Minimum Reynolds Number for the Relaxation of Similarity Requirements for Predicting Room Air Distribution

Nobuyuki KOBAYASHI* Yukio MATSUBARA**

Strict similarity for the prediction of velocity and temperature distributions in an air-conditioned room requires the equalities of both the Reynolds number and Archimedes number in both model and prototype. In actual model experiment, however, it is very difficult to satisfy these strict similarity requirements.

In the present paper, a minimum Reynolds number is suggested above which it is possible to neglect the Reynolds number from the strict similarity requirements without considerable error. This minimum Reynolds number was determined by measuring velocity distributions and observing the air flow patterns in three geometrically similar models and found to be about 7000, within the limits of the experiment.

List of Symbols

D ···· hydraulic diameter of supply inlet $(4Wh/2(W+h))$

e ···· degree of difference

g ···· acceleration due to gravity

H ···· room height

h ···· height of supply inlet

L ···· room length

n ···· total number of measuring point

$\overline{U}$ ···· mean air velocity in room model

$\overline{U}_0$ ···· mean air velocity at supply inlet

$\overline{U}^*$ ···· dimensionless velocity $(\overline{U}/\overline{U}_0)$

$\overline{U}_s^*$ ···· dimensionless velocity in velocity distribution considered to be standard

u ···· velocity fluctuation

$\sqrt{\overline{u^2}}/\overline{U}_0$ ···· turbulent intensity

W ···· room width

A_r ···· Archimedes number $(g\beta\Delta\theta L/\overline{U}_0{}^2)$

R_e ···· Reynolds number $(\overline{U}_0 D/\nu)$

R_{et} ···· turbulent Reynolds number $(\overline{U}_0 D/\kappa)$

β ···· coefficient of cubic expansion

$\Delta\theta$ ···· temperature difference

κ ···· eddy viscosity of air

ν ···· kinematic viscosity of air

* Asst. Professor, Department of Architecture, Faculty of Engineering, Tokyo Institute of Polytechnics

** So Associates for Building Development (formerly, Assistant of Tokyo Insitute of Polytechnics)

(Received Sep. 25, 1980)

1. Introduction

The similarity requirements for scale model studies of buoyant air flow (buoyant due to temperature differences) are the equalities of the Reynolds number (Re) and the Archimedes number (Ar) and, in addition, Prandtl number. Since air is used in this model experiment, the Prandtl numbers in model and prototye are identical. These dimensionless numbers are obtained by normalizing the Navie-Stokes equations of motion including a buoyancy term. In actual scale model studies, however, it is very difficult to satisfy both dimensionless numbers. For example, when a geometrically similar room model is constructed to a 1/10 scale, the representative velocity in the model must be ten times the velocity in the prototype because of the equality of Re. Furthermore, in order to make Ar in the model equal to that in the prototype, the representative temperature difference in the model must be a thousand times the temperature difference in the prototype, and this is almost impossible.

However, it has been reported that there is the region in which eddy viscosity is proportional to the represntative velocity and the length of the room under fully developed turbulent air flow and this region occupies a great part of the room space[1,2]. This can be expressed as follows

$$k \propto \overline{U}_0 L \cdots\cdots\cdots\cdots \quad (1)$$

If the flow in the room is fully turbulent, we can satisfy the similarity requirements by replacing Re with the turbulent Reynolds number (Ret), in which eddy viscosity is included. Namely, considering equation (1), it is obvious that the turbulent Reynolds numbers in the model and the prototype become automatically equal at each corresponding point. Therefore, the similarity requirements for the air flow in the room are reduced to only Ar and we can choose the representative velocity at our convenience.

Experimental studies using this similarity requirement have been reported [3], and good agreement of velocity and temperature distributions between the model and the prototype have been observed. Since these studies, the equality of the Archimedes number has been extensively adopted as the similarity requirement for predicting the velocity and temperature distributions in an air-conditioned room. Many applicable model studies using this similarity requirement for air-conditioning problems have been reported[4,5].

However, since the condition for fully developed turbulent flow in a room has not been clarified, these experiments may have been conducted under conditions when the air flow was not fully turbulent even though the equality of Ar was satisfied. This problem has been studied by many reseachers[6~10] but is still not thoroughly understood especially for air flow in a room.

