Two dimensional Fourier Transformation

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Abstract

Image processing is discussed for RAW format in digital cameras. The points are (1) suppression of noises, (2) amplifier of colors, (3) projection of dynamic range. Base technique is Fourier transformation in finite period. The expression is known as for multi-dimensional forms. However, if it is applied simply for image photographed in fields, he/she would get less information than his/her expectation. The reason is to make inappropriate preprocessing for Fourier transformation. We propose preprocessing about (1~3) points; and as several typical cases, we show the effects.

Figures and graphs are drawn by using full color. The true images will be got from Web-pages.

Keywords: preprocessing, color intention, color amplifier, color Fourier transformation, white noise, thermal noise, digital camera imaging, sRGB, RAW

1. Introduction

Peoples believe that the mass* of targets relates with the emission intensity. However, visualizing processing in camera-firmware and graphic software has non-linear relations about the mass-intensity**. The non-linearity is not only intensity but also is found on coloring***. We want to make physical photographic techniques under the linear responses.

*) Here, physical significance one about amount of targets under the gravity field is called "the mass".
**) It corresponds with density of the targets.
***) It relates to the difference of elements of the targets.

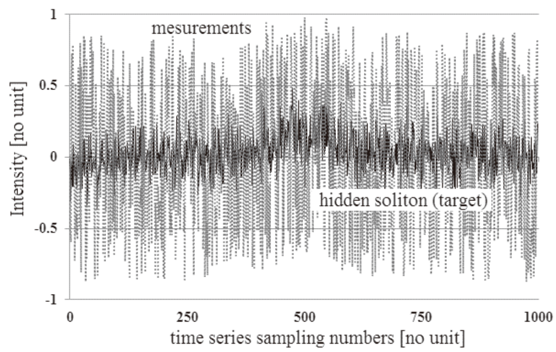

Figure 1: One dimensional model data generated from a soliton and uniform random numbers. Mixing ratio is 1:7. Our object is to get soliton (black line) wave in noises (purple dots).

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2. Objects and Noises

Targets in environmental fields are observed in noises; where signals of targets are weak and sometimes hidden in strong noises. The situation is displayed in one and two dimensional objects, as followings.
Our object is to detect the hidden targets and intensities in practicable computations. CPU time is less than 1 hour on i5-PC.

3. Fourier Transformation

Theory of Fourier Transformation (FT) is published [1]. A continuous function *F(t)* is expanded by the orthonormal set. Triangular functions {*sin(nt), cos(nt)*; *n*=0,1,...integer} is the set. $F(t)=\Sigma_0^\infty\{An\ sin(nt)+Bn\ cos(nt)\}$,

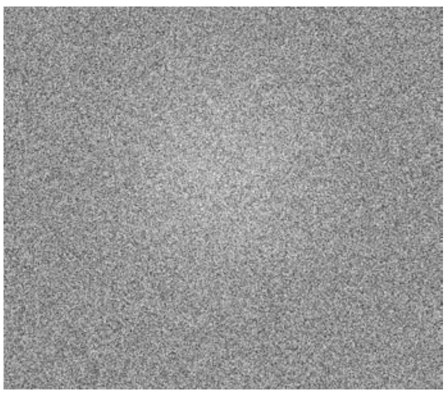

Figure 2: Two dimensional model data generated from a Gaussian and uniform random numbers. Mixing ratio is 1:7. The image is 721×721 pixels and monochrome.

$An=\int F(t)\sin(nt)dt,\ Bn=\int F(t)\cos(nt)dt.$ (1)

By sampling theory [2], Eq. (1) is limited within finite number N, $F(t)=\Sigma_0^N\{An\sin(nt)+Bn\cos(nt)\}$,

$A0=0,\ 2B0=\int_0^{2\pi}F(t)dt,\ 0\leq t\leq 2\pi,$

$\pi An=\int_0^{2\pi}F(t)\sin(nt)dt,\ \pi Bn=\int_0^{2\pi}F(t)\cos(nt)dt,$ (2)

Where Σ_0^N runs over n, and N is < sampling number/2. Using a function, $Ints(n)=Ln(An^2+Bn^2)^{0.5}$. (3)

The *Ints(n)* is used for detection the characteristics of target phenomena▲.

