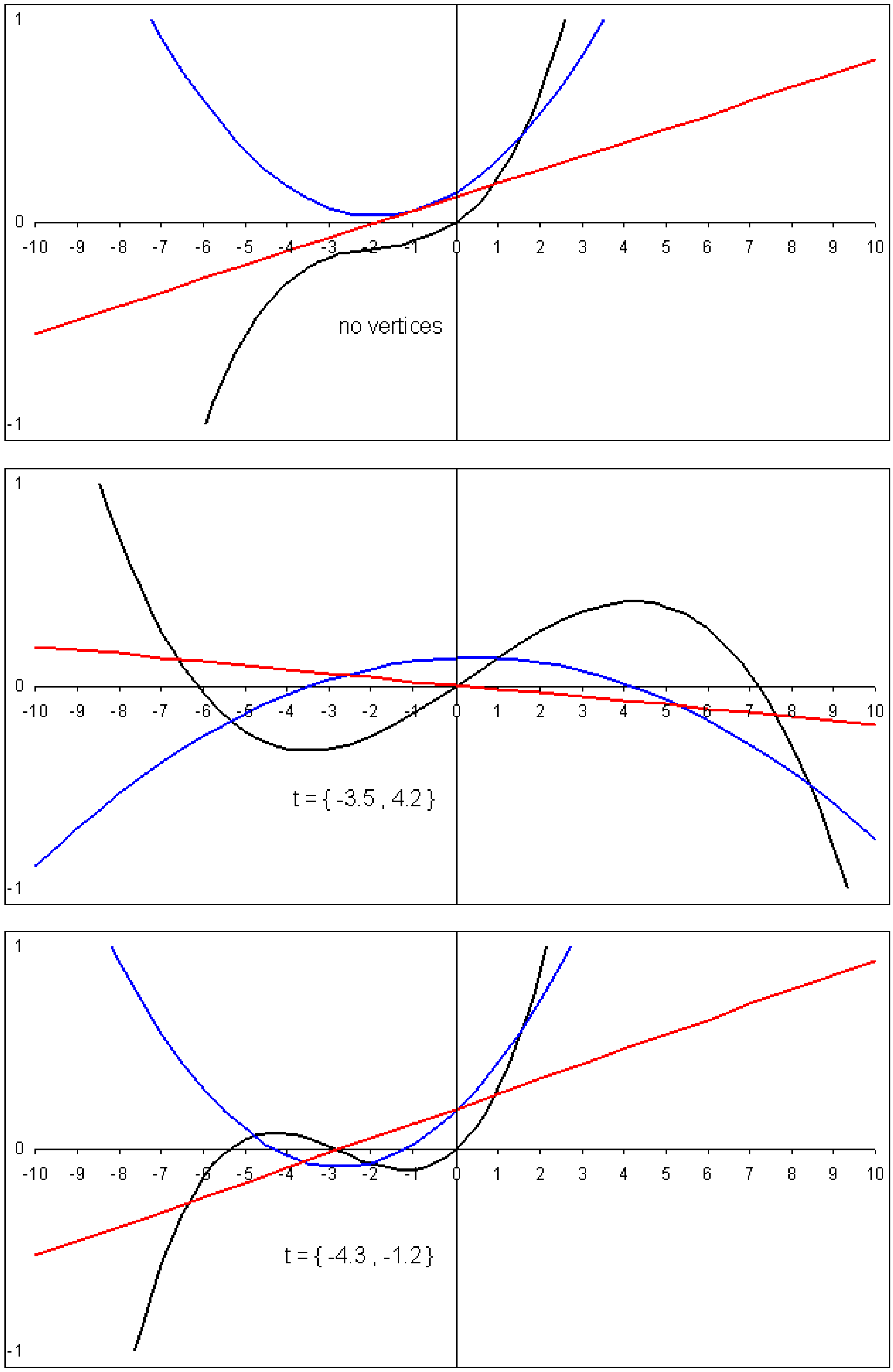


Figure 2.5: Vertex Indicator

Chapter 3

Methodology and Results

This chapter explains how to further interpret indications from charts such as Figure 2.4 in order to forecast short-term trends in share prices. A system of rules which serves to determine the best times to enter and exit the market is also established. It must be noted that the indicators described in chapter two are intended to be used in conjuction with sound fundamental analysis and money management.

Below are definitions of terms that are used in the following sections.

Slow EMA - for EMA's of a daily share price vector, a slow EMA length ranges from 50 to 200. For EMA's of a weekly share price vector, a slow EMA length ranges from 10 to 40. Slow EMA's provide a long-term perspective on trending in share price.

