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Neuropsychological Predictors of Math Calculation and Reasoning in School-Aged Children

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NEUROPSYCHOLOGICAL PREDICTORS OF MATH CALCULATION AND REASONING IN SCHOOL-AGED CHILDREN

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By

Dana Lynn Schneider

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ABSTRACT

NEUROPSYCHOLOGICAL PREDICTORS OF MATH CALCULATION AND REASONING IN SCHOOL-AGED CHILDREN

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Dissertation supervised by Ara Schmitt, Ph.D.

After multiple reviews of the literature, which documented that multiple cognitive processes may be involved in mathematics ability and disability, Geary (1993) proposed a model that included three subtypes of math disability: Semantic, Procedural, and Visuospatial. A review of the extant literature produced three studies that examined Geary's three subtypes, which provided some support for Geary's model. Given the paucity of research examining Geary's subtypes of math disability, this study aimed to add to the literature by exploring the presence of these three subtypes in a sample of school-aged children. The sample consisted of 60 participants (30 males, 30 females) ranging in age from 7 to 13. Participants were taken from the standardization sample of the NEPSY-II. Individuals with data on the NEPSY-II (Korkman, Kirk, & Kemp, 2007) and the Math Calculation and Math Reasoning subtests of the Wechsler Individual

Achievement Test Second Edition (WIAT-II; Wechsler, 2001) were included in the primary analysis. Confirmatory Factor Analysis (CFA) results showed that the model consisting of select NEPSY-II subtests used to create Geary's three domains was a good fitting model. After creating domain scores for each participant, regression analyses were conducted in order to determine if Geary's three subtypes accounted for a significant amount of variance in math reasoning and math calculation performance. Results showed that the Semantic subtype did not contribute a significant amount of variance to numerical operations performance when examined in isolation. Hierarchical regression analysis, which consisted of entering the Semantic domain first and then adding the Procedural and Visouspatial domains simultaneously, showed that the model did not account for any variance in numerical operations performance. When all three domains were entered simultaneously, the Visuospatial domain was the only domain to account for a significant amount of variance in numerical operations performance. In terms of math reasoning, the Semantic domain was the only one to account for a significant amount of variance. The results of the current study are discussed within the context of Geary's theory and previous research related to Geary's theory. Finally, limitations, directions for future research, and implications are considered.

DEDICATION

This dissertation is dedicated to my parents, Dan and Norma, my sister, Maria, and my husband, Bill, for their support throughout my educational endeavors. I would surely not be where I am today without all four of them. This document is also dedicated to my family and friends who provided unconditional encouragement and support.

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Each of the above-mentioned individuals holds a special place in my heart, and has been influential in helping me to achieve the goal of dissertation completion. I am deeply appreciative of their support throughout this journey, and undoubtedly could not have reached this point without all of them.

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Chapter I

Although the cognitive processes underlying reading problems are reasonably well understood, there is a paucity of research examining the cognitive processes underlying mathematical difficulties. Advancing research related to mathematical processes is critical, as data from the 2011 National Assessment of Educational Progress reveal that 35% of eighth-grade public school students perform within the "below basic" category on this national math exam (National Center for Education Statistics, 2011). Furthermore, math achievement gaps between races still exist, with the gap between White and Hispanic students growing larger. Another area of concern is the lower achievement of public school students when compared to private school students. Statistics also reveal that more students performed in the "below basic" range as eighth graders than as fourth graders. This trend may be partially explained by the increasing difficulty of math content from fourth to eighth grades; therefore, symptoms of math disabilities may not be apparent until students encounter more complex math concepts.

Students with math disabilities are considered to have a biologically-based disorder that manifests as deficiencies in one or more cognitive areas: language, working memory, executive functioning, and/or visuospatial functioning (Feifer & De Fina, 2005). Due to varying definitions, criteria, and statistical techniques employed in empirical studies, researchers have found different prevalence rates for dyscalculia (Shalev, 2007). Scientists have conservatively approximated the prevalence of dyscalculia in the general population to be 3% to 8% (Krasa & Shunkwiler, 2009); however, the prevalence rate for school-aged children is estimated to be 5% to 8% (Geary, 2007; Proctor, Floyd, & Shaver, 2005).

Barbaresi and colleagues (2005) examined the cumulative incidence of dyscalculia for individuals by age 19 using three formulas. Using these formulas, Barbaresi and colleagues (2005) found the cumulative percentages to range from 5.9% to 13.8%.

Significance of the Problem

Theoretical Basis. With the understanding that multiple cognitive processes may be involved in mathematics ability and disability (MD), Geary (1993) proposed three subtypes of math disability. First, semantic difficulties hinder a student's ability to retrieve over-learned math facts from memory. The regions of the brain associated with this subtype are hypothesized to be the posterior regions of the left hemisphere and subcortical structures (e.g., the thalamus; Geary, 1993). This region of the brain is also responsible for processing linguistic information (Dehaene & Cohen, 1997), which may explain the well-documented link between the semantic subtype and basic reading disabilities. The Procedural subtype is related to deficiencies in the processing and encoding of numeric information, as well as use of developmentally immature procedures (Geary, 1993). In terms of the regions of the brain, Dehaene and Cohen (1997) have implicated both the left and right inferior occipital-temporal regions. The final subtype involves visuospatial skill deficits that impair an individual's ability to represent and interpret numerical information, such as misaligning numbers in multicolumn math problems (Geary, 1993). Researchers hypothesize that visuospatial deficits are linked to the posterior regions of the right hemisphere. Even though only three research studies have directly examined Geary's theory, research studies have examined aspects of his theory.

Summary of research. Several studies have investigated the cognitive and neuropsychological processes involved in mathematics, which will be presented according to Geary's three subtypes.

Semantic subtype. Research (Dehaene & Cohen, 1997; Fuchs et al., 2005) has found the students with MD experience difficulty with reading skills, including phonological processing. For instance, Fuchs and colleagues (2005) found that the phonological processing domain accounted for unique variance in addition fact fluency (2.5%) even after pretreatment reading skills were included. When pretreatment reading skills were included in the equation for calculation and problem solving, the contribution of phonological processing was no longer significant. In a follow-up study, Fuchs and colleagues (2006) found that when phonological decoding, sight word efficiency, and language were removed from the regression models, the overall fit of the model significantly decreased. Swanson and Beebe-Frankenberger (2004) and Swanson (2006) also found that reading was a significant factor when predicting math performance.

Visuospatial subtype. Several researchers (e.g., Geary, 2011; Sortor & Kulp, 2003) have found that the visual-spatial sketchpad, visual perceptual, and visual processing contribute significantly to math performance. For instance, several studies (e.g., Geary et al., 2009; Swanson's, 2006) found that the visual-spatial sketchpad significantly contributed to variance in math performance. Sortor and Kulp's (2003) investigation showed that children in the upper quartile of mathematics achievement scored significantly higher on all measures of visual motor integration when compared to children in the lower quartile.

Contrary to the previous research studies, there are studies (e.g., Andersson, 2010; Floyd, Evans, & McGrew, 2003) that do not support the contribution of visual skills to math calculation performance. Andersson (2010) found that visual matrix span did not significantly contribute to the regression equation. Similarly, in a study by Floyd, Evans, and McGrew (2003), visual-spatial thinking did not demonstrate significant relations with math calculation.

Procedural subtype. The contribution of the aspects of the procedural domain (e.g., attention, processing, etc) to math performance vary depending on the areas of investigation, measurement of procedural domain, and measurement of math performance. For instance, there is conflicting evidence regarding the contribution of working memory and processing speed.

Swanson (2006) found that inhibition contributed significant variance to math performance, while processing speed and executive system did not significantly contribute to math performance. Fuchs and colleagues (2005) also failed to find a significant path between processing speed and math computation and calculation. Working memory was significant predictor for math computation but not math calculation. In a later study, Fuchs and colleagues (2006) failed to support both working memory and processing speed for math computation.

There are research studies (e.g., Geary et al., 2007; Geary, Hoard, & Hamson, 1999) that support the contribution of working memory and processing speed. Hale and colleagues (2008) found that children with math disabilities scored significantly lower on tasks of working memory and processing speed when compared to typical peers. Similarly, Geary (2011) found that processing speed and working memory contributed to math performance above and beyond the contributions of intelligence. Further, results from Swanson and Beebe-Frankenberger's (2004) study suggest that the influence of working memory on math performance across grades was stable.

Research specific to Geary's subtypes. Cirino, Morris, and Morris (2002, 2007) examined Geary's three subtypes and found conflicting results in terms of the constructs that contribute to mathematical abilities. In an older study, a significant proportion of variance in mathematical abilities was accounted for by semantic and procedural skills, and not visuospatial skills. The newer study found semantic and visuospatial skills to account for a significant percentage of mathematical abilities. The conflicting results could be a result of the difference between the two samples. The studies also utilized slightly different assessment measures, as updated versions of the tests were used in the later study. Lastly, the population studied was college students. In addition to a paucity of research examining Geary's subtypes, there is also limited research using the NEPSY-II (Korkman, Kirk, & Kemp, 2007) to examine children with math deficits.

Existing math studies using the NEPSY-II. In a study by Korkman, Kirk, and Kemp (2007), children with MD showed impairment with attention, executive functioning, visual memory, spatial memory, and visuospatial processing, while their language, sensorimotor functioning, and social perception functioning were relatively intact. These results, however, must be interpreted with caution, as there are many limitations with this study, including the small sample size. The sample size also varied by subtest and ranged from 15 to 20 children. Additional research is needed to examine Geary's theory within a school-aged population so that a more comprehensive understanding of the cognitive processes underlying mathematical abilities can be found.

Problem Statement

Research has shown that students with MD are not as proficient as their peers (Garnett & Fleischner, 1983); however, research studies have found different, and in some cases, conflicting results as to the cognitive processes involved with mathematical procedures. Research suggests that fluency of over-learned mathematical facts, such as addition, is linked to the left-hemisphere and frequently co-occur with reading disabilities (Bull & Johnston, 1997). Other research studies have implicated Crystallized Intelligence, Short-Term Memory, Visual Processing, Working Memory, Processing Speed, Attention, and Phonological Processing. Three studies have examined Geary's three subtypes, which found some support for this model. However, two studies that were similar in design found conflicting results (Cirino, Morris, and Morris, 2002, 2007). In addition to the conflicting results in terms of the processes that contribute to math performance, limitations of the studies restrict the generalizability of the results. Some studies failed to use advanced statistical procedures (e.g., regression), while others only examined college students. Researchers relied on teacher reports of attention, instead of directly measuring attention. Given the inconsistent results found in the extant literature, the paucity of research examining Geary's subtypes, and the paucity of research utilizing the NEPSY-II (Korkman et al., 2007) to measure math weaknesses, this study added to the literature base by examining Geary's three subtypes using the NESPY-II.

Research Questions and Hypotheses

Research question 1. What are the intercorrelations among select

NESPY-II subtests and the WIAT-II Numerical Operations and Math Reasoning subtests? It is hypothesized that the NEPSY-II subtests will have moderate correlations with the WIAT-II subtests.

Research question 2. Do NEPSY-II subtests load onto factors that mirror represent the three cognitive constructs (semantic retrieval, executive-procedural, and visuospatial) that Geary proposes are related to math? It is hypothesized that (a) Comprehension of Instruction, Narrative Memory, Phonological Processing, and Speeded Naming subtests of the NEPSY-II will load onto a factor that represents Geary's Semantic domain; (b) Auditory Attention, Response Set, and Inhibition subtests of the NEPSY-II will load onto a factor that represents Geary's Procedural domain; and (c) Arrows, Block Construction, Design Copying, Geometric Puzzles, Memory for Designs, Memory for Designs Delayed, Memory for Faces, and Memory for Faces Delayed subtests will load onto a factor that represents Geary's Visuospatial domain.

Research question 3. What are the intercorrelations among the confirmed or established factors of the select NEPSY-II subtests and the WIAT-II Numerical Operations and Math Reasoning subtests? It is hypothesized that factors will have moderate correlations with the math calculation and reasoning subtests.

Research question 4. When the Semantic domain score is considered a predictor variable, will it significantly predict or account for a significant amount of variance in numerical operations? When the Procedural and Visuospatial domain scores are included as predictor variables, will they significantly predict or account for a significant amount of variance in numerical operations above and beyond the variance already accounted for by the semantic domain score? Does the obtained regression equation resulting from a set of three predictor variables allow us to predict numerical operations performance reliably? It is expected that all three predictors will be included in the equation and that the obtained regression equation will reliably predict numerical operations performance.

Research question 5. When the Semantic, Procedural, and Visuospatial domain scores are considered predictor values simultaneously, which significantly predict or account for a significant amount of variance in Numerical Operations performance? Does the obtained regression equation resulting from a set of three predictor variables allow us to reliability predict Numerical Operations performance? It is expected that all three predictors will be included in the equation and that the obtained regression equation will reliably predict Numerical Operations performance.

Research question 6. When the Semantic, Procedural, and Visuospatial domain scores are considered predictor variables simultaneously, which significantly predict or account for a significant amount of variance in math reasoning performance? Does the obtained regression equation resulting from a set of three predictor variables allow us to predict Math Reasoning performance reliably? It is expected that all three predictors will be included in the equation and that the obtained regression equation will reliably predict Math Reasoning performance. The next section explores the current literature base.

Chapter II

Although the cognitive processes underlying reading problems are reasonably well understood, there is a paucity of research examining the cognitive processes underlying mathematical difficulties. Advancing research related to mathematical processes is critical as data from the 2011 National Assessment of Educational Progress (NAEP) reveal that only 35% of eighth-grade public school students perform at or above the proficient category on this national math exam (National Center for Education Statistics, 2011). NAEP statistics suggest that there has been an increasing trend in math achievement for fourth-and eighth-grade students since 1990; however, differences between groups of students still exist. For example, males continue to score higher than females, private school students score higher than those in public schools, students eligible for free or reduced-price lunch score poorer than students who were not eligible for free or reduced-price lunch, and minority students (African Americans, Hispanics, American Indians, and Alaska natives) score poorer than White students.

NAEP data also indicate that as students progress across grades, level of math achievement drops. For example, the percentage of public school students scoring below basic was higher for eighth-graders (28%) when compared to fourth-graders (18%), nationally. This trend is also observed in Pennsylvania with 26% of the eighth-graders performing at the below basic level, while only 13% of fourth-graders performed at the below basic level. This is likely due to the presentation of increasingly difficult math concepts that are harder to master. To illustrate, the fourth-grade test examines a student’s ability to solve simple real-world problems, perform simple computations with whole numbers, and to have a basic understanding of fractions and decimals. In contrast, the eighth-grade exam assesses a student's ability to solve problems with structural prompts (e.g., graphs), use algebraic concepts, and convert raw points to a percentage. In sum, socio-economic status, gender, racial/ethnic background, and decreasing achievement across grades speak to the need for additional research to aid in instruction and intervention planning. In order to understand what is required of students across grades, one must have a reference point of the typical/expected development of mathematical skills. Once typical development is explored, a discussion about students with math disabilities will occur.

Development of Mathematical Skills

The National Council of Teachers of Mathematics (NCTM, 2010) has outlined mathematics standards and expectations for students that schools, prekindergarten through grade 12, should use in their instructional programs to aid in the development of students' mathematical skills. NCTM divides these expectations into five content areas: (a) number and operations, (b) algebra, (c) geometry, (d) measurement, and (e) data analysis and probability. That being said, most research conducted in educational psychology classifies mathematics problems as either primarily math calculation or math reasoning. This is in part due to federal regulations that the guide the education of students with learning disabilities. The Individuals with Disabilities Education Improvement Act of 2004 (IDEA, 2004; PL 108-446) identifies a student as having a specific learning disability in either math calculation or problem solving. Educators and researchers generally view math calculation and problem solving as distinct areas due to the addition of linguistic information in math problem solving. For math calculation, the problem is already set up for solution; whereas, the latter requires a student to use the text of the problem to identify missing information, construct a number sentence, and subsequently solve the calculation problem. Research has shown that the cognitive processes attributable to math calculation and problem solving vary (e.g., Cirino, Morris, & Morris, 2002; Fuchs, et al., 2008). For instance, Fuchs and colleagues (2008) studied the cognitive profiles of math calculation and problem solving in third grade students. Results showed that attentive behavior and processing speed contributed to math calculation, while language contributed to math problem solving. Given the research supporting the separation of calculation and problem solving and the federal definition of specific learning disability in math, this research study examines math calculation and math problem solving. Therefore, the focus of the current discussion will be on the progression of skill development in the following two content areas: (a) number and operations and (b) data analysis and probability. The former content area best represents the development of math calculation skills and the latter best represents the development of math problem solving.

Number and operations. NCTM (2010) has outlined three skill areas under the number and operations domain that instructional programs should address: (a) understanding numbers, (b) understanding meanings of operations, and (c) computing fluently (NCTM, 2010). According to NCTM (2010), students should be able to comprehend and represent numbers and number systems, as well as understand the relationships among numbers. Students enrolled in pre-kindergarten through Grade 2 are required to count the number of objects in a series and understand the connections between numerals, number words, and the quantities they represent. They must develop an awareness of place values, base-ten number system, and fractions. For Grades 3 through 5, students should have an understanding of numbers less than zero and be able to compare whole numbers and decimals. By the end of fifth grade, students should know how fractions are a part of a whole. Students should also be able to use models to judge the size of fractions, and convert commonly used forms of fractions into decimals and percents. Sixth through eighth grade students are required to solve problems involving fractions, decimals, and percents. Additionally, these students should be able to compare and order fractions, percents, and decimals. Representing quantitative relationships using ratios and proportions is another skill that should be mastered by the end of eighth grade. Students in ninth through twelfth grades develop an understanding of vectors, matrices, very small numbers, and very large numbers. By the end of twelfth grade, students are required to justify relationships involving whole numbers using number-theory arguments (NCTM, 2010).

Another skill area within the numbers and operations domain that students should obtain is an understanding of the various operations (e.g., addition, subtraction, etc.), and how these operations are related to one another (NCTM, 2010). Pre-kindergarten through second grade students increase their understanding of the meanings and effects of adding and subtracting whole numbers, as well as the connection between addition and subtraction. By the end of second grade, students are required to identify situations that would involve multiplication and division, as well as understand the meanings and effects of multiplying and dividing whole numbers. Second grade students should be able to use the relationships that exist between operations, such as multiplication as the inverse of division and the distributivity of division over subtraction. By the end of sixth grade, students must have an understanding of the inverse relationship of all operations (including square roots and squaring). Eighth graders should know how to calculate all operations using decimals, fractions, and integers. Twelfth grade students are required to judge the effects of operations (e.g., multiplication and roots) on the magnitude of quantities. By the end of twelfth grade, students should also understand how to add and multiply vectors and matrices, and use permutations and combinations as counting techniques.

Lastly, NCTM (2010) includes computation fluency and estimation skills within the number and operations domain competencies. Students in prekindergarten through second grade develop different strategies for whole-number computations, which include using a variety of tools (e.g., calculators, paper and pencil, etc.) when solving math problems. By the end of second grade, students develop fluency for whole number combinations of adding and subtracting, whereas, by the end of fifth grade they should be fluent with adding, subtracting, dividing, and multiplying. An expectation for fifth grade is to use strategies to estimate the results of whole-number computations, fractions, and decimals. Fifth grade students are also required to use visual models when adding and subtracting with fractions and decimals. Eighth graders, on the other hand, should be able to identify and use appropriate methods (e.g., calculator, estimation, etc.) when computing fractions and decimals. Eighth grade students are also developing strategies for estimating the results of rational-number computations. In addition to developing and analyzing methods for solving problems involving proportions, students are also required to explain these methods. By the end of eighth-grade, students should be able to compute algorithms involving fractions, decimals, and integers fluently. Twelfth graders are required to complete operations involving real numbers, vectors, and matrices, and judge the reasonableness of their results.

