jet INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2018, VOL. 64, NO. 2, PP. 229-242
Manuscript received February 3, 2017; revised April, 2018. DOI: 10.24425/119393

When Can a Narrowband Power Amplifier Be Considered to Be Memoryless and when Not?

Andrzej Borys

***Abstract*—This paper tries to get a response to the following question: When can a narrowband power amplifier (PA) be considered to be memoryless and when can it not be considered memoryless? To this end, a thorough and consistent analysis of the notions and definitions related with the above topic is carried out. In the considerations presented, two models of the narrowband PA are exploited interchangeably: the black box model widely used in the literature and a model developed here, which is based on the Volterra series. These two models complement each other. In this paper, the conditions for a linear or nonlinear narrowband PA to be memoryless or approximately memoryless or possessing memory are derived and illustrated. They are formulated in terms of the signal delay as well as in terms of the amplitude-to-phase (AM/PM) conversion of the amplifier. Furthermore, the two possible interpretations of the amplitude-to-amplitude (AM/AM) and AM/PM conversions are given a mathematical framework. That is these conversions are presented through some operations. One set of these operations allows to treat the AM/AM and AM/PM conversions as distortions of the modulating signals. Or equivalently as distortions of a given signal constellation when it passes through the PA. Finally, it is proved that the Saleh's and Ghorbani's models of the AM/AM and AM/PM conversions occurring in the PAs, which were published in the literature, are not memoryless ones.**

***Kyeywords*—Narrowband power amplifiers (PAs), modelling memory in PAs, conditions for PA to be memoryless.**

I. Introduction

WRITING of this paper was inspired by some imprecise formulations and views presented in a tutorial article [1] on challenges in design of reconfigurable transceivers, published recently in IEEE Circuits and Systems Magazine. These imprecise formulations and views regard power amplifiers (PAs) used in such transceivers. For further discussion, we quote them here in an original, full form. They are expressed on page 49 of [1], left column, where one reads:

1. "For a low-power PA and/or a narrowband PA, its characteristics can be regarded as memoryless. It is usually modeled as a nonlinear system where its output depends solely on its input at that particular time."
2. "More specifically, a memoryless PA can be characterized by using an amplitude-to-amplitude (AM/AM) conversion function and an amplitude-to-phase (AM/PM) conversion function."
3. "Various memoryless PA models have been proposed for different types of PAs. The Saleh model [2], based on two rational functions, is defined as (*these functions will be given here later when discussing the Saleh model*). This model is designed especially for traveling-wave tube amplifiers. The Rapp model [3] has a smooth saturation in the AM-AM function, which is suitable for solid state power amplifiers (SSPAs). The Ghorbani model [4] is customized to field-effect transistor SSPAs."

Regarding the first formulation cited above, note that it suggests that the circuit property of being memoryless is identical with operation of processing (working with) only one value of the input signal. In the next sections, we will show that this belief is not correct.

Moreover, the aforementioned first formulation can be also construed as suggesting that a selective narrowband characteristics of a PA can make it memoryless. This belief is also, as we will see, not correct.

Further, the above formulation seems to suggest that a relation between working with relatively low powers and the property of being memoryless exists. We confirm in this paper that such a connection really exists. However, as it will follow from a more detailed analysis presented in the next sections, this relation regards, precisely saying, the notion of being approximately memoryless. We will show that PAs working with less powers are naturally better candidates to satisfy the condition of being approximately memoryless circuits.

With regard to the second formulation quoted, we show here, once again, confirming thereby the results obtained by the author of this paper elsewhere [5-7] with the use of other approaches, that the property of having no memory by a PA excludes demonstrating by it nonzero values of the AM/PM conversion.

Finally, the third formulation states that all the three models mentioned: of Rapp, of Saleh, and of Ghorbani are memoryless ones. We show here, confirming by the way also the results [8] achieved by the author of this paper in with the use of another approach, that only the Rapp's model is memoryless. But, the models of Saleh and of Ghorbani are representations incorporating memory effect.

By the way, note that an incorrect classification of the Rapp, Saleh, and Ghorbani models is presented also in a recent tutorial on PAs [9].

The paper is organized as follows. The next section presents a thorough analysis of the PA behavior; the Volterra series [10] is used in it. The description developed in section II is applied in the next one to determine the conditions for the PA to be memoryless or approximately memoryless or such a one that possesses memory. Equivalently and complementary, these conditions are also obtained with the help of a black box model

The work presented in this paper was supported partly by the grant AMG DS/450/2018.

The author is with the Department of Marine Telecommunications, Faculty of Electrical Engineering, Gdynia Maritime University, Gdynia, Poland, email: a.borys@we.am.gdynia.pl.

used in the literature to describe the AM/AM and AM/PM conversions of the narrowband PA. In section IV, the two possible interpretations of the latter conversions is given a mathematical framework. In particular, it is shown that there is a set of operations, which allows to treat the AM/AM and AM/PM conversions as distortions of the modulating signals. The result presented in section V determines the dependence existing between the signal delay caused by the PA memory and its AM/PM conversion value. In section VI, the results achieved in the previous sections are used to show that the Saleh's model is not a memoryless one. Note that this allows to refine the example classifications mentioned above as well as others in which the Saleh's model is considered to be memoryless, as for example, those in the papers [1], [11], [12] and the book [13]. The paper concludes with some final remarks.

II. Modeling Low-Power and Narrowband Amplifier Characteristics with the Use of Volterra Series

Influence of PA nonlinearities upon their behavior is commonly characterized [2], [9], [13] by evaluating their AM/AM and AM/PM distortions. For this purpose, a PA is driven by a bandpass signal of the form

$$x(t) = r(t)\cos\left(\omega_c t + \psi(t)\right), \tag{1}$$

where the angular frequency $\omega_c = 2\pi f_c$ in which f_c means the carrier frequency. Moreover, *t* in (1) denotes the time variable. Furthermore, it is assumed that $x(t)$ in (1) contains a slowly varying real-valued baseband signal $r(t)$. The latter signal modulates the carrier amplitude, but the carrier phase changes with time according to the function $\psi(t)$. It is also assumed that the function $\psi(t)$, similarly as $r(t)$, represents a slowly varying baseband signal.

Many authors, using different mathematical tools, as for example those applied in [5] and [13], have shown that the PA output signal, when this amplifier is driven by the input signal given by (1), can be expressed in the following form:

$$y(t) = A\left(r(t)\right)\cos\left(\omega_c t + \psi(t) + \Phi\left(r(t)\right)\right), \tag{2}$$

where the functions $A\left(r(t)\right)$ and $\Phi\left(r(t)\right)$ are generally nonlinear functions of $r(t)$. Moreover, these functions are called in the literature [2], [9] the AM/AM characteristic (conversion) and AM/PM characteristic (conversion), respectively.

PAs behave as linear devices for small enough values of the input signal amplitudes. However, for large enough values of these amplitudes, they begin to behave as typical nonlinear circuits. Transition from the linear to the nonlinear region of operation of a PA can be recognized by appearance in it of the nonlinear products like signal compression, harmonic and/or intermodulation distortion, cross-modulation distortion, and/or of the other kinds of distortions. Amongst the latter ones are the AM/AM and AM/PM distortions (conversions), we consider in this paper. These are the specific types of distortions. However, their appearance is an indicator of the transition to the nonlinear regime of operation, similarly as in the case of all the other ones mentioned above. If this entering into the nonlinear region of operation is not very strong, one says that the circuit is working in a mildly (weakly) nonlinear [14-19] or moderately nonlinear regime (when the nonlinear products produced are a little bit stronger than in the previous case). This notion and understanding regards also the PAs.

It seems that the notion of a low-power PA used in [1] in the context of its memoryless models needs some explanation. From reading [1], one can guess that there a PA is understood, which is power efficient. That is a PA with a reduced power consumption from DC supply. In analog circuits, such power reduction is achieved by making the values of the circuit bias voltages and currents as small as possible [20]. This circuit design philosophy aims at getting the so-called low-voltage and low-current analog circuits [20]. However, it does not mean that such circuits are automatically memoryless. They still contain reactance elements as, for example, parasitic capacitors, which prevent the low-voltage and low-current analog circuits from being considered as fully memoryless. In other words, we see that a low-power PA cannot be identified with a memoryless one.

On the other hand, as a rule, the low-voltage and low-current analog circuits including low-power PAs work with evidently smaller values of power of the transferred and/or amplified signals than those processed by their counterparts possessing bias circuitry designed for larger values of supply voltages. So, because of this reason, the notion of a low-power amplifier can be also understood in the sense just described. That is as an amplifier working with smaller signal amplitudes than those usually worked with. And in this sense, this notion will be used here.

Obviously, any so-called low-power PA, as any other quasi-linear analog circuit, can work in its linear region of operation or mildly nonlinear or moderately nonlinear or even in a strongly nonlinear region of operation (in the latter case when it is admissible and/or required.) For a given frequency, it depends upon the value of the input signal amplitude [21]. And we say then that such a PA behaves, accordingly, as a linear, mildly nonlinear, moderately nonlinear, or highly nonlinear circuit.

In what follows, we will derive a general model for the behavior of any low-power and narrow-band PA suitable for the following regions of operation: linear, mildly (weakly) nonlinear, and moderately nonlinear. At the starting point, we will take also into account the reactance elements of a PA mentioned above, which are responsible for existence of any memory in it. Only afterwards, we will check whether and under what conditions they can be neglected making the model approximately memoryless.

