# Analysis of variance for high dimensional time series

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**Classical ANOVA works well for high dimensional time series. For example, this method can be applied for radioactive data.**

Analysis of variance (ANOVA) is tailored for independent observations. Recently, there has been considerable demand for ANOVA of high-dimensional and dependent observations in many fields. For example, it is important to analyze differences among industry averages of financial data. However, ANOVA for these types of observations has been inadequately developed. In this paper, we thus present a study of ANOVA for high-dimensional and dependent observations. Specifically, we present the asymptotics of classical test statistics proposed for independent observations and provide a sufficient condition for them to be asymptotically normal. Numerical examples for simulated and radioactive data are presented as applications of these results.

## 1 Theoretical results

### 1.1 Setting

Let $p$ vetor-valued series $\boldsymbol{X}_{i1}, \ldots \boldsymbol{X}_{in_i}$ ($p \to \infty$) be generated from

$$\boldsymbol{X}_{it} = \boldsymbol{\mu} + \boldsymbol{\alpha}_i + \boldsymbol{\epsilon}_{it},\ t = 1, \ldots, n_i,\ i = 1, \ldots, q,$$

where

- $\boldsymbol{\epsilon}_i \equiv \{\boldsymbol{\epsilon}_{it};\ t = 1, \ldots, n_i\}$, $i = 1, \ldots, q$, are stationary with mean $\mathbf{0}$, autocovariance matrix $\boldsymbol{\Gamma}(\cdot)$ and spectral density matrix $\boldsymbol{f}(\lambda)$,
- $\{\boldsymbol{\epsilon}_{it};\ t = 1, \ldots, n_i\}$, $i = 1, \ldots, q$, are mutually independent.

Consider the problem of testing

$$H : \boldsymbol{\alpha}_1 = \cdots = \boldsymbol{\alpha}_q.$$

**Assumption 1** (high dimensional large sample setting)**.**

$$\frac{p}{\sqrt{n}} \to 0 \ as\ n, p \to \infty,$$

$$\frac{n_i}{n} \to \rho_i > 0 \ as\ n \to \infty.$$

### 1.2 Method

- For independent observations, the following Lawley-Hotelling test (1), likelihood ratio test (2), and Bartlett-Nanda-Pillai test (3) have been proposed:

$$LH \equiv n\mathrm{tr}\{\hat{\mathcal{S}}_H \hat{\mathcal{S}}_E^{-1}\}, \quad (1)$$

$$LR \equiv -n \log\{|\hat{\mathcal{S}}_E| / |\hat{\mathcal{S}}_E + \hat{\mathcal{S}}_H|\}, \quad (2)$$

$$BNP \equiv n\mathrm{tr}\hat{\mathcal{S}}_H (\hat{\mathcal{S}}_E + \hat{\mathcal{S}}_H)^{-1}, \quad (3)$$

where

$$\hat{\mathcal{S}}_H \equiv \sum_{i=1}^{q} n_i (\hat{\boldsymbol{X}}_{i\cdot} - \hat{\boldsymbol{X}}_{\cdot\cdot})(\hat{\boldsymbol{X}}_{i\cdot} - \hat{\boldsymbol{X}}_{\cdot\cdot})',$$

$$\hat{\mathcal{S}}_E \equiv \sum_{i=1}^{q} \sum_{t=1}^{n_i} (\boldsymbol{X}_{it} - \hat{\boldsymbol{X}}_{i\cdot})(\boldsymbol{X}_{it} - \hat{\boldsymbol{X}}_{i\cdot})'.$$

- We can derive the stochastic expansion of the standardized versions $T_1, T_2, T_3$ of three tests $LH, LR, BNP$ respectively;

$$T_1 \equiv \frac{1}{\sqrt{2(q-1)}} \left\{ \frac{1}{\sqrt{p}} LH - \sqrt{p}(q-1) \right\},$$

$$T_2 \equiv \frac{1}{\sqrt{2(q-1)}} \left\{ \frac{1}{\sqrt{p}} LR - \sqrt{p}(q-1) \right\},$$

$$T_3 \equiv \frac{1}{\sqrt{2(q-1)}} \left\{ \frac{1}{\sqrt{p}} BNP - \sqrt{p}(q-1) \right\}.$$