Data analysis and probability. NCTM (2010) characterizes the data analysis and probability domain in terms of four skill areas: (a) formulating questions that can be analyzed with data, (b) selecting and using the appropriate statistical methods to analyze data, (c) using data to develop and evaluate inferences and predictions, and (d) utilizing basic concepts of probability. The first skill area states that students will formulate questions that can be analyzed with data. Within this skill area, students are also required to collect, organize, and display the data in such a way to answer the proposed questions. In terms of grade requirements, second grade students should be able to represent data with concrete objects, and sort and classify objects based on the objects' attributes. Second graders are required to pose questions and collect data using their surroundings, while an expectation for fifth graders is to design investigations based on their proposed questions and determine how the data-collection methods may affect the data set. By the end of fifth grade, students must recognize differences in categorical and numerical data. Another expectation by the end of fifth grade is to be able to collect data using various methods (e.g., observations, surveys, etc.) and represent that data visually with such methods as tables or graphs. Eighth graders, on the other hand, are required to identify and use the appropriate graphical displays of data, such as histograms and scatter plots. Another expectation by the end of eighth grade is to be able to design studies involving two different populations or at least two different characteristics within the same population. By the end of twelfth grade, students should be able to (a) understand the differences between various studies and which inferences can be drawn from each study,

(b) identify the characteristics needed for a well-designed study, and (c) understand the difference between statistics and parameters.

Another skill area under the data analysis and probability domain is selecting and using the appropriate statistical methods to analyze data (NCTM, 2010). The expectation by the end of second grade is for students to be able to describe the data as elements and a whole. Fifth graders are required to identify the important features of data and make comparisons between related sets of data. Another expectation by the end of fifth grade is being able to compare different representations of data and determine the important aspects of the data based on each representation. Fifth graders should understand and be able to use measures of central tendency (e.g., median), while eighth graders are expected to understand measures of central tendency and spread (e.g., interquartile range). Another expectation by the end of eighth-grade is to understand and discuss the relationship between data sets and their graphical representations (e.g., histograms, box plots, etc.). Twelfth-graders should be able to calculate, display, and describe summary statistics for univariate measurement data, as well as display and discuss bivariate measurement data. By the end of twelfth-grade, students must understand how the shape, center, and spread of univariate data are affected by linear transformations.

Using data to develop and evaluate inferences and predictions is the third skill area under the data analysis and probability domain (NCTM, 2010). By the end of second-grade, students are required to discuss events as likely or unlikely based on their experiences. Fifth grade students should master the ability to use data to propose and justify conclusion and predictions. Eighth graders must make conjectures about the population based on the difference between two or more samples. Further, eighth grade students should use these conjectures to formulate new questions and design a new study to answer them. By the end of twelfth grade, students are required to examine the variability of sample statistics in order to construct sampling distributions. Another expectation for twelfth graders is to understand how companies use statistical techniques in the workplace to monitor process characteristics. Lastly, by the end of twelfth grade, students' posses the skills needed to analyze published reports.

The final skill area under the data analysis and probability domain is a student's ability to utilize basic concepts of probability (NCTM, 2010). By the end of fifth-grade, students should posses the skills needed to describe the likelihood of events using words (e.g., equally likely and impossible) and numbers. Fifth graders are also required to predict the probability of an outcome for a simple experiment, while, by the end of eighth-grade, these students should be able to compute the probabilities of simple compound events. Another expectation for eighth-graders is to use the appropriate terminology when describing complementary and mutually exclusive events. Before entering ninth grade, students are required to use their knowledge of probability to make and test conjectures found in an experiment. By the end of twelfth grade, students should understand the concepts of conditional probability, sample space, and probability distribution. Twelfth graders are required to construct empirical probability distributions using simulations, compute the expected value of random variables, and compute the probability of a compound event.

In summary, NCTM (2010) has outlined standards and expectations for students prekindergarten through grade 12, which teachers and other school professionals use to develop instructional and intervention programs. Although understanding normal development is important in understanding the processes underlying mathematics performance, it is also important to understand the areas and processes that prove problematic for students with math disabilities. Furthermore, the literature examining the development of math has focused on individuals with brain injuries and/or students identified with a math disability. As such, the following sections will examine (a) the definition of math disabilities, (b) the prevalence rate, and (c) the theoretical models of math disabilities, in order examine the cognitive processes shown or hypothesized to contribute to math performance.

Math Disabilities

Great variability exists in the literature regarding terms used to describe disordered math skills. The terms math disability, math specific learning disability, and dyscalculia imply a biologically based disorder manifested in specific cognitive deficits (Mazzocco, 2007). For the purposes of this paper, the term math disability (MD) has been adopted. The Individuals with Disabilities Education Improvement Act of 2004 (IDEA, 2004; PL 108-446), defines specific learning disability as a "disorder in one or more of the basic psychological processes involved in understanding or in using language spoken or written, that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations, including conditions such as a perceptual disabilities, brain injury, minimal brain dysfunction, dyslexia, and developmental aphasia" (IDEA, 2004, 300.8[c][10]). In order for a child to qualify as having a math related specific learning disability, the child has to demonstrate deficiencies in mathematics calculation and/or problem solving. According to IDEA, a deficiency can be identified using one of three procedures: (a) a severe discrepancy between intellectual ability and mathematic achievement; (b) a child's lack of response to a scientific, research-based intervention; or (c) any other research-based procedure used to determine a specific learning disability.

The Diagnostic and Statistical Manual of Mental Disorders-IV-TR (*DSM-IV-TR*) defines individuals as having a mathematics disorder if their "mathematical ability, as measured by individually administered standardized tests, is substantially below that expected given the person's chronological age, measured intelligence, and age-appropriate education….[that] significantly interferes with academic achievement or activities of daily living that require mathematical ability" (American Psychiatric Association, 2000, p. 54). Furthermore, the manual indicates that the following skills may be impaired in individuals diagnosed with MD: linguistic skills such as naming mathematical terms; perceptual skills, for example recognizing number symbols; attention skills; and mathematical skills such as counting objects. Common characteristics between *DSM-IV-TR* and IDEA definitions of learning disability is that there is a) no evidence of emotional, cultural, primary sensory, motor, and other environmental causes; and b) no evidence of a global intellectual disability.

Prevalence rates. Due to varying definitions of MD, prevalence rates are inconsistent across studies (Shalev, 2007). According to the *DSM-IV-TR*, the prevalence rate for MD is estimated to be 1%. On the other hand, scientists have conservatively approximated the lifetime prevalence of MD to be 3% to 8% (Krasa & Shunkwiler, 2009); however, the estimated prevalence rate for school-aged children ranges from 5% to 8% (Geary, 2007; Proctor, Floyd, & Shaver, 2005). Two studies in particular

(Barbaresi, Katusic, Colligan, Weaver, & Jacobson, 2005; Mazzocco & Myers, 2003) illustrate how definitions influence the estimates of MD prevalence rates.

Mazzocco and Myers (2003) conducted a longitudinal study that investigated the prevalence rates of MD in kindergarten through third-grade students. Students were identified as having a MD based on one of two ways: (a) scores on individually administered, norm-referenced mathematical assessments, or (b) two or more years of poor math class performance. Of the 209 children that completed this four-year study, 45% of these students obtained a standard score of less than 86 on a norm-referenced mathematical assessment. The range of percentages of students over the four years was as low as 1% in first and second grades and as high as 45% in kindergarten. Overall, over half of this sample (53%) met at least one of the two restrictive criteria for math learning disability over the primary school age years. When using Test of Early Mathematics Ability – Second Edition (TEMA-2; Ginsburg & Baroody, 1990) scores, 63% of the sample identified as having a MD during kindergarten still exhibited math problems in first, second, and/or third grades, yielding a prevalence rate of 9.6%.

Unlike Mazzocco and Myers (2003), who only looked at kindergarten through third grades, Barbaresi and colleagues (2005) conducted an epidemiological study with a birth cohort born between 1976 and 1982. Barbaresi and colleagues used three different definitions to examine the cumulative incidence of MD for individuals by age 19:

(a) Minnesota regression: achievement score < full-scale IQ * 0.62 + 17.40

(b) Discrepancy formula: an achievement score that was a certain number of standard scores below the full-scale IQ

(c) Low achievement formula: an achievement standard score < 90 and full-scale IQ score > 80.

The cumulative percentages for the above criteria were as follows: (a) Minnesota regression formula = 5.9%, (b) the discrepancy formula = 9.8%, and (c) low achievement formula = 13.8%. Due to the variation in criteria, the cumulative incidence for 7-year-old children ranged from 1.3% to 2.1%. This range significantly increased for 13-year-old children (5.3% to 11%) and then slightly increased for 19-year-old children (5.9% to 13.8%; Barbesi et al., 2005). These findings suggest that most children are identified as having MD by early adolescence (Shalev, 2007). Overall, 791 students were identified by one or more of the formulas as having MD. Of the 791 identified students, only 257 (32%) met the criteria for all three formulas.

Even though the prevalence rate for MD remains open for discussion, studies investigating the prevalence of MD are important because students with MD are less likely to meet the NCTM standards previously discussed. Since these students are less likely to meet these standards, it is important to understand current theoretical models of math disability and the possible cognitive process/skills that are contributing to lower math performance.

Theoretical Subtypes of Mathematical Disabilities

There is a consensus among researchers that there are at least three subtypes of MD, but the terms used to identify the subtypes vary (Hale & Fiorello, 2004). For instance, Geary (1993) identified the Semantic subtype to be associated with poor number-symbol associations, while Feifer and De Fina (2005) use the term semantic to describe an inability to decipher magnitude representations among numbers. Even though the terms used to describe the three subtypes vary, there is a consensus about the definition and cognitive processes involved with each subtype. The following section examines the three subtypes in terms of observed deficits, cognitive areas of weakness, and researched based support for the brain structures associated with each subtype. Although a discussion about the observed deficits (i.e., counting errors) and brain structures are discussed in the next section, the focus of the current study is the cognitive areas that can be assessed using psychoeducational instruments.

Geary's model of MD. Given previous research in neuropsychological and cognitive correlates of math performance, Geary (1993) proposed three subtypes of math disabilities. After reviewing cognitive studies that examined addition skills of MD children, Geary suggested that the literature supported two distinct functional deficits, procedural and memory retrieval. The early procedural skills for those with MD included low accuracy rates, frequent errors, and immature problem solving strategies (i.e., such as counting fingers using the sum procedure) when compared to typical students (Garnett & Fleischner, 1983; Geary, 1990). Geary found that the performance of the students with MD approached those of their normal peers by the end of second grade. Given this pattern of skill development, Geary suggested that children with MD followed a developmental-delay model. Specifically, children with MD demonstrated a pattern of performance that was similar to that of younger, academically normal children.

The second deficit that emerged out of the cognitive studies was memory-retrieval deficits, which he stated appeared to persist throughout the elementary school years. Geary suggested that this deficit also followed a developmental-difference model, as the pattern of performance for children with MD was qualitatively distinct from peers who were the same age, as well as younger, academically normal children. Children with MD have higher error rates and retrieval speeds that are highly unsystematic when recalling arithmetic facts from long-term memory (Garnett & Fleischner, 1983; Geary & Brown, 1991). Given the divergent developmental trajectories of the procedural and memory-retrieval skills of MD children, Geary determined that those skills were functionally distinct.

After examining the neuropsychological studies, Geary suggested three relatively distinct types of basic lower order mathematical deficit: fact retrieval, procedural, and spatial representation. Individuals with fact-retrieval deficits often have coexisting language and reading disabilities. Geary hypothesized that these students experience difficulties in the retrieval of arithmetic facts from semantic memory, which he called Semantic subtype. Geary's investigation into the neuropsychological literature found that individuals sometimes demonstrate difficulties executing arithmetic procedures, which Geary identified as the Procedural subtype. In the neuropsychological literature, children with MD were also found to have visuospatial deficits (Badien, 1983); however, visuospatial deficits were not found in the cognitive research. Geary attributed the differences in research due to a lack of investigation of visuospatial skills and math performance in the cognitive literature. Geary theorized that visuospatial deficits affect an individual's ability to represent numerical information spatially, which is known as the Visuospatial subtype.

Semantic subtype. Individuals with semantic MD are not able to retrieve math facts efficiently from long-term memory, and even with extensive drilling, these individuals are still likely to have trouble memorizing arithmetic tables (Geary, 1990, 1993; Howell, Sidorenko, & Jurica, 1987). In addition to having difficulty retrieving math facts, the amount of time spent trying to retrieve the correct response tends to vary more when compared to individuals who do not have Semantic MD. Students with Semantic MD have difficulty semantically representing numbers and cannot link symbolic with nonsymbolic representations of numbers (Feifer, 2008). This results in inconsistent retrieval of arithmetic facts from long-term memory (Geary, 1993). For instance, they will exhibit difficulties counting, rapidly identifying numbers, and retrieving stored facts, specifically addition and multiplication facts (Feifer, 2008). Furthermore, when they are able to retrieve facts, there is usually a high error rate (Geary, 1993). Despite these deficits, numeric qualities, comparisons between numbers, understanding basic concepts, and visual spatial skills may be preserved (Feifer and De Fina, 2005). The two most notable characteristics are variable solution times for correct retrieval and high error rates (Geary, 1993).

According to most researchers (e.g., Geary, 1993), the regions of the brain associated with this subtype are hypothesized to be the posterior regions of the left hemisphere and subcortical structures (e.g., thalamus). Dehaene and Cohen (1997) identified the left-hemispheric perisylvian areas as the structure of the brain used for identifying or naming digits. This region of the brain is also responsible for processing linguistic information (Dehaene & Cohen, 1997); therefore, researchers have found that semantic MD and reading disabilities often covary, especially when there are phonetic deficits. Although the genetic feature of this subtype is unclear, Geary (1993) hypothesized that it may be heritable due to its relationship with certain forms of reading disabilities.

Procedural subtype. Unlike the Semantic subtype, which hinders a student's ability to retrieve over-learned math facts from memory, the Procedural subtype is related to deficiencies in the processing and encoding of numeric information (Geary, 1993; Hale & Fiorello, 2004; Feifer, 2008). Students with the Procedural subtype will have difficulty writing numbers from dictation, reading numbers aloud, using math computational procedures, performing division problems, regrouping in subtraction, applying syntactical rules to a numeric symbol system, and inhibiting irrelevant information (Feifer, 2008; Geary, 1993; Gersten, Clarke, & Mazzocco, 2007). Individuals with Procedural MD have higher rates of counting errors (Gersten et al., 2007) and make frequent errors when executing mathematical procedures (e.g., borrowing; Geary, 1993). An overreliance on developmentally immature procedures when solving problems was also noted (Geary, 1993; Gersten et al., 2007; Hale & Fiorello, 2004). For instance, students with MD were more likely to count on their fingers, while typical students used either semantic counting or retrieval strategies (Geary, Hamson, & Hoard, 2000). Even though difficulties applying syntax rules to a numeric symbol system are observed, individuals with MD may be able to apply syntax rules to the alphabetic symbol system used for reading (Feifer, 2008). Syntax rules in math include putting numbers in their correct position so that a one in the tens position indicates that there are ten objects, while a one in the ones position indicates that there is only one object. Individuals with Procedural MD will be able to retrieve over-learned facts and make comparisons between numbers and magnitudes (Feifer & De Fina, 2005). A portion of individuals with the Procedural subtype may not have an understanding about the concepts underlying procedural use

(Geary, 1993), so they not only have trouble executing the steps, they do not even know the correct order of the steps.

In terms of the regions of the brain, Geary (1993) highlighted data that suggests the left-hemisphere; however, involvement of the subcortical processes has yet to be determined. More recently, Dehaene and Cohen (1997) identified both the left and right inferior occipital-temporal regions as the areas of the brain associated with the procedural coding of numbers. Therefore, students with procedural MD will not exhibit difficulties in math until they encounter problems that require the execution of arithmetical procedures (Feifer, 2008). A student may have difficulty with long division since they cannot recall the sequence of steps necessary to complete the problem. Research by Passolunghi and colleagues (e.g., Passolunghi & Siegel, 2004) has shown that students with mathematics difficulties have a general working memory deficit. For instance, Passolunghi and Cornoldi (2008) compared third and fifth graders with mathematical difficulties to same-aged controls on a battery of working memory and speed of articulation tasks. They found that the children with math difficulties scored significantly lower on tasks requiring manipulation of information when compared to controls. However, on tasks where the students had to simply recall information in its original form (e.g., Digits Forward), there was no difference between the two groups.

Visuospatial subtype. According to Geary (1993), the final subtype involves visuospatial skill deficits that impair an individual's ability to represent and interpret (e.g., place value errors) numerical information. Specifically, individuals with Visuospatial MD confuse signs and omit numbers (Feifer & De Fina, 2005). Difficulties aligning a column of numbers, comparing magnitudes, perceiving numbers visually, and various spatial attributes, such as mental rotation, size, location, and orientation were also noted. Mental rotation is a complex cognitive skill that is supported by various neuropsychological functions, such as spatial reasoning and higher level problem solving (Feifer & De Fina, 2005). Some deficits from the procedural subtype could be a result of visuospatial deficits, such as borrowing or carrying. Skills that are likely to be preserved include reading numbers, retrieving stored facts, using semantic strategies, and completing math algorithms (Feifer & De Fina, 2005). In terms of rote memorization, visuospatial skills are not as important, especially when compared to problems that involve quantitative analysis and reasoning skills (Feifer & De Fina, 2005).

The Visuospatial subtype is linked to the posterior regions of the right hemisphere. According to Geary (1993), the Visuospatial subtype is not related to phonetic deficits or any other RD. In general, visuospatial skills are predictive of MD; however, the degree to which visuospatial skills influence MD varies for different groups of children. For instance, Gersten et al. (2007) hypothesized that visuospatial impairments may hinder mathematical achievement more in Grades 4 through 9 since the curriculum emphasizes procedures in geometry and intricate concepts involving numbers.

Other models of MD. Geary (1993) compared children's performance on mathematical tasks with associated neuropsychological profiles and proposed a model for MD, which includes the three subtypes (semantic, procedural, and visuospatial), that still dominates today. Even though some researchers have identified more than three subtypes, their models have evolved from Geary's model. For instance, Feifer and De Fina's (2005) model has four subtypes. The procedural and visuospatial subtypes are the same as Geary's model of MD. Like Geary's semantic subtype, Feifer and De Fina's

(2005) verbal subtype represents poor number-symbol associations. Lastly, Feifer and De Fina changed the meaning of the semantic subtype (see above) to represent an inability to decipher magnitudes (e.g., 5 is smaller than 10). This paper will use Geary's (1993) subtypes (semantic, procedural, and visuospatial) because most researchers and practitioners use this model or a variation of this model.

Research examining components of Geary's theoretical subtypes. Geary's (1993) model has three subtypes of MD model, which include the Procedural, Visuospatial, and Semantic subtypes. Research has investigated how cognitive processes contribute to mathematics performance. A summary of research that examines the cognitive processes hypothesized to underlay Geary's subtypes will be discussed. The difference in research studies will also be explored, as some findings contradict findings in other studies.

Semantic subtype.

Calculation. Research (Dehaene & Cohen, 1997; Fuchs et al., 2005) has found the students with MD experience difficulty with reading skills, including phonological processing. For instance, Fuchs and colleagues (2005) investigated 564 first graders to determine the contribution of seven cognitive abilities on computation and fluency. Results showed that the phonological processing domain accounted for unique variance in addition fact fluency (2.5%) even after pretreatment reading skills were included. When pretreatment reading skills were included in the equation for calculation, the contribution of phonological processing was no longer significant. Phonological processing remained a significant predictor for Addition Fact Fluency, even after controlling for pretreatment reading skills. Results showed that the language domain did not have a meaningful relationship with CBM Computation and Addition Fact Fluency. In a follow-up study, Fuchs and colleagues (2006) found that processing speed, phonological decoding, and attention accounted for 33% of variance in arithmetic performance. When phonological decoding, sight word efficiency, and language were removed from the models, it significantly decreased the overall fit.