There is a vast literature on modelling of nonlinear circuits and systems with the use of Volterra series; see, for example, books [10], [17], and [24] on these topics, and references cited therein. This shows that the Volterra series is a best suited mathematical tool to tackle with the PAs characterised as just above. That is with nonlinear circuits working in their linear or mildly or moderately nonlinear regions of operation, and possibly possessing memory. Then, for the linear region of operation, one takes into account only the first (linear) component in the Volterra series, for the mildly nonlinear region, additionally also the second and third ones, and finally, for the moderately nonlinear region also the components of higher orders like fourth, fifth, sixth ones, and so on. The number of the latter ones depends, for larger signal amplitudes, upon the accuracy requirements imposed on in a concrete case [18], [19], [25], [26].

So, let us now start with a wideband PA whose nonlinear impulse responses [5], [6], [10], similarly as in [8], are given by

$$h^{(1)}(t) = b_1 \cdot \exp(-t/a), \tag{3a}$$

$$h^{(2)}(t_1,t_2) = b_2 \cdot \exp(-t_1/a)\exp(-t_2/a), \tag{3b}$$

$$h^{(3)}(t_1,t_2,t_3) = b_3 \cdot \exp(-t_1/a)\exp(-t_2/a)\exp(-t_3/a), \tag{3c}$$

and so on, for the values of the time variables t and t_i, $i = 1,2,3,\ldots$, greater and equal to zero, and which are identically zero otherwise. Furthermore, the upper index in the successive responses in (3): $h^{(1)}(t)$, $h^{(2)}(t_1,t_2)$, $h^{(3)}(t_1,t_2,t_3)$, and so on, means their order (degree). In other words, they are, accordingly, the first order (linear), second order, third order, and so on, nonlinear impulse responses (Volterra kernels) of the PA considered. Moreover, the coefficients $a > 0$ and b_i, $i = 1,2,3,\ldots$, in (3) are some constants. Observe further that the constant a corresponds to the time constant RC of a simple low-pass RC filter. We assume here that this time constant is very small making the PA a wideband amplifier. Furthermore, note that the constant a can be considered as a measure of the amplifier memory length.

Observe that the PA nonlinear model described by the expressions (3) mirrors its most relevant characteristics we need to take into account in our analysis. More precisely, by assuming that the coefficient a is arbitrarily small, we ensure that the amplifier is a wideband one. But, on another hand, by having $a \neq 0$ we ensure that this property of being wideband does not extend into the infinity. Moreover, this model is simple and algebraically easy to work out.

In what follows, we proceed similarly as in [8]. However, since the details of the calculations are a little bit tedious, they are moved into the appendix. In the rest of this section, we only characterise shortly the derivations presented therein and repeat a final result modelling the nonlinear <u>narrowband</u> PA ((4) in this section identical with (A11) in the appendix).

In short, substituting (1) and (3) into the Volterra series given by (A1) in the appendix, and performing then, successively, the manipulations indicated in (A2-A5), we arrive at (A6). The latter models a moderately nonlinear <u>wideband</u> amplifier driven by a signal of the form (1).

Here, the narrowband PA is modelled as a wideband one followed by an inherent passband filter. This allows us to consider this series connection as a one structure. It is then called the <u>narrowband</u> PA.

As it has been shown in the appendix, by virtue of the inherent passband filter at the end of the narrowband PA, all the products in (A6) related to the frequencies different from a carrier frequency are filtered out in such an amplifier. Taking into account this effect, one gets (A7) from (A6). And after introducing, instead of the corresponding nonlinear impulse responses, the nonlinear transfer functions $H^{(n)}(f_1,..,f_n)$ of the wideband part of the narrowband PA model, which are defined by (A8), one arrives finally at

$$\begin{aligned} y_N(t) \cong \sum_{n=1,\,\text{odd}}^{N} \left(\frac{r(t)}{2}\right)^n \cdot \Big[& C\big(n,(n-1)/2\big) \cdot \\ & \cdot H^{(n)}\big(\chi_n^{+}(\pm\omega_c)\big)\exp\big(j\big(\omega_c t + \psi(t)\big)\big) + \\ & + C\big(n,(n+1)/2\big) \cdot \\ & \cdot H^{(n)}\big(\chi_n^{-}(\pm\omega_c)\big)\exp\big(-j\big(\omega_c t + \psi(t)\big)\big)\Big] \; . \end{aligned} \tag{4, A11}$$

For definitions of $\chi_n^{+}(\pm\omega_c)$, $\chi_n^{-}(\pm\omega_c)$, ω_c, $C\big(n,(n-1)/2\big)$, and $C\big(n,(n+1)/2\big)$ occurring in (4), see the appendix.

Observe now that both (2) and (4) represent the same signal that is the one occurring at the output of the narrowband PA. This allows us to identify the signal $y_N(t)$ given by (4) with that provided by (2). Further, by comparison of the components in (4) with the corresponding ones in (2), we can deduce the expressions describing the AM/AM and AM/PM conversions in any narrowband, moderately nonlinear PA. This was done in [5] and [6]. In this paper, we will use however these expressions and the related ones, which were derived therein, to clarify other aspects of modelling the PAs.

III. Necessary Condition for Narrowband Amplifier to Be Approximately Memoryless

Let us begin this section with the following observation. If a circuit processes only one value of its input signal to provide the value of its output signal this fact does not mean that it represents a circuit without memory. Note that this is opposite to the widespread opinion that the aforementioned facts mean the same. This opinion says that working with (on) only one input signal value to provide the output signal value for a given time instant is identical with saying that such kind of processing is memoryless, and a circuit performing it does not possess memory. This view is also followed by the authors of the paper [1], when they are saying: “[memoryless PA] is usually modeled as a nonlinear system where its output depends solely on its input at that particular time” or “for a PA with memory effects, the output does not depend only on its current input, but on its previous inputs as well”. At first glance, these statements seem to be true. However, there are cases in which the above is not valid. In what follows below, we illustrate this on an example of a linear circuit possessing memory and driven by a sinusoidal signal. Note that such a circuit has a complex-valued transfer function, say $H_l(f)$, or equivalently an impulse response $h_l(t)$ different from a constant. Moreover, the sinusoidal input signal can be expressed as

$$\begin{aligned} x_i(t) &= AMP \cdot \cos(\omega_0 t) = AMP \cdot \cos(2\pi f_0 t) = \\ &= \frac{1}{2} AMP\big[\exp(j2\pi f_0 t) + \exp(-j2\pi f_0 t)\big] \end{aligned} , \tag{5}$$

where f_0 and $\omega_0 = 2\pi f_0$ have the usual meaning of frequency and angular frequency, respectively, of this signal. *AMP* in (5) is its amplitude and $j = \sqrt{-1}$.

The circuit is illustrated in Fig. 1.

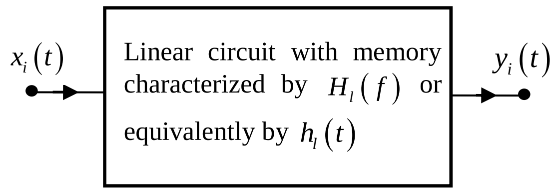

Fig. 1. Black-box scheme of a linear circuit.

Obviously, the signal $y_i(t)$ at the output of the circuit in Fig. 1 can be calculated in the following way:

$$\begin{aligned} y_i(t) &= \int_{-\infty}^{\infty} h_l(\tau)\, AMP \cos\left(2\pi f_0 (t-\tau)\right) d\tau = \frac{AMP}{2} \cdot \\ &\cdot \int_{-\infty}^{\infty} h_l(\tau)\left[\exp\left(j2\pi f_0 (t-\tau)\right) + \exp\left(-j2\pi f_0 (t-\tau)\right)\right] d\tau = \\ &= \frac{AMP}{2}\left[\exp\left(j2\pi f_0 t\right)\int_{-\infty}^{\infty} h_l(\tau)\exp\left(-j2\pi f_0 \tau\right) d\tau + \right. \\ &\left. + \exp\left(-j2\pi f_0 t\right)\int_{-\infty}^{\infty} h_l(\tau)\exp\left(j2\pi f_0 \tau\right) d\tau \right] = \\ &= \frac{AMP}{2}\left[\exp\left(j2\pi f_0 t\right) H_l(f_0) + \exp\left(-j2\pi f_0 t\right) \cdot \right. \\ &\left. \cdot H_l(-f_0)\right] = AMP\left|H_l(f_0)\right| \cos\left(2\pi f_0 t + \varphi_{H_l}(f_0)\right), \end{aligned} \tag{6}$$

where $\varphi_{H_l}(f_0)$ denotes phase shift of the transfer function $H_l(f)$ calculated at the frequency f_0. In derivation of the result given by (6), the definition of the Fourier transform and the fact that $\left(H_l(f)\right)^* = H_l(-f)$ were used. In the latter, the operation $(\cdot)^*$ means calculation of the conjugate of a complex number.

Comparison of the final result in (6) with (5) allows us to write the following

$$y_i(t) = \left|H_l(f_0)\right| x_i\left(t + \varphi_{H_l}(f_0)/(2\pi f_0)\right). \tag{7}$$

Observe now from (7) that the linear circuit with memory of Fig. 1 works with (processes) only one value of the input signal. This operation (mapping) can be described illustratively in the following way: It picks up the value equal to $x_i\left(t - \left(-\varphi_{H_l}(f_0)/(2\pi f_0)\right)\right)$ from the past of the input signal. The value chosen is scaled by $\left|H_l(f_0)\right|$ and attributed to the value of the circuit output signal for a considered time instant *t*.

