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Synthesis of the Magnetic Field Using Transversal 3D Coil System

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***Abstract*—Magnetic field is usually generated using magnets realized as a set of simple coils. In general, those magnets generate magnetic field with nonzero components in all directions. Usually during the design process only one component of the magnetic field is taken into account, and in the optimisation procedure the currents and positions of simple coils are found to minimize the error between the axial component of the magnetic field and the required magnetic field in the ROI. In this work, it is shown that if the high quality homogeneous magnetic field is generated then indeed one may neglect non-axial components. On the other hand, if the obtained magnetic field is not homogeneous either due to design requirements of too restrictive constrains, then all other components may severely deteriorate the quality of the magnetic field. In the second part of the paper, we show how to design a 3D transversal coil system to solve problems which are intractable in the 1D case.**

***Keywords*—gradient coils, homogeneous magnet design, optimization.**

I. Introduction

The problem of magnetic field synthesis is to find electric current density distribution over a limited volume which generates desired magnetic field in the given region of interest (ROI) [1], [2], [3]. Key parameters characterizing magnetic devices are the magnitude and the shape of the magnetic field generated in the region of interest (ROI) [4]. For example, two kinds of the magnetic field in the ROI are required for the Magnetic Resonance Imaging (MRI) purposes [5]. The main field has to be very strong and extremely homogeneous both in the value and the direction. The homogeneity requirement is very important since picture quality strongly depends on it. It follows, that ideally in a three-dimensional ROI the magnetic field vector should have a large constant component in one direction and zero components in the two remaining directions. This exquisitely uniform and very intense magnetic field is used to polarize the spin population of a sample so as to maximise the strength of the nuclear magnetic resonance (NMR) signal [6]. The second type of field is the gradient field applied to produce controlled variations in the main magnetic field. Three orthogonal gradient coils are used to produce controlled gradient magnetic fields, which superimposed on the main magnetic field force selective spatial excitations of the imaging volume.

Coil design (magnetic field synthesis) problem is described by a Fredholm equation of the first kind, which is known to be ill-posed [7]. Early approaches to coil design in MRI cancelled undesired spherical harmonic components of the magnetic field by symmetry and appropriate positioning of loops and arcs of wire [8] or by parametrized surface current densities [9]. In this work, the coil design problem is presented as the problem of finding the positions and/or currents of simple coils (loops with a current) that can generate a specified magnetic field in the ROI [10]. The ROI is usually a sphere or a cube located inside the coil. We will assume that simple coils are circular and centered along their common axis of symmetry. The 3D design problem is illustrated in Fig. 1, where three groups of coils are located along the three space directions. Each simple coil is defined by the position x_i, y_i or z_i of its center along its axis of symmetry, the radius r_i, and the current I_i.

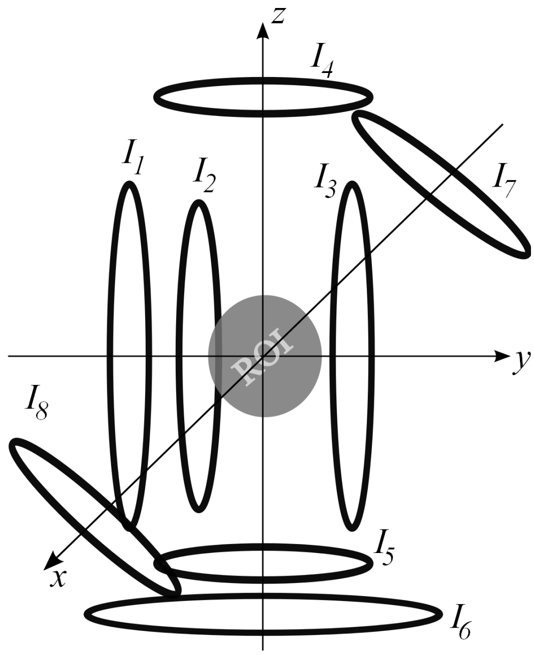

Fig. 1. The systems of eight transverse simple coils designed to generate magnetic field in the region of interest (ROI)