Math reasoning. Similar to math calculation, deficits in skills related to reading and language skills were found in math reasoning. Fuchs and colleagues (2005) examined math reasoning in first graders by using concepts/applications and story problems. They found that although phonological processing accounted for unique variance in concepts/applications, this unique contribution was no longer significant once pretreatment reading was entered into the equation. Fuchs and colleagues (2008) conducted a follow-up study that explored patterns of difficulty in domains of math problem solving in a group of third graders. The authors included the domains of language, semantic retrieval, concept formation, matrix reasoning, verbal working memory, numerical working memory, word identification, attention, and processing speed. When comparing students with problem-solving difficulty to students with no math difficulties, results showed that problem-solving difficulty was associated with language deficits. In another study by Fuchs and colleagues (2006), when language, phonological decoding, and sight word efficiency were removed from model, the overall fit of all of the models decreased significantly.

Other studies (e.g., Swanson, 2006) have also found reading deficits in students with MD. For instance, Swanson and Beebe-Frankenberger (2004) investigation found that reading significantly contributed to word problem solving performance.

Additionally, research by Swanson (2006) accounted for 46% of the variance in word problem solving when the following predictors were included in the model: math calculation, fluency, speed, inhibition, component knowledge, reading, phonological knowledge, vocabulary, age, phonological loop, sketchpad, and executive system. Reading and executive system were the only factors that contributed significant variance to the model.

Visuospatial subtype.

Calculation. Several researchers (e.g., Geary, 2011; Swanson, 2006) have found that the visual-spatial sketchpad, visual perceptual, and visual processing contribute significantly to math calculation performance. For instance, Swanson (2006) investigated math calculation skills in a group of second grade students. The predictors used in the regression formula were fluency, speed, inhibition, problem-solving component knowledge, reading, phonological knowledge, vocabulary, age, phonological loop, visual-spatial sketchpad, and executive system. When all factors were entered into the model, the predictors accounted for 74% of the variance in math calculation performance. The visual-spatial sketchpad significantly contributed to the variance, as did inhibition, reading, vocabulary, and age. Similar to Swanson's findings, Geary (2011) found that the visual-spatial sketchpad was uniquely predictive of math calculation performance.

Geary and colleagues (2009) also found evidence that supports the contribution of the visual-spatial sketchpad. Specifically, results showed that decreased visual-spatial sketchpad performance increased the likelihood of identification of math disability identification. A one standard deviation decrease in visual-spatial sketchpad scores, as measured by Block Recall subtest of the Working Memory Test Battery for Children

(WMTB-C; Pickering & Gathercole, 2001), resulted in a 2.78-fold increase in the odds of math disability identification.

In addition to the visual-spatial sketchpad, research supports the contribution of visual perception and visual processing to math calculation performance. Sortor and Kulp (2003) investigated the relationship between visual motor integration and mathematics in a group of 155 second through fourth grade students, ranging in age from 7 to 10. The testing battery consisted of the Beery-Buktenica Developmental Test of Visual-Motor Integration (VMI; Beery, 1997), the Visual Perception and Motor Coordination subtests of the VMI, the Stanford Achievement Test Series, Ninth Edition (Harcourt Brace Educational Measurement, 1996), and the Otis-Lennon School Ability Test, Seventh Edition (Otis & Lennon, 1995). Children in the upper quartile of mathematics achievement scored significantly higher on all measures of visual motor integration when compared to children in the lower quartile. Furthermore, all of the visual motor integration subtests significantly correlated with mathematics achievement, even after controlling for age and semantic cognitive abilities. Multiple linear regression analyses revealed a significant relationship between the Visual Perception subtest and mathematics achievement; however, the other visual motor integration tests were not significant. The Visual Perception subtest accounted for the most amount of variance (12%), followed by the VMI Test (8%), and Motor Coordination subtest (5%).

Hale, Fiorello, Kavanagh, Hoeppner, and Gaither (2001) investigated the mathematical computational skills of 174 children, ranging in age from 6 to 16. They used archival data covering a 6-year period to examine WISC-III (Wechsler, 1991) predictors of mathematical computation skills. Hale and colleagues used several assessments (e.g., WRAT-3; Wilkinson, 1993) to represent math computation. The theory underlying the WISC-III is the fluid-crystallized (Gf-Gc) model. The WISC-III measures six of the Gf-Gc factors: Processing Speed, Crystallized-Visual Abilities, Quantitative Knowledge, Short-term Memory, Visual Processing, and Crystallized Intelligence. Statistics showed that Quantitative Knowledge contributed the greatest amount of unique variance (10.7%) to math computation. Crystallized Intelligence, Short-Term Memory, and Visual Processing also contributed a significant amounts of variance in predicting math computation (4.1%, .2%, and .8%, respectively), while Processing Speed and Crystallized-Visual Abilities did not contribute significant amounts of variance.

Contrary to the previous research studies, there are studies (e.g., Andersson, 2010; Floyd, Evans, & McGrew, 2003) that do not support the contribution of visual skills to math calculation performance. Andersson (2010) found that visual matrix span did not significantly contribute to the regression equation. Similarly, in a study by Floyd, Evans, and McGrew (2003), visual-spatial thinking did not demonstrate significant relations with math calculation.

Math reasoning. There is a paucity of research supporting the visuospatial subtype for math reasoning. Andersson's (2010) investigation of one-step and multi-step word problem solving showed that visual matrix span accounted for a significant amount of variance in problem solving performance. Research by Floyd et al. (2003), on the other hand, failed to show significant relations between visual-spatial thinking and math reasoning. Research by Hale and colleagues (2008) suggest that students with MD attempt to solve problems visually; however, these students have difficulty translating the verbal content to written algorithms. Hale et al. found that factors examining spatial, working memory, verbal, and nonverbal reasoning skills contributed to math reasoning performance.

Procedural subtype.

Calculation. The contribution of the various aspects of the Procedural domain (e.g., attention, processing, etc) to calculation performance varied depending on what areas were investigated, how those areas were measured, and how math calculation was measured (e.g., standard achievement scores versus benchmark assessments).

Even though several research studies support the Procedural subtype, these studies failed to find a significant contribution for working memory and/or processing speed. Fuchs and colleagues (2005), discussed above, used the Calculation subtest of the WJ-III and CBM Computation to investigate this subtype. The seven predictors used were attention (teacher rated), language, nonsemantic problem solving, phonological processing, processing speed, executive function, and working memory. Overall, attention was the most robust predictor for both math outcome variables, which ranged from 1.4% for computation to 3.7% for calculation. Working memory was significant for computation but not calculation. Finally, processing speed was not a significant contributor for either math criterion. Swanson (2006) also found that processing speed, phonological loop, and executive system did not significantly contribute to math calculation performance, but results showed that inhibition contributed significant variance.

Fuchs and colleagues (2006) also found results that failed to support both working memory and speed. They studied arithmetic, algorithmic computation, and arithmetic word problems in a sample of 312 third graders using the following as predictors: language, nonverbal problem solving, concept formation, processing speed, long-term memory, working memory, phonological decoding, sight word efficiency, and attention (teacher reported). For arithmetic, the model consisting of working memory, long-term memory, processing speed, phonological decoding, and attention accounted for 33% of the variance. The significant predictors were processing speed, phonological decoding, and attention. When examining algorithmic computation, the model accounted for 47% of the variance, with arithmetic and attention as statistically significant paths. Working memory, long-term memory, and processing speed were not significant predictors.

Other research studies (e.g., Geary et al., 2007; Geary, Hoard, & Hamson, 1999) have shown that working memory significantly contribute to calculation performance. Swanson and Beebe-Frankenberger (2004) investigated the contribution of fluid intelligence, algorithm knowledge, reading, semantic processing, inhibition, speed, phonological processing, age, short-term memory, and working memory to math calculation in first through third grade students. When Swanson and Beebe-Frankenberger entered all of variables into the regression equation, the total variance accounted for was 73%, with working memory, age, reading, and algorithm knowledge contributing significant variance. Further, results suggest that the influence of working memory on math performance across grades was stable. These results are consistent with Swanson (2006) given the lack of processing speed contribution; however, the results are contrary, as Swanson showed that inhibition contributed significant variance, but working memory did not.

Hale and colleagues (2008) also found inconsistent contribution of working memory and processing speed to math calculation. Hale et al. (2008) investigated neuropsychological processing differences between typical children and children with MD using the Differential Ability Scales – Second Edition (DAS-II; Elliott, 2007) standardization sample. Results showed that children with MD scored significantly lower on the working memory and processing speed factors (Hale, et al., 2008). Even though the students with MD scored lower on these factors, the contribution of each factor differed for each group (control vs. MD). When the DAS-II factors and subtests were used as predictors for math performance, all of the factors and subtests significantly contributed to math calculation and reasoning performance for the typical students. The factors that significantly contributed to the performance on the math calculation subtest for the students with MD were the Verbal, Spatial, Processing Speed, and Visual-Verbal Memory factors. The results failed to find a significant contribution for the Nonverbal Reasoning factor. Hale and colleagues indicated that these results suggest that students with MD have difficulty carrying out sequential computational steps, especially in a quick and efficient manner.

Similar to the previous study (Hale et al., 2008), additional studies (Floyd, Evans, & McGrew, 2003; Geary et al., 2007; Geary et al., 1999) have determined that working memory and speed of processing contribute to group differences. Overall, students who are at risk for a math disability tend to perform significantly lower on tasks of working memory and processing when compared to those not at risk. Geary (2011) found that processing speed and working memory contributed to math performance above and beyond the contributions of intelligence. Further, results from Geary’s study showed that the central executive system of working memory was an important predictor for math calculation performance. Geary used three recall tasks (i.e., digit, counting, and listening) to represent the central executive system.

Floyd et al. (2003) also found evidence supporting processing and working memory deficits to poor math calculation performance. Floyd et al. (2003) used a nationally representative sample of children ages 6 to 19 from the WJ-III standardization sample to determine which measures of select Cattell-Horn-Carroll broad and narrow cognitive abilities significantly contributed to math calculation and math reasoning. For math calculation, Comprehension-Knowledge demonstrated moderate relations after the early school-age years. Fluid Reasoning, Short-term Memory, and Working Memory demonstrated moderate relations across ages, while Processing Speed demonstrated moderate to strong relations across ages.

Additional research studies have examined other aspects of the executive system, including shifting. Andersson (2010) investigated 274 third and fourth graders across three points in time, stretching into sixth grade. The sample consisted of four groups, children receiving special instruction in mathematics (MD-only), reading (RD-only), or both mathematics and reading (MD-RD), as well as a control group. Hierarchical regression analyses for arithmetic calculation showed that 45% of the variance was accounted for when the researchers added IQ, the difference between MD-RD and control, and difference between MD-only and control into the equation. There was an 8% increase (53% total) when number matching, trail making, visual matrix span, and digit span were entered into the equation. All variables contributed significant variance with the exception of visual matrix span and digit span. Taken together, the research supports the existence of the procedural domain; however, the exact components of this domain for math calculation remain open for debate.

Math reasoning. Similar to the previous discussion, the contribution of the various aspects of the Procedural domain (e.g., attention, processing, etc) varied for math reasoning. Fuchs and colleagues (2005; 2006; 2008) investigated math reasoning using various cognitive and executive functioning domains. In an older study, Fuchs and colleagues (2005) examined math reasoning by using concepts/applications and story problems. Attention (2.3%) and working memory (1.4%) contributed unique variance to story problems performance. Additionally, attention (2.8%) and working memory (1.4%) also accounted for unique variance for concepts/applications. In a more recent study, Fuchs and colleagues (2006) found that a model predicting word problem performance accounted for 52% of the variance, with arithmetic, language, nonverbal problem solving, concept formation, attention, and sight word efficiency showing statistically significant paths. The following variables did not significantly contribute to the model: working memory, long-term memory, and arithmetic computation. In a follow-up study, Fuchs and colleagues (2008) the examined differences in students who had computation difficulty only, problem-solving difficulty only, or computation and problem-solving difficulty. Results showed that attention and processing speed distinguished the specific computational difficulty group from the specific problem-solving difficulty group, with attention more attributable to the problem-solving difficulty group and processing speed to the computational difficulty group.

Swanson and Jerman (2006) found evidence to support the contribution of working memory and attention. Swanson and Jerman (2006) conducted a meta-analysis of the literature to investigate the differences of cognitive functioning of children who have math disabilities (MD) to children who are average achieving, children who have reading disabilities, and children who have co-morbid disabilities. Swanson and Jerman's (2006) search of the literature included publications between 1970 and June 2003, which yielded more than 300 articles. The researchers narrowed the pool of literature to 28 studies using various criterion measures (e.g., studies from peer-reviewed journals that used norm-referenced standardized measure of intelligence). They collapsed the cognitive tasks into 10 broader domains (literacy-reading, problem solving-semantic, naming speed, problem solving-visual spatial, long-term memory, short-term memory-words, short-term memory-numbers, working memory-semantic, working memory-visual-spatial, and attention). Results showed that the RD group performed better than the MD group on measures of working memory-semantic and visual-spatial working memory. When compared to the MD+RD group, the MD group performed better on measures of literacy, visual-spatial problem solving, semantic working memory, and short-term memory for words. However, the MD+RD group performed better than the MD group on measures of attention and short-term memory for numbers. The average group performed significantly better on all 10 measures when compared to the MD group.

The following two studies (Hale and colleagues, 2008; Swanson and Beebe-Frankenberger, 2004) found a significant contribution for working memory, but failed to find a significant contribution for processing speed. Swanson and Beebe-Frankenberger (2004) found that 7 of the 11 variables contributed significant variance to word problem solving performance. When fluid intelligence, algorithm knowledge, math calculation, reading, semantic processing, inhibition, speed, phonological processing, age, short-term memory, and working memory were entered into the regression equation, it accounted for 69% of the variance, with working memory, short-term memory, speed, reading, algorithm knowledge, and fluid intelligence significantly contributing to the variance. The model consisting of only working memory contributed approximately 30% variance to problem-solving accuracy. The significant relationship between working memory and problems solving continued to be significant even after controlling for the influence of phonological processing, inhibition, speed, math calculation, and reading skill. Results from Hale and colleagues (2008) showed that the following factors, Verbal, Nonverbal Reasoning, Spatial, and Working Memory, significantly contributed to math reasoning performance for the students with MD. Processing Speed and Visual-Verbal Memory did not contribute significantly to the model.

In Floyd, Evans, and McGrew's (2003) study, previously discussed, Comprehension-Knowledge showed moderate relations until age 10 for math reasoning, which increased to strong relations for the remainder of the age groups (up to age 19). Fluid Reasoning, Short-term Memory, and Working Memory generally demonstrated moderate gains across all ages. Processing Speed exhibited moderate relations up until age 14. Swanson's (2006) study, on the other hand, did not support the contribution of processing speed, but results showed that the executive system contributed significant variance to word problem solving performance in a group of second grades students. The predictors used in the regression formula were math calculation, fluency, speed, inhibition, problem-solving component knowledge, reading, phonological knowledge, vocabulary, age, phonological loop, visual-spatial sketchpad, and executive system.

When entered at the same time, the factors accounted for 46% of the variance, with reading and the executive system significantly contributing to the variance. The executive system consisted of digit and listening span tasks.

Even though the previous studies do not directly measure all of Geary's subtypes, they provide some evidence as to the existence of the various subtypes. The next section details research studies that have investigated Geary's three subtypes.

Research specific to Geary's theory of MD. After examining the extant literature, three articles were found that directly examined Geary's model. In addition to prevalence rates previously discussed, Mazzocco and Myers (2003) examined the possible subtypes of MD by investigating children's reading ability and performance on visual perceptual tasks as they related to mathematical performance. Reading ability was obtained from a measure of rapid automatized naming (RAN; Denckla & Rudel, 1976), while visual perceptual skills were assessed with the Position in Space subtest of the Developmental Test of Visual Perception – second edition (DTVP-2; Hammil, Pearson, & Voress, 1993). Participants were classified into one of three groups: low math performers, borderline math performers, or average or above average performers. Significantly more children in the low math performers group had a reading disability and visual perceptual difficulties when compared to the other two groups. Mazzocco and Myers' (2003) findings support the notion of semantic and visual-spatial deficits in mathematical difficulties. Participants with a reading disability had a greater frequency (ranging from 33% to 63%) of math disability than those without a reading disability (ranging from 5% to 10%) when examining kindergarten through third grades.

In an older study by Cirino, Morris, and Morris (2002), semantic and procedural skills accounted for approximately 17% of the variance in college students' calculation skills, whereas visuospatial skills did not contribute any unique variance. Cirino, Morris, and Morris (2007) reanalyzed the earlier data with the model from the more recent study and found results similar to the earlier study. All three domains significantly predicted Calculation scores; however, the Procedural domain contributed unique variance while the Visuospatial domain did not. Further analyses found that six variables correlated differently between the two studies on the Calculations subtest. Five subtests, which included Visual Discrimination, Closure, Comprehension, Information, and Matrix Reasoning, correlated more strongly, while the Trailmaking Test had a significantly lower correlation in the more recent study when compared to the earlier study. The math calculation scores in the earlier study were not normally distributed.

Cirino, Morris, and Morris conducted a follow up study in 2007 and investigated the mathematical reasoning of 337 referred college students by looking at their procedural, semantic, and visuospatial abilities. Mathematical abilities were measured using the Calculations and Applied Problems subtests of the Woodcock Johnson-Revised (WJ-R; Woodcock & Johnson, 1990). The procedural skills investigated were attention, sequencing, and working memory, which were measured using the following measures: the Trailmaking Test Part B (Reitan & Wolfson, 1985); the Visual Search and Attention Test (VSAT; Trenerry, Crosson, DeBoe, & Leber, 1990); the Semantic Fluency Test (Spreen & Benton, 1969); and the Picture Arrangement, Digit Span, and Digit Symbol subtests of the WAIS-III (Wechler, 1997). Cirino, Morris, and Morris did not include tests that measure the participants' ability to plan or problem solve.

Semantic skills included tasks that examined the retrieval of previously learned information. Cirino and colleagues used the following five measures: the Boston Naming Test (BNT; Kaplan, Goodglass, & Weintraub, 1983); the Peabody Picture Vocabulary Test-III (PPVT-III; Dunn & Dunn, 1997); and the Information, Vocabulary, and Comprehension subtests of the Wechsler Adult Intelligence Scale-III (WAIS-III; Wechsler, 1997). Lastly, the participants' visual processing skills were examined using tasks measuring perceptual, spatial, motor, and reasoning skills. The following subtests were utilized to represent Geary's Visuospatial subtype: Visual Discrimination, Figure Ground, and Closure subtests of the Test of Visual Perceptual Skills-Upper Level (TVPS-UL; Gardener, 1992a); the Test of Visual Motor Skills-Upper Level (TVMS-UL; Gardener, 1992b); and the Block Design, Matrix Reasoning, and Picture Completion subtests of the WAIS-III (Wechsler, 1997).

Factor analysis showed that all of the measures loaded onto the hypothesized construct/domain with the exception of Picture Arrangement, which loaded on the Visuospatial domain instead of the Procedural domain; and Semantic Fluency, which loaded onto both the Semantic and Procedural domains. All three domains were significantly correlated with both mathematical subtests. When examining the math calculation, the three domains accounted for 30% of the variance. The Semantic (5.7%) and Visuospatial (4.3%) domains each contributed a significant amount of variance to math calculation, while the Procedural domain did not contribute unique variance. For math reasoning, the three domains predicted 50% of the variance. Similar to the math calculation, only the Semantic (6.7%) and Visuospatial (12.5%) domains contributed significant unique variance. Cirino, Morris, and Morris failed to find a significant connection between math and the Procedural domain; however, aspects of executive functioning (e.g., planning) were not examined, which may explain why this research differed from other studies that have found significant connections between these concepts.