▲We recommend trial calculations for a triangular function and the combinations. Spectrum is sharp delta functions. It is another representation of triangular functions. Next, trying a Gaussian, the spectrum is condensed in lower frequency part. *That is*, so many waves are not required to express Gaussian.

Ln() is the natural logarithm. Spectrum of *F(t)* can be checked. Uniform random numbers, a sine and a Gaussian are constant values, a delta-function, and Gaussian, respectively. We extract a target from noises by using difference of the spectrum. An example is shown in Figure 3.

This result is get in case of target waves constructed with one triangular function. If the target is not triangular, and changes slowly; selection rule of *Ints(n)* of Eq. (3) must be modified by using conception of Lock-in amplifier. It is required to predict the future (*cf.*, Appendix).

Eq. (2) is rewritten as for 2D.

$F(x,y)=\Sigma_0^N\Sigma_0^M\{An\sin(nx)+Bn\cos(nx)\}$

$\{Cm\sin(my)+Dm\cos(my)\}.$ (4)

Since N and M of images are 500 ~ 1000, a calculation of Eq. (4) requires large CPU. We use major contributions,

$F(x,y)\sim[\Sigma_0^M\Sigma_0^N\{AnC0\sin(0)\sin(nt)+BnD0\cos(nt)\}$

$+\Sigma_0^N\Sigma_0^M\{B0Cm\sin(mt)+A0Dm(x)\sin(0)\cos(mt)\}]/2^*$

$=\Sigma_0^N B'n\cos(nt)+\Sigma_0^M C'm\sin(mt)$, *exchange M to N,*

$=\Sigma_0^N\{B'n\cos(nt)+C'n\sin(nt)\}=$※,

$C'n=\int F(x,y)\sin(nt)dt=C'n(y)$, *... therefore,*

※ $=\Sigma_0^N\Sigma_y\{B'n(y)\cos(nt)+C'n(y)\sin(nt)\}$,

under square region, using equivalent for x,y-direction,

$F(x,y)\sim[\Sigma_0^N\Sigma_y\{B'n(y)\cos(nt)+C'n(y)\sin(nt)\}$

$+\Sigma_0^M\Sigma_x\{A'm(x)\cos(mt)+D'm(x)\sin(mt)\}]/2,$ (5)

Eq. (5) shows that an image is constructed with FT-lines and amplitudes. The FT-lines are the scanning line. The pattern is swayed by noises. An effect of the approximation is displayed in below.

*) The "2" corresponds two sigma-operations, which express expansions for horizontal and vertical directions. These expansions are equivalent.

When we adopt $n\sim$N/10, Eq. (5) can be used for dispersion images of small particles in solvent. The effects are tested in sea water photography now.

4. Dynamic range of images

Dynamic range of a photo diode (PD) is $10^{4\sim5}$ (14~16 bits). The range is larger than 2^8 of human; therefore, we select a suitable image-format to satisfy the PD-spaces. We use 32 bits real in the Flexible Image Transport System (FITS [3]), which has 4.4T colors based on 14bits. This is too large color spaces, but to visualize the information, we must project them into 8-bits color spaces. It is dangerous; sometimes, plural significant phenomena wouldn't be detected. To avoid the defects, we must do FT in restricted zones (called as tiles).

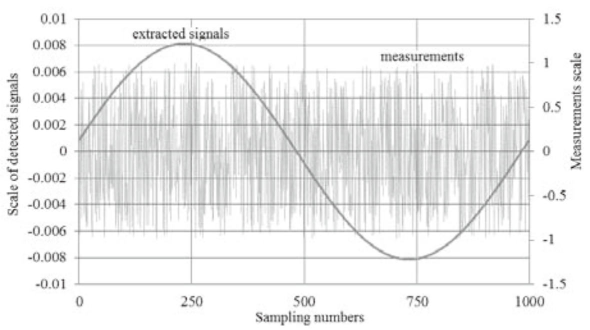

Figure 3: By using FT, a sine-wave of 1/128 in noises is detected. Detected intensity is complete, and the phase has small error.