Fast EMA - for EMA's of a daily share price vector, a fast EMA length ranges from 5 to 49. For EMA's of a weekly share price vector, a fast EMA length ranges from 1 to 10. Fast EMA's provide a short-term perspective on trending in share price.

Slow Velocity(Acceleration) - a vector of current velocity(acceleration) generated by applying the velocity(acceleration) indicator to a slow EMA. Also referred to as long-term velocity(acceleration), or LTV(LTA).

Fast Velocity(Acceleration) - a vector of current velocity(acceleration) generated by applying the velocity(acceleration) indicator to a fast EMA. Also referred to as short-term velocity(acceleration), or STV(STA).

Average Velocity(Acceleration) - a fast EMA of slow or fast velocity(acceleration). Applying an EMA to velocity and acceleration output is used to reduce noise in STV and LTV.

3.1 Entering the Market

The axiom that most stock market participants base their trading decisions upon is the common oberservation that a company's share price will have experienced positive performance at the end of its time as a publically traded company. The *positive performance* axiom can be translated in terms of current velocity as described in chapter two. If $\vec{\nu}$ was a full life sequence of a share price's current velocity, then the mean of $\vec{\nu}$ would almost always be positive. The positive performance axiom and the notion that fundamentally healthy companies most often should exhibit sustained positive slow velocity are two assumptions made when deciding when to enter the market.

Fundamentally healthy share prices often exhibit patterns of movement that can be objectively analyzed with indicators. The top chart in Figure 3.1 shows the weekly closing share price and its two EMA's of Valmont Industries Inc. (VMI) from late May, 2005 to early May, 2006. During this time period, VMI enjoyed a share price performance of more than 100%, mostly due to high fundamental ratings.

Short and long-term velocity are first calculated using the two EMA's. To reduce noise in STV and LTV, average STV and average LTV are calculated with an EMA of length 2. The values of the two EMA's and the velocity calculations are shown in their respective columns in Figure 3.2. Determining potential points of entry is done using various *shift operators*. Rows of the column labeled *s.o.#1* in Figure 3.2 display a 1 when both LTV>0 and STV<0. Rows of the column labeled *s.o.#2* display a 1 when both average STV<0 and LTV>0. When both shift operators identify the same time, a potential entry point is declared. Twelve potential entry points are displayed in the shaded rows of Figure 3.2 and in Figure 3.1.

Further analysis of potential entry points is done by observing the weekly current accelertion computed with an adaptive EMA. The potential point of entry examined in this section is the closing share price of $33.39 of the week of 27-Dec-05. The adaptive EMA length is 10 and the n and b parameters as described in Section 2.1 are 6 and 3 respectively.

Because current acceleration oscillates, it is assumed that the longer current acceleration remains positive or negative, the more likely it is that the close of the next week will yield a current acceleration indication of the opposite sign. The length of time current acceleration remains positivie or negative can be analyzed by evaluating average values of current acceleration over the past two months. Below are the weekly averages of current acceleration for VMI at the close of the week of 27-Dec-2005.