Cirino, Morris, and Morris' two studies varied in terms of the sample (e.g., the earlier study had a higher percentage of individuals with mood and/or anxiety disorders) and assessment instruments, such as newer test versions and normative differences (e.g., the earlier study used the WAIS-R while the latter used the WAIS-III). The differences between these two studies highlight the importance of replicating studies to see if the results previously found still apply, especially following the release of updated/revised assessments. In addition to the paucity of research directly examining Geary's three subtypes, there is also limited research using the NEPSY- Second Edition (NESPY-II; Korkman, Kirk, & Kemp, 2007).

Existing NEPSY-II mathematics studies. Korkman, Kirk, and Kemp (2007) conducted a research study with a sample of 20 children diagnosed with a mathematics disorder. The researchers matched the MD group to a control sample taken from the NEPSY-II normative group. The groups were matched on age, sex, race/ethnicity, and parent education level. Out of the six domains of the NEPSY-II, the MD sample exhibited significantly poorer performance on three domains: Attention and Executive Functioning, Memory and Learning, and Visuospatial Processing. Within the Attention and Executive Functioning domain, the MD group obtained significantly lower scores on the Response Set Total Correct (d = .86), Response Set Combined scaled score (d = .99), Auditory Attention versus Response Set Contrast scaled score (d = .94), Inhibition-

Inhibition Combined scaled score (*d* = 1.14), Inhibition total errors (*d* = .99), and Inhibition Naming versus Inhibition Contrast scaled score (*d* = 1.12). Overall, the MD group performed significantly poorer than matched controls on measures of attention and cognitive flexibility.

In addition to executive deficits, the MD group also obtained significantly lower scores on tasks of memory when compared to the control group. High effect sizes were found on the Memory for Designs (ranged from MD Spatial score *d* = .96 to MD Total score 1.27), Memory for Designs Delayed (ranged from MDD Spatial score *d* = .88 to MDD Total score *d* = 1.18), and Memory for Faces Delayed subtests (*d* = .88). Performance on the Narrative Memory Free and Cued Recall Total score (*d* = .75) and Word List Interference Recall Total score (*d* = .71) showed moderate effect sizes.

Deficits with visuospatial processing were also found. The MD group scored significantly lower on the Block Construction (*d* = .88), Geometric Puzzle (*d* = 1.23), and Picture Puzzle (*d* = 1.00) subtests. Even though significant differences between the two groups were found on the executive, visual, and memory domains, the two groups obtained similar scores in the Language, Sensorimotor, and Social Perception domains. Taken together, children with mathematics disorders showed impairment with attention, executive functioning, visual memory, spatial memory, and visuospatial processing, but their language, sensorimotor functioning, and social perception functioning were relatively intact. These results, however, must be interpreted with caution, as there are many limitations with this study, including the small sample size. The sample size also varied by subtest and ranged from 15 children for the Auditory Attention versus

Response Set Contrast Scaled score to 20 children for Phonological Processing Total Score.

Summary. Research has shown that students with MD are not as proficient as their peers (Garnett & Fleischner, 1983); however, research studies have found different, and in some cases, conflicting results as to the cognitive processes involved with math performance. Research suggests that fluency of over-learned mathematical facts, such as addition, is linked to the left-hemisphere and frequently co-occurs with reading disabilities (Bull & Johnston, 1997). Other research studies have implicated Crystallized Intelligence, Short-Term Memory, Visual Processing, Working Memory, Processing Speed, Attention, and Phonological Processing. Geary proposed a theory of math disabilities, which includes the Semantic, Procedural, and Visuospatial subtypes. Three studies have examined Geary's three subtypes, which found some support for this model. However, two studies that were similar in design found conflicting results (see the discussion on Cirino, Morris, and Morris, 2002, 2007). In addition to the conflicting results regarding the processes that contribute to math performance, study limitations restrict the generalizability of the results. Some studies failed to use advanced statistical procedures (e.g., regression), while others only examined college students. Researchers did not directly measure attention; instead using teacher-rating scales as a measure of attention. Given the inconsistent results found in the extant literature, the paucity of research examining Geary's subtypes, and the paucity of research utilizing the NEPSY-II (Korkman et al., 2007) to measure math weaknesses, this study seeks to add to the literature base by examining Geary's three subtypes using the NESPY-II.

Chapter III

Participants

Data from the standardization sample of the NEPSY – Second Edition (NEPSY-II; Korkman, Kirk, & Kemp, 2007) was utilized to examine the relationship between select neuropsychological subtests and the Numerical Operations and Math Reasoning subtests of the Wechsler Individual Achievement Test – Second Edition (WIAT-II; Wechsler, 2001). A total of 81 children, ranging in age from 5 to 12 (M = 8.83, SD = 2.23), were administered the NEPSY-II and WIAT-II, with a testing interval of 1 to 49 days. For the current study, the 5- and 6-year old participants were removed from the math sample due to age restrictions on select subtests (e.g., Response Set). Therefore, the sample was reduced to 60 participants ranging in age from 7.04 to 12.72 (M = 9.80, SD = 1.71). The math sample was composed of 50% females and 50% males. The majority of the sample was Caucasian (67%), followed by Hispanic (17%), African American (12%), and Other (5%). Preliminary analysis will be conducted on 206, 7- to 17-year old children taken from the NEPSY-II standardization sample. This sample will include the 60 children previously described plus an additional randomly selected 146 participants, aged 7 to 17.

One of the statistical techniques being used in this research study is confirmatory factor analysis. According to Stevens (1992), factor analysis can be conducted on data that has components with four or more loadings above .60 regardless of the sample size. If the components have low loadings (i.e., <|.40|), a sample of 150 is suggested. It is expected that at least one domain will have low loadings, thus the present sample of 206 meets this criteria. Further, regression analysis will be performed to examine the amount of variance explained in math calculation and reasoning using the created factors. In order for a reliable regression equation to be created, Stevens (1992) recommends at least 15 participants be present for every independent variable. Tabachnick and Fidell (2007) suggest the following rules of thumb: when assuming a medium-size relationship (α = .05; β = .20) between the independent and dependent variables: "$N \geq 50 + 8m$ (where m is the number of independent variables) for testing the multiple correlation and $N \geq 104 + m$ for testing individual predictors" (p. 123). Based on these criteria, the proposed sample size of 60 will produce a reliable regression equation using Steven's criteria (15 * 3 = 45), but not Tabachnick and Fidell's multiple correlation criteria (50 + 8 * 3 = 74). Caution must also be exercised when interpreting individual predictors, as the sample size of 60 does not meet Tabachnick and Fidell's suggested sample size of 107 (104 + 3). Overall, the sample sizes explained above are sufficient for factor analysis; however, the results of the regression analyses must be interpreted with caution.

Measures

NEPSY – Second Edition. Korkman, Kirk, and Kemp developed the NEPSY in 1998 to address the need for neuropsychological assessments specifically designed for use with children. Prior to the NEPSY, neuropsychological assessment of children relied on tests originally created for adults with brain damage. Even though Korkman, Kirk, and Kemp designed the NEPSY for children, they used Lurian's theory (Luria, 1973, 1962/1980) as the major underlying theory and foundation for the measure. It is interesting to note that Lurian built his theory upon cognitive assessment techniques used in populations of adults with brain damage. Lurian's theory states that complex cognitive functions are influenced by multiple brain systems. In addition to the complex cognitive functions, subcomponents of these functions must also be assessed using carefully focused tests. Korkman first created a Finnish version of the NEPSY (Korkman, 1980, 1988), and then in 1998, Korkman, Kirk, and Kemp released the English version of the NEPSY. The English version consisted of many of the subtests corresponding to the 1988 Finish version. The second edition builds upon the original edition by improving psychometric properties and increasing content coverage. The NEPSY-II is an individually administered test that measures the neuropsychological functioning of individuals aged 3 through 16. This edition of the instrument includes 32 subtests and 4 delayed tasks, which are divided into six content domain areas: 1) Attention and Executive Functioning, 2) Language, 3) Memory and Learning, 4) Social Perception, 5) Sensorimotor, and 6) Visuospatial Processing.

Given the multidimensionality and common subprocesses of attention and executive functions, the Attention and Executive Functioning domain consists of multiple key constructs. Specifically, this domain consists of tasks hypothesized to measure inhibition, initiation, cognitive flexibility, planning, sustained attention, distractibility, divided attention, and working memory. Inhibitory control is the process of stopping oneself from engaging in enticing and/or automatic behaviors. Initiation is the ability to initiate behavior, while cognitive flexibility consists of changing one's behavior, approach to a task or situation, or problem-solving strategy. Although separate, cognitive flexibility and initiation are components of planning. Planning is defined as one's ability to create a strategy to solve a problem or accomplish a task. Sustained attention consists of focusing on a task, activity, or situation while simultaneously suppressing irrelevant stimuli. Conversely, distractibility is the inability to sustain attention to a task, activity, or situation due to the presence of irrelevant stimuli. Divided attention requires a person to hold stimuli in one's mind while simultaneously attending to other stimuli. Working memory is the ability to actively hold and manipulate stimuli in one's mind. The Attention and Executive Functioning domain consists of the following subtests: Animal Sorting, Auditory Attention, Response Set, Clocks, Inhibition, and Statue. In summary, the Attention and Executive Functioning domain of the NEPSY-II measures many of the hypothesized underlying components of executive functioning.

Another domain of the NEPSY-II is Language, which measures one's ability to effectively express and understand verbal communication. The Language domain consists of the Comprehension of Instructions, Phonological Processing, Speeded Naming, Word Generation, and Body Part Naming. The NEPSY-II also consists of a Learning and Memory domain. According to the NEPSY-II Clinical and Interpretive Manual (2007), "learning refers to the acquisition of new information," while "memory is a more technical term describing the particular forms of acquisition and retrieval" (p. 9). The Learning and Memory domain consists of the following subtests: Memory for Designs, Memory for Faces, Memory for Names, Narrative Memory, Sentence Repetition, and Word List Interference. The Sensorimotor domain of the NEPSY-II examines fine-motor coordination and speech production. Fine-motor coordination is one's capacity to control hand movements quickly, smoothly, and with adequate precision when required to write or draw. Speech production is within the Sensorimotor domain given the oromotor functioning required when speaking. The Sensorimotor domain consists of the Fingertip tapping and Visuomotor Precision subtests.

The Visuospatial Processing domain consists of tasks measuring visual perception, spatial processing, and visuoconstructional skills. Visual perception, as defined by the authors of the NEPSY-II, is one's ability to match visual patterns or identify visual gestalts in pictures. One's capacity to understand the orientation of visual information across both two- and three- dimensional space is spatial processing. Lastly, visuoconstructional skills are a combination of visual, spatial processing, and manual skills, such as copying shapes. The subtests that make up the Visuospatial Processing domain are Arrows, Block Construction, Design Copy, Geometric Puzzles, and Picture Puzzles. An additional revision to the NEPSY-II was the addition of the Social Perception domain. The Social Perception domain examines the cognitive processes hypothesized to facilitate social interactions, such as encoding, recalling, and recognizing faces. The two subtests that make up the Social Perception domain are Affect Recognition and Theory of Mind. In summary, the NEPSY-II consists of 6 domains, 32 subtests, and 4 delayed tasks across various domains of functioning.

All subtests of the NEPSY-II are measured with scaled scores (M = 10, SD = 1). The authors eliminated domain scores for the second edition, as they determined that variable factor loading of the subtests to the domains negatively influenced the domain scores, making the domain scores less valid. Even though critical cognitive functions of each domain are assessed, Korkman, Kirk, and Kemp acknowledge their inability to assess all cognitive skills within each domain using a single battery.

The authors measured the reliability of the NEPSY-II scores using internal consistency, test-retest reliability, and inter-rater reliability. Internal consistency reliability coefficients ranged from 0.50 (Memory for Faces Total Score, age 5 to 6) to

0.98 (Fingertip Tapping Sequences Combined Scaled Score, age 7 to 12). The test-retest stability coefficients ranged from 0.18 (Imitating Hand Positions, age 7 to 8) to 0.94 (Inhibition Switching Total Completion Time, age 11 to 12). Lastly, there was a high degree of reliability between raters, which ranged from 93% to 99% (Word Generation and Memory for Names, respectively).

Korkman and colleagues (2007) measured the validity of the NEPSY-II scores by using three types of validity: content, construct, and concurrent. They claim evidence of content validity is provided by test revisions based on the performance of children with no known neurological disabilities, as well as comparisons of the performance of children with known neurological conditions to normal controls. Korkman and colleagues also gathered feedback and recommendations from customers, and they reviewed data from previous pilot studies and tryout phases to address content gaps. The authors reevaluated testing items for content and bias after the standardization phase. Validity was also evidenced by the authors' attempts to modify items based on response processes, such as examining frequently occurring incorrect responses to determine the plausibility of that answer.

Construct validity was assessed with intercorrelations of subtest scores across all ages and the special group sample, which ranged from -0.09 to 0.82. The intercorrelations show high correlations between subtests of the same domain, moderate correlations across domains, and higher correlations for the special group sample when compared to the normative sample. Concurrent validity of the NEPSY-II was assessed based on its relationships to other measures (e.g., individual intelligence and achievement tests), as well as special group studies. Correlations between the NEPSY-II and WIAT-II were strong, suggesting a link between neuropsychological processes and academic achievement. The strongest correlation between Numerical Operations and the NEPSY-II was CL Total (.51), while the weakest correlation was Geometric Puzzle Total Score (.00). For Math Reasoning, the strongest correlation is Sentence Recall (.64) and the weakest correlation is Affect Recognition (-0.01). As discussed in Chapter 2, an investigation of 20 children diagnosed with math disabilities suggested that these children had impairment when compared to matched control groups in the following areas: attention, executive functioning, visuospatial processing, and visual and spatial memory.

Wechsler Individual Achievement Test – Second Edition. The Wechsler Individual Achievement Test – Second Edition (WIAT-II) is an individually administered test that measures the academic achievement of individuals aged 4 through adulthood, or who are in grades pre-kindergarten through 16. According to the test manual, the WIAT-II consists of four domains (reading, written language, oral language, and mathematics) and nine subtests. Domain and subtest scores are reported as standard scores (i.e., mean of 100 and standard deviation of 15).

The Reading domain of the WIAT-II is used to assess a student's basic reading, reading comprehension, and phonological abilities. The domain score is calculated using the three reading subtests: Word Reading, Reading Comprehension, and Pseudoword Decoding. The Written Language domain assesses a students' ability to spell regular and irregular words, as well as his/her ability to write a narrative. Performance on the Spelling and Written Expression subtests are used to calculate the Written Language domain score. The Oral Language domain, on the other hand, is a student's ability to receive oral language, as well as express him/herself orally. The Oral Language domain score consists of performance from the Listening Comprehension and Oral Expression subtests.

Lastly, the Math domain measures math performance through the Numerical Operations and Math Reasoning subtests. The Numerical Operations subtest is defined as "the ability to identify and write numbers, count using 1:1 correspondence, and solve written calculation problems and simple equations involving the basic operations of addition, subtraction, multiplication, and division" (Wechsler, 2001, p. 561). The Math Reasoning subtest presents problems using both verbal and visual prompts, which are designed to assess an individual's ability to reason mathematically. Specifically, examinees are expected to count, solve single- and multi-step problems, identify geometric shapes, interpret graphs, solve problems related to statistics and probability, and identify mathematical patterns.

Reliability of the WIAT-II was accomplished using interitem comparisons, test-retest comparisons, and interscorer agreement. Interitem comparisons were calculated using the Spearman-Brown formula. The manual reported a split-half reliability for the Numerical Operations subtest of 0.91 and 0.92 for the Math Reasoning subtest, which reflects strong interitem consistency. Test-retest stability was calculated using Fisher's z transformation. On average, test-retest reliability for Numerical Operations was 0.92, while the average reliability for Math Reasoning was 0.94.

The test manual reported that three types of validity were measured: (a) content-related, (b) construct-related, and (c) criterion-related validity. Content-related validity was measured by expert judgments and empirical item analysis. First, the development team determined the curriculum objectives of each domain using the Individuals with

Disabilities Education Act (IDEA) Amendments of 1997. To ensure that the items of each subtest corresponded to the curriculum objectives, experts in each domain (e.g., mathematics) reviewed each subtest. The subtests were also compared to other achievement and diagnostic instruments, various state standards, school textbooks, teacher surveys, national item tryouts, and emerging curricular trends. The subtests items were then examined by expert reviewers. In addition to the previously described methods, content validity was also examined by conventional and item response theory (IRT). The item-total correlations were inspected for each subtest and age, such that any correlation less than .20 was considered for revision and replacement. Further, the item-level data were fit on a 1-parameter IRT model in order to examine the item difficulty, thereby indicating the correct order of the items (i.e., lower level to higher level). Items that did not fit the model were deleted. Items responses were also compared between examinees identified with a learning disability and those who were not identified. Any items that did not accurately discriminate between these two populations were deleted. Given the aforementioned results, the authors stated that the items on the subtests are internally consistent and representative of the content areas. Further, the items are reported to be free of significant gender and ethnic bias, and each subtest was described as content-homogeneous.

Construct-related validity of the WIAT-II subtests included intercorrelations of the subtests, studies of group differences, and correlations with measures of ability and other achievement tests. The intercorrelations revealed higher correlations when the mathematics subtests were compared with each other than when they were compared with the reading subtests. For instance, the intercorrelation between the Numerical

Operations and Math Reasoning subtests ranged from .53 (age 7) to .72 (age 12), when examining the intercorrelations from ages 5 to 12. Studies of group differences have shown that raw scores increase age to age and grade to grade. Further, sizeable differences between various clinical groups (e.g., gifted) and the standardization sample also verify the construct validity of the WIAT-II. In terms of correlations with measures of ability, the correlation between the Full Scale Intelligence Quotient (FSIQ) of the WISC-III and the Numerical Operations subtest ranged from .37 (age 9) to .71 (age 7) when examining ages 5 to 12. The range of correlations for ages 5 to 12 between the FSIQ and Math Reasoning subtest was .67 (age 9) to .81 (ages 6 and 7).

Criterion-related validity was examined by analyzing the correlation of the WIAT-II subtests to other achievement tests and grades in school. The mathematics subtests of the WIAT-II had high correlations coefficients with the corresponding subtests of the WIAT, Wide Range Achievement Test – Third Edition (WRAT-3; Wilkinson, 1993), and Differential Ability Scales (DAS; Elliott, 1990). Specifically, the Math Reasoning subtest of the WIAT-II had the following correlations: WIAT Math Reasoning = .86 and WRAT-3 Arithmetic= .65. The correlations for the Numerical Operations subtest were as follows: WIAT Numerical Operations = .81, WRAT-3 Arithmetic= .60, and DAS Basic Number= .75. The mathematics composite of the WIAT-II also had high correlations with group-administered achievement tests, such as the Stanford Achievement Tests, Ninth Edition (SAT-9; Harcourt Educational Measurement, 1996) and the Metropolitan Achievement Tests, Eighth Edition (MAT-*8*; Harcourt Educational Measurement, 1999). The correlation between the mathematics composite of the WIAT-II and Total Math score of the SAT-9 was .75, while it ranged from .68 (Levels 6-10) to .77 (Levels 3-5) on the Total Math score of the *MAT-8*. In terms of WIAT-II subtests and school grades, the subtests and school grades that assess the same constructs had the highest correlations. For instance, the correlation between the Math Reasoning subtest and math grades was .49, and the correlation between the Numerical Operations subtest and math grades was .36.