When a simple coil (for example centered along the z axis) is considered, it is convenient to use the cylindrical coordinates (z, r, φ). Due to symmetry the magnetic fields at the point (z_j, r_j, φ_j) does not depend on the φ_j coordinate. Since the coils are circular and coaxial, it follows that the B_φ component is zero. The components B_z and B_r will be called the *axial* and the *radial* component, respectively. The magnetic field at the point (z_j, r_j) generated by a simple coil with parameters

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(z_i, r_i, I_i) is equal to [5]:

$$B(z_j, r_j, z_i, r_i, I_i) = \frac{\mu_0 I_i}{2\pi\sqrt{(r_i + r_j)^2 + (z_j - z_i)^2}} \cdot \left[\frac{(z_i - z_j)}{r_j} \left(K(k) - \frac{r_i^2 + r_j^2 + (z_j - z_i)^2}{(r_i - r_j)^2 + (z_i - z_j)^2} E(k) \right) a_r + \right. \left. + \left(K(k) + \frac{r_i^2 - r_j^2 - (z_j - z_i)^2}{(r_i - r_j)^2 + (z_i - z_j)^2} E(k) \right) a_z \right]. \quad (1)$$

where $E(\cdot)$ and $K(\cdot)$ are the complete elliptic integrals of the first kind, $k^2 = 4r_i r_j/((r_i + r_j)^2 + (z_i - z_j)^2)$, and a_z and a_r are the unit vectors in the z and r directions. Using the superposition principle the total magnetic field at (z_j, r_j) can be computed as a sum of fields generated by all simple coils.

Under some specific conditions the radial component is very small when compared to the axial component due to the effect called "quadrature suppression". Let us assume that the generated field can be expressed as the required field B_0 and error terms in the form

$$B_z = B_0 + \varepsilon_z B_0, \quad B_r = \varepsilon_r B_0. \quad (2)$$

If ε_z and ε_r are small with respect to 1 then the relative error between the magnitude of the magnetic field $|B| = \sqrt{B_z^2 + B_r^2}$ and B_0 is

$$\delta = \frac{||B| - B_0|}{|B_0|} = |\sqrt{(1 + \varepsilon_z)^2 + \varepsilon_r^2} - 1| \approx \varepsilon_z + 0.5\varepsilon_r^2. \quad (3)$$

In [11], it has been shown that if the generated magnetic field is homogeneous then ε_z and ε_r have the same order of magnitude. It follows that in this case ε_r has negligible influence on the total error and that the radial component B_r can be neglected. In the following sections we will discuss this problem for different types of generated magnetic fields.

II. Shaping magnetic field using 1D coil system

In this section, we study the possibility of solving the problem of magnetic field design using coils located coaxially along one axis (1D coil system). We will investigate the influence of non-axial field components in situations when the desired field is homogeneous and linearly increasing (gradient field).

A. Homogeneous magnetic field

Let's study the coil design problem for the case when the goal is to generate a homogeneous magnetic field along the z axis [10], i.e. $B = B_0 a_z + 0a_x + 0a_y$, where a_x, a_y, a_z are the unit vectors in the three space direction.

In [13], the design problem is defined as a linear programming problem that minimizes the power consumption under constrains that at each of m target points located in the ROI, the axial component of the magnetic field satisfies the condition $|B_z - B_0| \leq \varepsilon B_0$. The quality parameter ε is usually in the the range 1 to 10 ppm.

One of the problems studied in [13] is the minimum-power bi-planar mammography magnet, where simple coils