In the present paper, we describe an experimental study in which the turbulent Reynolds number becomes automatically equal in both model and prototype. This codition, necessary for the relaxation of the strict similarity requirements, is investigated by measuring the velocity distributions and by visualization of the air movement using three different scale models.

2. Design of Experiment

To clarify the condition necessary for relaxation of strict similarity, we have only to look for the condition under which equation (1) is valid. This should be examined by direct measurement of the eddy viscosity, however, in the present experiment the prediction of velocity and temperature distributions in an air-conditioned room was considered of more practical use, so that we examined the condition by measurement of velocity distribution rather than eddy viscisity.

A series of experiments was conducted by supplying isothermal air to a model room through a thin supply inlet. The Reynolds number at the supply inlet was varied and the velocity distributions were measured.

For isothermal flow, the velocity distribution should not change and should be independent of the Reynolds number if equation (1) is valid. There should be a minimum Reynolds number above which the velocity distribution dose not change. In order to obtain this minimum Reynolds number a model experiment was conducted over a range of Reynolds numbers.

3. Outline of Experiment

Three scale size room models were used in the experiment. Figure 1 shows the configuration and the dimensions of the largest model, which was 1.8 m×1.2 m×0.8 m (length×width ×height) and had a slot-type supply inlet. The other models were geometrically similar to the largest one and were constructed to 1/2 and 1/4 scales. The models were made of plywood with one wall made from an acrylic resin plate for visualization.

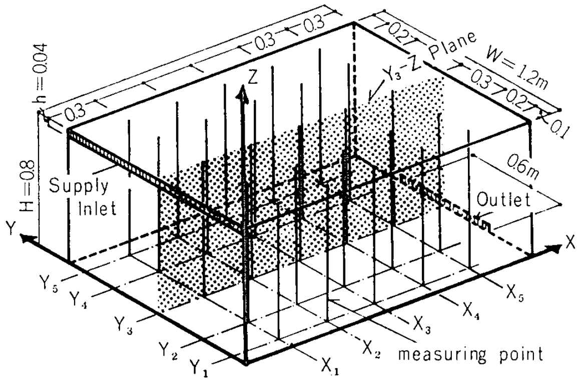

Fig. 1 Configuration of the largest model and the measuring points.

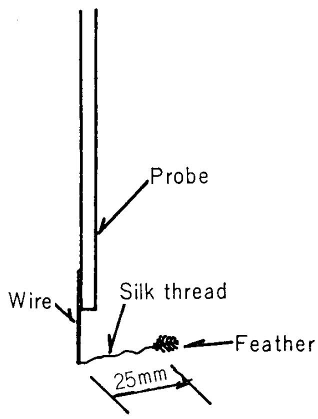

Fig. 2 Vertical probe for observing the air flow direction.

Air volumes supplied to the model were measured using a circular nozzle and a capacitance manometer. Air velocities in the model were measured by a thermistor anemometer at the measuring points shown in Figure 1. Twenty-five positions in the horizontal plane and eighteen positions in the vertical plane were chosen as the measuring points, for a total number of 450 measuring points in the model, however, there were cases for which the velocities at only ninety points in the Y_3-Z plane were measured, as shown in Figure 1.

The air flow direction at each measuring point was determined by observing the movement of a feather tied to a silk thread, which was attached to the end of a vertical probe inserted in the models. Figure 2 shows the method of observing the air flow direction.

The experimental conditions are listed in Table 1. The Reynolds numbers at supply inlet in Table 1 were calculated from the mean air velocity at the supply inlet, the hydraulic diameter of the inlet, and the kinematic viscosity at a temperature of 10°C. The Reynolds number was varied from 500 to 11000.

Air flow visualization was conducted in the largest and the 1/4 scale models by mixing metaldehyde particles with the supply air[11,12]. The experimental conditions of the air flow visualization are listed in Table 2.