Research Design

This purpose of this research is to investigate the factors that contribute to mathematics performance using Geary's theoretical subtypes of mathematical disabilities. The dependent variables in this study are the standard scores on the Numerical Operations and Math Reasoning subtests of the WIAT-II (Wechsler, 2001).

The independent variables are the three subtypes of Geary's theory (semantic retrieval, executive-procedural, and visuospatial), which are being created using certain subtests of the NEPSY-II. The subtests of the NEPSY-II are hypothesized to load onto Geary's domain measures. The first analyses will examine the independence of these domains, to verify that the subtests load onto one of the three latent constructs. The results of this analysis will be used to determine the predictive power of these constructs on math reasoning and calculation performance.

Four NEPSY-II subtests are used to represent the Semantic domain: Comprehension of Instructions, Narrative Memory, Phonological Processing, and Speeded Naming. The Comprehension of Instructions subtest is a measure of an individual's ability to receive, process, and execute oral instructions, such that the individual points to the appropriate stimuli based on the oral instructions. The Narrative Memory subtest examines memory for organized verbal material by having the individual recall a story that was presented to him/her. After the recall, the child is asked questions to elicit missing details. The Phonological Processing subtest assesses phonemic awareness by having individuals repeat a word, and then create a new word by omitting or substituting a syllable or phoneme. The Speeded Naming subtest is a timed subtest that assesses the individual's ability to retrieve and produce names of colors, shapes, sizes, letters, or numbers by having the individual name an array of colors and shapes; shapes and sizes; or letters and numbers as quickly as possible.

The Procedural domain is made up of three NEPSY-II subtests: Auditory Attention, Response Set, and Inhibition. The Auditory Attention subtest is designed to measure selective and sustained auditory attention by having the child listen to a series of words and touch the appropriate circle. The Response Set subtest assesses an individual's ability to shift and maintain complex sets by inhibiting previously learned responses and responding to matching or contrasting stimuli. The Inhibition subtest measures inhibition of automatic response in favor of novel responses, as well as switching between response sets. For this subtest, the individual looks at a series of shapes or arrows and names the shape or direction or an alternate response.

The Visuospatial domain consists of eight NEPSY-II subtests: Arrows, Block Construction, Design Copy, Geometric Puzzles, Memory for Designs, Memory for Designs Delayed, Memory for Faces, and Memory for Faces Delayed. The Arrows subtest measures an individual's ability to judge line orientation by having the individual look at an array of arrows and indicate which arrow(s) point to the center of the target. The Block Construction subtest combines visuospatial and visuomotor by having individuals construct three-dimensional models from two-dimensional drawings. The

Design Copy subtest assesses motor and visual-perceptual skills by having individuals copy two-dimensional geometric figures. The Geometric Puzzles subtest assesses visual discrimination, spatial localization, and visual scanning by presenting the individual with a large picture divided by a grid, as well as four smaller pictures taken from the larger picture. The individual must identify the location on the grid of the larger picture that the smaller picture was taken from. The Memory for Designs subtest measures spatial memory for novel visual material. The individual is briefly exposed to a grid with four to ten designs, and then the individual must select and place the designs in the grid. Memory for Designs Delayed is administered after a 30-minute delay and is designed to assess long-term visuospatial memory. The Memory for Faces subtest measures an individual's ability to encode facial features, as well as his/her ability to recognize faces. The child is exposed to a series of faces, and then is presented with three faces at time, one of which he/she selects as having previously seen. The last subtest for this domain is Memory for Faces Delayed subtest, which is administered 30-minutes after Memory for Faces and is designed to assess long-term memory for faces.

This approach was selected to allow for a representative sample of neuropsychological features that may significantly predict math achievement. When calculating the domain scores, first the subtests scores will be multiplied by the structural coefficients then summed. This will produce a weighted average score, which will then be transformed into a standardized score (to z score).

Procedures

After acceptance of the dissertation proposal, the formal Institutional Review Board (IRB) procedures were followed, which were approved by the IRB. The faculty dissertation chair contacted the Permissions Specialist within the Clinical Assessment Division of Pearson to obtain the NEPSY-II standardization data necessary to complete this study. Per Pearson requirements, the faculty dissertation chair completed a data request form that included the following information: name of test products, exact list of data requested, full study name, summary of research study results use (e.g., purpose, anticipated outcomes, and disposition of results), and requestor information (e.g., organization name and address). Prior to obtaining the de-identified standardization data in electronic format, a licensing agreement was signed.

Data Analysis

Research question 1. What are the intercorrelations among select NESPY-II subtests, and WIAT-II Numerical Operations and Math Reasoning subtests? It is hypothesized that the NEPSY-II subtests will have moderate correlations with the WIAT-II subtests. Pearson Product-Moment Correlation analysis will be completed on the sample of 60 participants. The two assumptions underlying Pearson Product-Moment Correlation are the variables are bivariately normally distributed and independent of each other.

Research question 2. Do NEPSY-II subtests load onto factors that mirror represent the three cognitive constructs (semantic retrieval, executive-procedural, and visuospatial) that Geary proposes are related to math? It is hypothesized that (a) Comprehension of Instruction, Narrative Memory, Phonological Processing, and Speeded Naming subtests of the NEPSY-II will load onto a factor that represents Geary's Semantic domain; (b) Auditory Attention, Response Set, and Inhibition subtests of the NEPSY-II will load onto a factor that represents Geary's Procedural domain; and (c)

Arrows, Block Construction, Design Copying, Geometric Puzzles, Memory for Designs, Memory for Designs Delayed, Memory for Faces, and Memory for Faces Delayed subtests will load onto a factor that represents Geary's Visuospatial domain. This research question was answered via confirmatory factor analysis.

Confirmatory factor analysis is a special type of structural equation modeling (SEM) used to test a theory about latent processes that are hypothesized to occur among variables (Green & Salkind, 2008; Mertler & Vannatta, 2005; Spicer, 2005; Tabachnick & Fidell, 2007). The assumptions underlying confirmatory factor analysis are that the variables are linearly related to the factors and multivariately normally distributed (Green & Salkind, 2008).

The extraction technique that will be utilized is the maximum likelihood factoring. The goal of maximum likelihood factoring is to "estimate factor loadings for population that maximize the likelihood of sampling the observed correlation matrix" (Tabachnik & Fidell, 2007, p. 633). The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy which will be used to determine if the structure of the factor analysis is acceptable by requiring a KMO value of greater than 0.50 (Spicer, 2005). Once the model is extracted, several choices are available. If the model is found to fit the data and there is a significant correlation between factors, then the results can be reported (Tabachnick & Fidell, 2007). In most cases though, several additional models are examined in an attempt to improve the fit of the model. According to Green & Salkind (2008), the factors should be made of four or more measures. Factor loadings inform the degree to which a variable and factor are correlated and will be used to determine if any parameters should be added or deleted (Spicer, 2005). Confirmatory factor analysis may reduce multicollinearity since highly correlated independent variables are combined into factors (Spicer, 2005).

Research question 3. What are the intercorrelations among the confirmed or established factors of the select NEPSY-II subtests and WIAT-II Numerical Operations and Math Reasoning subtests? It is hypothesized that factors will have moderate correlations with the math calculation and reasoning subtests. This analysis will be completed on the sample of 60 participants and will be calculated using Pearson Product-Moment Correlation. As previously discussed, the two assumptions are that the variables are bivariately normally distributed and independent of each other.

Research question 4. When the Semantic domain score is considered a predictor variable, will it significantly predict or account for a significant amount of variance in numerical operations? When the Procedural and Visuospatial domain scores are included as predictor variables, will they significantly predict or account for a significant amount of variance in numerical operations above and beyond the variance already accounted for by the Semantic domain score? Does the obtained regression equation resulting from a set of three predictor variables allow us to predict numerical operations performance reliably? It is expected that all three predictors will be included in the equation and that the obtained regression equation will reliably predict numerical operations performance. Sequential multiple regression (also known as hierarchical regression) will be used to determine the effects of the predictor variables on numerical operations performance.

For sequential regression, the researcher uses logic or theory to determine the order that the independent variables will be entered. Independent variables can be entered one at a time or in blocks (Tabachnick & Fidell, 2007). Previous research

(Cirino, Morris, & Morris, 2002, 2007) has shown the Semantic domain to be the most predictive, so that will be entered into the equation first. The Procedural and Visuospatial domains will be entered at the same time since there is conflicting evidence as to their contribution to math performance.

The assumptions for multiple regression can be broken down into two categories: assumptions about raw scale variables and those about the residuals (Mertler & Vannatta, 2005). The assumptions about raw scale variables are that the independent variables are fixed and measured without error. In addition, there has to be a linear relationship between the independent and dependent variables. In terms of residuals (the difference between obtained DV scores and predicted DV scores), they have to be normally distributed and not correlated with the independent variables (Mertler & Vannatta, 2005; Tabachnick & Fidell, 2007). The variance of the residuals must be constant across all values of the independent variables (Mertler & Vannatta, 2005). Another assumption is that there is a straight-line relationship between the residuals and predicted dependent variable scores (Tabachnick & Fidell, 2007). The variance of residuals is the same for all predicted dependent variables and independent of one another. Multicollinearity is an issue when the intercorrelations among independent variables are moderate to high and the reason why factor analysis was conducted prior to conducting the regression analyses (Spicer, 2005). Analyses are limited when the correlations between the independent variables are higher between each other than with the dependent variable.

Research question 5. When the Semantic, Procedural, and visuospatial domain scores are considered predictor values simultaneously, which significantly predict or account for a significant amount of variance in numerical operations? Does the obtained regression equation resulting from a set of three predictor variables allow us to reliability predict numerical operations performance? It is expected that all three predictors will be included in the equation and that the obtained regression equation will reliably predict numerical operations performance. Step-wise multiple regression will be used to determine the effects of the predictor variables on numerical operations performance.

For stepwise regression, the computer program determines the order of entry into the formula based on a mathematical equation. The assumptions stepwise regression are the same as those for multiple regression (see above) and include linearity, homogeneity of variance amongst residuals, normality of residuals, linearity between the residuals and predicted dependent variable scores, and independence of residuals from the independent variable scores.

Research question 6. When the Semantic, Procedural, and Visuospatial domain scores are considered predictor variables simultaneously, which significantly predict or account for a significant amount of variance in math reasoning performance? Does the obtained regression equation resulting from a set of three predictor variables allow us to predict math reasoning performance reliably? It is expected that all three predictors will be included in the equation and that the obtained regression equation will reliably predict math reasoning performance. The procedure used in the previous research question will be used to answer this research question. The data will be analyzed using stepwise multiple regression.

Chapter IV

Research studies have found different, and in some cases, conflicting results as to the cognitive processes involved with math performance. Research studies have implicated Crystallized Intelligence, Short-Term Memory, Visual Processing, Working Memory, Processing Speed, Attention, and Phonological Processing. With the understanding that multiple cognitive processes may be involved in mathematics ability and disability, Geary (1993) proposed three subtypes of math disability: Semantic, Procedural, and Visuospatial. Three studies have examined Geary's three subtypes, which found some support for this model. However, two studies that were similar in design found conflicting results (see the discussion on Cirino, Morris, and Morris, 2002, 2007). In addition to the conflicting results regarding the processes that contribute to math performance, study limitations restrict the generalizability of the results. Given the inconsistent results found in the extant literature, the paucity of research examining Geary's subtypes, and the paucity of research utilizing the NEPSY-II (Korkman et al., 2007) to measure math weaknesses, the purpose of this study was to examine Geary's three subtypes using the NESPY-II. This section describes the sample in further detail, including the processes used to obtain the final sample. After discussing the sample, the section will focus on the statistical procedures and results used to answer the six research questions proposed in the previous chapter.

Characteristics of the Sample

This study utilized two samples. A total of 81 children were administered the NEPSY-II and WIAT-II during the standardization of the NEPSY-II. This sample will be referred to as the math sample, as all of the participants were administered the math subtests of the WIAT-II. For the current study, the 5- and 6-year old participants were removed from the math sample due to age restrictions on select subtests (e.g., Response Set). Therefore, the sample was reduced to 60 participants ranging in age from 7.04 to 12.72 (*M* = 9.80, *SD* = 1.71). The math sample was composed of 50% females and 50% males. The majority of the sample was Caucasian (67%), followed by Hispanic (17%), African American (12%), and Other (5%). Given the research questions, the math sample was used to conduct regression analysis. Regression analyses will determine the amount of variance accounted for in math performance by Geary's model. Due to the nature of confirmatory factor analysis (CFA), a larger sample size is required; therefore, the sample used for the confirmatory factor analysis includes the 60 children previously described plus an additional randomly selected 146 participants from the standardization sample of the NEPSY-II. The sample of 206 will be referred to as the Total sample. The range of ages for the total sample was 7.04 to 16.98 (*M* = 10.85, *SD* = 2.58). The sample consisted of 50% females and 50% males. As with the math sample, the majority of participants in the total sample were Caucasian (61%), followed by Hispanic (18%), African American (14%), Other (4%), and Asian (3%).

Normality of Original Data

Normality of the total sample was explored using tests of normality (i.e., Kolmogorov-Smirnov and Shapiro-Wilk), skewness, kurtosis, and histograms. Analyses showed that Kolmogorov-Smirnov and Shapiro-Wilk test statistics were significant ($p < .05$) for all subtests, with the exception of the Design Copying score (Shapiro-Wilk = .99, $p > .05$). Normality was also assessed using standardized kurtosis and skewness values. Kline (2005) suggests that standardized kurtosis values greater than 3.0 indicate positive kurtosis, while values less than 3.0 indicate negative kurtosis. Obtained standardized kurtosis values all fell between -3.00 and +3.00, suggesting normality. Similarly, Kline noted that absolute skew values of greater than 3.0 are described as "extremely" skewed by some authors. For the total sample, all standardized values fell between -3.00 and +3.00, with the exception of Auditory Attention (z = -3.63). For the Auditory Attention subtest of the total sample, most of the scores for the Auditory Attention subtest were above the mean. An investigation of the histograms was consistent with the previous findings.

For the math sample (n = 60), Kolmogorov-Smirnov values were significant at the less than .05 level for the following subtests: Inhibition, Phonological Processing, Memory for Faces Delayed, Narrative Memory, Block Construction, Auditory Attention and Response Set. Shapiro-Wilk values were significant at the less than .05 level for Memory for Designs Delayed, Narrative Memory, Arrows, Auditory Attention and Response Set. The histograms were consistent with the previous findings.

Missing Data

The obtained samples from Pearson contained a significant amount of missing data; therefore, the researcher conducted missing data analyses before conducting further analyses. Missing data analyses allows for researchers to determine if there is a pattern to the missing data, or if the data is missing at random (Kline, 2005). Frequencies of missing data, as well as Separate Variance t-tests were used to determine the significance of missing data. A review of the frequency of missing data (Table 1) shows that the Inhibition subtest contained the greatest amount of missing data from both samples (16%

for the total sample and 18% for the math sample), followed by Memory for Designs Delayed (5% for the total sample and 7% for the math sample).

Table 1

Frequency of Missing Data

	Total Sample			Math Sample		
Subtest	N	Missing	%	n	Missing	%
Executive-Procedural						
Inhibition	173	33	16.0	49	11	18.3
Auditory Attention	201	5	2.4	60	0	0.0
Response Set	199	7	3.4	58	2	3.3
Semantic						
Comprehension of Instructions	206	0	0.0	60	0	0.0
Phonological Processing	206	0	0.0	60	0	0.0
Narrative Memory	203	3	1.5	59	1	1.7
Speeded Naming	203	3	1.5	58	2	3.3
Visuospatial						
Memory for Designs	200	6	2.9	58	2	3.3
Memory for Designs Delayed	196	10	4.9	56	4	6.7
Memory for Faces	204	2	1.0	58	2	3.3
Memory for Faces Delayed	201	5	2.4	58	2	3.3
Arrows	205	1	0.5	60	0	0.0
Block Construction	206	0	0.0	60	0	0.0
Design Copying	204	2	1.0	59	1	1.7
Geometric Puzzles	206	0	0.0	60	0	0.0

Note. *n* = number of participants; % = percentage of participants missing from the sample.

Separate Variance t-tests were calculated for subtests consisting of more than 5% missing data. For the math sample, only two subtests had more than 5% missing data: Inhibition and Memory for Designs Delayed (see Table 1). Therefore, Separate Variances tests were only conducted for the Inhibition and Memory for Designs Delayed subtests of the math sample. For Separate Variance t-tests, the grouping variable is present versus missing data. For instance, the participants with missing data for the Inhibition subtest were identified as part of the "missing" data group, while those with data were classified as the “present” data group. The computer then calculates the means for each group (missing and present) on the remaining subtests (e.g., Response Set). The means of each group are then compared to each other to see if the groups significantly vary from each other. Significant t-tests suggest a pattern to the missing data. Results showed that the two groups (missing and present) did not significantly differ for either subtest (Inhibition and Memory for Designs Delayed).

For the total sample, the only subtest with more than 5% missing data was Inhibition (see Table 1). The Separate Variance t-tests (see Table 2) showed that there were significant differences between the participants with missing data on the Inhibition subtest and the participants with present data on the Inhibition subtest. The present data group ($M = 10.29$) scored significantly higher than the missing data group ($M = 8.55$) on the Auditory Attention subtest, $t_{(36)} = 2.4$, $p < .05$. Similarly, the present data group scored significantly higher than the missing data group on the Response Set, $t_{(36)} = 2.2$, $p < .05$, Comprehension of Instructions, $t_{(59)} = 2.3$, $p < .05$, Phonological Processing, $t_{(49)} = 2.4$, $p < .05$, and Design Copying subtests, $t_{(43)} = 2.5$, $p < .05$. Only the significant t-test results are presented in Table 2. All other t-tests for the total sample were not significant. Given the results of the missing data analyses, there was a pattern to the missing data, especially for the Inhibition subtests. Given that the incomplete cases significantly differed from the cases with complete records, the findings from this study may not generalize to the whole population (Kline, 2005).

Table 2

Comparison of the Means for those with and without data on the Inhibition subtest

Subtest	Mean score for the missing data group	Mean score for the present data group	t	p
Auditory Attention	8.55	10.29	2.4	.023
Response Set	8.79	10.17	2.2	.034
Comprehension of Instructions	8.97	9.95	2.3	.025
Phonological Processing	8.94	10.02	2.4	.022
Design Copying	8.39	9.91	2.5	.018

Note. n = 206; Only significant results shown.

In order to conduct the regression analyses, a factor score for each subtype (Semantic, Procedural, and Visuospatial) was required for each participant. Participants needed score on each subtest in order to create the three factor scores. Therefore, missing data was replaced using the expectation-maximization (EM) algorithm of a model-based imputation method using LISREL. The EM algorithm consists of two steps: estimation and maximization. In the first step, estimation, SPSS calculated the missing observations by conducting a series of regressions, during which, the missing variable was regressed on the remaining variables for a particular case (Kline, 2005). Maximization was the second step, which consisted of submitting the whole imputed data for maximum likelihood estimation. SPSS repeated steps one and two until a stable solution was reached across the maximizations steps. When replacing data using the EM algorithm, SPSS conducts a series of regression analyses in order to determine the most stable/reliable estimate of the missing data. Larger sample sizes lead to more reliable estimates of missing data; therefore, the total sample (n = 206) was used for imputing missing data. The cases with math data were then copied to a separate database in order to obtain the data for the math sample.