Now, for more concrete illustration, consider the circuit of Fig. 1 to be a first-order low-pass passive filter consisting of one resistor of resistance *R* and one capacitor of capacitance *C*. As well known [27], its transfer function is given by

$$H_l(f) = \frac{1}{1 + j2\pi fRC}. \tag{8}$$

From (8), we get the following values of $\left|H_l(f_0)\right|$ (scaling factor) and $-\varphi_{H_l}(f_0)/(2\pi f_0)$ (delay t_D) occurring in (7)

$$\left|H_l(f_0)\right| = \frac{1}{\sqrt{1 + \left(2\pi f_0 RC\right)^2}} \tag{9a}$$

and

$$t_D = -\varphi_{H_l}(f_0)/(2\pi f_0) = \text{arctg}\left(2\pi f_0 RC\right)/(2\pi f_0). \tag{9b}$$

Moreover, because the value of $\text{arctg}\left(2\pi fRC\right) > 0$, the time instant $t - \left(-\varphi_{H_l}(f_0)/(2\pi f_0)\right) = t - \text{arctg}\left(2\pi f_0 RC\right)/(2\pi f_0) < t$. That is this time instant belongs, with regard to *t*, to the past of the input signal.

It follows clearly from the above example that the fact of processing only one value of the input signal is not sufficient for saying that a circuit is a memoryless one. Relevant is, as in the above example, whether this value is taken from the input signal past or it is its present value. Note that in our example this was the value of $x_i(t - t_D)$. That is the value taken from the input signal past. And only on the basis of the value of $t_D > 0$, we could recognize that we had to do with a circuit with memory.

Further, observe that on the basis of the above explanation we could make the following classification of circuits with regard to the property of possessing memory or not: without memory (pure memoryless), approximately memoryless (almost memoryless), and with memory. More precisely, the proposed classification could look like:

1. without memory when $t_D \equiv 0$,
2. approximately memoryless when $0 < t_D \le D$, where D means a chosen admissible value for which we assume that for the picked up value $x_i(t - t_D) \cong x_i(t)$ holds for every *t*,
3. with memory when $t_D > D$.

Pure memoryless circuits are such ones whose transfer functions are real-valued. So, their phase shifts $\varphi_{H_l}(f_0)$ are identically equal to zero for any frequency f_0. After the left-hand side expression in (9b), this means then that $t_D \equiv 0$.

Both circuit categories mentioned above in points 2 and 3 are with memory. However, at some circumstances, these circuits can behave approximately as memoryless ones depending whether they fulfil the condition $0 < t_D \le D$ or do not. In what follows now, we take a closer look at it. To this end, using the left-hand side expression of (9b), we rewrite it in the following form:

$$0 < t_D = -\varphi_{H_l}(f_0)/(2\pi f_0) \le D. \tag{10}$$

Observe that fulfillment of the condition given by (10) depends upon the following three factors:

1. the way of realization of the transfer function $H_l(f)$,
2. frequency f_0 of the sinusoidal signal applied at the circuit input,
3. the tolerance on the signal shift in time we assume not to distinguish from the zero value shift; it is denoted here by the capital letter *D*.

Note that these observations can be used in designing approximately memoryless amplifiers and/or checking whether they really possess the above property. For example, given a frequency of the input sinusoidal signal and its tolerance on the shift in time assumed not to be distinguished from the zero, we design a PA circuitry (mirrored in $H_l(f)$) accordingly to fulfil the requirement (10).

Let us now illustrate the above explanations on the previous example of a simple low-pass filter with turnover frequency $f_t = 100\ \text{kHz}$. We will check whether its behavior can be assumed to be approximately memoryless or does not, in the following environment: $f_0 = 1\ \text{kHz}$ and $D = 100\ \mu\text{s}$. To this end, we invoke the following relation between the time constant *RC* and its turnover frequency: $RC = 1/f_t$ [27]. Using it in (9b), we get $t_D = \text{arctg}(2\pi f_0/f_t)/(2\pi f_0)$. Next, observe that in our case $2\pi f_0/f_t = 2\pi \cdot 1/100 \cong 0.06$. And this allows us to use the standard polynomial expansion for the function $\text{arctg}(x) \cong x$ restricted to the first component only. Hence, we get $t_D \cong (2\pi f_0/f_t)/(2\pi f_0) = 1/f_t$. By substituting this into (10), we arrive at $0 < 1/f_t \le D$. Rearranging the latter, we receive $0 < 1/D \le f_t$. And finally, we check whether the last inequality is fulfilled for the values of our example. The substitution gives $0 < 10\ \text{kHz} \le 100\ \text{kHz}$. So, it is fulfilled. Therefore, we conclude that the low-pass filter in our example behaves approximately as a memoryless circuit at the environment characterized by the aforementioned parameters.

We will show now that similar arguments as those used in the example just presented can be also applied for description and explanation of a relation (mapping) between the input and output signals of a narrowband PA given by (1) and (2), respectively. To this end, let us start with writing down formally the mapping existing between these signals as

$$\begin{aligned} &A(r(t))\cos(\omega_c t + \psi(t) + \Phi(r(t))) = \\ &= PA(r(t)\cos(\omega_c t + \psi(t))) \end{aligned}, \qquad (11)$$

where the notation *PA* is used for this mapping. We draw here attention to the fact that the mapping $y(t) = PA(x(t))$ given by (11) is a specific one. It is defined only for the input signals having the form given by (1) and results in the output signals possessing the form expressed in (2).

The mapping given by (11) is illustrated in Fig. 2.

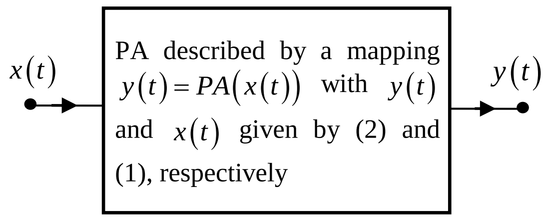

Fig. 2. PA description by a mapping $y(t) = PA(x(t))$ relating the signals having the form given by (1) and (2).

In the next step, let us rewrite (1) and (2) in the following form:

$$x(t) = r(t)\cos\left(2\pi f_c\left(t + \psi(t)/(2\pi f_c)\right)\right) \qquad (12)$$

and

$$y_N(t) = \frac{A(r(t))}{r(t)} r(t) \cos\left(2\pi f_c\left(t + \frac{\psi(t)}{2\pi f_c} + \frac{\Phi(r(t))}{2\pi f_c}\right)\right). \qquad (13)$$

Note that we have applied the notation $y_N(t)$ in (13) instead of $y(t)$, which was used before in (2). This is done now here because introducing in the meantime the notation $y_N(t)$ (see (4)) was for denoting the PA output signal after passing through its bandpass output filter. That is for such a PA that produces a narrowband output signal. But, see that (2) is nothing else than just the description of such the PA. So, thereby, alignment of notation has been achieved.

Further note that comparison of (12) with (13) allows to rewrite (13) in such a form

$$y_N(t) \cong \frac{A(r(t))}{r(t)} x\left(t + \frac{\Phi(r(t))}{2\pi f_c}\right) \qquad (14a)$$

if the following conditions:

$$r\left(t + \frac{\Phi(r(t))}{2\pi f_c}\right) \cong r(t) \quad \text{and} \quad \psi\left(t + \frac{\Phi(r(t))}{2\pi f_c}\right) \cong \psi(t) \qquad (14b)$$

are satisfied.

Note that (14a) resembles (7), and this fact will be the basis for our further discussion in the context of the PA model derived in section II. However, before starting this discussion, we draw the reader's attention to the fact that we will consider now a definitely more complicated case than that considered in our example discussed at the beginning of this section. In detail, we have assumed in the latter case that the input signal amplitude as well as its phase – that is *AMP* and the phase equal to zero in (5) - were not modulated. This is opposite to the case we consider now, in which both the input signal amplitude and its phase are modulated. The amplitude modulation goes according to the function $r(t)$, but the phase according to the function $\psi(t)$ - see (1).

To continue now, let us rewrite (4) in the following form:

$$
\begin{aligned}
y_N(t) \cong{} & \frac{r(t)}{2} \cdot \sum_{n=1,\ \text{odd}}^{N} \left(\frac{r(t)}{2} \right)^{n-1} \cdot \Big[C\big(n,(n-1)/2\big) \cdot \\
& \cdot H^{(n)}\big(\chi_n^{+}(\pm\omega_c)\big) \exp\big(j(\omega_c t + \psi(t))\big) + C\big(n,(n-1)/2\big) \cdot \\
& \cdot \Big(H^{(n)}\big(\chi_n^{+}(\pm\omega_c)\big)\Big)^{*} \exp\big(-j(\omega_c t + \psi(t))\big) \Big] \ .
\end{aligned} \tag{15}
$$

In derivation of (15), the equalities

$$C\big(n,(n-1)/2\big) = C\big(n,(n+1)/2\big) \tag{16a}$$

and

$$H^{(n)}\big(\chi_n^{-}(\pm\omega_c)\big) = \Big(H^{(n)}\big(\chi_n^{+}(\pm\omega_c)\big)\Big)^{*} \tag{16b}$$

have been applied.