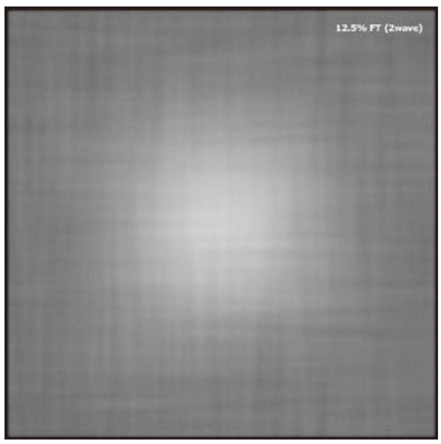

Figure 4: Image got from Figure.2 and Eq. (5). A Gaussian is found, which is hidden in noises.

In a tile, values of both terminals are different. The gradient is omitted by a linear-bias, and the both terminals are zeros. By introducing the linear-bias, it is enable to calculate FT; but, discontinuity lines are arisen at tile-boundary.

In a tile of N+1 pixels, a scanning line is a finite vector, $\{f_0, f_1,..f_N\}=\{f_i\}$. Where, scaling is introduced,

$f_i'=f_i-(m\ i+f_0),\ m=(f_N-f_0)/N$:=const., (6)

The $\{f_i'\}$ has zeros at both terminals. If large pulses exist near forced zero-terminals, the differences are very large; *i.e.*, it requires high frequency expansion-waves. It is possible that the pulse is arisen from noises. To suppress unnatural affects, some windows are adopted [4].

After FT, $\{f_i'\}$ is converted to $\{f_i\}$.

$f_i=f_i'+(m\ i+f_0)$. (7)

5. Colors

In sRGB space, the color is expressed by *F(x,y,r,g,b)*, where $\{x,y\}$ is the location of pixels, and $\{r,g,b\}$ is a color-intensity in interval of $[0,2^8-1]$. This is human's visual recognition world. The $\{r,g,b\}$ corresponds detected-intensities of [400,500], [500,600], [600,650] wavelength intervals in [nm]. It is not {r,g,b} in Jpeg-format, in which Jpeg-{r,g,b} is transformed from physical-$\{r,g,b\}$ by using 3×3 matrix*.

*) The matrix is not published as company's technical secret. An example is in the technical manual of StellaImage® software. The matrix is a mixing function among $\{r,g,b\}$ of physical-coloring.

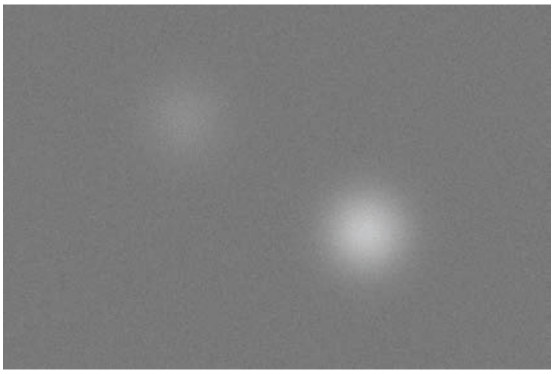

Figure 5: Image for testing color-intensity version of 2D FT. The size is 3018×2010 pixels.
There are 2 pale blue and red Gaussians in noises. The nose seems as plain gray; but the close-up is random dots of {r,g,b} 3 pure colors.

F(x,y,r,g,b) is converted into the average brightness and the displacements.

$B(x,y)=\{F(x,y,r)+F(x,y,g)+F(x,y,b)\}/3,$

$dR(x,y)=B(x,y)-F(x,y,r)$, *likewise g, b.* (8)

Each expression is 2D matrix data, which are processed by FT; and the noises can be suppressed.