Independent t-tests were used to examine the mean differences between the original sample and the sample with the imputed data. Being that further statistical analyses are required for both samples, independent t-tests were calculated for both samples (total and math). For the total sample, the Inhibition subtest had more than 5% of missing data. T-tests (see Table 3) comparing the original data and the imputed data showed that the two groups did not significantly differ on the Inhibition subtest, $t_{(377)}$ = 0.38, $p > .05$. The Inhibition and Memory for Designs Delayed subtests had more than 5% of missing data for the math sample. Independent t-tests (see Table 4) showed that the mean scores for the original data and the imputed data did not significantly on both the Inhibition, $t_{(107)} = 0.11$, $p > .05$, and Memory for Designs Delayed subtests, $t_{(114)} = -0.13$, $p > .05$. All t-tests revealed non-significant results (see Tables 3 and 4), meaning that the imputed data did not significantly differ from the original data for both the total and math samples.

The missing data analyses showed that there were significant differences between those with missing data and those with data present on the Inhibition subtest. Furthermore, in order to compute the factor scores, participants need a score on each of the subtests. Consequently, estimates of the missing data were calculated using the EM algorithm. The independent t-tests comparing the original and imputed data were not significant; as such, the remaining analyses were calculated using the imputed data. To reiterate, the imputed data consisted of the original data, plus the computer-generated data that was estimated for the missing data.

Table 3

T-tests Comparing Original Data and Imputed Data Regarding the Total Sample

	Original Data		Imputed Data			
Subtest	n	M(SD)	n	M(SD)	t	p
Executive-Procedural						
Inhibition	173	10.34 (3.07)	206	10.22 (2.89)	0.38	.703
Auditory Attention	201	10.02 (3.06)	206	10.01 (3.04)	0.05	.960
Response Set	199	9.97 (2.83)	206	9.91 (2.81)	0.20	.838
Semantic						
Comprehension of Instructions	206	9.80 (2.82)	206	9.80 (2.82)	0.00	1.000
Phonological Processing	206	9.85 (2.64)	206	9.85 (2.64)	0.00	1.000
Narrative Memory	203	10.23 (2.82)	206	10.23 (2.81)	-0.02	.982
Speeded Naming	203	10.11 (2.79)	206	10.09 (2.78)	0.09	.925
Visuospatial						
Memory for Designs	200	9.81 (3.08)	206	9.78 (3.06)	0.11	.913
Memory for Designs Delayed	196	10.00 (3.18)	206	9.94 (3.18)	0.18	.854
Memory for Faces	204	9.92 (2.84)	206	9.90 (2.83)	0.05	.961
Memory for Faces Delayed	201	9.73 (2.97)	206	9.68 (2.97)	0.16	.874
Arrows	205	10.42 (2.92)	206	10.42 (2.91)	0.01	.994
Block Construction	206	10.00 (2.78)	206	10.00 (2.78)	0.00	1.000
Design Copying	204	9.67 (3.13)	206	9.66 (3.12)	0.02	.983
Geometric Puzzles	206	9.78 (2.87)	206	9.78 (2.87)	0.00	1.000

Table 4

T-tests Comparing Original Data and Imputed Data Regarding the Math Sample

	Original Data		Imputed Data			
	n	M(SD)	n	M(SD)	t	p
Executive-Procedural						
Inhibition	49	10.16 (2.30)	60	10.12 (2.16)	0.11	.914
Auditory Attention	60	9.90 (2.89)	60	9.90 (2.89)	0.00	1.000
Response Set	58	10.29 (2.70)	60	10.25 (2.70)	0.09	.930
Semantic						
Comprehension of Instructions	60	9.88 (2.76)	60	9.88 (2.76)	0.00	1.000
Phonological	60	9.67 (2.74)	60	9.67 (2.74)	0.00	1.000

Processing						
Narrative Memory	59	10.12 (2.71)	60	10.12 (2.69)	0.00	.997
Speeded Naming	58	10.40 (3.07)	60	10.33 (3.04)	0.11	.911
Visuospatial						
Memory for Designs	58	9.79 (2.82)	60	9.80 (2.78)	-0.01	.989
Memory for Designs	56	9.66 (3.05)	60	9.73 (3.04)	-0.13	.898
Delayed						
Memory for Faces	58	9.74 (3.01)	60	9.70 (2.98)	0.08	.940
Memory for Faces	58	9.79 (3.17)	60	9.70 (3.18)	0.16	.874
Delayed						
Arrows	60	10.30 (2.70)	60	10.30 (2.70)	0.00	1.000
Block Construction	60	10.30 (2.55)	60	10.30 (2.55)	0.00	1.000
Design Copying	59	9.37 (2.99)	60	9.38 (2.96)	-0.02	.985
Geometric Puzzles	60	9.63 (3.09)	60	9.63 (3.09)	0.00	1.000

Normality of Imputed Data

Given the addition of missing data, tests of assumptions were investigated again. For the total sample, normality was explored using tests of normality (i.e., Kolmogorov-Smirnov and Shapiro-Wilk), skewness, kurtosis, and histograms. As with the original data, analyses showed that Kolmogorov-Smirnov and Shapiro-Wilk test statistics were significant ($p < .05$) for all subtests, with the exception of the Design Copying subtest (Shapiro-Wilk = .988, $p > .05$). Similar to the original sample, all standardized values for skewness fell between -3.00 and +3.00, with the exception of Auditory Attention (z = -3.59). In terms of kurtosis, Memory for Designs Delayed was the only subtest that fell outside of the -3.00 and +3.00 range (z = -3.14). This value was slightly lower than the one obtained with the original sample (z = -2.98). Histogram analysis was consistent with the previous results.

For the math sample (n = 60), Kolmogorov-Smirnov values were significant at the less than .05 level for the following subtests: Inhibition, Phonological Processing, Memory for Designs Delayed, Memory for Faces Delayed, Narrative Memory, Block

Construction, Auditory Attention and Response Set. Shapiro-Wilk values were significant at the less than .05 level for Memory for Designs Delayed, Narrative Memory, Arrows, Auditory Attention and Response Set. The histograms were consistent with the previous findings.

Additional screening analysis was completed on the total sample. Subtest scores were converted to z-scores, with scores less than -3.00 or greater than +3.00 identified as outliers. Based on that criterion, there were two outliers on the Speeded Naming subtest (z = 3.21 and 3.21). The following subtests had one outlier: Inhibition (z = 3.04), and Arrows (z = -3.24). The outliers were not deleted from the sample.

Descriptive Statistics

Subtest mean and standard deviations for both samples are presented in Table 5. For the total sample, the Arrows subtest had the highest mean (*M* = 10.42; *SD* = 2.91), while Design Copying had the lowest (*M* = 9.66; *SD* = 3.12). The largest NEPSY-II subtest mean for the math sample was Speeded Naming (*M* = 10.33; *SD* = 3.04), which is not consistent with the total sample. Similar to the total sample, the Design Copying mean (*M* = 9.38; *SD* = 2.96) was the lowest of the NEPSY subtests for the math sample.

Table 5

Descriptive Statistics Regarding Study Variables

	Total Sample		Math Sample	
Subtest	M	SD	M	SD
Procedural				
Inhibition	10.22	2.89	10.12	2.16
Auditory Attention	10.01	3.04	9.90	2.89
Response Set	9.91	2.81	10.25	2.66
Semantic				
Comprehension of Instructions	9.80	2.82	9.88	2.76
Phonological Processing	9.85	2.64	9.67	2.74

Narrative Memory	10.23	2.81	10.12	2.69
Speeded Naming	10.09	2.78	10.33	3.04
Visuospatial				
Memory for Designs	9.78	3.06	9.80	2.78
Memory for Designs Delayed	9.94	3.18	9.73	3.04
Memory for Faces	9.90	2.83	9.70	2.98
Memory for Faces Delayed	9.68	2.97	9.70	3.18
Arrows	10.42	2.91	10.30	2.70
Block Construction	10.00	2.78	10.30	2.55
Design Copying	9.66	3.12	9.38	2.96
Geometric Puzzles	9.78	2.87	9.63	3.09
Math				
Math Reasoning	---	---	105.73	13.11
Numerical Operations	---	---	103.53	12.81

Research Question 1

What were the intercorrelations among select NESPY-II subtests, and WIAT-II Numerical Operations and Math Reasoning subtests? It was hypothesized that the NEPSY-II subtests would have moderate correlations with the WIAT-II subtests. Pearson Product-Moment Correlation analyses were completed on the math sample (see Table 6). Cohen (1992) provides the following suggestions as to what constitutes a large or small effect: $r = .10$ is a small effect; $r = .30$ is a medium effect; and $r = .50$ is a large effect. For the Math Reasoning correlations, Comprehension of Instructions, $r = .47$, $p < .01$, and Block Construction, $r = .47$, $p < .01$ had large effects. Medium effects were found with Phonological Processing, $r = .39$, $p < .01$, Narrative Memory, $r = .30$, $p < .01$, Design Copying, $r = .34$, $p < .01$, and Inhibition, $r = .27$, $p < .05$. All other correlations between the NEPSY-II subtests and Math Reasoning subtest were not significant ($p > .05$). For Numerical Operations, three subtests showed medium effect sizes: Comprehension of Instructions, $r = .30$, $p < .01$, Design Copying, $r = .38$, $p < .01$, and Inhibition, $r = .30$, $p < .05$. The remaining correlations between the NEPSY-II subtests and Numerical Operations subtests were not significant (*p* >.05). Correlation analyses revealed significant correlations between the Math Reasoning subtest and three of the four subtypes hypothesized to load onto the Semantic subtype. Further, significant correlations were found between the Inhibition, Comprehension of Instructions, and Design Copying subtests and both math measures. This finding is relevant, as the correlations showed a relationship between both math subtests and at least one subtest of each hypothesized subtype.

Research Question 2

Do NEPSY-II subtests load onto factors that mirror the three cognitive constructs (Semantic retrieval, executive-Procedural, and Visuospatial) that Geary proposed were related to math? It was hypothesized that (a) Comprehension of Instruction, Narrative Memory, Phonological Processing, and Speeded Naming subtests of the NEPSY-II will load onto a factor that represents Geary's Semantic domain; (b) Auditory Attention, Response Set, and Inhibition subtests of the NEPSY-II will load onto a factor that represents Geary's Executive Procedural domain; and (c) Arrows, Block Construction, Design Copying, Geometric Puzzles, Memory for Designs, Memory for Designs Delayed, Memory for Faces, and Memory for Faces Delayed subtests will load onto a factor that represents Geary's Visuospatial domain. As previously discussed, Stevens (1992) recommends a sample of at least 150 participants when the components have low loadings; therefore, the total sample was used in order to conduct the factor analysis.

Table 6

Summary of Intercorrelations for Scores on the NEPSY-II and Math Subtests of the WIAT-II for the Math Sample

Subtest	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Procedural																
1. INH	-															
2. AA	.15	-														
3. RS	.20	.45**	-													
Semantic																
4. CI	.25	24**	.24	-												
5. PP	.08	.26*	.13	.56**	-											
6. NM	.05	.03	-.07	.49**	.25*	-										
7. SN	.21	.15	-.06	.15	.30*	.17	-									
Visuospatial																
8. MD	.14	.19	.22	.21	.41**	.16	.08	-								
9. MDD	.19	.11	.35**	.35**	.48**	.22	.09	.77**	-							
10. MF	-.17	.11	-.01	.15	.08	.27*	.08	.36**	.27*	-						
11. MFD	-.11	.10	.04	-.03	.07	.07	.08	.34**	.17	.62**	-					
12. AR	.02	.08	-.12	.11	.06	.04	-.15	.12	.04	.21	.24	-				
13. BC	.10	.17	.07	.30*	.29*	.48**	.09	.17	.19	.15	.08	.16	-			
14. DC	-.19	.01	.01	.22	.17	.29*	.08	.37**	.28*	.52**	.17	.18	.25	-		
15. GP	.12	.04	-.08	.22	.12	.33*	.06	.03	.03	.12	.16	.13	.12	.13	-	
Math																
16. MR	.27*	.21	.14	.47**	39**	.30**	.08	.19	.25	.09	-.01	.24	.47**	.34**	.14	-
17. NO	.30*	.01	.11	.30**	.08	.16	.18	.11	.16	.16	.03	.19	.23	.38**	-.00	.61**

Notes. INH = Inhibition; AA = Auditory Attention; RS = Response Set; CI = Comprehension of Instructions; PP = Phonological Processing; NBM = Narrative Memory; SN = Speeded Naming; MD = Memory for Designs; MDD = Memory for Designs Delayed; MF = Memory for Faces; MFD = Memory for Faces Delayed; AR = Arrows; BC = Block Construction; DC = Design Copy; GP = Geometric Puzzles; MR = Math Reasoning; NO = Numerical Operations.

* $p < .05$, two-tailed. ** $p < .01$, two-tailed.

Pre-analysis required to answer question 2. Pearson Product-Moment Correlation analyses (see Table 7) were conducted first in order to examine the relationships among the NEPSY-II subtests before conducting the Confirmatory Factor Analysis (CFA). Significant correlations were expected on the subtests that were theorized to load onto the same factors, while non-significant relationships were expected for the subtests that loaded onto separate factors. For the Procedural subtype, all subtests correlated significantly and demonstrated medium effect sizes with each other (Inhibition-Auditory Attention, $r = .21$, $p <.01$, Inhibition-Response Set, $r = .26$, $p < .01$, Auditory Attention-Response Set, $r = .37$, $p < .01$). The six correlations within the Semantic subtype were all significant. Lastly, 24 of the 28 correlations on the Visuospatial subtype were significant. These results show significant correlations within each hypothesized subtype.

Correlations between the subtypes were also examined. For the Procedural subtype, the Inhibition subtest showed significant correlations with all of the subtests composing the Semantic subtype. The Inhibition subtest also showed significant correlations with 6 of 8 correlations on the Visuospatial subtype. Comprehension of Instructions and Phonological Processing also showed significant correlations with all of the Procedural subtype subtests. Consistent with the Auditory Attention results, Comprehension of Instructions and Phonological Processing showed significant correlations with the same 6 subtests on the Visuospatial subtype. The Narrative Memory subtest also correlated significantly with 5 of the 8 subtests used to create the Visuospatial subtype. These results show that significant correlations were present between the hypothesized subtypes.

Table 7

Summary of Intercorrelations for Scores on the NEPSY-II Subtests for the Total Sample

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Procedural															
1. INH	–														
2. AA	.21**	–													
3. RS	.26**	.37**	–												
Semantic															
4. CI	.27**	.27**	.18**	–											
5. PP	.24**	.26**	.15*	.48**	–										
6. NM	.15*	.11	.07	.31**	.26**	–									
7. SN	.21**	.13	.05	.14*	.24**	.25**	–								
Visuospatial															
8. MD	.24**	.13	.08	.23**	.35**	.25**	.12	–							
9. MDD	.20**	.18*	.17*	.22**	.35**	.20**	.11	.81**	–						
10. MF	-.08	-.02	-.04	-.03	-.11	.05	.06	.14*	.15*	–					
11. MFD	.07	.06	.11	.03	.04	.12	.19**	.28**	.22**	.59**	–				
12. AR	.23**	.15*	.06	.21**	.30**	.13	-.03	.24**	.20**	.12	.16*	–			
13. BC	.19**	.12	.11	.21**	.23**	.20**	.12	.21**	.20**	.05	.07	.22**	–		
14. DC	.14*	.10	.12	.22**	.35**	.21**	.18**	.40**	.33**	.22**	.21**	.33**	.25**	–	
15. GP	.22**	.03	.04	.21**	.29**	.21**	.11	.34**	.31**	.10	.19**	.31**	.20**	.30**	–

Note. INH = Inhibition; AA = Auditory Attention; RS = Response Set; CI = Comprehension of Instructions; PP = Phonological Processing; NM = Narrative Memory; SN = Speeded Naming; MD = Memory for Designs; MDD = Memory for Designs Delayed; MF = Memory for Faces; MFD = Memory for Faces Delayed; AR = Arrows; BC = Block Construction; DC = Design Copy; GP = Geometric Puzzles.
* $p < .05$, two-tailed. ** $p < .01$, two-tailed.

Analyses answering question 2. After examining these intercorrelations, the models were analyzed using CFA. Initial model identification for the three latent variables was based on previous research and the theoretical model of the NEPSY-II, as described above and shown in Appendix A. The initial model was evaluated using multiple fit indices based on Sivo, Fan, Witta, and Willse's (2006) recommended optimal cut-off values for correct models using a sample size of 150 and 250 (see Table 8). For the current study, the sample size was 206; therefore, the cut-off values for a sample size of 150 and 250 are presented in order to determine the cut-off values for the current study.

Table 8

Recommended Optimal Cut-Off Values for Correct Models Using N = 150 and N = 250

LISERL Fit Index	*N* = 150	*N* = 250
GFI	0.89	0.93
AGFI	0.87	0.91
CFI	0.95	0.97
NFI	0.88	0.92
PNFI	0.72	0.75
RMR	0.14	0.12
SRMR	0.12	0.10
RMSEA	0.06	0.05

Note. High values indicate better model fit for all except the 3 indexes at the bottom (*RMR, SRMR, RMSEA*). GFI = goodness-of-fit index; AGFI = adjusted goodness-of-fit index; CFI = comparative fit index; NFI = normed fit index; PNFI = parsimonious normed fit index; RMR = root mean square residual; SRMR = standardized RMR; RMSEA = root mean square error of approximation.

Table 9

LISREL Analysis for Model 1

LISERL Fit Index	Value
GFI	0.87
AGFI	0.82
CFI	0.84
NFI	0.78

PNFI	0.64
RMR	0.75
SRMR	0.09
RMSEA	0.09

Note. High values indicate better model fit for all except the 3 indexes at the bottom (*RMR, SRMR, RMSEA*). GFI = goodness-of-fit index; AGFI = adjusted goodness-of-fit index; CFI = comparative fit index; NFI = normed fit index; PNFI = parsimonious normed fit index; RMR = root mean square residual; SRMR = standardized RMR; RMSEA = root mean square error of approximation.

Results from the initial model (Model 1), showed that most of the fit index values failed to meet the cut-off values (see Table 9). For instance, the Goodness-of-Fit Index for Model 1 was 0.87, which does not meet the less restrictive cut-off of 0.89 for a sample of 150 participants. Given that most of the values failed to meet the cut-off values, adjustments were made to the model using two of the recommended modifications. Two error covariance paths were added to the model (see Appendix B). One path was added between the Memory for Designs and Memory for Designs Delayed, and the other path was added between Memory for Faces and Memory for Faces Delayed. Of all of the modification indices provided in the output, adding a covariance path between those subtests statistically and theoretically enhanced the original model. Results for Model 2 are provided in Table 10.

Table 10

LISREL Analysis for Model 2

LISERL Fit Index	Value
GFI	0.94
AGFI	0.91
CFI	0.97
NFI	0.90
PNFI	0.73
RMR	0.47
SRMR	0.06
RMSEA	0.03

Note. High values indicate better model fit for all except the 3 indexes at the bottom (*RMR, SRMR, RMSEA*). GFI = goodness-of-fit index; AGFI = adjusted goodness-of-fit index; CFI = comparative fit index; NFI = normed fit index; PNFI = parsimonious normed fit index; RMR = root mean square residual; SRMR = standardized RMR; RMSEA = root mean square error of approximation.

With the exception of the root mean square residual (RMR) value, all obtained values fall within the cut-off value for the suggested values using a sample of 150. The GFI, AGFI, CFI, SRMR, and RMSEA all meet the more stringent cut-off values for the sample size of 250. The obtained NFI (.90) and PNFI (.73) values failed to meet the cut-off values off .92 and .75, respectively. The minimum fit function chi-square was significant, _ = 110.16, $p < .05$, indicating that the final model adequately fit the data. The standardized structural coefficients for the final model are provided in Table 11. Domain scores for each participant were created using the standardized structural coefficients. Once the domain scores were created, the correlations between the domain scores and the math subtests were investigated.