Introducing next a complex-valued function

$$
\begin{aligned}
& H_{nl}(r(t), N, H^{(1\div N)}, \omega_c) = \\
& = \sum_{n=1,\ \text{odd}}^{N} \left(\frac{r(t)}{2} \right)^{n-1} C\big(n,(n-1)/2\big) H^{(n)}\big(\chi_n^{+}(\pm\omega_c)\big) \ ,
\end{aligned} \tag{17}
$$

we can rewrite (15) in the following form:

$$
\begin{aligned}
y_N(t) \cong{} & \frac{r(t)}{2} \cdot \Big[H_{nl}(r(t), N, H^{(1\div N)}, \omega_c) \cdot \\
& \cdot \exp\big(j(\omega_c t + \psi(t))\big) + \Big(H_{nl}(r(t), N, H^{(1\div N)}, \omega_c)\Big)^{*} \cdot \\
& \cdot \exp\big(-j(\omega_c t + \psi(t))\big) \Big] = \\
& = r(t) \left| H_{nl}(r(t), N, H^{(1\div N)}, \omega_c) \right| \cdot \\
& \cdot \cos\Big(\omega_c t + \psi(t) + \varphi_{H_{nl}}\big(r(t), N, H^{(1\div N)}, \omega_c\big)\Big) \ ,
\end{aligned} \tag{18}
$$

where $|H_{nl}(\cdot)|$ and $\varphi_{H_{nl}}(\cdot)$ mean the magnitude and phase, respectively, of the function $H_{nl}(\cdot)$.

Note now that the function $H_{nl}(r(t), N, H^{(1\div N)}, \omega_c)$ defined by (17) plays a role of a describing function [28] of the nonlinear PA shown in Fig. 2. Admittedly, its definition applied here is a little bit different from that used for the formulation of describing functions in [28] or [29]. However, its action and usage is the same. It depends upon the amplitude modulating signal $r(t)$ and the angular frequency ω_c (or frequency f_c). The fact that it is an approximation obtained with the use of a Volterra series is also shown. This is done by indicating its dependence upon the number N of the components of the Volterra series taken into account. Moreover, the symbol $H^{(1\div N)}$ means that the PA nonlinear transfer functions of orders from 1 to N are taken to build up the approximation.

Equivalent PA model of the one of Fig. 2, which uses the describing function $H_{nl}(r(t), N, H^{(1\div N)}, \omega_c)$ (denoted in this paper also as $H_{nl}(r(t), N, H^{(1\div N)}, f_c)$), is visualized in Fig. 3.

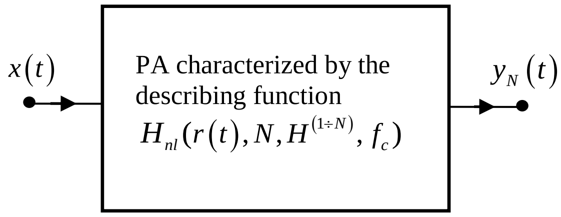

Fig. 3. PA description using the describing function $H_{nl}(r(t), N, H^{(1\div N)}, f_c)$.

Finally, see that in view of (1) equation (18) can be rewritten as

$$
\begin{aligned}
y_N(t) \cong{} & \left| H_{nl}(r(t), N, H^{(1\div N)}, \omega_c) \right| \cdot \\
& \cdot x\Big(t + \varphi_{H_{nl}}\big(r(t), N, H^{(1\div N)}, \omega_c\big) \big/ \omega_c\Big)
\end{aligned} \tag{19a}
$$

or

$$
\begin{aligned}
y_N(t) \cong{} & \left| H_{nl}(r(t), N, H^{(1\div N)}, f_c) \right| \cdot \\
& \cdot x\Big(t + \varphi_{H_{nl}}\big(r(t), N, H^{(1\div N)}, f_c\big) \big/ (2\pi f_c)\Big)
\end{aligned} \tag{19b}
$$

showing the frequency f_c instead of ω_c if the conditions (14b) are satisfied.

Furthermore, comparison of (14a) with (19b) shows that the following relations:

$$\frac{A\big(r(t)\big)}{r(t)} \cong \left| H_{nl}(r(t), N, H^{(1\div N)}, f_c) \right| \tag{20a}$$

and

$$\Phi\big(r(t)\big) \cong \varphi_{H_{nl}}\big(r(t), N, H^{(1\div N)}, f_c\big) \tag{20b}$$

hold.

Now, the more detailed considerations regarding (19b) follow. First, we show that the case when (19b) reduces to the linear case ($N = 1$) differs significantly from the truly nonlinear one ($N > 1$). To this end, assume $N = 1$ in (17). Then, we obtain

$$H_{nl}(r(t), N = 1, H^{(1)}, f_c) = H_l(f_c) = H^{(1)}(f_c) \ , \tag{21}$$

which shows that $H_{nl}(r(t), N = 1, H^{(1)}, f_c)$ reduces to the first order (linear) transfer function of the PA. This is clear because it was assumed to be a linear device. And, because of this reason, the subscript *nl* in $H_{nl}(\cdot)$ was changed to *l* in (21). Moreover, note also that this notation is consistent with the one used in description of the linear circuit with memory of Fig. 1.

Furthermore, note that in the case of a purely linear PA the expressions (20a) and (20b) reduce to

$$\left.\frac{A\big(r(t)\big)}{r(t)}\right|_l = \left|H_l(f_c)\right| = \left|H^{(1)}(f_c)\right| \quad (22a)$$

and

$$\Phi\big(r(t)\big)\Big|_l = \varphi_{H_l}(f_c) = \varphi_{H^{(1)}}(f_c) \ , \quad (22b)$$

where the subscript *l* is added in the left-hand side quantities to emphasise the fact that they regard the linear PA. Also, we see that (19b) assumes then the following form:

$$y_N(t) = \left|H_l(f_c)\right| x\left(t + \frac{\varphi_{H_l}(f_c)}{2\pi f_c}\right). \quad (23)$$

The result achieved in (22) and (23) is illustrated in Fig. 4.

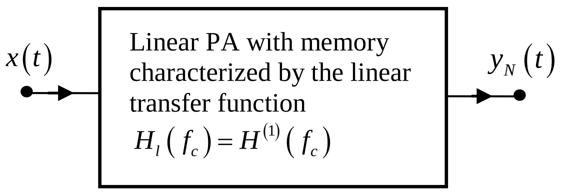

Fig. 4. Black-box scheme of a linear PA characterized by the linear transfer function $H_l(f_c)$ calculated at the carrier frequency f_c.

Note now that the form of the expression given by (23) is identical with the one given by (7). So, this allows us to say that the whole discussion and conclusions regarding the behaviour of the circuit of Fig. 1, which were presented above, are also fully applicable to the behaviour of the linear PA scheme presented in Fig. 4. Because of this reason, they are not repeated at this place, with only one exception - regarding the condition for the linear PA to be approximately memoryless. In view of (10), the latter condition is given by

$$0 < t_D = -\varphi_{H_l}(f_c)\big/(2\pi f_c) \le D_x \ . \quad (24)$$

where D_x means the tolerance on the signal shift in time assumed not to be distinguished from the zero value shift and that is now referred to as the PA input signal given by (1).

It is also worth noting that we could express shortly our last conclusions presented above in the following way: Processing the sinusoidal signal by a linear PA does not depend at all of whether the amplitude and/or phase of this signal are modulated or are not. It is identical in both the cases. The modulation operations have no influence on it.

As well, it is worth mentioning at this place that the conditions (14b) can be reformulated using the introduced notion of the tolerance *D* on the signal shift in time we assume not to distinguish from the zero value shift (see the definition presented at the beginning of this section). Such a reformulation can be carried out along the following lines: We assume that

$$r\left(t + \frac{\Phi\big(r(t)\big)}{2\pi f_c}\right) = r\left(t - t_{D\Phi}\right) \cong r(t) \quad (25a)$$

holds approximately for all times *t* if the delay $t_{D\Phi}$ defined in (25a) fulfils the following inequality:

$$t_{D\Phi} = -\Phi\big(r(t)\big)\big/(2\pi f_c) \le D_\Phi \quad (25b)$$

for all times *t*, where D_Φ means the tolerance on the signal shift in time assumed not to be distinguished from the zero value shift and that is now referred to the modulating signal $r(t)$.

Similarly, we assume that

$$\psi\left(t + \frac{\Phi\big(r(t)\big)}{2\pi f_c}\right) = \psi\left(t - t_{D\psi}\right) \cong \psi(t) \quad (26a)$$

holds approximately for all times *t* if the delay $t_{D\psi}$ defined in (26a) fulfils the following inequality:

$$t_{D\psi} = -\Phi\big(r(t)\big)\big/(2\pi f_c) \le D_\psi \quad (26b)$$

for all times *t*, where D_ψ means the tolerance on the signal shift in time assumed not to be distinguished from the zero value shift and that is now referred to as the modulating signal $\psi(t)$.

Note further that because the maximal frequencies in the spectrum amplitude characteristics of the baseband signals $r(t)$ and $\psi(t)$, say f_{mr} and $f_{m\psi}$, respectively, fulfil the corresponding inequalities: $f_{mr} << f_c$ and $f_{m\psi} << f_c$ the following relations hold:

$$D_x << D_r \ \text{ and } \ D_x << D_\psi \ . \quad (27)$$

Observe that it follows immediately from (24) and (27) that if a PA can be assumed to be approximately memoryless when processing a sinusoidal carrier signal, then it possesses automatically this property also with respect to the modulating signals $r(t)$ and $\psi(t)$. Maybe the above statement seems to be obvious, nevertheless, it is worth reminding.