### 1.3 Results

**Assumption 2** (Brillinger condition)**.** *Given a $p$-vector stationary process $\boldsymbol{\epsilon}_{it} = (\epsilon_{it}^{(1)}, \cdots, \epsilon_{it}^{(p)})'$ for each $k = 2, 3, \cdots$, and $j = 1, \cdots, k-1$, there exists an $m > 0$ with*

$$\sum_{t_1, \ldots, t_{k-1} = -\infty}^{\infty} \{1 + |t_j|\}^m |c_{a_1, \cdots, a_k}(t_1, \cdots, t_{k-1})| < \infty$$

*uniformly for $a_1, \cdots, a_k$, where $c_{a_1, \cdots, a_k}(t_1, \cdots, t_{k-1}) = \mathrm{cum}\{\epsilon_{it_1}^{(a_1)}, \cdots, \epsilon_{it_k}^{(a_k)}\}$.*

**Assumption 3** (Uncorrelated disturbance)**.**

$$\boldsymbol{\Gamma}(j) = \mathbf{0} \ for\ all\ j \neq 0.$$

**Remark 1.** *Assumption 3 is not severe because vector GARCH model (very practical nonlinear time series model) satisfies it.*

**Theorem 1.** *Suppose Assumptions 1-3. Then, under the null hypothesis $H$,*

$$T_i \xrightarrow{d} N(0, 1), \quad i = 1, 2, 3.$$

## 2 Simulation results

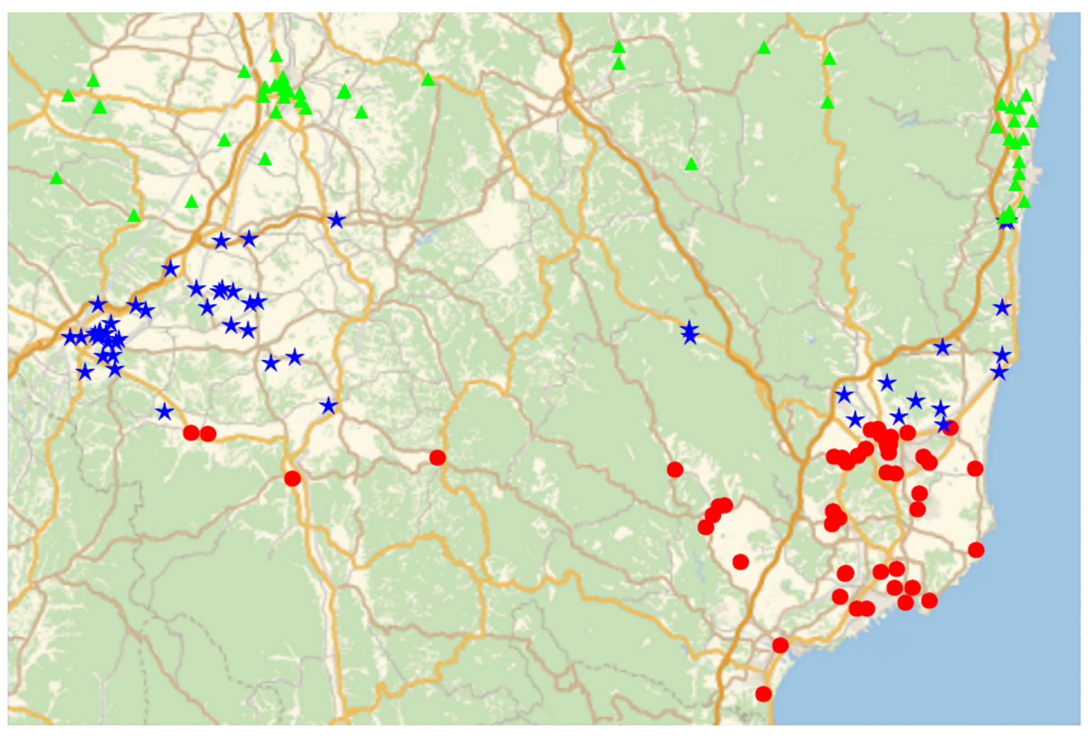
| Sample size of each group | Test Statistic | Significance Level 10% | 5% | 1% |
|---|---|---|---|---|
| 50 | $T_1$ (LH) | 0.906 | 0.879 | 0.781 |
| | $T_2$ (LR) | 0.615 | 0.493 | 0.313 |
| | $T_3$ (BNP) | 0.089 | 0.037 | 0.010 |
| 100 | $T_1$ (LH) | 0.564 | 0.460 | 0.286 |
| | $T_2$ (LR) | 0.323 | 0.212 | 0.098 |
| | $T_3$ (BNP) | 0.107 | 0.060 | 0.015 |
| 500 | $T_1$ (LH) | 0.161 | 0.100 | 0.032 |
| | $T_2$ (LR) | 0.134 | 0.082 | 0.022 |
| | $T_3$ (BNP) | 0.101 | 0.055 | 0.015 |
| 2500 | $T_1$ (LH) | 0.114 | 0.062 | 0.015 |
| | $T_2$ (LR) | 0.108 | 0.058 | 0.014 |
| | $T_3$ (BNP) | 0.100 | 0.057 | 0.014 |
| 7500 | $T_1$ (LH) | 0.109 | 0.058 | 0.012 |
| | $T_2$ (LR) | 0.107 | 0.055 | 0.012 |
| | $T_3$ (BNP) | 0.105 | 0.054 | 0.012 |