$extF(x,y,r)=B(x,y)-AF(x,y,r),\ 1\le A.$ (9)

The A is a color-amplify constant, and it is same values for *g* and *b*. The *extF(x,y,r,g,b)* is in an extended-space from sRGB. Since it is invisible, it is necessary to project images to sRGB space. On requirement of the linearity, we select affine map;

$F(x,y,r)=255(extF(x,y,r)-x_{min})/(x_{max}-x_{min}),$

$x_{max}=max(extF(x,y,r)_{max}, extF(x,y,g)_{max}, extF(x,y,b)_{max}),$

$x_{min}=min(extF(x,y,r)_{min,....}),$ (10)

We test the approach in case of model calculation. The model is that there are 2 Gaussians in white noises, the peaks are 0.5, 0.125, 0.5. Full width at half maximum

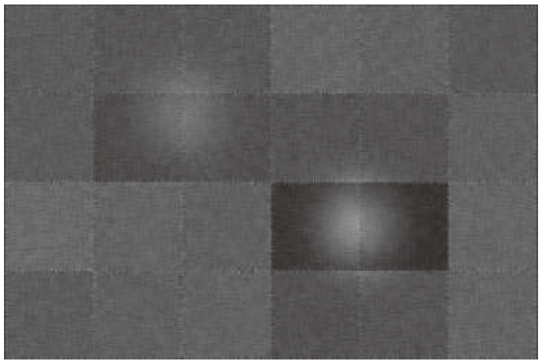

Figure 6: Image processed by 2D FT with color-intensity function. It is clear that there are 2 Gaussian, whose color is blue and red. Back ground noise (gray) is suppressed more dark. It is black in case of the complete calculation (Figure 7).

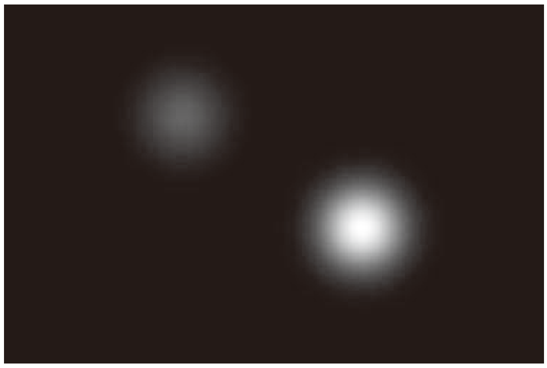

Figure 7: Expected image. This is noises are eliminated artificially, and Gaussians are amplified by 2 times. It is enabled, when we obtain true-equations of phenomena.

(FWHM) of the major Gaussian is 160, 180, 200 [pixel], and another is 200, 180, 160. They are pale blue and red (in Figure 5).

2D FT with color-intensity function gives Figure 6. The FT is processed under 500×500 pixel tiles, 10 times color-intensity, 5-expansion waves.

We test more difficult 2 images listed in Figure 8 and 11. Background level of Figure 8 is flat, and Figure 11 decreases along curve of 4^{th} power of cosine. They include 50 Gaussians in white noises, the peaks are 0.1~0.9 (average 0.5) and 0.5. FWHM is 10~90 (average 50) [pixel]. Colors depend on the difference among $\{r,g,b\}$-peaks of Gaussian. There is no complex color in one Gaussian. The image is Figure 8 and 11.

6. Conclusion

We discuss (1) suppression of noises, (2) intention of colors, (3) projection of dynamic range, and propose expressions to execute those objects. They are confirmed and tested on getting correct results and predicted images in reasonable CPU time. By using RAW format photography and 2D FT, we can research until invisible phenomena beyond human vision.