Consider now the nonlinear PA described by (14a) or by (19b) (if using the Volterra series). These expressions show first that the nonlinear PA represents a nonlinear circuit <u>with memory</u>. This is so because the delay of the input signal is not identically equal to zero. Second, the PA works with (processes) <u>only one value</u> of the input signal taken from its past. However, now, the instantaneous amplification factor $A\big(r(t)\big)\big/r(t)$ of this value and its delay $\Phi\big(r(t)\big)\big/(2\pi f_c)$ depend upon the modulating signal $r(t)$ (its instantaneous amplitude). And this obviously makes a significant difference between the nonlinear case discussed now and the previous linear one (see (22a) and (22b)).

The latter fact can be interpreted more practically as the dependence of the amplification factor $A\big(r(t)\big)/r(t)$ and of the signal delay $\Phi\big(r(t)\big)/(2\pi f_c)$ upon the energy sent during the symbol (bit) duration, E_{s_i}. This is so because we have

$$E_{s_i} = b\int_0^{T_s}\big(r(t)\big)^2 dt = b\int_0^{T_s}\big(AMP_{s_i}\big)^2 dt = b\big(AMP_{s_i}\big)^2 T_s \,, \qquad (28)$$

where *b* means some constant, T_s denotes the symbol duration, and AMP_{s_i} is the amplitude of the signal $r(t)$ during duration of the symbol s_i. This amplitude is assumed not to change in duration period of a given symbol. Obviously, if the next symbol differs from the previous one, its amplitude assumes a new value.

Equivalently, we can say that the amplification factor $A\big(r(t)\big)/r(t)$ and the signal delay $\Phi\big(r(t)\big)/(2\pi f_c)$ are dependent upon the power transmitted, $P_{s_i} = E_{s_i}/T_s$. It follows from (28) that the latter is given by

$$P_{s_i} = \frac{E_{s_i}}{T_s} = \frac{b\int_0^{T_s}\big(r(t)\big)^2 dt}{T_s} = b\big(AMP_{s_i}\big)^2 \,, \qquad (29)$$

Finally, the condition for the nonlinear PA to be approximately memoryless is the following:

$$0 < t_D = -\Phi\big(r(t)\big)/(2\pi f_c) \le D_x \,. \qquad (30)$$

Observe first that this condition depends upon the energy transmitted. As the rule is that the function $\Phi\big(r(t)\big)$ is non-decreasing one, it will be harder to satisfy the inequality (30) for larger values of the energy transmitted. Second, because this energy changes, as explained above, it is reasonable to consider the worst case, which will then have the following form:

$$0 < t_D = -\Phi\Big(\max_i\big(AMP_{s_i}\big)\Big)/(2\pi f_c) \le D_x \qquad (31a)$$

or

$$0 < t_D = -\Phi\Big(\max_i\Big(\sqrt{P_{s_i}/b}\Big)\Big)/(2\pi f_c) \le D_x \,. \qquad (31b)$$

Third, it follows from (31b) that in the nonlinear case the notion of the approximately memoryless PA depends upon the level of the power transmitted P_{s_i}.

IV. Interpretation of AM/AM and AM/PM Conversions as Distortions of Modulating Signals

In this section, we would like to draw attention to a certain duality existing in the telecommunications literature regarding interpretation of the results of the mapping from (1) to (2) (illustrated in Fig. 2). To explain this duality, let us define first the following vectors:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) = r(t) \\ x_2(t) = \psi(t) \\ x_3(t) = r(t)\cos\big(2\pi f_c t + \psi(t)\big) \end{bmatrix}, \qquad (32a)$$

$$\mathbf{y}_1(t) = \begin{bmatrix} y_1(t) = r(t) \\ y_2(t) = \psi(t) \\ y_3(t) = A\big(r(t)\big)\cos\big(2\pi f_c t + \psi(t) + \Phi(t)\big) \end{bmatrix}, \qquad (32b)$$

and

$$\mathbf{y}_2(t) = \begin{bmatrix} y_1(t) = A\big(r(t)\big) \\ y_2(t) = \psi(t) + \Phi(t) \\ y_3(t) = r(t)\cos\big(2\pi f_c t + \psi(t)\big) \end{bmatrix}. \qquad (32c)$$

Moreover, let us define also the following mappings (operations):

$$x(t) = X\big(\mathbf{x}(t)\big) = r(t)\cos\big(2\pi f_c t + \psi(t)\big), \qquad (33a)$$

$$y(t) = Y_1\big(\mathbf{y}_1(t)\big) = A\big(r(t)\big)\cos\big(2\pi f_c t + \psi(t) + \Phi(t)\big), \qquad (33b)$$

and

$$y(t) = Y_2\big(\mathbf{y}_2(t)\big) = A\big(r(t)\big)\cos\big(2\pi f_c t + \psi(t) + \Phi(t)\big). \qquad (33c)$$

Defining the inverse operations to X, Y_1, and Y_2 determined above will be helpful in our further considerations, too. We denote the inverse mappings here in the following way:

$$\mathbf{x}(t) = X^{-1}\big(x(t)\big), \qquad (34a)$$

$$\mathbf{y}_1(t) = Y_1^{-1}\big(y(t)\big), \qquad (34b)$$

and

$$\mathbf{y}_2(t) = Y_2^{-1}\big(y(t)\big). \qquad (34c)$$

Observe now that the vectors $\mathbf{y}_1(t)$ and $\mathbf{y}_2(t)$ given by (32b) and (32c), respectively, constitute two possible interpretations of the PA output signal. The first one says that both the modulating signals $r(t)$ and $\psi(t)$ are transferred from the PA input to its output unchanged. Contrary to the above, the amplitude of the carrier signal changes from $r(t)$ to $A\big(r(t)\big)$ and its phase from $\psi(t)$ to $\psi(t)+\Phi(t)$. Further, note that this process can be schematically described in the following way:

$$\begin{bmatrix} \mathbf{x}(t) = X^{-1}(x(t)) \\ \Downarrow \\ \mathbf{y}_1(t) = Y_1^{-1}(y(t)) \end{bmatrix} = \begin{bmatrix} r(t) \to r(t) \\ \psi(t) \to \psi(t) \\ r(t)\cos(2\pi f_c t + \psi(t)) \to \\ A(r(t))\cos(2\pi f_c t + \psi(t) + \Phi(t)) \end{bmatrix}. \quad (35)$$

Observe from the above that in this interpretation the modulating signals $r(t)$ and $\psi(t)$ appear at the PA output undistorted. But, opposite to the above, the amplitude of the carrier signal appears at the PA output as distorted if $A(r(t))$ is not a linear function of $r(t)$. And, similarly, the phase of the carrier signal appears at the PA output as distorted if $\Phi(r(t)) \neq 0$.

This description corresponds with what really happens by transferring the input signal given by (2) to the PA output. It shows how the parameters of the carrier signal change while the modulating signals remain unchanged.

Consider now the second case related with the vector $\mathbf{y}_2(t)$. We see here both the modulating signals $r(t)$ and $\psi(t)$ through transferring from the PA input to its output change. More precisely, $r(t)$ turns into $A(r(t))$, and it is distorted in the case when the latter does not represent a linear function of $r(t)$. Further, $\psi(t)$ turns into $\psi(t)+\Phi(t)$. So, it can be assumed to be distorted when $\Phi(r(t)) \neq 0$. However, in this case, we assume the carrier signal passes through the PA unchanged. This process can be schematically described in the following way:

$$\begin{bmatrix} \mathbf{x}(t) = X^{-1}(x(t)) \\ \Downarrow \\ \mathbf{y}_2(t) = Y_2^{-1}(y(t)) \end{bmatrix} = \begin{bmatrix} r(t) \to A(r(t)) \\ \psi(t) \to \psi(t) + \Phi(t) \\ r(t)\cos(2\pi f_c t + \psi(t)) \to \\ r(t)\cos(2\pi f_c t + \psi(t)) \end{bmatrix}. \quad (36)$$

Observe that the latter interpretation attributes all the changes and distortions the modulating signals $r(t)$ and $\psi(t)$, not the carrier signal. Obviously, this does not correspond to the physical reality in the PA, but it is a convention used widely by the telecommunications engineers. The reason for doing so is, we think, the kind of modelling and description of the digital modulation schemes used in telecommunications exploiting the so-called signal constellations [30]. This point of view and interpretation enables observations of changes (distortions) in a given signal constellation when it passes through the PA [31]. However, we are going to develop this topic in more detail in the next paper.

At the end of this section, note that formally both the interpretations described above are acceptable from the point of view of the PA considered as a black box. This is so because the following equalities are satisfied:

$$\begin{aligned} y(t) &= Y_1(\mathbf{y}_1(t)) = Y_2(\mathbf{y}_2(t)) = \\ &= PA(X(\mathbf{x}(t))) = PA(x(t)) \end{aligned} . \quad (37)$$

The relations (37) are also illustrated in Fig. 5.

