ACADEMIC REPORTS Fac. Eng. Tokyo Polytech. Univ. Vol. 35 No.1 (2012)

Stochastic Analysis of Charging and Recombination in Double-Hetero Tunnel-Junction Quantum Dot Semiconductor Laser

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The double-hetero tunnel-junction quantum dot semiconductor laser model is presented in this paper to analyze a charging process by tunneling to an isolated quantum dot in an active area. The double-hetero tunnel-junction structure including a low band gap quantum dot prevents tunneling transitions of electrons and holes to *p* and *n* type sides and enhances the recombination in the dots, while a high band gap quantum dot can be used for an electron or hole current selectable transistor. A photon contained in a laser output is produced by a recombination of an electron and a hole in the quantum dot. Although the output contains many photon quanta, each charging of the electron and hole is discrete process, and the recombination is occurred in a probability manner. The laser output analysis is based on the stochastic process analysis appropriate to the treatment of the problem. The analytical solution for the probability to the charging number is given, and numerical calculations of the output characteristics are presented.

Introduction

Quantum devices are regarded as being the most suitable key component for next-generation advanced optical and electronic technology. Among the quantum devices, the quantum dot devices in particular is considered to be a fundamental building block for quantum optic technologies, and has been intensively explored both experimentally and theoretically [1]. Technologies for fabricating quantum dot devices have now reached such a level that the optical characteristics can be precisely controlled by the accurate structure of the device configuration processes. Furthermore the double-hetero tunnel-junction quantum dot device are particularly important, and the fundamental physical properties are researched in detail [2][3].

As one of the novel functional devices, electron and hole current switch n-i-p type semiconductor quantum dot transistor is suggested [4]. The theory for the n-i-p type semiconductor quantum dot device can be applied for the semiconductor laser. The double-hetero tunnel-junction quantum dot devise is particularly useful for not only a transistor but also a laser. Therefore the double-hetero tunnel-junction n-i-p type semiconductor device should be researched more detail and possibilities of the device have to be considered.

Double-Hetero Tunnel-Junction n-i-p device

The n-i-p type semiconductor quantum dot device was suggested as a transistor which can select an electron or a hole current by positive or negative gate voltage. The device consists of n type area, i type dot, and p type area. Depending on the band gap energy of the quantum dot to the n and p type areas, the device can be used for a laser and a transistor. In the dot, a recombination of an electron and a hole produces a photon. The recombination characteristic is controlled by the structure of the band energy in the device.

The homo tunnel-junction n-i-p type quantum dot device is shown in Fig. 1 and the device was suggested recently for the transistor [4]. As shown in the figure, all the band gap energies of the source, the drain, and the dot have a same value. This structure is simple for fabrication because a same semiconductor material can be used for each of the n area, p area, and dot. Therefore the structure is appropriate for a first

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Received sept. 18, 2012

step model for the electron and hole recombining n-i-p type quantum dot device.

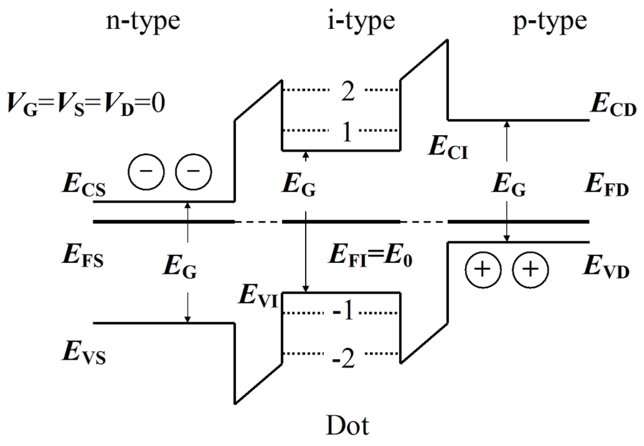

Fig. 1 A homo tunnel-junction device applied no bias voltage.

When appropriately adjusted n and p area bias voltages are applied to the homo tunnel-junction n-i-p type device, an electron becomes to be able to flow-in to the dot as shown in Fig.2. Immediately after flowing-in of an electron to the dot, potential energy of the dot is pulled up to $\boldsymbol{e}\times\boldsymbol{e}/(2\boldsymbol{C}_\Sigma)$ where $\boldsymbol{e}$ and $\boldsymbol{C}_\Sigma$ mean an elementary electric charge and the total capacitance around of the dot, and simultaneously a hole becomes to be able to flow-in to the dot from the drain to recombine to the electron in the dot.