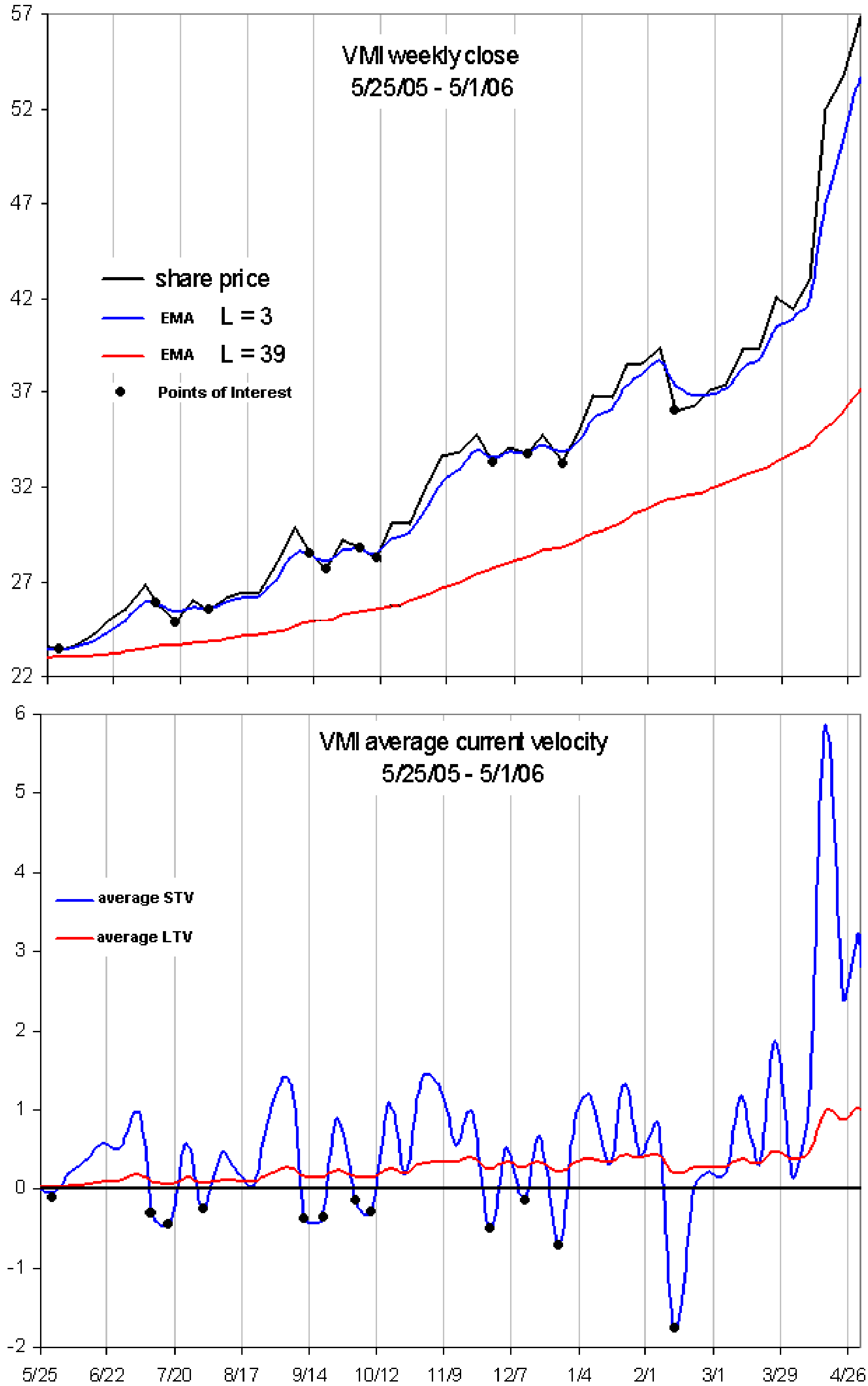


Figure 3.1: VMI

$$
\begin{aligned}
2\ week\ average &= -0.96 \\
3\ week\ average &= -0.92 \\
4\ week\ average &= -0.60 \\
5\ week\ average &= -0.16 \\
6\ week\ average &= -0.02 \\
7\ week\ average &= -0.26 \\
8\ week\ average &= -0.01
\end{aligned}
$$

A geometric approach to analyzing the length of time current acceleration remains positive or negative is illustrated in Figure 3.3. Figure 3.3 shows the current acceleration of VMI from the close of the week of 7-Nov-05 to the close of the week of 27-Dec-05, a timeframe of two months. If the line segments joining consecutive indications in Figure 3.3 comprised a parameterized curve $s(t)$ for $t \in [-7, 0]$, then clearly

$$\int_{-7}^{0} s(t)\ dt < 0.$$

In addition to the seven weekly averages of negative values, this observation increases the likliehood that the close of the next week will yield a positive current acceleration indication and thus a positive change in share price. Thus, a trader can decide to enter the market at this point in time or wait for a more suitable time within the next week.

A timeframe of daily share price is used to determine if an optimal day to buy appears within the next trading week. By using similiar shift operators as before, times of negative STV and postive LTV can be located. The top chart in Figure 3.4 displays the daily close of VMI share price and its two EMA's from 27-Dec-05 to 13-Jan-06. Shown in the bottom chart of 3.4 are the corresponding velocity indications. The length of the EMA used to generate the average STV is 5. The first shift operator indicates when both LTV>0 and STV<0. The second shift operator indicates when both LTV>0 and average STV<0. As before, when both shift operators coincide, an entry point is declared.

The column labeled Vertex in Figure 3.4 shows the vertex of smaller magnitude that is computed by applying the vertex indicator to the EMA of length 15. The vertex for the close of 4-Jan-2006 and the preceding five all have a magnitude of 1.1 or less. This observation indicates share price is not trending by a short-term perspective, or *trading flat*, which is favorable since it is hoped to enter the market before a trend develops. Within a month from 4-Jan-2005, VMI increases approximately 17%.