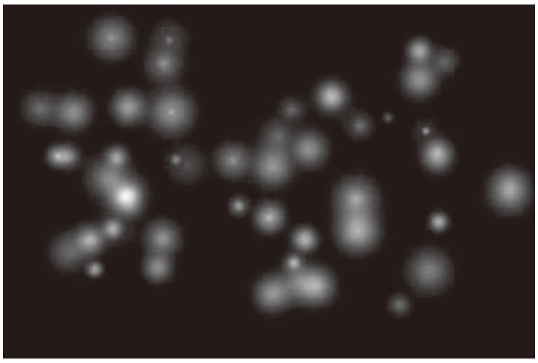

Figure 10: Expected image of 2D FT. White has not color signal; thus, the Gaussian cannot be intensified.

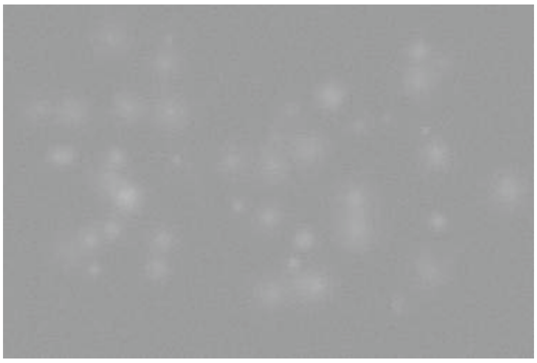

Figure 8: Test image of 50 Gaussians (background flat). Image is 3018×2010 pixels. Location, peak intensity, and colors of the Gaussians are calculated by uniform random numbers.

Figure 11. Image included optical-dimming whose intensity at the corner is 0.003 for center=1. The original image is in Figure 8.

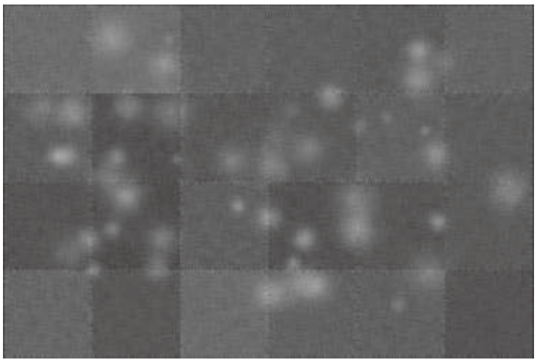

Figure 9: Image processed by 2D FT with color-intensity function. Whole Gaussians can be detected, whose colors are recognized.

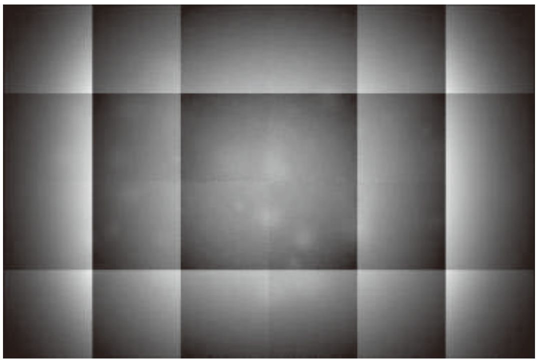

Figure 12: Image processed by 2D FT with color-intensity function. About 16 Gaussians can be detected with colors, where are in 2nd tile from the center.

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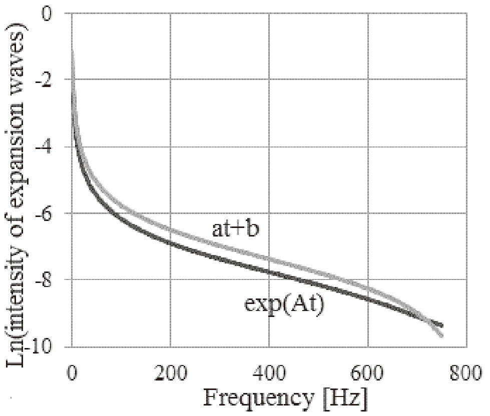

Figure A1. Fourier Transform spectra of the linear and exponential functions, where {a,b}={1,0}, A=Ln(1/3). The spectrum is condensed in low frequency area, and is decreased continuously. The character should be introduced in Fourier Transformation of targets.