Table 1. Experimental conditions for measuring velocity distribution

(a) for the largest model

No.	$\overline{U}_0$ (m/s)	R_e
1	9.2×10^{-2}	500
2	3.0×10^{-1}	1650
3	3.4×10^{-1}	1900
4	4.6×10^{-1}	2500
5	8.8×10^{-1}	4800
6	1.1×10^{0}	6000
7	1.3×10^{0}	7350

(b) for the 1/2 scale model

No.	$\overline{U}_0$ (m/s)	R_e
1	1.8×10^{-1}	500
2	3.6×10^{-1}	1000
3	1.7×10^{0}	4800
4	2.1×10^{0}	6600
5	3.1×10^{0}	8600
6	3.3×10^{0}	9000
7	4.0×10^{0}	11000

(c) for the 1/4 scale model

No.	$\overline{U}_0$ (m/s)	R_e
1	3.6×10^{-1}	500
2	5.8×10^{-1}	800
3	7.3×10^{-1}	1000
4	8.7×10^{-1}	1200
5	1.2×10^{0}	1650
6	1.5×10^{0}	2000
7	3.5×10^{0}	4800
8	4.8×10^{0}	6600
9	6.3×10^{0}	8600

Table 2. Experimental conditions for visualization.

(a) for the largest model

No.	$\overline{U}_0$ (m/s)	R_e
1	1.8×10^{-1}	1000
2	3.6×10^{-1}	1650
3	3.4×10^{-1}	1900
4	8.8×10^{-1}	4800
5	1.1×10^{0}	6000
6	1.3×10^{0}	7350

(b) for the 1/4 scale model

No.	$\overline{U}_0$ (m/s)	R_e
1	7.3×10^{-1}	1000
2	1.2×10^{0}	1650
3	1.5×10^{0}	2000
4	3.5×10^{0}	4800
5	5.1×10^{0}	7010

4. Results and Discussion

Velocity distribution at the various Reynolds numbers

Figure 3 shows the normalized vertical velocity distributions in the Y_3-Z plane of the largest model for various Reynolds numbers.

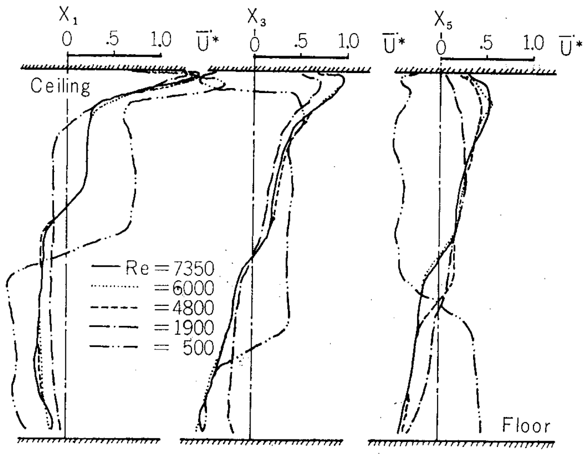

Fig. 3 Velocity distribution in the Y_3-Z plane of the largest model at various Reynolds number.

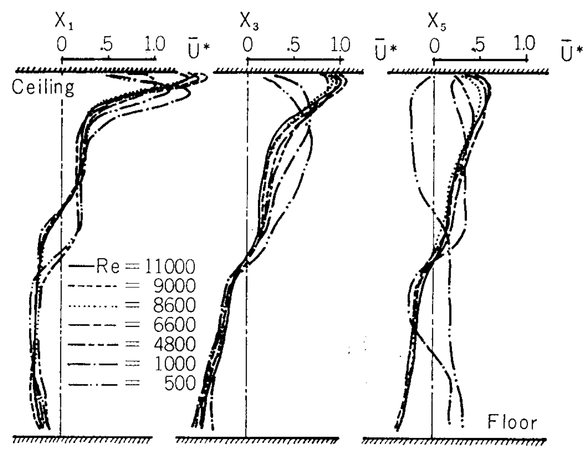

Fig. 4 Velocity distribution in the Y_3-Z Plane of the 1/2 scale at various Reynolds number