Date	**ema L=3**	STV	ave.STV	s.o.#1	s.o.#2	**ema L=39**	LTV	ave.LTV
1-May-06	53.54	3.5379	3.1633			37.11	1.1019	1.0260
24-Apr-06	50.37	0.6909	2.4140			36.07	0.8239	0.8741
17-Apr-06	46.99	8.1734	5.8602			35.14	1.2247	0.9744
10-Apr-06	41.98	1.7769	1.2337			34.26	0.5223	0.4737
3-Apr-06	40.97	-0.7129	0.1473	**1**		33.80	0.3253	0.3766
27-Mar-06	40.43	2.6508	1.8677			33.39	0.5569	0.4792
20-Mar-06	38.83	-0.1366	0.3015	**1**		32.94	0.3000	0.3237
13-Mar-06	38.28	1.6784	1.1779			32.60	0.4185	0.3710
6-Mar-06	37.23	0.1601	0.1769			32.24	0.2723	0.2758
27-Feb-06	36.99	0.3185	0.2104			31.97	0.2866	0.2828
21-Feb-06	36.89	0.8704	-0.0057		**1**	31.70	0.3139	0.2752
13-Feb-06	37.38	-3.0375	-1.7580	**1**	**1**	31.45	0.0790	0.1978
6-Feb-06	38.69	0.9866	0.8010			31.21	0.4548	0.4354
30-Jan-06	37.97	-0.0134	0.4297	**1**		30.78	0.3741	0.3966
23-Jan-06	37.35	1.8232	1.3159			30.37	0.4912	0.4415
17-Jan-06	36.21	-0.1486	0.3013	**1**		29.94	0.3173	0.3421
9-Jan-06	35.65	1.3178	1.2011			29.58	0.4190	0.3916
3-Jan-06	34.46	1.8205	0.9678			29.20	0.3982	0.3367
27-Dec-05	33.84	-1.4373	-0.7377	**1**	**1**	28.89	0.1519	0.2137
19-Dec-05	34.30	1.0637	0.6613			28.65	0.3671	0.3374
12-Dec-05	33.80	-0.4775	-0.1435	**1**	**1**	28.33	0.2494	0.2780
5-Dec-05	33.86	1.0600	0.5246			28.04	0.3685	0.3354
28-Nov-05	33.63	-1.3000	-0.5462	**1**	**1**	27.72	0.2079	0.2691
21-Nov-05	33.92	1.1616	0.9616			27.43	0.4168	0.3913
14-Nov-05	33.05	0.1848	0.5615			27.04	0.3262	0.3402
7-Nov-05	32.29	1.2680	1.3149			26.68	0.3907	0.3683
31-Oct-05	30.90	2.0260	1.4086			26.31	0.3844	0.3234
24-Oct-05	29.68	-0.2829	0.1737	**1**		26.01	0.1765	0.2012
17-Oct-05	29.29	1.7775	1.0870			25.80	0.3091	0.2508
10-Oct-05	28.49	-0.3767	-0.2940	**1**	**1**	25.57	0.1185	0.1341
3-Oct-05	28.75	-0.6433	-0.1288	**1**	**1**	25.43	0.1305	0.1653
26-Sep-05	28.67	1.5467	0.9002			25.25	0.2865	0.2349
19-Sep-05	28.11	-0.4133	-0.3928	**1**	**1**	25.04	0.1170	0.1319
12-Sep-05	28.55	-1.2082	-0.3519	**1**	**1**	24.90	0.1031	0.1616
6-Sep-05	28.51	1.5219	1.3608			24.71	0.3158	0.2788
29-Aug-05	27.16	1.5239	1.0385			24.44	0.2524	0.2049
22-Aug-05	26.30	-0.0156	0.0678	**1**		24.25	0.1044	0.1099
15-Aug-05	26.18	0.1354	0.2346			24.14	0.1187	0.1209
8-Aug-05	25.89	0.7892	0.4330			24.01	0.1522	0.1254
1-Aug-05	25.59	-0.7115	-0.2794	**1**	**1**	23.90	0.0397	0.0717
25-Jul-05	25.69	1.0986	0.5847			23.81	0.1745	0.1357
18-Jul-05	25.38	-0.5028	-0.4429	**1**	**1**	23.70	0.0416	0.0579
11-Jul-05	25.86	-0.9623	-0.3232	**1**	**1**	23.64	0.0461	0.0906
5-Jul-05	25.91	1.1720	0.9551			23.52	0.2115	0.1797
27-Jun-05	25.02	0.4941	0.5214			23.35	0.1252	0.1160
20-Jun-05	24.45	0.6798	0.5760			23.23	0.1156	0.0976
13-Jun-05	23.90	0.4496	0.3686			23.14	0.0738	0.0614
6-Jun-05	23.57	0.3291	0.2066			23.08	0.0477	0.0368
31-May-05	23.42	-0.0984	-0.0384	**1**	**1**	23.05	0.0104	0.0149
23-May-05	23.40	-0.3802	0.0818	**1**		23.03	0.0013	0.0240

