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Features of the earthquake source process simulated by Vere-Jones' branching crack model

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【Abstract】

Vere-Jones' branching crack model was developed in 1970s. In this model, the earthquake source is regarded as the result from the total population of crack elements in a critical or near-critical branching process, where the crack propagates through a series of steps. At each step, each crack element simply terminates or generates several other crack elements nearby. Regarding the total number of steps (generations) as the duration time and the total number of crack elements as the total energy released, the following features of earthquake source processes can be simulated by this model: (a) the magnitude-frequency relation, (b) the source time function, and (c) the rupture duration-seismic moment relation.

1. the Gutenberg-Richter magnitude-frequency relation holds as an asymptote for more general cases of the GW branching processes. The distribution of energies is asymptotically a Pareto distribution (power law) for the critical case, or a tapered Pareto distribution (tapered power law, Kagan distribution) for the subcritical case.
2. The variance of rupture durations is relatively larger for small magnitude events and gives a scaling relation of $T \sim M^{1/2}$ between the rupture duration T and the seismic moment M of an earthquake.
3. The number of crack elements at each generation (time step) show similar patterns of earthquake source time functions. Vere-Jones' branching crack model explains why the EQ magnitude cannot be determined until the whole source process is completed.

【On earthquake sources】

- ◆ Summary statistics: Magnitude, Seismic moment, Energy, Moment tensor
- ◆ Models: Slip distribution (spatial or spatiotemporal), Fault geometry, Source time function, Stress drop distribution
- ◆ Complexity: Accompanying (sub) faults, Fault thickness
- ◆ Interaction between the earthquake source and the surrounding environments:

The barrier model (Aki, 1984) and the asperity model (Lay and Kanamori, 1981). Both models refer to strong patches of the fault that are resistive to rupture or slip. In the barrier model, smoothly slipped patches are surround by relatively strong patches, and so earthquake is regarded as a stress-roughen process that can propagate leaving unbroken patches. In the asperity model, highly stressed strong patches are surrounded by the regions where stress has already been released by aseismic slips.

【 Vere-Jones' branching crack model – Galton-Watson process】

- Each earthquake starts from an individual crack.
- Each crack triggers further cracks independently in the same manner (probability distribution).

Y: Number of cracks triggered by a crack

$p_i = \Pr\{i \text{ cracks are triggered by a given crack}\},\ i = 1, 2, \ldots$

$\mathbf{E}\,Y = <Y> = \sum_{i=1}^{\infty} i\, p_i = \mu$

$\mathbf{Var}\,Y = \mathbf{E}Y^2 - (\mathbf{E}Y)^2 = \sigma^2 < \infty.$

- The energy (moment) of the earthquake can be emulated by the total number of cracks.
- The rupture duration time can be emulated by the total number of generations.

【 Seismic moments】

- ◆ The moment M of the earthquake can be emulated by the total number X of cracks in Vere-Jones' branching crack model.
- ◆ (***Dwass theorem***) For a general branching process with a single time 0 ancestor and offspring distribution Y and total population size X:

$$\Pr\{X = k\} = \frac{1}{k}\Pr\{Y_1 + Y_2 + \cdots + Y_k = k - 1\}$$

where $Y_1, Y_2, \ldots, Y_k$ are independent copies of Y.

- ◆ Using the Dwass theorem and central limit theorem, we can prove

$$\Pr\{X = n\} \sim \frac{1}{n\sqrt{2\pi n\sigma^2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right] \sim n^{-\frac{3}{2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right]$$

When $\mu < 1$, $\Pr\{X = n\} \sim n^{-\frac{3}{2}} \exp\left[-\frac{(1-\mu)^2}{2\sigma^2} n\right]$ (subcritical regime)

When $\mu = 1$, $\Pr\{X = n\} \sim n^{-\frac{3}{2}}$ (critical regime)

- ◆ Asymptotes of the survival functions of magnitudes (m) and moments (M)