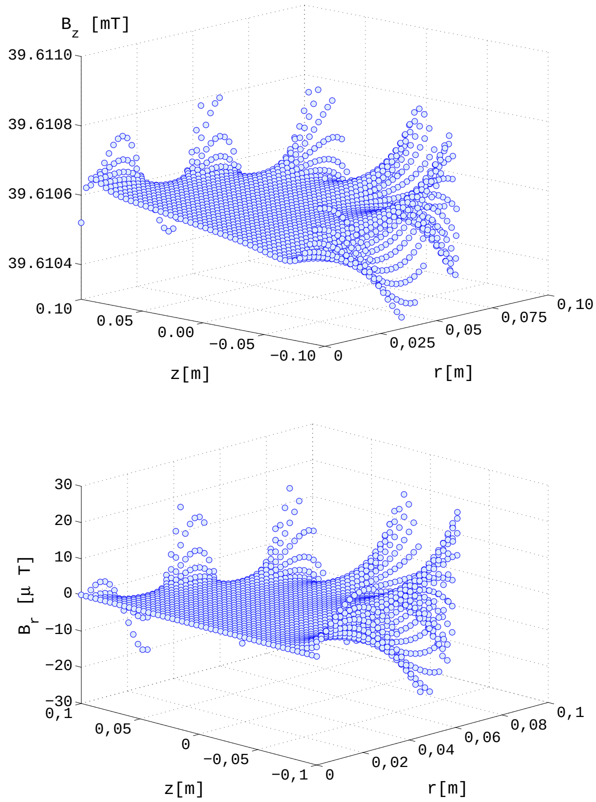

Fig. 2. The axial component B_z and the radial component $B_r = \sqrt{B_x^2 + B_y^2}$ of the magnetic field over the circular ROI with diameter 20 cm for the solution presented in [13]

are placed in the plane 30 cm below the ROI center and in the plane 15 cm above the ROI center. A solution with 6 simple coils (2 below and 4 above the ROI) is found. Fig. 2 presents the axial and radial components of the magnetic field computed at the border of the circular ROI with the diameter 20 cm for the solution found in [13] (compare Table II of [13]). The minimum and maximum values of the axial component over the ROI are $3.9610321 \cdot 10^{-2}$ T and $3.9610814 \cdot 10^{-2}$ T. The difference is $4.93 \cdot 10^{-7}$ T, and the homogeneity of the magnetic field is at the order of 10 ppm which is a standard value for MRI purposes. Note that the other two components B_x and B_z are much smaller then the value of B_z. Maximum value of $|B_x|$ and $|B_y|$ are at the level of $2.5 \cdot 10^{-7}$ T. The maximum relative error δ between $|B| = \sqrt{B_z^2 + B_x^2 + B_y^2}$ and B_z is approximately 10^{-12} so the radial component has practically no influence on the magnitude of the magnetic field, and hence it can be neglected in the design process.

Fig. 3 presents the radial and axial components of magnetic field at the border of the ROI for the solution proposed in [14] (compare Table II of [14]). The B_z component of the magnetic field is at the homogeneity level of 21 ppm. Note that the B_r component is a couple of orders of magnitude smaller than the B_z component. Fig. 4 shows the ratio B_r/B_z. Its maximum is below 10^{-5}. It follows that the maximum relative error δ between $|B| = \sqrt{B_z^2 + B_r^2}$ and B_z is below $5 \cdot 10^{-10}$.

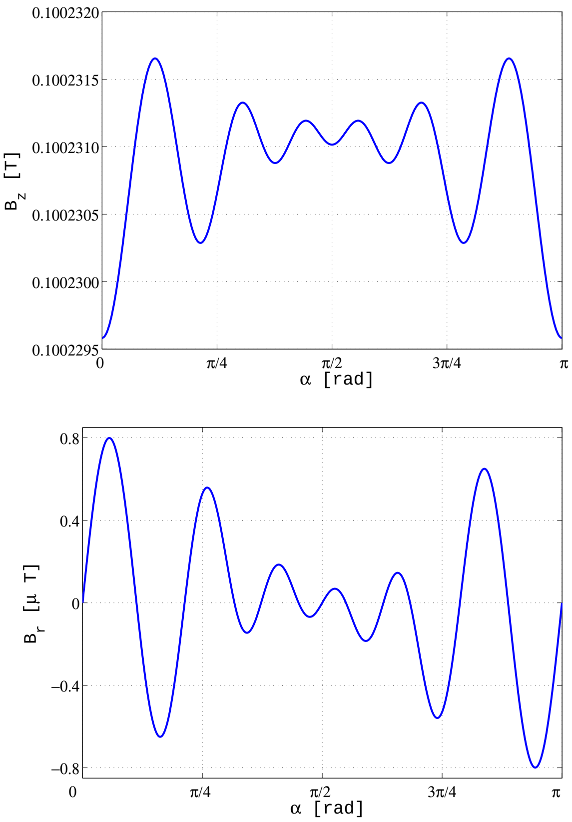

Fig. 3. The axial component B_z and the radial component B_r of the magnetic field over the circular ROI with radius 9 cm for the solution presented in [14]