For the flow-in process of a hole, a positive charging energy $\boldsymbol{e}\times\boldsymbol{e}/(2\boldsymbol{C}_\Sigma)$ is required for returning to zero charge (one electron and one hole) state of the dot, however, the required compensation energy is given by an energy released by an electron's pulling down. There is a possibility to induce direct tunnel passing of electron and hole from n (p) to p (n) area, and to give considerable recombination performance degradation.

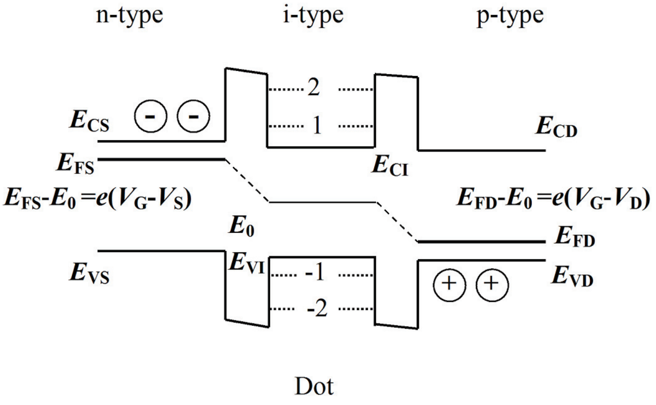

Fig. 2 Mixture process of tunneling and recombination of electrons and holes in the homo tunnel-junction n-i-p structure.

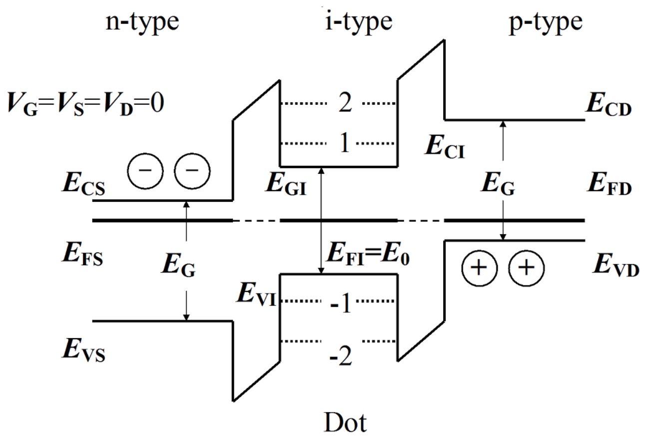

Fig. 3 The double-hetero tunnel-junction device applied no bias voltage.

To prevent the direct tunnel passing, a double-hetero tunnel-junction transistor shown in Fig. 3 is introduced. As shown in the figure, the band gap energy E_{GI} of the dot is smaller than the band gap energy E_G of the n and p areas. The type of a junction consisting of two different band gap energies is called a hetero junction. Therefore the device introduced here can be called a double-hetero tunnel-junction quantum dot device because the both sides configure hetero junctions.

As shown in Fig.4, the difference between the band gap of the dot and the band gap of the source (or drain) configures pooling wells to electrons and holes to enhance recombination in the dot. Therefore it is considered that an electron flowed from n area into the dot can recombine without flow-out to the p area, and the double-hetero tunnel-junction structure is useful for an improvement of an electron and hole recombination enhancing.

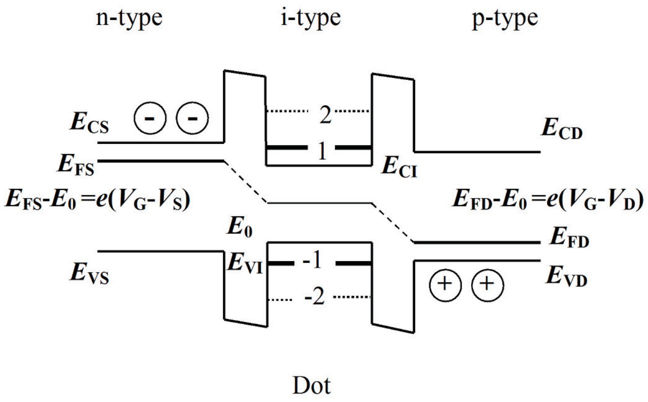

Fig. 4 Recombination enhancing by double-hetero tunnel-junction structure.