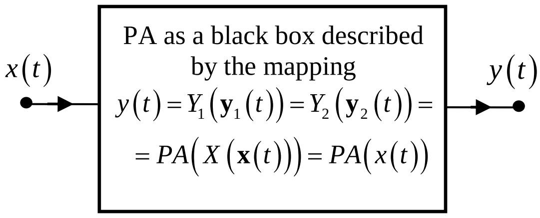

Fig. 5. Illustration of relations (37) for the PA viewed as a black box.

V. Relation Between AM/PM Conversion and Memoryless Property

First, we recall here that the AM/PM conversion of the PA has been defined in the literature [2], [9] as the function $\Phi(r(t))$ occurring in (2). Second, it follows from (30) that the relation between the delay t_D, which is the parameter determining whether a circuit possesses memory or does not, and the function $\Phi(r(t))$ is the following:

$$t_D = -\Phi(r(t)) / (2\pi f_c) \ . \quad (38)$$

An third, let us also recall our definition of possessing memory or not possessing it that was formulated in the three points below (9b).

Considering now the above, we deduce easily that

1. a circuit possesses no memory when $\Phi(r(t)) \equiv 0$;
2. a circuit can be assumed to be approximately memoryless if the values of $\Phi(r(t))$ fulfil the inequality $0 < -\Phi(r(t))/(2\pi f_c) \leq D_x$;
3. a circuit has evidently a memory when the values of $-\Phi(r(t))/(2\pi f_c)$ are greater than the assumed value D_x.

Note also here that the second condition from point 2 above, expressing the condition for a circuit to be approximately memoryless (almost memoryless), can be formulated for the worst case - with help of (31) - as

$$0 < -\Phi\left(\max_i \left(AMP_{s_i}\right)\right) / (2\pi f_c) \leq D_x \quad (39a)$$

or

$$0 < -\Phi\left(\max_i \left(\sqrt{P_{s_i}/b}\right)\right) / (2\pi f_c) \leq D_x \ . \quad (39b)$$

Finally in this section, observe that the results presented therein confirm that the property of having exactly no memory by a PA excludes demonstrating by it nonzero values of the AM/PM conversion.

VI. Model of Saleh and Related Ones Incorporate Memory Effect

In [2], Saleh developed a model, which approximates the AM/AM and AM/PM conversions of the PA by the following functions:

$$\text{AM/AM} = A\left(r\left(t\right)\right) = \frac{\alpha_1 r\left(t\right)}{1+\alpha_2\left(r\left(t\right)\right)^2} \tag{40a}$$

and

$$\text{AM/PM} = \Phi\left(r\left(t\right)\right) = \frac{\beta_1\left(r\left(t\right)\right)^2}{1+\beta_2\left(r\left(t\right)\right)^2}\ , \tag{40b}$$

where the coefficients α_1 and α_2 as well as β_1 and β_2 assume real values. They are adjusted to the measured data for a given amplifier.

Note that it follows evidently from (40b) that the values of the AM/PM conversion of the PA differ from zero for $r\left(t\right) \neq 0$. Therefore, in view of the results of the previous section, the Saleh's model cannot be considered as a memoryless one. Possibly, it can be only regarded as an approximately (almost) memoryless one when the conditions specified for this PA class in the previous section are fulfilled.

The model developed by Ghorbani [4] behaves similarly in the sense that its AM/PM function differs from zero for $r\left(t\right) \neq 0$. Hence, the conclusions drawn for the Saleh's model just before are valid also in this case.

Contrary to the above, the Rapp's model [3] assumes $A\left(r\left(t\right)\right) \equiv 0$, what means that this model is exactly memoryless. Obviously, another thing is how exactly this model, incorporating the above assumption, describes reality.

VII. Concluding Remarks

This paper shows that there are still topics, in which doubtful and imprecise notions and definitions are used. As seen, they are repeated and disseminated by a new generation of researchers, without any critical reviewing.

We show here that some critical reviewing is in many cases desirable. One of the examples is the memoryless narrowband PA, which is assumed in the literature to possess the nonzero values of the AM/PM conversions. This example is considered in this paper and we show here that the two properties mentioned above are mutually exclusive. A thorough analysis of the problem is presented. As a result, a useful and consistent platform for understanding the behavior of the PA is developed.

We hope this platform will be also utilized for re-thinking the current principles the designers of the PAs use.

Appendix
Derivation of Equation (4) for the Nonlinear Passband Power Amplifier

Let us substitute (1) and (3) into the Volterra series defined as [10]

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} h^{(1)}(\tau)x(t-\tau)d\tau + \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(2)}(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)\cdot \\ &\cdot x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(4)}(\tau_1,\tau_2,\tau_3,\tau_4)x(t-\tau_1)x(t-\tau_2)\cdot \\ &\cdot x(t-\tau_3)x(t-\tau_4)d\tau_1 d\tau_2 d\tau_3 d\tau_4 + \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(5)}(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5)x(t-\tau_1)x(t-\tau_2)\cdot \\ &\cdot x(t-\tau_3)x(t-\tau_4)x(t-\tau_5)d\tau_1 d\tau_2 d\tau_3 d\tau_4 d\tau_5 + \ldots\ldots\ . \end{aligned} \tag{A1}$$

As the result, we obtain

$$\begin{aligned} y(t) &= b_1 \int_{-\infty}^{\infty} \exp\left(-\tau/a\right)\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau\right)\exp\left(j\left(\omega_c\left(t-\tau\right)+\psi\left(t-\tau\right)\right)\right)\right\}d\tau + \\ &+ b_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(-\tau_1/a\right)\exp\left(-\tau_2/a\right)\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_1\right)\exp\left(j\left(\omega_c\left(t-\tau_1\right)+\psi\left(t-\tau_1\right)\right)\right)\right\}\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_2\right)\exp\left(j\left(\omega_c\left(t-\tau_2\right)+\psi\left(t-\tau_2\right)\right)\right)\right\}d\tau_1 d\tau_2 + \\ &+ b_3 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(-\tau_1/a\right)\exp\left(-\tau_2/a\right)\exp\left(-\tau_3/a\right)\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_1\right)\exp\left(j\left(\omega_c\left(t-\tau_1\right)+\psi\left(t-\tau_1\right)\right)\right)\right\}\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_2\right)\exp\left(j\left(\omega_c\left(t-\tau_2\right)+\psi\left(t-\tau_2\right)\right)\right)\right\}\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_3\right)\exp\left(j\left(\omega_c\left(t-\tau_3\right)+\psi\left(t-\tau_3\right)\right)\right)\right\}\cdot \\ &\cdot d\tau_1 d\tau_2 d\tau_3 + b_4 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left(-\tau_1/a\right)\exp\left(-\tau_2/a\right)\cdot \\ &\cdot \exp\left(-\tau_3/a\right)\exp\left(-\tau_4/a\right)\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_1\right)\exp\left(j\left(\omega_c\left(t-\tau_1\right)+\psi\left(t-\tau_1\right)\right)\right)\right\}\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_2\right)\exp\left(j\left(\omega_c\left(t-\tau_2\right)+\psi\left(t-\tau_2\right)\right)\right)\right\}\cdot \\ &\cdot \operatorname{Re}\left\{r\left(t-\tau_3\right)\exp\left(j\left(\omega_c\left(t-\tau_3\right)+\psi\left(t-\tau_3\right)\right)\right)\right\}\cdot \end{aligned} \tag{A2}$$

$$\cdot \operatorname{Re}\left\{ r(t-\tau_4)\exp\left(j\left(\omega_c (t-\tau_4) + \psi(t-\tau_4) \right) \right) \right\} \cdot$$
$$\cdot d\tau_1 d\tau_2 d\tau_3 d\tau_4 + b_5 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp(-\tau_1/a)\exp(-\tau_2/a)\cdot$$
$$\cdot \exp(-\tau_3/a)\exp(-\tau_4/a)\exp(-\tau_5/a)\cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_1)\exp\left(j\left(\omega_c (t-\tau_1) + \psi(t-\tau_1) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_2)\exp\left(j\left(\omega_c (t-\tau_2) + \psi(t-\tau_2) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_3)\exp\left(j\left(\omega_c (t-\tau_3) + \psi(t-\tau_3) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_4)\exp\left(j\left(\omega_c (t-\tau_4) + \psi(t-\tau_4) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_5)\exp\left(j\left(\omega_c (t-\tau_5) + \psi(t-\tau_5) \right) \right) \right\} \cdot$$
$$\cdot d\tau_1 d\tau_2 d\tau_3 d\tau_4 d\tau_5 + \ldots \quad .$$

In the next step, let us restrict consideration of the characteristics of the PA discussed to only the range involved approximately in its memory. This can be done by neglecting these parts of the impulse responses (3) that lie outside the time interval $<0,\ a>$. That is by equating them to zero. In other words, it means that we exploit then the following approximations:

$$h^{(1)}(t) \cong \begin{cases} b_1 \cdot \exp(-t/a) \text{ for } t \in <0,\ a>, \\ 0 \ \text{ otherwise} \end{cases} \tag{A3a}$$

$$h^{(2)}(t_1,t_2) \cong \begin{cases} b_2 \cdot \prod_{i=1}^{2} \exp(-t_i/a) \text{ for } t_1,t_2 \in <0,\ a>, \\ 0 \ \text{ otherwise} \end{cases} \tag{A3b}$$

$$h^{(3)}(t_1,t_2,t_3) \cong \begin{cases} b_3 \cdot \prod_{i=1}^{3} \exp(-t_i/a) \text{ for } t_1,t_2,t_3 \in <0,\ a>, \\ 0 \ \text{ otherwise} \end{cases} \tag{A3c}$$

$$h^{(4)}(t_1,\ldots,t_4) \cong \begin{cases} b_4 \cdot \prod_{i=1}^{4} \exp(-t_i/a),\ t_i \in <0,\ a>,\ i=1,..,4 \\ 0 \ \text{ otherwise} \end{cases}, \tag{A3d}$$

$$h^{(5)}(t_1,\ldots,t_5) \cong \begin{cases} b_5 \cdot \prod_{i=1}^{5} \exp(-t_i/a),\ t_i \in <0,\ a>,\ i=1,..,5 \\ 0 \ \text{ otherwise} \end{cases}, \tag{A3e}$$

and so on.