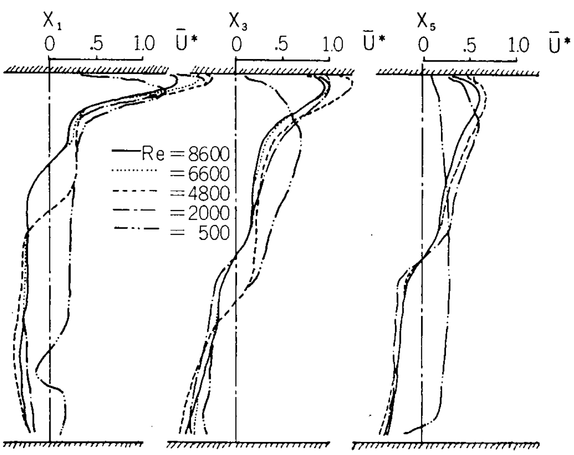

Fig. 5 Velocity distribution in the Y_3–Z plane of the 1/4 scale model at various Reynolds number.

As can be seen from the figure, the velocity distributions at each X position appear similar for a Reynolds number larger than 4800, though there is a little difference in the velocity distributions near the ceiling and at the center of the X_5 position (where the flow direction becomes adverse because of the circular flow generated in the model).

Figures 4 and 5 show the velocity distributions in the 1/2 scale and the 1/4 scale model. In these figures, it can be seen that the velocity distributions become more alike as the Reynolds number becomes larger. In Figure 4, the velocity distributions seem to be similar for Re>6600. In Figure 5, the velocity distributions appear to be similar for Re>6600.

To judge whether the velocity distribution should be considered similar, a degree of difference (e) was defined as follows

$$e=\sum|(\bar{U}_s^*-\bar{U}^*)/\bar{U}_s^*|/n \qquad (2)$$

The degree of difference signifies the mean ratio of the difference per unit measuring point to the velocity distribution considered to be standard. If the shapes of the distribution were similar and the degree of difference smaller than 0.15, the velocity distributions were considered equal. This judging method is usefui for the *practical* prediction of velocity and temperature distributions in an air-conditioned room. Table 3 illustrates the degree of difference in each model. The conditions under which the degree of difference becomes smaller than 0.15 are R_e>4800 in the largest model, Re>6600 in the 1/2 scale model and R_e>6600 in the 1/4 scale model.

Figure 6 shows the comparison of the velocity distributions under the condition that e<0.15 in each model. Figure 6 shows that the distrib-

Table 3. Degree of difference

(a) for the largest model

No.	R_e	e
1	500	0.63
3	1900	0.42
5	4800	0.10
6	6000	0.07
7*	7350	0

(b) for the 1/2 scale model

No.	R_e	e
1	500	0.51
2	1000	0.51
3	4800	0.22
4	6600	0.14
5	8600	0.08
6	9000	0.10
7*	11000	0

(c) for the 1/4 scale model

No.	R_e	e
1	500	0.53
6	2000	0.20
7	4800	0.23
8	6600	0.07
9*	8600	0

* denotes the conditions under which the velocity distribution was considered to be standard in each model.

utions are almost the same.

The turbulent intensity at the center of the supply inlet was measured for the largest and the 1/4 scale model by using a hot-wire anemometer. Table 4 shows the turbulent intensity at the center of the supply inlet. It is in the range of 10～20% excluding the $R_e=500$ case.

Air flow patterns at the various Reynolds numbers

Observations of the air flow pattern in the model were conducted under the conditions listed in Table 2. The flow was photographed and the movement of the metaldehyde particles sketched in the Y_3-Z plane. The flow was illuminated by a narrow light beam. Some

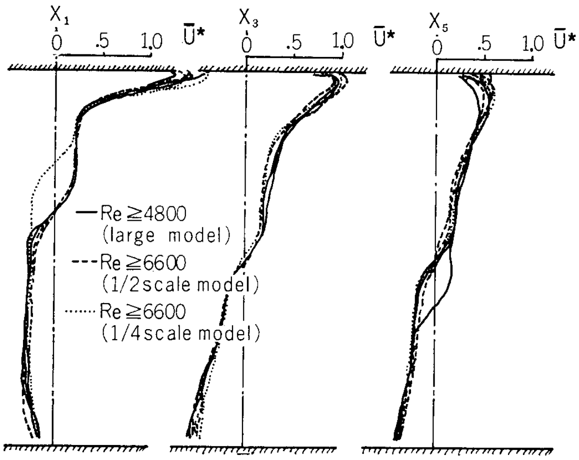

Fig. 6 Comparison of $\overline{U}^*$ in the largest model, the 1/2 scale model and the 1/4 scale model in case of large Reynolds numbers.