Analyses

Using the probability p_0 as an initial term solution for the probability p_N which represents the state of the dot for positive and negative values of the charging number of electrons N are respectively given by:

$$p_N = p_0 \prod_{r=1}^{N} \left(\frac{\lambda_{r-1}}{\mu_r} \right) \qquad (1 \le N < \infty) \tag{1a}$$

$$p_{-N} = p_0 \prod_{r=1}^{N} \left(\frac{\mu_{-r+1}}{\lambda_{-r}} \right) \qquad (1 \le N < \infty) \tag{1b}$$

In the above equations a negative value of N represents positively charged states of the dot, corresponds to an excess of N holes in the dot, and λ_r, μ_r are flow in rate and flow out rate respectively. The undetermined probability p_0 in the above equations can be fixed by using the axiom in probability theory that requires the sum of all probabilities to be equal to unity.

$$\sum_{N=-\infty}^{\infty} p_N \equiv \mathbf{1}$$

The application of the formula gives the following expression for p_0.

$$p_0 = \frac{1}{1 + \sum_{N=1}^{\infty} \prod_{r=1}^{N} \left(\frac{\lambda_{r-1}}{\mu_r} \right) + \sum_{N=1}^{\infty} \prod_{r=1}^{N} \left(\frac{\mu_{-r+1}}{\lambda_{-r}} \right)} \tag{2}$$

In the above analyses, the negative charge flow-in rate λ_N is given by

$$\boldsymbol{\lambda}_N = \boldsymbol{\lambda}_{\mathrm{S}N} + \boldsymbol{\lambda}_{\mathrm{D}N} \tag{3a}$$

For a positive value of N, a negative charge flow-in rate to the dot $\boldsymbol{\lambda}_N$ is regarded as a sum of electron flow-in rates from the source $\boldsymbol{\lambda}_{\mathrm{S}N}$ and the drain $\boldsymbol{\lambda}_{\mathrm{D}N}$ to the dot. The two terms on the right-hand side of Eq. (3a) can be estimated in a similar manner to the metal quantum dot transistor, and are obtained by simply extending the orthodox theory giving

$$\begin{aligned} \lambda_{\mathrm{S}N} &= \frac{1}{e^2 R_{\mathrm{Se}}} F_{\mathrm{S}}^{+}(E_{\mathrm{CS}}) \\ \lambda_{\mathrm{D}N} &= \frac{1}{e^2 R_{\mathrm{De}}} F_{\mathrm{D}}^{+}(E_{\mathrm{CD}}) \end{aligned} \tag{3b}$$

$$(0 \le N < \infty)$$

where $F_{\mathrm{S}}^{+}(X)$ is given by

$$F_{\mathrm{S}}^{+}(X)=\int_{X}^{\infty}\frac{1}{1+\mathrm{e}^{\frac{E-E_{\mathrm{FS}}}{kT}}}\left(1-\frac{1}{1+\mathrm{e}^{\frac{E-E_{\mathrm{Qe}}-(N+1/2)e^{2}/C_{\Sigma}}{kT}}}\right)dE$$

In the above equations, R_{Se} and R_{De} are the tunneling resistances for an electron through the n and p area side barriers, respectively, and E_{Qe} is the lowest state of electrons in the conduction band of the dot biased by $-eV_{\mathrm{G}}$ and E_{FS} is the Fermi level in the n area biased by $-eV_{\mathrm{S}}$. The equation for $F_{\mathrm{S}}^{+}(X)$ is derived in a similar manner based on the concept for the derivation of Eq. (3b). The integrant in the equation consists of two parts; the first part is the Fermi-Dirac distribution in the n area and the second part is the empty state distribution in the dot. While the Fermi level in the dot, when no electron is charged, is given by E_0, the Fermi level in the dot when an electron is charged, however, becomes the electron energy level of the charged electron, since the highest level of electron energies in the dot at temperature 0 K is the quantum energy level in the conduction band occupied by the last charged electron. Therefore the Fermi level when N electrons are charged into the dot is given by $E_{\mathrm{Qe}}+(N+1/2)e^2/C_{\Sigma}$.