Figure 3.2: VMI shift operator

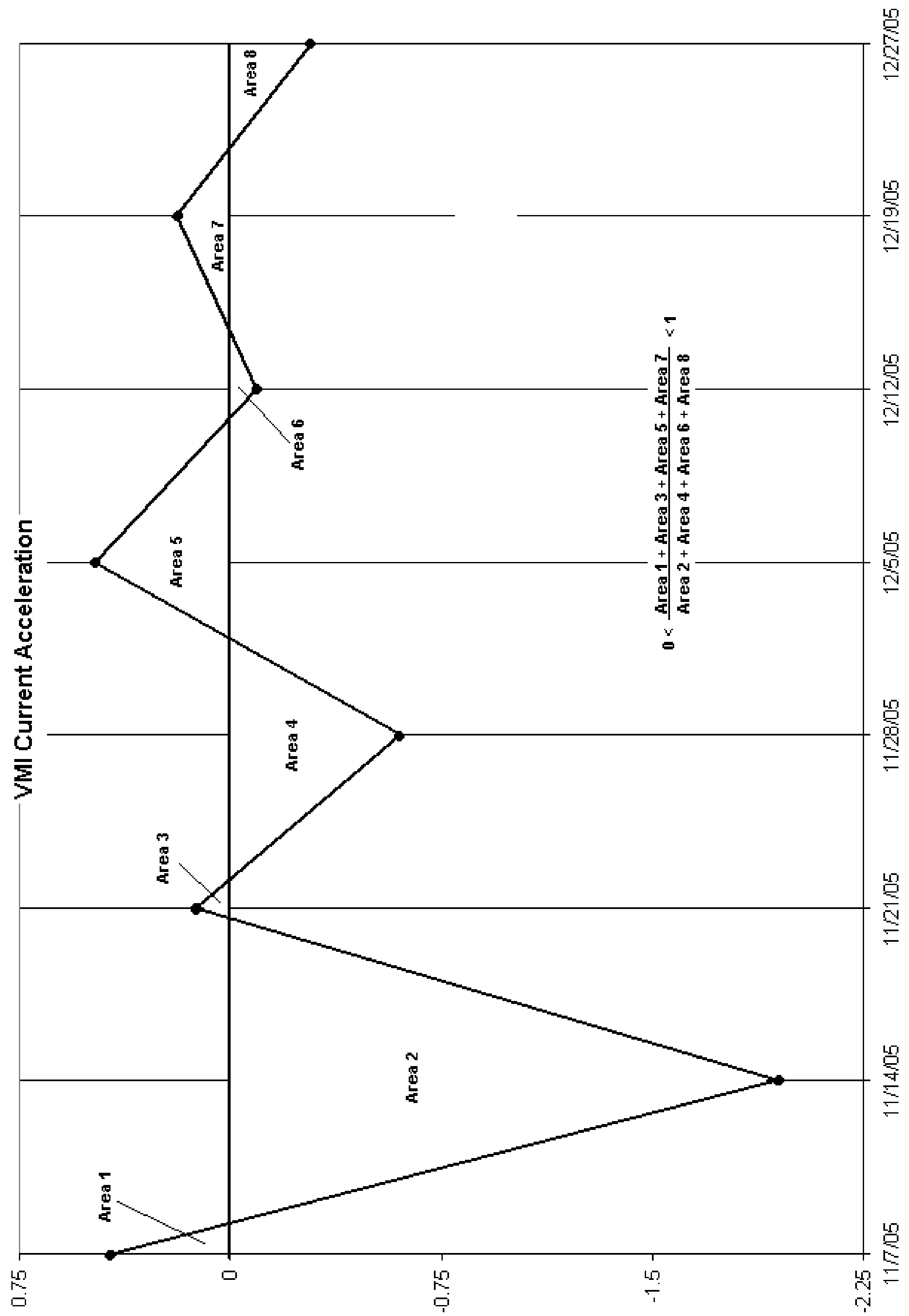
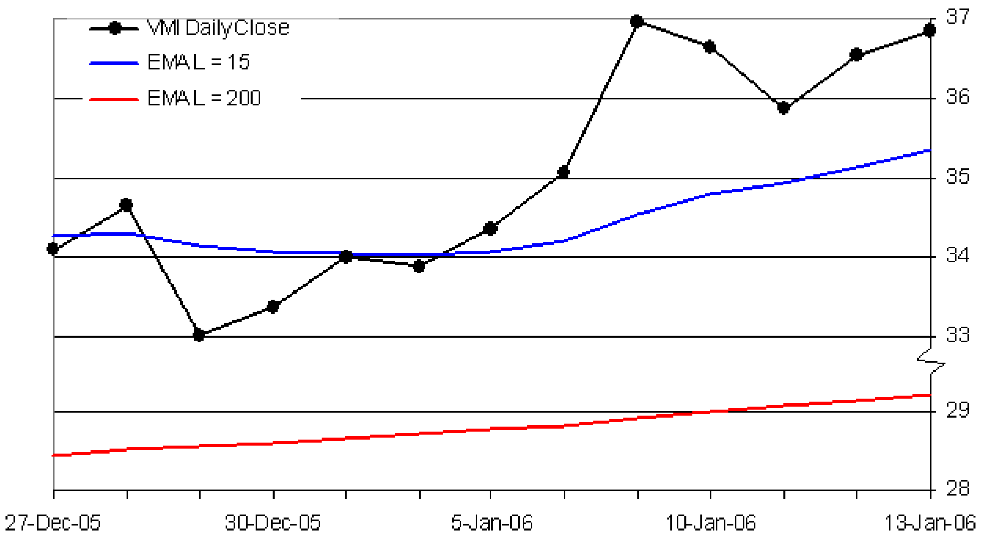