Table 4. Turbulent intensities at the center of the supply inlet.

R_e	for the largest model	for the 1/4 scale model
500	4.1 (%)	4.6 (%)
1650	12.4	....
1900	15.8	....
2000	....	9.8
4800	13.7	16.0
6000	13.5	....
6600	....	18.6
7350	12.6	....

examples of these photos and sketches are shown in Figures 7, 8 and 9.

The air flow pattern varies with the Reynolds number, but when R_e became larger than 6000 in the largest model, the air flow pattern did not change. In the 1/4 scale model, the air flow patterns were entirely different, but the air flow pattern for $R_e=7010$ was very similar to that of the largest model with $R_e>6000$.

Considering the abovementioned results of the visualization, it is expected that the air flow pattern in the 1/4 scale model for $R_e>7010$ will not change. Therefore, the conditions underwhich the air flow pattern did not chage were $R_e>6000$ in the largest model and $R_e>7010$ in the 1/4 scale model.

To summarize from the measurement of the velocity distributions, the conditions under which the velocity distribution does not change are $R_e>4800$ in the largest model and $R_e>6600$ in the 1/2 scale model and the 1/4 scale model. From the results of the visualization, the air flow patterns are similar for $R_e>6000$ in the largest model and $R_e>7010$ in the 1/4 scale model.

Though there appears some difference among the models for the velocity distributions and air flow patterns, it appears that the minimum Reynolds number for which the velocity distributions are independent of R_e and scale is $R_e>7000$ (within the limits of our experimental error). Terefore, if a scale model experiment to predict the velocity and temperature distribution is conducted under the condition of $R_e>7000$, the turbulent Reynolds numbers in both the model and the prototype become automatically equal and we can neglect the Reynolds number from the strict similarity requirements, leaving only the Archimedes

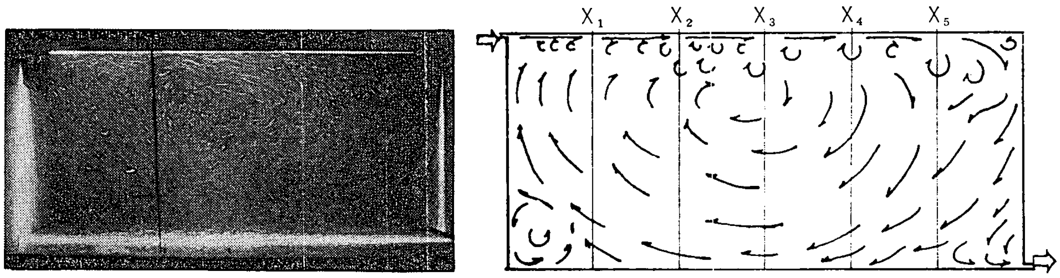

Fig. 7 Photograph and sketch of air flow pattern in the largest model for $R_e=6000$.

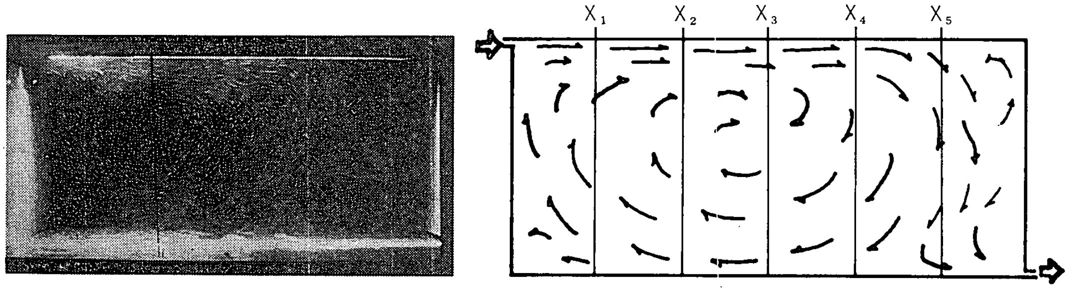

Fig. 8 Phothgraph and sketch of air flow pattern in the largest model for $R_e=7350$.