Appendix: Detection of small slopes in normal random numbers

We show detections of a small slope hidden in random perturbations. It is an important technique to predict the future. The approach is based on the conception of Lock-in amplifier [B1].

We use computer normal random numbers instead of the perturbations, and the slopes are linear and exponential functions.

These expressions are, $f(t)=at+b$ for the linear function, and $g(t)=\exp(At)$, $A<0$, for the exponential function, and the normal random numbers are $h(t)=\{\Sigma_{[1,12]}rand(t)\}-6$, derived by the central limit theorem [B2].

Where the "t" is in period [0,L], the L is 2π and 1 for $f(t)$ and $g(t)$. The $\{a,b\}$ is a constant, $A=Ln(1/3)/L$, and $rand(t)$ is the uniform random numbers built in computer languages. Mixing of the perturbation is $W_L(t)=Cf(t)+(1-C)h(t)$ or $W_E(t)=Cg(t)+(1-C)h(t)$, in which C is a constant, and period is $0<C<1$. The sampling number is fixed to 1600. If we use more large numbers, better results are got. Under those conditions, we go to the next step.

We calculate the Fourier Transform spectra of the linear and exponential functions. It is following functions.

The spectra are represented as vectors of 15 elements*, {-1.14473, -1.83788, -2.24335, -2.53104, -2.7542, -2.93654, -3.0907, -3.22425, -3.34206, -3.44744, -3.54278, -3.62982, -3.7099, -3.78404, -3.85307}:=*Lv(1:15) for the linear-function*, and {-1.56519, -2.24709, -2.65045, -2.9374, -3.16021, -3.34236, -3.49641, -3.62989, -3.74765, -3.85299, -3.9483, -4.03532, -4.11538, -4.18951, -4.25853}:=*Ev(1:15)*

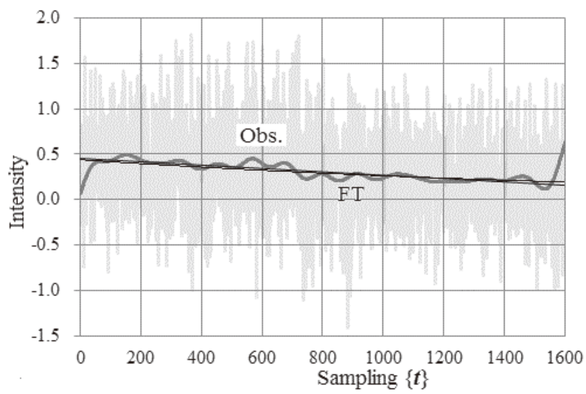

Figure A2. Fourier Transformation of $W_E(t)$ of C=0.5, and the inverse transformation until 15 expansion waves.

for the exponential function. (*The limit is for small encoding routines.) Using *Lv()* and *Ev()*, Eq. (3), $Ints(n)=Ln(An^2+Bn^2)^{0.5}$, is written as;

$Df=Ints(1)-Lv(1),\ Cut(n)=Lv(n)-Df,\ 1<n<16,$ (A1)

$If\ \{Ints(n) < Cut(n)\}Ints(n)'=Ints(n),$ (A2)

$If\ \{Ints(n) > Cut(n)\}Ints(n)'=Cut(n),$ (A3)

$If\ \{n>15\}Ints(n)'=Ints(n).$ (A4)

Same equations are derived in case of *Ev(1:15)*.

$Rf=exp\{Ints(n)'\}/exp\{Intes(n)\},$ (A5)

$An'=AnRf^2,\ Bn'=BnRf^2,$ (A6)

By using the rewritten {*An', Bn'*}, inverse Fourier Transformation is executed. As overview of those equations, the effect cuts vibrating waves in high frequency.

We show a case, $W_E(t)$ of *C*=0.5, in Figure A2.