Table 11

CFA Standardized Structural Coefficients

Subtest	Procedural	Semantic	Visuospatial
Procedural			
Inhibition	0.49		
Auditory Attention	0.58		
Response Set	0.53		
Semantic			
Comprehension of Instructions		0.63	
Phonological Processing		0.73	
Narrative Memory		0.43	
Speeded Naming		0.32	
Visuospatial			
Memory for Designs			0.62
Memory for Designs Delayed			0.55
Memory for Faces			0.17

Memory for Faces Delayed	0.32
Arrows	0.49
Block Construction	0.40
Design Copying	0.62
Geometric Puzzles	0.54

Research Question 3

What were the intercorrelations among the established factors of the select NEPSY-II subtests and the WIAT-II Numerical Operations and Math Reasoning subtests? It was hypothesized that the factors would show moderate correlations with the math calculation and math reasoning subtests. For the Math Reasoning subtest (see Table 12), a large correlation was found for the Semantic domain, $r = .48$, $p < .01$, while a medium to large correlation was found for the Visuospatial domain, $r = .40$, $p < .01$. A medium, but significant correlation was found between Math Reasoning and the Procedural Domain, $r = .28$, $p < .05$. Of the three Geary domains (Procedural, Semantic, and Visuospatial), only the Visuospatial domain significantly correlated with Numerical Operations, $r = .28$, $p < .05$. A stronger correlation than was expected was found between the math calculation and reasoning subtests, $r = .61$, $p < .01$.

Table 12

Intercorrelations of Domain Scores and Math Achievement

Domain	Procedural	Semantic	Visuospatial	Math Reasoning
Procedural	–			
Semantic	.29*	–		
Visuospatial	.17	.47**	–	
Math Reasoning	.28*	.48**	.40**	–
Numerical Operations	.18	.24	.28*	.61**

* $p < .05$, two-tailed. ** $p < .01$ level, two-tailed.

Research Question 4

Will the Semantic domain score significantly predict or account for a significant amount of variance in Numerical Operations performance? Will the executive-Procedural and Visuospatial domains account for significantly more variance in Numerical Operations performance beyond the variance already accounted for by the Semantic domain score? Does the obtained regression equation resulting from a set of three predictor variables reliably predict Numerical Operations performance? It was expected that all three predictors would be included in the equation and that the obtained regression equation would reliably predict Numerical Operations performance.

Table 13

Hierarchical Regression Results for Numerical Operations

Subtype	β	t	p	F or F change	R^2	R^2 Change	R^2 Total Adjusted
Semantic	.24	1.90	.062	3.61	.06	.06	.04
Visuospatial	.21	1.47	.146	--	--	--	--
Procedural	.12	.91	.369	--	--	--	--

Note: β = Beta; t = t-test; p = probability; F = F distribution; and R^2 = multiple correlation squared.

Multicollinearity was assessed before interpreting the regression results for each model. For all regression models, the Variance Inflation Factor scores were below the suggested cut-off value of 10, and all of the tolerance statistics where above the suggested value of .2 (Field, 2005). These results indicated that collinearity was not present for any of the regression models.

Given the results of previous research studies, the only predictor entered in the first regression model was the Semantic subtype. Regression results (see Table 13) indicated that the Semantic subtype did not significantly predict Numerical Operations performance, $R^2_{adj} = .04$, $F_{(1, 58)} = 3.61$, $p = .062$. For the next regression model, the Semantic subtype was entered in block one, and then the Visuospatial and Semantic domains were entered simultaneously in block two. When the Visuospatial and Semantic subtypes were added to the equation, they did not significantly contribute to Numerical Operations performance. Regression results show that these three subtypes did not significantly contribute to Numerical Operations performance.

Research Question 5

When the Semantic, Procedural, and Visuospatial domain scores were considered predictor values simultaneously, which domains will significantly predict or account for a significant amount of variance in Numerical Operations performance? Will an obtained regression equation resulting from a set of three predictor variables allow for the reliable prediction of Numerical Operations performance? It was expected that all three predictors would be included in the equation and that the obtained regression equation would reliably predict Numerical Operations performance.

Table 14

Stepwise Regression Results for Numerical Operations

Subtype	β	t	p	F or F change	R^2	R^2 Change	R^2 Total Adjusted
Visuospatial	.28	2.20	.031	4.86	.08	.08	.06
Semantic	.14	1.00	.323	--	--	--	--
Procedural	.14	1.08	.285	--	--	--	--

Note: β = Beta; t = t-test; p = probability; F = F distribution; and R^2 = multiple correlation squared.

When all three subtypes were included in the regression analysis simultaneously (see Table 14), the Visuospatial subtype was the only one that contributed significantly to

Numerical Operations performance, R^2_{adj} = .06, $F_{(1, 58)}$ = 4.86, p =.031. Although this model was significant, it only accounted for 6% of variance in Numerical Operations performance.

Research Question 6

When the Semantic, executive-Procedural, and Visuospatial domain scores were considered predictor variables simultaneously, which will significantly predict or account for a significant amount of variance in Math Reasoning performance? Will an obtained regression equation resulting from a set of three predictor variables allow for the reliable prediction of Math Reasoning performance? It was expected that all three predictors would be included in the equation and that the obtained regression equation would reliably predict Math Reasoning performance.

Table 15

Stepwise Regression Results for Math Reasoning

Domains	β	t	p	F or F change	R^2	R^2 Change	R^2 Total Adjusted
Semantic	.48	4.15	<.001	17.19	.23	.23	.22
Visuospatial	.23	1.78	.08	--	--	--	--
Procedural	.16	1.33	.19	--	--	--	--

Note: β = Beta; t = t-test; p = probability; F = F distribution; and R^2 = multiple correlation squared.

Unlike the Math Calculation, research related to Math Reasoning performance is limited; therefore, all domains were entered simultaneously. When all three subtypes were included in the regression analysis simultaneously (see Table 15), the Semantic subtype was the only one that contributed significantly to Math Reasoning performance, R^2_{adj} = .22, F (1, 58) = 17.19, p <.001. The Semantic subtype accounted for 22% variance in Math Reasoning performance.

Summary

The original sample contained missing data; therefore, the researcher imputed data using the EM algorithm. Analyses showed that the imputed data did not significantly differ from the original data. Results from the statistical analyses showed moderate correlations between three of the NEPSY-II subtests (i.e., Comprehension of Instructions, Design Copy, and Inhibition) and both math subtests. For the intercorrelations within each of the theorized domains, all correlations were significant for the Procedural and Semantic domains. For the Visuospatial domain, 24 of 28 correlations were significant. Four NEPSY-II subtests (Inhibition, Comprehension of Instruction, Narrative Memory, and Phonological Processing) showed significant correlations for most of the correlations across the other domains. For instance, of the 12 other subtests not hypothesized to load onto the Procedural subtype, the Inhibition subtest showed significant correlations with 10 of those subtests. After examining the intercorrelations amongst the subtests, the researcher investigated two CFA models. The initial model with a single path between each subtest and the hypothesized construct did not adequately fit the data. After adding two covariance paths (one between Memory for Designs and Memory for Designs Delayed, and other between Memory for Faces and Memory for Faces Delayed) to the original model, the obtained values suggested that the final model adequately fit the data. The researcher then used the final model to conduct regression analysis for Numerical Operations and Math Reasoning subtests. For the Numerical Operations subtest, the Semantic domain was added first, and then the Procedural and Visuospatial domains were added simultaneously. That regression model did not significantly predict Numerical Operations performance. When all three domains were entered simultaneously, the only significant predictor was Visuospatial, which accounted for only 6% of Numerical Operations performance. When all three domains were entered simultaneously for Math Reasoning, the Semantic domain accounted for 22% of variance in Math Reasoning performance. The next chapter will explore the results in the context of previous research studies, discuss the limitations of the current study, and offer suggestions as to future research studies.

Chapter V

After multiple reviews of the literature documenting that multiple cognitive processes may be involved in mathematics ability and disability, Geary (1993) proposed a model that included three subtypes of dyscalculia, or math disability. The Semantic subtype hinders a student's ability to retrieve over-learned math facts from memory. Regarding Geary's Procedural subtype, students demonstrate deficiencies in the processing and encoding of numeric information, as well as use of developmentally immature procedures. Lastly, the Visuospatial subtype consists of deficits that impair an individual's ability to represent and interpret numerical information, such as misaligning numbers in multicolumn math problems. Three studies (Cirino, Morris, & Morris, 2002, 2007; Mazzocco & Myers, 2003) have examined Geary's three subtypes and provide support for Geary's model. However, two studies that were similar in design found conflicting results (Cirino, Morris, & Morris, 2002, 2007). For instance, in the earlier study (2002), semantic and procedural skills accounted for a significant amount of math calculation skills, whereas semantic and visuospatial skills were significant predictors in the more recent study (2007). In addition to the conflicting results found regarding the processes that contribute to math performance, limitations of these studies restrict the extent to which the results can be generalized to other aged students. Given the paucity of research examining Geary's subtypes of math disability, this study aimed to add to the literature by exploring the presence of these three subtypes in a sample of school-aged children.

Students with math disabilities are considered to have biologically-based disorders that manifest as deficiencies in language, working memory, executive functioning, and/or visuospatial functioning (Feifer & De Fina, 2005). Research suggests that fluency of over-learned mathematical facts, such as addition facts, is linked to the left-hemisphere (Bull & Johnston, 1997). Additionally, individuals with math fact difficulties tend to have co-occurring reading disabilities (Bull & Johnston, 1997). Other research studies have implicated Crystallized Intelligence, Short-Term Memory, Visual Processing, Working Memory, Processing Speed, Attention, and Phonological Processing as contributors to math performance. Therefore, researchers have yet to identify the neuropsychological correlates underlying math functioning. Another goal of this study was to use a neuropsychological instrument commonly used in the schools, the NEPSY-II (Korkman et al., 2007), to study Geary's theory of math disability subtypes. The six research questions posed as part of this study are presented in the next section, followed by a brief description of the findings and associated discussion.

Summary of Results and Relevant Literature

The first research question sought to determine the intercorrelations among select NESPY-II subtests, and WIAT-II Numerical Operations and Math Reasoning subtests. It was hypothesized that the NEPSY-II subtests would have moderate correlations with the WIAT-II subtests. Math performance showed significant correlations with aspects of semantic, procedural, and visuospatial skills; however, the correlations varied by domain subtests and math subtests. Regarding the Semantic domain, a medium effect was found between math calculation performance and the Comprehension of Instructions subtest. Numerical Operations performance did not correlate significantly with the remaining subtests of the Semantic domain. A large effect was found between Comprehension of Instructions and Math Reasoning performance. Additionally, the Math Reasoning subtest showed medium correlations with the Phonological Processing and Narrative Memory subtests. The correlation between Speeded Naming and Math Reasoning was not significant. An examination of the hypothesized Procedural domain subtests shows that the Inhibition subtest had medium correlations with both the Math Reasoning and Numerical Operations subtests. The remaining correlations between the math subtests and Semantic domain subtests were not significant. Lastly, for the Visuospatial domain, the Design Copying subtest showed a moderate correlation with both Calculation and Math Reasoning. The Math Reasoning subtest also showed a large correlation with Block Construction.

Only three of the correlations between Numerical Operations and the NEPSY-II subtests were significant; however, each of those correlations represented a different domain. Even though it was hypothesized that significant correlations would be found between all of the NEPSY-II subtests and the math subtests,, one subtest from each domain was significant with Numerical Operations performance. Similar to the correlations between Numerical Operations and the NEPSY-II, the Math Reasoning subtest also significantly correlated with the same three subtests (Comprehension of Instructions, Inhibition, and Design Copying). The Math Reasoning subtest, however, also correlated significantly with two additional subtests from the Semantic domain (Phonological Processing and Narrative Memory) and one from the Visuospatial domain (Block Construction). The results are expected, as math reasoning requires semantic, procedural, and visuospatial skills.

The results of the current study were consistent with the studies conducted by Cirnio, Morris, and Morris (2002, 2007), as those studies showed significant relationships between math performance and semantic skills. This study adds to the existing literature base, as it shows a relationship between math performance and semantic skills. Confirming the connection between math performance and semantic skills advances the understanding of math performance/disabilities, as it highlights the importance of language-related skills when completing math problems.

In a previous research study (Korkman, Kirk, & Kemp, 2007), students with MD demonstrated poorer performance on the Inhibition, Narrative Memory, and Block Construction subtests when compared to students without MD. The results of the current study are consistent with Kirkman, Kirk, and Kemp's (2007) study, which found significant correlations between the Inhibition, Narrative Memory, and Block Construction subtests to math performance. Together, these results suggest a relationship between each of Geary's hypothesized domains and math performance, such that Inhibition represents the Procedural domain, Narrative Memory the Semantic domain, and Block Construction the Visuospatial domain. Korkman, Kirk, and Kemp did not find a significant difference between those with MD and those without MD on the Language subtests of the NEPSY-II. Inconsistent with Korkman, Kirk, and Kemp's (2007) study, the current results showed a significant relationship between math performance and performance on the Comprehension of Instruction, Phonological Processing, and Narrative Memory subtests of the NEPSY-II. As previously described, the contribution of language skills to math performance have been found in other research studies, as well as the current study. Korkman, Kirk, and Kemp's study was limited in sample size, which may account for the insignificant relationship between language skills and math performance.

The purpose of this research was to investigate the relationship between Geary's three hypothetical subtypes and math calculation and math reasoning performance in a school-aged population. Therefore, the second research question sought to investigate whether the subtests of the NEPSY-II load onto factors that mirror Geary's three cognitive constructs (Semantic, Procedural, and Visuospatial). Comprehension of Instruction, Narrative Memory, Phonological Processing, and Speeded Naming subtests of the NEPSY-II were hypothesized to load onto a factor that represents Geary's Semantic domain. The Auditory Attention, Response Set, and Inhibition subtests of the NEPSY-II were hypothesized to load onto a factor that represents Geary's Executive Procedural domain. Finally, Arrows, Block Construction, Design Copying, Geometric Puzzles, Memory for Designs, Memory for Designs Delayed, Memory for Faces, and Memory for Faces Delayed subtests were hypothesized to load onto a factor that represents Geary's Visuospatial domain. The correlations between the NEPSY-II subtests were investigated before creating the factors. Correlation analyses for the Procedural subtype showed that all subtests hypothesized to load onto the Procedural domain correlated significantly and demonstrated medium effect sizes with each other. The Inhibition subtest showed significant correlations with most, if not all, of the subtests composing the Visuospatial and Semantic subtypes. For the Semantic subtype, all correlations within that domain were significant. Across domains, Comprehension of Instructions and Phonological Processing showed significant correlations with most, if not all, of the subtests for the Procedural and Visuospatial domains. Lastly, 24 of the 28 correlations on the Visuospatial subtype were significant. As hypothesized, these results show significant correlations within each hypothesized subtype; however, the number of significant correlations across domains was higher than expected.

For the initial model (i.e., a single path between each subtest and the theorized construct), most of the fit index values failed to meet the cut-off values, meaning that the model did not adequately fit the data. In order to conduct regression analyses to determine the contribution of Geary's subtypes to math performance, the proposed model had to adequately fit the data. As a result, two adjustments were made to the model using two of the recommended modification indices. Two error covariance paths between the Memory for Designs and Memory for Designs Delayed and another between Memory for Faces and Memory for Faces Delayed were added to the model. The addition of the two covariance paths theoretically and statistically enhanced the model. The final model, which consisted of the 15 single paths and 2 error covariance paths, enhanced the model theoretically, as each of the covariance paths included subtests that required examinees to recall the same information immediately after exposure to the stimuli and after a time delay. The results of the factor analysis found that the final model adequately fit the data; therefore, the results supported the use of the NEPSY-II subtests to represent Geary's three subtypes. Similar to a previous research study (Cirino, Morris, & Morris, 2007), adjustments were made to make a model that fit the data. However, unlike previous research, which required adjustments to the paths between the subtests and the constructs, this study only required the addition of two covariance error paths.

In addition to supporting the use of the NEPSY-II subtests to represent Geary's subtypes, the CFA results also confirmed the construct validity of the NEPSY-II, as the subtests measuring similar skills (language, visuospatial, and executive functioning) grouped together with each other. After obtaining the final model, the relationships between the created constructs and math subtests were investigated.

The third research question sought to explore the intercorrelations among the established factors of the select NEPSY-II subtests and the WIAT-II Numerical Operations and Math Reasoning subtests. It was hypothesized that the three subtypes would have moderate correlations with the Numerical Operations and Math Reasoning subtests. Of the three Geary domains, only the Visuospatial domain correlated significantly with Numerical Operations. For the Math Reasoning subtest, moderate correlations were found for the Semantic and Visuospatial domains, while a small correlation was found with the Procedural domain. Even though there was a small correlation between the Procedural domain and Math Reasoning performance, all three domains significantly correlated with Math Reasoning, which was expected. Math Calculation, on the other hand, did not show a significant correlation with the Procedural and Semantic domains, which was not expected because all three subtypes were hypothesized to show significant correlations with Numerical Operations performance.

The math calculation results from the current study are not consistent with Geary's (1993) theory, as he implicated semantic skills as the primary cognitive skill underlying math calculation performance. Even though a significant correlation was not found between semantic skills and math calculation, Geary based his theory on basic math facts, which although assessed using the Numerical Operations subtest, were not the only math skills measured. The math skills examined on the Numerical Operations subtest extend beyond basic math facts and include higher-level math skills, such as algebra and geometry. One hypothesis for the differences between the current study and

Geary's theory could be the reliance on visuospatial skills when completing higher-order math problems. For math reasoning, all three domains correlated significantly, which provides support for Geary's theory. The results from all of the correlations analyses showed the significant relationship between math problem solving and language/semantic skills. When investigated individually, the Comprehension of Instructions subtest demonstrated the strongest correlation, which is not surprising given the language demands required when completing math reasoning problems.

The results of the current correlation analysis are, at times, consistent with previous research studies. In a study by Mazzocco and Myers (2003), results showed that significantly more children in the low math performers group had a reading disability and visual perceptual difficulties when compared to the average and above average groups. The results from the current study are comparable to Mazzocco and Myers study, as the current study showed a significant relationship between visuospatial skills and Math Calculation and Math Reasoning. The research studies discussed in the literature review found contradictory results in terms of the contribution of visuospatial skills to math performance; however, the results of the current study suggest a significant contribution of visuospatial skills to math performance. As such, visuospatial skills should be assessed when examining a student with math difficulties to determine the contribution of visuospatial deficits to poor math performance. Additionally, language skills were significantly related with Math Reasoning skills; therefore, language skills should also be assessed when a student is demonstrating difficulties with Math Reasoning skills.

After examining the correlations between math performance and Geary's domains, the next research question sought to determine if the Semantic domain score would significantly predict or account for a significant amount of variance in Numerical Operations. Additionally, would a model that included the Procedural and Visuospatial domain scores account for a significant amount of variance in Numerical Operations above and beyond the variance already accounted for by the Semantic domain score? Lastly, this research question sought to determine if the regression equation resulting from a set of three predictor variables would reliably predict Numerical Operations performance.

Regression results indicated that the Semantic subtype did not significantly predict Numerical Operations performance. Further, when the Visuospatial and Semantic domains were added to the regression equation, they did not significantly contribute to Numerical Operations performance. Regression results showed that these three subtypes did not significantly contribute to Numerical Operations performance. The results are contrary to the hypothesis, which assumed that a regression equation consisting of all three constructs would reliably predict Numerical Operations performance.