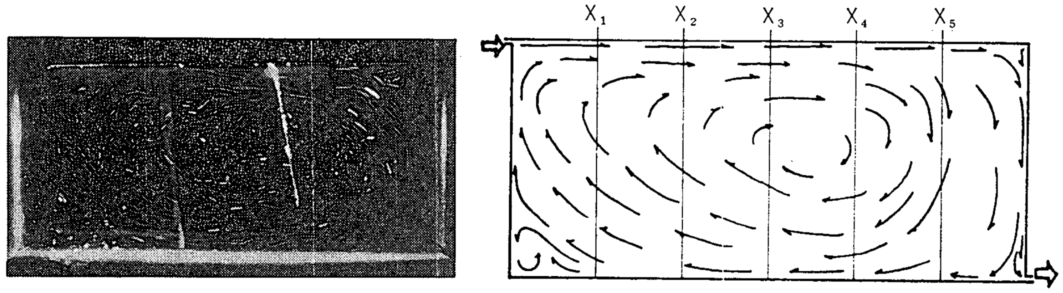

Fig. 9 Photograph and sketch of air flow pattern in the 1/4 scale model for $R_e=7010$.

number as the similarity requirement.

5. Conclusion

Considering the results of velocity distribution measurement and visualization of the air flow, the minimum Reynolds number above which it is possible to neglect the Reynolds number from strict similarity without considerable error is 7000 for use in the prediction of velocity and temperature distribution in an air-conditioned room.

References

1) T. Shoda, S. Murakami and N. Kobayashi, "Study on Air Flow in Halh-enclosed Spaces, Part 1", Transact. of the Architectural Institute of Japan, No. 234, August, 1975.

2) T. Tsuchiya, "Modeling Criteria for the Room Air Motion", Diss. Thesis, Unv. of Tokyo, March, 1969.

3) T. Shoda, T. Tsuchiya, T. Ichikawa and R. Shoda, "Field and Model Studies for Air Distribution in a Meeting Room", Journal of Institute of Industrial Science, Univ. of Tokyo,

October, 1969.

4) N. Nakahara, T. Goto and Y. Miyagawa, "Field and Model Studies for Room Air Distribution, Part 1", Journal of Soc. of Heating, Air-Conditioning and Sanitary Engineers of Japan, Vol. 45, No. 10, 1971.

5) M. Kobayashi, T. Yasuda, S. Ito and Y. Miyagawa, "Field and Model Studies for Room Air Distribution, Part 2", Journal of Soc. of Heating, Air-Conditioning and Sanitary Engineers of Japan, Vol. 48, No. 3, 1974.

6) W. Linke, "Eigenshaften der Strahllüftung", Kältetechnik Klimatisierung, 18 Jahrgang, Heft 3, 1966.

7) C. L. Burton and R. E. Willson, "Application of Model Studies to Industrial Gas-Flow System", ASME, paper No. 59-A-280, 1959.

8) K. I. Parczewski and P. N. Renzi, "Scale Model Studies of Temperature Distribution in Internally Heated Enclosures", ASHRAE transact., No. 1854, 1963.

9) Y. Kodama, K. Seike and N. Takada, "Influences of Shape of Room Space on Air Flow". Report of the Annual Meeting of the Architectural Institute of Japan, September, 1977.

10) N. Kobayashi and Y. Matsubara, "The Necessary Condition for Air Flow in Room being Fully Developed Turbulent", Report of the Annual Meeting of the Architectural Institute of Japan, September, 1979.

11) G. A. Carpenter and L. J. Moulsley, "A Visualization Technique for Studying Air Movement in Large Enclosures over a Wide Range of Ventilation Rates", J. I. H. V. E., Vol. 39, March, 1972.

12) N. Kobayashi, T. Shoda, S. Murakami and S. Togari, "Visualization of Air Flow in the Room under Warm Air Heating", 2 nd Symposium on Flow Visualization ISAS, Univ. of Tokyo, July, 1974.