Figure 3.3: VMI weekly accleration generated from adaptive EMA (L=10, n=6, b=3)



Date	Close	EMA L=15	STV	Vertex	ave STV	s.o. #1	s.o. #2	EMA L=200	LTV
13-Jan-06	36.84	35.35	0.200	2.0	0.194			29.21	0.077
12-Jan-06	36.54	35.14	0.299	N/A	0.191			29.13	0.082
11-Jan-06	35.87	34.94	0.052	0.3	0.137			29.06	0.063
10-Jan-06	36.64	34.81	0.120	0.3	0.180			28.99	0.068
9-Jan-06	36.95	34.55	0.499	N/A	0.209			28.91	0.094
6-Jan-06	35.07	34.21	0.172	-2.1	0.065			28.83	0.067
5-Jan-06	34.37	34.08	0.097	-1.0	0.011			28.77	0.060
4-Jan-06	33.90	34.04	-0.061	-0.8	-0.031	1	1	28.71	0.049
3-Jan-06	34.01	34.06	0.046	-0.4	-0.016		1	28.66	0.057
30-Dec-05	33.39	34.07	0.027	-0.1	-0.048		1	28.60	0.056
29-Dec-05	33.03	34.16	-0.360	-1.1	-0.085	1	1	28.56	0.029
28-Dec-05	34.65	34.33	0.138	-0.8	0.052			28.51	0.068
27-Dec-05	34.10	34.28	-0.104	-0.7	0.010	1		28.45	0.051