Introducing the above approximations into (A2), we can rewrite this relation in the following way:

$$y(t) \cong b_1 \int_0^a \exp(-\tau/a)\cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau)\exp\left(j\left(\omega_c (t-\tau) + \psi(t-\tau) \right) \right) \right\} d\tau +$$
$$+ b_2 \int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a)\cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_1)\exp\left(j\left(\omega_c (t-\tau_1) + \psi(t-\tau_1) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_2)\exp\left(j\left(\omega_c (t-\tau_2) + \psi(t-\tau_2) \right) \right) \right\} d\tau_1 d\tau_2 +$$
$$+ b_3 \int_0^a\int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a)\exp(-\tau_3/a)\cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_1)\exp\left(j\left(\omega_c (t-\tau_1) + \psi(t-\tau_1) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_2)\exp\left(j\left(\omega_c (t-\tau_2) + \psi(t-\tau_2) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_3)\exp\left(j\left(\omega_c (t-\tau_3) + \psi(t-\tau_3) \right) \right) \right\} \cdot$$
$$\cdot d\tau_1 d\tau_2 d\tau_3 + b_4 \int_0^a\int_0^a\int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a)\cdot$$
$$\cdot \exp(-\tau_3/a)\exp(-\tau_4/a)\cdot \tag{A4}$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_1)\exp\left(j\left(\omega_c (t-\tau_1) + \psi(t-\tau_1) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_2)\exp\left(j\left(\omega_c (t-\tau_2) + \psi(t-\tau_2) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_3)\exp\left(j\left(\omega_c (t-\tau_3) + \psi(t-\tau_3) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_4)\exp\left(j\left(\omega_c (t-\tau_4) + \psi(t-\tau_4) \right) \right) \right\} \cdot$$
$$\cdot d\tau_1 d\tau_2 d\tau_3 d\tau_4 + b_5 \int_0^a\int_0^a\int_0^a\int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a)\cdot$$
$$\cdot \exp(-\tau_3/a)\exp(-\tau_4/a)\exp(-\tau_5/a)\cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_1)\exp\left(j\left(\omega_c (t-\tau_1) + \psi(t-\tau_1) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_2)\exp\left(j\left(\omega_c (t-\tau_2) + \psi(t-\tau_2) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_3)\exp\left(j\left(\omega_c (t-\tau_3) + \psi(t-\tau_3) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_4)\exp\left(j\left(\omega_c (t-\tau_4) + \psi(t-\tau_4) \right) \right) \right\} \cdot$$
$$\cdot \operatorname{Re}\left\{ r(t-\tau_5)\exp\left(j\left(\omega_c (t-\tau_5) + \psi(t-\tau_5) \right) \right) \right\}$$
$$\cdot d\tau_1 d\tau_2 d\tau_3 d\tau_4 d\tau_5 + \ldots \quad .$$

Now, we assume that the slowly varying baseband signals $r(t)$ and $\psi(t)$ occurring in (1) do not approximately change in the interval $<0,\ a>$ of the integrations indicated in the definite integrals in (A4) for the time variables $\tau,\ \tau_i,\ i=1,2,3,\ldots$. In other words, we assume that

$$r(t-\tau_i) \cong r(t) \quad \text{and} \quad \psi(t-\tau_i) \cong \psi(t) \tag{A5}$$

for $\tau = \tau_1,\ \tau_i,\ i = 1, 2, 3, \ldots$, taking on the values from the range $< 0,\ a >$.

Note that in the frequency domain the above assumption can be interpreted in the following way: (A5) is valid if the maximal frequency in the amplitude characteristics of the baseband signals $r(t)$ and $\psi(t)$, say f_m, fulfils the inequality: $f_m << 1/a$.

So, taking into account (A5) in (A4), we can rewrite the latter as

$$\begin{aligned}
y(t) \cong\ & b_1 \cdot r(t) \int_0^a \exp(-\tau/a) \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau)+\psi(t)\right)\right)\right\} d\tau + \\
& + b_2 \cdot (r(t))^2 \int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a) \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_1)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_2)+\psi(t)\right)\right)\right\} d\tau_1 d\tau_2 + \\
& + b_3 \cdot (r(t))^3 \int_0^a\int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a)\exp(-\tau_3/a) \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_1)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_2)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_3)+\psi(t)\right)\right)\right\} d\tau_1 d\tau_2 d\tau_3 + \\
& + b_4 \cdot (r(t))^4 \int_0^a\int_0^a\int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a) \cdot \\
& \cdot \exp(-\tau_3/a)\exp(-\tau_4/a)\,\mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_1)+\psi(t)\right)\right)\right\} \cdot \\
& \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_2)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_3)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_4)+\psi(t)\right)\right)\right\} d\tau_1 d\tau_2 d\tau_3 d\tau_4 + \\
& + b_5 \cdot (r(t))^5 \int_0^a\int_0^a\int_0^a\int_0^a\int_0^a \exp(-\tau_1/a)\exp(-\tau_2/a) \cdot \\
& \cdot \exp(-\tau_3/a)\exp(-\tau_4/a)\exp(-\tau_5/a) \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_1)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_2)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_3)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_4)+\psi(t)\right)\right)\right\} \cdot \\
& \cdot \mathrm{Re}\left\{\exp\left(j\left(\omega_c(t-\tau_5)+\psi(t)\right)\right)\right\} d\tau_1 d\tau_2 d\tau_3 d\tau_4 d\tau_5 + \ldots .
\end{aligned} \qquad \text{(A6)}$$

In what follows, we restrict ourselves to retaining only the first N components in the Volterra series description (A6). Also, we include a passband filter with the centre frequency $f_c = \omega_c/(2\pi)$ at the PA output into its structure. That is we treat this filter as a part of the PA model. And, because of this reason, we can say about such a PA that it is as a narrowband one.

Obviously, without including the passband filter into the PA model or lack of such a filter at all, the relation (A6) models a moderately nonlinear wideband amplifier driven by a signal of the form (1).

In what follows now, we consider modelling narrowband amplifiers. To this end, observe that by virtue of the passband filter all the products in (A6) related to the frequencies different from $\pm f_c$ are filtered out in the narrowband amplifier. So, in effect, we obtain the following expression:

$$\begin{aligned}
y_N(t) \cong & \sum_{n=1,\ \text{odd}}^{N} \left(\frac{r(t)}{2}\right)^n \underbrace{\int_0^a \ldots \int_0^a}_{n\ \text{times}} b_n \Big[C\left(n,(n-1)/2\right) \cdot \\
& \cdot \exp\left(j\left(\left(\omega_c t + \psi(t)\right) - \omega_c \sum_{i=1;C(n,(n-1)/2)}^{n} \tau_i \right)\right) + C\left(n,(n+1)/2\right) \cdot \\
& \cdot \exp\left(-j\left(\left(\omega_c t + \psi(t)\right) + \omega_c \sum_{i=1;C(n,(n+1)/2)}^{n} \tau_i \right)\right)\Big] \prod_{i=1}^{n} \exp(-\tau_i/a)\, d\tau_i ,
\end{aligned}$$
(A7)

where $y_N(t)$ denotes the PA output signal after passing through its bandpass output filter (narrowband output signal). Furthermore, the symbol $C(n,m) = n!/(m!(n-m)!)$ in (A7) is the so-called binomial coefficient and the symbol $\sum_{i=1;C(n,m)}^{n} \tau_i$ stands for a sum of the time variables τ_i related with one of the distinct product frequencies. Note that there are in each case $C(n,m)$ of such combinations what is indicated here by using this symbol beneath the summation symbol $\sum$. The detailed derivations can be found in [5] and [6].

To get rid of the integrals in (A7), we apply now the multidimensional Fourier transforms [10] to the nonlinear impulse responses occurring in (A7). These transforms are called the nonlinear transfer functions $H^{(n)}(f_1,..,f_n)$ of the corresponding orders $n = 1$ (linear case), $2, 3, \ldots$, and are given by

$$\begin{aligned}
H^{(n)}(f_1,..,f_n) = & \underbrace{\int_{-\infty}^{\infty} .. \int_{-\infty}^{\infty}}_{n\ \text{times}} h^{(n)}(\tau_1,..,\tau_n) \cdot \\
& \cdot \exp(-j2\pi f_1 \tau_1) \cdot\cdot \exp(-j2\pi f_n \tau_n)\, d\tau_1 \cdot\cdot d\tau_n
\end{aligned} , \qquad \text{(A8)}$$

where $f_1,.., f_n$ mean the frequencies forming the n-th dimensional frequency space [10]. In what follows below, we use also, for simplicity, the notation $H^{(n)}(\omega_1,..,\omega_n)$ with $\omega_i = 2\pi f_i, \; i=1,2,...,n,$ instead of $H^{(n)}(f_1,..,f_n)$. Generally in this paper, the notations $H^{(n)}(f_1,..,f_n)$ and $H^{(n)}(\omega_1,..,\omega_n)$ are used interchangeably.