The pale blue zigzag curve is Obs. $W_E(t)$. The blue bold curve (FT) is inverse transformation; two black lines are regression lines for the FT and Obs, whose expressions are,

$g(t)=0.439exp(-5.17\times10^{-4}t)$, $R^2=0.506$,

$f(t)=-1.87\times10^{-4}t+0.455$, $R^2=0.0286$.

A regression line is got from Obs. $W_E(t)$ directly. Exponential function is not derived. Because, $W_E(t)$ near *t*=0 vibrates between plus and minus. The determination coefficient is small. Thus, no reliable prediction is got. An exponential function hidden in random noises is got from Fourier Transformation. The determination coefficient is reasonable value because of noises is 50%. The expectation of A is -6.88×10^{-4}, and a coefficient of the exponential function is 0.500. Regression exponential function is reasonable.

Thus; the Fourier Transformation approach is a superior technique to evaluate hidden function. It is a fact certainly; however, we consider that the difference between the linear and exponential functions is little even in case of random noise of 50%. If there is more difficult condition, is it enable that the two functions are distinguished? We believe the difference of two functions is no means, only gradient in measurement period has information for the future. If the prediction is done under following exponential smoothing method [B3], the gradient is significant factor.

$C(i+1)=\omega C(i)+(1-\omega)D(i),\ 0<\omega<1,\ \omega=dumping\ factor,$ (A7)

Where, *{C(i)}* is prediction, and *{D(i)}* is observations. The "*i*" is current time, index after "*i+1*" is the future. The expression predicts one step future, and gives good trace for disturbing phenomenon; however, more steps cannot predict.

Eq. (A7) is expanded in [*i-j,i*]:={*k*} period,. Using weight *W(k)* and least squares method, we get;

$C(i+j)=\sim C(i\text{-}j:\ i)+j\ Div(i\text{-}j:\ i),\ 0<j<N,$ (A8)

$\sim C(i\text{-}j:\ i)=\Sigma_{k=(i-j,\ i)}C(k)W(k)/\Sigma_{k=(i-j,\ i)}W(k),$ (A9)

$Div(i\text{-}j:\ i)=\{N\Sigma_{k=(i-j,\ i)}jC(k) - (\Sigma_{k=(i-j,\ i)}j)(\Sigma_{k=(i-j,\ i)}C(k))\}/\{N\Sigma_{k=(i-j,\ i)}j^2 -(\Sigma_{k=(i-j,\ i)}j)^2\}.$ (A10)

The Eq. (A10) is stable in random perturbations, but Eq. (A9) is not. It depends on {*W(i)*}. If elements to determine the future are dispersed homogeneously in the period, all *W(i)*s are equivalent. It is Simple Moving Average (SMA). If elements near the last have more information, Triangular Moving Average (TMA) or Gaussian Moving Average (GMA) is appropriate. GMA equals to Kolmogorov-Zurbenko filter.

We wonder those averages have explicit information of the gradient near the last point. It is introduced as {*W(i)*} implicitly. The defect should be improved. If an approximation gradient is got from Eq. (A10), as it is *Div(i-j: i)*, Eq. (A9) is rewritten;

$\sim CC(i\text{-}j:\ i)= \sim C(i\text{-}j:\ i)+\Sigma_{k=(i-j,i)}Div(k)(k-i)\ W(k)/\Sigma_{k=(i-j,\ i)}W(k).$ (A11)

Using Eq. (A11), we get Table A1.

Table A1: Comparison with some averages

Rand Ratio*	SMA	TMA	GMA
0.5	0.499	0.500	0.500
0.9	0.104	0.101	0.100

* At the same time, it is the expectation.

Detecting phenomenon of one-shot, you develop equipment. The measurements are hidden in strong noises. The governing equation is known by theoretical derivations. Such a condition is often found now. To detect the phenomenon in high precision, the appendix is written.

Even if there is another phenomenon, it is not detected. But, you develop the equipment to detect phenomenon under derivative governing equation. On statistical visions, the contrary phenomenon must be discussed. The determination coefficients should be compared. There are infinitive contrary phenomena; thus the approach cannot be executed.