$m = \frac{2}{3}\log_{10} M - 2.9$

- When $\mu < 1$ (subcritical regime),

$\Pr\{\text{magnitude} \geq m\} \sim \text{const} \cdot 10^{-0.75m - c\, 10^{-d\, m}}$

$\Pr\{\text{moment} \geq M\} \sim \text{const} \cdot M^{-0.5} \exp(-M/M_C)$ (tapered Pareto distribution)

- When $\mu = 1$ (critical regime),

$\Pr\{\text{magnitude} \geq m\} \sim \text{const} \cdot 10^{-0.75m}$ i. e., $b = 0.75$

$\Pr\{\text{moment} \geq M\} \sim \text{const} \cdot M^{-0.5}$ (Pareto distribution, G-R law)

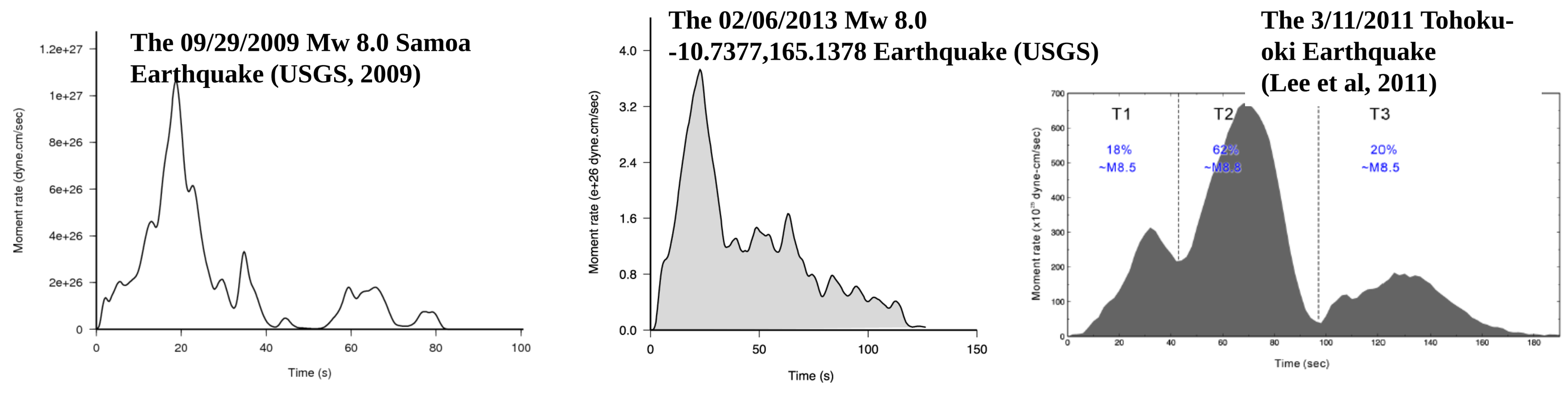
M_C: corner magnitude c, d: constants

- The critical case gives the Gutenberg-Richter magnitude-frequency relation, and the subcritical gives the tapered Partodistribution for earthquake sizes (Kagan, 1993).