We apply (A8) in (A7) in the following way:

$$\underbrace{\int_0^a .. \int_0^a}_{n \text{ times}} b_n \cdot \exp\left(-j\omega_c \sum_{i=1;C(n,(n-1)/2)}^{n} \tau_i\right) \prod_{i=1}^{n} \exp(-\tau_i/a)\, d\tau_i \cong$$
$$\cong \underbrace{\int_{-\infty}^{\infty} .. \int_{-\infty}^{\infty}}_{n \text{ times}} b_n \cdot \exp\left(-j\omega_c \sum_{i=1;C(n,(n-1)/2)}^{n} \tau_i\right) \prod_{i=1}^{n} \exp(-\tau_i/a)\, d\tau_i \cong \quad \text{(A9)}$$
$$\cong H^{(n)}\left(\chi_n^+(\pm\omega_c)\right) = b_n \cdot \frac{a}{1+j\chi_{n,1}^+(\pm\omega_c)a} \cdot ... \cdot \frac{a}{1+j\chi_{n,n}^+(\pm\omega_c)a},$$

where $\chi_n^+(\pm\omega_c)$ and $\chi_n^-(\pm\omega_c)$ denote such the angular frequency sets $\{\omega_1,..,\omega_n\}$ whose elements $\omega_i, \; i=1,2,...,n,$ can assume only the values $+\omega_c$ or $-\omega_c$, and whose sums give the value $+\omega_c$ or $-\omega_c$, respectively. Furthermore, $\chi_{n,1}^+(\pm\omega_c)$ in (A9) means the first element of the set $\chi_n^+(\pm\omega_c)$, and so on.

Obviously, the components in (A7) related with the expressions involving the sums $\sum_{i=1;C(n,(n+1)/2)}^{n} \tau_i$ can be transformed in the same way as those in (A9) with $H^{(n)}\left(\chi_n^-(\pm\omega_c)\right)$ meaning

$$H^{(n)}\left(\chi_n^-(\pm\omega_c)\right) = b_n \cdot \frac{a}{1+j\chi_{n,1}^-(\pm\omega_c)a} \cdot ... \cdot \frac{a}{1+j\chi_{n,n}^-(\pm\omega_c)a} \quad . \qquad \text{(A10)}$$

Further, taking into account this and (A9) in (A7), we get

$$y_N(t) \cong \sum_{n=1,\text{ odd}}^{N} \left(\frac{r(t)}{2}\right)^n \cdot \Big[C\big(n,(n-1)/2\big) \cdot$$
$$\cdot H^{(n)}\left(\chi_n^+(\pm\omega_c)\right) \exp\big(j(\omega_c t + \psi(t))\big) + C\big(n,(n+1)/2\big) \cdot \quad \text{(A11)}$$
$$\cdot H^{(n)}\left(\chi_n^-(\pm\omega_c)\right) \exp\big(-j(\omega_c t + \psi(t))\big)\Big] \quad .$$

For more details, see also [8].

REFERENCES

[1] Z. Zhu, H. Leung, and X. Huang, “Challenges in reconfigurable radio transceivers and application of nonlinear signal processing for RF impairment mitigation,” *IEEE Circuits and Systems Magazine*, vol. 13, pp. 44-65, 2013.

[2] A. A. M. Saleh, “Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers,” *IEEE Trans. on Communications*, vol. 29, pp. 1715-1720, 1981.

[3] C. Rapp, “Effects of HPA-nonlinearity on a 4-DPSWOFDM signal for a digital sound broadcasting system,” in *Proc. Second European Conf. on Satellite Commun.*, pp. 179–184, 1991.

[4] A. Ghorbani and M. Sheikhan, “The effect of solid state power amplifiers (SSPAs) nonlinearities on MPSK and M-QAM signal transmission,” in *Proc. Sixth Int. Conf. Digital Process. Signal Comm.*, pp. 193–197, 1991.

[5] A. Borys and W. Sieńko, “On modelling AM/AM and AM/PM conversions via Volterra series,” *Int. Journal of Telecommunications and Electronics (JET)*, vol. 62, pp. 267-272, 2016.

[6] A. Borys, “Quadrature mapping, Saleh’s representation, and memory models,” *Int. Journal of Telecommunications and Electronics (JET)*, vol. 62, pp. 389-394, 2016.

[7] A. Borys, “Saleh’s model of AM/AM and AM/PM conversions is not a model without memory – another proof,” *Int. Journal of Telecommunications and Electronics (JET)*, vol. 63, pp. 73-77, 2017.

[8] A. Borys, “On correct understanding and classification of Saleh’s and related models of AM/AM and AM/PM conversions,” *Int. Journal of Telecommunications and Electronics (JET)*, accepted for publication, 2017.

[9] J. Joung, C. K. Ho, K. Adachi, and S. A. Sun, “Survey on power-amplifier-centric techniques for spectrum- and energy-efficient wireless communications,” *IEEE Communications Surveys & Tutorials*, vol. 17 pp. 315-333, 2015.

[10] M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems*, New York: John Wiley & Sons, 1980.

[11] M. O’Droma, S. Meza, and Y. Lei, “New modified Saleh models for memoryless nonlinear power amplifier behavioural modelling,” *IEEE Commun. Lett.*, vol. 13, pp. 399–401, 2009.

[12] R. Pasricha and S. Kumar, “Power amplifier memory-less nonlinear modeling,” *International Journal of the Physical Sciences*, vol. 6, pp. 2644-2648, 2011.

[13] M. C. Jeruchim, P. Balaban, and K. S. Shanmugan, *Simulation of Communication Systems: Modeling, Methodology, and Techniques*. New York: Springer, 2000.

[14] A. A. M. Saleh, “Matrix analysis of mildly nonlinear, multiple-input, multiple-output systems with memory,” *The Bell System Technical Journal*, vol. 61, pp. 2221-2243, 1982.

[15] I. W. Sandberg, “The mathematical foundations of associated expansions for mildly nonlinear systems,” *IEEE Transactions on Circuits and Systems*, vol. 30, pp. 441-445, 1983.

[16] I. W. Sandberg, “Existence and evaluation of almost periodic steady-state responses of mildly nonlinear systems,” *IEEE Transactions on Circuits and Systems*, vol. 31, pp. 689-701, 1984.

[17] D. D. Weiner and J. Spina, *Sinusoidal Analysis and Modeling of Weakly Nonlinear Circuits*. New York: Van Nostrand, 1980.

[18] Y. L Kuo, “Frequency-domain analysis of weakly nonlinear networks. "Canned" Volterra analysis, part 1,” *IEEE Circuits and Systems Magazine*, vol. 11, pp. 2-8, August 1977.

[19] Y. L Kuo, “Frequency-domain analysis of weakly nonlinear networks. "Canned" Volterra analysis, part 2,” *IEEE Circuits and Systems Magazine*, vol. 11, pp. 2-6, October 1977.

[20] J. Huijsing, R. van de Plassche, and W. Sansen (Editors), *Analog Circuit Design: Chapter 1.2. CMOS Low-Power Analog Circuit Design*. Dordrecht: Kluwer Academic Publishers, 1999.

[21] A. Borys, “An analysis of slew-induced distortion in single-amplifier active filters using the Volterra-Wiener series technique,” *Int. Journal of Circuit Theory and Applications*, vol. 10, pp. 81-94, 1982.

[22] M. O’Droma, S. Meza, and Y. Lei, “New modified Saleh models for memoryless nonlinear power amplifier behavioural modelling,” *IEEE Commun. Lett.*, vol. 13, pp. 399–401, 2009.

[23] R. Pasricha and S. Kumar, “Power amplifier memory-less nonlinear modeling,” *International Journal of the Physical Sciences*, vol. 6, pp. 2644-2648, 2011.

[24] W. J. Rugh, *Nonlinear System Theory: The Volterra/Wiener Approach*. Baltimore: Johns Hopkins Univ. Press, 1981.

[25] L. O. Chua and Y. Liao, “Measuring Volterra kernels (II)”, *Int. Journal of Circuit Theory and Applications*, vol. 17, pp. 151-190, 1989.

[26] L. O. Chua and Y. Liao, “Measuring Volterra kernels (III): How to estimate the highest significant order”, *Int. Journal of Circuit Theory and Applications*, vol. 19, pp. 189-209, 1991.

[27] S. Mitra, *Analysis and Synthesis of Linear Active Networks*. New York: John Wiley & Sons, 1969.

[28] A. Gelb and W. E. Vander Velde, *Multiple-Input Describing Functions and Nonlinear System Design*. New York: McGraw-Hill, 1968.

[29] D. P. Atherton, *Nonlinear Control Engineering: Describing Function Analysis and Design.* London: Van Nostrand Reinhold, 1975.

[30] J. G. Proakis and M. Salehi, *Digital Communications*. New York: McGraw-Hill, 2008 (5th edition).

[31] S. Golara, S. Moloudi, and A. A. Abidi, “Processes of AM-PM distortion in large-signal single-FET amplifiers”, *IEEE Trans. on Circuits and Systems—I: Regular Papers*, vol. 64, pp. 245-260, 2017.