Geary (1993) theorized that semantic skills were the primary cognitive correlates underlying math calculation performance. Previous research (Cirino, Morris, & Morris, 2002, 2007) also showed that a significant amount of math calculation performance was accounted for by Geary's Semantic subtype. The results from the current study do not support the significant contribution of semantic/language skills to math calculation performance. The subtest used in the current study to represent math calculation skills extended beyond basic math facts, which is the math skill emphasized in Geary's theory. One limitation of this study involves the combination of math skills on the Numerical Operations subtest. Future research studies should use math measures that isolate discrete skills, such as Curriculum Based Measures (CBM) of math fluency.

The results from the previous research question failed to find a significant relationship between Numerical Operations and the Semantic domain; therefore, the fifth research question sought to determine if the three domain scores accounted for a significant amount of variance in Numerical Operations when all domains were considered predictor variables simultaneously. This research question also sought to determine if the given regression equation was able to reliably predict Numerical Operations performance. It was hypothesized that all three predictors would be included in the equation, with the obtained regression equation reliably predicting Numerical Operations performance.

When all three subtypes were included in the regression analysis simultaneously, the Visuospatial subtype was the only domain that contributed significant variance to Numerical Operations performance. Although this model was significant, it only accounted for 6% of variance in Numerical Operations performance. These results are not consistent with predictions, as it was hypothesized that all domains would account for a significant amount of variance in math calculation performance. These results were also not consistent with Geary's (1993) theory and previous research studies, as the current study did not support the significant contribution of semantic/language skills to math calculation performance. The inconsistent findings are likely the result of the math measures used. As previously discussed, Geary's theory focused on basic math facts, which was not consistent with the math calculation subtest used in the current study.

In a previous study by Cirino, Morris, and Morris (2002), semantic and procedural skills accounted for approximately 17% of the variance in college students' calculation skills, whereas visuospatial skills did not contribute any unique variance. The Procedural domain was entered into the regression equation first and accounted for 12% of total variance in calculation skills. When the Semantic domain was entered into the regression equation, the total amount of variance increased to 17%.

Cirino, Morris, and Morris (2007) conducted a follow up study, which showed that the three domains accounted for 30% of the variance in calculation skills. Contrary to the earlier study, the Semantic (5.7%) and Visuospatial (4.3%) domains each contributed a significant amount of variance to math calculation, while the Procedural domain did not contribute unique variance. Cirino, Morris, and Morris then reanalyzed the earlier data (2002) with the model from the more recent study (2007) and found results similar to the earlier study. Taken together, the three domains significantly predicted math calculation performance; accounting for 26% of total variance. Similar to the earlier results, only the Procedural and Semantic domains contributed unique variance. Further analyses showed that the Semantic domain contributed 3% unique variance, while the Procedural domain contributed 7% of the variance. Combined, the Semantic and Procedural domains only accounted for 10% of variance, which is less than half of the total variance accounted for when all three domains are included in the equation. These results were consistent with the results from the more recent study (Cirino, Morris, & Morris, 2007), such that the Semantic and Visuospatial domains only accounted for 10% of unique variance.

Results from the current study are contrary to the results found in the Cirino, Morris, and Morris (2002, 2007) studies. First, the current study failed to obtain significant results when all three domains were entered into the regression equations.

Further, the previous findings consistently showed significant results for the Semantic domain. For the current study, the Semantic domain did not account for any variance in math calculation, even when the Semantic domain was the only variable in the regression equation. When all three domains were added simultaneously, the Visuospatial domain was the only domain to account for a significant amount of variance in math calculation. In the current study, the Visuospatial domain accounted for 6% of variance, which is slightly higher than the 4.3% previously described (see Cirino, Morris, & Morris, 2007). The inconsistent findings were not unexpected given the differences between the studies. For instance, the current study examined school-aged students, while the Cirino, Morris, and Morris (2002, 2007) studies examined college-aged students. Further, the assessment instruments utilized varied across the studies. In the current study, the NEPSY-II was the primary measure of cognitive and neuropsychological functioning, while the other studies utilized various measures of neuropsychological functioning (e.g., Trailmaking Test Part B; Reitan & Wolfson, 1985; and the Visual Search and Attention Test; Trenerry, Crosson, DeBoe, & Leber, 1990).

The focus of the final research question was on Math Reasoning performance. Specifically, the last research question sought to determine if the Semantic, Procedural, and Visuospatial domain scores accounted for a significant amount of variance in Math Reasoning performance. It was hypothesized that all three predictors would be included in the equation, with the obtained regression equation reliably predicting Math Reasoning performance.

When all three subtypes were included in the regression analysis simultaneously, the Semantic subtype was the only domain that contributed significantly to Math

Reasoning performance. The Semantic subtype accounted for 22% variance in Math Reasoning performance. These results were contrary to the hypothesis, as the regression equation consisted of only one domain instead of all three domains. Consistent with Geary's (1993) theory, the Semantic domain accounted for a significant amount of variance in math reasoning performance; however, contrary to Geary's theory, visuospatial and executive functioning skills did not contribute a significant amount of variance. The contribution of language skills to math reasoning performance was expected, as participants are required to answer orally presented questions. Geary implicated various executive functioning skills, such as attention and cognitive flexibility in his theory, but the current study did not find a significant contribution of the Procedural domain to math reasoning performance. Previous research studies have incorporated several measures of executive procedural skills; however, the Procedural domain for the current study only consisted of three subtests. The discrepancy between previous research and the current study may be attributable to the executive procedural subtests used.

The results for the current research question were, at times, consistent with previous findings. Contrary to the current study, results from Cirino, Morris, and Morris (2007) showed that the three domains predicted 50% of the variance. The Semantic (6.7%) and Visuospatial (12.5%) domains contributed significant unique variance, while the Procedural domain did not. For the current study, the Semantic domain was the only domain to contribute a significant amount of variance. The Semantic domain accounted for 22% of the variance in math reasoning, which is significantly higher than the 6.7% found in the Cirino, Morris, and Morris study but lower than the 50% of total variance accounted for by all three domains. It was expected that the Procedural domain would account for a significant amount of variance in math reasoning skills; however, the current study did not isolate and investigate each aspect of executive functioning. For instance, Cirino, Morris, & Morris (2007) used the Trailmaking Test Part B (Reitan & Wolfson, 1985) to assess sequencing and the Digit Span subtest of the WAIS-III (Wechler, 1997) to examine working memory. It is hypothesized that significant relationships between the Procedural subtype and math performance would have been found if additional tests examining executive functioning would have been added to the current study. Additional limitations of the current study are explored in the following section.

Limitations

Some of the limitations of the present study pertain primarily to the sample and the methods of measuring and defining Geary's subtypes, as well as math performance. The sample was limited because of the number of children who were administered the NEPSY-II and WIAT-II. The sample also contained missing data, which had to be imputed in order to run the regression analyses. Given the small sample size, the contribution of each subtest to math performance could not be examined. Additionally, the examiner could not assess gender, racial, and age differences due to the limited sample size.

In terms of the instruments, the math subtests used contain a variety of skills; thereby, limiting the analysis of specific math skills (e.g., math fluency). Furthermore, each math skill (calculation and math reasoning) was only measured using one subtest. Additionally, this study was limited by only examining cognitive and neuropsychological skills using one instrument (NEPSY-II). In addition to the study being limited to one neuropsychological instrument, the Semantic and Visouspatial domains contained several subtests, while the Procedural domain only consisted of three subtests (Inhibition, Auditory Attention, and Response Set). The Procedural domain is theorized to include numerous cognitive abilities (Geary 1993), which were not sufficiently represented with the three subtypes used in the current study. Results from Swanson and Beebe-Frankenberger's (2004) study implicate working memory as a contributor to math performance across grades; however, the current study did not use a direct measure of working memory, such as Digits Backward. Given the limitations of the current study, suggestions for future direction follow.

Recommendations for Future Research

Advancing research related to mathematical processes is critical, as data from the 2011 National Assessment of Educational Progress reveal that 35% of eighth-grade public school students perform within the "below basic" category (National Center for Education Statistics, 2011). Furthermore, math achievement gaps between races still exist, with the gap between White and Hispanic students growing larger. Another area of concern is the lower achievement of public school students when compared to private school students. Statistics also reveal that more students performed in the "below basic" range as eighth graders than as fourth graders. Results of national examinations, such as the one just described, underscore the importance of research related to math abilities and disabilities. Even though the current study adds to the literature base examining math, further research is critical to determining the cognitive and neuropsychological components attributable to math performance.

Results of the current study do not support the applicability of Geary's three domains to math calculation and problem solving skills; however, the limitations previously described limit the generalizability of the current results. The current results were consistent with a previous study (Cirino, Morris, & Morris, 2007) regarding the contribution of language/semantic skills to problem solving abilities. Unlike the previous study, the current study failed to find a significant contribution of visuospatial skills to problem solving performance. Additionally, research suggests that executive functioning skills, such as working memory, are essential components of math problem solving. Therefore, future researcher studies should further examine the cognitive and neuropsychological factors implicated in math problem solving by using more than one subtest for each theorized executive function component (e.g., attention, shifting, and inhibition). Additionally, future researchers should examine math performance at each age since developmental and physiological changes occur throughout the lifespan. Specifically, how do changes with executive procedural skills, such as working memory, affect math performance?

The current study, as well as previous studies (e.g., Mazzocco & Myers, 2003), used mathematical assessments that consisted of various skills. The Numerical Operations subtest of the WIAT-II ranges from items measuring basic addition and subtraction to more complex questions requiring knowledge of geometry and algebra. As such, future researchers should examine discrete skills, such as long division or math fact fluency, to determine the contribution of Geary's subtypes to discrete skills.

Implications for School Psychologists

As students continue to demonstrate below basic mathematical performance (National Center for Education Statistics, 2011), school psychologists play an integral role in identifying the strengths and needs of students and assisting in development and implementation of interventions to remediate student needs. School psychologists and other assessment professionals need to consider the cognitive and neuropsychological skills implicated in math performance when selecting instruments to identify mathematical difficulties. School psychologists should consider including reliable measures of visuospatial and semantic skills in their assessment batteries when examining mathematical difficulties. As suggested by the current research, the Comprehension of Instructions, Inhibition, and Design Copying subtests of the NEPSY-II correlated significantly with both math calculation and math reasoning; therefore, school psychologist should consider adding these subtests to their test batteries when examining math deficits. The current research also highlighted the contribution of language skills to math reasoning performance. Given the relationship between language skills and math reasoning, school psychologists should examine various aspects of language, such as receptive language, when examining math reasoning deficits.

In addition to identifying strengths and weaknesses using a neuropsychological battery, school psychologists also need to be able to identify appropriate interventions given the student's profile. Results from the current research suggest the influence of language abilities on math performance; therefore, interventions aimed at language deficits may prove beneficial to improving math performance.

Conclusion

Results from this study implicated visuospatial skills to mathematical calculation performance, while semantic skills contributed significantly to mathematical reasoning performance. Continued research in the area of mathematics performance is needed, as the extant literature has failed to determine the cognitive and neuropsychological skills implicated in mathematics performance. A better understanding of the specific cognitive and neuropsychological deficits related to mathematical difficulties is needed in order for educators to be able identify interventions that address areas of weakness.

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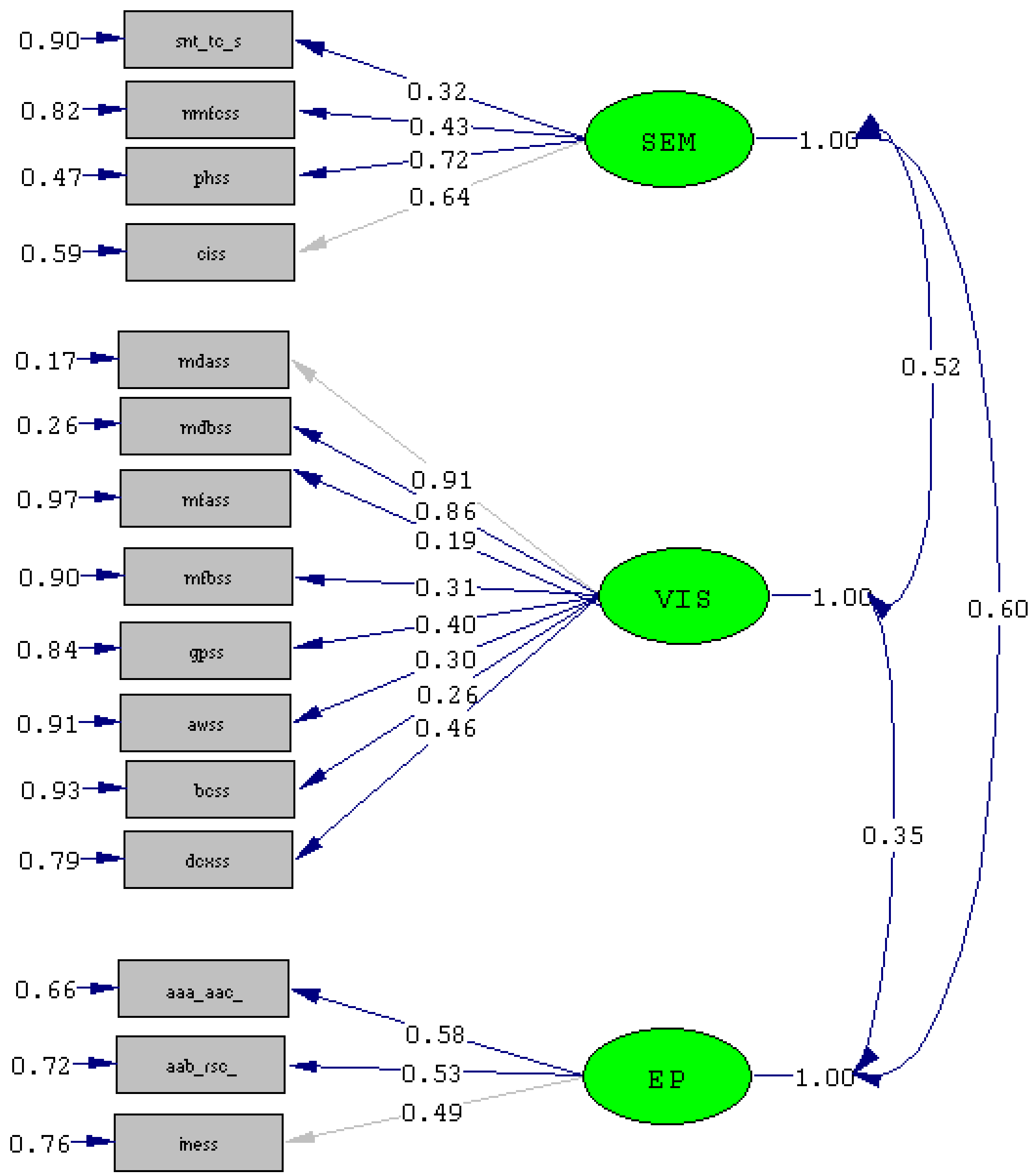
Appendix A

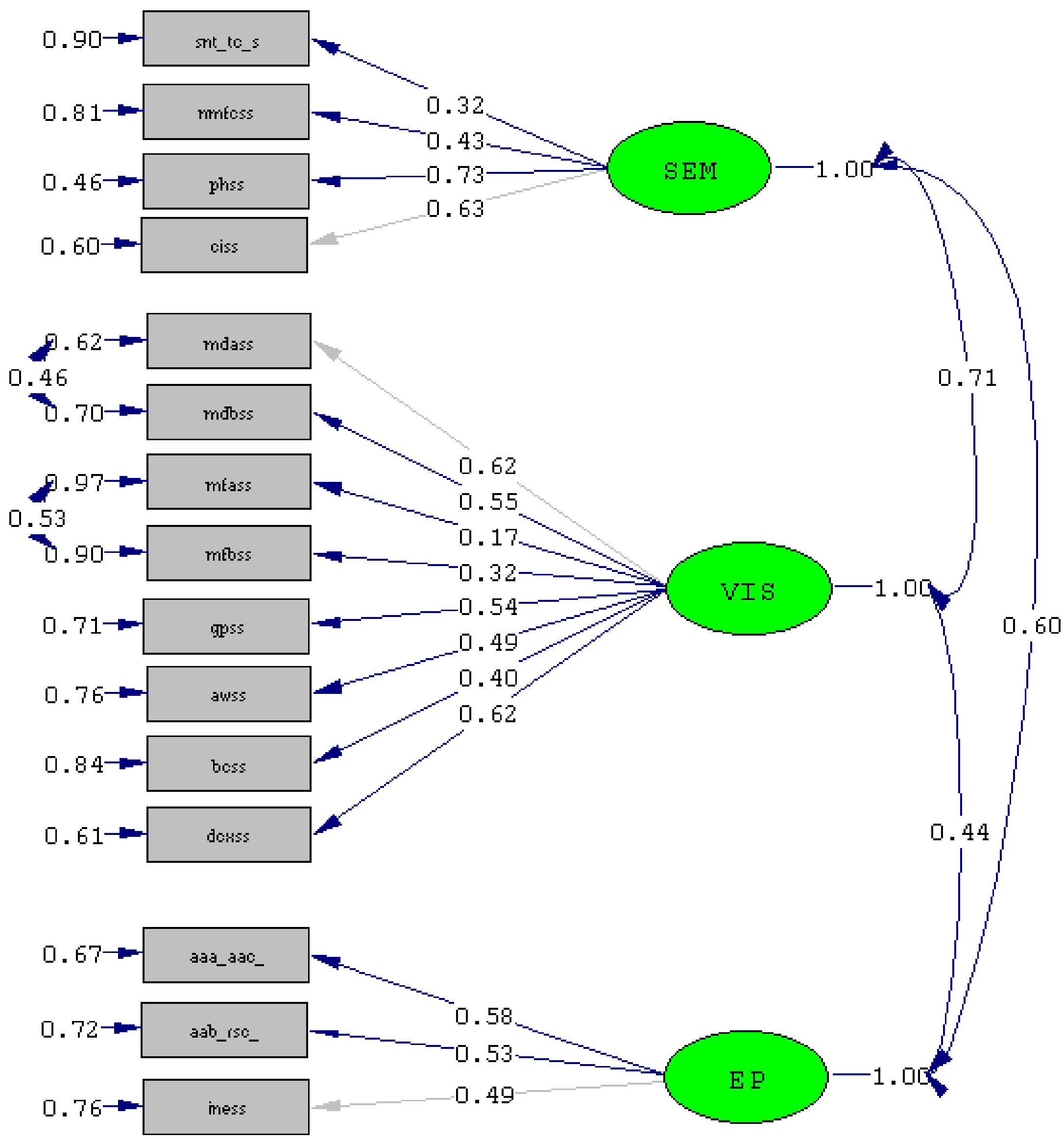
Path Model with Standardized Coefficients for the Initial Model



Note: snt_tc_s = Speeded Naming; nmfcss = Narrative Memory; phss = Phonological Processing; ciss = Comprehension of Instruction; mdass = Memory for Designs; mdbss = Memory for Designs Delayed; mfass = Memory for Faces; mfbss = Memory for Faces Delayed; gpss = Geometric Puzzles; awss = Arrows; bcss = Block Construction; dcxss = Design Copy; aaa_aac = Auditory Attention; aab_rsc = Response Set; iness = Inhibition; SEM = Semantic; VIS = Visuospatial; EP = Procedural.

Appendix B

Path Model with Standardized Coefficients for the Final Model



Note: snt_tc_s = Speeded Naming; nmfcss = Narrative Memory; phss = Phonological Processing; ciss = Comprehension of Instruction; mdass = Memory for Designs; mdbss = Memory for Designs Delayed; mfass = Memory for Faces; mfbss = Memory for Faces Delayed; gpss = Geometric Puzzles; awss = Arrows; bcss = Block Construction; dcxss = Design Copy; aaa_aac = Auditory Attention; aab_rsc = Response Set; iness = Inhibition; SEM = Semantic; VIS = Visuospatial; EP = Procedural.