- ◆ Kagan (2010): ***Yes.*** *b*=0.75 ***is universal for earthquakes!***

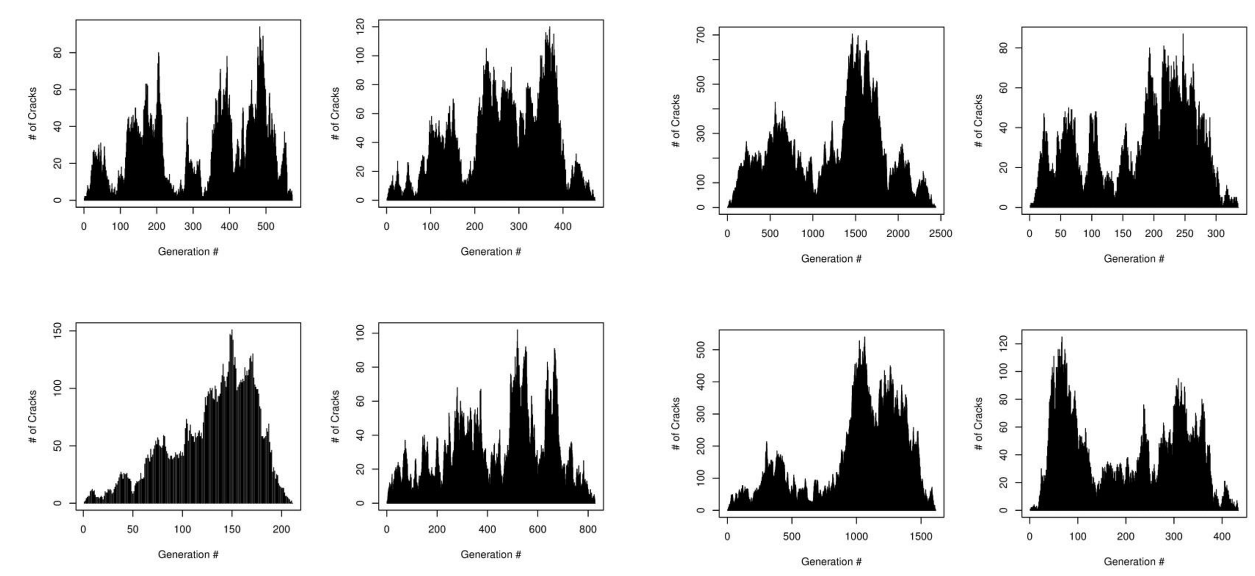
【Source-time functions】

- ◆ The source time function represents the release rate of seismic moment as a function of time.
- ◆ In Vere-Jones' branching crack model, it is assumed: Each generation is a time step, and energy at each time step is the number of cracks at this generation.
- ◆ Some examples of source time functions.



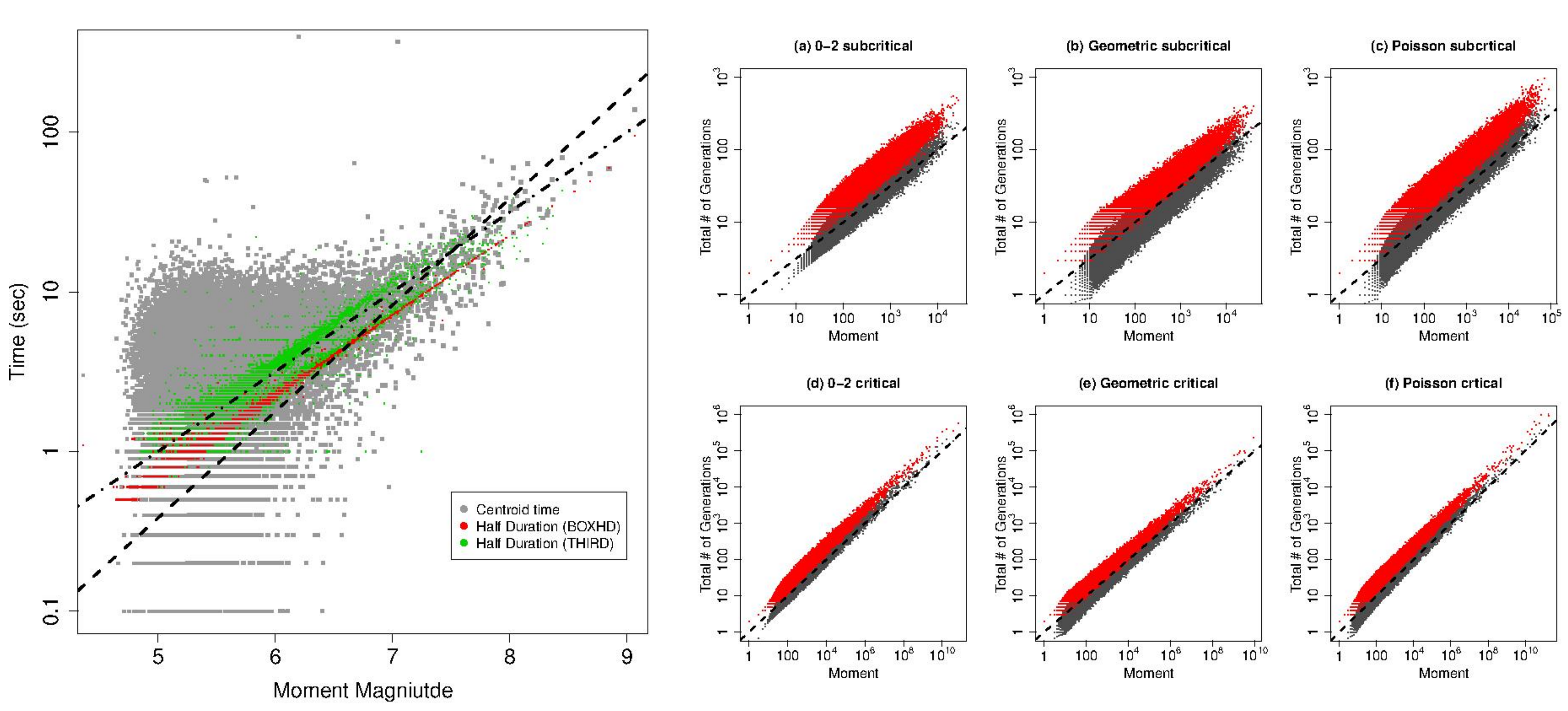
Plots of some typical source time functions. ***They are all different!***