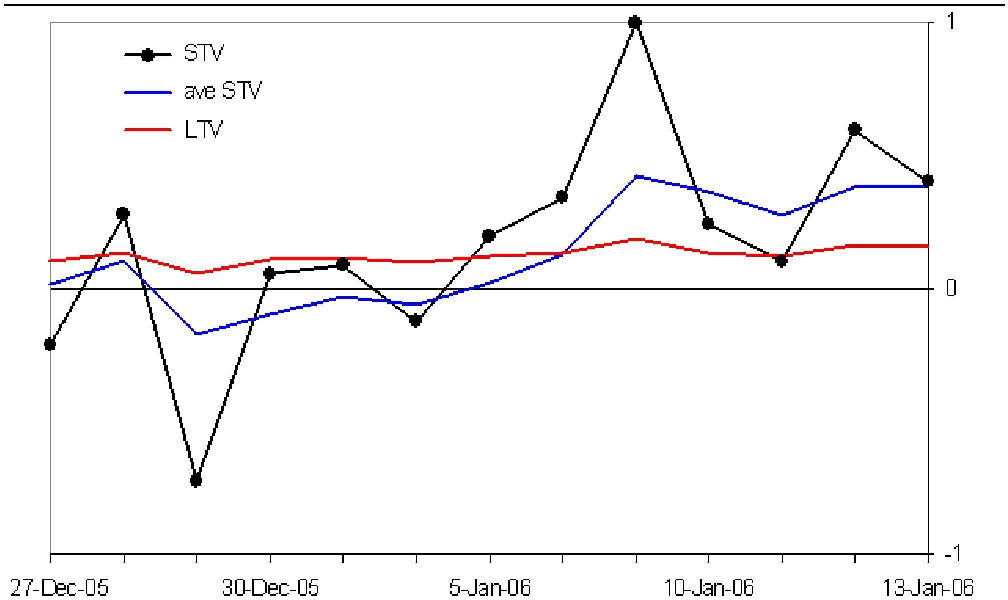


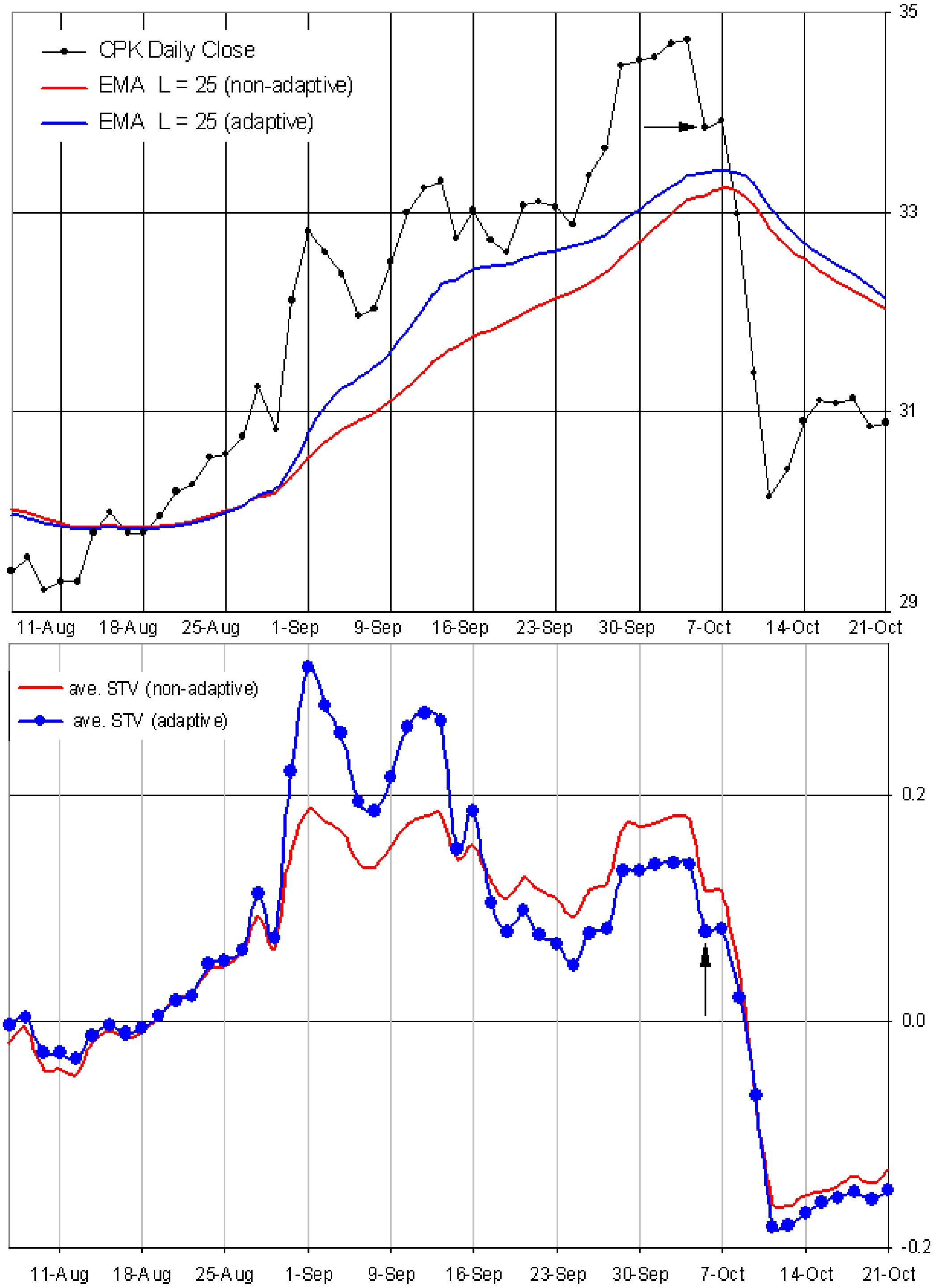
Figure 3.4: VMI daily shift operator

3.2 Exiting the Market

Exiting the market is emphasized less than entry since fundamentally healthy share prices should increase rather than decrease over the long-term. However, in technical analysis, a situation can occur in which the share price has risen to such a degree that an oscillating indicator has reached an upper bound. This is generally interpreted as an indication that share price is becoming overvalued and may experience a decline.