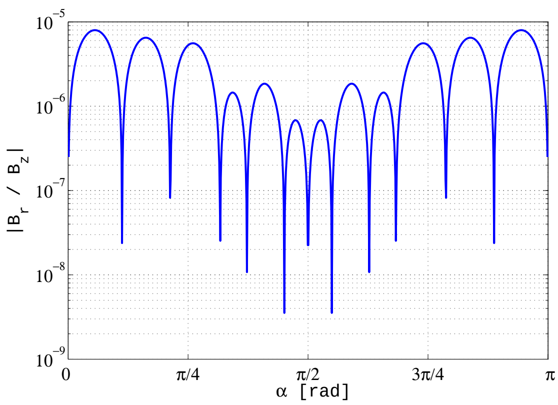

Fig. 4. The ratio B_r/B_z over the circular ROI with radius 9 cm for the solution presented in [14]

Summarizing, we have shown that indeed, if high quality homogeneous magnetic field is obtained then one may neglect the radial component in the design process and this has no practical effect on the magnitude of the magnetic field. The problem when homogeneous field has to be generated in the direction which is not perpendicular to coils planes is studied later in the paper.

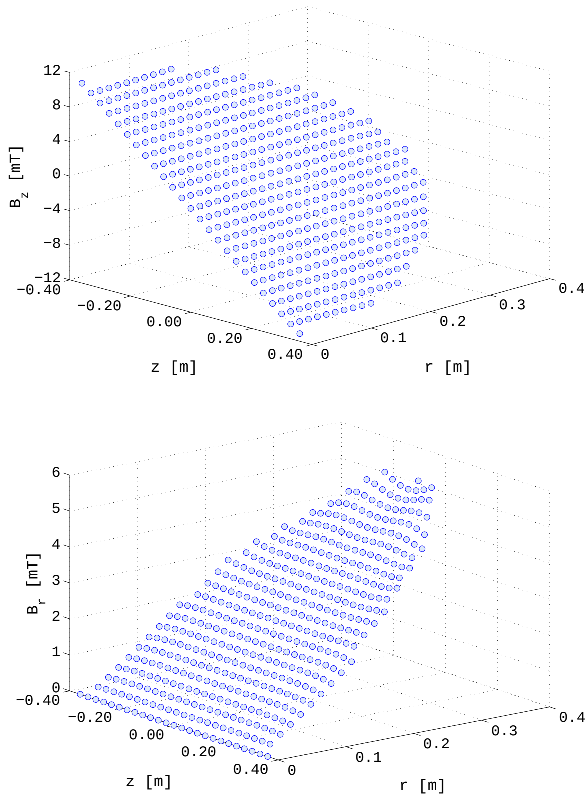

Fig. 5. The axial component B_z and the radial component $B_r = \sqrt{B_x^2 + B_y^2}$ of the magnetic field over the ROI with radius 36 cm for the solution presented in [15]

B. *Gradient magnetic field*

The z-gradient coil is the coil that in the region of interest excites magnetic field with the constant value of the derivative $G_z = \frac{\partial B_z}{\partial z}$, i.e. the axial component grows linearly with z [10]. The simplest way to achieve this goal is to use the Maxwell coil, i.e. a pair of simple coils with radius r at positions $z = \pm 0.87r$ with currents flowing in opposite directions. Maxwell coil produces magnetic field with the gradient of B_z varying less than 5% with the ROI being a sphere of radius $0.5r$. An improved solution with two pairs of simple coils is presented in [15]. Parameters of the simple coils are following

$$z_1 = 0.44r, \quad z_2 = 1.19r, \quad I_1 = 7.47I_2.$$

This solution provides variance of the gradient not larger than 5% within the sphere of radius $0.8 \cdot r$.