- ◆ Simulation examples of big earthquakes



- ◆ Vere-Jones' branching crack model has a similar source time function as an earthquake. If the branching process does not stop at a certain time step, any number of cracks could be produced in its continuation. This randomness explains why the EQ magnitude cannot be determined unless the recorded waveforms contain information of the whole source process.
- ◆ This above conclusion is different from Olson (2005, 2006, *Nature*), which declared that the earthquake magnitude can be determined by the first several P wave phases.

【The rupture duration-seismic moment relation 】



(LEFT) Centroid times (gray dots) and half durations (red and green dots) of seismic events in the global CMT catalog. The centroid time is the time at which an earthquake has released half its energy. The rupture duration is generally estimated by using an empirically determined relationship such that the duration increases as the cube root of the seismic moment. TRIHD indicates a triangular moment-rate function, and BOXHD indicates a boxcar moment-rate function. The dashed and dot-dashed lines represent slopes of 1/2 and 1/3, respectively.
(RIGHT) Centroid times (black dots) and durations (red dots) of simulated earthquakes by 3 branching cracking models, the 0-2 model ($p_0 = 1 - a, p_2 = a, p_1 = p_3 = p_5 = p_6 = p_7 = \cdots = 0$, with $a = 0.5$ for the critical case and $a < 0.5$ for the subcritical cases), the Poisson branching model ($p_i = \lambda^i e^{-\lambda}/i!$, $i = 0,1,\cdots$, with $\lambda = 1$ for the critical case and $\lambda < 1$ for the subcritical case), and the geometric branching model ($p_i = a(1-a)^i$, $i = 0,1,\cdots$, with $a = 0.5$ for the critical case and $a > 0.5$ for the subcritical case), all approximately indicating that the scaling law in the relation between the duration and the energy is $T \sim M^{1/2}$.

- ◆ The conventional 2D (circular or rectangle crack) model for earthquake rupture supposes that, once the rupture starts from a certain point on the fault plane, it propagates straightly to the boundary of the source area without a stop. With the assumptions that the rupture duration is related to the source dimension L through the rupture velocity V_S (which is assumed to be a constant and proportional to the shear wave velocity $T \propto L/V_S$ and that the stress drop is close to a constant for all the earthquakes, $T \sim M^{1/3}$ can be obtained for the conventional 2D source model.
- ◆ Simulations of Vere-Jones branching crack models show a relationship $T \sim M^{1/2}$, which cannot be disproved by observations.

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