Comparing current velocity indications that are generated with an adaptive and a non-adaptive EMA of equal length can help determine when share price is overvalued. The top chart in Figure 3.5 shows the daily closing price of Chesapeake Utilities Corp. (CPK) from 8-August-2005 to 21-October-2006. The parameters for the adaptive EMA are $n = 28$ and $b = 12$ and the lengths of both EMA's are 25. Share price trends upward from 1-September-2005 to 7-October-2005 and then three large consecutive decreases occur. The bottom chart in Figure 3.5 shows average STV that is generated by an EMA of length 5. As share price is trending upward from 1-September-2005 to 7-October-2005, both accounts of average STV are not trending upward. In fact, the average STV according to the adaptive EMA has been trending downward and approaching zero since 1-September-2005. In technical analysis, observations of this type are referred to as divergences, which often indicate the end of trending in share price.

The table in Figure 3.5 displays the STV, STA, and vertex calculations generated from the two EMA's. All STV and STA indications at the close of 6-October-2005 are negative. Furthermore, both vertex indications are negative and nearly zero which indicates the trend has reversed according to the adaptive and non-adaptive EMA's. Thus, a considerable time to exit the market is at the close of 6-October-2005.



Date and Close		non-adaptive EMA L = 25				adaptive EMA L = 25			
		ema	STV	STA	Vertex**	ema	STV	STA	Vertex**
6-Oct-05	33.84	33.17	-0.01	-0.15	-0.05	33.39	-0.03	-0.14	-0.21
** of the two possible vertices calculated, the vertex of smaller magnitude is shown									

Figure 3.5: exit for CPK

3.3 Forecasting and Error Analysis

Forecasting prices with a cubic polynomial of the form

$$x_t = at^3 + bt^2 + ct + d$$

requires the polynomial to be expressed as a Maclaurin series of the third degree,

$$x_t = x_0 + [\frac{d}{dt}x_t|_{t=0}]t + [\frac{d^2}{dt^2}x_t|_{t=0}]\frac{t^2}{2!} + [\frac{d^3}{dt^3}x_t|_{t=0}]\frac{t^3}{3!}.$$

The first three terms of the Maclaurin series comprise a parabolic approximation in which both velocity and acceleration are accounted for, which simplifies to $d + ct + bt^2$. The last term of the series is regarded as the error term which simplifies to at^3. From Section 2.2, a is defined as a linear combination of the four most recent data observations,

$$a = \frac{1}{6}x_0 - \frac{1}{2}x_{-1} + \frac{1}{2}x_{-2} - \frac{1}{6}x_{-3}.$$

If x_0, x_{-1}, and x_{-2} were expressed as $x_{t-1} + e_t$ where $e_i \sim \mathrm{N}(\mu_i, \sigma_i)$ then a can be expressed as a function of the distance between consecutive data points,

$$\begin{aligned} a &= \tfrac{1}{6}(x_{-3} + e_1 + e_2 + e_3) - \tfrac{1}{2}(x_{-3} + e_1 + e_2) + \tfrac{1}{2}(x_{-3} + e_1) - \tfrac{1}{6}x_{-3} \\ &= \tfrac{1}{6}e_1 - \tfrac{1}{3}e_2 + \tfrac{1}{6}e_3. \end{aligned}$$

Although e_t need not be normally distributed, clearly the magnitude of a is directly influenced by the magnitude of change between each x_t. If $\vec{x}$ represented an EMA of share price data, then the magnitude of change between each x_t is directly related to the α parameter of the EMA used because as $\alpha \to 0$, the distance between adjacent smoothed data observations approaches zero. Therefore, the last term of the Maclaurin series can be expressed as an error function

$$\mathrm{E}(t, \alpha) = a(\alpha)t^3, \quad 0 < \alpha < 1$$

where a is the coefficient of the cubic term of the polynomial and a function of α. Therefore, as $\alpha \to 0$, $|\mathrm{E}(t, \alpha)| \to 0$.

Conclusion

A trading system based on a piecewise cubic polynomial is restricted by its limited ability to extensively forecast share price levels. However with the various indications(moving averages, velocity, acceleration, vertex), the present condition of a share price and its degree of trending can be assessed. Any attempt to time share price movement can be highly subjective, therefore the objective is only to determine a likely outcome.

Testing the entry and exit techniques with sector analysis and in markets that are more volatile than the U.S. stock market would help to further evaluate the extent of applicability for this trading system. For example, in futures markets, commodity price data can be modeled with a piecewise cubic function. However, entry and exit strategies would be equally emphasized since a commodity price is not expected to experience long term positive performance as a share price is. In other words, a great decrease in commodity price is of more concern than a great decrease in share price.

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