Fig. 5 shows the radial and axial components of magnetic field over the ROI for the coils with radius $r = 45$ cm. The radius of the ROI is $0.8r = 36$ cm. It is interesting to note that the radial component in the ROI increases almost linearly along the r axis. The maximum value of the radial component is almost half of the maximum value of the axial component. This example shows that radial component is quite significant and has a large impact on the magnitude of the magnetic field in the region of interest.

As a conclusion we note that generation of magnetic field with constant gradient in one direction influences significantly magnetic fields in the orthogonal direction. One way to solve this problem could be using simultaneously three orthogonal coils to minimize non-axial components of the magnetic field.

III. Shaping magnetic field using transversal 3D coil system

In the previous section, we have shown that when the desired magnetic field needs to be generated in the direction perpendicular to the coils planes the other components generated by the achieved coils system do not influence the desired field. In this section, we study the problem when the homogeneous field needs to be generated in an arbitrary direction using transversal 3D coil system. We will assume that the homogeneous magnetic field B_0 of 10 [mT] needs to be generated in the radial direction for $\varphi = \pi/4$ and $\theta = [0, \pi/2]$ in the spherical coordinates.

A. The Least Squares Method

Let us assume that there are n simple coils with currents $i_1,\ i_2, \ldots, i_n$ located coaxially along three axis of the coordinate system (see Fig. 1). Let us choose m target points evenly distributed in the ROI. The desired values of each component of the magnetic field are $\mathbf{b}_{\text{req}} = (b_{x1}, b_{x2}, \ldots, b_{xm}, b_{y1}, b_{y2}, \ldots, b_{ym}, b_{z1}, b_{z2}, \ldots, b_{zm})^T$. The problem can be presented as the least-squares (LSQ) optimisation problem [16]. The LSQ solution is the one that minimizes the sum of squares of residual errors for all target points. Since the relation between the field at target point and the current for a given coil is linear (compare (1)), one can formulate the problem as an overdetermined set of linear equations:

$$\mathbf{A} \cdot \mathbf{i} = \mathbf{b}_{\text{req}}, \tag{4}$$

where $\mathbf{A} \in \mathbb{R}^{3m,n}$ is the coefficient matrix, $\mathbf{i} \in \mathbb{R}^n$ is a vector of the currents to be found, and $\mathbf{b} \in \mathbb{R}^{3m}$ is a vector describing the required field at the target points. Solution of (4) can be expressed by

$$\hat{\mathbf{i}} = \arg\min_{\mathbf{i}} \|\mathbf{A}\mathbf{i} - \mathbf{b}_{\text{req}}\|_2^2, \tag{5}$$

where $\|\cdot\|_2$ denotes the Euclidean norm. It is well known that minimum of (5) can be found solving the set of normal equations:

$$\mathbf{A}^T\mathbf{A}\hat{\mathbf{i}} = \mathbf{A}^T\mathbf{b}_{\text{req}}. \tag{6}$$

The matrix A is usually ill-conditioned (especially for large n), which may lead to propagation of numerical errors, and in consequence to wrong solutions. One can use regularisation methods to avoid such problems [17]. For the test problems in the paper the coil numbers n is limited ($n \leq 18$) and additionally the QR method is used to solve the set of normal equations. This allows us to obtain acceptable solutions without the necessity of using regularization techniques.

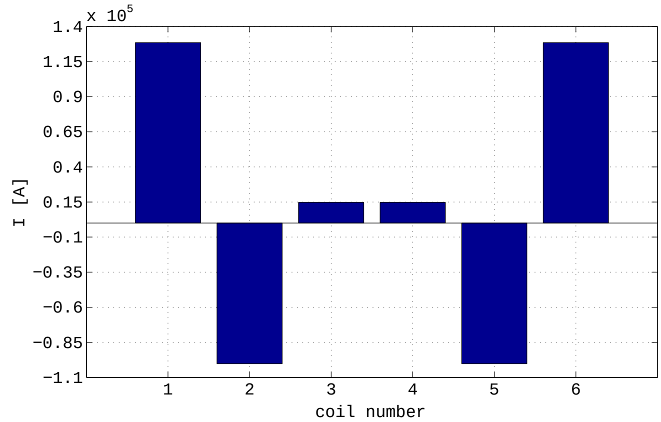

Fig. 6. Current distribution in each coil when $\theta = 0$ and the $1D$ coil system is used.