For derivation of Eq. (3b) to the metal, the value of the energy $\boldsymbol{E}$ for an electron can take any value between $-\infty$ and ∞. For the semiconductor, however, the integration has to be limited to the conduction band for an electron. $X=\max\{E_{\mathrm{CS}}, E_{\mathrm{Qe}}\}$ should then be assumed as the lower bound of the integration because the lowest energy level of tunneling electrons is a higher value among both the lower bound levels in the source and the dot. $F_{\mathrm{D}}^{+}(X)$ is given by an equation similar to the equation for $F_{\mathrm{S}}^{+}(X)$, where the subscript $_{\mathrm{S}}$ is replaced with $_{\mathrm{D}}$. All other necessary rates can be calculated in a similar manner.

Computation results

Analytical procedure is constructed at a basis of stochastic theory, and probability equations for the number states of elementary charges in the dot and an equilibrium solution to the equations are derived in previous section. In this procedure, Coulomb blockade energy is treated as a type of capacitive insertion energy. Therefore the charging energy is regarded as an energy which pull-up or pull-down a potential energy of the dot like a gate voltage energy. The energy has to be exactly discriminated from a type of quantum energies which are decided by Schrödinger wave function for an electron in the dot as a classical potential box.

In this treatment, all the energies considered here except recombination energy are classical types, and the transition between quantum state energies, namely recombination in the dot, produce a photon. The detail effects of quantum energy states can be included in a simple extension of this treatment, however, the treatment will be complex in some level and make unclear the recombination effects. Therefore the secondary quantum state effects are excluded here.

Fig. 5 shows the characteristics of a laser output (a recombination current consisting of an electron current flowing-into the dot of the negative state and a hole current flowing-into the dot of the positive states), a direct tunneling electron current to the p area, and a direct tunneling hole current to the n area at 300K. As shown in the figure, the direct tunneling electron and hole currents to the p and n areas take large values, and approximately equivalent to the recombination current. The fact means that light emission based on the recombination of electron and hole is not performed clearly. The tendency is appeared equivalently for both the direct tunneling electron and hole currents because tunnel-junction resistances to an electron and a hole are assumed to be a same value. Furthermore several spike current peaks are seen. the spikes are caused by the rapid current increasing based on the voltages for dismissing of Coulomb blockade. .

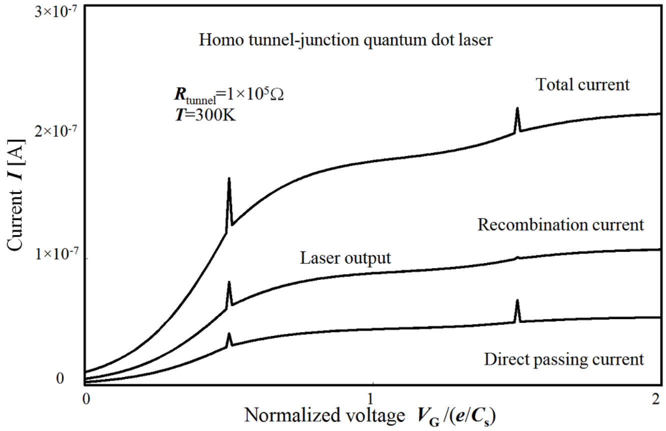

Fig. 5 Laser output produced by recombination current at 300K, and direct tunneling electron and hole currents and in a homo tunnel-junction laser. Both the electron and hole current have same value, and they are expressed by the curve of direct passing current.

Characteristics of laser output produced by the recombination, and the direct passing electron and hole currents at 30K are shown in Fig. 6. For the tunnel resistances to an electron and a hole are assumed to take symmetrical values. The characteristics show clear stair current tendencies, however, the direct passing current is still large although the current is partially reduced. As a remarkable property, the recombination current is growing up at normalized voltage 0.5, and has a value higher than that of the direct passing current. In this characteristic lower threshold of laser is not expected.

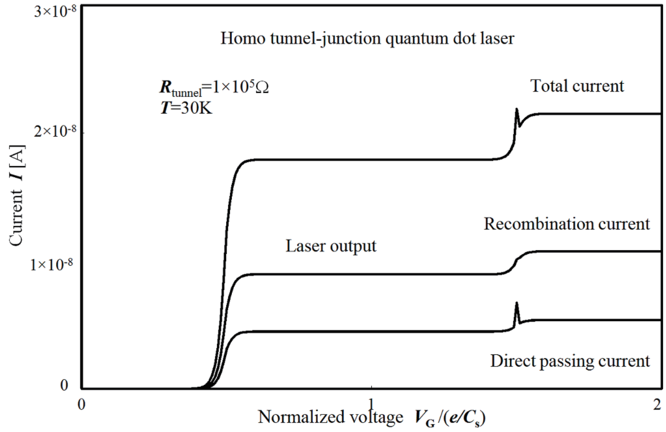

Fig. 6 Laser output produced by recombination current at 30K, and electron and hole currents in homo tunnel-junction laser.