Table 1: Rejection rate of the test statistics.

| Sample size of each group | Test Statistic | Significance Level 10% | 5% | 1% |
|---|---|---|---|---|
| 50 | $T_1$ (LH) | 0.960 | 0.939 | 0.890 |
| | $T_2$ (LR) | 0.768 | 0.684 | 0.520 |
| | $T_3$ (BNP) | 0.200 | 0.108 | 0.020 |
| 100 | $T_1$ (LH) | 0.844 | 0.770 | 0.606 |
| | $T_2$ (LR) | 0.636 | 0.518 | 0.346 |
| | $T_3$ (BNP) | 0.352 | 0.230 | 0.086 |
| 500 | $T_1$ (LH) | 0.978 | 0.959 | 0.907 |
| | $T_2$ (LR) | 0.972 | 0.950 | 0.864 |
| | $T_3$ (BNP) | 0.958 | 0.934 | 0.821 |
| 2500 | $T_1$ (LH) | 1.000 | 1.000 | 1.000 |
| | $T_2$ (LR) | 1.000 | 1.000 | 1.000 |
| | $T_3$ (BNP) | 1.000 | 1.000 | 1.000 |
| 7500 | $T_1$ (LH) | 1.000 | 1.000 | 1.000 |
| | $T_2$ (LR) | 1.000 | 1.000 | 1.000 |
| | $T_3$ (BNP) | 1.000 | 1.000 | 1.000 |

Table 2: Powers of the test statistics under the alternative hypothesis (ii) $\boldsymbol{\alpha}_1 = (-0.01, \cdots, -0.01)'$, $\boldsymbol{\alpha}_2 = \mathbf{0}$, $\boldsymbol{\alpha}_3 = (0.01, \cdots, 0.01)'$.

## 3 Application to radioactive data



- We apply $T_1, T_2, T_3$ to the radioactive data of Fukushima.
- Data: This data set consists of three groups with 50 dimensions and about 8000 cell lines.
  - 3 groups, (i) Green area, (ii) Blue area, and (iii) Red area.
  - Very low autocorrelation;

→ All of the tests reject hypothesis $H$, and their P-values are all around 0.