B. Test results and discussion

Table I presents the geometrical specification of coils positions used in tests. Two cases are considered. In the first one the total number of 18 simple coils is used—six in each directions, while in the second case 12 coils are used— in each directions. Coils are symmetrically positioned in all spatial directions. The ROI is the sphere of diameter 20 cm located in the center of the coordinate system. During the simulations, $m = 1419$ test points evenly distributed in the ROI are used.

TABLE I
Geometrical specification of the coils used for the design tests. All dimensions are in cm and represents the one of three sets of coils in all directions.

coil no	case 1		case 2	
	radius	axis position	radius	axis position
1	80	-120	80	-80
2	60	-100	45	-80
3	40	-80	45	80
4	40	80	80	80
5	60	100	–	–
6	80	120	–	–

Fig. 6 presents the solution (6) obtained for the case when $\theta = 0$ (field is perpendicular to the coils) using 1D coil system with six coils. Fig. 7 presents the corresponding magnetic field in the plane $y = 0$. The value of the residual error $\|B - B_{\text{req}}\|$ is $1.447228381491 \cdot 10^{-7}$. The maximum value of B_z over the whole ROI is 10.00002520002 [mT] and the minimum value is 9.999990976816 [mT]. Hence, the inhomogeneity is kept on the level of 34.2 ppm. When instead of six coils only four coils are used the residual error increases to $4.161372969709 \cdot 10^{-5}$ and inhomogeneity is at the level of 6.73 ppm.

Now, let us consider the case when the direction of the homogeneous magnetic field is not along the z axis. For the 1D case one can observe strong deterioration of the solution. This is presented in Fig. 8, where the value of the residual error increases very fast with the angle θ. Even for small θ the solution becomes useless. It is necessary to use transversal 3D coil system to improve the solution. Solution with the coils

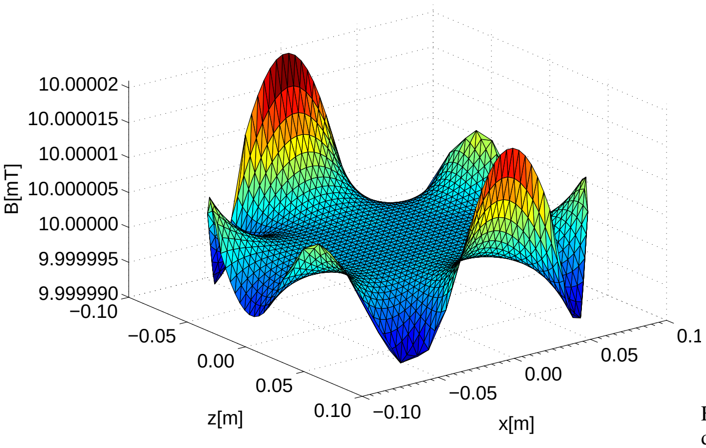

Fig. 7. Magnetic field distribution in ROI on the plane $y = 0$ for the solution presented in Fig. 6.

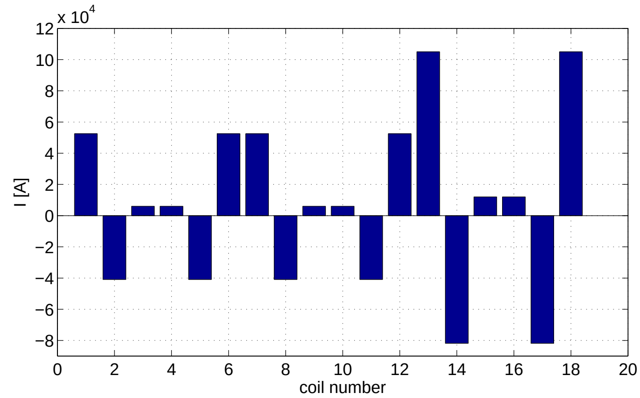

Fig. 9. Currents distribution for 3D system of 18 coils for the case of directional angle $\theta = 35.264°$. First six bars stand for the coils located coaxially along the x-axis, bars numbered $7 - 12$ represent coils located along y-axis and the rest stand for coils along z-axis.