Lower shift of growing up point of the recombination are shown in Fig. 7. For the n-i-p semiconductor quantum dot device, double-hetero tunnel-junction structure is applied in this case. Because the energy of the dot is assumed to have a value smaller than the band gaps of the n and p areas, the recombination of a hole current from the p and an electron current from the n flowing are improved.

Therefore the threshold for lasing can be significantly made lower. The improvement means that a recombination of an electron current or a hole in the dot is performed effectively. The result shows the introduced novel double-hetero tunnel-junction structure leads high performance lower threshold laser oscillation. For this model, since symmetrical tunnel resistances for an electron and a hole are assumed, the characteristics show symmetrical current dependency on the bias voltages.

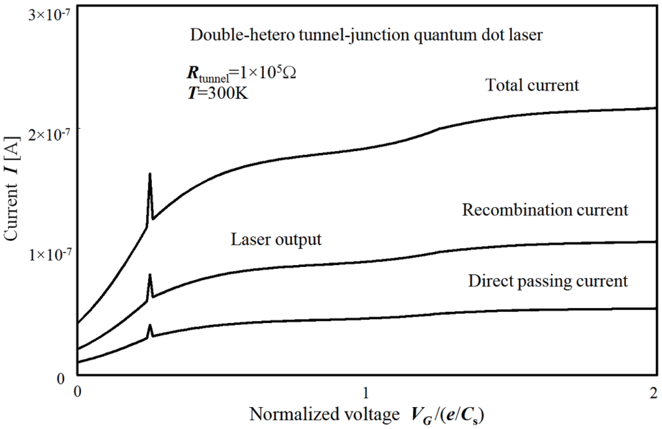

Fig. 7 Shifted laser output by enhanced recombination current in double-hetero tunnel-junction at 300K.

Lower temperature characteristics at 30K of the enhanced recombination current, the electron current, and the hole current are shown in Fig. 8. The model in this case adopts the double-hetero tunnel-junction structure. The growing up point of the recombination to the bias voltage is clear and still has lower value. For the tunnel resistances, symmetrical values are assumed. The results obtained in this research demonstrate the double-hetero tunnel-junction structure is useful for developing the n-i-p type quantum dot laser, and offer a possibility for various applications of the device.

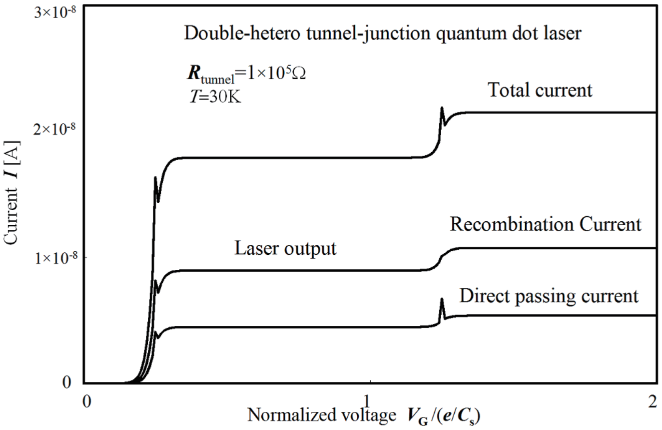

Fig. 8 Shifted laser output by enhanced recombination current in double-hetero tunnel-junction at 30K.

Conclusion

The analysis for charging process by tunneling to the isolated quantum dot in the active region in the double-hetero tunnel-junction quantum dot semiconductor laser is presented. The double-hetero tunnel junction structure including a low band gap quantum dot be able to suppress direct tunneling of electrons and holes with no recombination to *p* and *n* type sides and to enhance the recombination in the dots, while a high band gap quantum dot can be used for an electron or hole current selectable transistor [4]. A photon contained in a laser output is produced by a recombination of an electron and a hole in the quantum dot, and the stochastic solution for each charging of the electron and hole in discrete process is given. Based on the solution, the numerical calculations of the output characteristics to the applied voltage are presented.

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