震源過程とVere-Jonesの 分枝モデル間の類似特性

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【Abstract】

Vere-Jones' branching crack model was developed in 1970s. In this model, the earthquake source is regarded as the result from the total population of crack elements in a critical or near-critical branching process, where the crack propagates through a series of steps. At each step, each crack element simply terminates or generates several other crack elements nearby. Regarding the total number of steps (generations) as the duration time and the total number of crack elements as the total energy released, the following similarities are found between earthquake sources and this model:

1. The distribution of energies is asymptotically a Pareto distribution (power law) for the critical case, or a tapered Pareto distribution (tapered power law, Kagan distribution) for the subcritical case.
2. VJ's branching crack model explains why the EQ magnitude cannot be determined or predicted before the whole source process finishes.
3. The number of crack elements at each generation (time step) show similar patterns of earthquake source time functions.

【On earthquake sources】

◆ Summary statistics
- Magnitude
- Seismic moment, Energy
- Moment tensor

◆ Models
- Slip distribution (spatial or spatiotemporal)
- Fault geometry
- Source time function
- Stress drop distribution

◆ Complexity
- Accompanying (sub) faults
- Fault thickness

◆ ….

【 Vere-Jones' branching crack model 】

- Each earthquake starts from an individual crack.
- Each crack triggers further cracks independently in the same manner (probability distribution).

  Y: Number of cracks triggered by a crack

  $p_i = \Pr\{i \text{ cracks are triggered by a given crack}\},\ i = 1, 2, \ldots$

  $\mathbf{E}\, Y = \langle Y \rangle = \sum_{i=1}^{\infty} i\, p_i = \mu$

  $\mathbf{Var}\ Y = \mathbf{E}Y^2 - (\mathbf{E}Y)^2 = \sigma^2 < \infty.$
- The energy (moment) of the earthquake can be emulated by the total number of cracks.
- The duration time can be emulated by the total number of generations.

【 Total Energies】

◆ It is reasonable to assume the energy M of the earthquake can be emulated by the total number X of cracks in VJ's branching model.

◆ (***Dwass theorem***) For a general branching process with a single time 0 ancestor and offspring distribution Y and total population size X:

$$\Pr\{X = k\} = \frac{1}{k}\Pr\{Y_1 + Y_2 + \cdots + Y_k = k - 1\}$$

where $Y_1, Y_2, \ldots, Y_k$ are independent copies of Y.

◆ Using the Dwass theorem and central limit theorem, we can prove

$$\Pr\{X = n\} \sim \frac{1}{n\sqrt{2\pi n\sigma^2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right] \sim n^{-\frac{3}{2}} \exp\left[-\frac{(n(1-\mu)-1)^2}{2n\sigma^2}\right]$$

When $\mu < 1$, $\Pr\{X = n\} \sim n^{-\frac{3}{2}} \exp\left[-\frac{(1-\mu)^2}{2\sigma^2} n\right]$ (subcritical regime)

When $\mu = 1$, $\Pr\{X = n\} \sim n^{-\frac{3}{2}}$ (critical regime)

◆ Asymptotes of the survival functions of magnitudes (m) and moments (M)

$m = \frac{2}{3}\log_{10} M - 2.9$

• When $\mu < 1$ (subcritical regime),

$\Pr\{\text{magnitude} \ge m\} \sim \text{const} \cdot 10^{-0.75m - c\, 10^{-d\, m}}$

$\Pr\{\text{moment} \ge M\} \sim \text{const} \cdot M^{-0.5} \exp(-M/M_C)$ (tapered Pareto distribution)

• When $\mu = 1$ (critical regime),

$\Pr\{\text{magnitude} \ge m\} \sim \text{const} \cdot 10^{-0.75m}$ i. e., $b = 0.75$

$\Pr\{\text{moment} \ge M\} \sim \text{const} \cdot M^{-0.5}$ (Pareto distribution, G-R law)

M_C: corner magnitude c, d: constants

• The critical case gives the famous Gutenberg-Richter magnitude-frequency relation, and the subcritical gives the more proper distribution for earthquake sizes (Kagan, 1993).

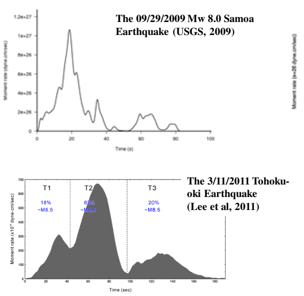
◆ Kagan (2010): b=0.75 ***is universal for earthquakes!***

【Source-time functions】

◆ The source time function represents the release rate of seismic moment as a function of time.

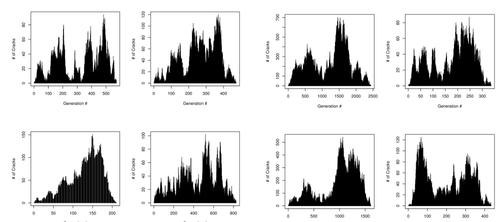
◆ In Vere-Jones's branching crack model, it is assumed: Each generation is a time step, and energy at each time step is the number of cracks at this generation.

◆ Some examples of source time functions.



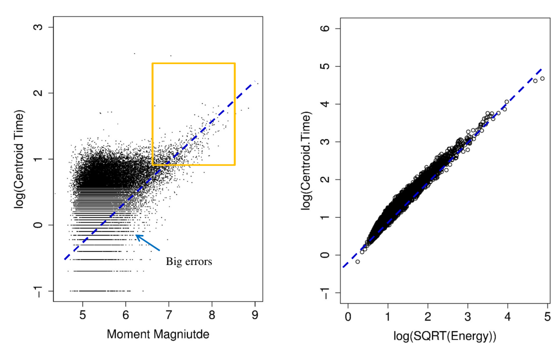
Plots of some typical source time functions. *They are all different!*

◆ Simulation examples of big earthquakes



◆ Regarding "generation" as time step, VJ's branching crack model has a similar source time function as an earthquake. If the branching process does not stop at a certain time step, any number of cracks could be produced in its continuation. This randomness of VJ's branching model explains why the EQ magnitude cannot be determined unless the recorded waveforms contain information of the whole source process.

【The duration-moment relation 】



◆ The centroid time is the time at which an earthquake has released half its energy.

◆ VJ's branching crack model has a similar relationship between source duration time and magnitude as earthquakes.

【Conclusions】

Please see Abstract.

【Acknowledgements】

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【Main references】

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