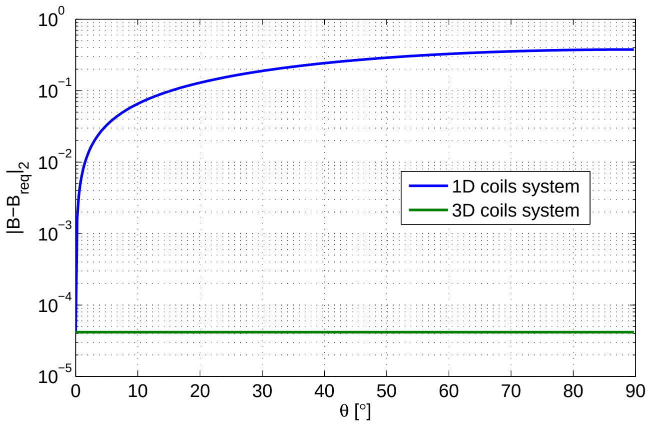

Fig. 8. Changes of the residual norm of the solution in the function of the magnetic field direction angle θ for the 1D and 3D coils system, with four coils in each spatial directions.

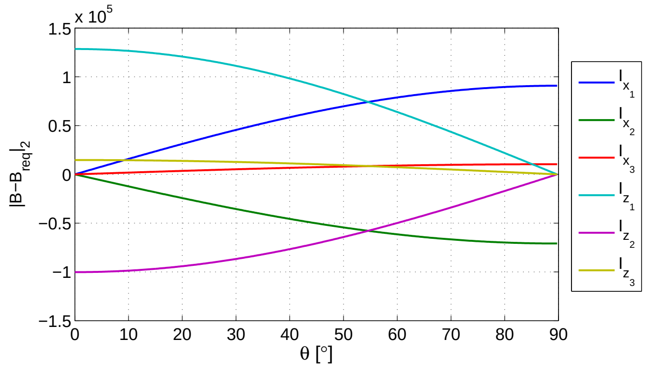

Fig. 10. The graph of the currents values in the coils in the function of the magnetic field direction angle θ.

in all three directions guarantees the quality of the solution at the same level as before.

As an example let us consider the case $\theta = 35.264°$, which corresponds to the direction of the magnetic field being at the same angle to all three axes. The current distribution for the case of 18 coils (six in each directions) is presented on fig 9. The residual error is at the low level of $\|B - B_{\text{req}}\| = 1.447228381514 \cdot 10^{-7}$, while in the 1D case with 6 coils the residual error is $\|B - B_{\text{req}}\|_2 = 0.2174856317093$.

Fig. 10 presents the graph of the values of currents in the 3D coil system of 18 coils as a function of the requested homogeneous magnetic field direction angle θ. Due to the symmetry of the system the following conditions are fulfilled:

$$\begin{cases} I_{x_1} = I_{x_6} = I_{y_1} = I_{y_6}, \\ I_{x_2} = I_{x_5} = I_{y_2} = I_{y_5}, \\ I_{x_3} = I_{x_4} = I_{y_3} = I_{y_4}, \\ I_{z_1} = I_{z_6}, \\ I_{z_2} = I_{z_5}, \\ I_{z_3} = I_{z_4} \end{cases} \tag{7}$$

The graph gives us the specific guideline for controlling the requested homogeneity of the magnetic field in each direction. Similar graphs can be generated for each specific geometrical structure of 3D coils system, not necessarily symmetric.

IV. Conclusion

In the paper we have shown that the non-axial components of the magnetic field can be neglected in the case when high quality homogeneous magnetic field is generated. On the other hand, when a nonhomogeneous magnetic field is obtained the non-axial components significantly influence the magnitude of the magnetic field. Therefore, in the design process one has to take into account all components of the magnetic field.

When homogeneous magnetic field needs to be generated in an arbitrary direction one cannot use coils coaxially located along a single axis only. Even very small deviation from the axial direction of the requested magnetic field makes it necessary to use transversal 3D coils structure. It has been shown that 3D system can generate a high quality homogeneous magnetic field in an arbitrary directions without changing the coil system structure.

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